On the Detection of QCD Axion Dark Matter by Coherent Scattering

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In collaboration with
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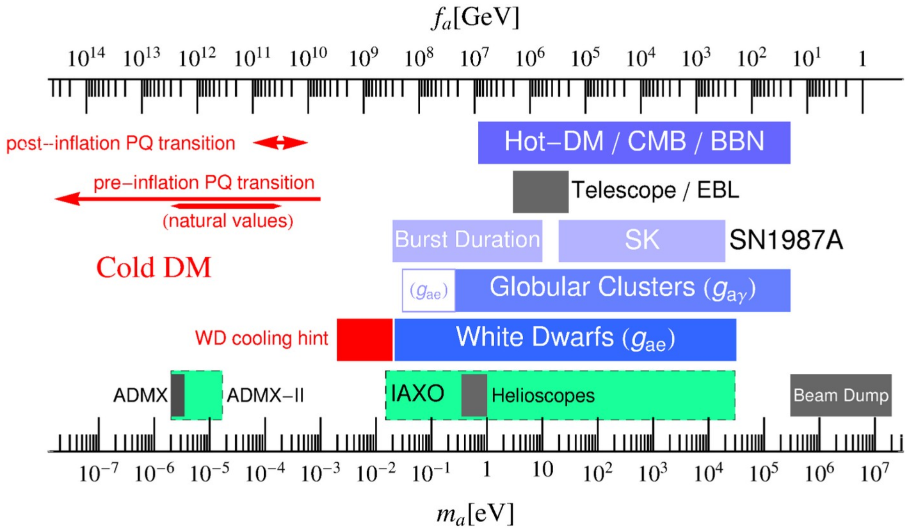
Based on
arXiv:2112.13536

Outline

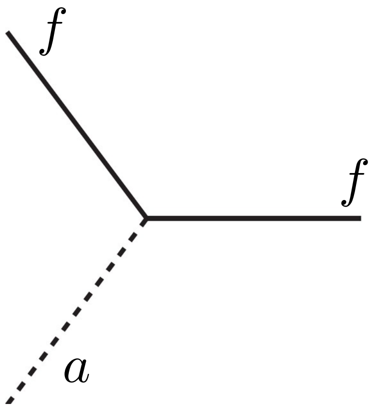
- Axion coherent scattering with matter.
  Derivation of spin-independent interaction.
  Torsion balance from axion DM wind.
- Refraction: f_a^{-2} Contribution.
  Forward scattering. Modified dispersion relation in matter.
- Hard scattering: f_a^{-4} Effect.
  Coherent effect, stimulation emission.
- Conclusion.

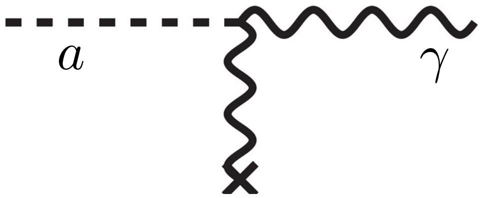
Axions

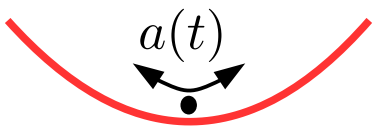
- A Nambu-Goldston boson to solve the strong CP problem.
- Light CP odd particle which couples to gauge bosons.
- Most important candidate for dark matter (DM).
- Coupling between SM and axion is suppressed by axion decay constant fa .
- fa > O(10^8) GeV , ma < O(0.1) eV is prime target region.



