# Nucleon Decay and Pati-Salam

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Based on TPD, J. Gargalionis: 2211.02054

# Pati-Salam Basics

# Pati-Salam

A theory where quarks and leptons are unified in a left-right symmetric fashion (need to introduce RHN).

$$SU(4) \otimes SU(2)_L \otimes SU(2)_R$$

$$f_L = \begin{pmatrix} Q \\ \ell \end{pmatrix}_L , \quad f_R = \begin{pmatrix} Q \\ \ell \end{pmatrix}_R \qquad \times 3$$

$$f_L \sim (\mathbf{4},\, \mathbf{2},\, \mathbf{1}) \qquad f_R \sim (\mathbf{4},\, \mathbf{1},\, \mathbf{2})$$

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$$SU(4) \otimes SU(2)_L \otimes SU(2)_R$$

$$f_L = \begin{pmatrix} u_r & d_r \\ u_b & d_b \\ u_g & d_g \\ \nu_e & e \end{pmatrix}_L \quad , \quad f_R = \begin{pmatrix} u_r & d_r \\ u_b & d_b \\ u_g & d_g \\ \nu_e & e \end{pmatrix}_R \qquad \times 3$$

# Pati-Salam

A theory where quarks and leptons are unified in a left-right symmetric fashion (need to introduce RHN).

**Lepton number as the fourth "color"**

Jogesh C. Pati*
*Department of Physics and Astronomy, University of Maryland, College Park, Maryland 20742*

Abdus Salam
*International Centre for Theoretical Physics, Trieste, Italy*
*and Imperial College, London, England*

(Received 25 February 1974)

$$\begin{pmatrix} \nu_e & e \end{pmatrix}_L \qquad \begin{pmatrix} \nu_e & e \end{pmatrix}_R$$

# Breaking Pati-Salam

Need a scalar to spontaneously break Pati-Salam...

We require:

$$SU(4) \otimes SU(2)_R \xrightarrow{\langle S \rangle} SU(3)_c \otimes U(1)_Y$$

such that

$$Y = T_{3R} + \frac{1}{2} T_{15}$$

is unbroken.

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$$S \in \{(\mathbf{4}, \mathbf{1}, \mathbf{2}), (\mathbf{10}, \mathbf{1}, \mathbf{3}), (\mathbf{36}, \mathbf{1}, \mathbf{2}), (\mathbf{20''}, \mathbf{1}, \mathbf{4}), (\mathbf{35}, \mathbf{1}, \mathbf{5}), \ldots\}$$

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Difficult to Yukawa couple easily...which we want

# Light Neutrinos

High-scale Pati-Salam

$$\Delta \sim (\mathbf{10}, \mathbf{1}, \mathbf{3})$$

$$m_\nu \simeq \frac{(m_D^\nu)^2}{M_R^2} \propto \frac{m_u^2}{Y_\Delta v_\Delta}$$

$$v_\Delta \gtrsim 10^{12}\ \mathrm{GeV}$$

Type-I Seesaw

Low-scale Pati-Salam

$$\xi \sim (\mathbf{1}, \mathbf{1}, \mathbf{1}), \quad \chi \sim (\mathbf{4}, \mathbf{1}, \mathbf{2})$$

$$\mathcal{L} \supset y_R \bar{\xi} \chi^\dagger f_R + \frac{1}{2} \mu_\xi \bar{\xi} \xi^c$$

$$v_\chi \gtrsim ??$$

Inverse Seesaw

Pati-Salam breaking can naturally solve the neutrino mass problem.

# Gauge Bosons of PS

$$SU(4) \qquad \begin{pmatrix} \hat{G}_\mu - \frac{1}{3}\mathbb{1}B_\mu & \hat{X}^+_\mu \\ \hat{X}^-_\mu & B_\mu \end{pmatrix}$$

$$SU(2)_{L/R} \qquad \begin{pmatrix} W_3 & W^+ \\ W^- & -W_3 \end{pmatrix}_{L/R}$$

# Gauge Bosons of PS

$$X_\mu \sim (\mathbf{3}, \mathbf{1})_{\frac{2}{3}}$$

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# Gauge Boson Leptoquark

$$SU(4) \quad \begin{pmatrix} u_r & d_r \\ u_b & d_b \\ u_g & d_g \\ \nu_e & e \end{pmatrix}$$

$$\mathcal{L}_X = \frac{g_4}{\sqrt{2}} \left( \overline{d'_i} \, (K^{de}_{L/R})_{ij} \, \not{X} P_{L/R} e'_j + \overline{u'_i} \, (K^{u\nu}_{L/R})_{ij} \, \not{X} P_{L/R} \nu'_j \right) + \text{ H.c.}$$

**No diquark couplings**, for the (broken) SU(4) bosons -> **no B-violating processes**

In fact, like in the SM, **B is an accidental symmetry** of the theory.

