The Edge of Precision in Simulations for the LHC

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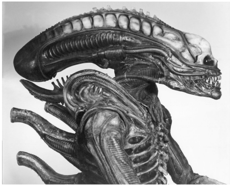
which alien am I?

cuddly E.T.

or



evil alien?

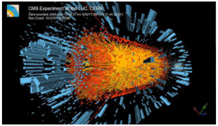
- why precision tools?
- current precision
- better parton showers
- outlook

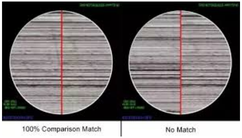
why precision

(carrying coal to Newcastle)

motivation: the need for (more) accurate tools

- to date no discovery of new physics (BSM) (a pity, but that's Nature)
- hope for "simple" discoveries is waning (don't expect anything glaringly obvious)
- push into precision tests of the Standard Model (find it or constrain "subtle"!)
- statistical uncertainties approach zero (because of fantastic work of accelerator, DAQ, etc.)
- systematic exp. uncertainties decrease (because of ingenious experimental work)
- theoretical uncertainties are or become dominant (obstacle to full explitation of LHC)





CSI LHC: need precise & accurate tools for precision physics

how to build an event generator

- paradigm: "divide et impera"
- divide simulation in distinct phases, with (logarithmically) separated scales
- start with signal event

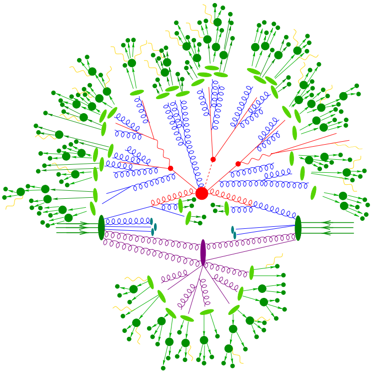
  (fixed order perturbation theory)
- dress partons with parton shower

  (resummed perturbatkon theory)
- add underlying event

  (phenomenological models)
- hadronize partons

  (phenomenological models)
- decay hadrons

  (effective theories, simple symmetries & data)



current precision

(where we are)

fixed-order accuracy

(apologies for any omissions in active field with $\approx$ 100 publications/past 5 years)

- N^{3}LO for single-boson production (1503.06056 . . . 1802.00833, 1807.11501)
  for DIS, and for VBF H-production in double DIS (1803.09973; 1606.00840)
- NNLO for practically all $2 \to 2$ (and some $2 \to 3$) processes:
  - jj (1705.10271, 1905.09047, . . .)
  - Vj, γj, Hj (1408.5325, 1504.02131, 1504.07922, 1505.03893, 1705.04664, 1901.11041, 1905.13738, . . .)
  - tt & single top (1303.6254, 1511.00549; 1404.7116, . . .)
  - VV and $\gamma\gamma$ (1408.5243, 1504.01330, 1507.06257, 1604.08576, 1605.02716, 1708.02925, 1711.06631, . . .)
  - VBF (1506.02660, 1802.02445, . . .)
  - dijets in DIS (1804.05663, . . .)
- virtual $2 \to \geq 3$ amplitudes (1511.05409, 1511.09404, 1604.06631, 1712.02229, 1811.11699, . . .)
- relative size argument: $\alpha_s^2 \approx \alpha_W$:
  must include NLO EW corrections for $\mathcal{O}(1-10\%)$ accuracy
  $\Longrightarrow$ automated in OpenLoops, Recola, aMc@Nlo _ MadGraph

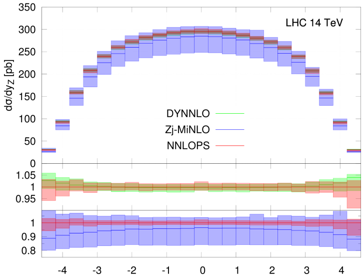
(1705.00598, 1704.05783, 1405.0301)

