SM and BSM predictions of M_W using different renormalization schemes — FlexibleSUSY implementation and the example of the MRSSM

Dominik Stöckinger

TU Dresden

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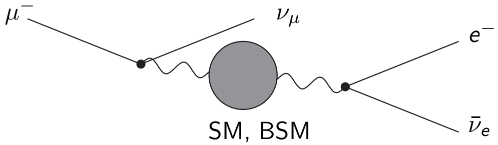
recent work with: [Peter Athron, Markus Bach, Douglas Jacob, Wojciech Kotlarski, Alexander Voigt]

Outline

1. SM calculation (intro/review)
2. BSM calculations (Problem, solution)
3. MRSSM prediction of M_W
4. Conclusions

How to calculate M_W?

$$\mu\text{-lifetime} \quad \tau_\mu = f(M_W; \alpha, M_Z, \ldots)$$



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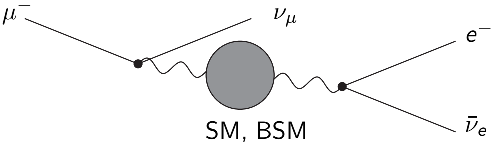
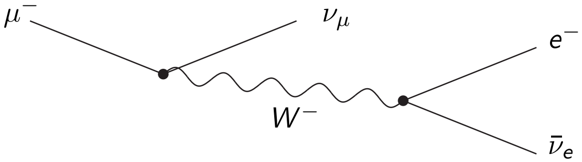


invert $\Rightarrow$ theory prediction for M_W

μ-lifetime $\Leftrightarrow$ G_μ

Convenient: use G_μ: effective matrix element/4-fermion coupling:

$$\tau_\mu^{-1} \equiv G_\mu^2 \underbrace{\frac{m_\mu^5}{192\pi^2}\left(1 + \mathcal{O}\left(\frac{m_e^2}{m_\mu^2}, \frac{m_\mu^2}{M_W^2}\right) + \Delta_{\rm QED}\right)}_{\text{known, model-independent}}$$



$$G_\mu \overset{\rm SM\_tree}{=} \frac{\alpha\pi}{\sqrt{2} M_W^2 s_W^2}$$

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Muon-decay and M_W: SM on-shell scheme

(Input) parameters:

$$\alpha = \frac{1}{137....} \qquad M_{W,Z} = \text{pole masses} \qquad s_W^2 \equiv 1 - \frac{M_W^2}{M_Z^2}$$

Central relation:

$$G_\mu = \frac{\pi\alpha}{\sqrt{2} M_W^2 s_W^2}\left(1 + \Delta r\right)$$

$\Rightarrow$ Δr= loop corrections (self-energy, vertex, box, counterterms)

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$\Rightarrow$ model-prediction for M_W in terms of M_Z, α, G_μ, Δr

$$G_\mu = \frac{\pi\alpha}{\sqrt{2} M_W^2 s_W^2}\left(1 + \Delta r\right)$$

Leading loop effects:

- $\Delta\alpha$: shift of finestructure constant (via δZ_e) $\longrightarrow \log \frac{m_{\text{light}}}{M_W}$
- $\Delta\rho$: shift of ρ parameter[$\sim T$-parameter] (via δs_W^2) $\longrightarrow \frac{m_t^2}{M_W^2}$

$$\Delta r = \Delta\alpha - \frac{c_W^2}{s_W^2}\Delta\rho + \Delta r_{\text{remainder}}$$

$$6\% \qquad -3\% \qquad +1\%$$

full 2-loop + leading 3-loop available

[Sirlin. . . . . . Freitas, Hollik, Walter, Weiglein, Awramik, Czakon. . .]

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Possible resummation of leading 1-loop terms [Consoli, Hollik, Jegerlehner '89]

$$1 + \Delta r = \frac{1}{(1 - \Delta\alpha)(1 + \frac{c_W^2}{s_W^2}\Delta\rho)}(1 + \Delta r_{\text{remainder}})$$

