



## W-BOSON MASS, SMEFT AND FUTURE TESTS AT E+E- COLLIDER

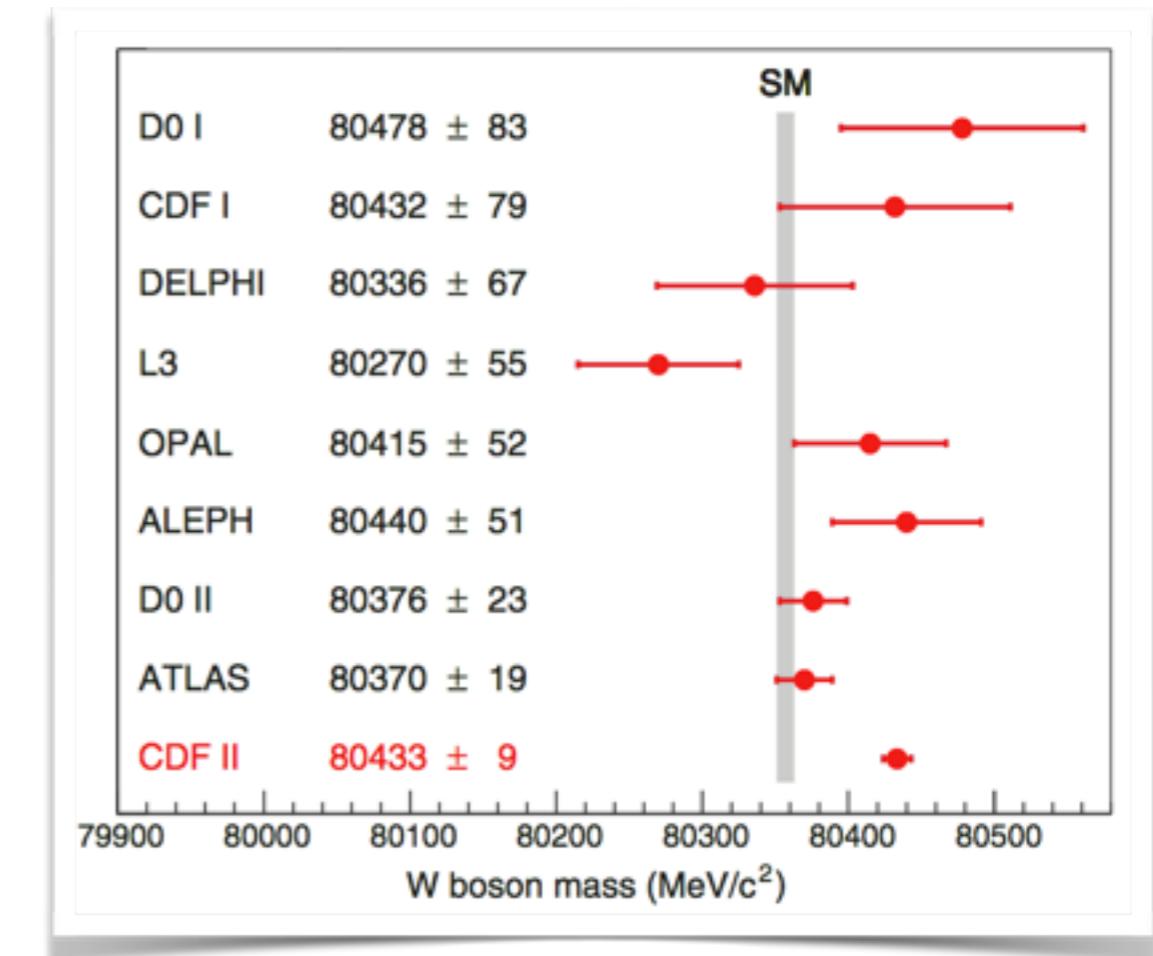
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Based on arXiv:2204.04805, with Ji-Ji Fan, Ling-Feng Li, Kun-Feng Lyu





# CDF II W-Boson Mass



[Science 376 (2022)]

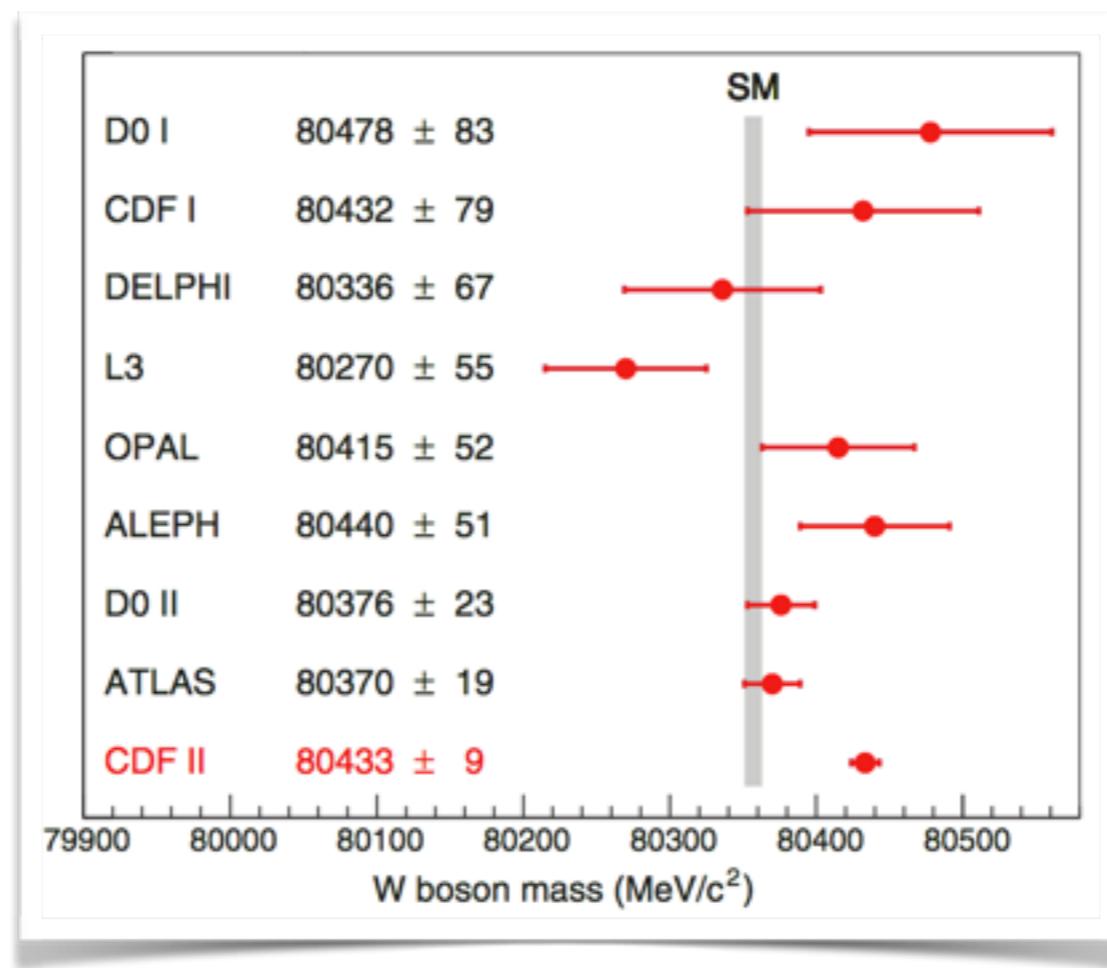
$$M_W = 80,433.5 \pm 6.4_{\text{stat}} \pm 6.9_{\text{syst}} = 80,433.5 \pm 9.4 \text{ MeV}/c^2$$

VS.

$$M_W = 80,357 \pm 4_{\text{inputs}} \pm 4_{\text{theory}} \text{ MeV}$$



## CDF II W-Boson Mass



[Science 376 (2022)]

While making any NP-interpretation, one needs to be cautious, given that significant tension exists between this CDF II result and the direct measurements performed by the D0 collaboration at the Tevatron and the ATLAS/LHCb collaboration at the Large Hadron Collider (LHC) and especially the latter ones are in good agreement with the SM prediction



