Axion Quality Problem and its solution

Seong Chan Park (Yonsei)

The 2nd "Asian-European-Institutes Workshop for BSM" and the 10th "KIAS Workshop on Particle Physics and Cosmology", Nov. 14, 2022, The Grand Sumorum, Jeju, Korea

3 Main players

Dhong Yeon Cheong

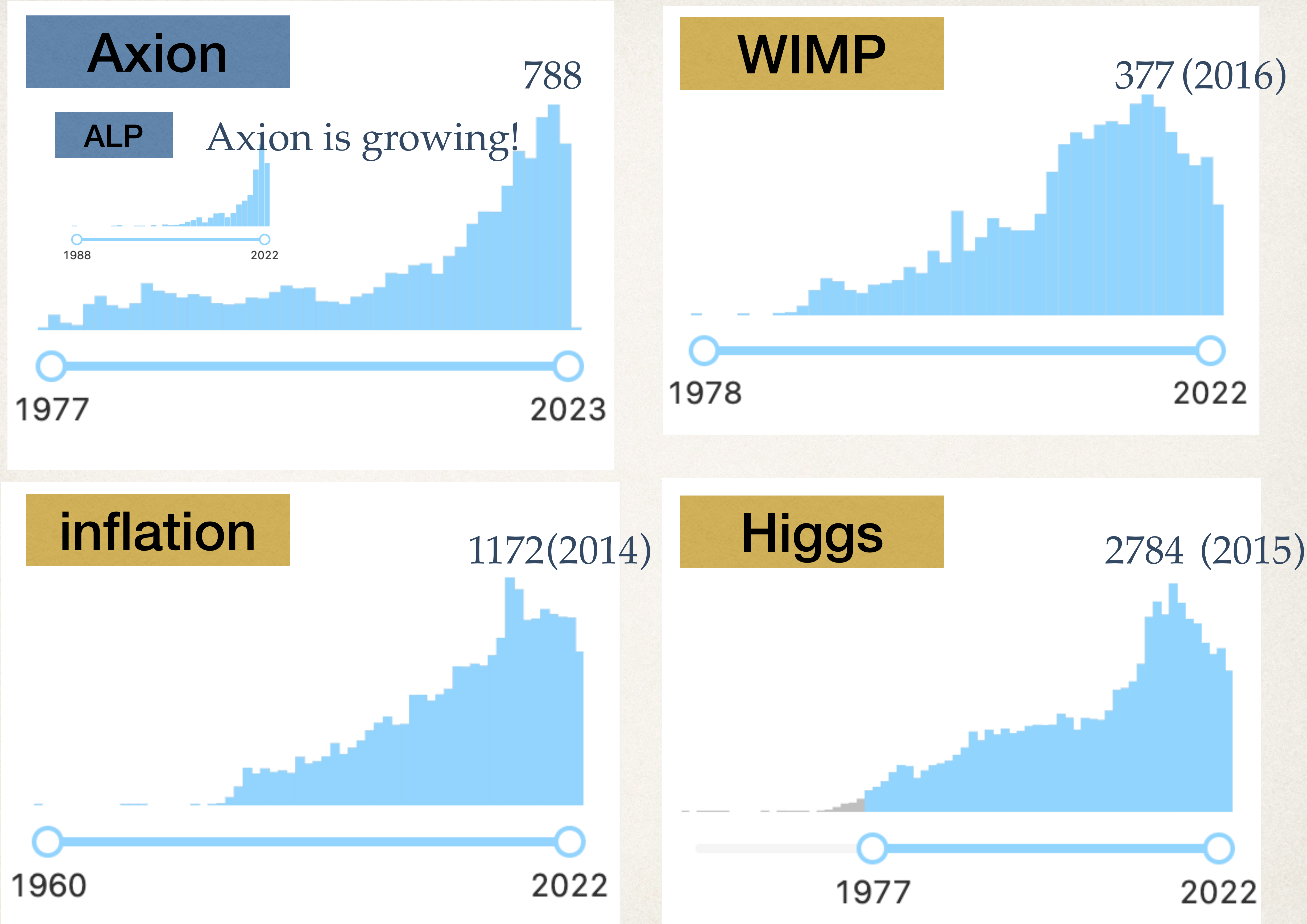
Sung Mook Lee

Yoshiki Kanazawa

Yonsei students

Univ. of Tokyo student

D. Y. Cheong, K. Hamaguchi, Y. Kanazawa, S.M.Lee, N. Nagata, SCP [2210.11330]



inspireHEP.net

- Question about Global $U(1)$ symmetry

- No global symmetry is allowed in a Gravitating system
- … such as our Universe!

See, eg. [T. Banks, N. Seiberg, Phys. Rev. D 83, 084019 (2011)]
[E. Witten, Nature Phys. 14, 116 (2018)]
[D. Harlow, H. Ooguri, Phys. Rev. Lett. 122, 191601 (2019)]

Axion Quality Problem

Axion is the most compelling solution to the Strong CP problem with Global $U(1)_{PQ}$

[R. D. Peccei, H. R. Quinn, Phys. Rev. Lett. 38, 1440 (1977)]
[R. D. Peccei, H. R. Quinn, Phys. Rev. D 16, 1791 (1977)]
[S. Weinberg, Phys. Rev. Lett. 40, 223 (1978)]
[F. Wilczek, Phys. Rev. Lett. 40, 279 (1978)]

[J. E. Kim, PRL (1979)] [M. A. Shifman, V. I. Vainstein, V. I. Zakharov (1980)]
[A. P. Zhitnitskii (1980)] [M. Dine, W. Fischler, M. Srednicki, PLB (1981)]

"No global symmetries allowed by gravity"

Gravitational ~~$U(1)_{PQ}$~~ potential shift

→ ***Axion Quality Problem***

[H. M. Georgi, L. J. Hall, M. B. Wise, Nucl. Phys. B 192, 409 (1981)]
[M. Dine, N. Seiberg, Nucl. Phys. B 273, 109 (1986)] [R. Holman, *et. al.*, Phys. Lett. B 282, 132 (1992)]
[R. Kallosh, A. D. Linde, D. A. Linde, L. Susskind, PRD (1995)] [E.J.Chun, A. Lucas, *et. al.*, PLB(1995)]