| LQ Quantum Numbers | Minimal Pati-Salam Embedding |
| --- | --- |
| $\mathrm{S}_3 \sim (\overline{\mathbf{3}},\ \mathbf{3})_{1/3}$ | $(10, 3, 1)$ |
| $\mathrm{R}_2 \sim (\mathbf{3},\ \mathbf{2})_{7/6}$ | $(15, 2, 2)$ |
| $\widetilde{R}_2 \sim (\mathbf{3},\ \mathbf{2})_{1/6}$ | $(15, 2, 2)$ |
| $\widetilde{S}_1 \sim (\overline{\mathbf{3}},\ \mathbf{1})_{4/3}$ | $(10, 1, 3)$ |
| $\mathrm{S}_1 \sim (\overline{\mathbf{3}},\ \mathbf{1})_{1/3}$ | $(4, 1, 2)/(10, 1, 3)$ |
| $\overline{S}_1 \sim (\overline{\mathbf{3}},\ \mathbf{1})_{-2/3}$ | $(4, 1, 2)/(10, 1, 3)$ |
| $\mathrm{U}_3^\mu \sim (\mathbf{3},\ \mathbf{3})_{2/3}$ | - |
| $\mathrm{V}_2^\mu \sim (\overline{\mathbf{3}},\ \mathbf{2})_{5/6}$ | - |
| $\widetilde{V}_2^\mu \sim (\overline{\mathbf{3}},\ \mathbf{2})_{-1/6}$ | - |
| $\widetilde{U}_1^\mu \sim (\mathbf{3},\ \mathbf{1})_{5/3}$ | - |
| $\mathrm{U}_1^\mu \sim (\mathbf{3},\ \mathbf{1})_{2/3}$ | $\widehat{X}_\mu$ |
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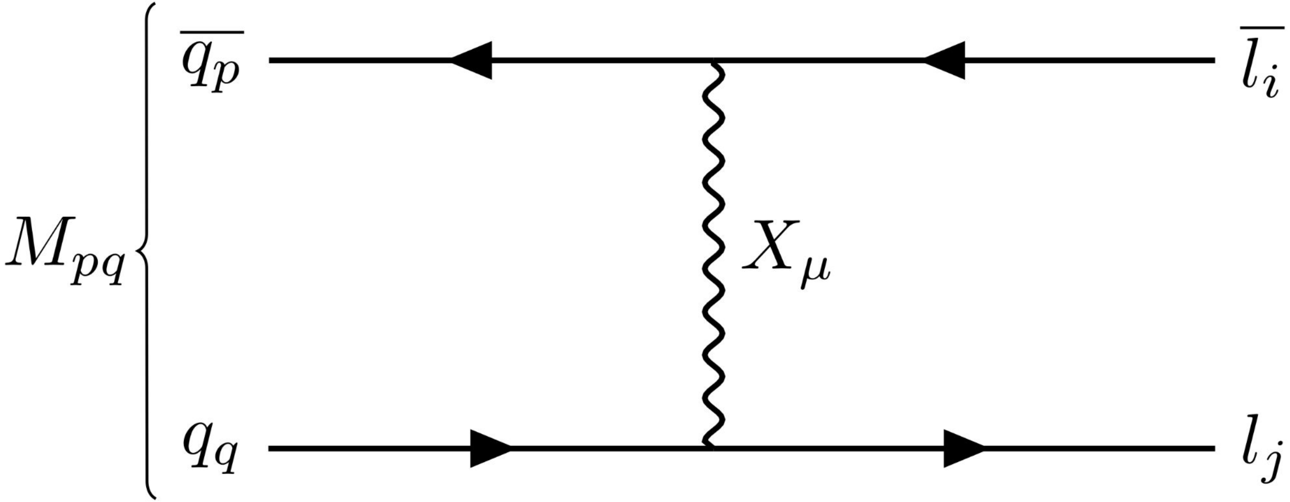
PS automatically generates scalar and vector LQs. Diquark couplings not generated -> can be light.

# Experimental constraints on PS breaking
# (How low can we go?)

# Prediction of PS

Gauge bosons don't mediate proton decay (accidental B).

They do mediate rare meson decays.



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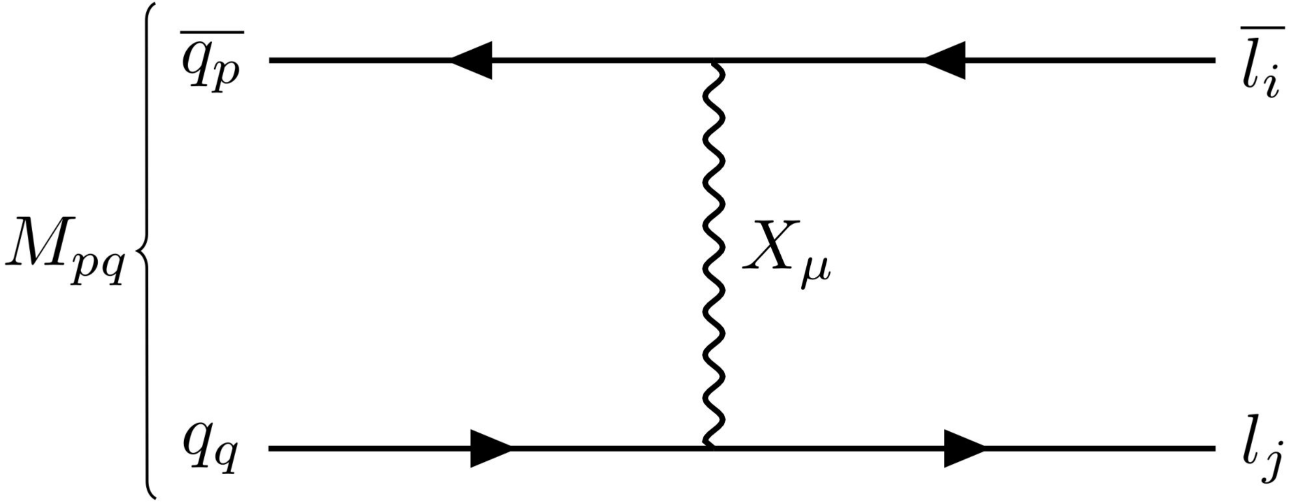


$\overline{u}_p u_q/\overline{d}_p d_q/\overline{u}_p d_q$ $\qquad$ $\nu_\ell \overline{\nu}_\ell/\ell^+\ell^-/\overline{\nu}_\ell\, \ell^-$

| Hadronic Process | Measured value or upper limit | $(p,i)(q,j)$ |
|---|---|---|
| $\mathcal{B}(K_L^0 \to ee)$ | $9.0^{+6.0}_{-4.0} \times 10^{-12}$ | (1,1)(2,1) |
| $\mathcal{B}(K_L^0 \to \mu\mu)$ | $6.84^{+0.11}_{-0.11} \times 10^{-12}$ | (1,2)(2,2) |
| $\mathcal{B}(K_L^0 \to \mu e)$ | $< 4.7 \times 10^{-12}$ | (1,2)(2,1)/(1,1)(2,2) |
| $\mathcal{B}(B_d^0 \to ee)$ | $< 2.5 \times 10^{-9}$ | (1,1)(3,1) |
| $\mathcal{B}(B_d^0 \to \mu\mu)$ | $< 7^{+13}_{-11} \times 10^{-11}$ | (1,2)(3,2) |
| $\mathcal{B}(B_d^0 \to \tau\tau)$ | $< 2.1 \times 10^{-3}$ | (1,3)(3,3) |
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| $\mathcal{B}(B_d^0 \to \tau\mu)$ | $< 1.4 \times 10^{-5}$ | (1,3)(3,2)/(1,2)(3,3) |
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| $\mathcal{B}(B_s^0 \to \tau\tau)$ | $< 6.8 \times 10^{-3}$ | (2,3)(3,3) |
| $\mathcal{B}(B_s^0 \to \mu e)$ | $< 5.4 \times 10^{-9}$ | (2,2)(3,1)/(2,1)(3,2) |
| $\mathcal{B}(B_s^0 \to \tau e)$ | $-$ | (2,3)(3,1)/(2,1)(3,3) |
| $\mathcal{B}(B_s^0 \to \tau\mu)$ | $< 4.2 \times 10^{-5}$ | (2,2)(3,3)/(3,2)(2,3) |

$$\mathcal{L}_{Xde} = \frac{g_4}{\sqrt{2}} \left( \overline{d'_i}\,(K_L^{de})_{ij}\,\not{X} P_L e'_j + \overline{d'_i}\,(K_R^{de})_{ij}\,\not{X} P_R e'_j \right) + \text{ H.c.}$$