(generates correct 2-loop terms of $\mathcal{O}(\Delta\alpha^2, \Delta\alpha\Delta\rho, \Delta\rho^2)$)
(somewhat awkward to combine with full 2-loop computation)

$$G_\mu = \frac{\pi\alpha}{\sqrt{2}M_W^2 s_W^2}\left(1+\Delta r\right)$$

$$\leftrightarrow$$

$$G_\mu = \frac{\pi\alpha(M_Z)}{\sqrt{2}M_W^2\left(1-\frac{M_W^2}{\rho M_Z^2}\right)}\left(1+\Delta r_{\text{remainder}}\right)$$

achieves same resummation, with

$$\alpha(M_Z) \equiv \frac{\alpha}{1-\Delta\alpha} \qquad \rho \equiv \frac{1}{1-\Delta\rho}$$

via $\Pi^\gamma(M_Z^2) - \Pi^\gamma(0)$ via $\Sigma_{W,Z}(0)$

$\Rightarrow$ absorbs leading effects in new parameters! $\longrightarrow \log\frac{m_{\text{light}}}{M_W}, \frac{m_t^2}{M_W^2}$

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Muon-decay and M_W: SM $\overline{\text{MS}}$-scheme

- $\overline{\text{MS}}$-scheme with $\mu = M_Z$
- simpler prescription for counterterms, but obscured relation to observables
- here: possibility to similarly absorb leading effects! [Degrassi, Fanchiotti, Sirlin '89]

Muon-decay and M_W: SM $\overline{\text{MS}}$-scheme

(Input) parameters:

$\hat{\alpha}, \hat{s}^2 = \overline{\text{MS}}$ via gauge couplings $\qquad$ $M_{W,Z}$ = pole masses

Central relation:

$$G_\mu = \frac{\pi\hat{\alpha}(M_Z)}{\sqrt{2}M_W^2\hat{s}^2}\left(1 + \Delta\hat{r}_W\right)$$

Further relations to observables:

$$\hat{\alpha}(M_Z) \equiv \frac{\alpha}{1-\Delta\hat{\alpha}} \qquad \hat{\rho} \equiv \frac{M_W^2}{M_Z^2\hat{c}^2} = \frac{c_W^2}{\hat{c}^2} \qquad \hat{s}^2 = \left(1 - \frac{M_W^2}{\hat{\rho}M_Z^2}\right)$$

via $\Pi^\gamma(0)_{fin}$ $\qquad$ via $\delta M_{W,Z\,fin}$

$$G_\mu = \frac{\pi\hat\alpha(M_Z)}{\sqrt{2}M_W^2\hat s^2}\left(1+\Delta\hat r_W\right) \qquad \hat s^2 = \left(1-\frac{M_W^2}{\hat\rho M_Z^2}\right)$$

Same structure as resummed form of on-shell scheme!

Contains the same leading loop effects:

- $\Delta\hat\alpha$: $\longrightarrow \log\frac{m_{\text{light}}}{M_W}$
- $\hat\rho$: $\longrightarrow \frac{m_t^2}{M_W^2}$

$\overline{\text{MS}}$-scheme calculation promises higher accuracy but involves more intermediate quantities. Status=on-shell:

full 2-loop + leading 3-loop available [...Degrassi, Gambino, Giardino...]

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Definitions and remarks

$$\alpha, s_W^2 \qquad \hat{\alpha}(M_Z), \hat{\rho} \qquad \hat{\alpha}(M_Z), \hat{s}^2$$

$$\Delta\alpha \equiv \Pi^{\gamma}_{\text{light}}(0) - \Pi^{\gamma}_{\text{light}}(M_Z^2) \qquad \Delta\rho \equiv \frac{\Sigma_Z(0)}{M_Z^2} - \frac{\Sigma_W(0)}{M_W^2}$$

$$\Delta\hat{\alpha} \equiv \Pi^{\gamma}(0)|_{\text{fin}} \qquad \frac{1}{\hat{\rho}} = \left(1 + \frac{\delta M_W^2}{M_W^2} - c^2_{\text{bare}} \frac{\delta M_Z^2}{M_Z^2}\right)_{\text{fin}}$$

Remarks:

- relate low-E (G_μ, α) to high-E ($M_{W,Z}$) [QED running] $\longrightarrow \log \frac{m_{\text{light}}}{M_W}$
- breaking of custodial symmetry $\longrightarrow \frac{m_t^2}{M_W^2}$

Current status as in [Degrassi, Gambino, Giardino '14/Athron, Bach, Jacob, Kotlarski, Voigt '22]

$$M_W = \begin{cases} 80.357(3) & \overline{\text{MS}}\text{-scheme} \\ 80.351(3) & \overline{\text{MS}}\text{-scheme (update)} \\ 80.363(4) & \text{on-shell scheme} \\ 80.355(3) & \text{on-shell scheme (update)} \end{cases}$$

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   - FlexibleSUSY approach

Overview BSM contributions

typically dominant contributions via $\Delta\rho \sim T$-parameter. E.g.