## 6D SMEFT

Consider totally six self-contained operators in relation to EWPOs

$$\begin{aligned}\mathcal{O}_{WB} &= gg' H^\dagger \sigma^a H W_{\mu\nu}^a B^{\mu\nu}, & \mathcal{O}_T &= \frac{1}{2} (H^\dagger \overset{\leftrightarrow}{D}_\mu H)^2, \\ \mathcal{O}_L^{(3)l} &= (i H^\dagger \sigma^a \overset{\leftrightarrow}{D}_\mu H) (\bar{L}_L \gamma^\mu \sigma^a L_L), & \mathcal{O}_{LL}^{(3)l} &= (\bar{L}_L \gamma_\mu \sigma^a L_L) (\bar{L}_L \gamma^\mu \sigma^a L_L), \\ \mathcal{O}_L^l &= (i H^\dagger \overset{\leftrightarrow}{D}_\mu H) (\bar{L}_L \gamma^\mu L_L), & \mathcal{O}_R^e &= (i H^\dagger \overset{\leftrightarrow}{D}_\mu H) (\bar{l}_R \gamma^\mu l_R).\end{aligned}$$

Fit with W-boson mass (CDF/PDG), the EWPOs, and one Higgs observable

$$\begin{aligned}R_b &= \frac{\Gamma_b}{\Gamma_{\text{had}}}, & R_\ell &= \frac{\Gamma_{\text{had}}}{\Gamma_\ell}, & A_f, & A_{FB}^f = \frac{3}{4} A_e A_f \quad (f = b, \ell), \\ \sin^2 \theta_{\text{eff}}^{\text{lep}} &= \frac{1}{4} \left( 1 - \frac{g_V^l}{g_A^l} \right), & \Gamma_Z, & \sigma_{\text{had}}^0 = \frac{12\pi}{m_Z^2} \frac{\Gamma_e \Gamma_{\text{had}}}{\Gamma_Z^2}, & \Gamma_W, & \text{BR}_{W \rightarrow \text{had}}\end{aligned}$$



# 6D SMEFT

$$\frac{\Delta \mathcal{O}}{\mathcal{O}} = C_{\mathcal{O}}^{WB} \frac{c_{WB}}{\Lambda^2} + C_{\mathcal{O}}^T \frac{c_T}{\Lambda^2} + C_{\mathcal{O}}^{3L} \frac{c_L^{(3)l}}{\Lambda^2} + C_{\mathcal{O}}^{3LL} \frac{c_{LL}^{(3)l}}{\Lambda^2} + C_{\mathcal{O}}^L \frac{c_L^l}{\Lambda^2} + C_{\mathcal{O}}^R \frac{c_R^e}{\Lambda^2}$$

$\mathcal{O}$	$C_{\mathcal{O}}^{WB}$	$C_{\mathcal{O}}^T$	$C_{\mathcal{O}}^{3L}$	$C_{\mathcal{O}}^{3LL}$	$C_{\mathcal{O}}^L$	$C_{\mathcal{O}}^R$
$m_W$	-0.0111	0.0433	-0.0264	0.0264	/	/
$A_b$	-0.00781	0.0142	-0.0285	0.0285	/	/
$R_b$	0.00189	-0.00345	0.00691	-0.00691	/	/
$R_\ell$	-0.00969	0.0177	-0.159	0.0353	-0.124	0.109
$A_{FB}^\ell$	-1.01	1.84	-2.41	3.69	1.28	1.46
$A_\ell$	-0.583	1.06	-1.38	2.13	0.739	0.843
$A_{FB}^b$	-0.625	1.14	-1.50	2.28	0.784	0.894
$\Gamma_Z$	-0.0112	0.079	-0.121	0.158	-0.0113	-0.0113
$\sigma_{\text{had}}^0$	0.00142	-0.00259	0.0572	-0.00519	0.152	-0.0895
$\Gamma_W$	-0.0322	0.126	-0.174	0.193	/	/
$\text{BR}_{W \rightarrow \text{had}}$	/	/	-0.0200	/	/	/
$\sin^2 \theta_{\text{eff}}^{\text{lep}}$	0.0483	-0.0881	0.115	-0.176	-0.0612	-0.0698
$\mu_{ggh}^{\gamma\gamma}/\mu_{\text{SM}}$	5.8	/	/	/	/	/



## 2D Marginalization

- Case (1):  $\mathcal{O}_{WB}$  and  $\mathcal{O}_T$  only

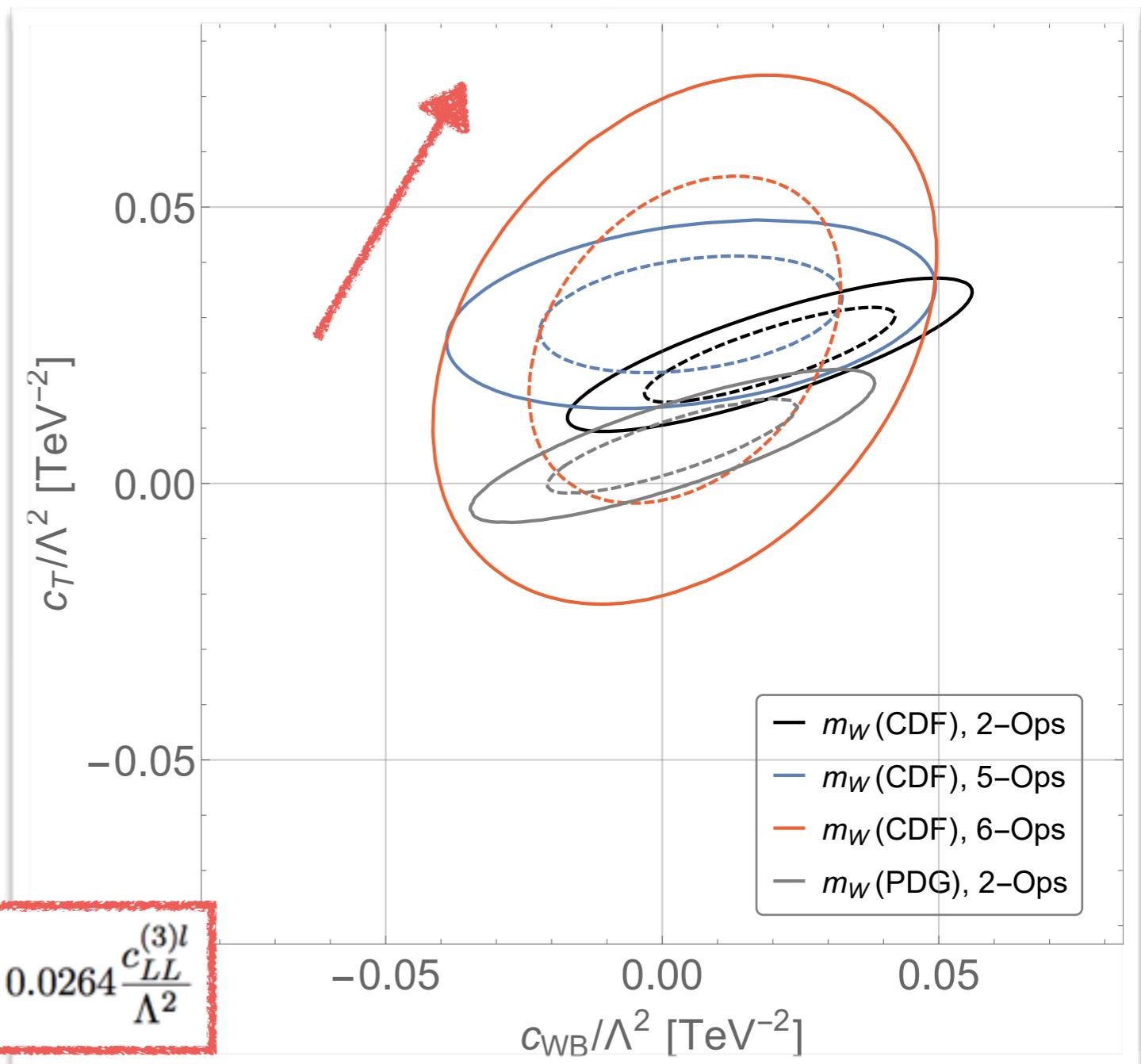
$$S = 16\pi \frac{c_{WB} v^2}{\Lambda^2} \approx 3c_{WB} \left( \frac{\text{TeV}}{\Lambda} \right)^2$$

$$T = \frac{c_T}{\alpha} \frac{v^2}{\Lambda^2} \approx 7.7 c_T \left( \frac{\text{TeV}}{\Lambda} \right)^2$$