Axion DM detection







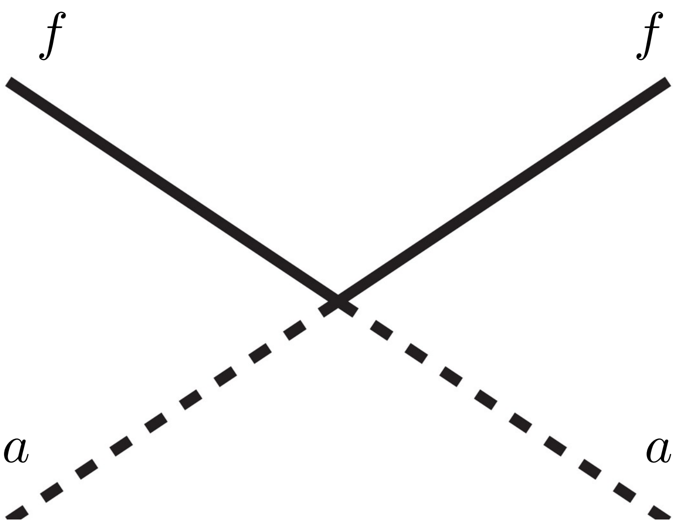
$$\propto \frac{1}{f_a}$$

Absorption

Conversion

Time varying parameter
from state DM VEV

Elastic Scattering of Axion



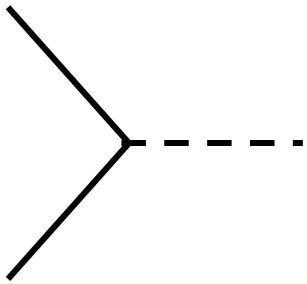
$$\propto \frac{1}{f_a^2}$$

Naively small, but we may utilize coherent effect with spin-independent interaction

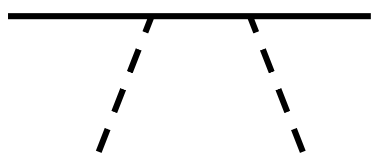
$$\mathcal{L} = \frac{g_{aN}}{f_a^2} a a \bar{N} N$$

N: nucleon

Leading-order Interaction



$$\propto \frac{1}{f_a}\partial_\mu a\bar{N}\gamma^\mu\gamma^5 N$$



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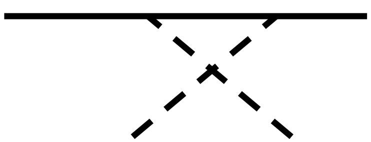


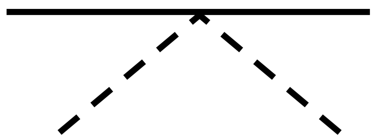
$$\mathcal{O}\left(\frac{m_a^2}{f_a^2 m_N}\right)$$

Spin-independent but highly suppressed.

c.f., soft pion theorem

Higher-order Interaction

Higher-order chiral perturbation leads another spin-independent interaction:



$$O\left(\frac{m_a}{f_a f_\pi}\right)$$

$$\mathcal{L} = \frac{g_{aN}}{f_a^2} a a \bar{N} N$$

$$g_{aN} = \frac{m_N}{2(m_u + m_d)^2}(m_d^2 f_{Tu}^N + m_u^2 f_{Td}^N)$$

$$f_{Tq}^N = \frac{\langle N \left| m_q \bar{q} q \right| N \rangle}{m_N}$$

c.f., pion sigma term

In the limit $m_q \to 0$, the axion is massless and this interaction is zero.

Elastic Scatter of Axion DM

$$\mathcal{L} \sim \frac{m_N}{1000 f_a^2} a a \bar{N} N$$

fa: decay constant
N: nucleon field

- The interaction is doubly suppressed by the decay constant.
- But quantum mechanical effect is helpful?
- As the DM density is 0.3 GeV/cm^3 and velocity ~ 100 km/s

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Macroscopic Compton length

$$(m_a v_{\rm DM})^{-1} \sim \frac{0.1\,{\rm eV}}{m_a}\,{\rm cm}$$

→ Coherent enhancement?

