A Large N Expansion for Minimum Bias

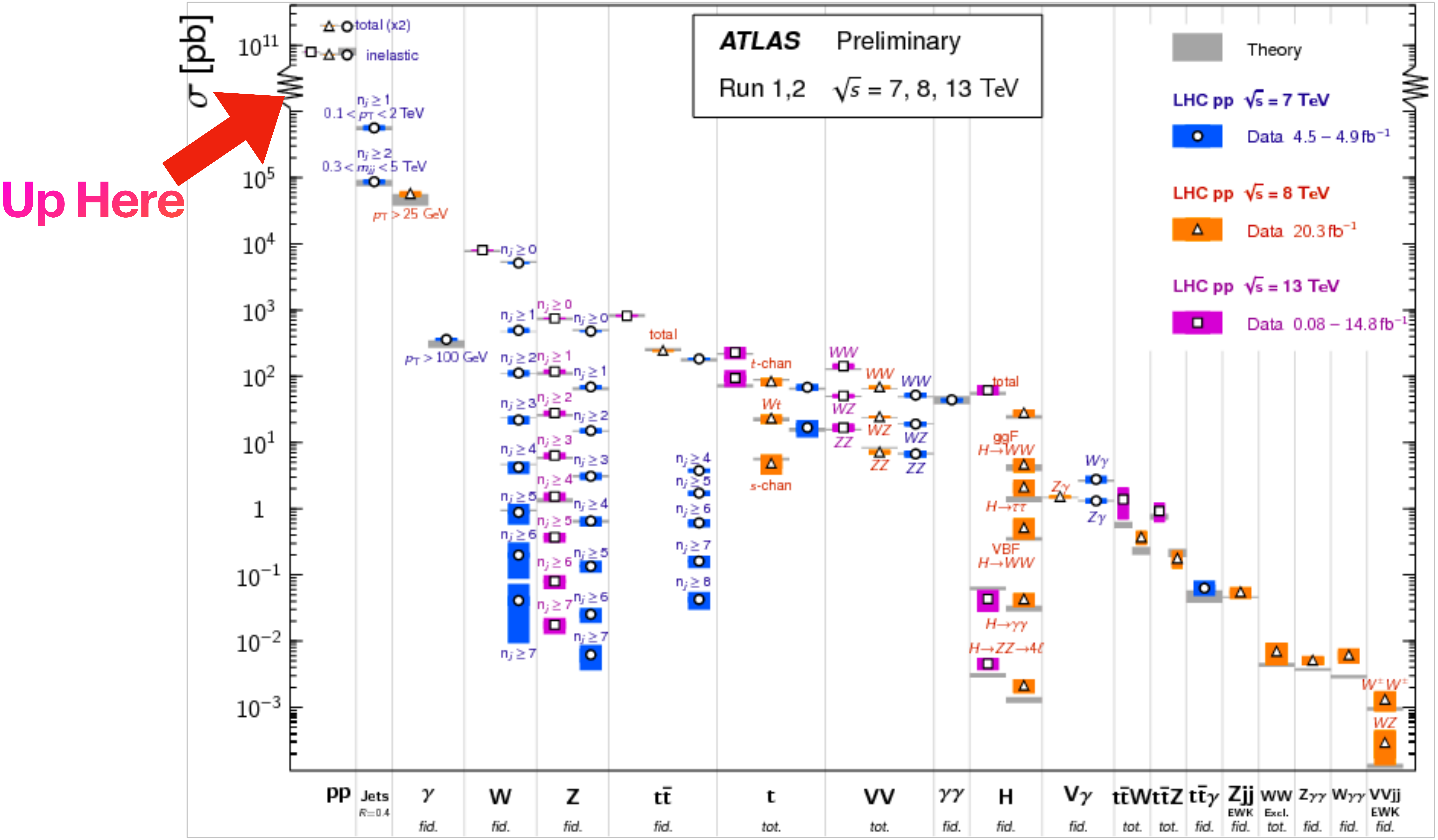
Based on: Andrew Larkoski, TM, JHEP 2110 094 (2021) [arXiv:2107.04041]

2nd AEI Workshop for BSM and 10th KIAS Workshop on Particle Physics and Cosmology

Propose and discuss a framework that can provide a first principles effective description of minimum bias events

Minimum bias: experimentally, some minimal trigger, typically some forward calorimeter activity

Soft QCD, where strong nature of interactions dominate. *Ergodic*



Up Here
ATLAS Preliminary
Run 1,2 $\sqrt{s}$ = 7, 8, 13 TeV
Theory
LHC pp $\sqrt{s}$ = 7 TeV
Data 4.5 – 4.9 fb^{-1}
LHC pp $\sqrt{s}$ = 8 TeV
Data 20.3 fb^{-1}
LHC pp $\sqrt{s}$ = 13 TeV
Data 0.08 – 14.8 fb^{-1}
σ [pb]
total (x2)
inelastic
pp
Jets
γ
W
Z
t$\bar{t}$
t
VV
γγ
H
Vγ
t$\bar{t}$W
t$\bar{t}$Z
t$\bar{t}$γ
Zjj EWK
WW Excl.
Zγγ
Wγγ
VVjj EWK

From first principles?

EFT is a powerful symmetry based approach

This one power counts using more unusual expansion parameter 1/N, with N number of particles in the event

Shift symmetry (goldstone boson story?)

Fractional dispersion (non-locality?)

Reasons to seek first principles approach

From the theory side connect to large ‘N’ (~large charge) approaches; asymptotic analyses; bootstrap (not this talk)

From the pheno side

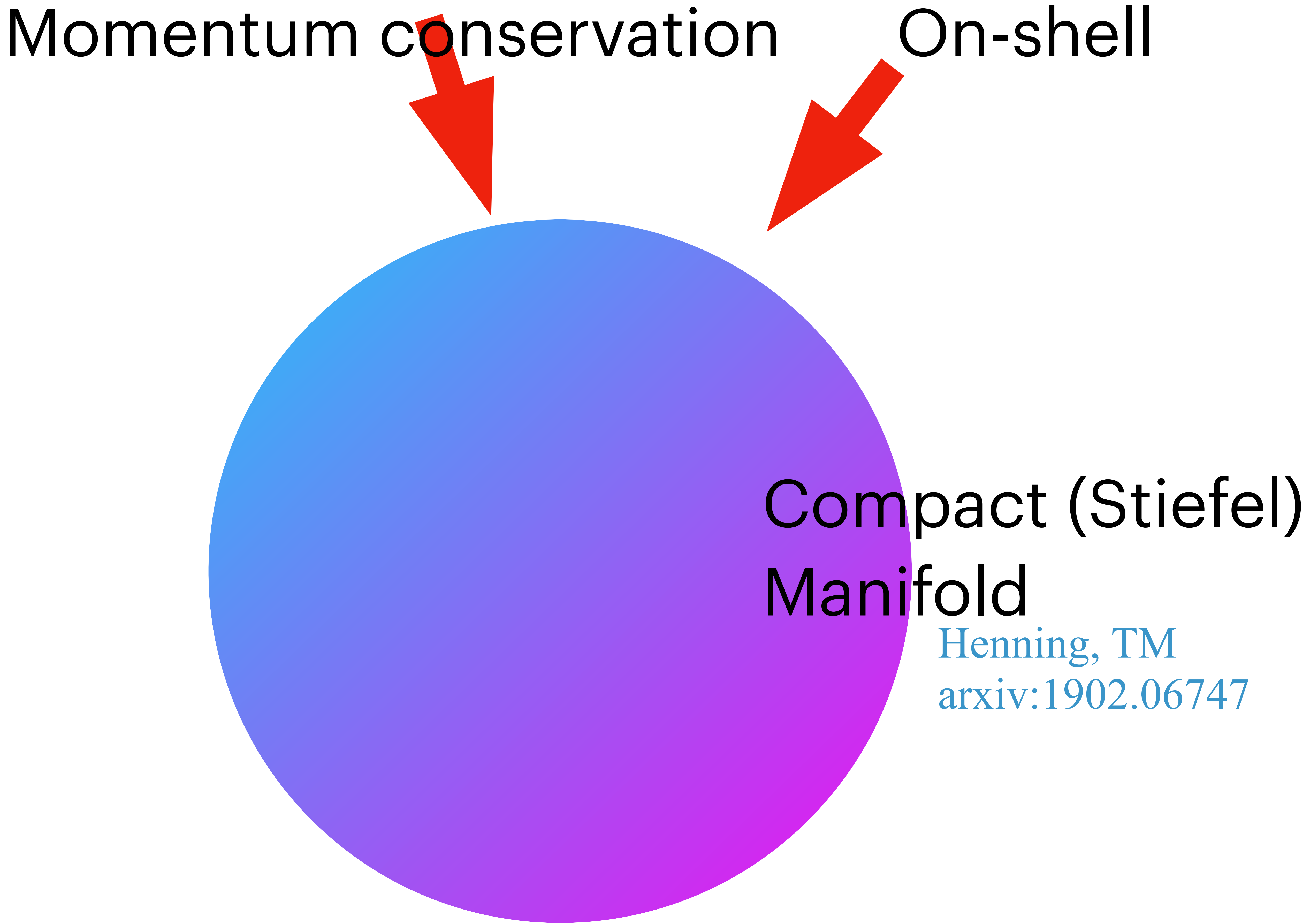
Equal footing

Treat both small and large systems, at both low and high energy, all within the same framework.