- Case (2): five operators except  $\mathcal{O}_R^e$
- Case (3): all six operators

To fit the CRF  $m_W$ ,  $T \sim 0.1$  is favored!

$$\frac{\Delta m_W}{m_W} = -0.0111 \frac{c_{WB}}{\Lambda^2} + 0.0433 \frac{c_T}{\Lambda^2} - 0.0264 \frac{c_L^{(3)l}}{\Lambda^2} + 0.0264 \frac{c_{LL}^{(3)l}}{\Lambda^2}$$





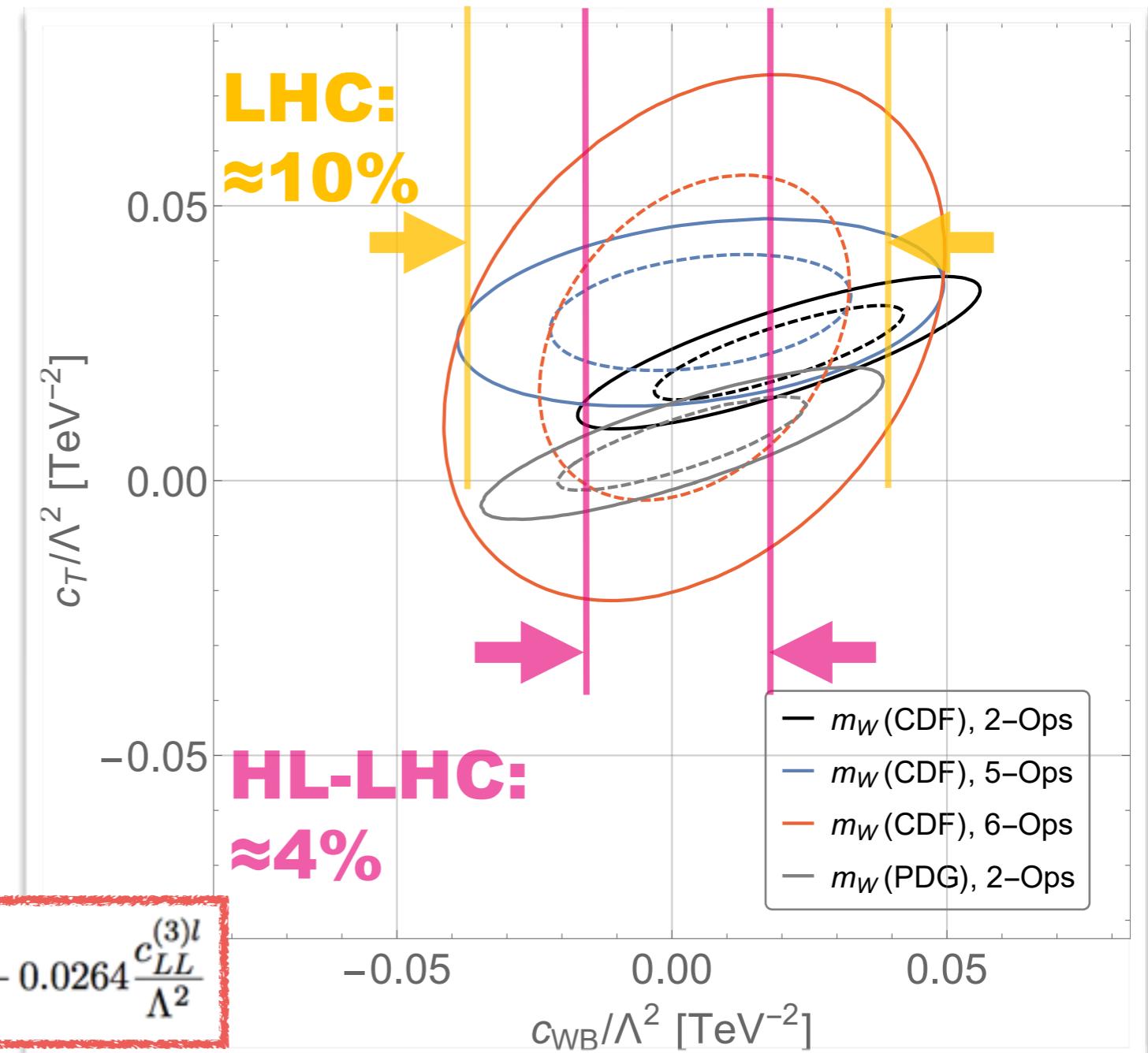
## 2D Marginalization

- Include the signal strength of Higgs di-photon decay to constrain OwB

$$\frac{\mu_{ggh}^{\gamma\gamma}}{\mu_{\text{SM}}} = \frac{5.8}{\Lambda^2} C_{WB}$$

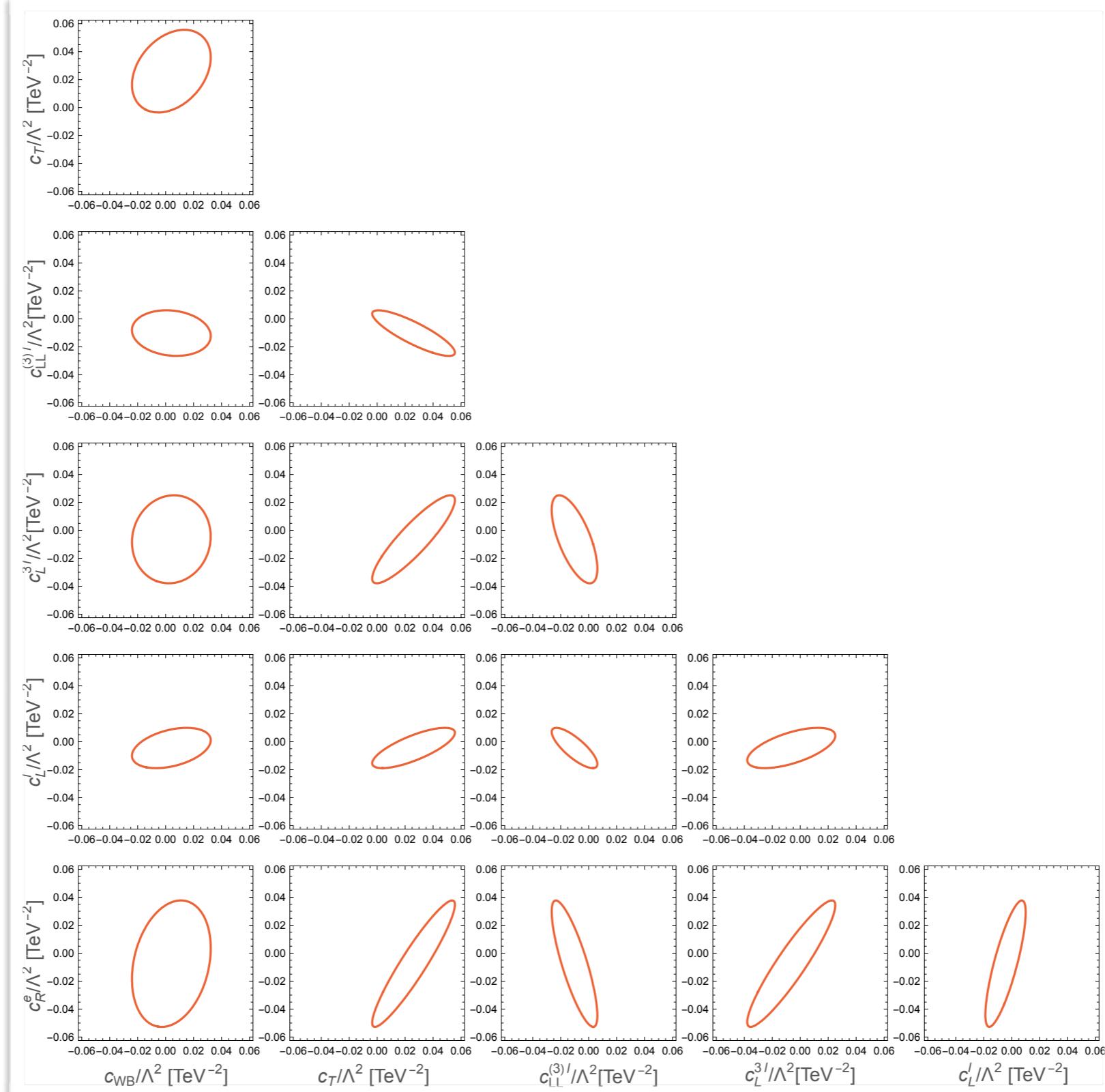
- Help to constrain  $O_T$  to some extent

