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All the covariant tensor currents of massless particles in the covariant formulation

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Motivation

In the Standard Model (SM)

Unsolved problems: Dark matter, Neutrino oscillation, Matter antimatter asymmetry, $\cdots$

But, at the LHC : No new particles and phenomena...

In this situation with no decisive evidence for new physics beyond the SM

$\downarrow$

One powerful strategy : Studying all the allowed effective interactions of particles including high-spins in a model-independent way

High-spin massless particles

Realizations

- ✓ With conventional high-spin massless fields
- ✓ With light-cone coordinates
- ✓ In SUSY
- ⋮

VS

Restrictions

No-go theorems

- ✓ Soft photon theorem
- ✓ LY theorem
- ✓ WW theorem
- ⋮

P. A. Zyla *et al.*, PTEP (2020)

M. Fierz and W. Pauli, Proc. Roy. Soc. Lond. A (1939)
B. de Wit and D. Z. Freedman, Phys. Rev. D (1980)
T. Curtright, Phys. Lett. B (1979)

S. Weinberg, Phys. Rev. (1964)
L. D. Landau, Dokl. Akad. Nauk SSSR. (1948)
C. N. Yang, Phys. Rev. (1950)
S. Weinberg and E. Witten, Phys. Lett. B. (1980)

Motivation

No-go theorems $\rightarrow$ Complete absence of high-spin massless particles in nature?

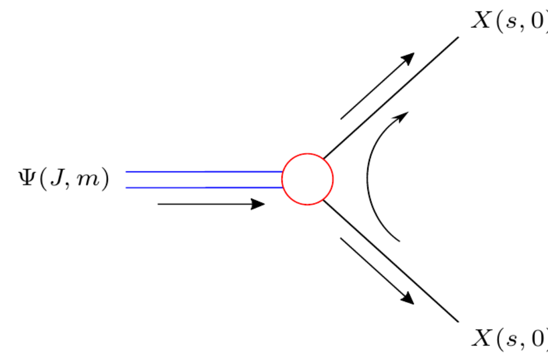
LY theorem

✓ Spin 1 $\rightarrow \gamma\gamma$ ×

✓ Spin 1 $\rightarrow gg$ ∘

✓ Spin $J = 0, 2, \cdots \rightarrow \gamma\gamma$ or gg ∘

$\vdots$



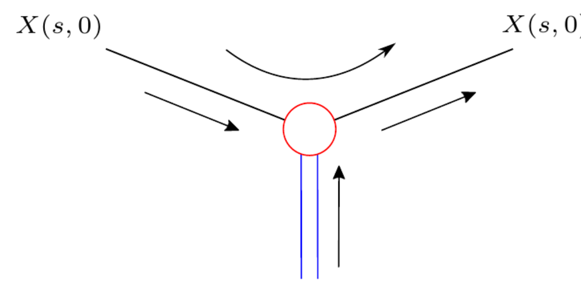
WW theorem

✓ All theories allowing a covariant conserved current J_μ

Forbid $\rightarrow$ spin-$s > 1/2$ massless particles with $Q = \int d^3\vec{x}\, J_0$

✓ All theories allowing an energy-momentum tensor $T_{\mu\nu}$

Forbid $\rightarrow$ spin-$s > 1$ massless particles with $P_\mu = \int d^3\vec{x}\, T_{0\mu}$



Exceptions

Spin-1 massless particles, γ and g with $0 = \int d^3\vec{x}\, J_0$

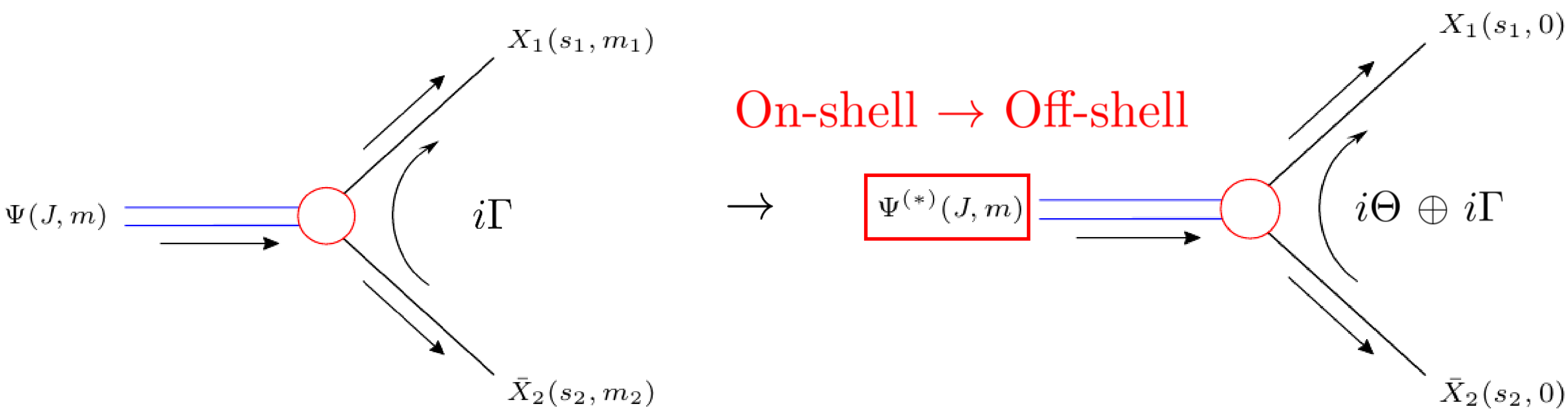
and

Discovery of gravitational waves
$\rightarrow$ a spin-2 massless particle called graviton

All the covariant tensor currents of massless particles in the covariant formulation

$$\Theta^{[J;s_1,s_2]}_{\mu_1\cdots\mu_J}$$

The first extention of [S. Y. Choi and J. H. Jeong, Phys. Rev. D (2022)]



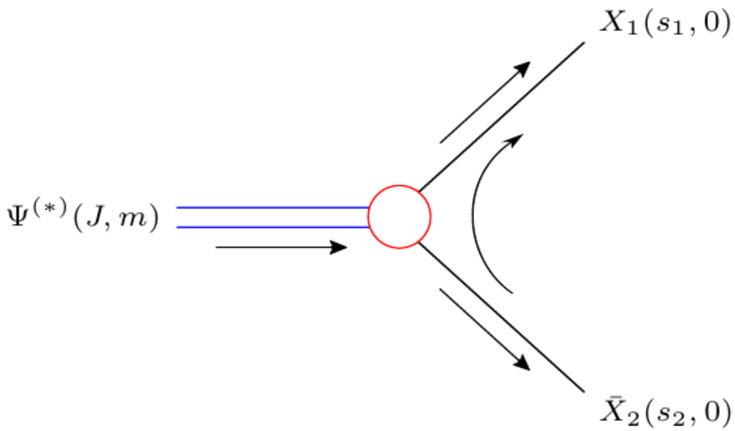
General assumption : Symmetric tensor current

coupled to $\Pi^{\mu_1\cdots\mu_J\nu_1\cdots\nu_J}(p) = \int d^4x\, e^{ip\cdot x}\, \langle 0|T\{\Psi^{\mu_1\cdots\mu_J}(x)\Psi^{\dagger\nu_1\cdots\nu_J}(0)\}|0\rangle$

with the symmetric free field $\Psi^{\mu_1\cdots\mu_J}$

Outline

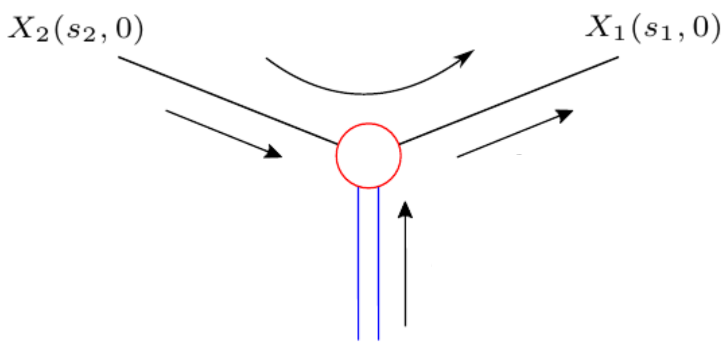
1. Algorithm



Decay matrix elements

$$\langle X_1 \bar{X}_2 | \Theta^{[J;s_1,s_2]}_{\mu_1\cdots\mu_J} | 0 \rangle$$

2. Crossing symmetry and identical-momentum limits



Scattering matrix elements

$$\langle X_1 | \Theta^{[J;s_1,s_2]}_{\mu_1\cdots\mu_J} | X_2 \rangle$$

Only integer $s_{1,2}$ and off-shell Ψ^*

3. Summary

<Matrix elements>

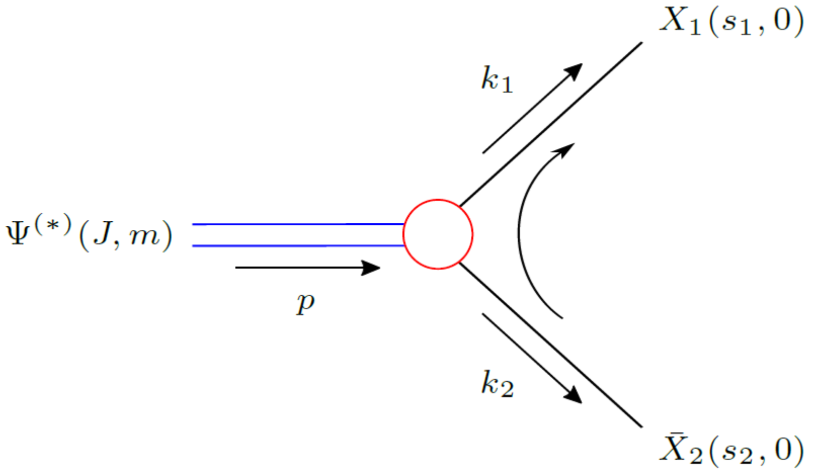
$$\Theta^{[J;s_1,s_2]}_{(\lambda_1,\lambda_2)\mu_1\cdots\mu_J}(k_1,k_2) = \varepsilon^{*\alpha_1\cdots\alpha_{s_1}}(k_1,\lambda_1)\varepsilon^{*\beta_1\cdots\beta_{s_2}}(k_2,\lambda_2)\boxed{\Gamma^{[J;s_1,s_2]}_{\alpha_1\cdots\alpha_{s_1},\beta_1\cdots\beta_{s_2};\mu_1\cdots\mu_J}(k_1,k_2)}$$