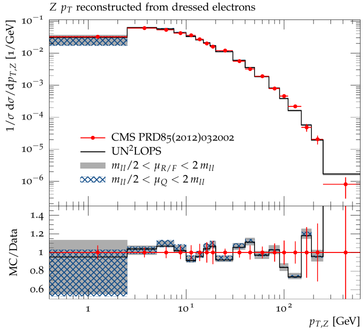
SM precision simulation in a nutshell: Drell-Yan

- current "accuracy standard(s)":
  - fixed-order: $\mathrm{N^3LO}$ for inclusive, NNLO for Vj
  - matching: NNLOPS for inclusive V
  - merging: MEPS@NLO for $V+\leq 2$ jets at NLO $V+\geq 3$ jets at LO
- dominating QCD effects: $\mathcal{O}(10\text{-}30\%)$
  - low-$p_\perp$ region dominated by parton shower
  - high-$p_\perp$ region dominated by (multi-) jet topologies
  - higher accuracy in rate (and some shapes) through NNLO matching
- must add EW corrections for %-level precision
  - EW correction at large scales $\mathcal{O}(10\%)$
  - QED FSR + EW for V line shapes at $\mathcal{O}(1\%)$

NNLOPS for Z production: MINLO & UNNLOPS

(1407.2904, 1405.3607)

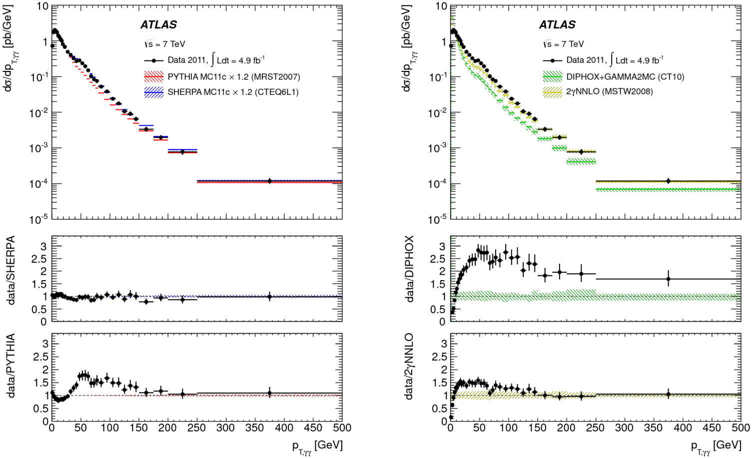




- different logic of achieving NNLO precision
- available for H, V production (both) and VV production (MINLO)

merging example: $p_{\perp,\gamma\gamma}$ in MEPS@LO vs. NNLO

(arXiv:1211.1913 [hep-ex])



better parton showers?

(the story never gets old)

colour coherence, encore

- collinear:

$$
{}_n\langle 1,\ldots,n|1,\ldots,n\rangle_n \xrightarrow{i\|j} \sum_{\lambda,\lambda'=\pm} {}_{n-1}\left\langle 1,\ldots,\not{i}(ij),\ldots,\not{j},\ldots,n\left|\frac{8\pi\alpha_s}{2p_ip_j}P^{\lambda\lambda'}_{(ij)i}(z)\right|1,\ldots,\not{i}(ij),\ldots,\not{j},\ldots,n\right\rangle_{n-1},
$$

  with spin-dependent splitting function $P^{\lambda\lambda'}_{(ij)i}(z)$

- soft:

$$
{}_n\langle 1,\ldots,n|1,\ldots,n\rangle_n \xrightarrow{p_j\to 0} -8\pi\alpha_s\sum_{i,k\neq j} {}_{n-1}\langle 1,\ldots,\not{j},\ldots,n|\mathbf{T}_i\mathbf{T}_k\, w_{ik,j}|1,\ldots,\not{j},\ldots,n\rangle_{n-1}
$$

  with colour-insertion operators $\mathbf{T}_{i,k}$ & soft eikonal

$$
w_{ik,j} = \frac{p_ip_k}{(p_ip_j)(p_jp_k)} = \frac{W_{ik,j}}{E_j^2} = \frac{1}{E_j^2}\,\frac{1-\cos\theta_{ik}}{(1-\cos\theta_{ij})(1-\cos\theta_{jk})}
$$