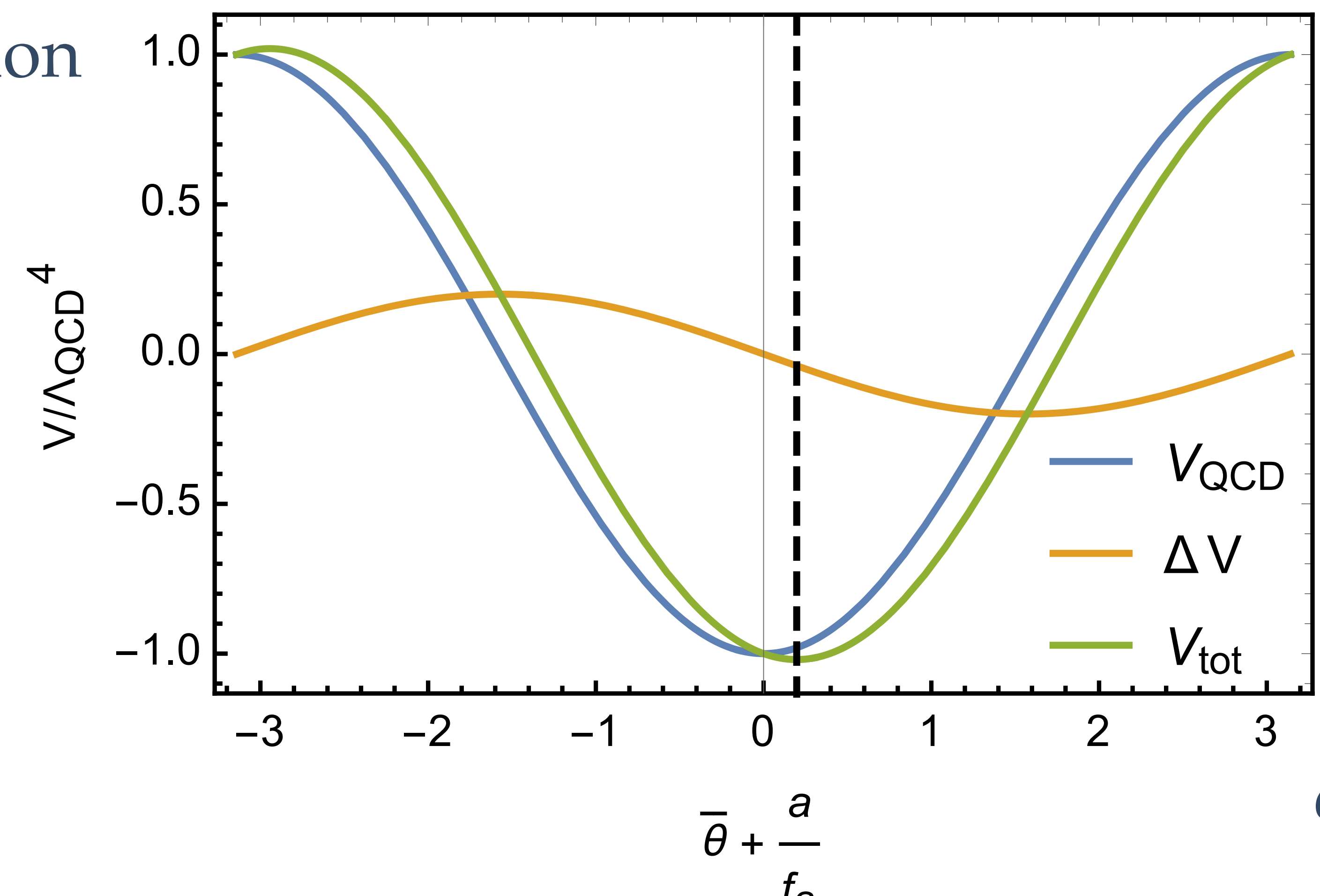
QCD+ Gravitation

$$V_{QCD}(\theta) = -\Lambda_{QCD}^4 \cos\theta \rightarrow \theta_{eff} = 0$$

$$V_{QCD} + \Delta V_{Grav} \rightarrow \theta_{eff} \neq 0$$

↑

Gravitational instanton
(Euclidean wormhole)



Giddings-Strominger Action (1988)

$U(1)$ PQ field $\Phi = \frac{f_a}{\sqrt{2}} e^{i\theta(r)}$

(Gravity + Axion)

$$S = \int d^4x \sqrt{|g|} \left(-\frac{M_P^2}{2} R + \frac{f_a^2}{2} (\partial_\mu \theta)^2 \right)$$

spherically symmetric spacetime $ds^2 = dr^2 + R(r)^2 d\Omega_3^2$

→solve Einstein's equation with B.C. $R'(0) = 0$

→wormhole solution

→ S_{wh} wormhole action

[S. B. Giddings, A. Strominger, Nucl. Phys. B 306, 890 (1988)]