Potential to aid in elucidation of nature of small scale (p p collision) collective phenomena in QCD; jet quenching. Not relying on any particular model

Potential to aid in elucidation of nature of small scale collective phenomena in QCD; jet quenching

$$d\sigma(p_a, p_b, p_1, \ldots, p_N) = \sigma_N(p_a, p_b, p_1, \ldots, p_N)\, \delta^{(4)}(p_a + p_b - p_1 - \ldots - p_N) \prod_{i=1}^{N} \delta(p_i^2 - m_i^2)$$



Henning, TM
arxiv:1902.06747

Potential to aid in elucidation of nature of small scale collective phenomena in QCD; jet quenching

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Momentum conservation

On-shell

Nice to have

$$= 1 + \sum_{l=1} c_l\, Y_l(\{p_i\})$$

Harmonics

Compact (Stiefel) Manifold

Henning, TM
arxiv:1902.06747

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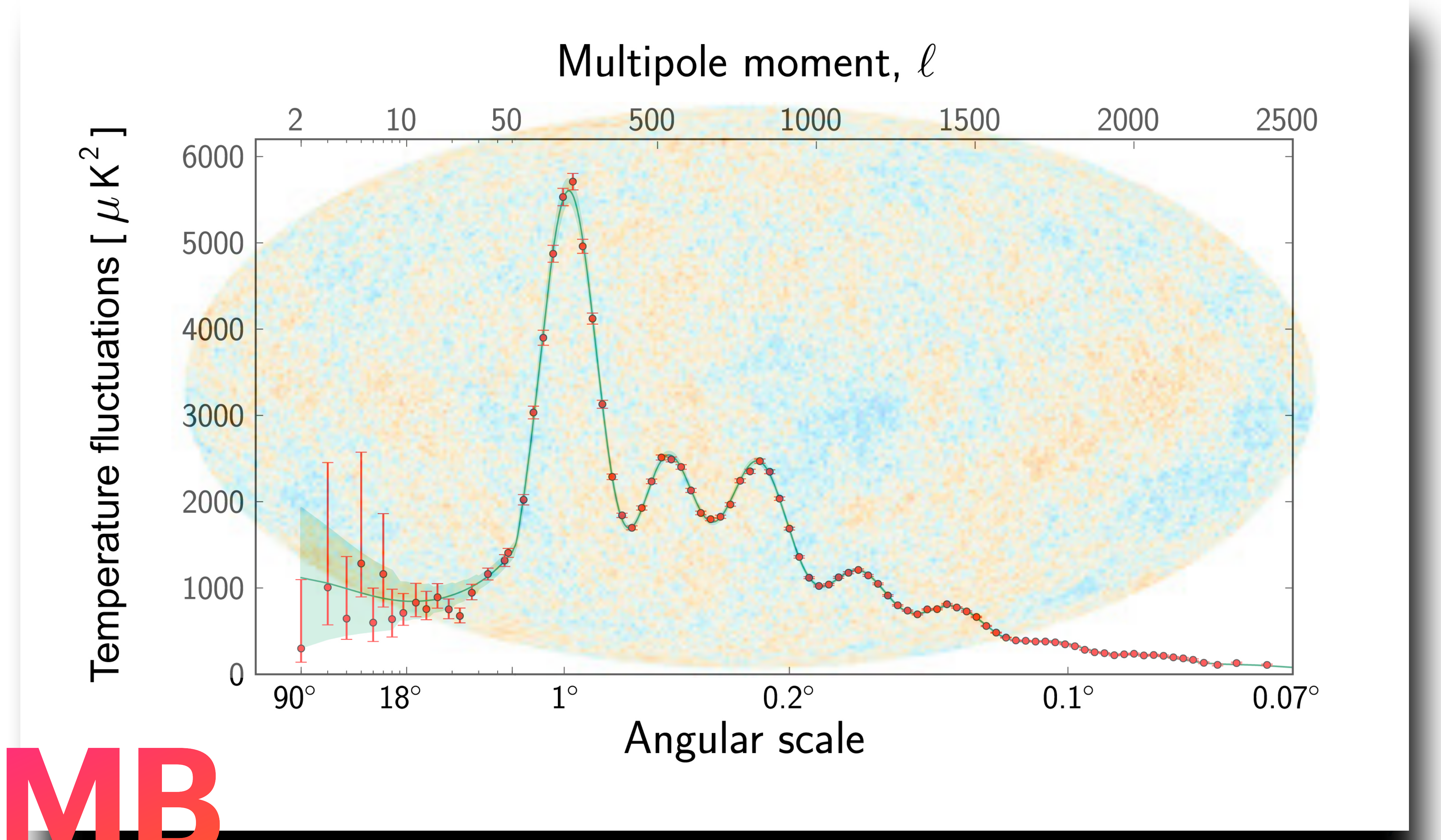
Harmonics



C.f. the CMB

Reasons to seek first principles approach

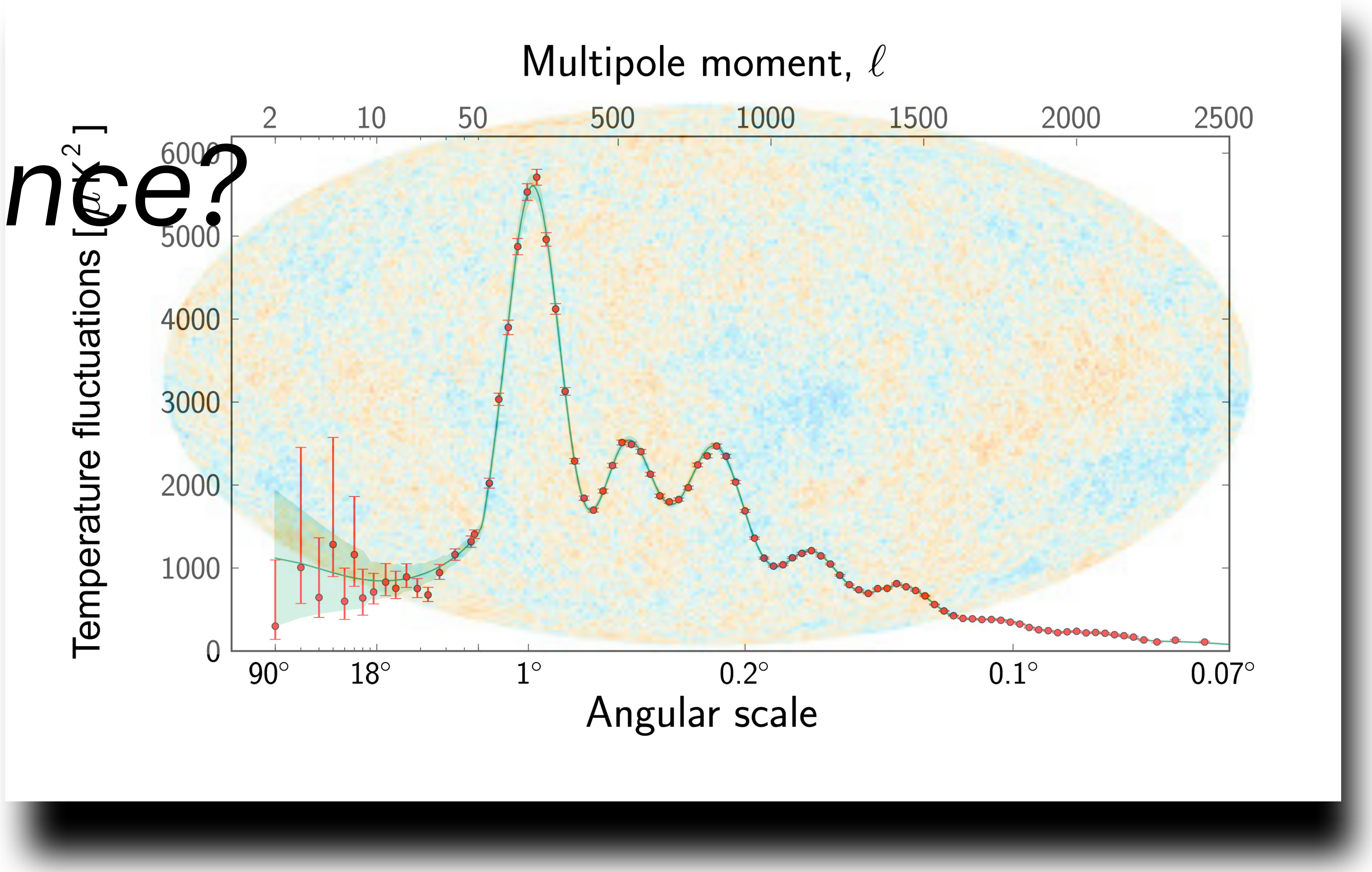
Equal footing

Treat both small and large systems, at both low and high energy, all within the same framework.