$$\equiv \varepsilon^{*\alpha}(k_1,\lambda_1)\varepsilon^{*\beta}(k_2,\lambda_2)\,\Gamma^{[J;s_1,s_2]}_{\alpha,\beta;\mu}(k_1,k_2)$$

Integer $s_{1,2}$

$$\varepsilon^{*\alpha_1\cdots\alpha_{s_1}}(k_1,\pm s_1) = \varepsilon^{*\alpha_1}(k_1,\pm 1)\cdots\varepsilon^{*\alpha_{s_1}}(k_1,\pm 1)$$

$$\varepsilon^{*\beta_1\cdots\beta_{s_2}}(k_2,\pm s_2) = \varepsilon^{*\beta_1}(k_2,\pm 1)\cdots\varepsilon^{*\beta_{s_2}}(k_2,\pm 1)$$



Repeated structures
for the spin-1 polarization vector

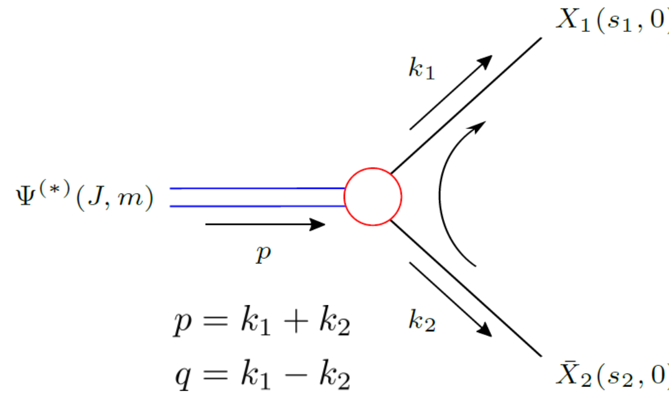
↓

Key guideline
in constructing the Form-factor and basic operators

R. E. Behrends and C. Fronsdal, Phys. Rev. (1957)

Algorithm

<Form-factor and basic operators>



$[J \to 0 + 0]$ $\quad$ $(s_{1,2} = 0)$

+1 +1

$$\Theta^{[J;s_1,s_2]}_{(\lambda_1,\lambda_2)\mu}(k_1, k_2) \equiv \varepsilon^{*\alpha}(k_1, \lambda_1)\varepsilon^{*\beta}(k_2, \lambda_2)\, \Gamma^{[J;s_1,s_2]}_{\alpha,\beta;\mu}(k_1, k_2)$$

Symmetric covariant tensor currents

$\to$ q_μ, p_μ, and $g_{\mu\nu}$

Compact square bracket operator form

$$p_{\mu_1} \cdots p_{\mu_n} \to [\,p\,]^n$$

$$q_{\mu_1} \cdots q_{\mu_n} \to [\,q\,]^n$$

$$g_{\mu_1\mu_2} \cdots g_{\mu_{2n-1}\mu_{2n}} \to [\,g\,]^n$$

Off-shell Ψ^*

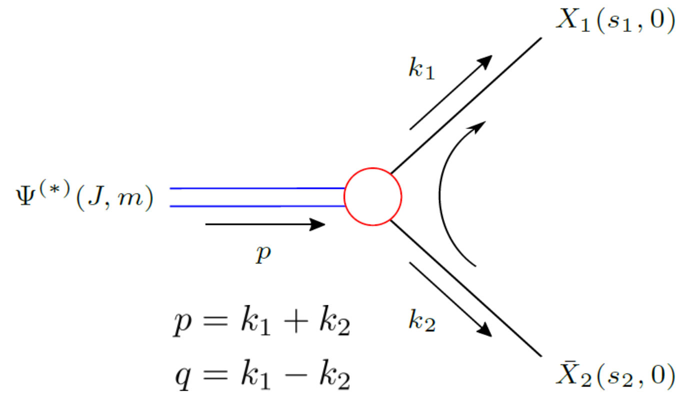
$$[\Theta^{[J;0,0]}_{(0,0)}(k_1, k_2)] = \sum_{n=0}^{J} \left[F^{[J;0,0]}_{(0,0)}(k_1, k_2)\right]^n$$

$$\sim \sum_{n,m,l} A^{[J;0,0]}_{(0,0)n,m,l}[\,q\,]^n[\,p\,]^m[\,g\,]^l \text{ with } J = n + m + 2l$$

Form-factor operators

$$\left[F^{[J;s_1,s_1]}_{(\lambda s_1,\mp\lambda s_2)}(k_1, k_2)\right]^n$$

Algorithm



<Form-factor and basic operators>

$[J \to s_1 + s_2]$ $\quad (s_1 \neq 0 \;\text{ or }\; s_2 \neq 0)$

$$\Theta^{[J;s_1,s_2]}_{(\lambda_1,\lambda_2)\mu}(k_1, k_2) \equiv \boxed{\varepsilon^{*\alpha}(k_1, \lambda_1)\varepsilon^{*\beta}(k_2, \lambda_2)}\,\Gamma^{[J;s_1,s_2]}_{\alpha,\beta;\mu}(k_1, k_2)$$

Boosts of spin-1 polarization vectors of massless particles

$$\varepsilon_{\alpha,\beta}(k_{1,2}) \to \varepsilon_{\alpha,\beta}(k'_{1,2}) + \eta\, k'_{1\alpha,2\beta}$$

Polarization-covariant operators

$$\langle k_1\rho\alpha\mu\rangle_+ = i(k_{1\rho}g_{\alpha\mu} - k_{1\alpha}g_{\rho\mu} - k_{1\mu}g_{\rho\alpha}) \quad \to \quad F^\dagger_{1\rho\mu} = \partial_\rho X^\dagger_{1\mu} - \partial_\mu X^\dagger_{1\rho}$$

$$\langle k_2\sigma\beta\nu\rangle_+ = i(k_{2\sigma}g_{\beta\nu} - k_{2\beta}g_{\sigma\nu} - k_{2\nu}g_{\sigma\beta}) \quad \to \quad F_{2\sigma\nu} = \partial_\sigma X_{2\nu} - \partial_\nu X_{2\sigma}$$

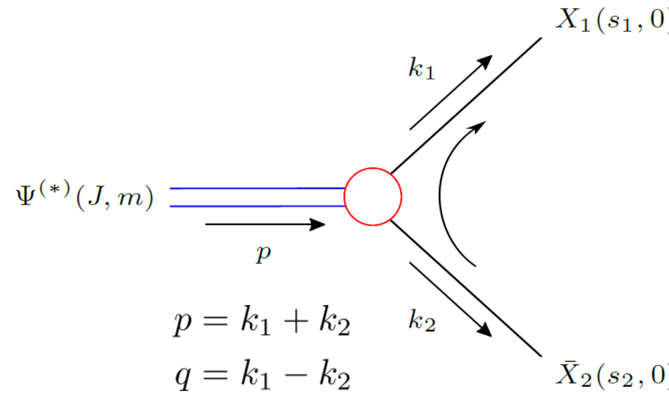
$$\langle k_1\rho\alpha\mu\rangle_- = \varepsilon_{\gamma\rho\alpha\mu}\, k_1^\gamma \quad \to \quad -i\tilde{F}^\dagger_{1\rho\mu} = -i\varepsilon_{\rho\mu\gamma\alpha}F^{\dagger\gamma\alpha}/2$$

$$\langle k_2\sigma\beta\nu\rangle_- = \varepsilon_{\delta\sigma\beta\nu}\, k_2^\delta \quad \to \quad -i\tilde{F}_{2\sigma\nu} = -i\varepsilon_{\rho\mu\delta\beta}F^{\delta\beta}/2$$

Contraction symbols

$$\langle k_1 b cd\rangle_i \langle k_2 f gh\rangle_j = \langle k_1 bcd\rangle_i \langle k_2 fgh\rangle_j\, g^{bf}$$