$$\frac{\Delta m_W}{m_W} = -0.0111 \frac{c_{WB}}{\Lambda^2} + 0.0433 \frac{c_T}{\Lambda^2} - 0.0264 \frac{c_L^{(3)l}}{\Lambda^2} + 0.0264 \frac{c_{LL}^{(3)l}}{\Lambda^2}$$



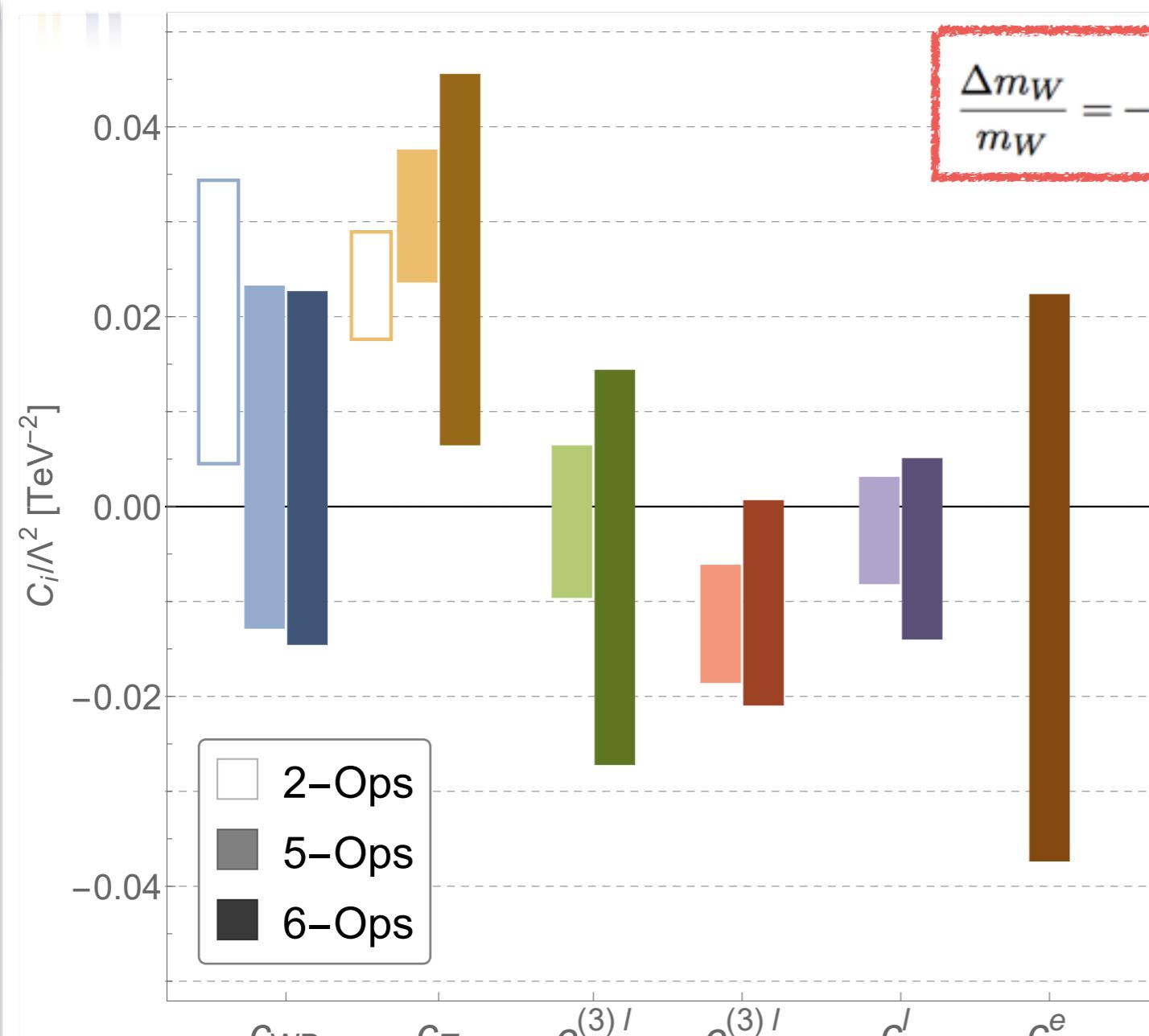


## 2D Marginalization: Case (3)





# 1D Marginalization



$$\frac{\Delta m_W}{m_W} = -0.0111 \frac{c_{WB}}{\Lambda^2} + 0.0433 \frac{c_T}{\Lambda^2} - 0.0264 \frac{c_L^{(3)l}}{\Lambda^2} + 0.0264 \frac{c_{LL}^{(3)l}}{\Lambda^2}$$

A positive  $C_T$  is strongly favored:  
at 68% C.L.

$$C_T > (0.005 - 0.015) \times \frac{\Lambda^2}{\text{TeV}^2}$$



- Tree-level NP ( $C_T \sim \mathcal{O}(1)$ ):  
 $\Lambda \lesssim \mathcal{O}(10)\text{TeV}$
- Loop-level NP ( $C_T \sim \mathcal{O}(10^{-2})$ ):  
 $\Lambda \lesssim \mathcal{O}(1)\text{TeV}$



# One Representative NP Scenario

$$\rho \equiv \frac{m_W^2}{m_Z^2 \cos^2 \theta_W} = \frac{\sum_i [4T_i(T_i + 1) - Y_i^2] |v_i|^2 c_i}{\sum_i 2Y_i^2 |v_i|^2}$$

$T_i$ :SU(2)<sub>L</sub> isospin  
 $Y_i$ :hypercharge  
 $v_i$ :v.e.v.  
 $c_i$ :1 for complex representation  
1/2 for real representation

- T=0 (or rho=1): NP with  $4T_i(T_i + 1) = 3Y_i^2$ 
  - SM + singlet (1,1,0)
  - SM + doublet (0,2,1)
- T>0 (or rho>1): NP with  $4T_i(T_i + 1) > 3Y_i^2$ 
  - SM + A hypercharge-free triplet ( $\Sigma$ ) (1,3,0)

$$\mathcal{L} \supset -\lambda H^\dagger \Sigma H - \frac{\lambda_3}{2} (H^\dagger H) \Sigma \Sigma, \Rightarrow T \simeq \frac{1}{\alpha} \frac{2\langle \Sigma \rangle^2}{v^2}$$