- Higgs triplet (tree-level ρ)
- 2HDM
- stop/sbottom, other SUSY particles
- 4th generation (t', b')

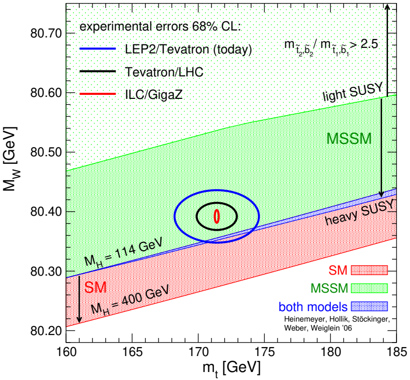
In general, heavy BSM decouples

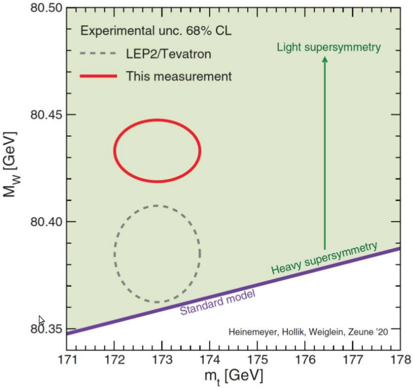
$$M_W^{\text{BSM}} \to M_W^{\text{SM}} \qquad (\text{if } M_{\text{BSM}} \to \infty)$$

in particular, no change of QED running below M_W

Example: on-shell calculations in MSSM and 2HDM

MSSM: full 1-loop Δr, 2-loop $\Delta\rho$ [Haestier, Heinemeyer, Hollik, DS, Weber, Weiglein '06... Zeune]





2HDM: full 1-loop Δr and 2-loop $\Delta\rho$ [Lopez-Val, Sola... Hessenberger, Hollik]

How to implement M_W in generic spectrum generators?

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$\overline{\text{MS}}$-scheme looks natural $\rightsquigarrow$ applied in

- “classical” calculations [Bagger,Matchev,Pierce,Zhang’96]
- Softsusy, Spheno, FlexibleSUSY...

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The non-decoupling problem

In hindsight obvious, but initially obscured! E.g. by light spectra and "compensating bug" in some programs

$$G_\mu = \frac{\pi\hat{\alpha}(M_Z)}{\sqrt{2}M_W^2\hat{s}^2}\left(1+\Delta\hat{r}_W\right) \qquad \hat{s}^2 = \left(1-\frac{M_W^2}{\hat{\rho}M_Z^2}\right)$$

$$M_{\text{BSM}} \gg M_W \gg m_{\text{light}} \text{ !!}$$

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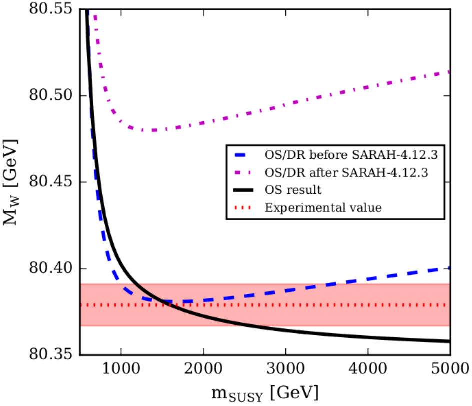
- $\Delta\hat{\alpha}$: $\longrightarrow$ log $\frac{m_{\text{light}}}{M_W}$ (physical), log $\frac{M_{\text{BSM}}}{M_W}$ (unphysical, must drop out)
- $\hat{\rho}$: $\longrightarrow \frac{m_t^2}{M_W^2}$, BSM sources of custodial sym. break. (physical)

Example of the non-decoupling problem [Diessner, Weiglein '19]



Origin: 1-Loop implementation of $\overline{\text{MS}}$ formula leads to

$$\frac{\cdots}{1-\Delta\hat{\alpha}^{1L}}(1+\Delta\hat{r}_W^{1L}) \sim 1 + \underbrace{\Delta\hat{\alpha}^{1L} + \Delta\hat{r}_W^{1L} + \ldots}_{\text{non-dec. cancels}} + \underbrace{(\Delta\hat{\alpha}^{1L})^2 + \ldots}_{\text{incomplete 2L, non-decoup.}}$$

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Implementation in FlexibleSUSY [Athron, Bach, Jacob, Kotlarski, DS, Voigt '22]

FlexibleSUSY:

- spectrum generator generator
- e.g. MSSM, NMSSM, MRSSM, 2HDM, other non-SUSY models...
- precise computation of BSM spectrum and observables (M_H, a_μ, FlexibleDecay...)