(obviously, frame-dependent when expressed by energies & angles)

soft eikonals, decomposed

- textbook decomposition (pink bible): $W_{ik,j} = \tilde{W}^i_{ik,j} + \tilde{W}^k_{ki,j}$
  with "radiator functions" $\tilde{W}^i_{ik,j}$: (identify "splitters" to combine with collinear terms)

$$\tilde{W}^i_{ik,j} = \frac{1}{2}\left(\frac{1-\cos\theta_{ik}}{(1-\cos\theta_{ij})(1-\cos\theta_{jk})} + \frac{1}{1-\cos\theta_{ij}} - \frac{1}{1-\cos\theta_{jk}}\right)$$

- express θ_{jk} for use in i-splitter term:

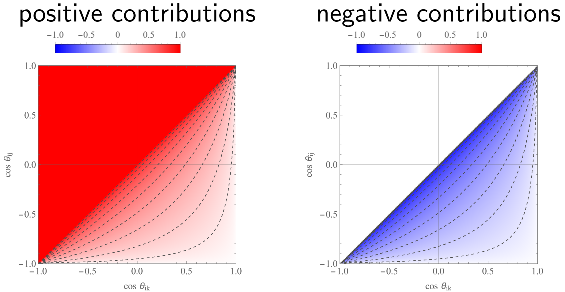
$$\cos\theta_{jk} = \cos\theta_{ij}\cos\theta_{ik} + \sin\theta_{ij}\sin\theta_{ik}\cos\phi^i_{jk} \ \ldots$$

- ...and average over azimuth ϕ^i_{jk}:

$$\frac{1}{2\pi}\int_0^{2\pi}\mathrm{d}\phi^i_{jk}\,\tilde{W}^i_{ik,j} = \frac{\tilde{I}^i_{ik,j}}{1-\cos\theta^i_j}\,, \qquad \text{where} \qquad \tilde{I}^i_{ik,j} = \begin{cases} 1 & \text{if} \quad \theta^i_j < \theta^i_k \\ 0 & \text{else} \end{cases}$$

(this is the well-known source of angular ordering)

- azimuthally integrated radiator function (normalised to 2π):



- need to include azimuthal modulation, if observables sensitive to it
- but: naive inclusion bound to fail (MC efficiency $\to 0$)

soft eikonals, decomposed again

(F.Herren, S.Hoche, FK, D.Reichelt, M.Schonherr, 2208.06057)

- define **positive definite** radiators:

  (borrowing from Catani & Seymour, Nucl. Phys. **B485** (1997) 291)

$$\bar{W}^i_{ik,j} = \frac{1-\cos\theta_{ik}}{(1-\cos\theta_{ij})(2-\cos\theta_{ij}-\cos\theta_{jk})}$$

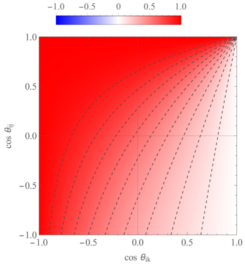
- same result after azimuthal averaging, but $\tilde{I}^i_{ik,j} \longrightarrow \bar{I}^i_{ik,j}$ with

$$\bar{I}^i_{ik,j} = \frac{1}{\sqrt{(\bar{A}^i_{ij,k})^2-(\bar{B}^i_{ij,k})^2}}$$

where

$$\bar{A}^i_{ij,k} = \frac{2-\cos\theta^i_j(1+\cos\theta^i_k)}{1-\cos\theta^i_k}\,,\; \bar{B}^i_{ij,k} = \frac{\sqrt{(1-\cos^2\theta^i_j)(1-\cos^2\theta^i_k)}}{1-\cos\theta^i_k}$$

integrated radiator, $\bar{I}^i_{ik,j}$



matching with collinear terms

- collinear limit of eikonal factors:

$$w_{ik,j} \xrightarrow{i||j} w_{ik,j}^{(\mathrm{coll})}(z) = \frac{1}{2p_ip_j}\,\frac{2z}{1-z}\,, \qquad \text{where} \qquad z \xrightarrow{i||j} \frac{E_i}{E_i+E_j}$$