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no events of the variety $K_L \to \mu^{\pm} + e^{\mp}$ have yet been observed, there is a lower limit on the mass of $X$ in the "basic" model given by $f^2/m_X{}^2 < G_F \alpha^2$. For $f^2/4\pi \approx 1$, this implies that $X$ must be superheavy ($m_X > 3 \times 10^4$ GeV). For variants to the "basic"

$$\mathcal{L}_{Xde} = \frac{g_4}{\sqrt{2}} \left( \overline{d'_i} \, (K_L^{de})_{ij} \, \not{X} P_L e'_j + \overline{d'_i} \, (K_R^{de})_{ij} \, \not{X} P_R e'_j \right) + \text{ H.c.}$$

# Kaon Decays

For two generations, Kaon decays are inevitable from unitarity:

$$\mathcal{L}_{X\,de} = \frac{g_4}{\sqrt{2}} \left( \overline{d'_i}\,(K_L^{de})_{ij}\, \not{X} P_L e'_j + \overline{d'_i}\,(K_R^{de})_{ij}\, \not{X} P_R e'_j \right) + \text{ H.c.}$$

$$K_{L/R}^{de} = \begin{pmatrix} \cos_x & \sin_x \\ -\sin_x & \cos_x \end{pmatrix}$$

For three generations, flavour can conspire to suppress these decays e.g.

$$K_{L/R}^{de} = \begin{pmatrix} 0 & 0 & 1 \\ \cos_x & \sin_x & 0 \\ -\sin_x & \cos_x & 0 \end{pmatrix}$$

Dominant limits arise from B-meson decays in this case -> less severe constraints.

| | $K_L^{de} = K_R^{de} = \mathbb{1}_{3\times 3}$ | SCENARIO 1 | SCENARIO 2 | SCENARIO 3 | SCENARIO 4 |
|---|---|---|---|---|---|
| $\mathcal{B}(K_L^0 \to ee)$ | 0 | $13\,\kappa_1^{K^{ee}}$ TeV | 0 | $1817\,\kappa_3^{K^{ee}}$ TeV | 0 |
| $\mathcal{B}(K_L^0 \to \mu\mu)$ | 0 | $177\,\kappa_1^{K^{\mu\mu}}$ TeV | 0 | 0 | $1900\,\kappa_4^{K^{\mu\mu}}$ TeV |
| $\mathcal{B}(K_L^0 \to \mu e)$ | $\boxed{2467\text{ TeV}}$ | $230\,\kappa_1^{K^{\mu e}}$ TeV | 0 | 0 | 0 |
| $\mathcal{B}(B_d^0 \to ee)$ | 0 | $39.7\,\kappa_1^{B^{ee}}$ TeV | 0 | 0 | 0 |
| $\mathcal{B}(B_d^0 \to \mu\mu)$ | 0 | $151\,\kappa_1^{B^{\mu\mu}}$ TeV | 0 | 0 | 0 |
| $\mathcal{B}(B_d^0 \to \tau\tau)$ | 0 | 0 | 0 | 0 | 0 |
| $\mathcal{B}(B_d^0 \to \mu e)$ | 0 | $140\,\kappa_1^{B^{\mu e}}$ TeV | 0 | $140\,\kappa_3^{B^{\mu e}}$ TeV | $140\,\kappa_4^{B^{\mu e}}$ TeV |
| $\mathcal{B}(B_d^0 \to \tau e)$ | 12.1 TeV | $10.6\,\kappa_1^{B^{\tau e}}$ TeV | $10.6\,\kappa_2^{B^{\tau e}}$ TeV | 0 | $10.6\,\kappa_4^{B^{\tau e}}$ TeV |
| $\mathcal{B}(B_d^0 \to \tau\mu)$ | 0 | $11.3\,\kappa_1^{B^{\tau\mu}}$ TeV | $11.3\,\kappa_2^{B^{\tau\mu}}$ TeV | $11.3\,\kappa_3^{B^{\tau\mu}}$ TeV | 0 |
| $\mathcal{B}(B_s^0 \to ee)$ | 0 | $29.5\,\kappa_1^{B_s^{ee}}$ TeV | $29.5\,\kappa_2^{B_s^{ee}}$ TeV | 0 | 0 |
| $\mathcal{B}(B_s^0 \to \mu\mu)$ | 0 | $90.0\,\kappa_1^{B_s^{\mu\mu}}$ TeV | $90.0\,\kappa_2^{B_s^{\mu\mu}}$ TeV | 0 | |
| $\mathcal{B}(B_s^0 \to \tau\tau)$ | 0 | 0 | 0 | 0 | 0 |
| $\mathcal{B}(B_s^0 \to \mu e)$ | 0 | $92.3\,\kappa_1^{B_s^{\mu e}}$ TeV | $92.3\,\kappa_2^{B_s^{\mu e}}$ TeV | $92.3\,\kappa_3^{B_s^{\mu e}}$ TeV | $92.3\,\kappa_4^{B_s^{\mu e}}$ TeV |
| $\mathcal{B}(B_s^0 \to \tau e)$ | 0 | – | 0 | 0 | – |
| $\mathcal{B}(B_s^0 \to \tau\mu)$ | – | – | 0 | – | 0 |

$$K_{L/R}^{de} \sim \quad \begin{pmatrix} 0 & 0 & 1 \\ \cdot & \cdot & 0 \\ \cdot & \cdot & 0 \end{pmatrix} \qquad \begin{pmatrix} \cdot & 0 & \cdot \\ \cdot & 0 & \cdot \\ 0 & 1 & 0 \end{pmatrix} \qquad \begin{pmatrix} 0 & \cdot & \cdot \\ 0 & \cdot & \cdot \\ 1 & 0 & 0 \end{pmatrix}$$

M. Dolan, R. Volkas, TPD: 2012.05976

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| $\mathcal{B}(B_s^0 \to \mu\mu)$ | 0 | $90.0\,\kappa_1^{B_s\mu\mu}$ TeV | $90.0\,\kappa_2^{B_s\mu\mu}$ TeV | 0 | |
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| $\mathcal{B}(B_s^0 \to \tau e)$ | 0 | — | 0 | 0 | — |
| $\mathcal{B}(B_s^0 \to \tau\mu)$ | — | — | 0 | — | 0 |

$$K_{L/R}^{de} \sim \begin{pmatrix} 0 & 0 & 1 \\ \cdot & \cdot & 0 \\ \cdot & \cdot & 0 \end{pmatrix} \quad \begin{pmatrix} \cdot & 0 & \cdot \\ \cdot & 0 & \cdot \\ 0 & 1 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & \cdot & \cdot \\ 0 & \cdot & \cdot \\ 1 & 0 & 0 \end{pmatrix}$$