Wormhole connects two asymptotically flat spaces

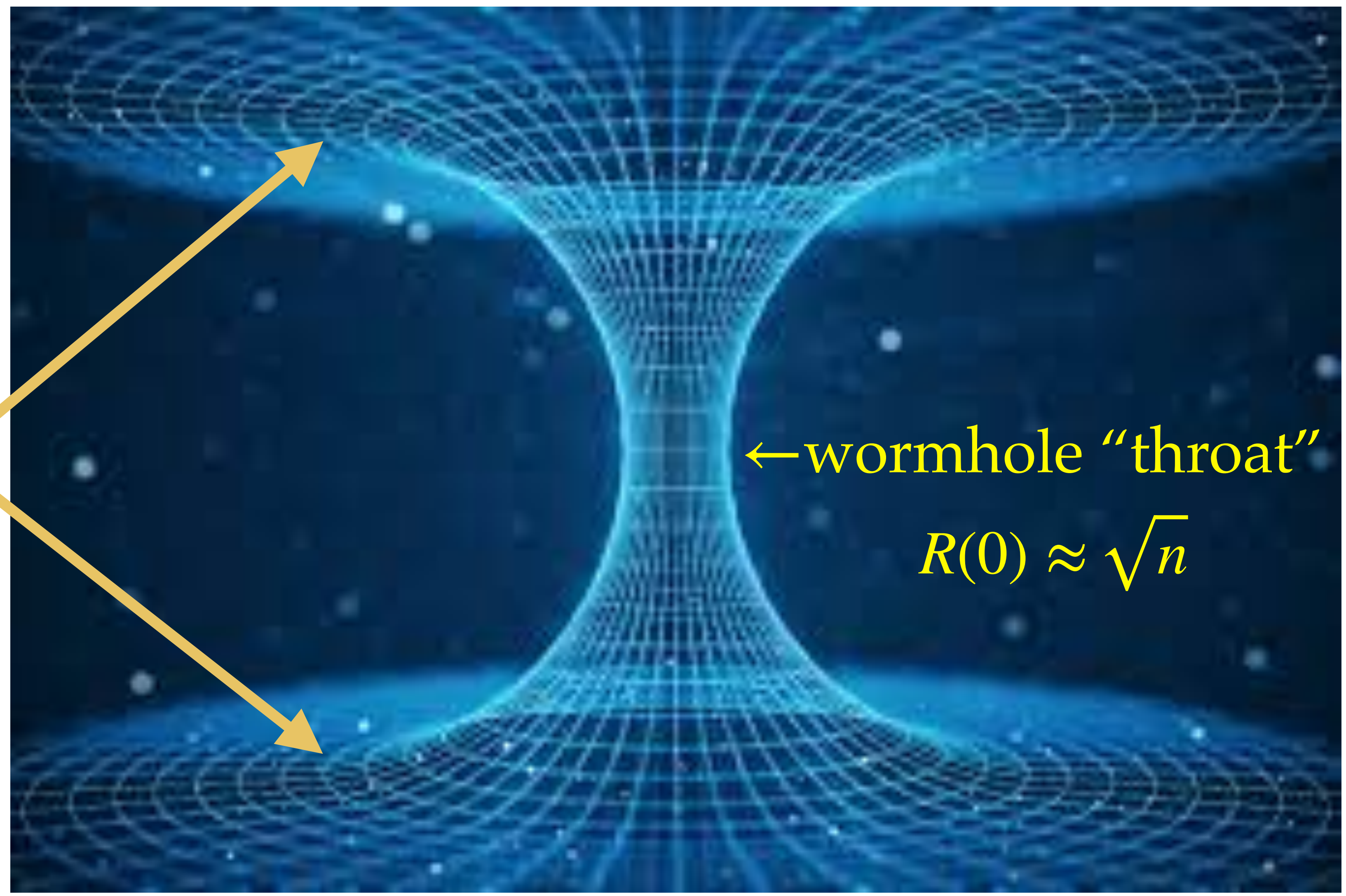
$$ds^2 = dr^2 + R(r)^2 d\Omega_3^2$$

Non-perturbative sol. to Einstein's eq. in
Euclidean, Spherically symmetric spacetime

Euclidean Time

Asymptotically flat spacetime

$R(r) \sim r \quad (r \to \infty)$



Conserved charge

$n = 2\pi^2 R^3(r) f_a^2\, \theta'(r) \in \mathbb{Z}$

Gravitational Instanton potential due to WH

$$\Delta V_{wh} \simeq \frac{1}{R_0^4} e^{-S_{wh}} \cos\left(\frac{a}{f_a} + \delta\right)$$

R_0 : Wormhole throat

S_{wh} : Wormhole action

Presence of this operator shifts the effective θ angle

$$\theta_{eff} \simeq \frac{e^{-S_{wh}}}{(\Lambda_{QCD} R_0)^4}$$

↑ wormhole action

Requesting $\theta_{eff} \lesssim 10^{-10}$ (neutron EDM), we demand
$S_{wh} \gtrsim 190$

for $\Lambda_{QCD} = 200 \text{ MeV}, R_0 \simeq M_P^{-1}$

A page for undergrads

QCD+Gravitation

$$V = -\Lambda_{QCD}^4 \cos(\theta) - \frac{e^{-S}}{R_0^4}\cos(\theta+\delta)$$
$$= -\Lambda_{QCD}^4 \left[\cos\theta + \frac{e^{-S}}{(\Lambda_{QCD}R_0)^4}\cos(\theta+\delta)\right]$$

$$\frac{dV}{d\theta} \propto \sin\theta + A\sin(\theta+\delta) = 0 \qquad A = \frac{e^{-S}}{(\Lambda_{QCD}R_0)^4}$$
$$= A\sin\delta + (1+\cos\delta)\theta + \mathcal{O}(\theta^2)$$

$$\longrightarrow \quad V' = 0 \to \theta \sim \frac{A\sin\delta}{1+\cos\delta} \sim A$$ @minimum of potential

$$\boxed{A = \frac{e^{-S}}{(\Lambda_{QCD}R_0)^4} < 10^{-10}}$$ for neutron EDM bound

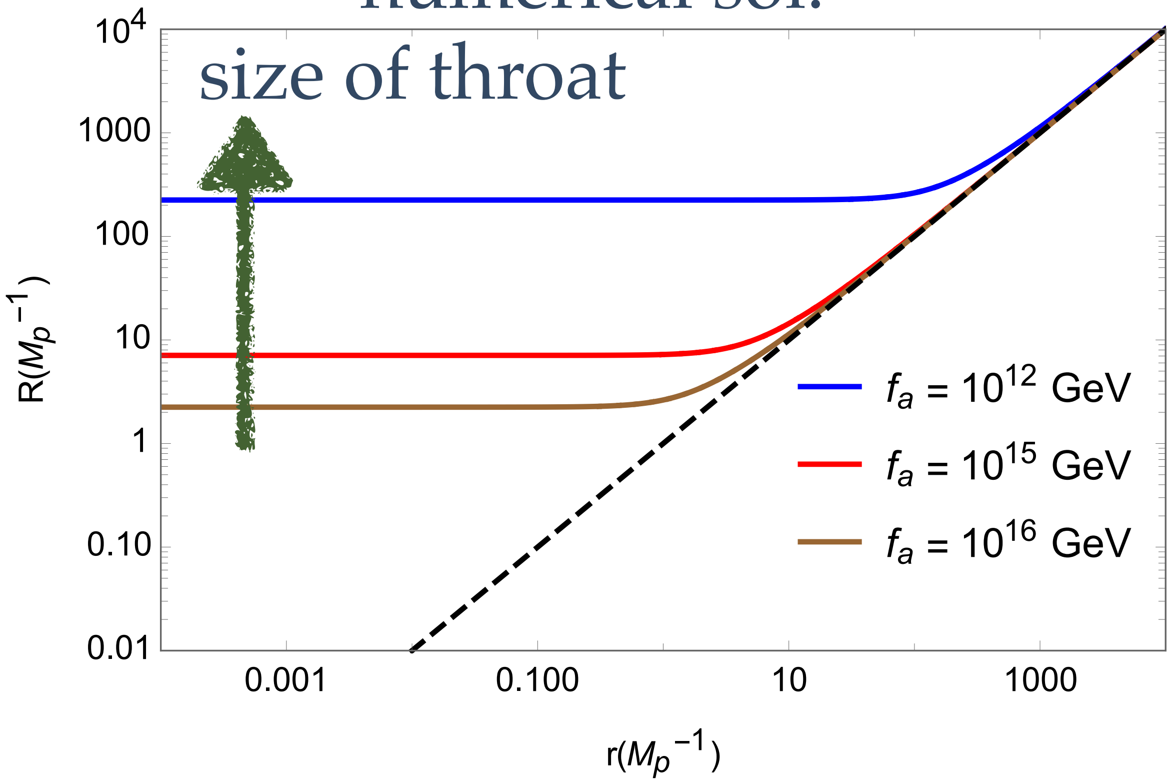
$$\Lambda_{QCD} = 200\ \text{MeV}, R_0 \simeq 1/M_P \quad \longrightarrow \quad S \gtrsim 190$$