- compare with leading $(1-z)$-terms of splitting functions

($1/z$ term in $g \longrightarrow gg$ captured with other "dipole")

$$\begin{aligned} P_{qq}(z) &= C_F\left(\frac{2z}{1-z}+(1-z)\right)\,, \\ P_{gg}(z) &= C_A\left(\frac{2z}{1-z}+z(1-z)\right)\,, \\ P_{gq}(z) &= T_R\,(1-2z(1-z))\,. \end{aligned}$$

$\longrightarrow$ defines "collinear remnant"

kinematics mapping: birds-eye view

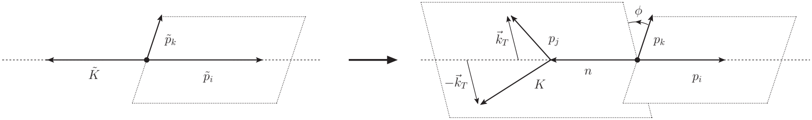
(F.Herren, S.Hoche, FK, D.Reichelt, M.Schonherr, 2208.06057)

- kinematics as main obstacle to NLL accuracy in dipole showers: recoil of subsequent soft emissions may change "NLL history"
- construct new mapping $\{\tilde{p}_l\} \longrightarrow \{p_l\}$

(inspired by Catani & Seymour's treatment of identified hadrons)

- logic: disentangle colour spectator $\tilde{p}_k$ and recoil partner $\tilde{K}$

(*i.e.* define a **global** recoil scheme, use spectator for eikonal/azimuth)



constructing the kinematics

- splitter $\tilde{p}_i \to p_i = z\tilde{p}_i$, spectator $\tilde{p}_k \to p_k$ (splitter and spectator keep direction)
- introduce orientation n to define splitting variable $z = \frac{p_i n}{(p_i+p_j)n}$
- with recoil momentum $\tilde{K}$: $n = \tilde{K} + (1-z)\tilde{p}_i$
- construct emitted momentum and recoil partner after splitting:

(demand $\tilde{K}^2 = K^2$ & $p_j^2 = m_j^2 = 0$)

$$\begin{aligned} p_j &= v\,\bar{n} + \frac{1}{v}\frac{\mathrm{k}_\perp^2}{2\tilde{p}_i\tilde{K}}\,\tilde{p}_i - k_\perp \\ K &= (1-v)\,\bar{n} + \frac{1}{1-v}\frac{\mathrm{k}_\perp^2+\tilde{K}^2}{2\tilde{p}_i\tilde{K}}\,\tilde{p}_i + k_\perp\,. \end{aligned}$$

with $v = \frac{p_i p_j}{p_i\tilde{K}}$ and additional direction $\bar{n} = n - \frac{n^2}{2\tilde{p}_i n}\tilde{p}_i$

- transverse momentum vanishes for $p_i \parallel p_j$

$$\mathrm{k}_\perp^2 = v(1-v)\,2p_jK - v^2K^2 = v(1-v)(1-z)\,2\tilde{p}_i\tilde{K} - v^2\tilde{K}^2$$

constructing the kinematics

- boost every momentum in recoil partner to new system $\tilde{K} \to K$:

$$p_l^\mu \to \Lambda^\mu_\nu(K,\tilde{K})\, p_l^\nu \text{ with } \Lambda^\mu_\nu(K,\tilde{K}) = g^\mu_\nu - \frac{2(K+\tilde{K})^\mu (K+\tilde{K})_\nu}{(K+\tilde{K})^2} + \frac{2\tilde{K}^\mu K_\nu}{K^2}$$

- construct emission phase space:
  - obtain by factorising 3-body phase space, result:

$$\mathrm{d}\Phi^{(\mathrm{FI})}_{+1}(-\tilde{K};\tilde{p}_1,\ldots,\tilde{p}_{j-1},\tilde{p}_{j+1},\ldots,\tilde{p}_n;p_j) = \frac{-2\tilde{p}_i\tilde{K}}{16\pi^2}\,\mathrm{d}v\,\mathrm{d}z\,z\,\frac{\mathrm{d}\phi}{2\pi}$$