M. Dolan, R. Volkas, TPD: 2012.05976

# PS Breaking Limits

Depending on the flavour structure of 'de' mixing, gauge LQ mass limits vary between:

$$m_X \gtrsim \mathcal{O}(80 - 2500)\ \text{TeV}$$

M. Dolan, R. Volkas, TPD: 2012.05976

Leading to direct constraints on the PS breaking scale:

$$m_X \propto g_4 \max(v_\chi, v_\Delta, \dots)$$

Not *that* low, if lower masses are desired modifications need to minimal model can be made to further suppress these meson decay limits.

J. Pati, A. Salam: 1974
R. Foot: hep-ph/9708205
G. Filewood, R. Foot: hep-ph/9903374

L. Luzio, A. Greljo, M. Nardecchia: 1708.08450
C. Cornella, J. Fuentes-Martin, G. Isidori: 1903.11517
J. Fuentes-Martin, P. Strangl: 2004.11376

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In this talk I only focus on the minimal 'low-scale' model: $\xi \sim (\mathbf{1}, \mathbf{1}, \mathbf{1}), \quad \chi \sim (\mathbf{4}, \mathbf{1}, \mathbf{2})$

## Pati-Salam and UV Physics
## (*Can* we go lower?)

# B-conservation in PS

Baryon number is an **accidental** symmetry in Pati-Salam (unlike B-L)

Therefore there are **higher-dimensional** operators which will violate B.

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Baryon number is an **accidental** symmetry in Pati-Salam (unlike B-L)

Therefore there are **higher-dimensional** operators which will violate B.

Particularly important if such operators exist at **dimension-5.**

Even a Planck-scale suppression of such an operator can generate rapid proton decay for light (TeV) leptoquarks.

J. Arnold, M. Wise: 1304.6119
N. Assad, B. Fornal, B. Grinstein: 1708.06350
C. Murgui, M. Wise: 2105.14029

High-scale Pati-Salam

$$\Delta \sim (\mathbf{10}, \mathbf{1}, \mathbf{3})$$

$$m_\nu \simeq \frac{(m_D^\nu)^2}{M_R^2} \propto \frac{m_u^2}{Y_\Delta v_\Delta}$$

$$v_\Delta \gtrsim 10^{12}\ \mathrm{GeV}$$

Low-scale Pati-Salam

$$\xi \sim (\mathbf{1}, \mathbf{1}, \mathbf{1}), \quad \chi \sim (\mathbf{4}, \mathbf{1}, \mathbf{2})$$

$$\mathcal{L} \supset y_R \bar{\xi} \chi^\dagger f_R + \frac{1}{2} \mu_\xi \bar{\xi} \xi^c$$

$$v_\chi \gtrsim 80 \times 10^3\ \mathrm{GeV}\ ??$$

High-scale Pati-Salam

$$\Delta \sim (\mathbf{10}, \mathbf{1}, \mathbf{3})$$

$$m_\nu \simeq \frac{(m_D^\nu)^2}{M_R^2} \propto \frac{m_u^2}{Y_\Delta v_\Delta}$$

$$v_\Delta \gtrsim 10^{12}\ \mathrm{GeV}$$

With the bare-minimum particle content (for a realistic theory) there are **no dimension 5** operators whatsoever.

But the LQs here are expected to have large masses due to the large PS breaking.

## Dimension 5

| Field content | $J$ | Number of operators |
|---|---|---|
| $\Phi^\dagger\Phi^\dagger\xi\xi$ | 0 | $n_\xi(n_\xi+1)/2$ |
| $\Phi^\dagger\chi^\dagger f_L\xi$ | 0 | $n_\xi n_f$ |
| $\Phi^\dagger\Phi\xi\xi$ | 0 | $n_\xi(n_\xi+1)/2$ |
| $\chi^\dagger\chi^\dagger f_L f_L$ | 0 | $n_f(n_f+1)/2$ |
| $\chi^\dagger\chi^\dagger f_R f_R$ | 0 | $n_f(n_f+1)$ |
| $\Phi\chi^\dagger f_L\xi$ | 0 | $n_\xi n_f$ |
| $\chi^\dagger\chi\xi\xi$ | 0 | $n_\xi(n_\xi+1)/2$ |
| $\chi\chi f_L f_L$ | 4 | $n_f(n_f+1)/2$ |
| $\chi\chi f_R f_R$ | 4 | $n_f(n_f+1)/2$ |
| $\Phi\Phi\xi\xi$ | 0 | $n_\xi(n_\xi+1)/2$ |

## Low-scale Pati-Salam

$$\xi \sim (\mathbf{1},\mathbf{1},\mathbf{1}), \quad \chi \sim (\mathbf{4},\mathbf{1},\mathbf{2})$$

$$\downarrow$$

$$\mathcal{L} \supset y_R\bar{\xi}\chi^\dagger f_R + \frac{1}{2}\mu_\xi\bar{\xi}\xi^c$$

$$\downarrow$$

$$v_\chi \gtrsim 80 \times 10^3\,\mathrm{GeV}\,??$$

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| $\Phi\chi^\dagger f_L\xi$ | 0 | $n_\xi n_f$ |
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| $\boxed{\chi\chi f_L f_L}$ | 4 | $n_f(n_f+1)/2$ |
| $\boxed{\chi\chi f_R f_R}$ | 4 | $n_f(n_f+1)/2$ |
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### Low-scale Pati-Salam

$$\xi \sim (\mathbf{1},\mathbf{1},\mathbf{1}), \quad \chi \sim (\mathbf{4},\mathbf{1},\mathbf{2})$$

$$\Downarrow$$

$$\mathcal{L} \supset y_R\bar{\xi}\chi^\dagger f_R + \frac{1}{2}\mu_\xi\bar{\xi}\xi^c$$

$$\Downarrow$$

$$v_\chi \gtrsim 80\times 10^3\,\mathrm{GeV}\,??$$

B violation already occurs at dim 5! Operators are Weinberg-like.
Contain the **PS-breaking** scalar.

