Axion dark matter with thermal friction

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Outline

- Conventional Axion DM : overview
- Axion interactions with a hidden thermal bath
- Cosmological evolution of the axion field in the presence of thermal friction
- Implications to axion DM abundance and density perturbation

Strong CP problem and QCD axion

$$y_u H Q_L u_R^c + y_d H^* Q_L d_R^c + \frac{g_s^2}{32\pi^2}\theta G\tilde{G}$$

CPV in the QCD sector

$$\bar{\theta} = \theta + \arg\,\det\,(y_u y_d) < 10^{-10}$$

Non-observation of neutron EDM

[Abel et al '20]

while $\delta_{\rm CKM} = \arg\,\det\left[y_u y_u^\dagger, y_d y_d^\dagger\right] \sim \mathcal{O}(1)$

The QCD vacuum energy is minimized at the CP-conserving point ($\bar{\theta} = 0$).

[Vafa, Witten '84]

$$V_{\rm QCD} = -\Lambda_{\rm QCD}^4 \cos\bar{\theta}$$

Promote $\bar{\theta}$ to a dynamical field (=QCD axion) : $\frac{g_s^2}{32\pi^2}\left(\theta + \frac{\phi}{f}\right)G\tilde{G}$

[Peccei, Quinn '77, Weinberg '78, Wilczek '78]

QCD axion lagrangian

$$\frac{g_s^2}{32\pi^2}\left(\frac{\phi}{f}+\bar{\theta}\right)G^{a\mu\nu}\widetilde{G}^a_{\mu\nu}+\sum_A\frac{g_A^2}{32\pi^2}c_A\frac{\phi}{f}F^{A\mu\nu}\widetilde{F}^A_{\mu\nu}+\frac{\partial_\mu\phi}{f}\left(\sum_\psi c_\psi\psi^\dagger\bar{\sigma}^\mu\psi+c_H H^\dagger i\overleftrightarrow{D}^\mu H\right)$$

Below the QCD scale $\Downarrow$

$$\underbrace{-m_\pi^2 f_\pi^2\cos\left(\frac{\phi}{f}+\bar{\theta}\right)}_{\boxed{m_\phi\simeq 5.7\,\mu\mathrm{eV}\left(\frac{10^{12}\,\mathrm{GeV}}{f}\right)}}+\frac{e^2}{32\pi^2}(c_\gamma-1.92)\frac{\phi}{f}F\tilde{F}+c_e\frac{\partial_\mu\phi}{f}\bar{e}\gamma^\mu\gamma^5 e+c_N\frac{\partial_\mu\phi}{f}\bar{N}\gamma^\mu\gamma^5 N$$

$$c_\gamma=\begin{cases}\frac{E}{N}, & \text{KSVZ}\\ \frac{8}{3}, & \text{DFSZ}\end{cases}$$

[Kim ’79, Shifman, Vainshtein, Zakharov ’80]

[Dine, Fischler, Srednicki ’81, Zhitnitsky ’80]

Axion-Like Particle (ALP)

- Cousins of the QCD axion, not being involved in the strong CP problem
- Ubiquitous in many BSM scenarios, e.g. string theory

[Arvanitaki, Dimopoulos, Dubovsky, Kaloper, Marsh-Russell, '09]

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2} m_\phi^2 \phi^2 + \frac{\phi}{f} \sum_A \frac{g_A^2}{32\pi^2} c_A F^{A\mu\nu} \tilde{F}^A_{\mu\nu} + \frac{\partial_\mu \phi}{f} \left(\sum_\psi c_\psi \bar{\psi} \bar{\sigma}^\mu \psi + c_H H^\dagger i \overleftrightarrow{D}^\mu H \right)$$

i) approximate shift symmetry $U(1)_{PQ}$ $\quad \phi(x) \to \phi(x) + c$

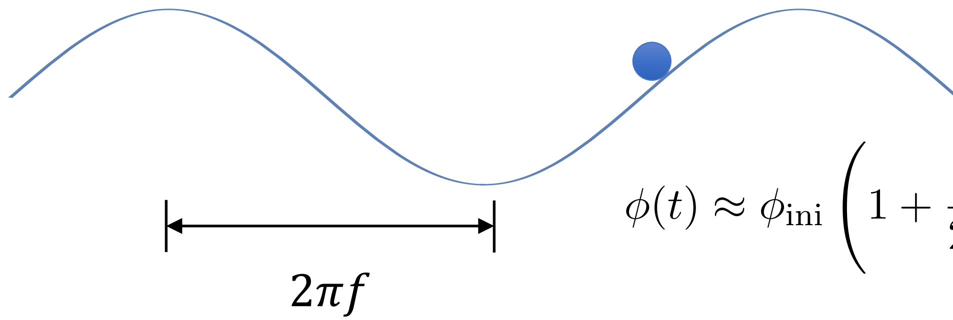
: ALP can be naturally light.

ii) periodicity $\quad \dfrac{\phi(x)}{f} \equiv \dfrac{\phi(x)}{f} + 2\pi$

: f characterizes typical size of ALP couplings.

Conventional axion dark matter scenario

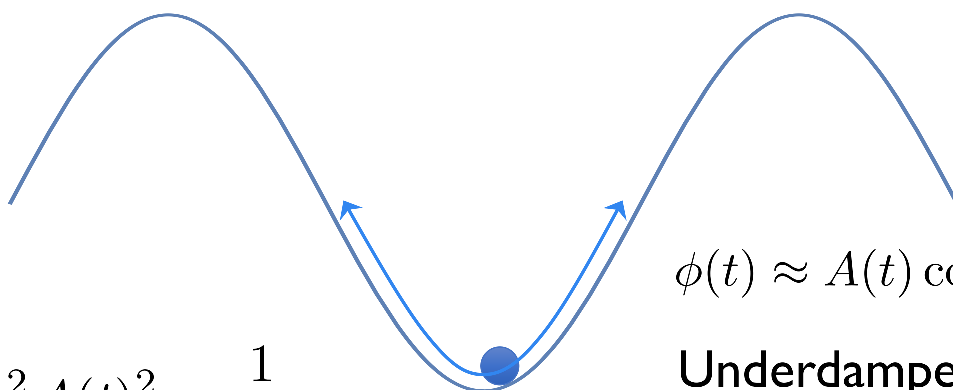
$H(t) \gg m_\phi(t)$



$$\phi(t) \approx \phi_{\rm ini}\left(1 + \frac{1}{20}\left[\frac{m_\phi^2}{H_{\rm ini}^2} - \frac{m_\phi^2}{H(t)^2}\right]\right)$$