<Form-factor and basic operators>



$$p = k_1 + k_2$$
$$q = k_1 - k_2$$

$[0 \to 1+1]$

$$\Theta^{[0;1,1]}_{(\lambda_1,\lambda_2)}(k_1,k_2) \equiv \varepsilon^{*\alpha}(k_1,\lambda_1)\varepsilon^{*\beta}(k_2,\lambda_2)\,\Gamma^{[0;1,1]}_{\alpha,\beta}(k_1,k_2)$$

Even-parity scalar operator

$$\frac{1}{2}\langle k_1\rho\alpha\mu\rangle_{\pm}\langle k_2\sigma\beta\nu\rangle_{\pm} = i\langle k_1k_2\alpha\beta\rangle_{+} \quad \to \quad \begin{array}{l}\Theta^{[0;1,1]}(\pm1,\pm1) = (k_1\cdot k_2)\\ \Theta^{[0;1,1]}(\pm1,\mp1) = 0\end{array}$$

Odd-parity scalar operator

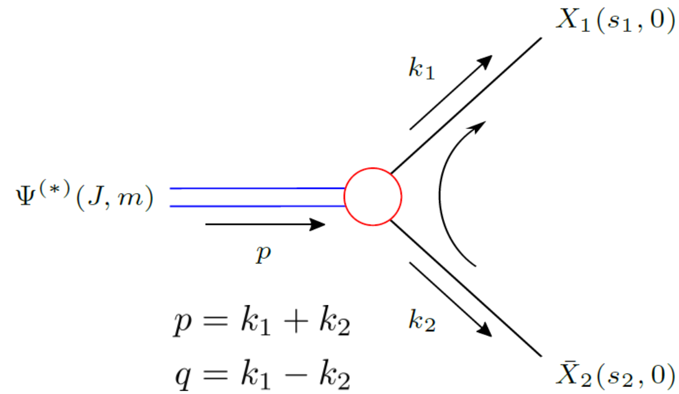
$$\frac{1}{2}\langle k_1\rho\alpha\mu\rangle_{\pm}\langle k_2\sigma\beta\nu\rangle_{\mp} = i\langle k_1k_2\alpha\beta\rangle_{-} \quad \to \quad \begin{array}{l}\Theta^{[0;1,1]}(\pm1,\pm1) = \pm(k_1\cdot k_2)\\ \Theta^{[0;1,1]}(\pm1,\mp1) = 0\end{array}$$

Basic bosonic scalar operators

$$S^{\pm}_{\alpha,\beta} = \frac{i}{2}\left[\langle k_1k_2\alpha\beta\rangle_{+} \pm \langle k_1k_2\alpha\beta\rangle_{-}\right] \quad \to \quad \Theta^{[0;1,1]}(\pm1,\pm1) = (k_1\cdot k_2)$$

Algorithm

<Form-factor and basic operators>



$$[1 \to 1+0] \oplus [1 \to 0+1]$$

$$\Theta^{[1;1,0]}_{(\lambda_1,0)\mu}(k_1,k_2) = \varepsilon^{*\alpha}(k_1,\lambda_1)\,\Gamma^{[1;1,0]}_{\alpha;\mu}(k_1,k_2)$$

$$\Theta^{[1;0,1]}_{(0,\lambda_2)\mu}(k_1,k_2) \equiv \varepsilon^{*\beta}(k_2,\lambda_2)\,\Gamma^{[1;0,1]}_{\beta;\mu}(k_1,k_2)$$

Even-parity vector operator

$$i\langle k_1 k_2 \alpha\mu\rangle_+ \quad \to \quad \Theta^{[1;1,0]}_{\mu}\boxed{(\pm 1,0)} = -(k_1\cdot k_2)\,\varepsilon^*_{\perp\mu}(k_1,\pm;k_2)$$

$$i\langle k_2 k_1 \beta\nu\rangle_+ \quad \to \quad \Theta^{[1;0,1]}_{\mu}\boxed{(0,\pm 1)} = -(k_1\cdot k_2)\,\varepsilon^*_{\perp\nu}(k_2,\pm;k_1)$$

Odd-parity vector operator

$$i\langle k_1 k_2 \alpha\mu\rangle_- \quad \to \quad \Theta^{[1;1,0]}_{\mu}\boxed{(\pm 1,0)} = \mp\,(k_1\cdot k_2)\,\varepsilon^*_{\perp\mu}(k_1,\pm;k_2)$$

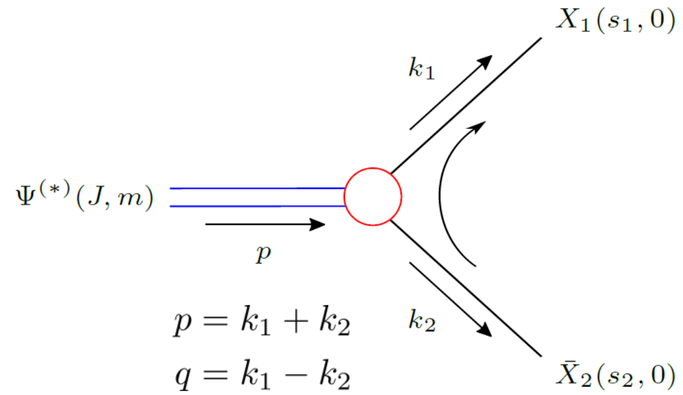
$$i\langle k_2 k_1 \beta\nu\rangle_- \quad \to \quad \Theta^{[1;0,1]}_{\mu}\boxed{(0,\pm 1)} = \mp\,(k_1\cdot k_2)\,\varepsilon^*_{\perp\nu}(k_2,\pm;k_1)$$

Covariant polarization vectors

$$\varepsilon_\perp(k_{1,2},\pm;k_{2,1}) = \varepsilon(k_{1,2},\pm) - \left(\frac{k_{2,1}\cdot\varepsilon(k_{1,2},\pm)}{k_1\cdot k_2}\right)k_{1,2} \equiv \varepsilon_{1,2\perp}(\pm)$$

Algorithm

<Form-factor and basic operators>



$$[1 \to 1 + 0] \oplus [1 \to 0 + 1]$$

$$\Theta^{[1;1,0]}_{(\lambda_1,0)\mu}(k_1, k_2) = \varepsilon^{*\alpha}(k_1, \lambda_1)\, \Gamma^{[1;1,0]}_{\alpha;\mu}(k_1, k_2)$$

$$\Theta^{[1;0,1]}_{(0,\lambda_2)\mu}(k_1, k_2) \equiv \varepsilon^{*\beta}(k_2, \lambda_2)\, \Gamma^{[1;0,1]}_{\beta;\mu}(k_1, k_2)$$

Even-parity vector operator

$$i\langle k_1 k_2 \alpha\mu\rangle_+ \quad \to \quad \Theta^{[1;1,0]}_{\mu}\boxed{(\pm 1, 0)} = -(k_1 \cdot k_2)\, \varepsilon^*_{\perp\mu}(k_1, \pm; k_2)$$

$$i\langle k_2 k_1 \beta\nu\rangle_+ \quad \to \quad \Theta^{[1;0,1]}_{\mu}\boxed{(0, \pm 1)} = -(k_1 \cdot k_2)\, \varepsilon^*_{\perp\nu}(k_2, \pm; k_1)$$

Odd-parity vector operator

$$i\langle k_1 k_2 \alpha\mu\rangle_- \quad \to \quad \Theta^{[1;1,0]}_{\mu}\boxed{(\pm 1, 0)} = \mp(k_1 \cdot k_2)\, \varepsilon^*_{\perp\mu}(k_1, \pm; k_2)$$

$$i\langle k_2 k_1 \beta\nu\rangle_- \quad \to \quad \Theta^{[1;0,1]}_{\mu}\boxed{(0, \pm 1)} = \mp(k_1 \cdot k_2)\, \varepsilon^*_{\perp\nu}(k_2, \pm; k_1)$$

Basic bosonic vector operators

$$V^{\pm}_{1\alpha;\mu} = \frac{i}{2}\left[\langle k_1 k_2 \alpha\mu\rangle_+ \pm \langle k_1 k_2 \alpha\mu\rangle_-\right] \quad \to \quad \Theta^{[1;1,0]}_{\mu}\boxed{(\pm 1, 0)} = -(k_1 \cdot k_2)\, \varepsilon^*_{\perp\mu}(k_1, \pm; k_2)$$

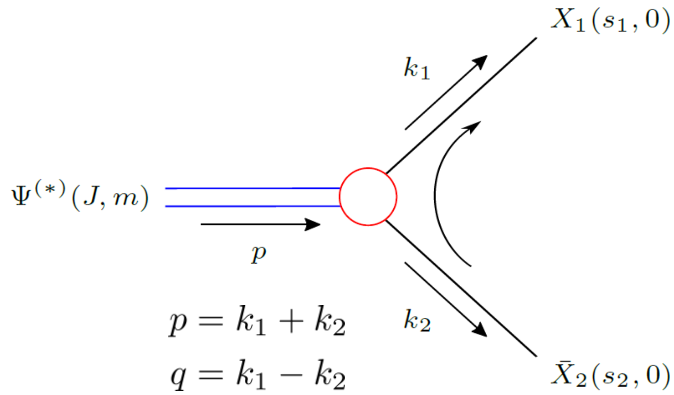
$$V^{\pm}_{2\beta;\nu} = \frac{i}{2}\left[\langle k_2 k_1 \beta\nu\rangle_+ \pm \langle k_2 k_1 \beta\nu\rangle_-\right] \quad \to \quad \Theta^{[1;0,1]}_{\nu}\boxed{(0, \pm 1)} = -(k_1 \cdot k_2)\, \varepsilon^*_{\perp\nu}(k_2, \pm; k_1)$$