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# Implications

$$\mathcal{L} \supset \frac{1}{4\Lambda} \sum_X (C_X)^{pq} (\mathcal{O}_X)_{pq} + \text{H.c.}$$

Let's consider these B-violating dim-5 operators in more detail

$$(\mathcal{O}_X)_{pq} \equiv (\overline{f_X^c})_p^{\alpha i} (f_X)_q^{\beta j} \chi^{\gamma k} \chi^{\delta l} \epsilon_{\alpha\beta\gamma\delta} \epsilon_{ij} \epsilon_{kl}$$

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After PS breaking:

$\chi_d \sim (\mathbf{3},\mathbf{1})_{-\frac{1}{3}}$

$$(\mathcal{O}_X)_{pq} \supset \frac{4v_R}{\sqrt{2}}(\overline{d^c_X})^a_p (u_X)^b_q (\chi^d)^c \epsilon_{abc}$$

Di-quark couplings are now generated for the scalar LQ -> proton decay!

# Nucleon decay

$$\mathcal{L} \supset \overline{f_X^c} f_X \chi\chi + \bar{\xi}\chi^\dagger f_R + \mathrm{H.c}$$

In combination they generate nucleon decay at dim-6:

$$\mathcal{L}^{(6)} \supset -\sum_X \frac{C_X y_R v_R \sin\theta}{\sqrt{2}\Lambda m_{\chi^d}^2} \mathcal{O}_{udd}^{S,XR} + \mathrm{H.c.}$$

# Nucleon decay

$$\mathcal{L}^{(6)} \supset -\sum_X \frac{C_X y_R v_R \sin\theta}{\sqrt{2}\Lambda m_{\chi^d}^2} \mathcal{O}_{udd}^{S,XR} + \mathrm{H.c.}$$

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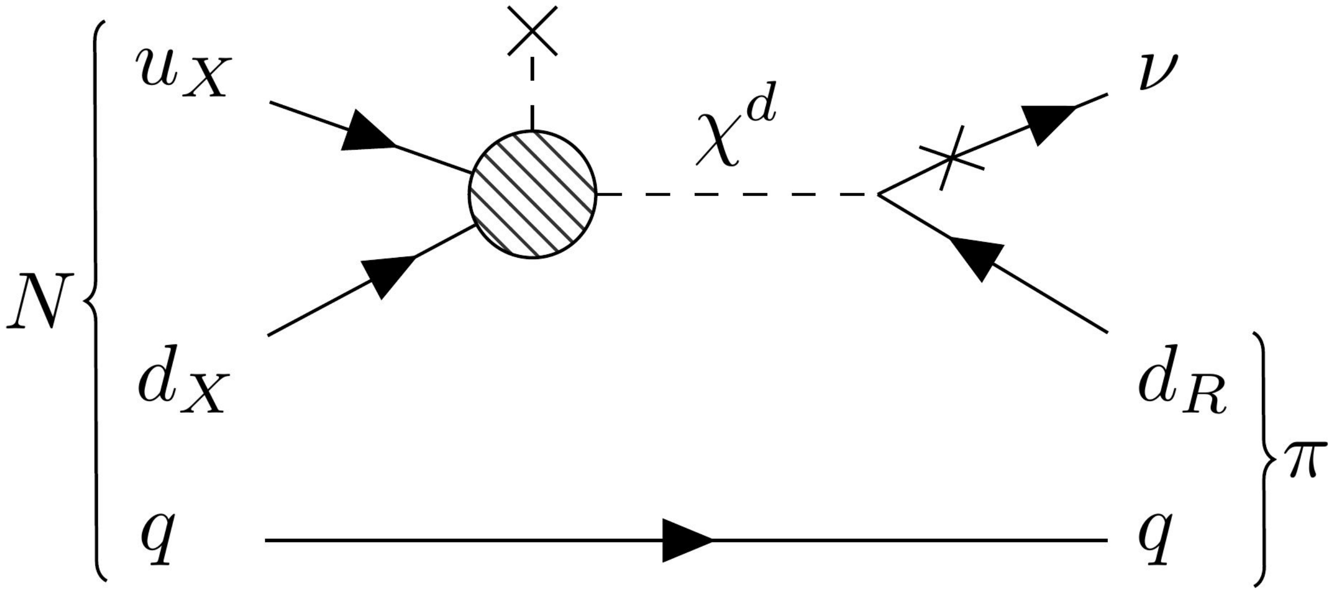
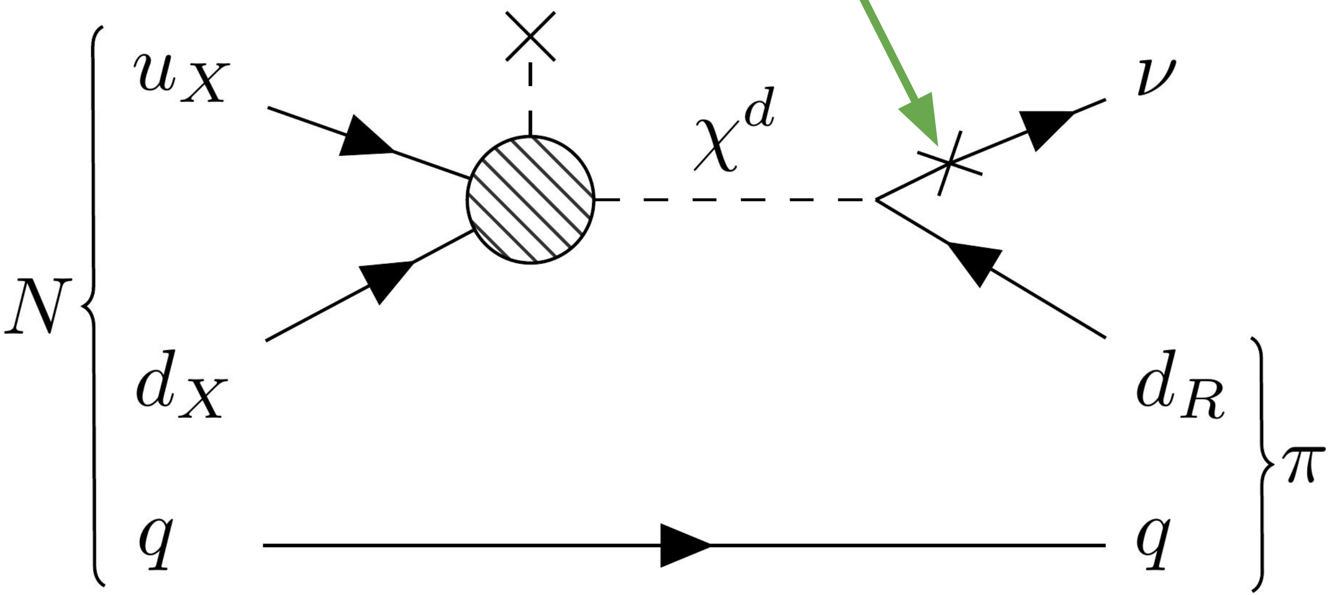