Potential to aid in elucidation of nature of small scale (p p collision) collective phenomena in QCD; jet quenching. Not relying on any particular model

"Track a harmonic" as evidence?

$$= 1 + \sum_{l=1} c_l Y_l(\{p_i\})$$



What will be addressed; what will not

Assume that events are binned in multiplicity, N

i.e. **Not** attempt a description of fluctuations in multiplicity

Therefore, can capture how *normalized* distributions, binned in N, change as a function of N, and as a function of Q

We take the large N limit at fixed Q, meaning we do not consider a scaling of Q and N such that Q/N (c.f. 't Hooft coupling) remains finite. (Although this could be interesting)

What will be addressed; what will not

Proto-EFT approach: power-counting, symmetries

But no sense of framework in which to calculate e.g. quantum corrections (yet)

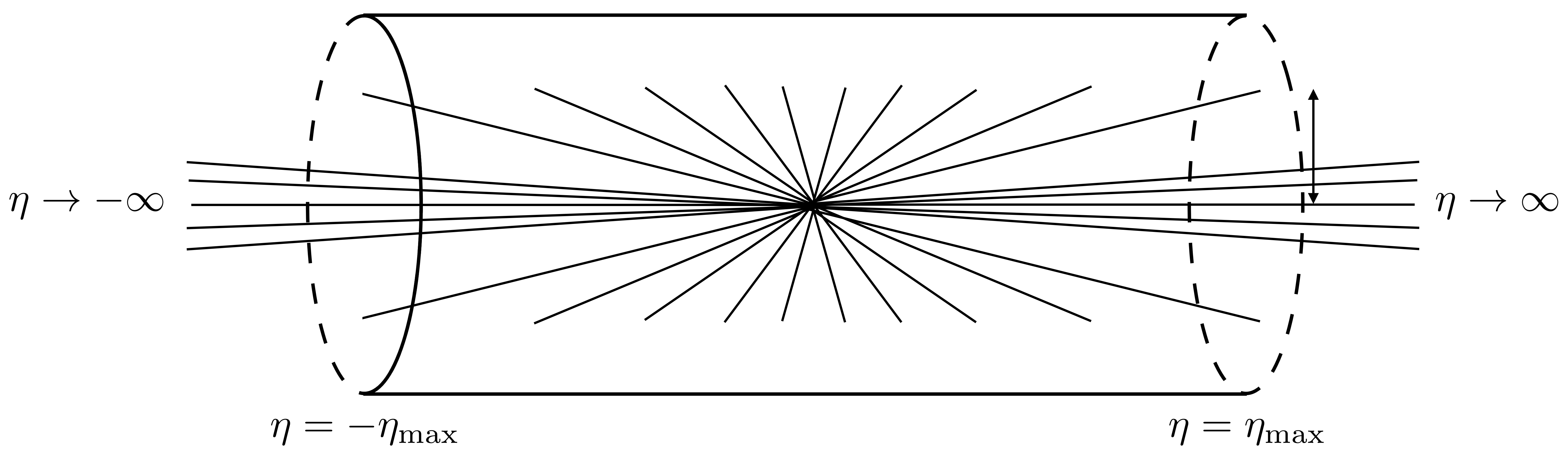
Testing self-consistency of assumptions, understanding their consequences to explain broad features of data

Physical / directly measurable quantities only (e.g. no ‘centrality’)

Outline

Power Counting and Symmetries

Simple Predictions, comparison to data



Power counting and symmetries for pp/AA min bias

1. We focus on $\eta \sim 1 \ll \eta_{\max}$
2. Everything massless $p_\perp \gg m_\pi$
3. Beam momentum is O(1) of CoM
4. Number of $\eta \ll \eta_{\max}$ particles $N \gg 1$
5. $\langle p_\perp \rangle \sim \sqrt{\langle p_\perp^2 \rangle}$

Mean transverse momentum representative of all particles' momentum

1. O(2) symmetry about beam
2. $\eta \to -\eta$ along the beam
3. S_N permutation sym in all detected particles

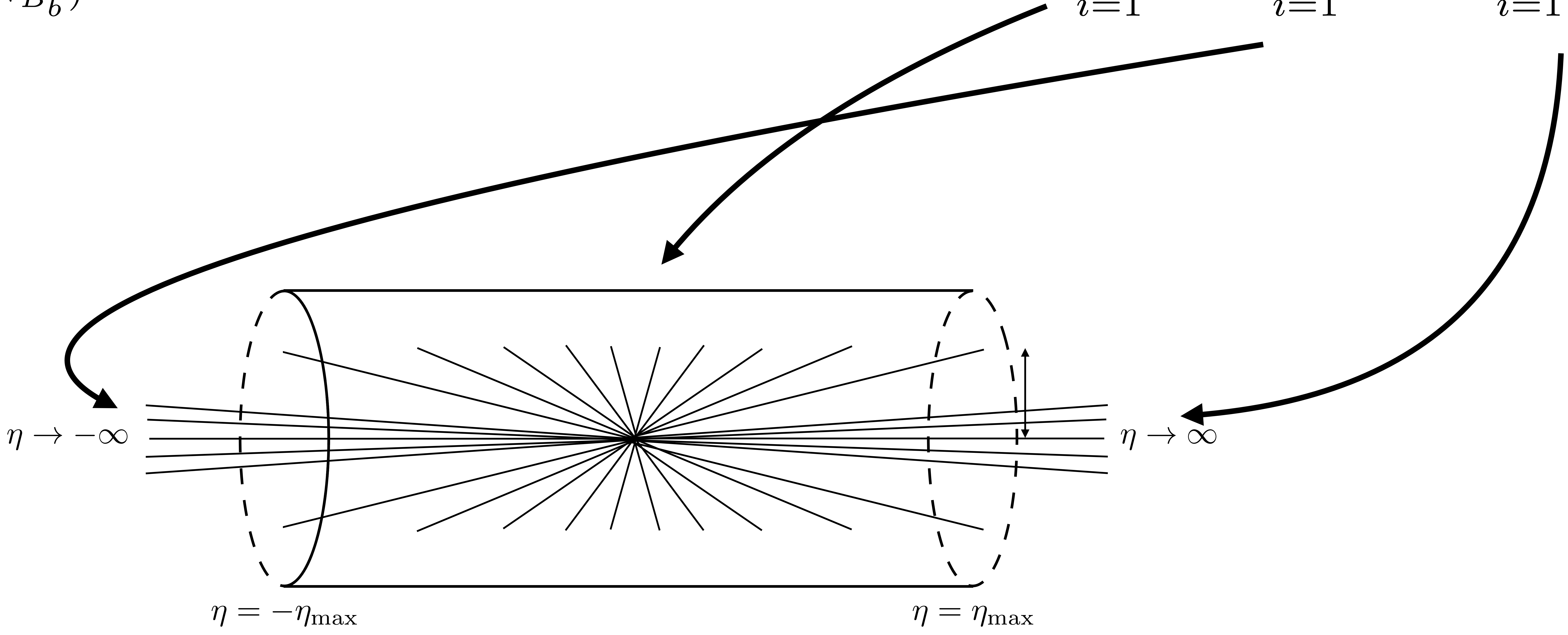
Blind to all but momentum

4. $\eta \to \eta + \Delta\eta$ symmetry

Never move particles out of detection region into beam, and vice versa

Effective matrix element

$$\sigma = \int_{\mathrm{LIPS}(N+N_{B_a}+N_{B_b})} \sigma_N(p_1,\ldots,p_N,\{p_{a_i}\},\{p_{b_i}\})\;\delta^{(4)}\Big(p_a+p_b-\sum_{i=1}^{N} p_i-\sum_{i=1}^{N_{B_a}} p_{a_i}-\sum_{i=1}^{N_{B_b}} p_{b_i}\Big)$$



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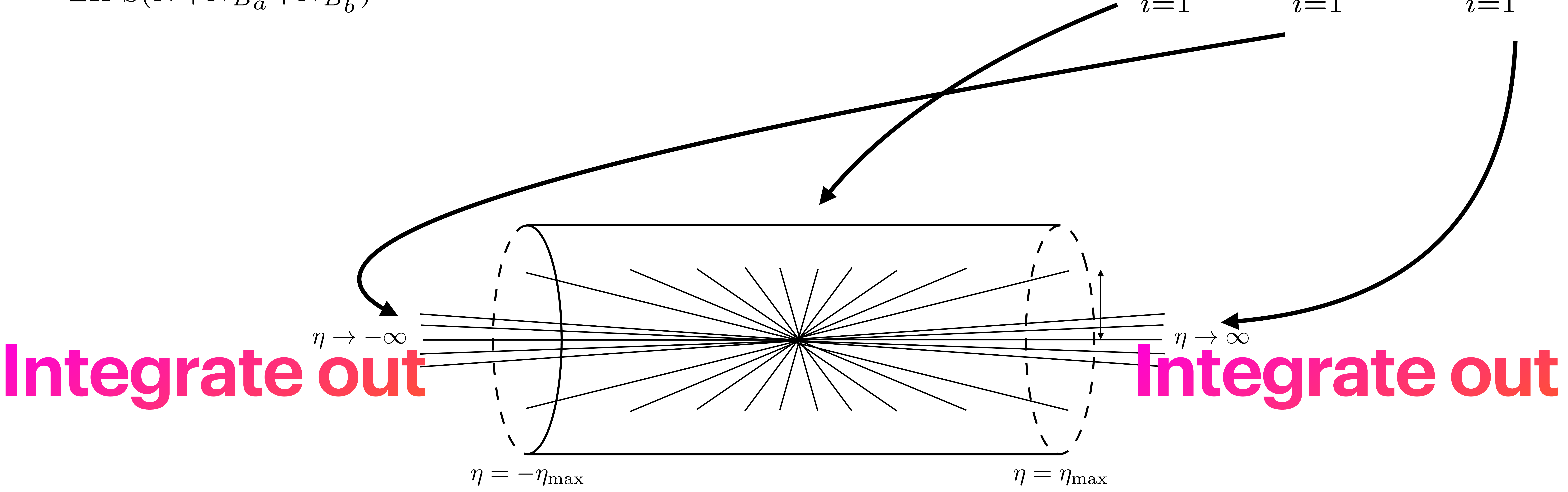

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$$=\int_0^Q dk^+\int_0^Q dk^-\int_{\mathrm{LIPS}(N)} f(k^+k^-)\widetilde{\sigma}_N(p_1,\ldots,p_N;k^+k^-)\delta(k^- - \sum_{i=1}^{N}k_i^-)\delta(k^+ - \sum_{i=1}^{N}k_i^+)\delta^{(2)}(\sum_{i=1}^{N}\vec{p}_{\perp_i})$$