Slow-roll : dark energy

$m_\phi(t) \gtrsim H(t)$

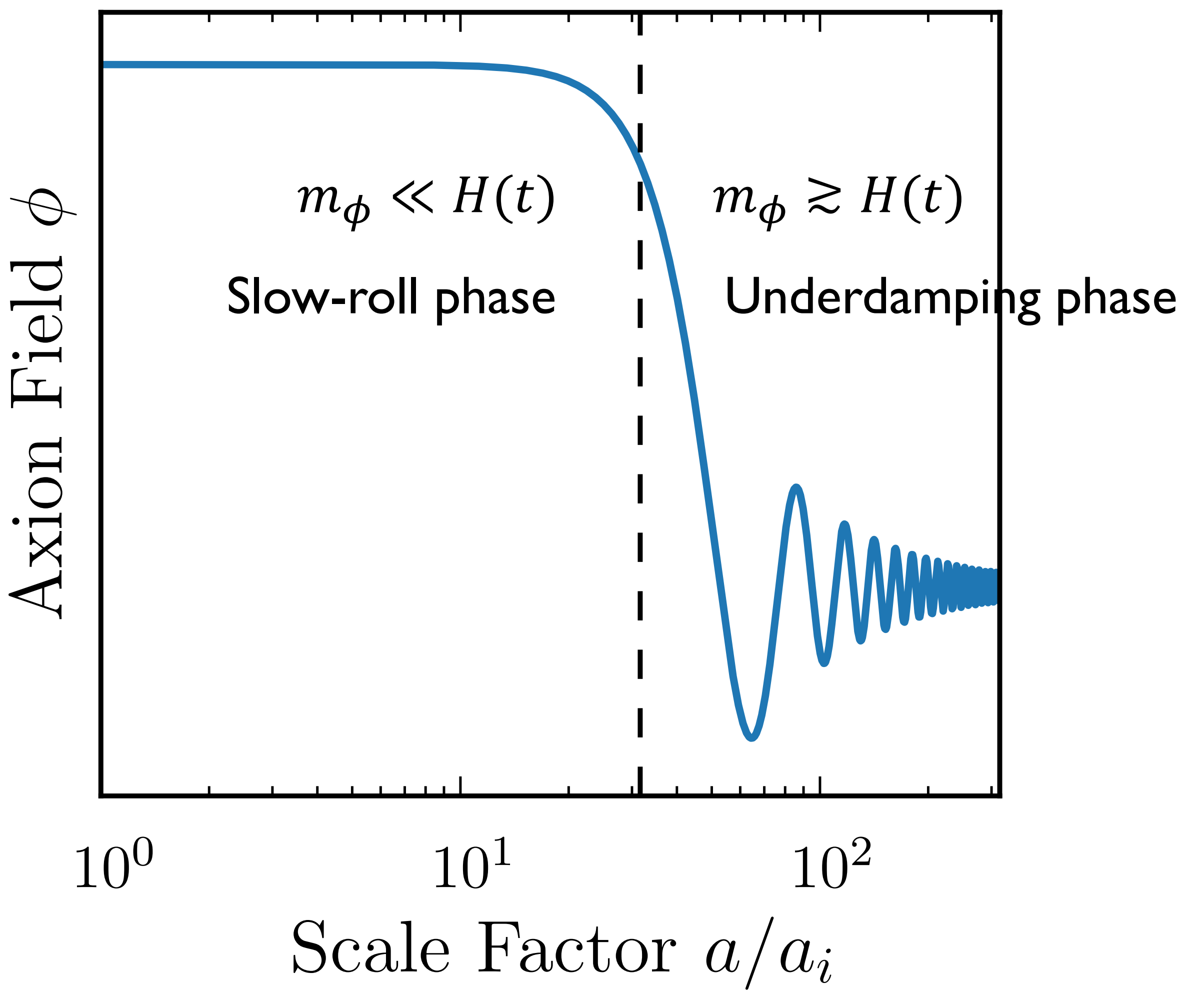


$$\phi(t) \approx A(t)\cos(m_\phi t)$$

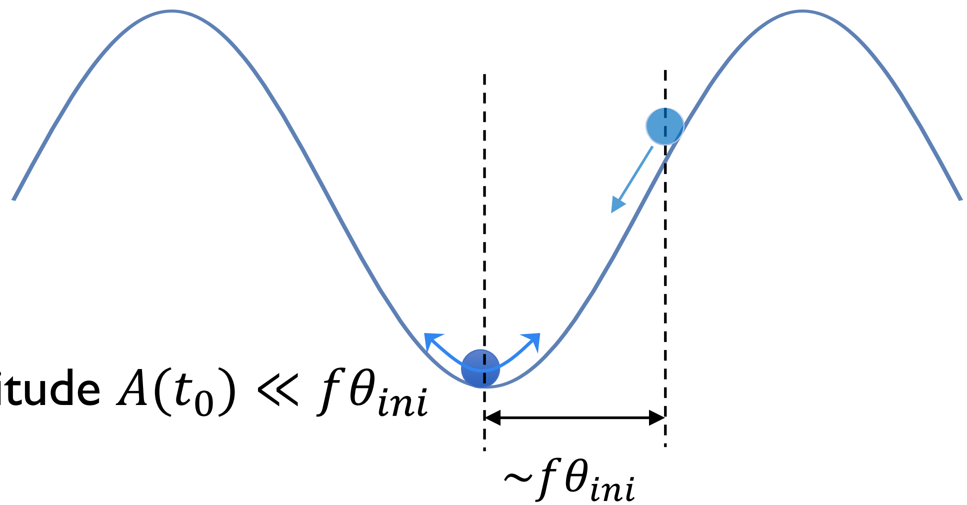
$$\rho_\phi \approx \frac{1}{2} m_\phi^2 A(t)^2 \propto \frac{1}{a^3}$$

Underdamped oscillation : cold dark matter

Time evolution of the axion field



Misalignment production of axion dark matter



Present oscillation amplitude $A(t_0) \ll f\theta_{ini}$

$$\phi(t,\vec{x}) \approx A(t_0)\cos m_\phi(t - \vec{v}\cdot\vec{x})$$

$|\vec{v}| \sim 10^{-3}$: virial velocity of the axion DM

$$\rho_\phi = \frac{1}{2} m_\phi^2 A(t_0)^2 \quad \Rightarrow \quad \frac{\rho_\phi}{\rho_{\rm DM}} \approx \sqrt{\frac{m_\phi}{1\,\mu{\rm eV}}}\left(\frac{f\theta_{\rm ini}}{10^{13}\,{\rm GeV}}\right)^2 \times \underbrace{\sqrt{\frac{m_\phi}{m_{\phi,{\rm osc}}}}}_{O(100)}$$

$O(100)$ for QCD axion

The present amplitude $A(t_0)$ is determined from the initial amplitude $\sim f\theta_{ini}$ and the axion mass m_ϕ & $m_{\phi,osc}$.