  - note: azimuthal angle expressed through scalar products $\longrightarrow$ PS Lorentz-invariant
- IS kinematics from FS through crossing relations

parton evolution

- define evolution parameter:

$$t = 2E_j^2\,(1-\cos\theta_j{}^i) = v\,(1-z)\,2\tilde{p}_i\tilde{K}$$

and therefore soft evolution given by

$$\mathrm{d}P_{ik,j}^{i\,(\mathrm{soft})}(t,z,\phi) = \mathrm{d}\Phi_{+1}(\{\tilde{p}\},p_j)\,8\pi\alpha_s\,C_i\,\bar{w}_{ik,j}^i = \mathrm{d}t\,\mathrm{d}z\,\frac{\mathrm{d}\phi}{2\pi}\,\frac{\alpha_s}{2\pi\,t}\,2C_i\,\bar{W}_{ik,j}$$

- same for collinear evolution, but could evolve in virtuality or similar

checking the maps: analytic considerations

(F.Herren, S.Hoche, FK, D.Reichelt, M.Schonherr, 2208.06057)

- analyse Lorentz boost (*i.e.* impact on previous emissions)
- decompose new recoil momentum as

$$K^\mu = \tilde{K}^\mu - X^\mu = \tilde{K}^\mu - [p_j - (1-z)\tilde{p}_i]^\mu$$

(X^μ will go to zero for soft/collinear emissions)

- write Lorentz transformation as

$$\Lambda^\mu_\nu(K,\tilde{K}) = g^\mu_\nu + \tilde{K}^\mu A_\nu + X^\mu B_\nu$$

with

$$A^\nu = 2\left[\frac{(\tilde{K}-X)^\nu}{(\tilde{K}-X)^2} - \frac{(\tilde{K}-X/2)^\nu}{(\tilde{K}-X/2)^2}\right], \qquad \text{and} \qquad B^\nu = \frac{(\tilde{K}-X/2)^\nu}{(\tilde{K}-X/2)^2}$$

- follow CAESAR formalism and analyse behaviour under scaling

(A. Banfi, G. P. Salam, & G. Zanderighi, JHEP **03** (2005) 073)

$$k_{t,l} \to k'_{t,l} = k_{t,l}\rho^{(1-\xi_l)/a+\xi_l/(a+b)}\,, \quad \eta_l \to \eta'_l = \eta - \xi_l \frac{\ln\rho}{a+b}\,, \text{where } \xi = \frac{\eta}{\eta_{\max}}$$

- impact of recoil in Lund plane under global rescaling must vanish
- boost in $\rho \to 0$ limit:

$$A^\nu \xrightarrow{\rho\to 0} 2\,\frac{\tilde{K}X}{\tilde{K}^2}\,\frac{\tilde{K}^\nu}{\tilde{K}^2} - \frac{X^\nu}{\tilde{K}^2} \text{ and } \quad B^\nu \xrightarrow{\rho\to 0} \frac{\tilde{K}^\nu}{\tilde{K}^2}$$

and

$$\Delta p_l^\mu = 2\,\frac{\tilde{K}X}{\tilde{K}^2}\,\frac{\tilde{p}_l\tilde{K}}{\tilde{K}^2}\,\tilde{K}^\mu - \frac{\tilde{p}_l X}{\tilde{K}^2}\,\tilde{K}^\mu + \frac{\tilde{p}_l\tilde{K}}{\tilde{K}^2}\,X^\mu$$