Algorithm

<Form-factor and basic operators>

[2 → 1 + 1]

$$\Theta^{[2;1,1]}_{(\lambda_1,\lambda_2)\mu_1\mu_2}(k_1,k_2) = \varepsilon^{*\alpha}(k_1,\lambda_1)\varepsilon^{*\beta}(k_2,\lambda_2) \times \Gamma^{[2;1,1]}_{\alpha,\beta;\mu_1\mu_2}(k_1,k_2)$$



Even-parity tensor operator

$$\langle k_1 \rho\alpha\mu\rangle_+ \langle k_2 \sigma\beta\nu\rangle_+ - \langle k_1 \rho\alpha\mu\rangle_- \langle k_2 \sigma\beta\nu\rangle_- \;\to\; g^{\rho\sigma}F^{\dagger}_{1\mu\rho}F_{2\nu\sigma} - \frac{1}{4}g_{\mu\nu}F_1^{\dagger\rho\sigma}F_{2\rho\sigma}$$

Energy-momentum tensor of a spin-1 particle

$$\to \begin{array}{l} \Theta^{[2;1,1]}_{\mu\nu}(\pm1,\mp1) = -(k_1\cdot k_2)\left[\varepsilon^*_{1\perp\mu}(\pm)\varepsilon_{2\perp\nu}(\mp) + \mu\leftrightarrow\nu\right] \\ \Theta^{[2;1,1]}_{\mu\nu}(\pm1,\pm1) = 0 \end{array}$$

Odd-parity tensor operator

$$\langle k_1 \rho\alpha\mu\rangle_+ \langle k_2 \sigma\beta\nu\rangle_- - \langle k_1 \rho\alpha\mu\rangle_- \langle k_2 \sigma\beta\nu\rangle_+$$

$$\to \begin{array}{l} \Theta^{[2;1,1]}_{\mu\nu}(\pm1,\mp1) = \mp(k_1\cdot k_2)\left[\varepsilon^*_{1\perp\mu}(\pm)\varepsilon_{2\perp\nu}(\mp) + \mu\leftrightarrow\nu\right] \\ \Theta^{[2;1,1]}_{\mu\nu}(\pm1,\pm1) = 0 \end{array}$$

Basic bosonic tensor operators

$$T^{\pm}_{\alpha,\beta;\mu\nu} = \frac{1}{2}\sum_{\tau=\pm}\tau\left[\langle k_1 \rho\alpha\mu\rangle_\tau \langle k_2 \sigma\beta\nu\rangle_\tau \pm \langle k_1 \rho\alpha\mu\rangle_\tau \langle k_2 \sigma\beta\nu\rangle_{-\tau}\right]$$

$$\to \Theta^{[2;1,1]}_{\mu\nu}(\pm1,\mp1) = -(k_1\cdot k_2)\left[\varepsilon^*_{1\perp\mu}(\pm)\varepsilon^*_{2\perp\nu}(\mp) + \mu\leftrightarrow\nu\right]$$

Algorithm

Compact square bracket operator form

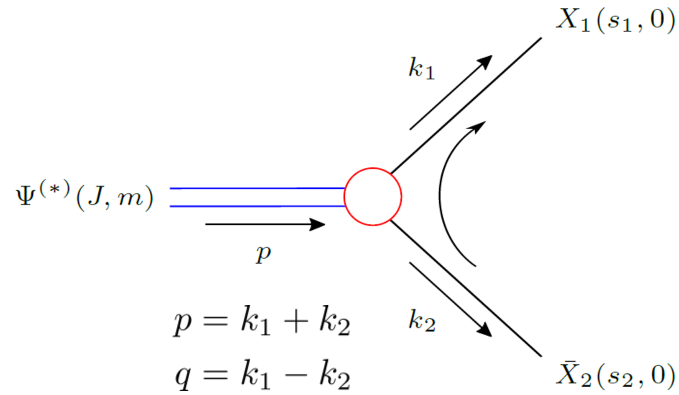
$$S^{\pm}_{\alpha_1,\beta_1}\cdots S^{\pm}_{\alpha_n\,\beta_n} \quad\rightarrow\quad [S^{\pm}]^n$$

$$V^{\pm}_{1\alpha_1;\mu_1}\cdots V^{\pm}_{1\alpha_n;\mu_n} \quad\rightarrow\quad [V_1^{\pm}]^n$$

$$V^{\pm}_{2\beta_1;\mu_1}\cdots V^{\pm}_{2\beta_n;\mu_n} \quad\rightarrow\quad [V_2^{\pm}]^n$$

$$T^{\pm}_{\alpha_1,\beta_1;\mu_1\mu_2}\cdots T^{\pm}_{\alpha_n,\beta_n;\mu_{2n-1}\mu_{2n}} \quad\rightarrow\quad [T^{\pm}]^n$$

$$\Gamma^{[J;s_1,s_2]}_{\alpha_1\cdots\alpha_{n_1},\beta_1\cdots\beta_{n_2};\mu_1\cdots\mu_J} \quad\rightarrow\quad [\Gamma^{[J;s_1,s_2]}]$$



$$S^{\pm}_{\alpha,\beta} \quad\rightarrow\quad (\pm,\pm)$$

$$V^{\pm}_{1\alpha;\mu} \quad\rightarrow\quad (\pm,0)$$

$$V^{\pm}_{2\beta;\mu} \quad\rightarrow\quad (\pm,0)$$

$$T^{\pm}_{\alpha,\beta;\mu\nu} \quad\rightarrow\quad (\pm,\mp)$$

<Constructing covariant three-point vertices>

Helicity configurations: $(\pm s_1, \pm s_2)$ and $(\pm s_1, \mp s_2)$

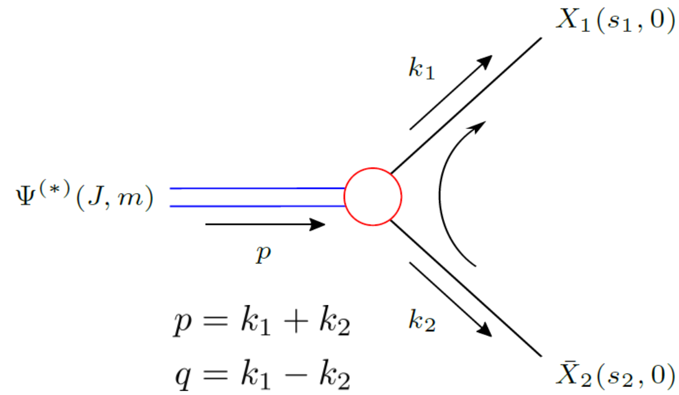
Ex) $\Psi^{(*)} \rightarrow X_1(k_1, \boxed{+s_1}) + \bar{X}_2(k_2, \boxed{+s_2})$ (Integer $s_{1,2} \neq 0$ and $s_1 \geq s_2$)

$$\rightarrow \sum_{n=0}^{J_-} [F^{[J;s_1,s_2]}_{(+s_1,+s_2)}]^n [S^+]^{s_2} [V_1^+]^{s_1-s_2}$$

with $J_- = J - (s_1 - s_2)$

1. $(+s_2,\ +s_2)$
2. $(+(s_1 - s_2) + s_2,\ +s_2)$
3. Complement μ indices

Algorithm



<Constructing covariant three-point vertices>

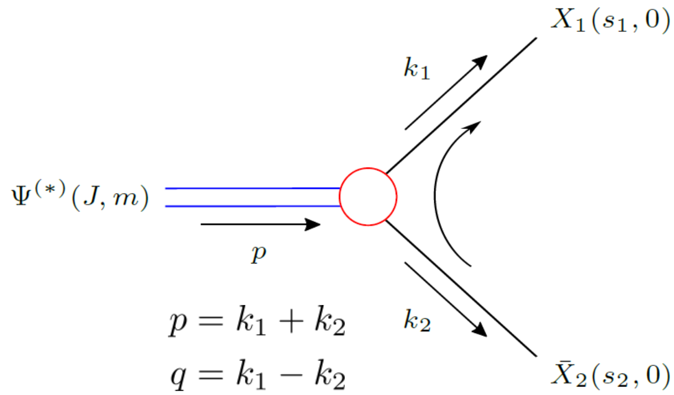
$$[\Gamma^{[J;s_1,s_2]}] = \sum_{\lambda=\pm}\Bigg\{\Theta(J_-)\sum_{n=0}^{J_-}[F^{[J;s_1,s_2]}_{(\lambda s_1,\lambda s_2)}]^n\,[S^{\lambda}]^{s_{\rm min}}\Big([V_1^{\lambda}]^{s_1-s_{\rm min}}+[V_2^{\lambda}]^{s_2-s_{\rm min}}\Big)$$
$$+\gamma_{s_{\rm min}}\,\Theta(J_+)\sum_{n=0}^{J_+}[F^{[J;s_1,s_2]}_{(\lambda s_1,-\lambda s_2)}]^n\,[T^{\lambda}]^{s_{\rm min}}\Big([V_1^{\lambda}]^{s_1-s_{\rm min}}+[V_2^{-\lambda}]^{s_2-s_{\rm min}}\Big)\Bigg\}$$