Axion interactions with a hidden thermal bath

$$\mathcal{L} \supset \frac{1}{2}\partial^\mu\phi\partial_\mu\phi - \frac{1}{2}m_\phi^2\phi^2 + \frac{\partial_\mu\phi}{f_h}J_h^\mu$$

: hidden sector current

Axion shift-symmetry ($\phi \to \phi + c$) conserving interaction for a naturally small axion mass

$$\phi(t,x) = \phi_{\rm cl}(t) + \hat{\phi}(t,x)$$

Macroscopically-occupied ground state (classical field)
+ Thermalized excited states (quantum field)

Axion interactions with a hidden thermal bath

$$\mathcal{L} \supset \frac{1}{2}\partial^\mu \phi \partial_\mu \phi - \frac{1}{2} m_\phi^2 \phi^2 + \frac{\partial_\mu \phi}{f_h} J_h^\mu$$

Eq. of motion $\quad \nabla^\mu \partial_\mu \phi_{\rm cl} + m_\phi^2 \phi_{\rm cl}^2 = -\frac{\langle \partial_\mu J_h^\mu \rangle_{\phi_{\rm cl}}}{f_h}$

: thermal expectation value under the ϕ_{cl}-background

By symmetries and power-counting,

$$\langle \partial_\mu J_h^\mu \rangle_{\phi_{\rm cl}} = \Gamma(T_h) u^\mu \frac{\partial_\mu \phi_{\rm cl}}{f_h} + \mathcal{O}(\partial_\mu \partial_\nu \phi_{\rm cl}/f_h, \partial_\mu \phi_{\rm cl} \partial_\nu \phi_{\rm cl}/f_h^2)$$

where $\quad \Gamma(T_h) \sim T_h^3 \qquad u^\mu$: 4-velocity vector of the hidden thermal fluid

In the rest frame of the thermal fluid, $u^\mu = (1,0,0,0)$

$$\ddot{\phi}_{\rm cl} + 3H\dot{\phi}_{\rm cl} + m_\phi^2 \phi_{\rm cl} = -\gamma_\phi(T)\dot{\phi}_{\rm cl}$$

where $\gamma_\phi \equiv \xi \dfrac{T^3}{f_h^2}$ with $\xi \propto \left(\dfrac{T_h}{T}\right)^3$

T : temperature of the universe

The axion interaction with a hidden thermal bath can give rise to a new friction term on top of the Hubble friction.

Motivated example :
Sphaleron processes in non-abelian gauge theories

$$\int d^4x \, \frac{\alpha_h}{8\pi} \frac{\phi}{f_h} G_h^{\mu\nu a} \widetilde{G}^a_{h\,\mu\nu}$$

$$= \int d^4x \, \partial_\mu \left(\frac{\alpha_h}{8\pi} \frac{\phi}{f_h} J^\mu_{\rm CS} \right) - \frac{\alpha_h}{8\pi} \frac{\partial_\mu \phi}{f_h} J^\mu_{\rm CS}$$

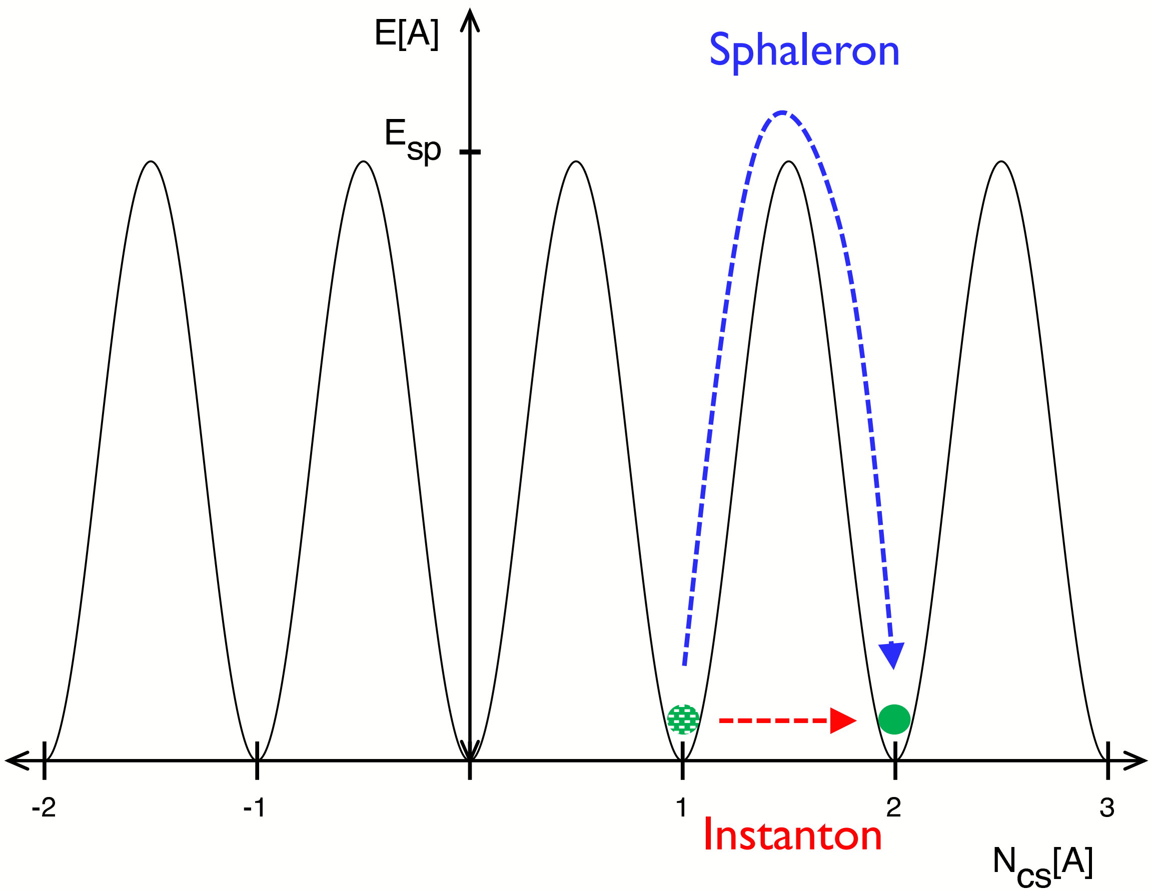
Axion mass via
Instantons
(assumed small)

Thermal friction via
Sphalerons

$$J^\mu_{\rm CS} \equiv 2\epsilon^{\mu\nu\rho\sigma} \mathrm{Tr} \left(A_\nu G_{\rho\sigma} + \frac{2}{3} i g_3 A_\nu A_\rho A_\sigma \right)$$ Chern-Simon current

$$\partial_\mu J^\mu_{\rm CS} = G_h^{\mu\nu a} \widetilde{G}^a_{h\,\mu\nu}$$

Vacuum structure of non-abelian gauge theories $N_{\rm CS} = \frac{\alpha_h}{8\pi} \int d^3x J^0_{\rm CS}$