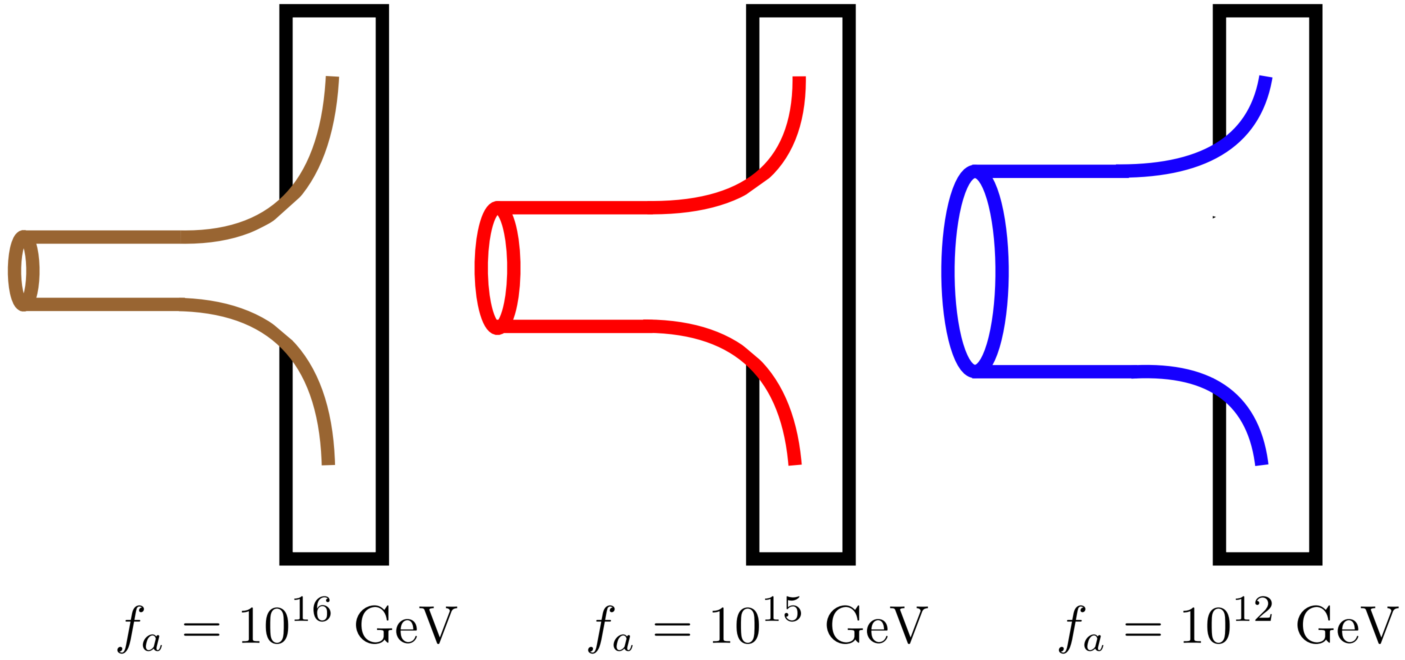
Giddings-Strominger Wormhole (1988)

[S. B. Giddings, A. Strominger, Nucl. Phys. B 306, 890 (1988)] [R. Alonso, A. Urbano, JHEP 02, 136 (2019)]

$$S = \int d^4x \sqrt{|g|} \left(-\frac{M_P^2}{2} R + \frac{f_a^2}{2} (\partial_\mu \theta)^2 \right) \qquad ds^2 = dr^2 + R(r)^2 d\Omega_3^2$$

numerical sol.





$$S_{wh} = \sqrt{\frac{3\pi^2}{8} \frac{M_P}{f_a}} \gtrsim 190 \quad \text{for} \quad f_a \lesssim 10^{16} \text{ GeV}$$

Q. Problem solved?

Wormholes with a Dynamical Radial Mode

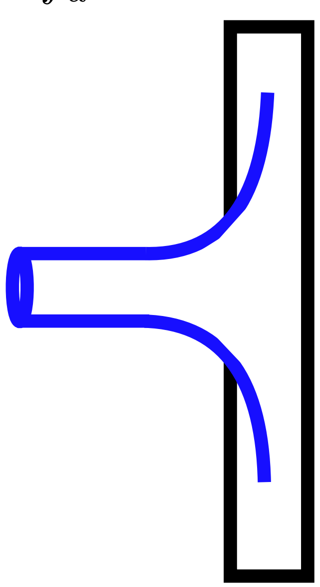
Dynamical $f(r)$: $\Phi = \frac{f_a}{\sqrt{2}} e^{i\theta(r)} \rightarrow \Phi = \frac{f(r)}{\sqrt{2}} e^{i\theta(r)}$

[L. F. Abbott, M. B. Wise, Nucl. Phys. B 325, 687 (1989)]
[R. Kallosh, A. D. Linde, D. A. Linde, L. Susskind, Phys. Rev. D 52, 912 (1995)]

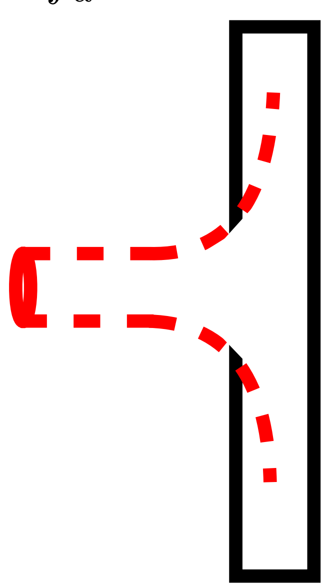
potential $V(f) = \frac{\lambda}{4}(f^2 - f_a^2)^2$

$$S = \int d^4x \sqrt{|g|} \left(-\frac{M_P^2}{2} R + \frac{f^2}{2}(\partial_\mu \theta)^2 + \frac{1}{2}(\partial_\mu f)^2 + V(f) \right)$$

$f_a = 10^{16}$ GeV



$f_a = 10^{12}$ GeV



radius doesn't grow when dynamics is taken into account

$$\rightarrow \quad S \simeq n \log(M_P/f_a) \ll 190$$

Axion Quality Problem remains!