$(J_\mp = J - |s_1 \mp s_2|)$, $(\gamma_{s_{\rm min}} = 1 - \delta_{s_{\rm min},0}$ with $s_{\rm min} = \min[s_1,s_2])$

$$S^{\pm}_{\alpha,\beta} \to (\pm,\pm)$$
$$V^{\pm}_{1\alpha;\mu} \to (\pm,0)$$
$$V^{\pm}_{2\beta;\mu} \to (\pm,0)$$
$$T^{\pm}_{\alpha,\beta;\mu\nu} \to (\pm,\mp)$$

First key result

Algorithm



<Constructing covariant three-point vertices>

$$[\Gamma^{[J;s_1,s_2]}] = \sum_{\lambda=\pm} \left\{ \Theta(J_-) \sum_{n=0}^{J_-} [F^{[J;s_1,s_2]}_{(\lambda s_1, \lambda s_2)}]^n [S^\lambda]^{s_{\min}} \left([V_1^\lambda]^{s_1 - s_{\min}} + [V_2^\lambda]^{s_2 - s_{\min}} \right) \right.$$

$$\left. + \gamma_{s_{\min}} \Theta(J_+) \sum_{n=0}^{J_+} [F^{[J;s_1,s_2]}_{(\lambda s_1, -\lambda s_2)}]^n [T^\lambda]^{s_{\min}} \left([V_1^\lambda]^{s_1 - s_{\min}} + [V_2^{-\lambda}]^{s_2 - s_{\min}} \right) \right\}$$

$$(J_\mp = J - |s_1 \mp s_2|), \quad (\gamma_{s_{\min}} = 1 - \delta_{s_{\min},0} \ \text{ with } \ s_{\min} = \min[s_1, s_2])$$

$S^{\pm}_{\alpha,\beta} \rightarrow (\pm, \pm)$

$V^{\pm}_{1\alpha;\mu} \rightarrow (\pm, 0)$

$V^{\pm}_{2\beta;\mu} \rightarrow (\pm, 0)$

$T^{\pm}_{\alpha,\beta;\mu\nu} \rightarrow (\pm, \mp)$

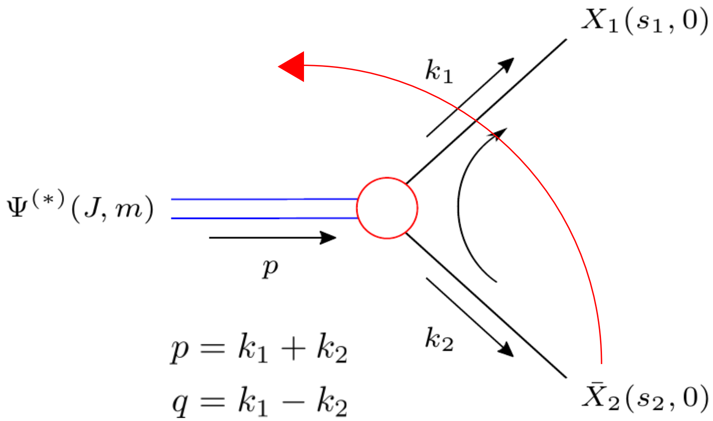
First key result

Constraints

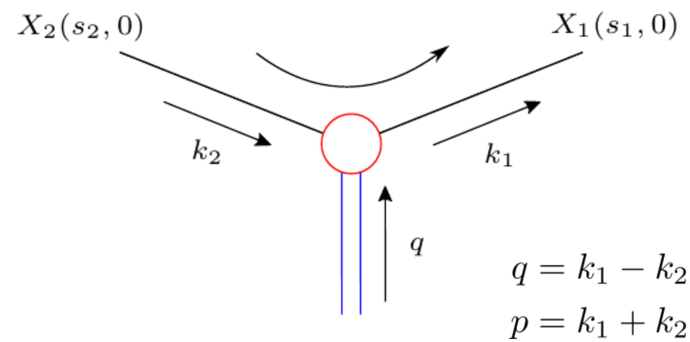
✓ $\Theta_{\lambda_1,\lambda_2} = 0 \quad \text{for} \quad J < |s_1 - s_2|$

✓ $\Theta_{\pm s_1,\mp s_2} = 0 \quad \text{for} \quad J < |s_1 + s_2|$

Crossing symmetry and identical-momentum limits



$\longrightarrow$



Decay matrix elements

$$\langle k_1,\lambda_1;k_2,\lambda_2|\left[\Theta^{[J;s_1,s_2]}\right]|\,0\,\rangle = \left[\Theta^{[J;s_1,s_2]}_{(\lambda_1,\lambda_2)}(k_1,k_2)\right] \longrightarrow$$

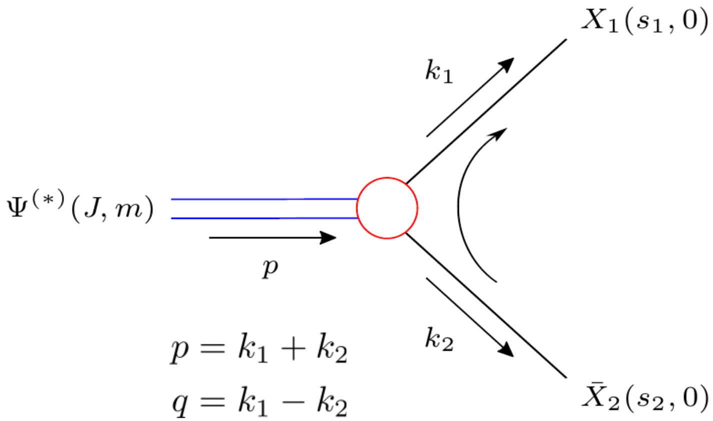
Scattering matrix elements

$$\langle k_1,\lambda_1|\left[\Theta^{[J;s_1,s_2]}\right]|k_2,\lambda_2\rangle = \left[\bar{\Theta}^{[J;s_1,s_2]}_{(\lambda_1,\lambda_2)}(k_1,k_2)\right]$$

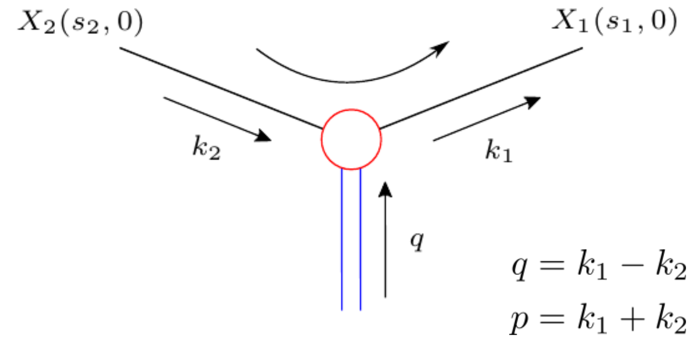
In the three-point vertices	$+k_2 \to -k_2$
In the X_2 wave tensors	$\varepsilon^*(k_2,\pm) \to -\varepsilon(k_2,\mp)$

$$\left[\Theta^{[J;s_1,s_2]}_{(\lambda_1,-\lambda_2)}\right] \longleftrightarrow \left[\bar{\Theta}^{[J;s_1,s_2]}_{(\lambda_1,+\lambda_2)}\right]$$

Crossing symmetry and identical-momentum limits



$\longrightarrow$



Decay matrix elements

Scattering matrix elements

$$\langle k_1,\lambda_1;k_2,\lambda_2|\left[\Theta^{[J;s_1,s_2]}\right]|\,0\,\rangle = \left[\Theta^{[J;s_1,s_2]}_{(\lambda_1,\lambda_2)}(k_1,k_2)\right] \longrightarrow \langle k_1,\lambda_1|\left[\Theta^{[J;s_1,s_2]}\right]|k_2,\lambda_2\rangle = \left[\bar{\Theta}^{[J;s_1,s_2]}_{(\lambda_1,\lambda_2)}(k_1,k_2)\right]$$

$$\left[\Theta^{[J;s_1,s_2]}_{(\lambda_1,-\lambda_2)}\right] \longleftrightarrow \left[\bar{\Theta}^{[J;s_1,s_2]}_{(\lambda_1,+\lambda_2)}\right]$$

Constraints		Constraints
✓ $\Theta_{\lambda_1,\lambda_2} = 0$ for $J < \lvert s_1 - s_2\rvert$		✓ $\bar{\Theta}_{\lambda_1,\lambda_2} = 0$ for $J < \lvert s_1 - s_2\rvert$
✓ $\Theta_{\pm s_1,\mp s_2} = 0$ for $J < \lvert s_1 + s_2\rvert$	$\longrightarrow$	✓ $\bar{\Theta}_{\pm s_1,\pm s_2} = 0$ for $J < \lvert s_1 + s_2\rvert$

Crossing symmetry and identical-momentum limits

$p = k_1 + k_2$

$q = k_1 - k_2$

Constraints

✓ $\bar{\Theta}_{\lambda_1,\lambda_2} = 0 \quad \text{for} \quad J < |s_1 - s_2|$