## Best-fit Point

Observables	Case (1)	Case (2)	Case (3)	Experimental Measurement
$m_W(\text{GeV})$	80.4182	80.4335	80.4335	$80.4335 \pm 0.0094$ [21]
$A_b$	0.934895	0.93481	0.934944	$0.923 \pm 0.020$ [5, 19]
$A_\ell (P_\tau)$	0.14889	0.14744	0.14736	$0.1465 \pm 0.0033$ [5, 19]
$A_\ell (\text{SLD})$	0.14889	0.14744	0.14736	$0.1513 \pm 0.0021$ [5, 19]
$R_b$	0.21587	0.21588	0.21587	$0.21629 \pm 0.00066$ [5, 19]
$R_\ell$	20.7510	20.7592	20.7634	$20.767 \pm 0.025$ [5, 19]
$A_{\text{FB}}^b$	0.10448	0.10340	0.10335	$0.0996 \pm 0.0016$ [5, 19]
$A_{\text{FB}}^\ell$	0.01657	0.01629	0.01627	$0.0171 \pm 0.0010$ [5, 19]
$\Gamma_Z(\text{GeV})$	2.49818	2.49515	2.49537	$2.4955 \pm 0.0023$ [5]
$\sigma_{\text{had}}^0(\text{nb})$	41.4915	41.4729	41.4771	$41.480 \pm 0.033$ [22]
$\Gamma_W(\text{GeV})$	2.09262	2.09109	2.09261	$2.085 \pm 0.042$ [5]
$\text{BR}_{W \rightarrow \text{had}}$	0.6748	0.6748	0.6749	$0.6741 \pm 0.0021$ [5]
$\sin^2 \theta_{\text{eff}}^{\text{lep}}(10^{-5})$	23127.7	23146.6	23147.7	$(23143 \pm 25)$ [6]
$\mu_{ggh}^{\gamma\gamma}/\mu_{\text{SM}}$	1.11	1.03	1.02	$1.02 \pm 0.11$ [23]
$\chi^2/\text{D.O.F}$	1.38	1.20	1.34	

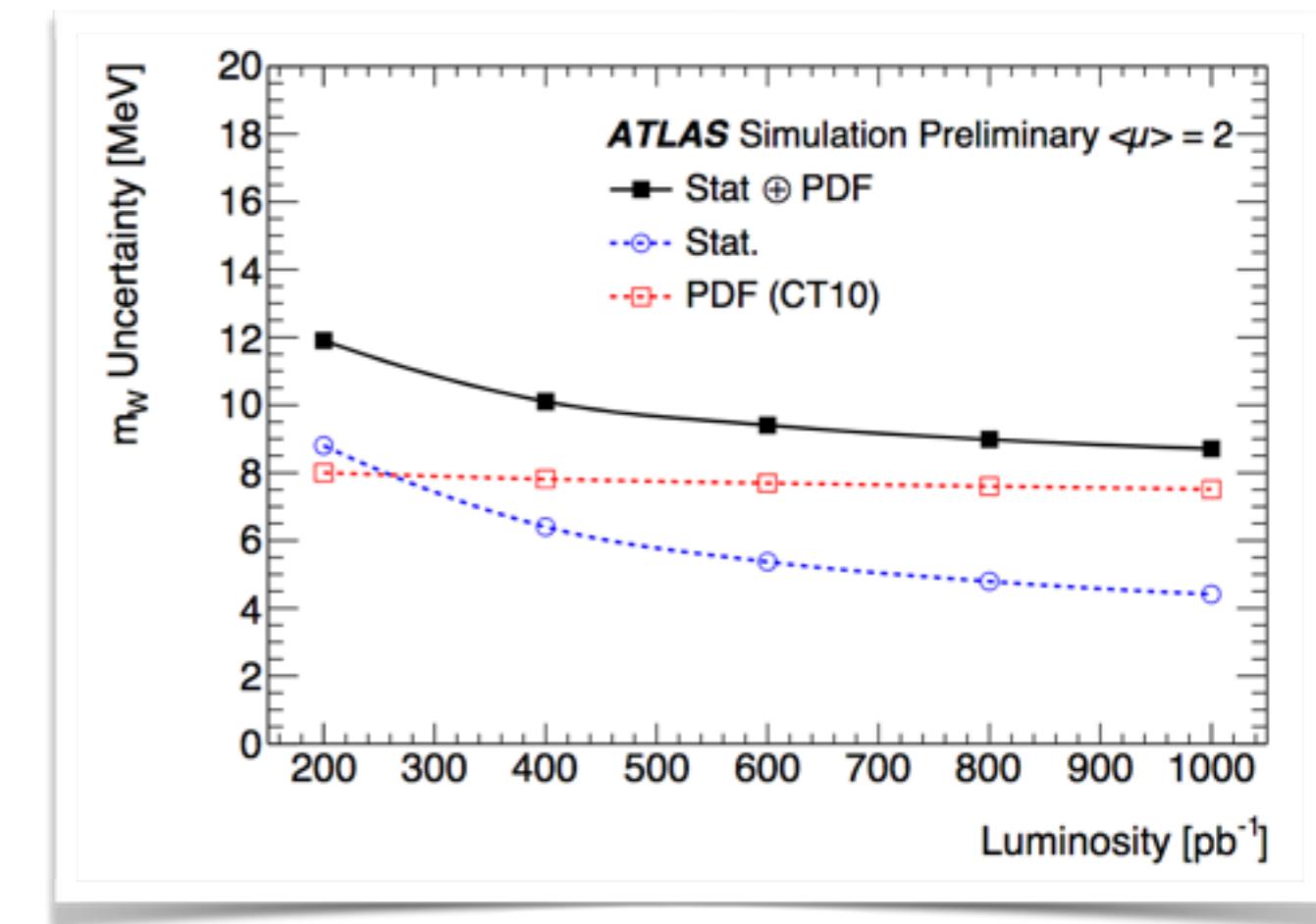
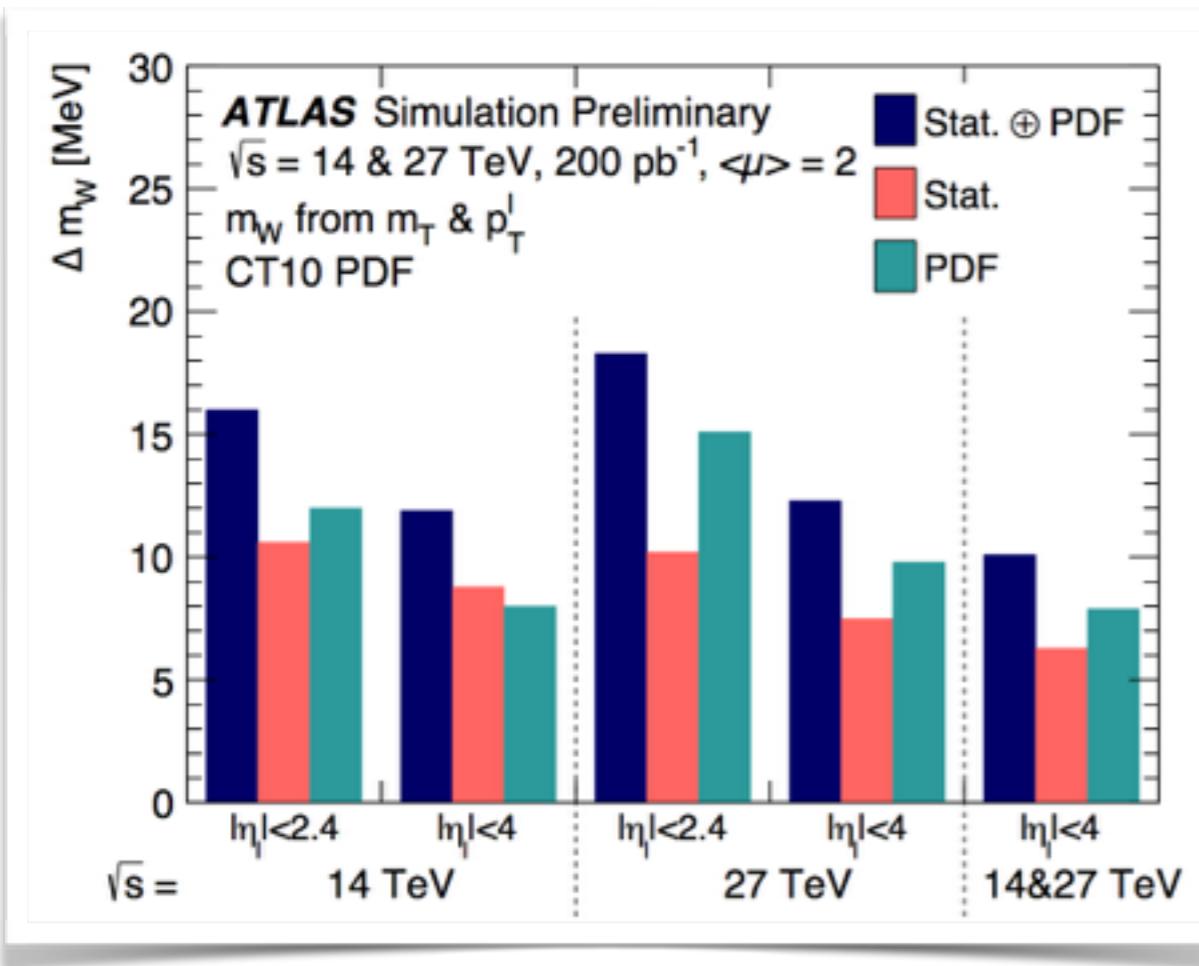
Mw is always well-fit,  
hence driving the whole fit