The sphaleron processes are real-time processes hopping the potential barrier by thermal fluctuation.

$$\int d^4x\,\frac{\alpha_h}{8\pi}\frac{\partial_\mu\phi_{\rm cl}}{f_h}J^\mu_{\rm CS} = \int dt\,\underbrace{\frac{\dot\phi_{\rm cl}}{f_h}}_{\text{Chemical potential for } N_{CS}} N_{\rm CS}$$ for spatially homogeneous $\phi_{cl}(t)$

Non-zero axion field velocity drives the sphaleron processes.

$$\frac{\alpha_h}{8\pi}\langle\partial_\mu J^\mu_{\rm CS}\rangle_{\phi_{\rm cl}} = \frac{\partial_t N_{\rm CS}}{V} \propto \frac{\dot\phi_{\rm cl}}{f_h}$$ in linear order

$$\Rightarrow \quad \ddot\phi_{\rm cl} + 3H\phi_{\rm cl} + m_\phi^2\phi_{\rm cl} = \underbrace{-\frac{\alpha_h}{8\pi f_h}\langle\partial_\mu J^\mu_{\rm CS}\rangle_{\phi_{\rm cl}}}_{\gamma_\phi\dot\phi_{cl}}$$

Thermal friction from sphalerons

$$\gamma_\phi \equiv \xi\frac{T^3}{f_h^2}$$

$$\xi \sim (N\alpha_h)^5\left(\frac{T_h}{T}\right)^3$$ for $SU(N)$ pure Yang-Mills

[McLerran, Mottola, Shaposhinikov '91, Arnold, Son, Yaffle '97]

Axion dynamics with thermal friction

Eq. of motion $$\ddot{\phi}_{\rm cl} + (3H + \gamma_\phi)\dot{\phi}_{\rm cl} + m_\phi^2 \phi_{\rm cl} = 0$$

$$\gamma_\phi = \xi \frac{T^3}{f_h^2}$$ with approximately constant ξ

$$H \sim \frac{T^2}{M_P}$$ in radiation domination

➡ $\gamma_\phi \gg H$ in the hot early universe

The thermal friction can be important for axion dynamics in the early universe.

Approximate solution to the equation of motion : overdamping phase

$$\ddot{\phi}_{\rm cl} + (3H + \gamma_\phi)\dot{\phi}_{\rm cl} + m_\phi^2 \phi_{\rm cl} = 0$$

When $\gamma_\phi \gg m_\phi, H$ (overdamping),

$$\dot{\phi}_{\rm cl} \simeq -\frac{m_\phi^2}{\gamma_\phi}\phi_{\rm cl}$$

$$\Rightarrow \quad \frac{\Delta\phi_{\rm cl}}{\phi_{\rm cl}} \sim \mathcal{O}(1) \quad \text{for} \quad \Delta t = (m_\phi^2/\gamma_\phi)^{-1}$$

A new time scale for axion dynamics

vs

$$\frac{\Delta a}{a} \sim \frac{\Delta T}{T} \sim \mathcal{O}(1) \quad \text{for} \quad \Delta t = H^{-1}$$

$$\ddot{\phi}_{\rm cl} + (3H + \gamma_\phi)\dot{\phi}_{\rm cl} + m_\phi^2 \phi_{\rm cl} = 0 \qquad m_\phi^2(T) \simeq m_0^2 \left(\frac{\Lambda_\phi}{T}\right)^\beta$$

When $\gamma_\phi \gg m_\phi, H$,

ϕ_{cl} slow-rolls as long as $\left(\frac{m_\phi^2}{\gamma_\phi}\right)^{-1} \gtrsim H^{-1}$ $(\gamma_\phi H \gtrsim m_\phi^2)$

ϕ_{cl} substantially moves after $\left(\frac{m_\phi^2}{\gamma_\phi}\right)^{-1} \lesssim H^{-1}$ $(\gamma_\phi H \lesssim m_\phi^2)$

$$\boxed{\phi(t) \simeq f\theta_i \exp\left(-\frac{1}{5+\beta}\frac{m_\phi^2(t)}{\gamma_\phi(t)H(t)}\right)\left[1 + \mathcal{O}\left(m_\phi^2/\gamma_\phi^2, H/\gamma_\phi\right)\right]}$$

[cf. K.V. Berghaus, T. Karwal '19]

The axion field exponentially decays before oscillation.

Approximate solution to the equation of motion : underdamping phase

$$\ddot{\phi}_{\rm cl} + (3H + \gamma_\phi)\dot{\phi}_{\rm cl} + m_\phi^2 \phi_{\rm cl} = 0$$

When $m_\phi \gg \gamma_\phi, H$ (underdamping), $\ddot{\phi}_{\rm cl} + m_\phi^2 \phi_{\rm cl} \approx 0$

$$\phi_{\rm cl}(t) = A(t)\cos\left(\int_{t_1}^{t} dt'\, m_\phi(t') + \theta_0\right)$$

$$A(t) \simeq \sqrt{2}\phi_1 \sqrt{\frac{m_1}{m_\phi(t)}} \left(\frac{t_1}{t}\right)^{3/4} e^{-(\gamma_1/2H_1)(1-\sqrt{t_1/t})} \times \left[1 + \mathcal{O}\left(\gamma_\phi^2/m_\phi^2, H^2/m_\phi^2\right)\right]$$

[SHI, KS Jeong, Y Lee '21]