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$+k_2 \to -k_2$ $\qquad$ $\varepsilon^*(k_2, \pm) \to -\varepsilon(k_2, \mp)$

$$\bar{S}^{\pm}_{\alpha,\beta} = -S^{\pm}_{\alpha,\beta} \quad \to \quad \bar{\Theta}^{[0;1,1]}(\pm 1, \mp 1) = +(k_1 \cdot k_2)$$

$$\bar{V}^{\pm}_{1\alpha;\mu} = -V^{\pm}_{1\alpha;\mu} \quad \to \quad \bar{\Theta}^{[1;1,0]}_{\mu}(\pm 1, 0) = +(k_1 \cdot k_2)\, \varepsilon^*_{1\perp\mu}(\pm)$$

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$$\bar{T}^{\pm}_{\alpha,\beta;\mu\nu} = -T^{\pm}_{\alpha,\beta;\mu\nu} \quad \to \quad \bar{\Theta}^{[2;1,1]}_{\mu\nu}(\pm 1, \pm 1) = -(k_1 \cdot k_2)\big[\varepsilon^*_{1\perp\mu}(\pm)\varepsilon_{2\perp\nu}(\pm) + \mu \leftrightarrow \nu\big]$$

$$F_{(\lambda_1,\lambda_2)} \sim (q_\mu), (p_\mu), (g_{\mu\nu}) \quad \to \quad \bar{F}_{(\lambda_1,-\lambda_2)} \sim (p_\mu), (q_\mu), (g_{\mu\nu})$$

Crossing symmetry and identical-momentum limits

$$k_1 = k^0(1, +\sin\theta_1, 0, \cos\theta_1)$$
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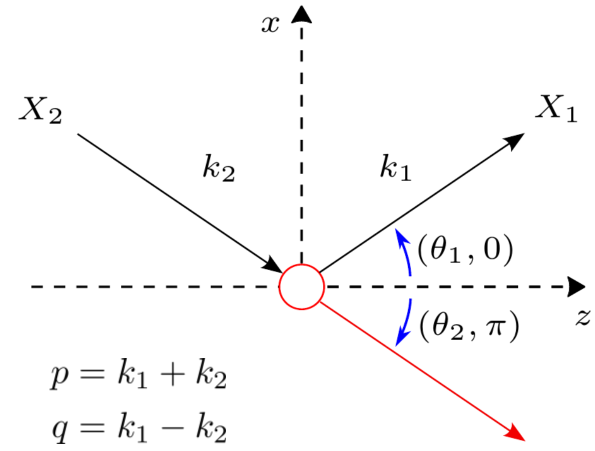

Constraints

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Crossing symmetry and identical-momentum limits

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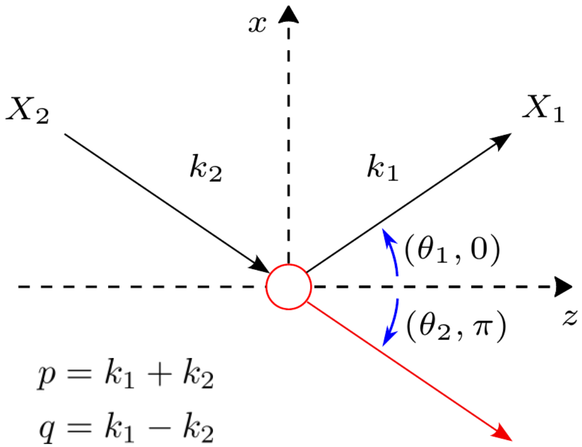

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✓ $\bar{\Theta}_{\lambda_1,\lambda_2} = 0$ for $J < |s_1 - s_2|$

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$$\bar{T}^{\pm}_{\alpha,\beta;\mu\nu} = -T^{\pm}_{\alpha,\beta;\mu\nu} \quad \to \quad \bar{\Theta}^{[2;1,1]}_{\mu\nu}(\pm1,\pm1) = -(k_1\cdot k_2)\big[\varepsilon^*_{1\perp\mu}(\pm)\varepsilon_{2\perp\nu}(\pm) + \mu\leftrightarrow\nu\big]$$
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Crossing symmetry and identical-momentum limits

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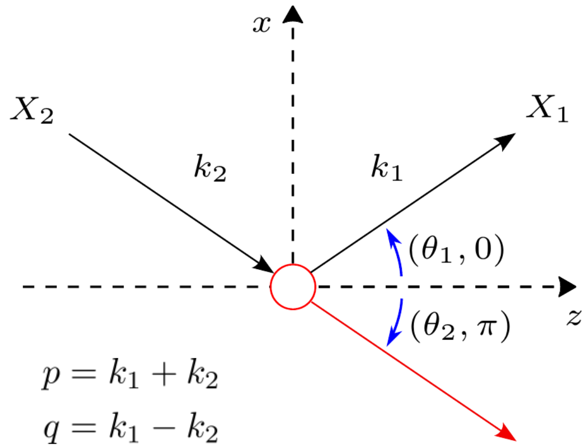

$$\lim_{\theta_{1,2}\to 0} \langle k_1, \lambda_1 | \left[\Theta^{[J;s_1,s_2]}\right] | k_2, \lambda_2 \rangle = \langle k, \lambda_1 | \left[\Theta^{[J;s_1,s_2]}\right] | k, \lambda_2 \rangle (-1)^{-\lambda_2}$$

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$$\bar{T}^{\pm}_{\alpha,\beta;\mu\nu} = -T^{\pm}_{\alpha,\beta;\mu\nu} \quad \to \quad \bar{\Theta}^{[2;1,1]}_{\mu\nu}(\pm 1, \pm 1) = -(k_1\cdot k_2)\left[\varepsilon^*_{1\perp\mu}(\pm)\varepsilon_{2\perp\nu}(\pm) + \mu \leftrightarrow \nu\right]$$
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Crossing symmetry and identical-momentum limits

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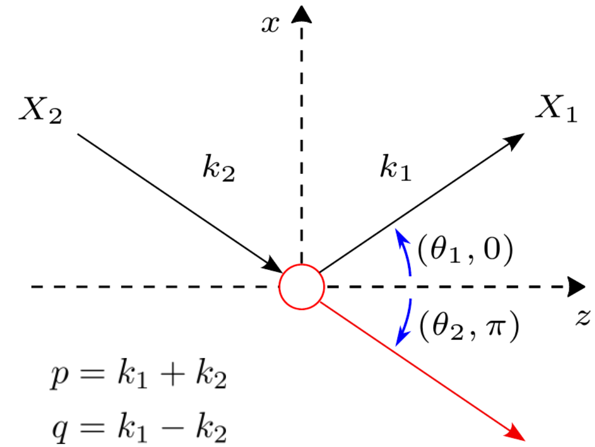

$$\lim_{\theta_{1,2}\to 0} \langle k_1, \lambda_1 | \left[\Theta^{[J;s_1,s_2]}\right] | k_2, \lambda_2 \rangle = \langle k, \lambda_1 | \left[\Theta^{[J;s_1,s_2]}\right] | k, \lambda_2 \rangle (-1)^{-\lambda_2}$$

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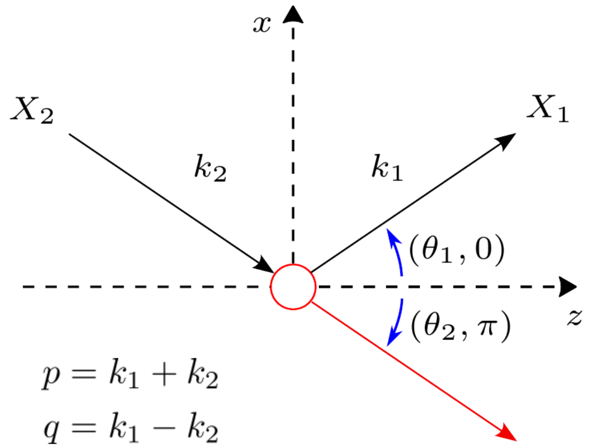
Involve only $(k_1 \cdot k_2)$ and $\varepsilon^{(*)}_{\perp}$

Crossing symmetry and identical-momentum limits

$$k_1 = k^0(1, +\sin\theta_1, 0, \cos\theta_1) \qquad k_2 = k^0(1, -\sin\theta_2, 0, \cos\theta_2) \xrightarrow{\theta_{1,2}\to 0} k_{1,2} = k = k^0(1,0,0,1)$$



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$$p = k_1 + k_2 \qquad q = k_1 - k_2$$

$$\lim_{\theta_{1,2}\to 0} \sqrt{k_1\cdot k_2}\, \varepsilon^*_{1\perp}(\pm) = \pm k$$

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Constraints

✓ $\bar\Theta_{\lambda_1,\lambda_2} = 0 \quad \text{for} \quad J < |s_1 - s_2|$

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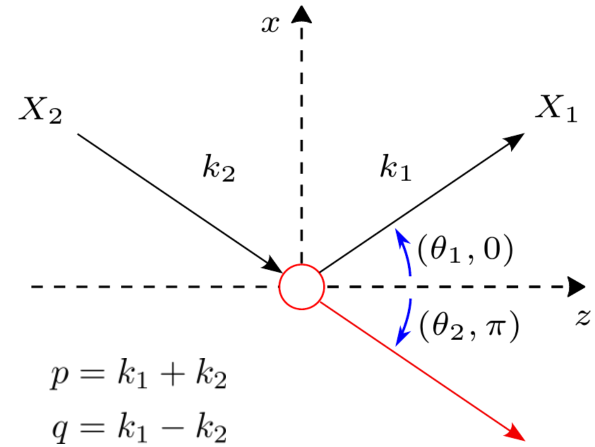