- colour-singlet decay or production ($e^-e^+ \to$ hadrons, $q\bar{q} \to V$)
  $\longrightarrow \tilde{K} =$ c.m.-momentum, only energy component (not rescaled)
- assume emitter $\tilde{p}_i$ is soft (and for ALARIC $a = 1$, $b = 0$)

$$\tilde{p}_l \tilde{K} \sim \rho^{1-\xi_l} \text{ and } \quad \tilde{p}_l X \sim \rho^{2-\xi_l-\max(\xi_i,\xi_j)}$$

  and therefore, in components

$$\begin{array}{lcccl} \Delta p_l^0 & \sim & \rho^{1-\xi_l} X^0 + \rho^{2-\xi_l-\max(\xi_i,\xi_j)} \tilde{K}^0 + \rho^{1-\xi_l} X^0 & \sim & \rho^{2-\xi_l-\max(\xi_i,\xi_j)} \\ \Delta p_l^3 & \sim & \rho^{1-\xi_l} X^3 & \sim & \rho^{2-\xi_l-\max(\xi_i,\xi_j)} \\ \Delta p_l^{1,2} & \sim & \rho^{1-\xi_l} X^{1,2} & \sim & \rho^{2-\xi_l} \end{array}$$

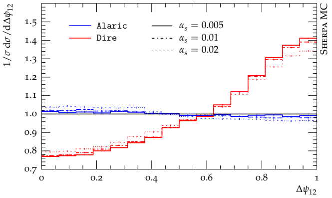
- therefore: impact of subsequent emissions vanishes with *rho*

set-up of numerical tests

(F.Herren, S.Hoche, FK, D.Reichelt, M.Schonherr, 2208.06057)

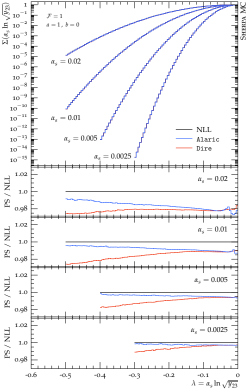
- compare results in $\alpha_S \to 0$ limit with NLL result
- set-up for checks
  - fixed α_S
  - leading colour $C_A = 2C_F = 3$
  - all partons massless
- example: azimuthal angle between two leading Lund-plane declusterings

(should be $\Delta\Psi_{12} = 0$)