$$N_{\rm target} = O(10^{23})$$

Large phase number density

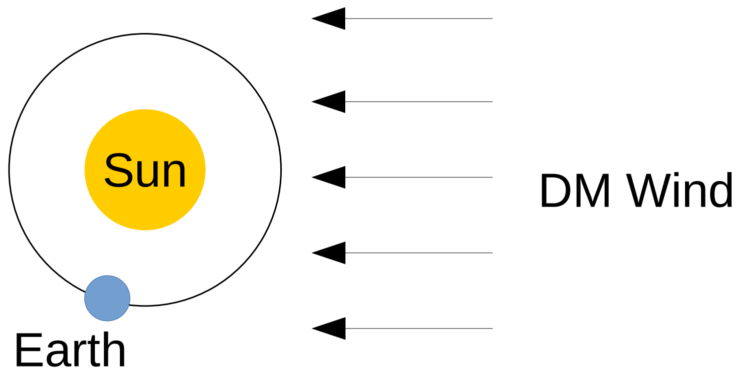
$$f_{\rm DM} \sim \frac{\rho_{\rm DM}}{m_{\rm DM}\,(m_{\rm DM} v_{\rm DM})^3} \sim 10^7 \left(\frac{m_{\rm DM}}{0.1\ {\rm eV}}\right)^{-4}$$

→ Stimulation effect?

$$\sigma(aN \to aN) \to \sigma(aN \to aN) \times (1 + f_{\rm DM})$$

Scattering Signature

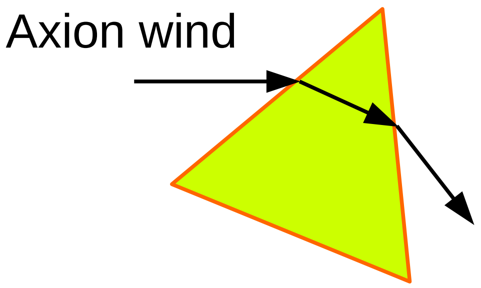
- Each axion has low momentum $p_a = m_a v, \quad (v \sim 10^{-3})$
- Single scatter of axion DM is hard to detect.
- Collective scattering of axion may provide visible signature.
- For proper motion of solar system, there is DM wind (~100 km/s)



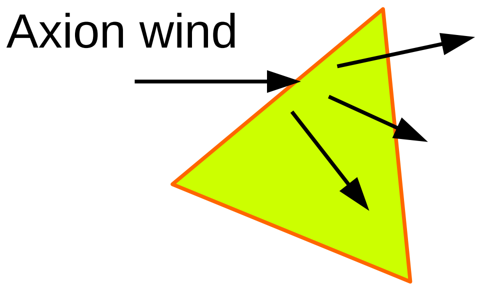
- "Friction" from DM wind causes additional acceleration on materials.
  - Torsion balance: ultimate sensitivity : 10^{-23} cm/s^2

Detection of Axion DM

Such axion can lead additional acceleration to detector, e.g., torsion balance

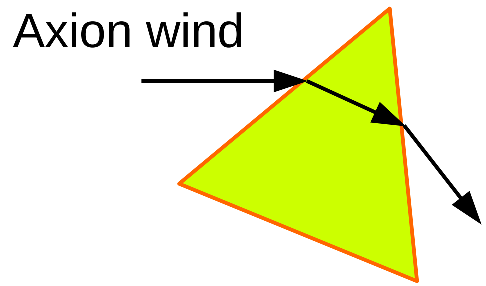


Refraction: $O(f_a^{-2})$ effect.



Scattering: $O(f_a^{-4})$ effect
+ coherent + stimulate effect?

Refraction



Refraction: $O(f_a^{-2})$ effect.

Modification of dispersion relation of axion.

$$\frac{m_N}{f_a^2} aa\langle \bar{N} N \rangle \to \frac{\rho_{\rm matter}}{f_a^2} aa$$

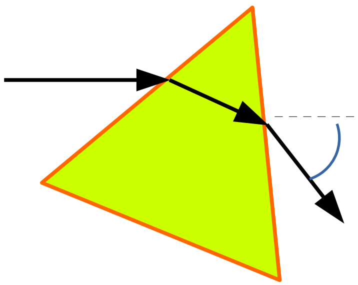
Shift of mass term of axion

Refraction index

$$n - 1 \simeq \frac{\rho_{\rm matter}}{v^2 m_a^2 f_a^2} \simeq 10^{-11} \left(\frac{\rho_{\rm matter}}{1\ g/{\rm cm}^3} \right)$$

independent on fa.