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Constraints

✓ $\bar\Theta_{\lambda_1,\lambda_2} = 0 \quad \text{for} \quad J < |s_1 - s_2|$

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$$F_{(\lambda_1,\lambda_2)} \sim (q_\mu), (p_\mu), (g_{\mu\nu}) \quad \to \quad \bar F_{(\lambda_1,-\lambda_2)} \sim (p_\mu), \cancel{(q_\mu)}, (g_{\mu\nu}) \quad \longrightarrow \quad 2(k_\mu), (g_{\mu\nu})$$

Crossing symmetry and identical-momentum limits

Form factors including $(k_1 \cdot k_2)$ in denominators

$$\bar{S}^{\pm}_{\alpha,\beta}/(k_1\cdot k_2) \quad\rightarrow\quad \bar{\Theta}^{[0;1,1]}_{(\pm1,\mp1)}(k,k) = +1$$

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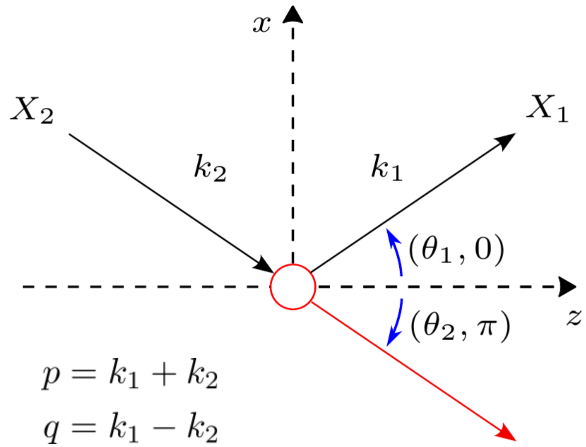

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Crossing symmetry and identical-momentum limits

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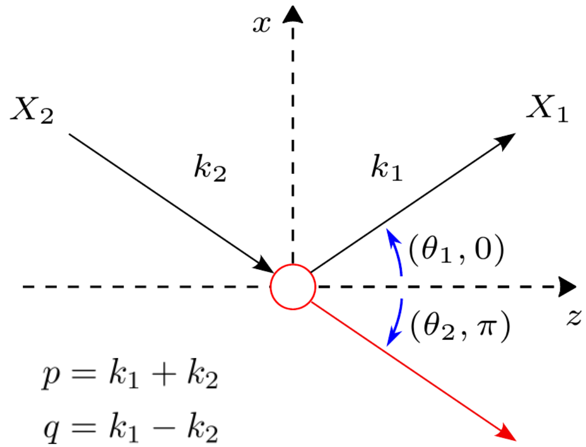
$$\bar{V}^{\mp}_{2\beta;\nu}/\sqrt{k_1 \cdot k_2} \quad \rightarrow \quad \bar{\Theta}^{[1;0,1]}_{(0,\mp 1)\nu}(k,k) = \pm k_\nu$$

$\rightarrow$ *Violate the covariance of tensor currents!*



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$$\bar{V}^{\mp}_{2\beta;\nu} = -V^{\pm}_{2\beta;\nu} \quad \rightarrow \quad \bar{\Theta}^{[1;0,1]}_{\nu}(0,\mp 1) = \cancel{-(k_1 \cdot k_2)}\,\varepsilon_{2\perp\nu}(\mp)$$

$$\bar{T}^{\pm}_{\alpha,\beta;\mu\nu} = -T^{\pm}_{\alpha,\beta;\mu\nu} \quad \rightarrow \quad \bar{\Theta}^{[2;1,1]}_{\mu\nu}(\pm 1,\pm 1) = -(k_1 \cdot k_2)\left[\varepsilon^*_{1\perp\mu}(\pm)\varepsilon_{2\perp\nu}(\pm) + \mu \leftrightarrow \nu\right] \quad \rightarrow \quad 2k_\mu k_\nu$$

$$F_{(\lambda_1,\lambda_2)} \sim (q_\mu), (p_\mu), (g_{\mu\nu}) \quad \rightarrow \quad \bar{F}_{(\lambda_1,-\lambda_2)} \sim (p_\mu), \cancel{(q_\mu)}, (g_{\mu\nu}) \quad \rightarrow \quad 2(k_\mu), (g_{\mu\nu})$$

Crossing symmetry and identical-momentum limits

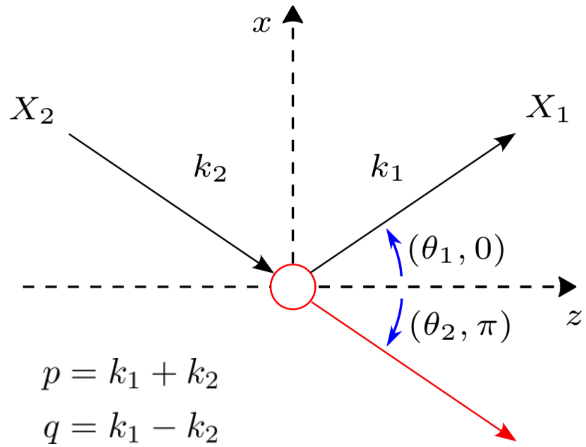
Form factors including $(k_1 \cdot k_2)$ in denominators

$$\bar{S}^{\pm}_{\alpha,\beta}/(k_1 \cdot k_2) \quad \rightarrow \quad \boxed{\bar{\Theta}^{[0;1,1]}_{(\pm 1,\mp 1)}(k,k)} = +1$$

$$\bar{V}^{\pm}_{1\alpha;\mu}/\sqrt{k_1 \cdot k_2} \quad \rightarrow \quad \bar{\Theta}^{[1;1,0]}_{(\pm 1,0)\mu}(k,k) = \pm k_\mu$$

$$\bar{V}^{\mp}_{2\beta;\nu}/\sqrt{k_1 \cdot k_2} \quad \rightarrow \quad \bar{\Theta}^{[1;0,1]}_{(0,\mp 1)\nu}(k,k) = \pm k_\nu$$

→ *Violate the covariance of tensor currents!*



Ex) $\bar{S}^{\pm}_{\alpha,\beta}/(k_1 \cdot k_2) \rightarrow$ A scalar current Θ

$$\langle k, \pm | R_z^\dagger(\varphi)\, \Theta\, R_z(\varphi) | k, \mp \rangle \quad \rightarrow \quad \boxed{\langle k, \pm |\, \Theta\, | k, \mp \rangle} = e^{\pm 2i\varphi} \langle k, \pm |\, \Theta\, | k, \mp \rangle = 0$$

→ *Contradiction!*

Constraints

✓ $\bar{\Theta}_{\lambda_1,\lambda_2} = 0 \quad$ for $\quad J < |s_1 - s_2|$

✓ $\bar{\Theta}_{\pm s_1,\pm s_2} = 0 \quad$ for $\quad J < |s_1 + s_2|$

$$\bar{S}^{\pm}_{\alpha,\beta} = -S^{\pm}_{\alpha,\beta} \quad \rightarrow \quad \bar{\Theta}^{[0;1,1]}(\pm 1, \mp 1) = \cancel{+(k_1 \cdot k_2)}$$

$$\bar{V}^{\pm}_{1\alpha;\mu} = -V^{\pm}_{1\alpha;\mu} \quad \rightarrow \quad \bar{\Theta}^{[1;1,0]}_{\mu}(\pm 1, 0) = \cancel{+(k_1 \cdot k_2)}\, \varepsilon^{*}_{1\perp\mu}(\pm)$$

$$\bar{V}^{\mp}_{2\beta;\nu} = -V^{\pm}_{2\beta;\nu} \quad \rightarrow \quad \bar{\Theta}^{[1;0,1]}_{\nu}(0, \mp 1) = \cancel{-(k_1 \cdot k_2)}\, \varepsilon_{2\perp\nu}(\mp)$$

$$\boxed{\bar{T}^{\pm}_{\alpha,\beta;\mu\nu} = -T^{\pm}_{\alpha,\beta;\mu\nu} \quad \rightarrow \quad \bar{\Theta}^{[2;1,1]}_{\mu\nu}(\pm 1, \pm 1) = -(k_1 \cdot k_2)\left[\varepsilon^{*}_{1\perp\mu}(\pm)\varepsilon_{2\perp\nu}(\pm) + \mu \leftrightarrow \nu\right]} \quad \rightarrow \quad 2k_\mu k_\nu$$