Light cone momentum

$$k^\pm = E \pm p_z$$
$$= p_\perp e^{\pm\eta}$$

Integrate over boosts and energy of available energy

Effective "cross section", pulled out factor f

Transverse momentum conservation in large N limit

Expansion of matrix element

$$\sigma = \int_0^Q dk^+ \int_0^Q dk^- \int_{\mathrm{LIPS}(N)} f(k^+k^-)\ \widetilde{\sigma}_N(p_1,\ldots,p_N)\ \delta(k^- - \sum_{i=1}^N p_{\perp i} e^{\eta_i})\,\delta(k^+ - \sum_{i=1}^N p_{\perp i} e^{-\eta_i})\,\delta^{(2)}(\sum_{i=1}^N \vec{p}_{\perp i})$$

$$= 1 + \frac{c_1^{(2)}}{Q^2} \sum_{i=1}^N p_{\perp i}^2 + \mathcal{O}(Q^{-4})$$

(After momentum conservation identities)

$$0 = \left(\sum_{i=1}^N \vec{p}_{\perp i}\right)^2 = \sum_{i=1}^N p_{\perp i}^2 + \sum_{i\neq j}^N p_{\perp i} p_{\perp j} \cos(\phi_i - \phi_j)\,,$$

$$k^+k^- = \left(\sum_{i=1}^N p_{\perp i} e^{-\eta_i}\right)\left(\sum_{j=1}^N p_{\perp j} e^{\eta_j}\right) = \sum_{i=1}^N p_{\perp i}^2 + \sum_{i\neq j}^N p_{\perp i} p_{\perp j} \cosh(\eta_i - \eta_j)$$

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In powers of 1/N

Ergodicity

$$p_\perp \sim Q/N$$

$$\Longrightarrow \frac{1}{Q^2}\sum_{i=1}^N p_{\perp i}^2 \sim \frac{1}{N}$$

N terms in the sum

Expansion of matrix element

$$\sigma = \int_0^Q dk^+ \int_0^Q dk^- \int_{\text{LIPS}(N)} f(k^+k^-)\ \widetilde{\sigma}_N(p_1,\dots,p_N)\ \delta(k^- - \sum_{i=1}^N p_{\perp i}e^{\eta_i})\,\delta(k^+ - \sum_{i=1}^N p_{\perp i}e^{-\eta_i})\,\delta^{(2)}(\sum_{i=1}^N \vec{p}_{\perp i})$$

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(After momentum conservation identities)

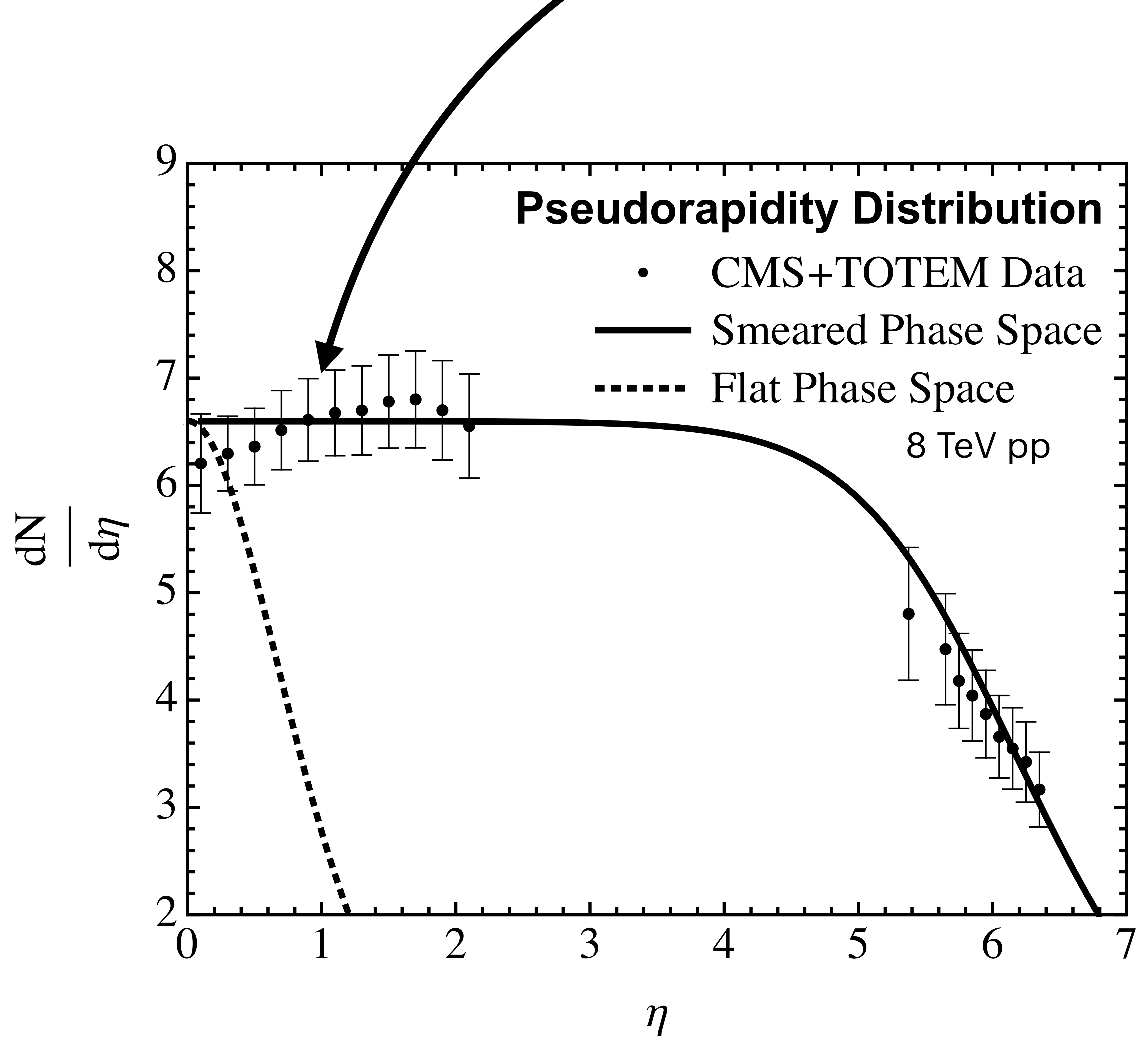
The inevitable 'flatness' of large N

$$\lim_{N\to\infty}\sum_{i=1}^N p_{\perp i}^2 \to N\langle p_\perp^2\rangle + \mathcal{O}\left(\sqrt{N}\langle p_\perp^2\rangle\right)$$

$$\lim_{N\to\infty}|\mathcal{M}(1,2,\dots,N)|^2 \to 1 + \frac{c_1^{(2)}}{Q^2}N\langle p_\perp^2\rangle + \cdots$$

Fixing the function f to give flat-in-rapidity

$$\sigma = \int_0^Q dk^+ \int_0^Q dk^- \int_{\text{LIPS}(N)} f(k^+k^-)\ \widetilde{\sigma}_N(p_1,\ldots,p_N)\ \ \delta(k^- - \sum_{i=1}^N p_{\perp i}e^{\eta_i})\,\delta(k^+ - \sum_{i=1}^N p_{\perp i}e^{-\eta_i})\,\delta^{(2)}(\sum_{i=1}^N \vec{p}_{\perp i})$$