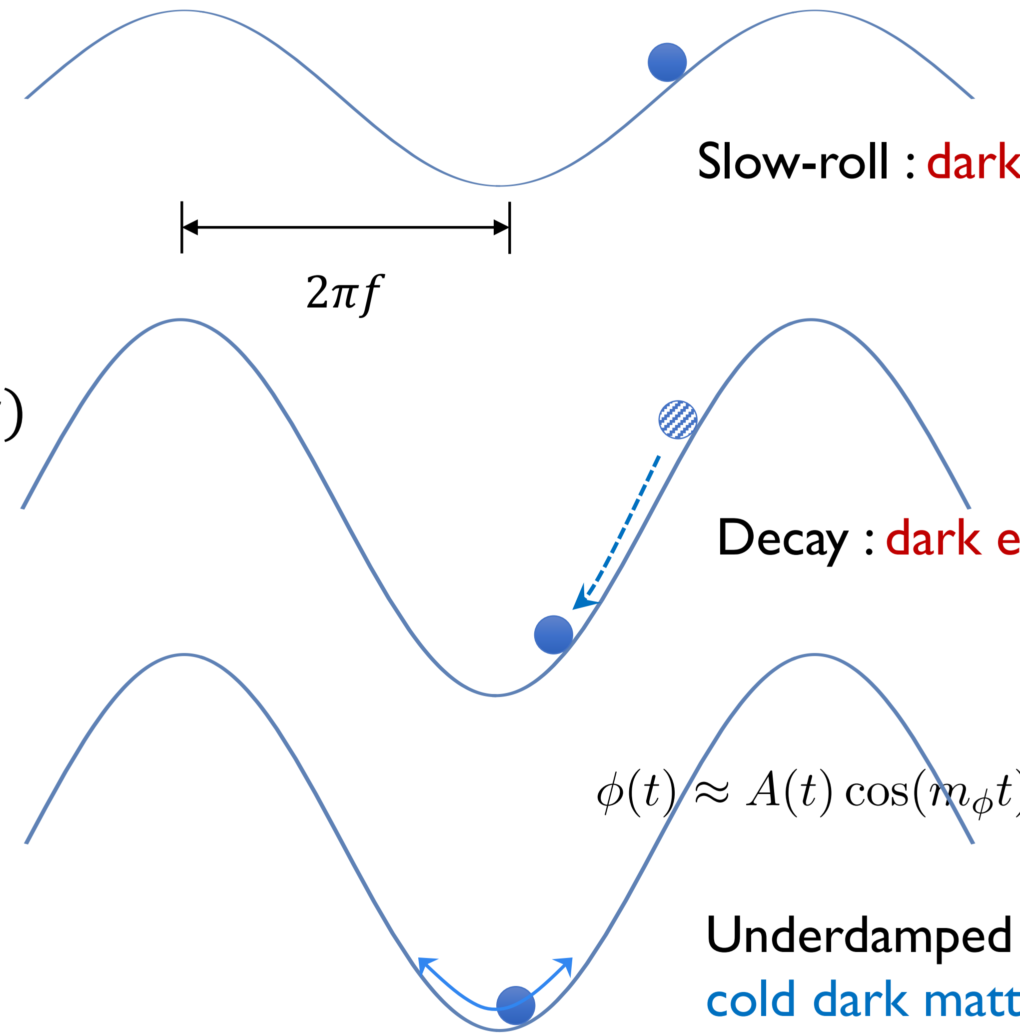
Strong damping by thermal friction around t_1

t_1 : oscillation starting time around $m_\phi \approx \gamma_\phi$

$$\phi_1 \equiv \phi(t_1),\ m_1 \equiv m_\phi(t_1),\ \gamma_1 \equiv \gamma_\phi(t_1)$$