R. Kallosh, A. Linde, D. Linde, and L. Susskind, 1995

$\xi|\Phi|^2 R$ model ⇒ This is our new addition

Spherically symmetric, Euclidean $ds^2 = dr^2 + R^2(r)d\Omega_3^2$

Wormhole action with PQ-field + R coupling

$$S = \int d^4x\sqrt{|g|}\left(-\frac{M^2}{2}R - \xi|\Phi|^2 R + |\partial_\mu\Phi|^2 + V(\Phi)\right)$$

$$= \int d^4x\,\sqrt{g}\left(-\frac{M_P^2}{2}\Omega^2(f)R + \frac{f^2}{2}(\partial_\mu\theta)^2 + \frac{1}{2}(\partial_\mu f)^2 + V(f)\right)$$

$$M_P^2 = M^2 + \xi|\Phi|^2 \to \xi \le \frac{M_P^2}{f_a}$$

$$\Phi(r) = \frac{f(r)}{\sqrt{2}}e^{i\theta(r)}$$

$$\Omega^2(f) = 1 + \frac{\xi}{M_P^2}\left(f^2 - f_a\right)^2$$

$$V(f) = \frac{\lambda}{4}(f^2 - f_a^2)^2$$

Boundary conditions $f'(0) = R'(0) = 0, f(\infty) = f_a$

then solve the Einstein equation

with $\xi \neq 0$, We solve the Einstein eq.
=> Wormhole geometry is modified
=> check if Quality problem is resolved or not!

Two gravitational formulations

Metric formalism $\mathcal{R}_{\mu\nu}(g_{\mu\nu})$

$$\bar{\Gamma}^{\lambda}_{\mu\nu} = \frac{1}{2} g^{\lambda\alpha}(g_{\mu\alpha,\nu} + g_{\alpha\nu,\mu} - g_{\mu\nu,\alpha})$$

(Einstein)

Palatini Formalism $\mathcal{R}_{\mu\nu}(\Gamma)$

metric and connections are independent a priori

metric rules!!

$\{g_{\mu\nu}\}$

metric & connection rule!!

$\{g_{\mu\nu}, \Gamma^{\mu}_{\rho\sigma}\}$

(cf) when minimally coupled limit ($\xi = 0$), two formulations are equivalent

Numerical solutions

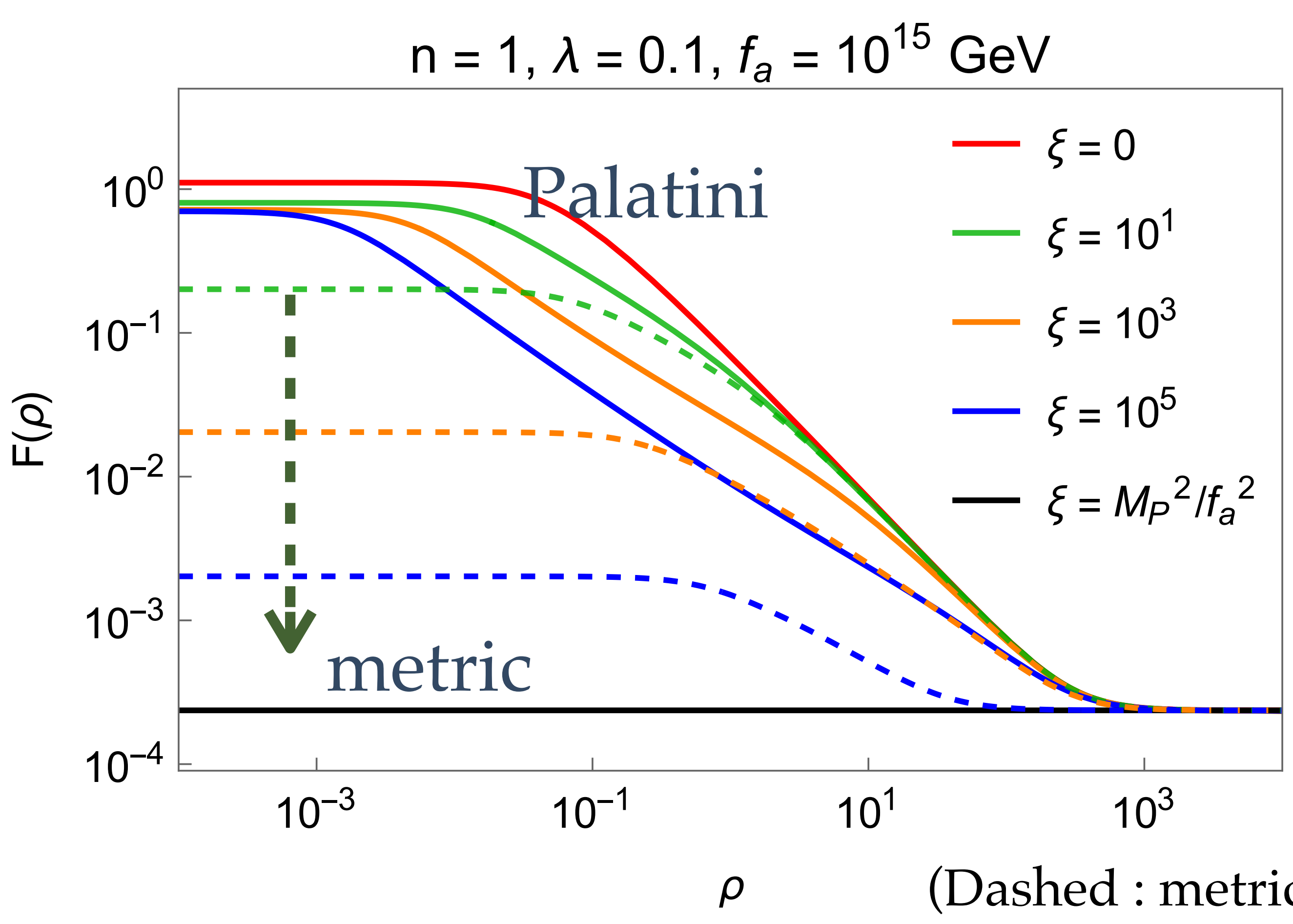
Einstein equations $\quad \zeta = 0,\ 1$ (metric, Palatini)