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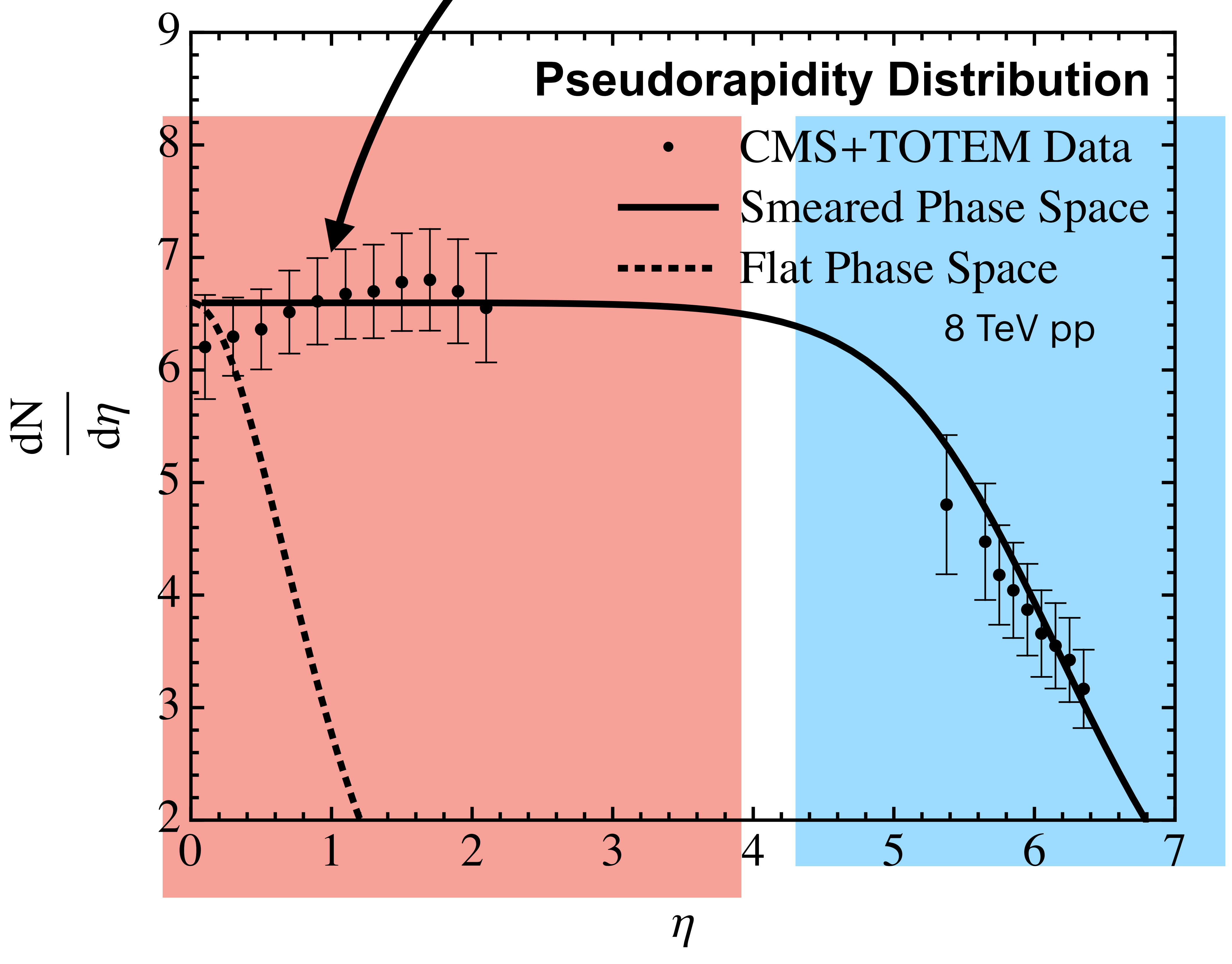
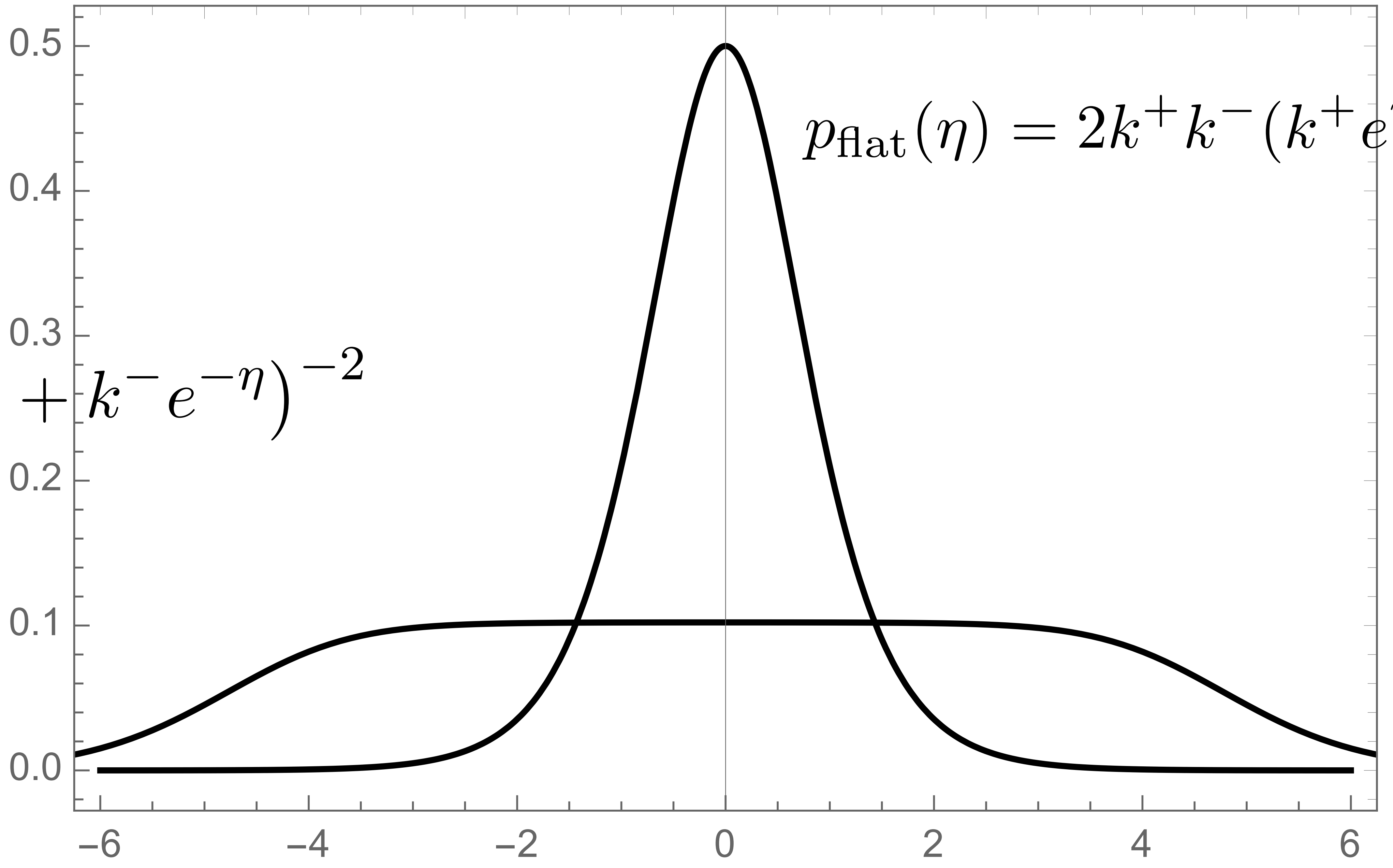


Any function f(x) that is analytic and highly peaked at x=0 produces the 'Feynman' plateau. Effective description is an Expansion around this

Fall-off can be fitted for useful self-consistency check, but it is <u>outside effective description</u>, so general results are agnostic to it

Flat phase space to flat rapidity

$$p(\eta) = \frac{1}{Q^2}\int_0^Q dk^+ \int_0^Q dk^- \, f\left(k^+k^-\right)\, p_{\text{flat}}(\eta)$$
$$= \frac{1}{Q^2}\int_0^Q dk^+ \int_0^Q dk^- \, f\left(k^+k^-\right)\, 2k^+k^-\left(k^+e^\eta + k^-e^{-\eta}\right)^{-2}$$
$$= \int_0^1 dx\, f(x)\, \frac{1-x^2}{1+x^2+2x\cosh(2\eta)}\,.$$



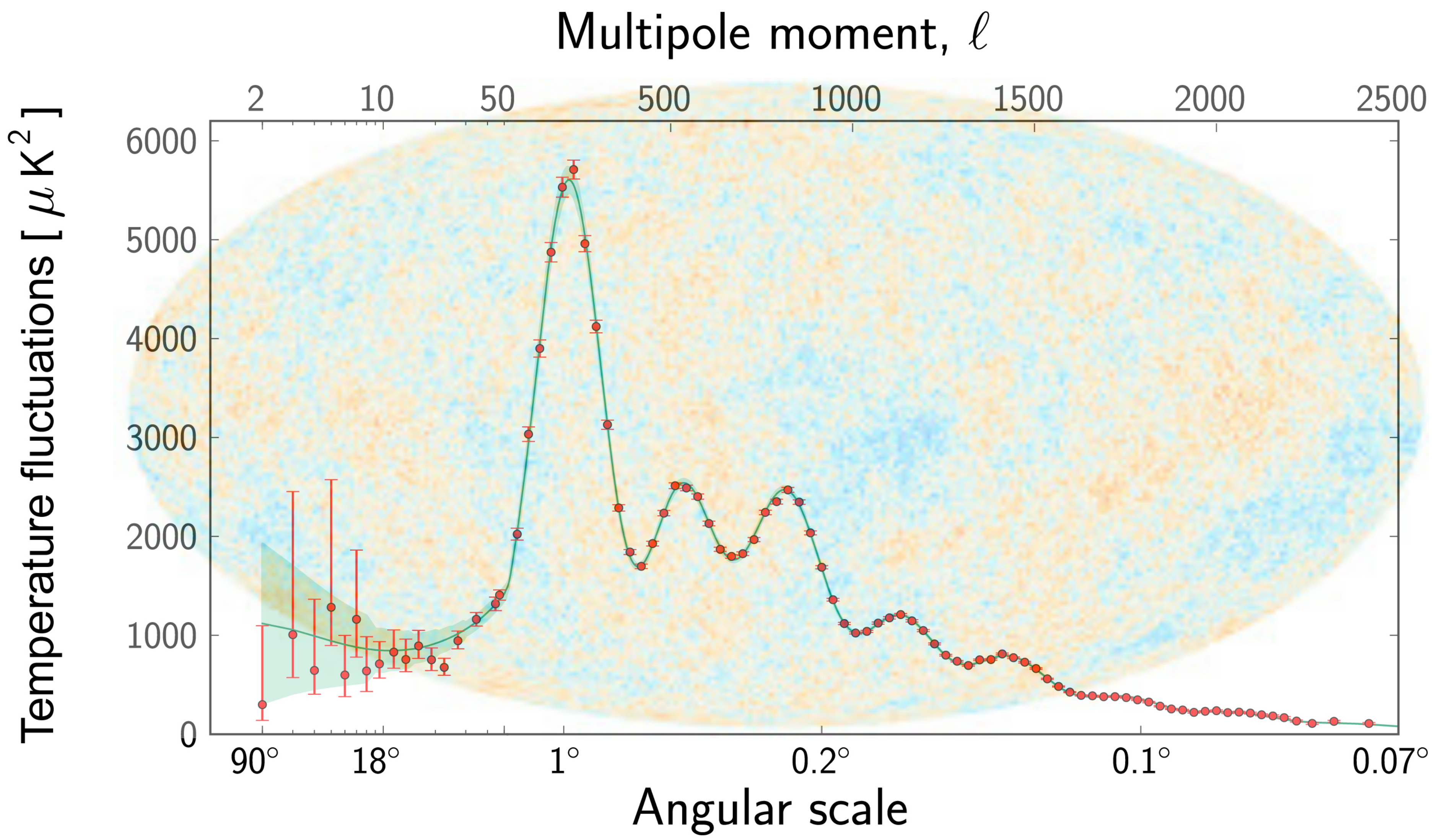
$$p_{\text{flat}}(\eta) = 2k^+k^-\left(k^+e^\eta + k^-e^{-\eta}\right)^{-2}$$