More operators may  
not yield a better fit



# W-boson Mass at (HL-)LHC

- Highly challenging in normal environment: systematics such as pileups takes over, making the kinematics reconstruction of W events inaccurate
- Special run with low ins-L and low pile-up: low detector occupancy => optimal reconstruction of missing pT, PDF + Stat becomes dominant



[ATL-PHYS-PUB-2018-026]



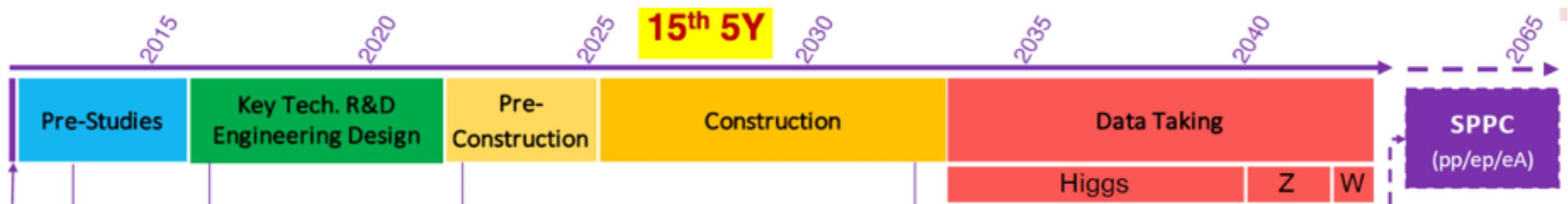
# Future e+e- Collider

Defines the precision frontier of next decades in Higgs and electroweak physics and has an inborn advantage to measure W-boson mass.



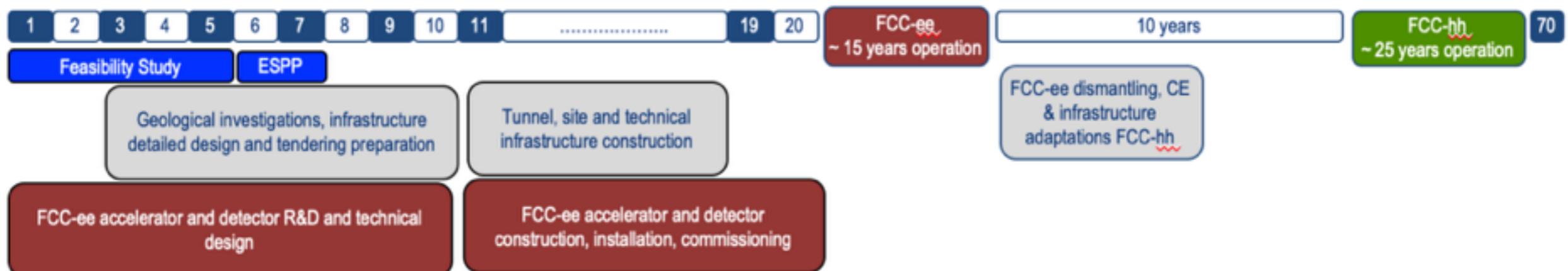


# Roadmap of CEPC-SppC/FCC



[From Xinchou Lou, HKUST IAS HEP program (2022)]

2020 +



[From Frank Zimmermann, HKUST IAS HEP program (2022)]



# Operation Modes of CEPC/FCC-ee

CEPC Operation mode	ZH	Z	W <sup>+</sup> W <sup>-</sup>	ttbar (new)
$\sqrt{s}$ [GeV]	~ 240	~ 91.2	~ 160	~ 360
Run time [years]	7	2	1	~7.7
CDR	$L / IP [\times 10^{34} \text{ cm}^{-2}\text{s}^{-1}]$	3	32	10
	$\int L dt [\text{ab}^{-1}, 2 \text{ IPs}]$	5.6	16	2.6
	Event yields [2 IPs]	$1 \times 10^6$	$7 \times 10^{11}$	$2 \times 10^7$
Latest	$L / IP [\times 10^{34} \text{ cm}^{-2}\text{s}^{-1}]$	5.0	115	15.4
	$\int L dt [\text{ab}^{-1}, 2 \text{ IPs}]$	9.3	57.5	4.0
	Event yields [2 IPs]	$1.7 \times 10^6$	$2.5 \times 10^{12}$	$3 \times 10^7$

[From Xinchou Lou, HKUST IAS HEP program (2022)]