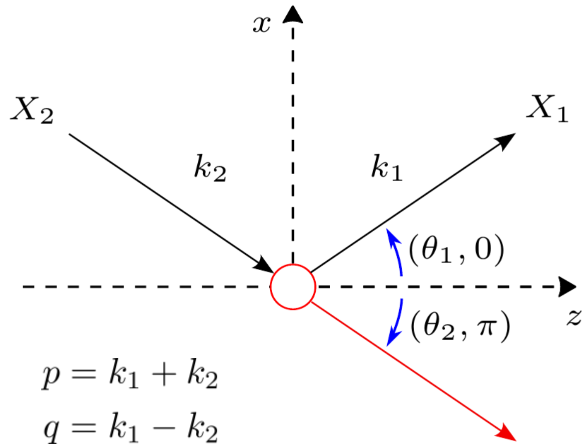
$$\boxed{F_{(\lambda_1,\lambda_2)} \sim (q_\mu), (p_\mu), (g_{\mu\nu}) \quad \rightarrow \quad \bar{F}_{(\lambda_1,-\lambda_2)} \sim (p_\mu), \cancel{(q_\mu)}, (g_{\mu\nu})} \quad \rightarrow \quad 2(k_\mu), (g_{\mu\nu})$$

Crossing symmetry and identical-momentum limits

Covariant vertices

$$[\bar{\Gamma}^{[J;s_1,s_2]}] = \sum_{\lambda=\pm} \Big\{ \Theta(J_-) \sum_{n=0}^{J_-} [\bar{F}^{[J;s_1,s_2]}_{(\lambda s_1,-\lambda s_2)}]^n \boxed{[\bar{S}^\lambda]^{s_{\min}} \Big([\bar{V}_1^\lambda]^{s_1-s_{\min}} + [\bar{V}_2^\lambda]^{s_2-s_{\min}} \Big)}$$

$$+\gamma_{s_{\min}} \Theta(J_+) \sum_{n=0}^{J_+} [\bar{F}^{[J;s_1,s_2]}_{(\lambda s_1,\lambda s_2)}]^n [\bar{T}^\lambda]^{s_{\min}} \boxed{\Big([\bar{V}_1^\lambda]^{s_1-s_{\min}} + [\bar{V}_2^{-\lambda}]^{s_2-s_{\min}} \Big)} \Big\}$$



$p = k_1 + k_2$

$q = k_1 - k_2$

<Covariance conditions on form factors>

$$\lim_{k_{1,2}\to k} \bar{A}^{[J;s_1,s_2]}_{(\lambda s_1,-\lambda s_2)n,m,l} \times (k_1 \cdot k_2)^{\frac{s_1+s_2}{2}} = 0$$

$$\lim_{k_{1,2}\to k} \bar{A}^{[J;s_1,s_2]}_{(\lambda s_1,+\lambda s_2)n,m,l} \times (k_1 \cdot k_2)^{\frac{|s_1-s_2|}{2}} = 0$$

Second key result

Constraints

✓ $\bar{\Theta}_{\lambda_1,\lambda_2} = 0 \quad \text{for} \quad J < |s_1 - s_2|$

✓ $\bar{\Theta}_{\pm s_1,\pm s_2} = 0 \quad \text{for} \quad J < |s_1 + s_2|$

Summary

- ✓ An efficient algorithm in the covariant formulation. $\boxed{\Theta^{[J;s_1,s_2]}_{\mu_1\cdots\mu_J}}$
- ✓ Covariance constraints on form factors.
- ✓ Covariant three-point vertices for all off-shell cases.
- ✓ Covariant four-point vertices.
- ✓ Program for generating covariant vertices and Lagrangian operators.
- ✓ Algorithm $\leftrightarrow$ Spinor helicity formalism.

Thank you

Back up

Three-point vertices for identical particles, X_1 and $\bar{X}_2$ ($X_1 = \bar{X}_2$)

$$\Big[\langle X_1; k_1, \lambda_1 | \langle \bar{X}_2; k_2, \lambda_2 |\Big] \Theta_{\mu_1 \cdots \mu_J} | 0 \rangle = \Big[\langle \bar{X}_2; k_1, \lambda_1 | \langle X_1; k_2, \lambda_2 |\Big] \Theta_{\mu_1 \cdots \mu_J} | 0 \rangle$$

$$\downarrow \quad k_1 \leftrightarrow k_2$$

$$\varepsilon^{*\alpha}(k_1, \lambda_1)\varepsilon^{*\beta}(k_2, \lambda_2)\ \Gamma_{\alpha,\beta;\mu}(k_1, k_2) = \varepsilon^{*\alpha}(k_2, \lambda_2)\varepsilon^{*\beta}(k_1, \lambda_1)\ \Gamma_{\alpha,\beta;\mu}(k_2, k_1)$$

$$\downarrow \quad \alpha \leftrightarrow \beta$$

Identical particle condition

$$\Gamma_{\alpha,\beta;\mu}(p, q) = \Gamma_{\beta,\alpha;\mu}(p, -q)$$

Selction rules

1. $(J = 1, 3, \cdots) \not\to (0 + 0)$
2. $(1) \to$ ~~$(0 + 0)$~~, $(1/2 + 1/2)$ ~~$(1 + 1)$~~, ~~$(\cdots)$~~

$$\vdots$$

$$
\begin{aligned}
B^{\pm}(0,5.2) &\rightarrow K^*(1,0.9)^{\pm} + \gamma(1,0) \\
H(0,125) &\rightarrow \gamma(1,0) + \gamma(1,0) \\
H(0,125) &\rightarrow g(1,0) + g(1,0) \\
H(0,125) &\rightarrow Z(1,91) + \gamma(1,0) \\
H(0,125) &\rightarrow Z^*(1,\text{virtual}) + Z(1,91) \\
t(1/2,173) &\rightarrow b(1/2,4) + W^+(1,80) \\
\tau(1/2,1.7) &\rightarrow \pi(0,0.15) + \nu_\tau(1/2,0) \\
\tau(1/2,1.7) &\rightarrow \rho(1,0.77) + \nu_\tau(1/2,0) \\
Z(1,91) &\rightarrow \tau(1/2,1.7) + \bar{\tau}(1/2,1.7) \\
V^*(1,\text{virtual}) &\rightarrow W^-(1,80) + W^+(1,80) \\
J/\psi(1,3) &\rightarrow a_2(1320)(2,1.3) + \rho(1,0.77) \\
J/\psi(1,3) &\rightarrow f_4(2050)(4,2) + \gamma(1,0)
\end{aligned}
$$

Matrix elements in the covariant formulation

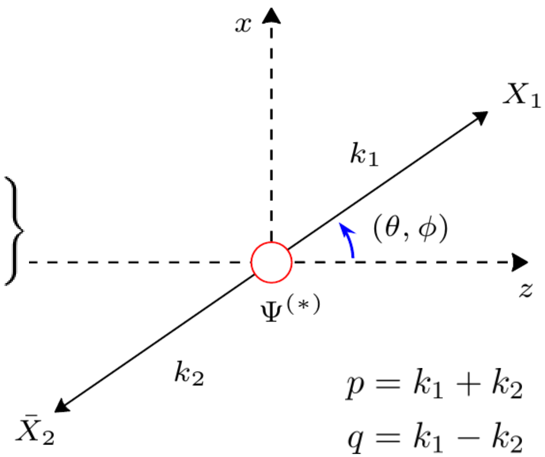
<Constructing covariant three-point vertices>

$$[\Gamma^{[J;s_1,s_2]}] = \sum_{\lambda=\pm}\Bigg\{\Theta(J_-)\sum_{n=0}^{J_-}[F^{[J;s_1,s_2]}_{(\lambda s_1,\lambda s_2)}]^n\,[S^\lambda]^{s_{\min}}\Big([V_1^\lambda]^{s_1-s_{\min}}+[V_2^\lambda]^{s_2-s_{\min}}\Big)$$

$$+\gamma_{s_{\min}}\Theta(J_+)\sum_{n=0}^{J_+}[F^{[J;s_1,s_2]}_{(\lambda s_1,-\lambda s_2)}]^n\,[T^\lambda]^{s_{\min}}\Big([V_1^\lambda]^{s_1-s_{\min}}+[V_2^{-\lambda}]^{s_2-s_{\min}}\Big)\Bigg\}$$

$(J_\mp = J-|s_1\mp s_2|)$, $(\gamma_{s_{\min}} = 1-\delta_{s_{\min},0}$ with $s_{\min}=\min[s_1,s_2])$

The $\Psi^{(*)}$ rest frame (ΨRF)



<How to construct covariant tensor currents>

Wave tensors			Momenta			Form factors		
$\varepsilon^{*\alpha}$	$\to$	$X_1^{\dagger\alpha}$	$ik_{1\rho}$	$\to$	$\partial_\rho X_1^{\dagger\alpha}$	$A[(k_1\cdot k_2)]$	$\to$	$(\partial_\gamma X_1^{\dagger\alpha}),(\partial^\gamma X_2^\beta)$
ε^{β}	$\to$	X_2^{β}	$ik_{2\sigma}$	$\to$	$\partial_\sigma X_2^{\beta}$			

$\alpha \equiv \alpha_1\cdots\alpha_{s_1}$

$\beta \equiv \beta_1\cdots\beta_{s_2}$

$S^\pm_{\alpha,\beta}$	$\to$	$(\pm,\pm)$
$V^\pm_{1\alpha;\mu}$	$\to$	$(\pm,0)$
$V^\pm_{2\beta;\mu}$	$\to$	$(\pm,0)$
$T^\pm_{\alpha,\beta;\mu\nu}$	$\to$	$(\pm,\mp)$