M_W:

- SM-contributions fully taken into account from [Degrassi, Gambino, Giardino '14]
- BSM-contributions: strictly expand $\overline{\text{MS}}$ formula to 1L-BSM order:

$$M_W^2 = M_W^{2\,\text{SM}}(1 + \Delta_W)$$

with

$$\Delta_W = \frac{\hat{s}^2}{\hat{c}^2 - \hat{s}^2}\left[\frac{\hat{c}^2}{\hat{s}^2}(\Delta\hat{\rho}_{\text{tree}} + \Delta\hat{\rho}) - \Delta\hat{r}_W - \Delta\hat{\alpha}\right]^{1\text{L}}_{\text{BSM}}$$

Outline

Technical summary of MRSSM

Motivated by new symmetry, $\theta \to e^{i\alpha}\theta$ (more predictive!)

- $\tilde{q}_L$: R=+1, $\tilde{q}_R$: R= -1, no LR-mixing! Majorana masses forbidden!

Dirac gauginos, new superfields $\hat{O}$, $\hat{T}$, $\hat{S}$

- new Higgs triplet!

Dirac Higgsinos, new superfields $\hat{R}_u$, $\hat{R}_d$

- New superpotential terms

$$W_{\text{MRSSM}} = \ldots + \mu_u \hat{H}_u \hat{R}_u + \Lambda_u \hat{H}_u \hat{T} \hat{R}_u + \lambda_u \hat{H}_u \hat{S} \hat{R}_u + y_u \hat{Q} \hat{H}_u \hat{U}$$

like top-Yukawa coupling!

Main effects in MRSSM

Higgs triplet VEV and Λ, λ-couplings $\Rightarrow$ impact on M_W (and M_h)

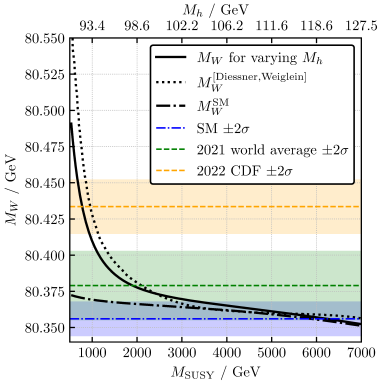
Tree-level

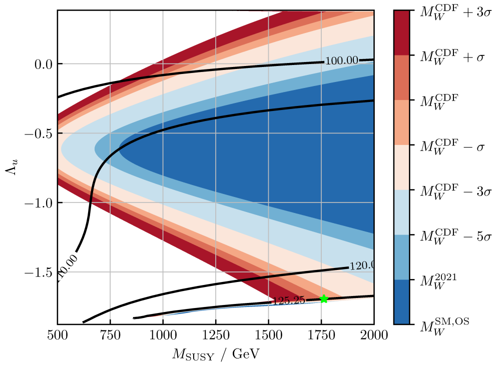
$$\hat{\rho}_{\text{tree}} = 1 + \frac{4v_T^2}{v_d^2 + v_u^2}$$

Main one-loop effect from Λ, λ's

$$T \approx \frac{1}{16\pi^2 s_W^2 M_W^2} \frac{v^4(4\lambda^4 + 5\Lambda^4)}{10 m_D^2}$$

MRSSM results [Athron, Bach, Jacob, Kotlarski, DS, Voigt '22]





$\tan\beta = 37.6$, $M_{\text{SUSY}} = 1727\text{GeV}$

$\Lambda_d = 2.41$, $\Lambda_u = 1.70$, $\lambda_d = 2.43$, $\lambda_u = 0.267$

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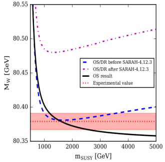
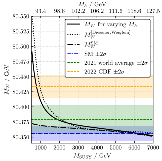
Conclusions

- SM theory prediction very advanced
  - on-shell and $\overline{\text{MS}}$ at 2-loop + beyond
  - uncertainties $\sim \pm 3 \ldots 10$ MeV



- BSM predictions tricky (extra scale!)
  - on-shell 2-loop $\Delta\rho$ in some models
  - SM-like $\overline{\text{MS}}$-scheme non-decoupling



- FlexibleSUSY & applications
  - FlexibleSUSY now implements full SM+strict BSM-1L
  - Decoupling works, accurate prediction
  - MRSSM is promising!