# Nucleon decay

$$\sin\theta \simeq \frac{m_u}{y_R v_R}$$

$$\mathcal{L}^{(6)} \supset -\sum_X \frac{C_X y_R v_R \sin\theta}{\sqrt{2}\Lambda m_{\chi^d}^2} \mathcal{O}_{udd}^{S,XR} + \text{H.c.}$$

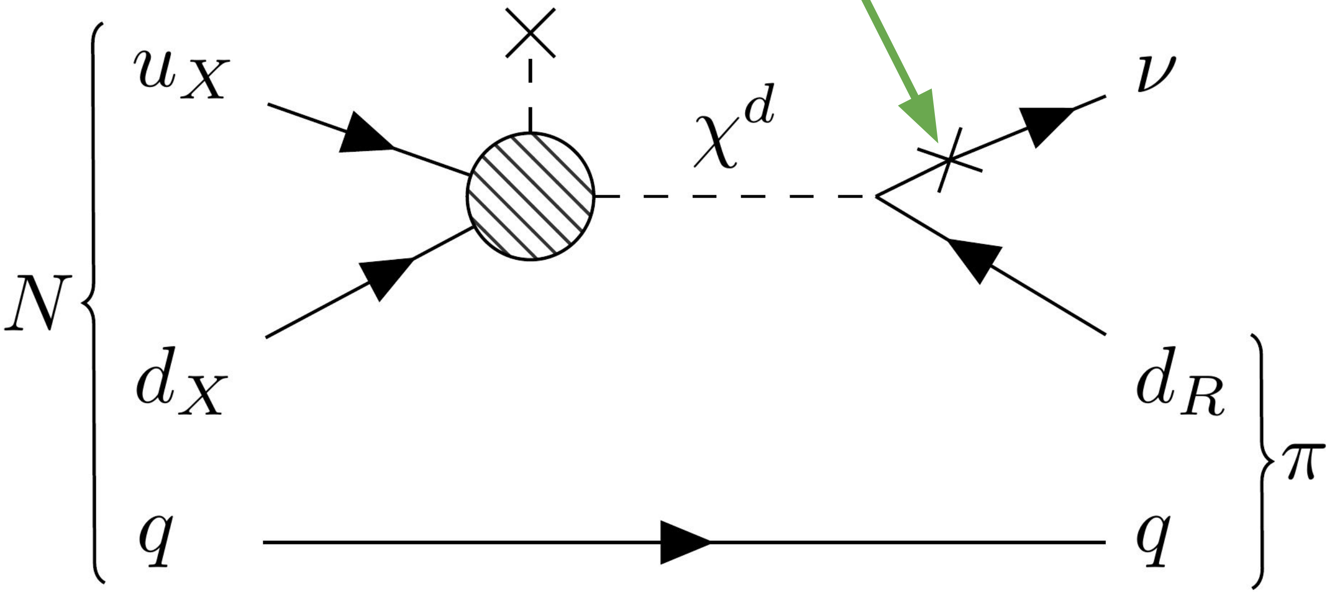


$$m_u \overline{\nu_L} \nu_R$$

$$\sin\theta \simeq \frac{m_u}{y_R v_R}$$

# Nucleon decay

$$\mathcal{L}^{(6)} \supset -\sum_X \frac{C_X y_R v_R \sin\theta}{\sqrt{2}\Lambda m^2_{\chi^d}} \mathcal{O}^{S,XR}_{udd} + \text{H.c.}$$



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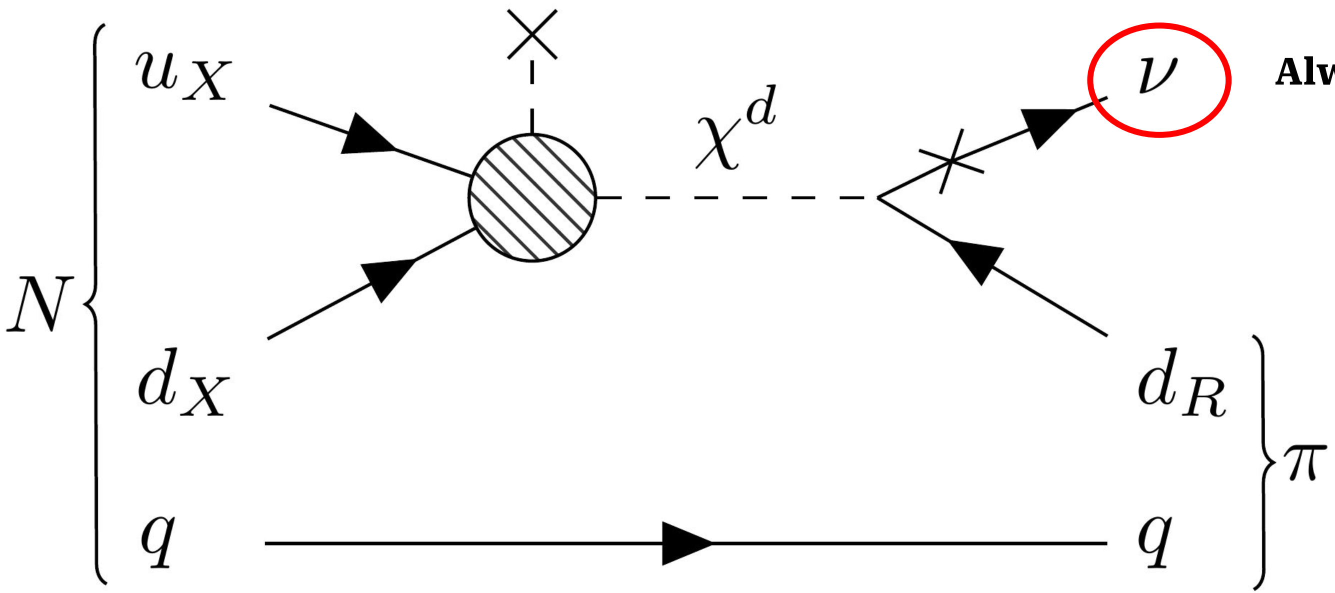


**Always** neutral.

# Nucleon decay

Therefore predict the nucleon decays:

$$\Gamma(n \to M^0 \nu)$$

$$\Gamma(p \to M^+ \nu)$$

$$m_{\chi^d} \gtrsim 8 \times 10^6 \,\mathrm{GeV} \sqrt{|C_L| \frac{10^{19}\mathrm{GeV}}{\Lambda} \frac{m_u}{171\mathrm{GeV}}}$$

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Constrained by the PS breaking scale.