Cosmological evolution of the axion field with thermal friction

$\gamma_\phi(t), H(t) \gg m_\phi(t)$



Slow-roll : dark energy

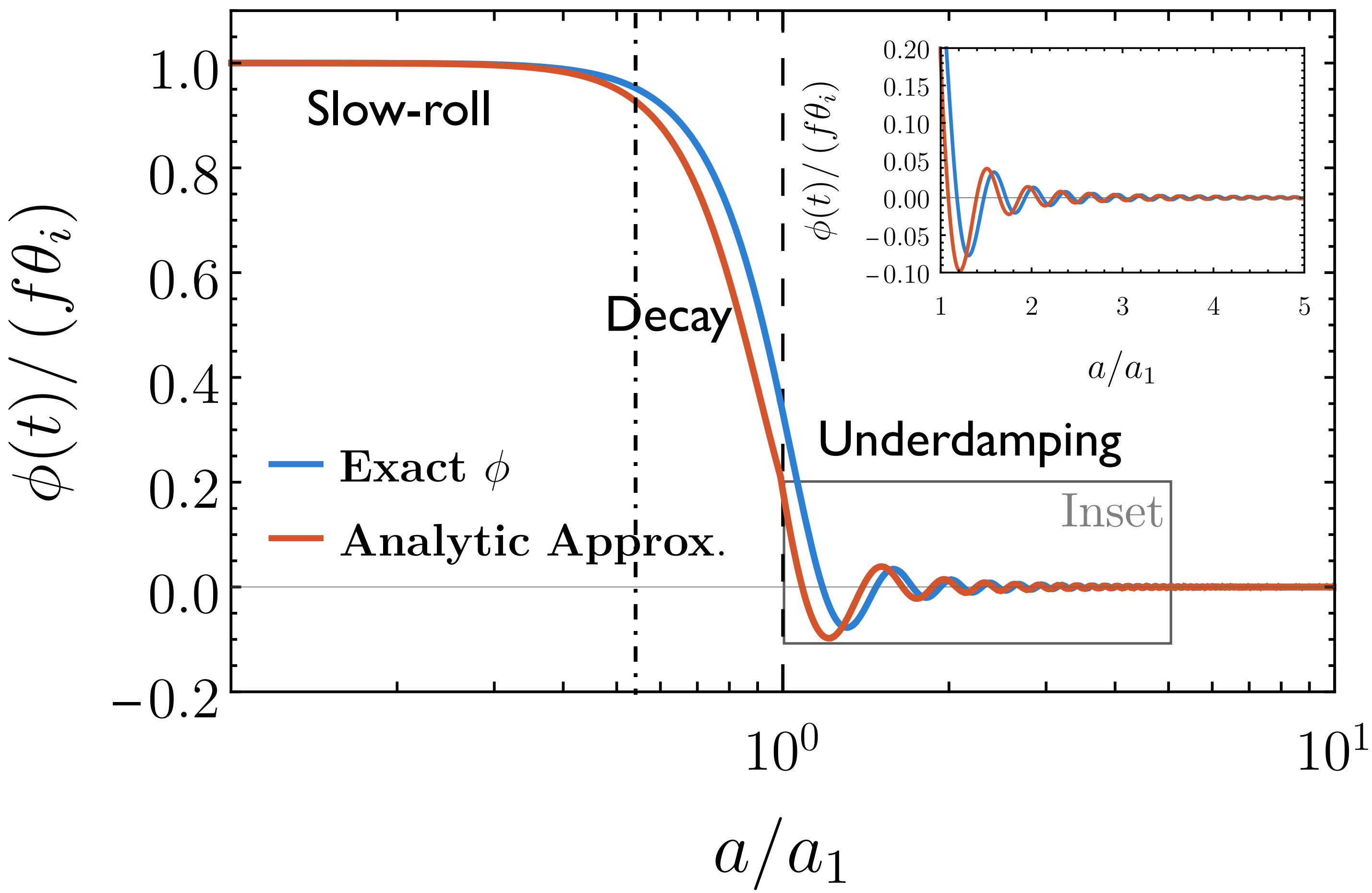
$\gamma_\phi(t) > m_\phi(t) > H(t)$

Decay : dark energy

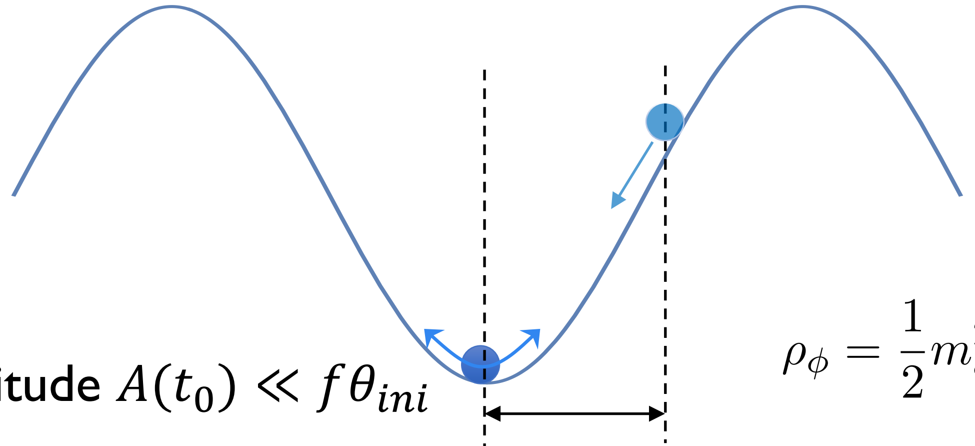
$m_\phi(t) \gtrsim \gamma_\phi(t), H(t)$

Underdamped oscillation : cold dark matter

Time evolution of the axion field in the presence of thermal friction



Misalignment production of axion dark matter with thermal friction



Present oscillation amplitude $A(t_0) \ll f\theta_{ini}$

$$\rho_\phi = \frac{1}{2} m_\phi^2 A(t_0)^2$$

$\sim f\theta_{ini}$

$$\frac{\rho_\phi}{\rho_{\rm DM}} \approx \left(\frac{m_\phi}{\mu{\rm eV}}\right)\left(\frac{f\theta_i}{10^{16}\,{\rm GeV}}\right)^2\left(\frac{e^{-(7+\beta_1)\gamma_1/(5+\beta_1)H_1}}{10^{-7}}\right)$$
$$\times\left(\frac{\xi}{10^{-7}}\right)\left(\frac{10^5\,{\rm GeV}}{f_h}\right)^2$$

$$\frac{\gamma_1}{H_1} \simeq 10\left(\frac{2.2\times 10^5\,{\rm GeV}}{f_h}\right)^{4/3}\left(\frac{m_\phi}{\mu{\rm eV}}\right)^{1/3}\left(\frac{\xi}{10^{-7}}\right)^{2/3} \quad \text{(for } \beta_1 = 0\text{)}$$

ρ_ϕ is exponentially sensitive to the axion-hidden bath coupling.

Implication to the QCD axion dark matter

$$\frac{\alpha_s}{8\pi}\frac{\phi}{f}G^{\mu\nu a}\widetilde{G}^a_{\mu\nu}$$

$$m_\phi^2 \simeq m_0^2\left(\frac{0.15\,\mathrm{GeV}}{T}\right)^8 \quad \text{for} \quad T \gtrsim 0.15\,\mathrm{GeV}$$

The QCD axion mass is determined by f.

with $$m_0 \simeq \frac{f_\pi m_\pi}{f}\frac{\sqrt{m_u m_d}}{m_u+m_d} \simeq 5.7\mu\mathrm{eV}\left(\frac{10^{12}\,\mathrm{GeV}}{f}\right)$$

$$(\Omega_\phi h^2)^{\rm conv}_{\rm QCD} \simeq 0.1\,\theta_i^2\left(\frac{f}{10^{12}\,\mathrm{GeV}}\right)^{7/6}$$

Conventional scenario with the Hubble damping only

For string theoretic QCD axions,

$$f \sim \frac{M_P}{8\pi^2} \sim 10^{16}\,\mathrm{GeV} \;\; (\Leftrightarrow\; m_0 \sim \mathrm{neV})$$

$$(\Omega_\phi h^2)^{\mathrm{conv}}_{\mathrm{QCD}} \simeq 0.1\,\theta_i^2 \left(\frac{f}{10^{12}\,\mathrm{GeV}}\right)^{7/6}$$

$$\Rightarrow \qquad \theta_i \ll 1$$

In order to not overproduce QCD axion dark matter, the initial misalignment angle has to be fine-tuned in the conventional scenario.

$$\frac{\alpha_s}{8\pi}\frac{\phi}{f}G^{\mu\nu a}\widetilde{G}^a_{\mu\nu} + \frac{\alpha_h}{8\pi}\frac{\phi}{f_h}G_h^{\mu\nu a}\widetilde{G}^a_{h\,\mu\nu}$$

Assuming axion coupling to hidden gluons, the thermal friction arises, and it can efficiently damp away the QCD axion dark matter.

$$\Omega_\phi h^2 \simeq 0.1\theta_i^2 \left(\frac{f}{10^{16}\,\mathrm{GeV}}\right)\left(\frac{e^{-15\gamma_1/13H_1}}{10^{-6}}\right) \times \left(\frac{0.5\times 10^5\,\mathrm{GeV}}{f_h}\right)^2\left(\frac{\xi}{10^{-7}}\right)$$

$$\frac{\gamma_1}{H_1} \simeq 12\left(\frac{0.5\times 10^5\,\mathrm{GeV}}{f_h}\right)^{12/7}\left(\frac{\xi}{10^{-7}}\right)^{6/7}\left(\frac{10^{16}\,\mathrm{GeV}}{f}\right)^{1/7}$$

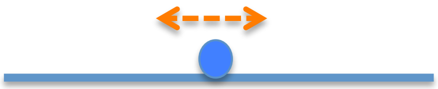
- *The string theoretic QCD axions are cosmologically viable without fine-tuning the initial condition.*
- *Theoretical motivation for the experimental search for QCD axion dark matter lighter than μeV.*

Implications to density perturbations

- Isocurvature bound in pre-inflationary scenario

Quantum fluctuation of axion field during inflation

$$\delta\phi_{\rm ini} = \frac{H_I}{2\pi}$$



CMB constraint on axion dark matter fluctuation

$$\frac{\delta\rho_\phi}{\rho_{\rm DM}} = 2\frac{\rho_\phi}{\rho_{\rm DM}}\frac{\delta\phi}{\phi} \lesssim 10^{-5}$$ [Planck '18]

$$\frac{\delta\phi}{\phi} = \frac{\delta\phi_{\rm ini}}{\phi_{\rm ini}}$$

The background field value and perturbations are equally damped away by thermal friction.

$$\boxed{H_I \lesssim 10^{-5}\pi\left(\frac{\rho_{\rm DM}}{\rho_\phi}\right) f\theta_i}$$

Implications to density perturbations

- Isocurvature bound in pre-inflationary scenario

$$\boxed{H_I \lesssim 10^{-5}\pi \left(\frac{\rho_{\rm DM}}{\rho_\phi}\right) f\theta_i}$$