Force from Refraction?



Indeed, single axion particle has momentum transfer $O((n-1)p_a)$

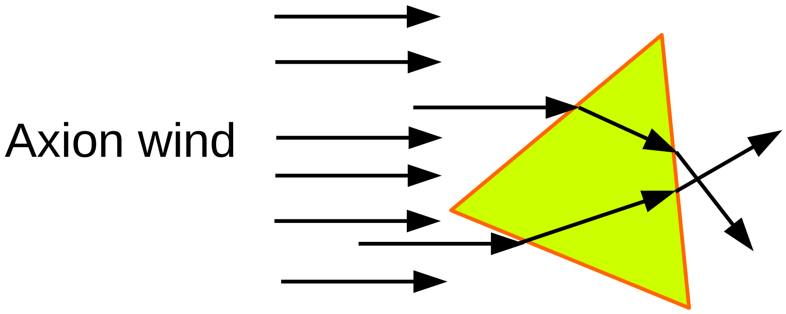
Naive guess:

$$a \overset{?}{\sim} \frac{\rho_{\rm DM} v_{\rm DM} |n-1| S}{M} = 10^{-17} \left(\frac{|n-1|}{10^{-10}}\right) \left(\frac{S}{\rm cm^2}\right) \left(\frac{\rm g}{M}\right) \ {\rm cm/s^2}.$$

(Area: $\frac{S}{\rm cm^2}$; Mass: $\frac{\rm g}{M}$)

$$\gg a_{\rm ultimate} = O(10^{-23}) \ {\rm cm/s^2}$$

No Net Refraction Force



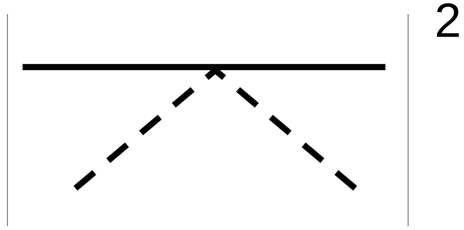
We need integrate over axion flux. However,

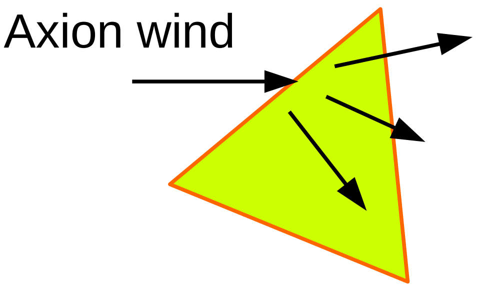
$$F \propto \int_S \vec{a} = 0$$

There is no force for uniform axion wind.

c.f., relic neutrino case:
[Cabibboa&Maiani, PLB 114 (1982) 115]

Axion Scattering

$\sigma \propto$ 



Scattering: $O(f_a^{-4})$ effect
+ coherent + stimulate effect?

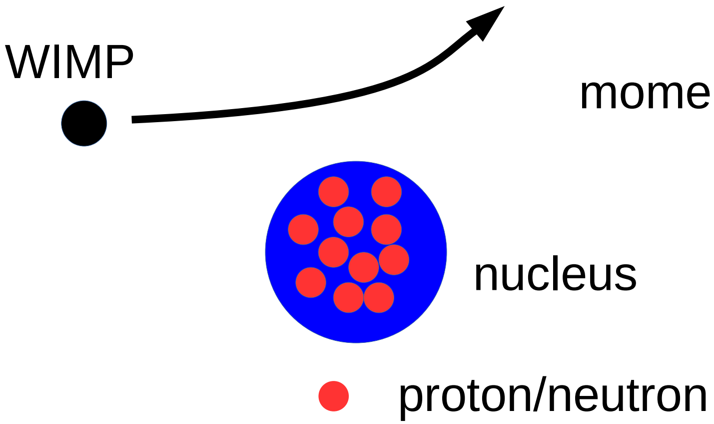
Coherent Effect

As the Compton length is macroscopic $(m_a v_{\mathrm{DM}})^{-1} \sim \frac{0.1\,\mathrm{eV}}{m_a}\,\mathrm{cm}$

Axion can scatter with macroscopic number $O(10^{24})$ of nucleons, coherently.