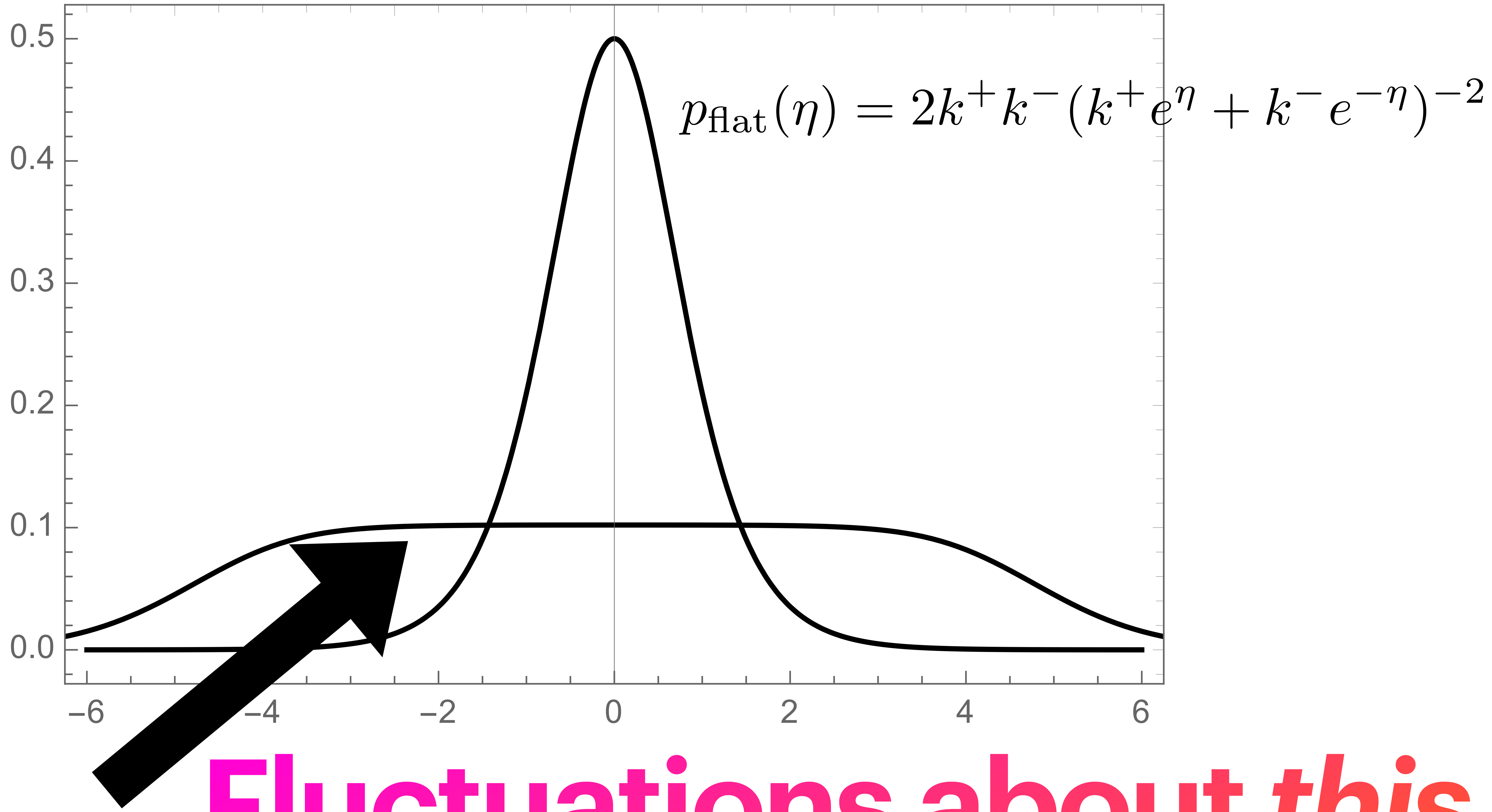
Take e.g. $$f(k^+k^-) = \frac{n}{\gamma_E + \log n}\, e^{-n\frac{k^+k^-}{Q^2}}$$

n now parameterises the ‘cutoff’ of the theory

Normalized prob $$1 = \frac{1}{Q^2}\int_0^Q dk^+ \int_0^Q dk^- \, f(k^+k^-) = \int_0^1 dx \log\frac{1}{x}\, f(x)$$

Flat phase space to flat rapidity





Fluctuations about *this*

$$= 1 + \sum_{l=1} c_l \, Y_l(\{p_i\})$$

Outline

Power Counting and Symmetries

Simple Predictions, comparison to data

The predictions include (From power counting and symmetries)

- In the $N \to \infty$ limit, the symmetries of min bias events and central limit theorem require the matrix element is exclusively a function of the total energy of the observed final state particles
- The distribution of particle transverse momentum is universal, and depends on a single parameter, with fractional dispersion relation
- Scaling of multiplicity with collider energy
- By a positivity condition, all azimuthal correlations vanish as $N \to \infty$ at fixed collision energy

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Transverse momentum distribution

The distribution on unsmeared phase space can be shown to be a Bessel function

$$p_{\text{flat}}(p_\perp) \;= p_\perp\, K_0\left(\frac{2Np_\perp}{\sqrt{k^+k^-}}\right) \qquad\qquad K_0(z) \to \sqrt{\frac{\pi}{2z}}\, e^{-z}$$

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The function f is now fixed, no wiggle-room

$$p(p_\perp) = \frac{1}{Q^2}\int_0^Q dk^+ \int_0^Q dk^- \, f\left(k^+k^-\right)\, p_{\text{flat}}(p_\perp)$$

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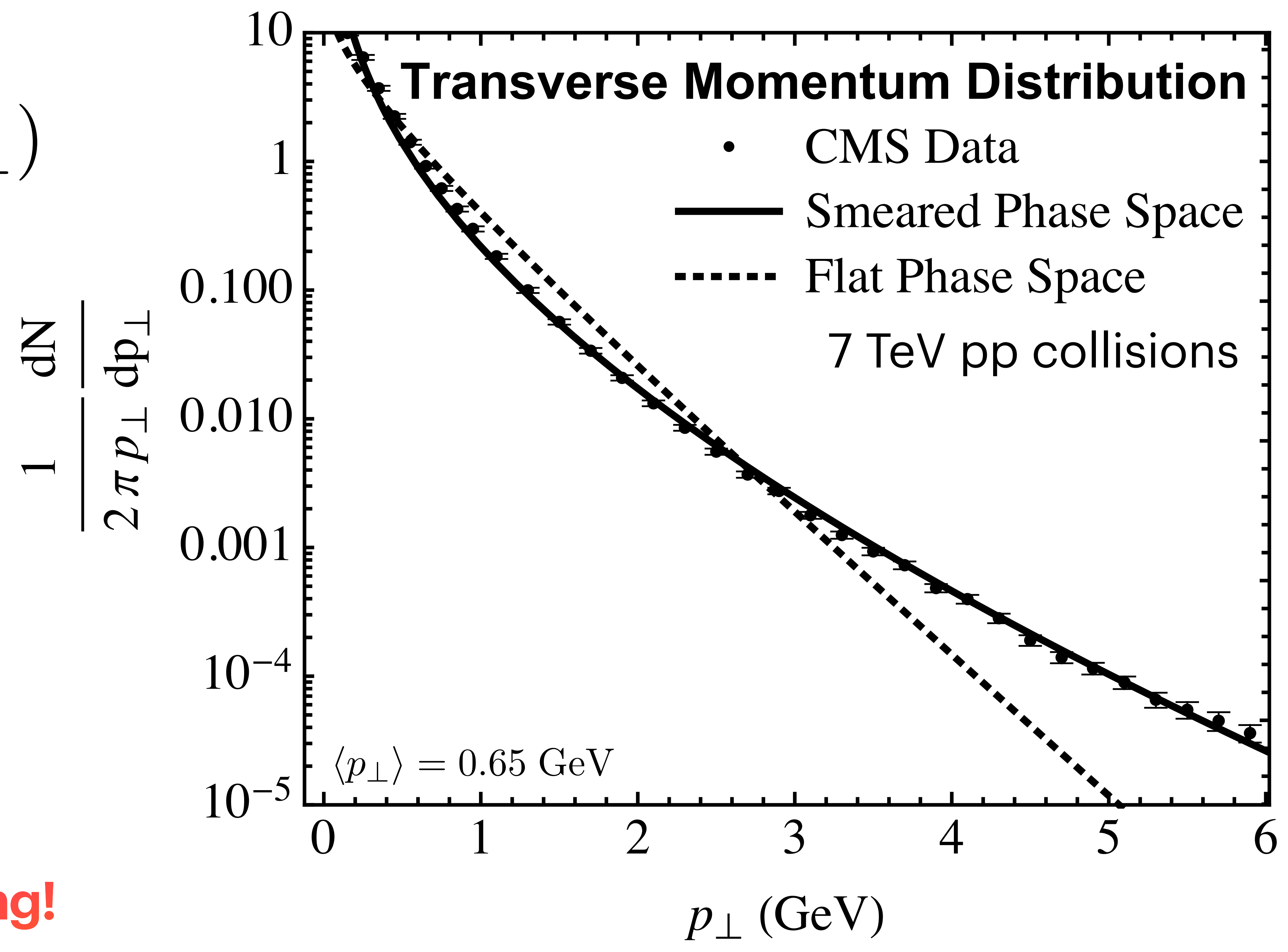
Expression for distribution depends only on variable = average pT

$$p(p_\perp) \sim e^{-\frac{3\pi}{4}\frac{p_\perp^{2/3}}{\langle p_\perp\rangle^{2/3}}}$$