# Nucleon decay

Therefore predict the nucleon decays:

$$\Gamma(n \to M^0 \nu)$$

$$\Gamma(p \to M^+ \nu)$$

$$\Lambda_{\text{Planck}} \qquad\qquad \Lambda_{\text{GUT}}$$

$$v_R \gtrsim \begin{pmatrix} 10^4\,\text{GeV} \\ 3\times 10^5\,\text{GeV} \\ 5\times 10^6\,\text{GeV} \end{pmatrix} \qquad v_R \gtrsim \begin{pmatrix} 4\times 10^5\,\text{GeV} \\ 9\times 10^6\,\text{GeV} \\ 10^8\,\text{GeV} \end{pmatrix} \quad \text{for} \quad \begin{matrix} m_u \\ m_c \\ m_t \end{matrix}$$



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# Nucleon decay

The full three-generational mixing case:

$$\mathcal{L}^{(6)} \supset -\sum \frac{(C_X)_{pq} v_R}{\sqrt{2}\Lambda m_{\chi^d}^2} \Omega_{rs} (\mathcal{O}_{udd}^{S,XR})_{pqrs} + \mathrm{H.c.}$$

$$\mathbf{\Omega} \equiv \mathbf{K}_L^{u\nu\dagger} \mathbf{M}_u [v_R \mathbf{Y}_R^{\mathsf{T}}]^{-1} \mathbf{Y}_R \mathbf{V}_{\mathrm{CKM}}^R$$

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Importantly:

$$\mathbf{\Omega} \text{ and } \mathbf{K}_{L/R}^{de} \textbf{ do not align}$$

# Nucleon

| ARIO 1 | SCENARIO 2 | SCENARIO 3 | SCENARIO 4 |
|---|---|---|---|
| $^{K^{ee}}$ TeV | 0 | $1817\,\kappa_3^{K^{ee}}$ TeV | 0 |
| $_1^{K^{\mu\mu}}$ TeV | 0 | 0 | $1900\,\kappa_4^{K^{\mu\mu}}$ TeV |
| $_1^{K^{\mu e}}$ TeV | 0 | 0 | 0 |
| $_1^{B^{ee}}$ TeV | 0 | 0 | 0 |
| $_1^{B^{\mu\mu}}$ TeV | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 |
| $_1^{B^{\mu e}}$ TeV | 0 | $140\,\kappa_3^{B^{\mu e}}$ TeV | $140\,\kappa_4^{B^{\mu e}}$ TeV |
| $_1^{B^{\tau e}}$ TeV | $10.6\,\kappa_2^{B^{\tau e}}$ TeV | 0 | $10.6\,\kappa_4^{B^{\tau e}}$ TeV |
| $_1^{B^{\tau\mu}}$ TeV | $11.3\,\kappa_2^{B^{\tau\mu}}$ TeV | $11.3\,\kappa_3^{B^{\tau\mu}}$ TeV | 0 |
| $_1^{B_s^{ee}}$ TeV | $29.5\,\kappa_2^{B_s^{ee}}$ TeV | 0 | 0 |
| $_1^{B_s^{\mu\mu}}$ TeV | $90.0\,\kappa_2^{B_s^{\mu\mu}}$ TeV | 0 | |
| 0 | 0 | 0 | 0 |
| $_1^{B_s^{\mu e}}$ TeV | $92.3\,\kappa_2^{B_s^{\mu e}}$ TeV | $92.3\,\kappa_3^{B_s^{\mu e}}$ TeV | $92.3\,\kappa_4^{B_s^{\mu e}}$ TeV |
| — | 0 | 0 | — |
| — | 0 | — | 0 |
| $K^{de}_{L/R} \sim$ | $\begin{pmatrix} 0 & 0 & 1 \\ \cdot & \cdot & 0 \\ \cdot & \cdot & 0 \end{pmatrix}$ | $\begin{pmatrix} \cdot & 0 & \cdot \\ \cdot & 0 & \cdot \\ 0 & 1 & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & \cdot & \cdot \\ 0 & \cdot & \cdot \\ 1 & 0 & 0 \end{pmatrix}$ |

The full three-generational mixing case:

$$\mathcal{L}^{(6)} \supset -\sum \frac{(C_X)_{pq} v_R}{\sqrt{2}\Lambda m^2_{\chi^d}} \Omega$$

$$\mathbf{\Omega} \equiv \mathbf{K}_L^{u\nu\dagger}\mathbf{M}_u[v_R \mathbf{Y}$$

Importantly:

$$\mathbf{\Omega} \text{ and } \mathbf{K}^{de}_{L/R} \textbf{ do not align}$$

# Nucleon decay

The full three-generational mixing case:

$$\mathcal{L}^{(6)} \supset -\sum \frac{(C_X)_{pq} v_R}{\sqrt{2}\Lambda m_{\chi^d}^2} \Omega_{rs} (\mathcal{O}_{udd}^{S,XR})_{pqrs} + \mathrm{H.c.}$$

$$\mathbf{\Omega} \equiv \mathbf{K}_L^{u\nu\dagger} \mathbf{M}_u [v_R \mathbf{Y}_R^{\mathsf{T}}]^{-1} \mathbf{Y}_R \mathbf{V}_{\mathrm{CKM}}^R$$

Importantly:

$$\mathbf{\Omega} \text{ and } \mathbf{K}_{L/R}^{de} \textbf{ do not align}$$

Consequence: You cannot suppress **both** the meson and nucleon decays at the same time.

# Quick Example

Assume that

$$K_{L/R}^{de} \sim \begin{pmatrix} 0 & 0 & 1 \\ \cdot & \cdot & 0 \\ \cdot & \cdot & 0 \end{pmatrix}$$

Chosen such that the constraints from rare meson decays (mediated by the gauge boson LQ) are minimised. This fixes:

# Quick Example

Assume that

$$K_{L/R}^{de} \sim \begin{pmatrix} 0 & 0 & 1 \\ \cdot & \cdot & 0 \\ \cdot & \cdot & 0 \end{pmatrix}$$

Chosen such that the constraints from rare meson decays (mediated by the gauge boson LQ) are minimised. This fixes:

$$i \in \{1, 2\} = \{d, s\}$$

$$\sum_j \Omega_{ji} \simeq 0.5\, m_u \tilde{\mathbf{V}}_{1i} + 1.3\, m_c \tilde{\mathbf{V}}_{2i} + 1.0\, m_t \tilde{\mathbf{V}}_{3i}$$

Limits dominated by top-mass unless more flavour structure is assumed, **at worst** the charm quark contribution dominates.