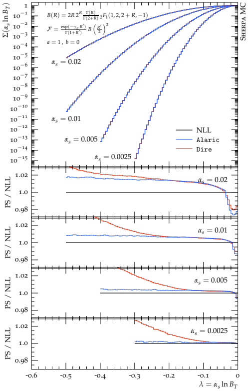


numerics: event shapes

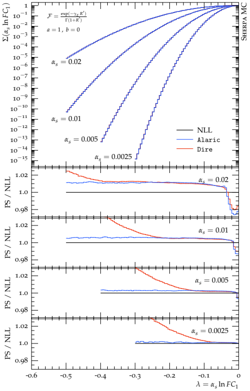
y_{23}



B_T

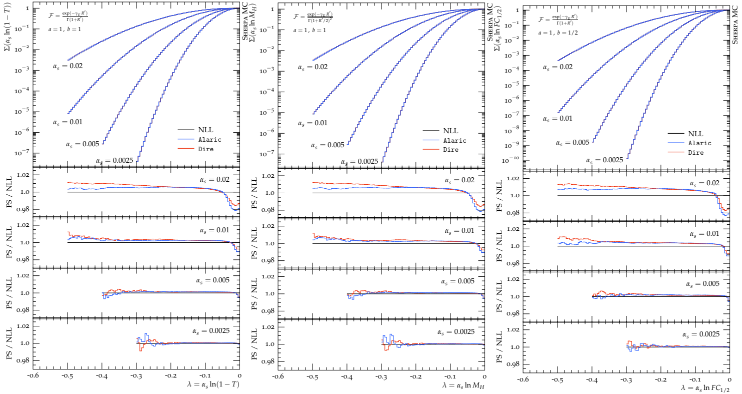


FC_1



more event shapes

$1-T$ $\quad M_H \quad$ $FC_{1/2}$



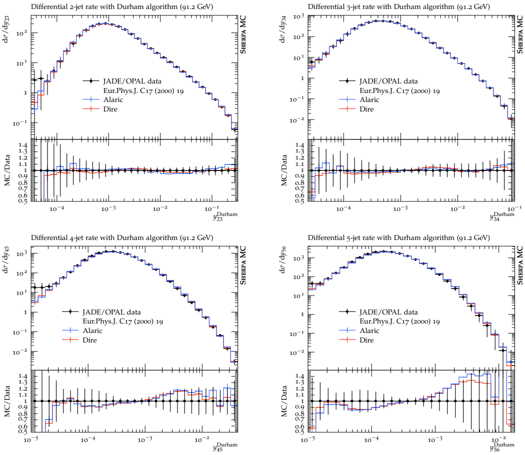
set-up of data comparison

- compare hadron-level results with LEP data
- perturbative set-up
    - no higher orders (no matching or merging)
    - running two-loop α_S with $\alpha_S(M_Z) = 0.118$
    - use CMW scheme for soft eikonal parts
    - all partons massless, masses emulated through simplistic thresholds
    - leading colour $C_A = N_c = 3,\ C_F = \frac{N_c^2-1}{2N_c}$
- non-perturbative set-up
    - need to use PYTHIA hadronization

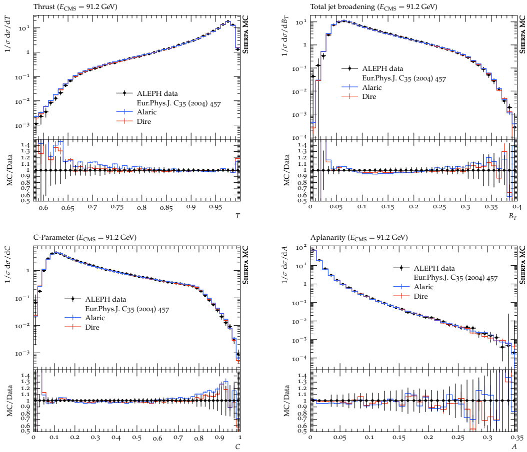
      (ALARIC not yet ready for heavy hadron decays)
    - default parameters of PYTHIA 6.4, but

$\mathtt{PARJ(21)} = 0.3$, $\mathtt{PARJ(41)} = 0.4$, $\mathtt{PARJ(42)} = 0.36$(ALARIC)/0.45(DIRE)

differential jet rates

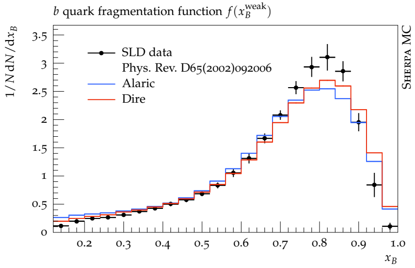


some event shapes: $1-T$, B_T, C, A



just for fun: *b*-quark fragmentation function

- emulation of quark masses with naive thresholds in $g \to q\bar{q}$: stop evolution if $t \leq (2m_q)^2$
  $\Longrightarrow$ must include quark masses also in $q \to qg$
- quick plausibility check: *b*-quark fragmentation function



including NLO splitting kernels

(Hoeche, FK & Prestel, 1705.00982, and Hoeche & Prestel, 1705.00742)

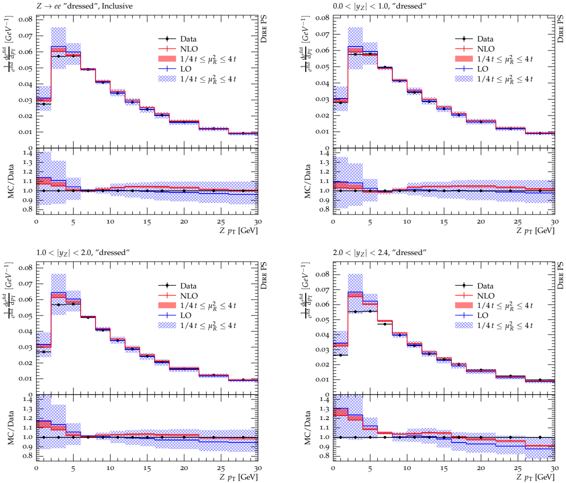
- expand splitting kernels as

$$P(z,\,\kappa^2)\;=\;P^{(0)}(z,\,\kappa^2)\,+\,\frac{\alpha_s}{2\pi}\,P^{(1)}(z,\,\kappa^2)$$