## FCC-ee collider parameters

K. Oide

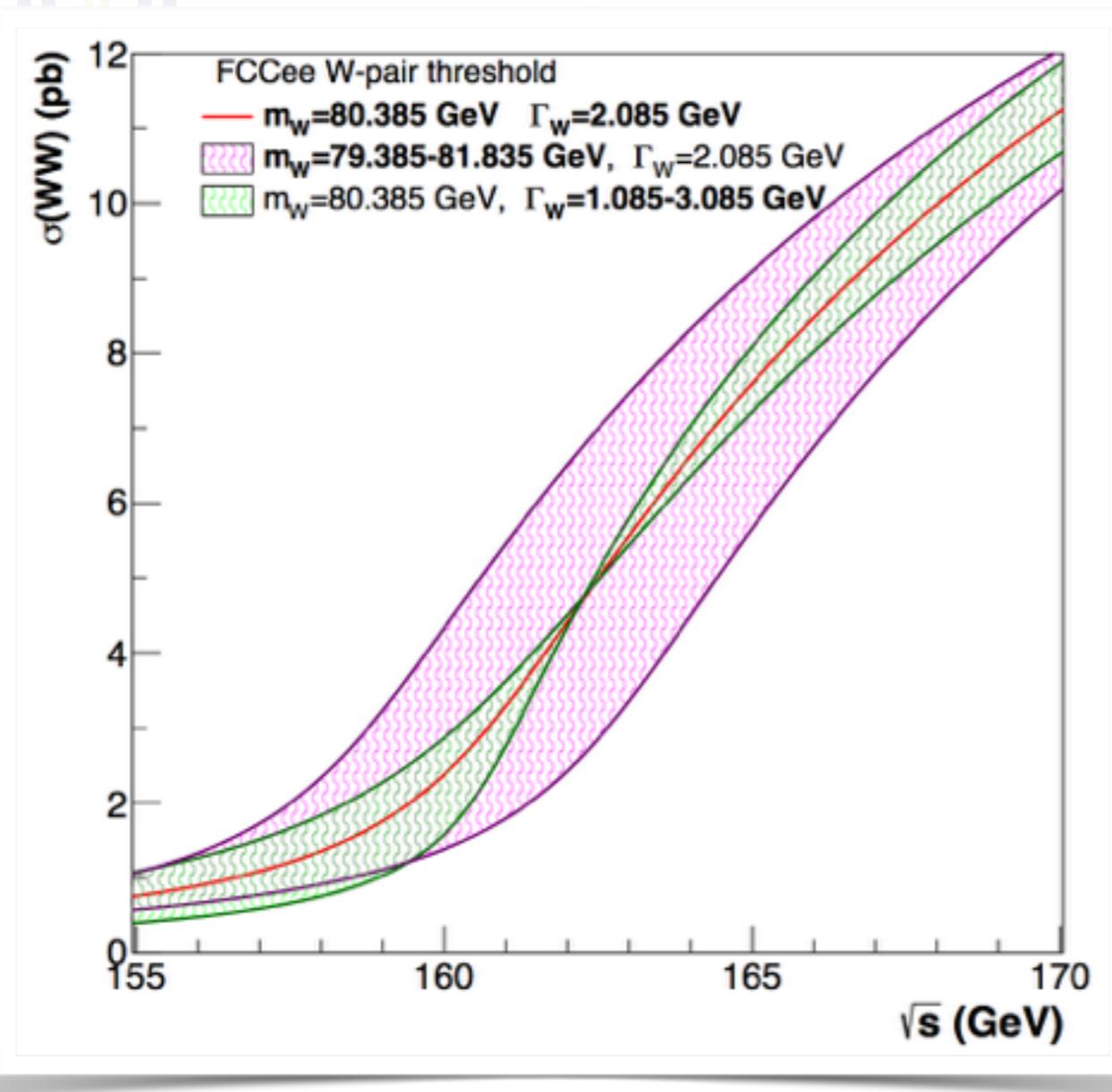
	Z	WW	H (ZH)	ttbar
beam energy [GeV]	45	80	120	182.5
beam current [mA]	1400	135	26.7	5.0
number bunches/beam	8800	1120	336	42
bunch intensity [ $10^{11}$ ]	2.76	2.29	1.51	2.26
SR energy loss / turn [GeV]	0.0391	0.37	1.869	10.0
total RF voltage 400/800 MHz [GV]	0.120/0	1.0/0	2.48/0	4.0/7.67
long. damping time [turns]	1170	216	64.5	18.5
horizontal beta* [m]	0.15	0.2	0.3	1
vertical beta* [mm]	0.8	1	1	1.6
horizontal geometric emittance [nm]	0.71	2.17	0.64	1.49
vertical geom. emittance [pm]	1.42	4.34	1.29	2.98
horizontal rms IP spot size [ $\mu\text{m}$ ]	10	21	14	39
vertical rms IP spot size [nm]	34	66	36	69
beam-beam parameter $\xi_x / \xi_y$	0.004/.159	0.011/0.111	0.0187/0.129	0.096/0.138
rms bunch length with SR / BS [mm]	4.32 / 15.2	3.55 / 7.02	2.5 / 4.45	1.67 / 2.54
luminosity per IP [ $10^{34} \text{ cm}^{-2}\text{s}^{-1}$ ]	181	17.3	7.2	1.25
total integrated luminosity / year [ $\text{ab}^{-1}/\text{yr}$ ]	86	8	3.4	0.6
beam lifetime rad Bhabha / BS [min]	19 / ?	20 / ?	10 / 19	12 / 46

[From Frank Zimmermann,  
HKUST IAS HEP program (2022)]





# Methods of Measuring W-boson Mass at e+e- Collider



- Threshold scan: relies on rapid rise of the W-pair production near its kinematic threshold.
  - Simple and clean: involving event selection and counting only
  - 12/ab data equally shared between 157.5-162.5 GeV  $\Rightarrow \Delta M_W \sim 0.5 \text{ MeV (stat)}$
- Kinematic reconstruction: exploits the W-pair final state reconstruction and kinematic fit.
  - Less influenced by statistical uncertainty
  - Combining data at WW threshold and 240-365 GeV  $\Rightarrow \Delta M_W \sim 0.5 \text{ MeV (stat)}$

[FCC CDR, vol. 1]



# Methods of Measuring W-boson Mass at e+e- Collider

Observable	LEP precision	CEPC precision	CEPC runs	CEPC $\int \mathcal{L} dt$
$m_Z$	2.1 MeV	0.5 MeV	$Z$ pole	$8 \text{ ab}^{-1}$
$\Gamma_Z$	2.3 MeV	0.5 MeV	$Z$ pole	$8 \text{ ab}^{-1}$
$A_{FB}^{0,b}$	0.0016	0.0001	$Z$ pole	$8 \text{ ab}^{-1}$
$A_{FB}^{0,\mu}$	0.0013	0.00005	$Z$ pole	$8 \text{ ab}^{-1}$
$A_{FB}^{0,e}$	0.0025	0.00008	$Z$ pole	$8 \text{ ab}^{-1}$
$\sin^2 \theta_W^{\text{eff}}$	0.00016	0.00001	$Z$ pole	$8 \text{ ab}^{-1}$
$R_b^0$	0.00066	0.00004	$Z$ pole	$8 \text{ ab}^{-1}$
$R_\mu^0$	0.025	0.002	$Z$ pole	$8 \text{ ab}^{-1}$
$m_W$	33 MeV	1 MeV	$WW$ threshold	$2.6 \text{ ab}^{-1}$
$m_W$	33 MeV	2–3 MeV	$ZH$ run	$5.6 \text{ ab}^{-1}$
$N_\nu$	1.7%	0.05%	$ZH$ run	$5.6 \text{ ab}^{-1}$