Fractional dispersion…interesting!

$$\langle p_\perp\rangle \simeq \frac{\pi^{3/2}Q}{8\sqrt{n}N}$$



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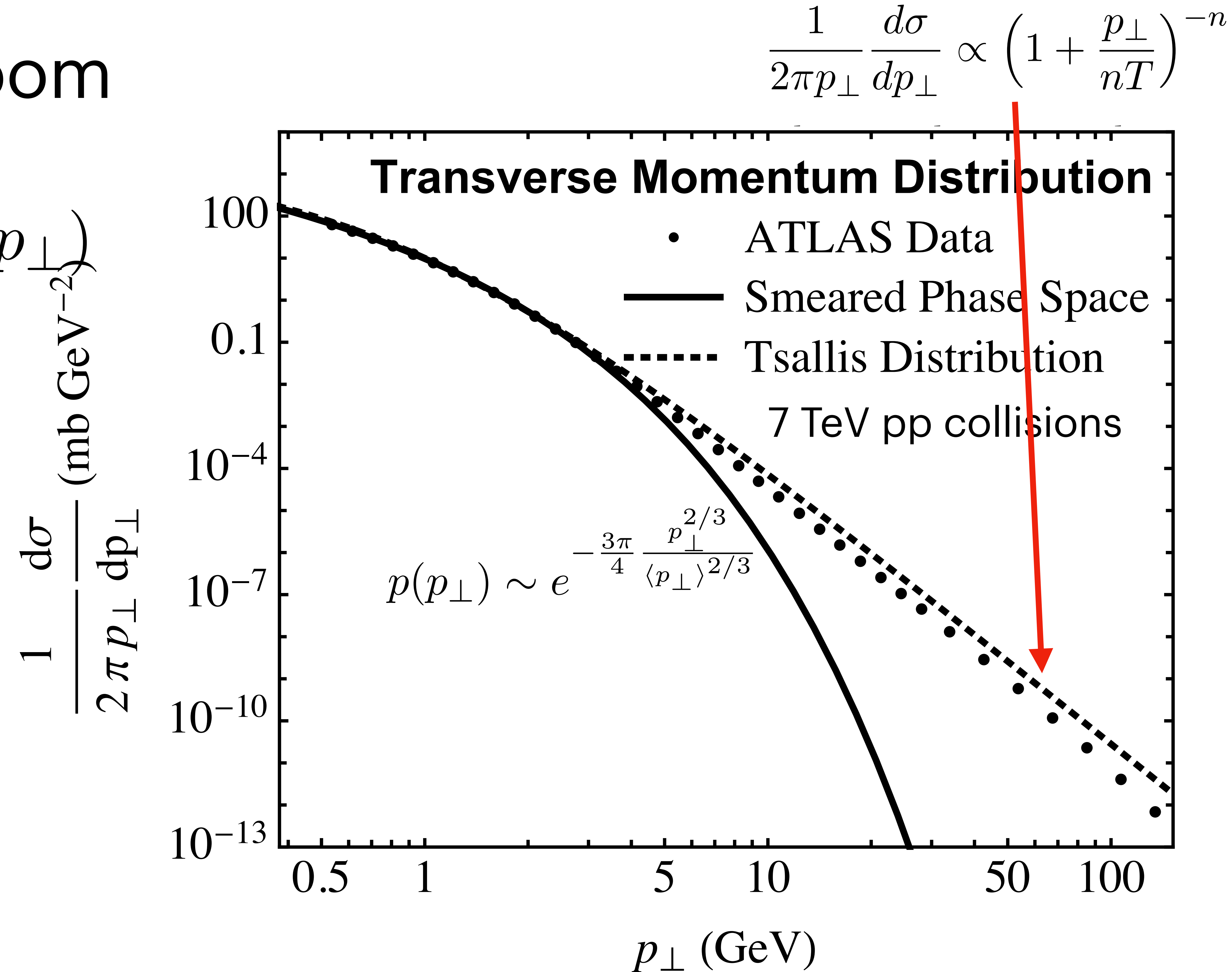
See edge of validity of the effective min bias description, does not agree at high pT as one would expect



Transverse momentum distribution

Consistency 1

$$\langle p_\perp \rangle \simeq \frac{\pi^{3/2} Q}{8\sqrt{n} N}$$

$$\langle p_\perp^2 \rangle = \int_0^\infty dp_\perp \, p_\perp^2 \, p(p_\perp) = \frac{Q^2}{nN^2}$$

Satisfies power counting $\sqrt{\langle p_\perp^2 \rangle} \sim \langle p_\perp \rangle$

$$\sqrt{\langle p_\perp^2 \rangle} = \frac{8}{\pi^{3/2}} \langle p_\perp \rangle \simeq 1.44 \langle p_\perp \rangle$$

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$$\sqrt{\langle p_\perp^2 \rangle} = \frac{8}{\pi^{3/2}} \langle p_\perp \rangle \simeq 1.44 \langle p_\perp \rangle$$

Consistency 2

$$N \simeq \frac{\pi^{3/2} Q}{8\sqrt{n} \langle p_\perp \rangle}$$

Eta fit $n = 1.6 \times 10^5$

$\langle p_\perp \rangle = 0.65\ \mathrm{GeV}$

$8\ \mathrm{TeV}$

➡ $N \simeq 21$

The predictions include (From power counting and symmetries)

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- The distribution of particle transverse momentum is universal, and depends on a single parameter, with fractional dispersion relation

- Scaling of multiplicity with collider energy

- By a positivity condition, all azimuthal correlations vanish as $N \to \infty$ at fixed collision energy

In conclusion

Min bias is theoretically interesting: there is a curious setup for an EFT (fractional dispersions/ partition functions/unusual expansion parameter)

Provide a collection of first principles predictions e.g.: particular scalings in N; dispersion relations; scalings in s

Extra

The predictions include (From power counting and symmetries)

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Scaling of multiplicity with collider energy

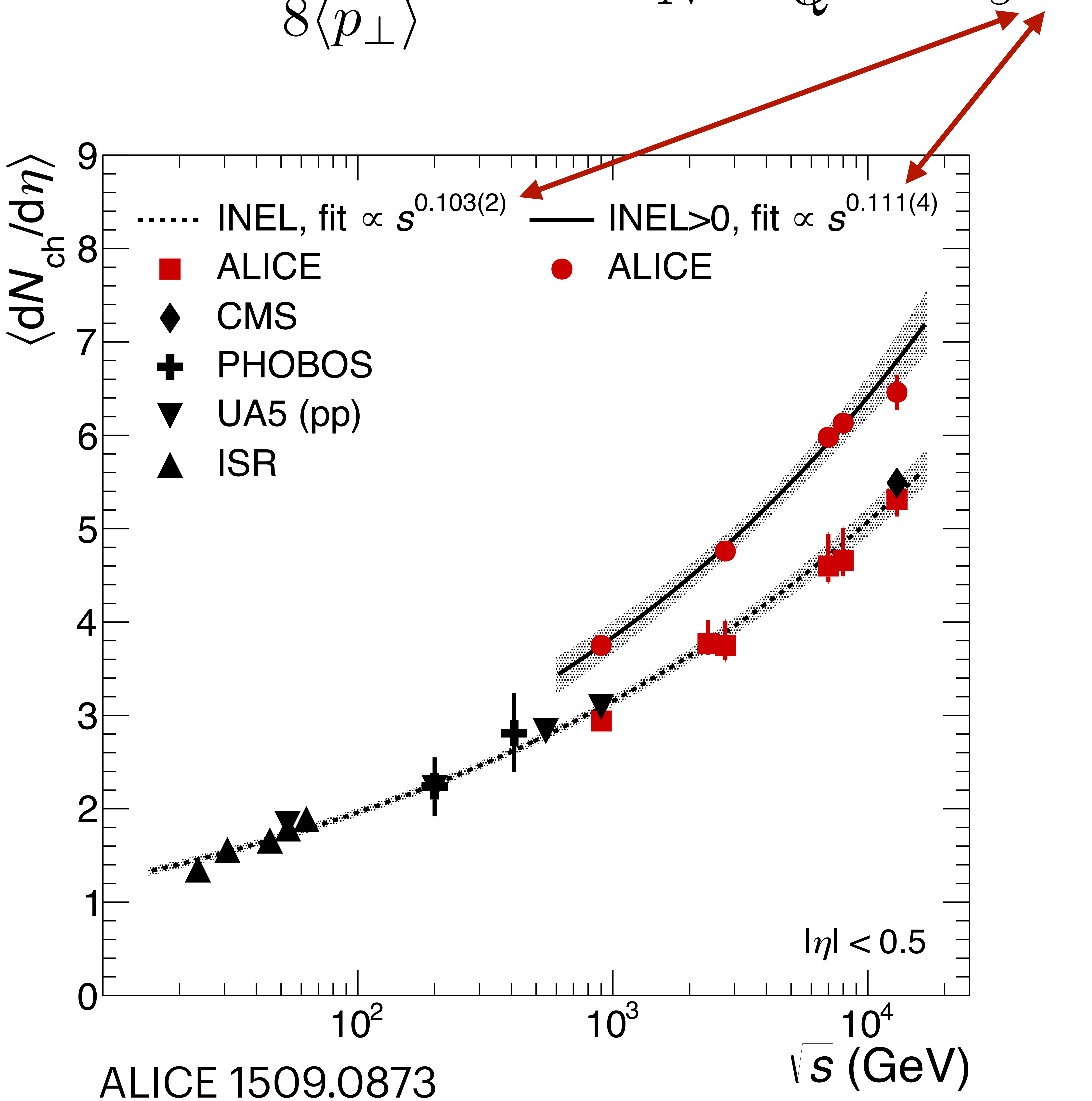
$$\langle p_\perp \rangle \simeq \frac{\pi^{3/2} Q}{8\sqrt{n} N} \qquad \Longrightarrow \quad N = \frac{\pi^{3/2} Q}{8\sqrt{n}\langle p_\perp \rangle}$$