# SO(10) connection

What (heavy) particles would generate the B-violating dim-5 operators?

Scalars:

$$S \sim (\mathbf{6}, \mathbf{1}, \mathbf{1})$$

Fermions:

$$\begin{aligned} F_R &\sim (\mathbf{6}, \mathbf{1}, \mathbf{1}) \\ &\sim (\mathbf{6}, \mathbf{1}, \mathbf{3}) \\ F_L &\sim (\mathbf{6}, \mathbf{2}, \mathbf{2}) \end{aligned}$$

# SO(10) connection

What (heavy) particles would generate the B-violating dim-5 operators?

Scalars:

$$S \sim (\mathbf{6}, \mathbf{1}, \mathbf{1})$$

$$SO(10): \quad \begin{matrix} \mathbf{10} \\ \mathbf{126} \end{matrix} \quad \supset (\mathbf{6}, \mathbf{1}, \mathbf{1})$$

**At least one** required in SO(10) theories for fermion mass generation. Generation of dim-5 operator seems inevitable with GUT scale suppression.

# Further Work

- While Pati-Salam is popular, modified theories of PS are usually considered in low-scale models to allow for even lighter (TeV) scale models.

  What are the implications (if any) that such dimension-5 B-violating operators have on the allowed breaking scale in such 'ultra-low-scale' models? Might be even more important here as even lighter breaking scales are desired.

  E.g. 4321, chiral PS.

- What are the minimum ingredients necessary in PS to allow for low breaking scales **without** generating these dim-5 operators?

  Tied to how neutrino mass is made small in the model.

  E.g. (10,1,3) scalar breaks PS without dim-5 operators however leads to type-1 seesaw which minimally requires high-scale breaking.

# Conclusions

- Constraining models with Pati-Salam symmetries is interesting. It predicts exotic particles as well as interesting observational decay channels.

- Recently Pati-Salam theories have seen a resurgence due to the ability of light LQs present in the theory as a candidate for various anomalous measurements.

- We have found that there are dimension-5 operators which can lead to sizeable levels of nucleon decay in the minimal model, even with a Planck scale suppression. These lead to limits comparable, or stronger, to the usual rare meson decays that are used to constrain PS.

- Cannot manipulate the flavour structure to suppress these nucleon decays as well as the rare meson decays. **Important to consider** possible dimension-5 operators when assessing various low-scale theories with a Pati-Salam symmetry.

# Fin

# Backup

# Decay Calculations

$$\Gamma(p \to \pi^+ \nu) = \frac{m_p v_R^2}{16\pi\Lambda^2 m_{\chi^d}^4} \left| \sum_{X,p} (C_X)_{11} \Omega_{p1} \langle \pi^+ | (ud)_X d_R | p \rangle \right|^2$$

$$\Gamma(n \to \pi^0 \nu) = \frac{1}{2} \Gamma(p \to \pi^+ \nu)$$

$$\Gamma(p \to K^+ \nu) = \frac{m_p v_R^2}{16\pi\Lambda^2 m_{\chi^d}^4} \left| \sum_{X,p} (C_X)_{12} \Omega_{p1} \langle K^+ | (us)_X d_R | p \rangle + \sum_{X,p} (C_X)_{11} \Omega_{p2} \langle K^+ | (ud)_X s_R | p \rangle \right|^2$$

# Mass generation in PS

$$f_L \sim (\mathbf{4},\ \mathbf{2},\ \mathbf{1}) \qquad\qquad f_R \sim (\mathbf{4},\ \mathbf{1},\ \mathbf{2})$$

Generating fermion masses requires mass term of the form:

$$\mathcal{L} \supset Y_{ij} (\overline{f_L})^i \langle A \rangle (f_R)^j$$

$$\implies A \in \{\phi \sim (\mathbf{1}, \mathbf{2}, \mathbf{2}),\ \Phi \sim (\mathbf{15}, \mathbf{2}, \mathbf{2})\}$$

$$\implies \{\phi : Y_{ij}^{d/u} = Y_{ij}^{e/\nu}, \Phi : Y_{ij}^{d/u} = -3 Y_{ij}^{e/\nu}\}$$

# Mass generation in PS

$$\mathcal{L} \supset \boxed{Y_1^{\phi}\,\overline{f_L}\phi f_R + Y_2^{\phi}\,\overline{f_L}\phi^c f_R} + \boxed{Y_\Delta \overline{f}^c_R \Delta f_R} + \text{ H.c.}$$

$$\phi \supset H$$

$$m_e \propto m_d$$

$$m_D^\nu \propto m_u$$

$$Y_\Delta v_\Delta \overline{\nu_R^c}\nu_R$$

$$m_\nu \simeq \frac{(m_D^\nu)^2}{M_R^2} \propto \frac{m_u^2}{Y_\Delta v_\Delta}$$

$$m_u \simeq 170 \text{ GeV} \implies v_\Delta \gtrsim 10^{12} \text{ GeV}$$

# Physical Mixing Matrices

$$\mathbf{K}^{u\nu}_{L/R} = (U^u_{L/R})^\dagger V^\nu_{L/R}$$

$$\mathbf{K}^{de}_{L/R} = (U^d_{L/R})^\dagger U^e_{L/R}$$

$$\mathbf{V}^{\mathrm{CKM}}_{L/R} = (U^u_{L/R})^\dagger U^d_{L/R}$$

$$\mathbf{U}^{\mathrm{PMNS}}_{L/R} = (U^e_{L/R})^\dagger V^\nu_{L/R}$$

$$\mathbf{K}^{u\nu}_{L/R} = \mathbf{V}^{L/R}_{\mathrm{CKM}} \mathbf{K}^{de}_{L/R} \mathbf{U}^{L/R}_{\mathrm{PMNS}}$$

So we predict that (with a single mass generating scalar):

$$m_u \propto m_\nu, \quad m_d \propto m_e$$

| | $\mu = m_Z$ | $\mu = 1$ TeV | $\mu = 10$ TeV | $\mu = 100$ TeV | $\mu = 1000$ TeV | $\mu = M_{\text{GUT}}$ |
|---|---|---|---|---|---|---|
| $m_e/m_d$ | 0.177 | 0.205 | 0.230 | 0.251 | 0.271 | 0.399 |
| $m_\mu/m_s$ | 1.891 | 2.195 | 2.454 | 2.688 | 2.902 | 4.265 |
| $m_\tau/m_b$ | 0.612 | 0.724 | 0.823 | 0.913 | 0.997 | 1.565 |