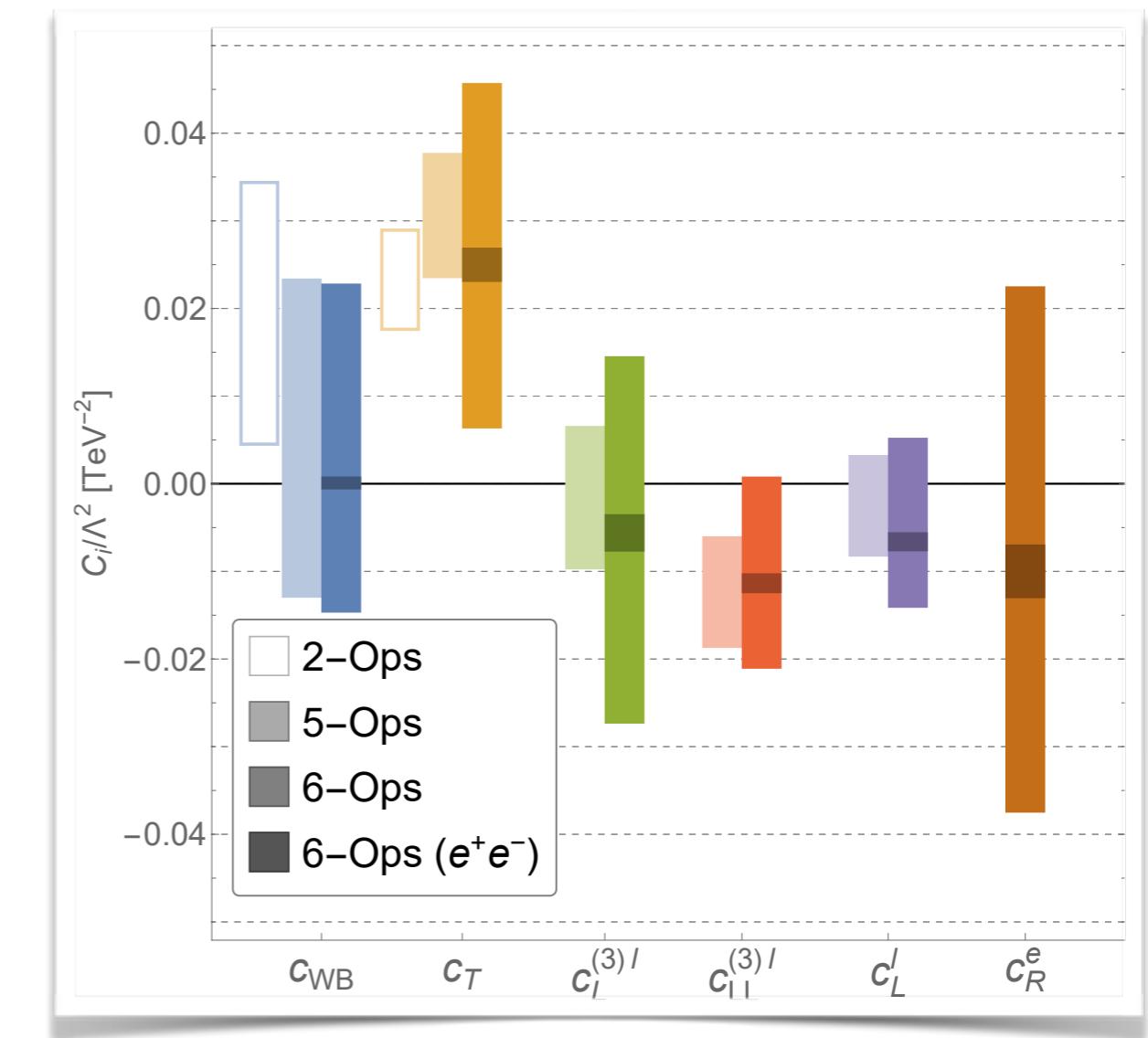
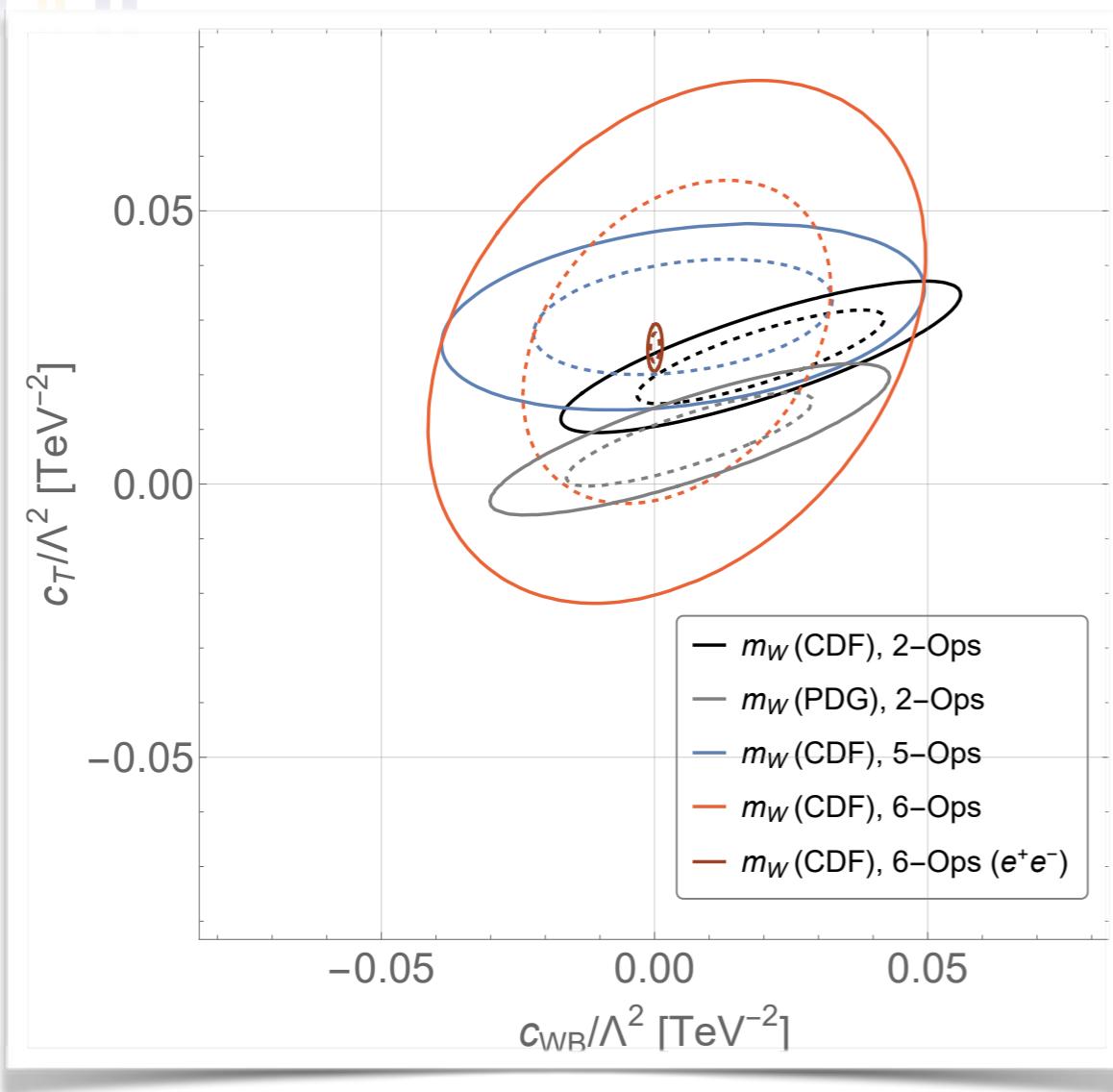
[FCC CDR, vol. 1]

[CEPC CDR, vol. II]

Observable	present value $\pm$ error	FCC-ee Stat.	FCC-ee Syst.	Comment and dominant exp. error
$m_W$ (MeV)	$80350 \pm 15$	0.6	0.3	From WW threshold scan Beam energy calibration
$\Gamma_W$ (MeV)	$2085 \pm 42$	1.5	0.3	From WW threshold scan Beam energy calibration
$\alpha_s(m_W)(\times 10^4)$	$1170 \pm 420$	3	small	from $R_\ell^W$ [43]
$N_\nu(\times 10^3)$	$2920 \pm 50$	0.8	small	ratio of invis. to leptonic in radiative Z returns



# W-boson Mass at Future e+e- Collider



Combination of the FCCee precisions and the current central values of the set of observables =>  $C_T$  can be tested with more than 10sigma away from its null limit



# Theoretical Uncertainties

Quantity	FCC-ee	future parametric unc.	Main source
$M_W$ [MeV]	0.5 – 1	1 (0.6)	$\delta(\Delta\alpha)$
$\sin^2 \theta_{\text{eff}}^\ell$ [ $10^{-5}$ ]	0.6	2 (1)	$\delta(\Delta\alpha)$
$\Gamma_Z$ [MeV]	0.1	0.1 (0.06)	$\delta\alpha_s$
$R_b$ [ $10^{-5}$ ]	6	< 1	$\delta\alpha_s$
$R_\ell$ [ $10^{-3}$ ]	1	1.3 (0.7)	$\delta\alpha_s$

Quantity	FCC-ee	Current intrinsic error	Projected intrinsic error
$M_W$ [MeV]	0.5–1 <sup>†</sup>	4 ( $\alpha^3, \alpha^2\alpha_s$ )	1
$\sin^2 \theta_{\text{eff}}^\ell$ [ $10^{-5}$ ]	0.6	4.5 ( $\alpha^3, \alpha^2\alpha_s$ )	1.5
$\Gamma_Z$ [MeV]	0.1	0.4 ( $\alpha^3, \alpha^2\alpha_s, \alpha\alpha_s^2$ )	0.15
$R_b$ [ $10^{-5}$ ]	6	11 ( $\alpha^3, \alpha^2\alpha_s$ )	5
$R_\ell$ [ $10^{-3}$ ]	1	6 ( $\alpha^3, \alpha^2\alpha_s$ )	1.5

[A. Freitas, S. Heinemeyer et. al., 1906.05379]

Question: Can the theory interpretation of  $m_W$  catch-up with the experiment progress by the time of the future e+e- collider runs?



## Summary

- In the SMEFT, an upward shift of  $m_W$  is mainly driven by the  $O_T$  operator
- The favored NP scale could be within the reach of current and near-future LHC searches
  - Tree-level NP:  $\Lambda \lesssim \mathcal{O}(10)\text{TeV}$
  - Loop-level NP:  $\Lambda \lesssim \mathcal{O}(1)\text{TeV}$
- Future  $e^+e^-$  colliders have an inborn advantage to measure  $W$  physics and test the relevant SMEFT
- We need to be open for all possibilities, to finally reveal the truth on this anomaly ... ...

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GRF under grant No. 16305219



*Thank you!*