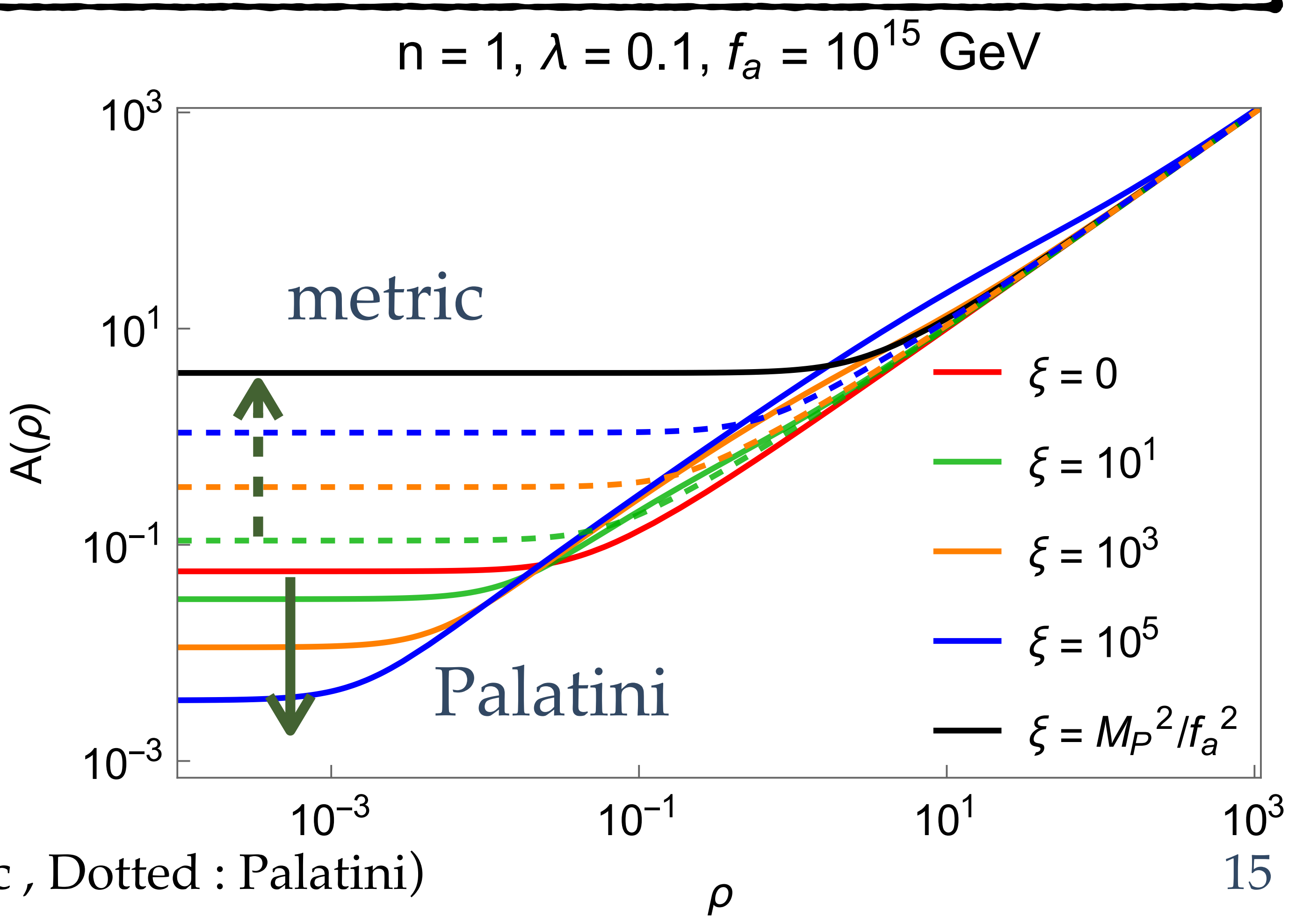
$$\Omega^2\left[R'^2 - 1 + \zeta\left(2RR'\omega' + R^2\omega'^2\right)\right] = -\frac{R^2}{3M_P^2}\left[-\frac{1}{2}f'^2 + V(f) + \frac{n^2}{8\pi^4 f^2 R^6}\right]$$

$$f'' + 3\frac{R'}{R}f' - \frac{dV}{df} + \frac{n^2}{4\pi^4 f^3 R^6} = 6\xi f\left[\frac{R''}{R} + \frac{R'^2}{R^2} - \frac{1}{R^2} + \zeta(\omega'^2 + \omega'' + 3\frac{R'}{R}\omega')\right]$$

Dimensionless: $\quad \rho \equiv \sqrt{3\lambda}M_P r, \quad A \equiv \sqrt{3\lambda}M_P R, \quad F \equiv f/\sqrt{3}M_P.$

Boundary Conditions $\quad R'(0) = 0, \quad f'(0) = 0, \quad f(\infty) = f_a.$





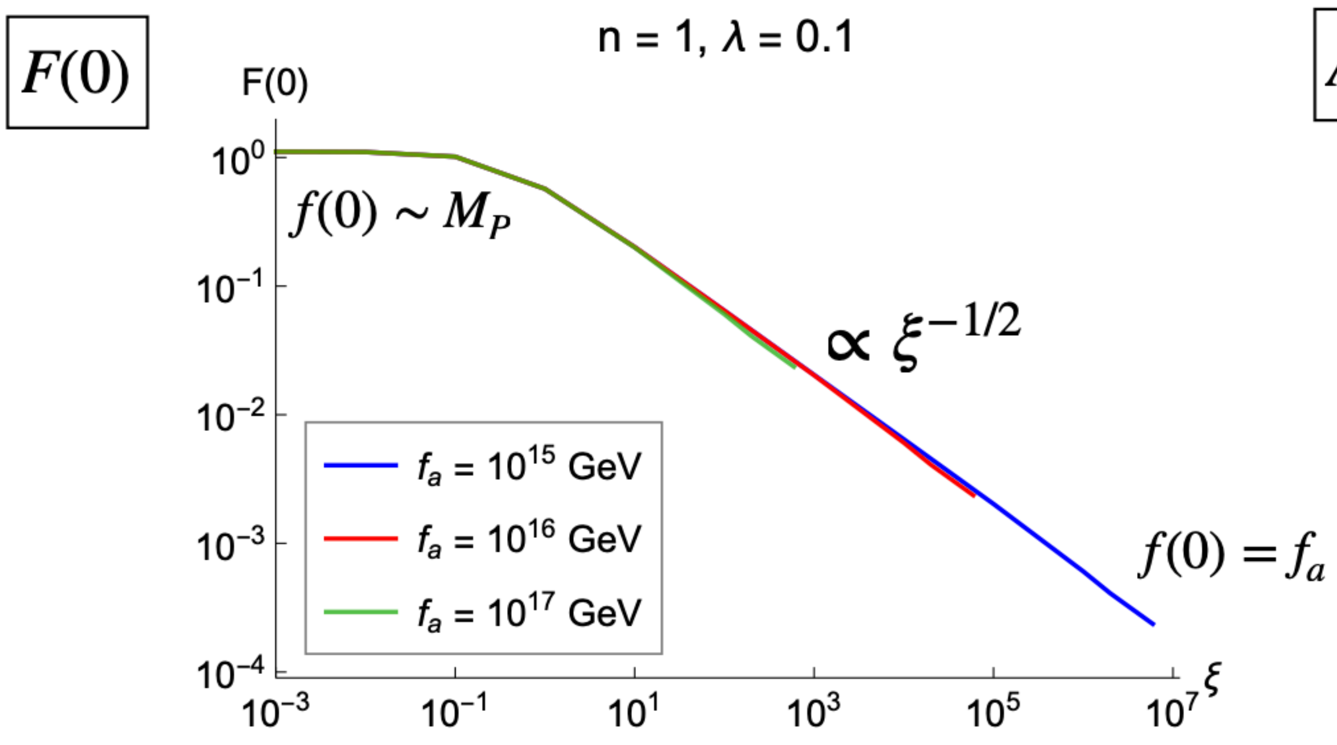
(Dashed : metric , Dotted : Palatini)