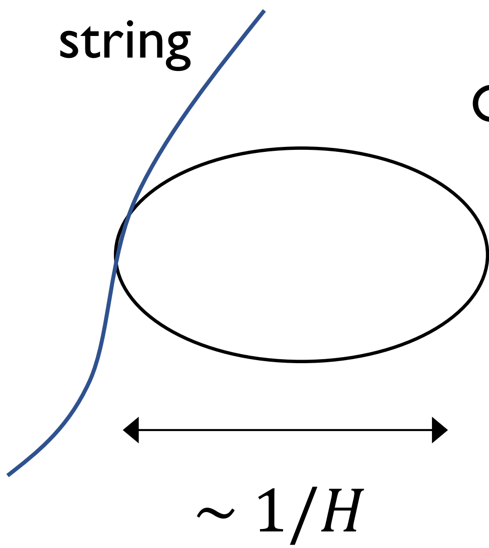
This bound on H_I for $\rho_\phi = \rho_{DM}$ can be significantly relaxed due to enlarged $f\theta_i$ because of the thermal friction.

$$f\theta_i \simeq 10^{12}\,\mathrm{GeV}\left(\frac{\rho_\phi}{\rho_{\rm DM}}\right)^{1/2} \exp\left(\frac{7+\beta_1}{2(5+\beta_1)}\frac{\gamma_1}{H_1}\right) \times \left(\frac{\mu\mathrm{eV}}{m_\phi}\right)^{1/2}\left(\frac{10^{-7}}{\xi}\right)^{1/2}\left(\frac{f_h}{10^5\mathrm{GeV}}\right)$$

Implications to density perturbations

- Axion minicluters in post-inflationary scenario

In the conventional scenario without thermal friction,



Collapses around t_{osc}

$O(1)$ density contrast
at the scale $1/H_{osc}$

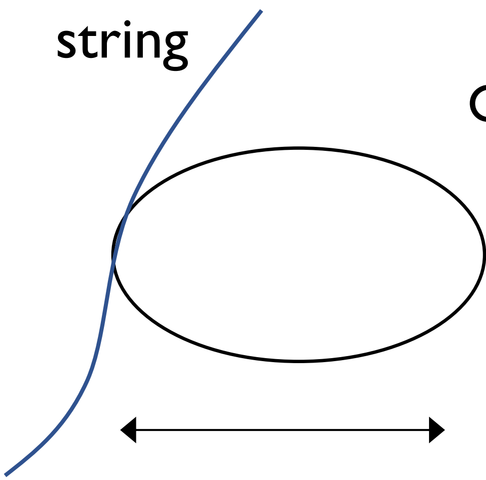
Gravitational collapse before
matter-radiation equality
[Vaquero, Redondo, Stadler '18]

Axion miniclusters $\quad r_{\mathrm{mc}} \lesssim \left(\frac{a_{\mathrm{eq}}}{a_{\mathrm{osc}}}\right) H_{\mathrm{osc}}^{-1}$

Implications to density perturbations

- Axion minicluters in post-inflationary scenario

In the presence of the thermal friction,



$$\sim \frac{1}{\sqrt{\gamma_\phi H}} < \frac{1}{H}$$

Collapses around t_* : $m_\phi(t_*)^2 = \gamma_\phi(t_*)H(t_*)$

$O(1)$ density contrast
at the scale $1/\sqrt{\gamma_* H_*}$

Gravitational collapse before
matter-radiation equality

Smaller axion miniclusters

$$r_{\rm mc} \lesssim \left(\frac{a_{\rm eq}}{a_*}\right)\left(\sqrt{\gamma_* H_*}\right)^{-1}$$

Conclusions

- Thermal friction is a general leading form of axion interactions with a thermal bath.
- A strongly motivated example is the axion-hidden YM interaction via sphaleron processes.
- The thermal friction makes a new decay phase in cosmological axion field evolution, so it can strongly damp away the axion density.
- As a consequence, neV light QCD axion is cosmologically viable without fine-tuning.
- The axion isocurvature bound can be substantially relaxed, and different axion minicluster size and mass are predicted.

Backup slides

Constraints from hidden YM

The axion mass correction from hidden YM instantons has to be small enough not to affect the axion dynamics discussed so far.

$$\frac{\Lambda_h^2}{f_h} \ll m_\phi \qquad \Lambda_h \simeq T_h \exp\left(-\frac{6\pi}{11 N \alpha_h(T_h)}\right)$$

Confinement scale for $SU(N)$ pure YM

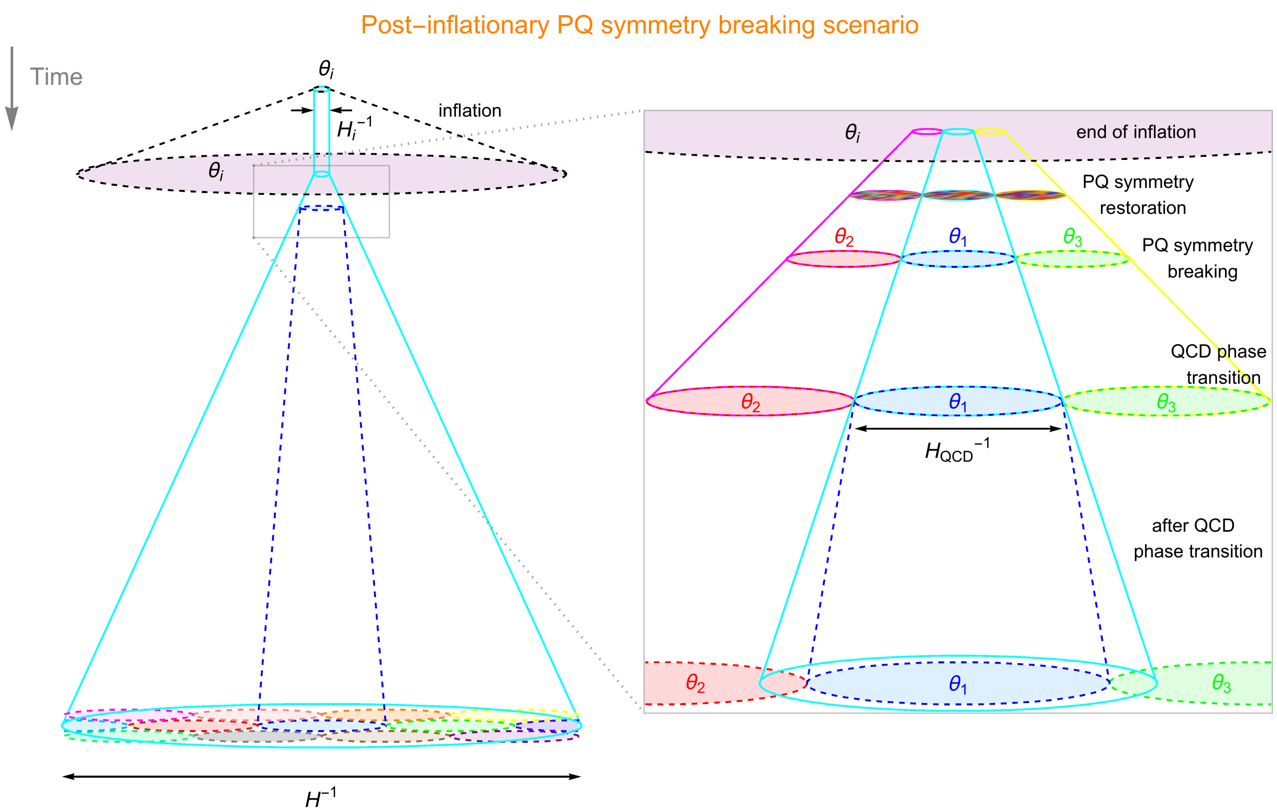
$$\Rightarrow \quad \alpha_h(T_1) \lesssim \frac{\pi}{N \ln(M_P/f_h)}$$