Similar to WIMP spin independent scattering:

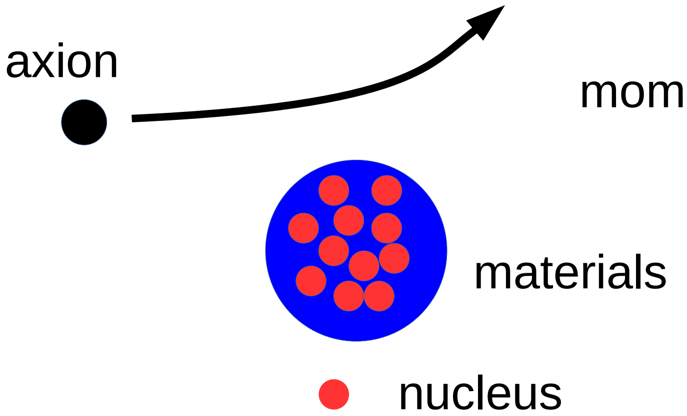
Coherent Scattering



momentum transfer < (nucleus size)$^{-1}$

$\sigma \propto A^2$ A: sum of proton/neutron inside a nucleus.

Coherent Scattering



momentum transfer $< (O(\mathrm{cm}))^{-1}$

$$\sigma \propto N_{\mathrm{avo}}^2 \sim 10^{48}$$

Stimulate Effect

Process: $X \to Y$ (X,Y: boson)

Initial state: # of X is 1 and # of Y is zero

Hamiltonian $H \sim H_{XY} a_Y^\dagger a_X$

$$\mathcal{M}_0 \sim \langle N_X = 0, N_Y = 1 \, | H | \, N_X = 1, N_Y = 0 \rangle$$

Stimulate Effect

Process: $X \to Y$ (X,Y: boson)

Initial state: # of X is 1 and # of Y is N_Y

Hamiltonian $H \sim H_{XY} a_Y^\dagger a_X$

$$\mathcal{M}_{N_Y} \sim \langle N_X = 0, N_Y + 1 \,|H|\, N_X = 1, N_Y \rangle = \sqrt{N_Y + 1}\mathcal{M}_0$$

$$|N_Y\rangle = \frac{1}{\sqrt{N_Y!}}(a_Y^\dagger)^{N_Y} |0\rangle$$

$$[a, a^\dagger] = 1$$

Stimulate Effect

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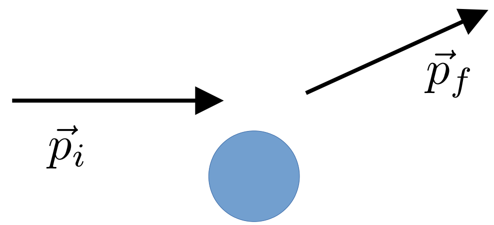
$$[a, a^\dagger] = 1$$

Due to large number of final state particle, the cross section is enhanced.

$$f(\vec{p}) \sim f_0 = \frac{\rho_{\rm DM}}{m_a (m_a v_{\rm DM})^3} \sim 10^{11} \left(\frac{\rho_{\rm DM}}{0.3\,{\rm GeV/cm^3}}\right) \left(\frac{m_{\rm DM}}{10^{-2}\ {\rm eV}}\right)^{-4}$$

Force of Axion DM



$$\vec{F} = \int d^3p_i d^3p_f \delta(|\vec{p_f}| - |\vec{p_i}|) \frac{|\vec{p_i}|}{m_a} (\vec{p_i} - \vec{p_f}) \frac{d\sigma}{d\Omega_{if}} f(\vec{p_i})(1 + f(\vec{p_f})).$$