WH solution w.r.t. ξ coupling

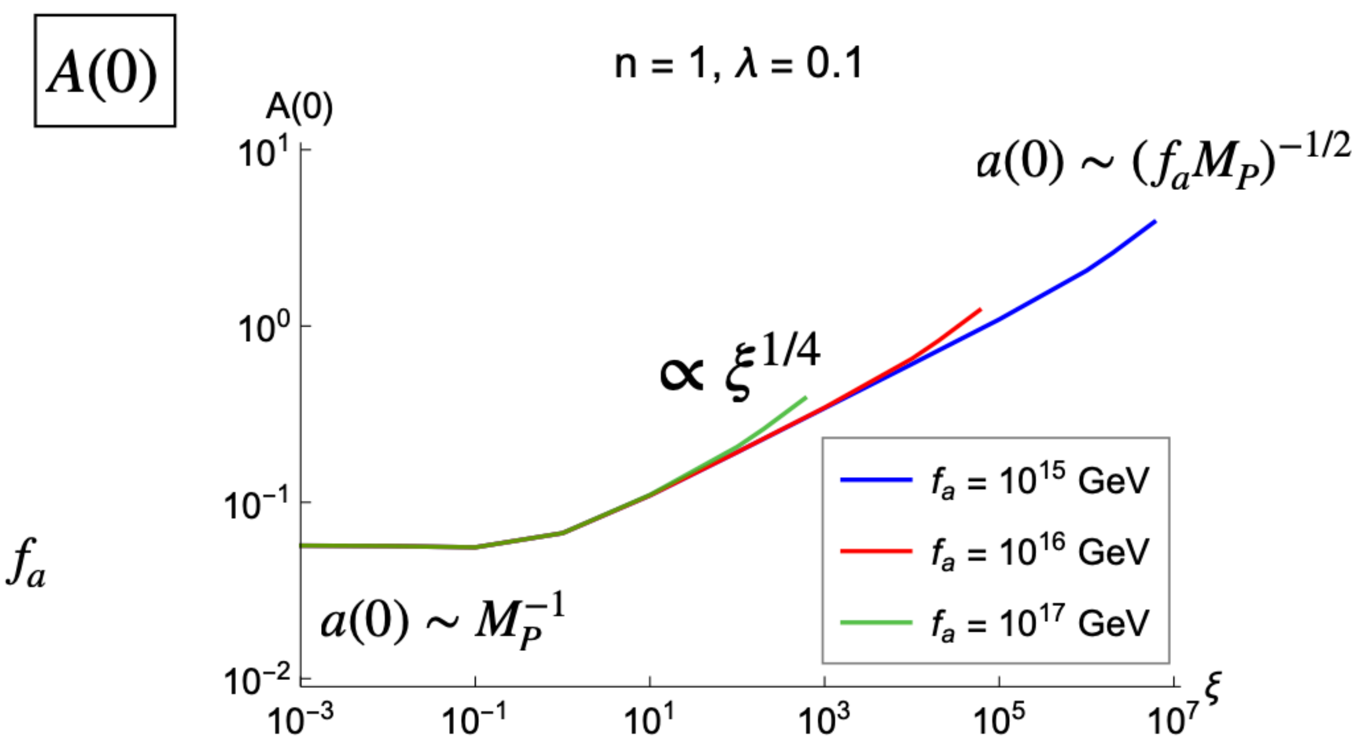
$$\rho \equiv \sqrt{3\lambda} M_P r, \qquad A \equiv \sqrt{3\lambda} M_P R, \qquad F \equiv f/\sqrt{3} M_P.$$