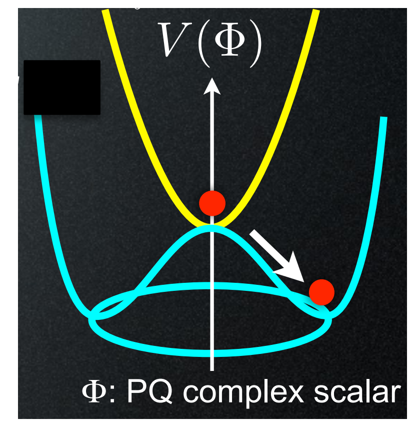
Dark Radiation from the hidden gluons and thermalized axions are constrained by the Big Bang Nucleosynthesis: [Yeh, Olive, Fields '21]

$$\Delta N_{\rm eff} = \frac{8}{7}\left(\frac{11}{4}\right)^{4/3}\left(\frac{T_h}{T}\right)^4 (N^2 - 1/2) < 0.124 \quad \Rightarrow \quad \frac{T_h}{T} \lesssim 0.3 \ \text{(for } SU(2)\text{)}$$

Thus $\boxed{\xi \sim (N\alpha_h)^5 \left(\frac{T_h}{T}\right)^3 \lesssim 10^{-7} \quad \text{for } f_h > \text{TeV}}$

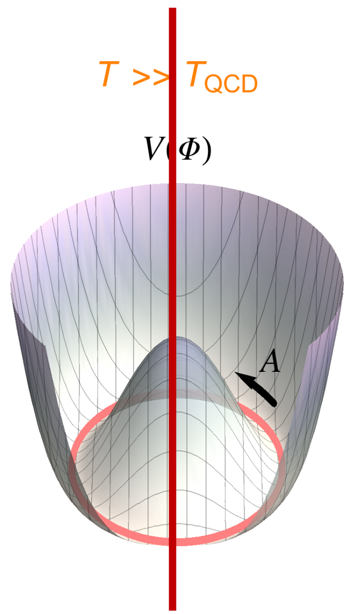
Conventional axion dark matter scenario II : string-wall network decay

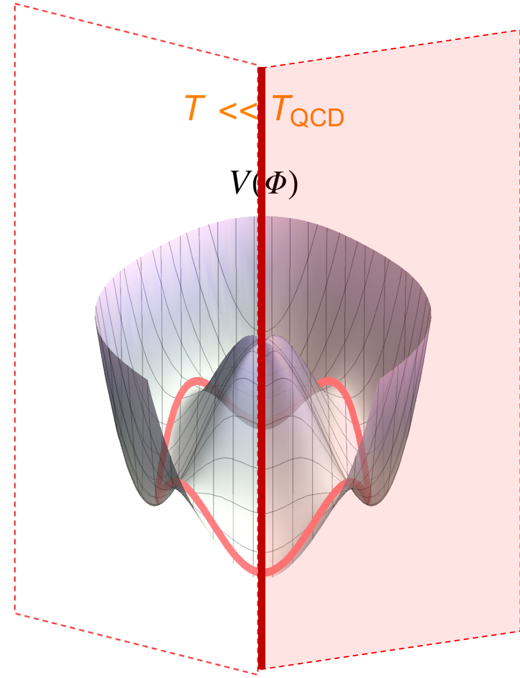




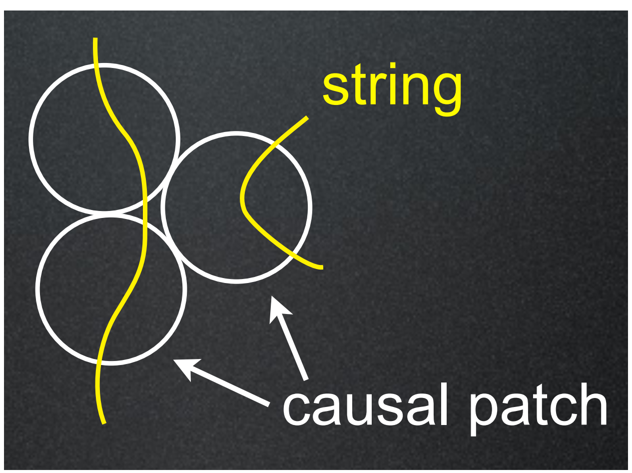
The axion field value can vary by O(1) over the horizon scale.

String





String +
domain walls



$$\mu_{\rm st} \sim f^2 \ln(ft)$$

$$\Rightarrow \quad \rho_\phi \propto \rho_{\rm mis} \ln\left(\frac{f}{m_\phi}\right)$$

The string-wall system collapses to axion dark matter when axion starts to oscillate.

Implications to density perturbations

- QCD axion dark matter in post-inflationary scenario

In the conventional scenario without thermal friction,

$$\frac{\Omega_a}{\Omega_{\rm DM}} \simeq (5 \pm 2) \times \left(\frac{f_a(t_0)}{10^{11}{\rm GeV}} \right)^{1.19}$$

Hiramatsu, Kawasaki, Saikawa, Sekiguchi (2012)

- f cannot be greater than 10^{10} GeV not to overclose the universe.
- If QCD axion minicluster exists, it implies *meV QCD axion*.

Implications to density perturbations

- QCD axion dark matter in post-inflationary scenario

  In the presence of the thermal friction,

  - f can be much greater than 10^{10} GeV thanks to the strong thermal damping.
  - The existence of QCD axion minicluster doesn't tell us axion mass.
  - The QCD axion minicluster size can be much greater than the conventional one.

$$r_{\rm mc} \lesssim \left(\frac{a_{\rm eq}}{a_*}\right) (\sqrt{\gamma_* H_*})^{-1} = \left(\frac{a_{\rm eq}}{a_*}\right) m_*^{-1}$$