- aim: reproduce DGLAP evolution at NLO
  include all NLO splitting kernels
- three categories of terms in $P^{(1)}$:
  - cusp (universal soft-enhanced correction) (already included in original showers)
  - corrections to $1\to 2$
  - new flavour structures (e.g. $q\to q'$), identified as $1\to 3$
- new paradigm: two independent implementations

physical results: DY at LHC

(untuned showers vs. 7 TeV ATLAS data, optimistic scale variations)



leading colour differential two-loop soft corrections

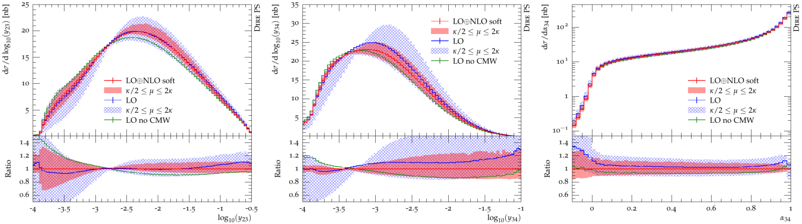
(Dulat, Hoeche & Prestel, 1805.03757)

- analyse two-emission soft contribution and compare with iterated single emissions
- subtract double-counted terms and endpoint contributions
- capture residual effect by reweighting original parton shower, with
  - accounting for finite recoil
  - including first $1/N_c$ corrections

    (another way to solve "problem" in Dasgupta et al., 1805.09327)
  - incorporating spin correlations

scale uncertainties

- varying κ in the soft-enhanced terms, including NLO explicit corrections



summary & next steps

(the now and the future)

summary

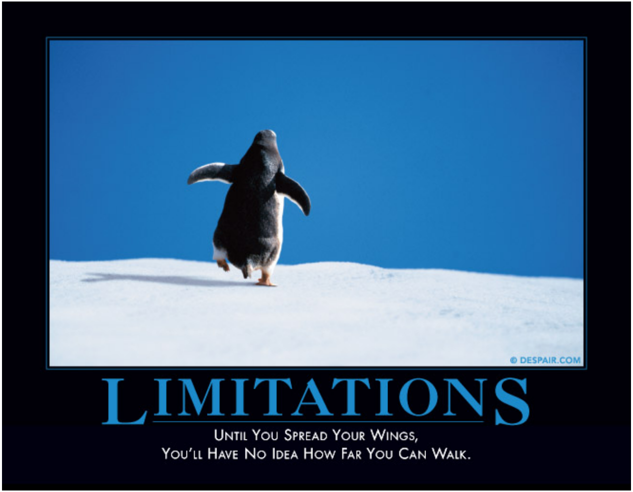
- reviewed status of precision for LHC
- re-visited color coherence, re-decomposed soft eikonal

  (correct in differential azimuthal angle)
- designed new kinematics, IS and FS radiation on same footing

  (disentangle color- and recoil-partners)
- analytics: kinematics doesn't spoil "NLL history"
  numerics: kinematics reproduces NLL for $e^-e^+ \to$ hadrons
- for more details, especially IS, *cf.* arXiv on Monday

next steps

- near future:
  - include quark masses ($\longrightarrow$ wanna do pheno!)
  - checks and simple pheno for pp collisions ($q\bar{q} \to V$ & $gg \to H$)
  - LO multijet merging (need to implement parton shower history & rejections)
  - NLO matching (we know the terms . . .)
- further future:
  - add double–soft terms (F. Dulat, S. Hoeche & S. Prestel, Phys. Rev. **D98** (2018) 074013)
  - add triple–collinear terms (S. Hoeche, FK, & S. Prestel, JHEP **10** (2017) 093)
- far future:
  - spin correlations (have to implements CK algorithm for showers)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LIMITATIONS
Until You Spread Your Wings,
You'll Have No Idea How Far You Can Walk.

alien? or predator?