Boundary Conditions $R'(0) = 0, \qquad f'(0) = 0, \qquad f(\infty) = f_a.$

Radial



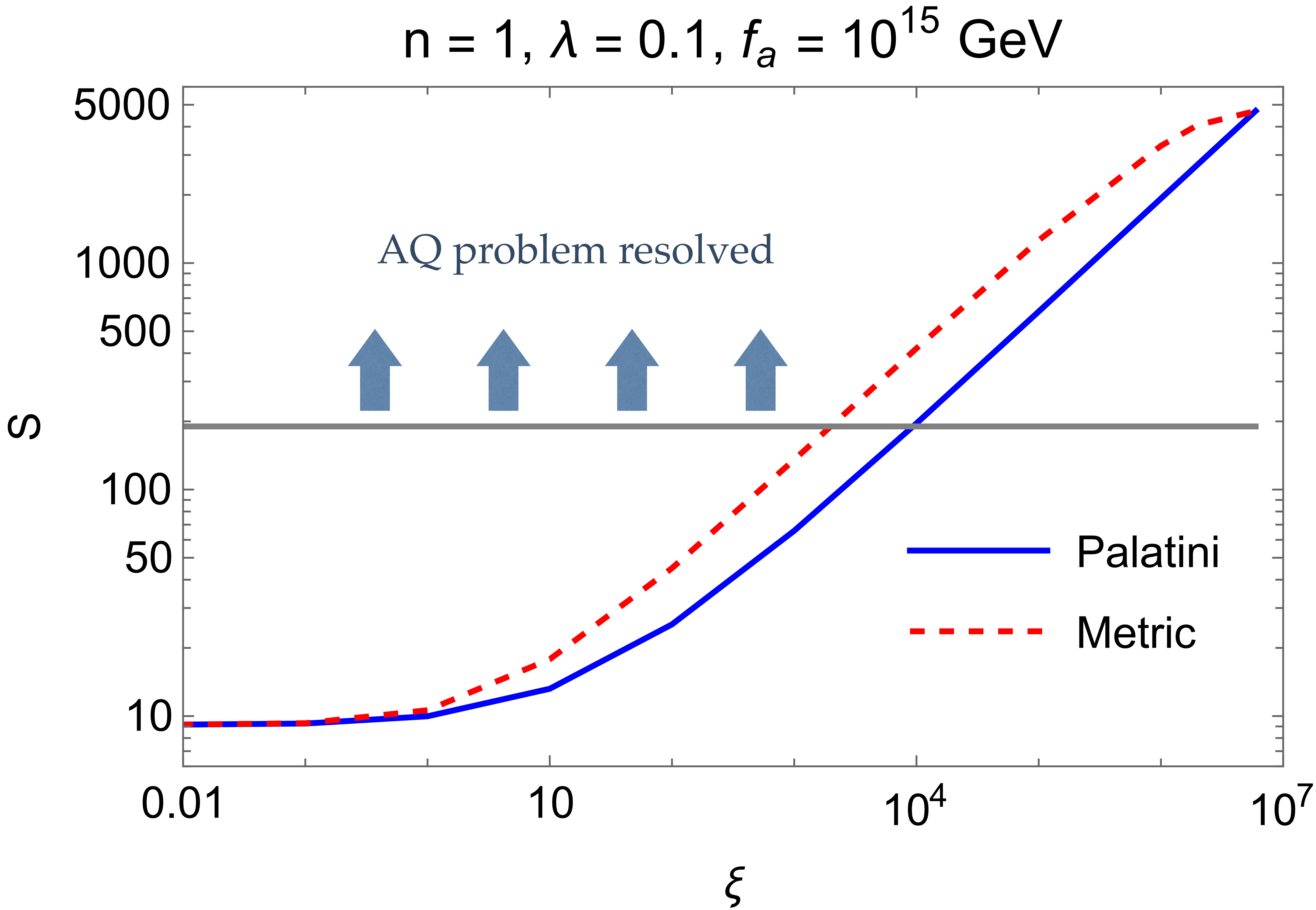
Throat size



behavior similar in both formulations of gravity

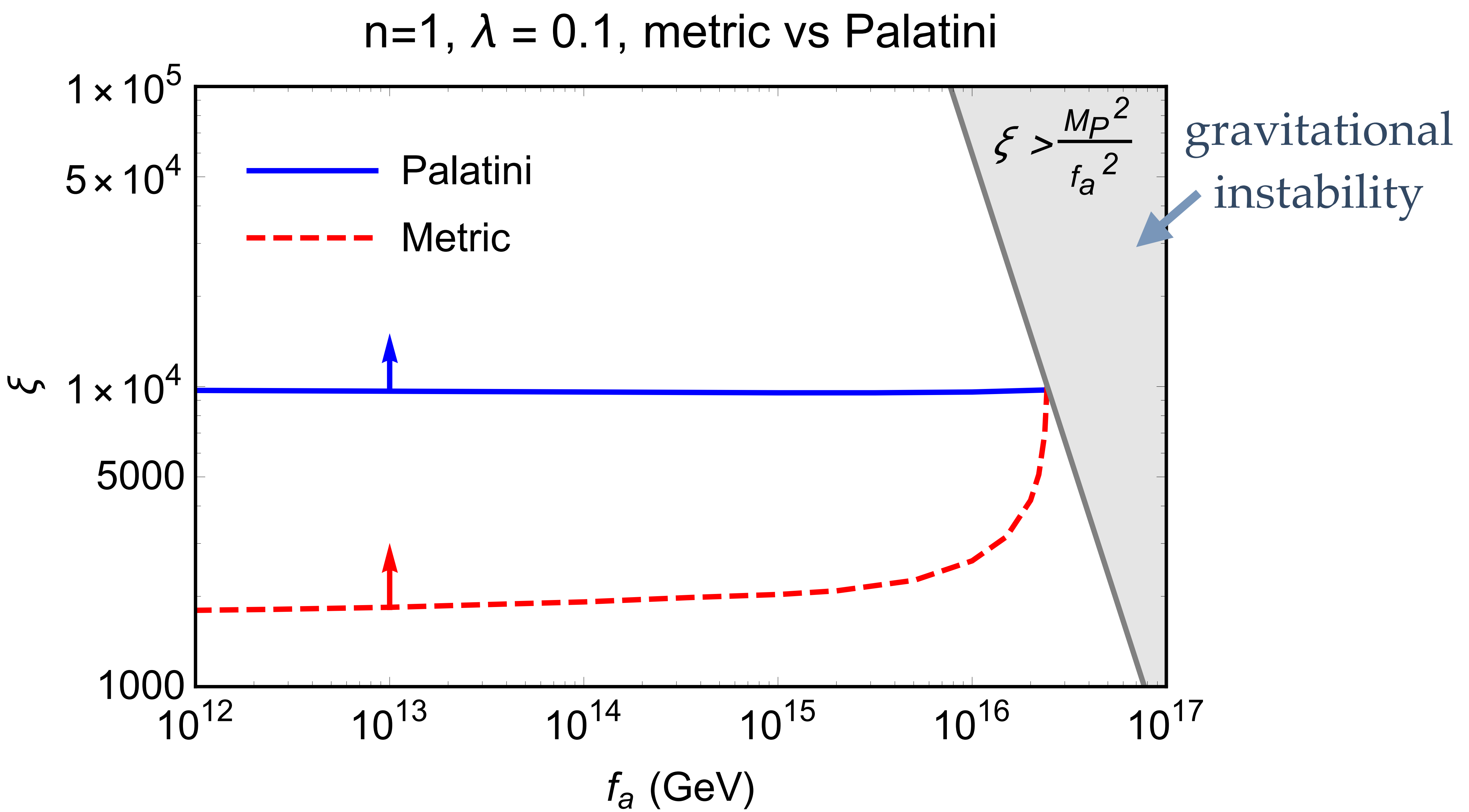
Axion Quality Problem and non-minimal gravity

[D. Y. Cheng, K. Hamaguchi, Y. Kanazawa, S.M.Lee, N. Nagata, S.C.Park, 2210.11330]



Conclusion: Quality problem solved with $\xi|\Phi|^2 R$

[D. Y. Cheng, K. Hamaguchi, Y. Kanazawa, S.M.Lee, N. Nagata, S.C.Park, 2210.11330]



Axion Quality Problem is solved in both formulations of gravity