Force of Axion DM

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$f(\vec{p_i}) f(\vec{p_f})$

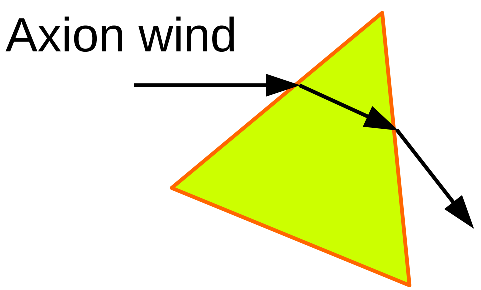
$p_i \leftrightarrow p_f$ +1 -1 +1 +1

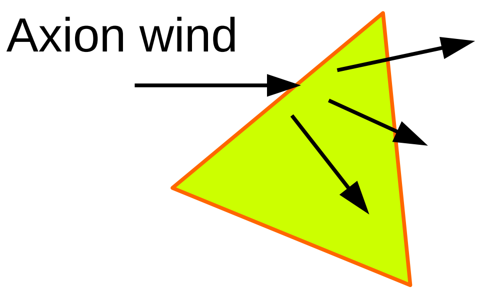
Stimulation effect part is odd, and no contribution to force.

$$a \simeq O(10^{-29})\,\mathrm{cm/s^2} \times \left(\frac{\rho_{\mathrm{DM}}}{0.3\,\mathrm{GeV/cm^3}}\right) \left(\frac{\rho}{1\,\mathrm{g/cc}}\right) \left(\frac{10^{-3}}{v_{\mathrm{DM}}}\right) \left(\frac{m_a}{10^{-1}\,\mathrm{eV}}\right)$$

Detection of Axion DM

Such axion can lead additional acceleration to detector, e.g., torsion balance





Refraction: $O(f_a^{-2})$ effect.

Scattering: $O(f_a^{-4})$ effect
+ coherent + stimulate effect?

No net effect for uniform DM density.

Stimulation effect is canceled.

c.f., relic neutrino case:
[Cabibboa&Maiani, PLB 114 (1982) 115]

After all...

- Direct detection of elastic scatter of axion DM is hard, even with help of quantum mechanical enhancement.

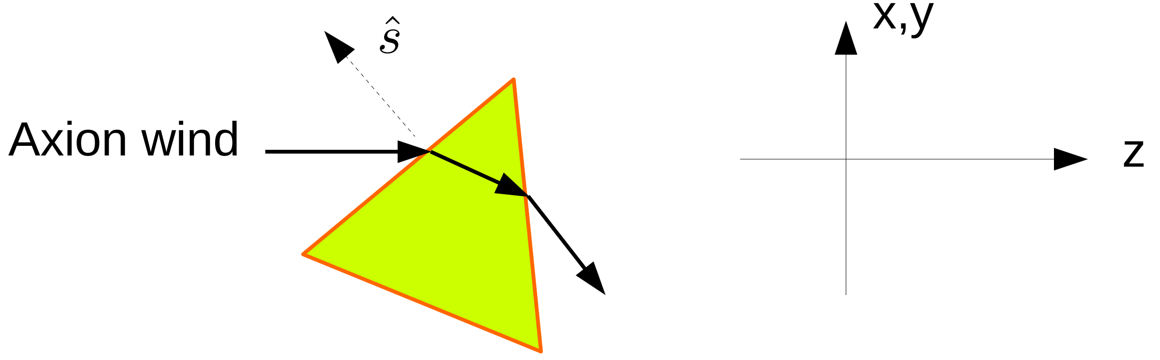
$$\Delta a_{\mathrm{axion}} \sim 10^{-30}\ \mathrm{cm/s^2} \left(\frac{m_a}{1\ \mathrm{meV}}\right)$$

- Not enough static force. Ultimate experimental reach ~ 10^{-23} cm/s^2

- With non-uniform DM density, time-varying force is possible.

- Axion can scatter off the Sun with large probability. Any chance?

Cancel of Refraction



At $O(n-1)$ level,

$$\vec{F} = (n-1)p\int_S d\vec{a} = 0 \qquad d\vec{a} = \frac{\hat{s}}{s_z}dxdy$$

$$\int_S da_i = \int_V d^3x\partial_j(\delta_{ij}) = 0$$

Gauss law