Little n was fixed by pseudorapidity falloff

$$\eta_{\max} \simeq \log \frac{Q}{p_{\perp \mathrm{cut}}} \simeq \log n \qquad \Longrightarrow \quad N \sim \frac{\pi^{3/2} \sqrt{p_{\perp \mathrm{cut}}} Q^{1/2}}{8 \langle p_\perp \rangle}$$

Scaling of multiplicity with collider energy

$$N \sim \frac{\pi^{3/2}\sqrt{p_{\perp\mathrm{cut}}}Q^{1/2}}{8\langle p_\perp \rangle} \qquad N \sim Q^{1/2} = s^{1/4}$$



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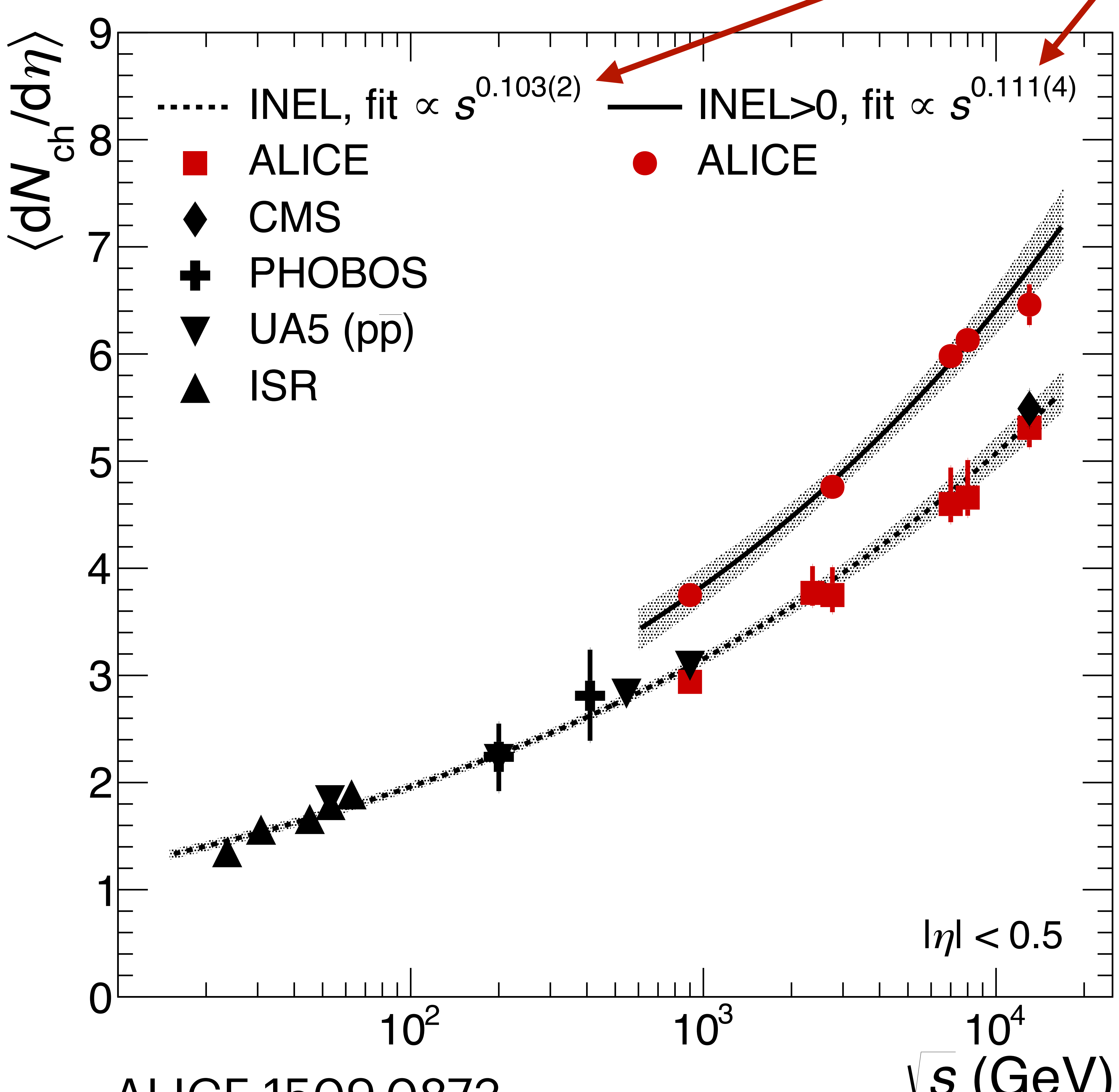
This framework connects the scaling of average pT with this measurement



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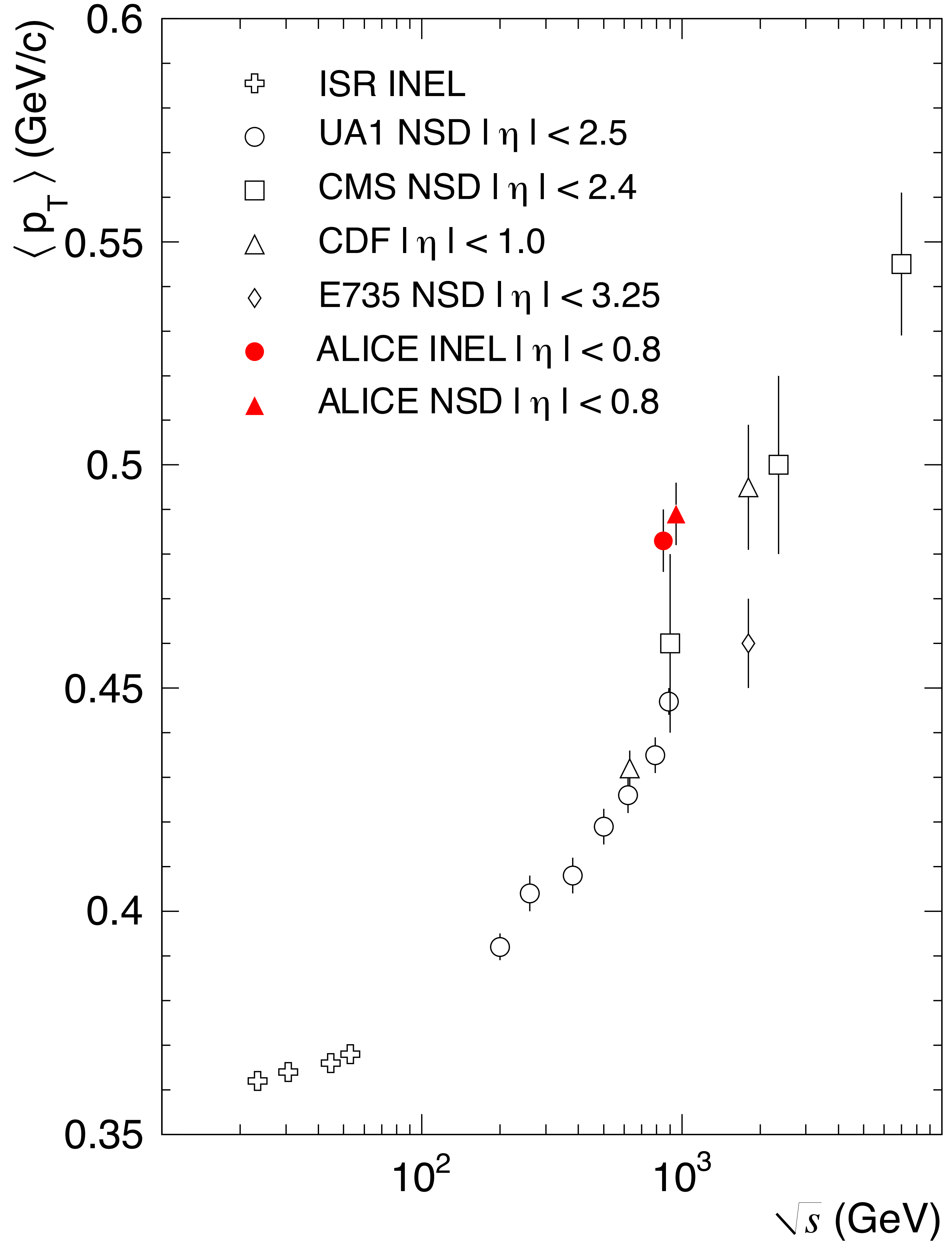

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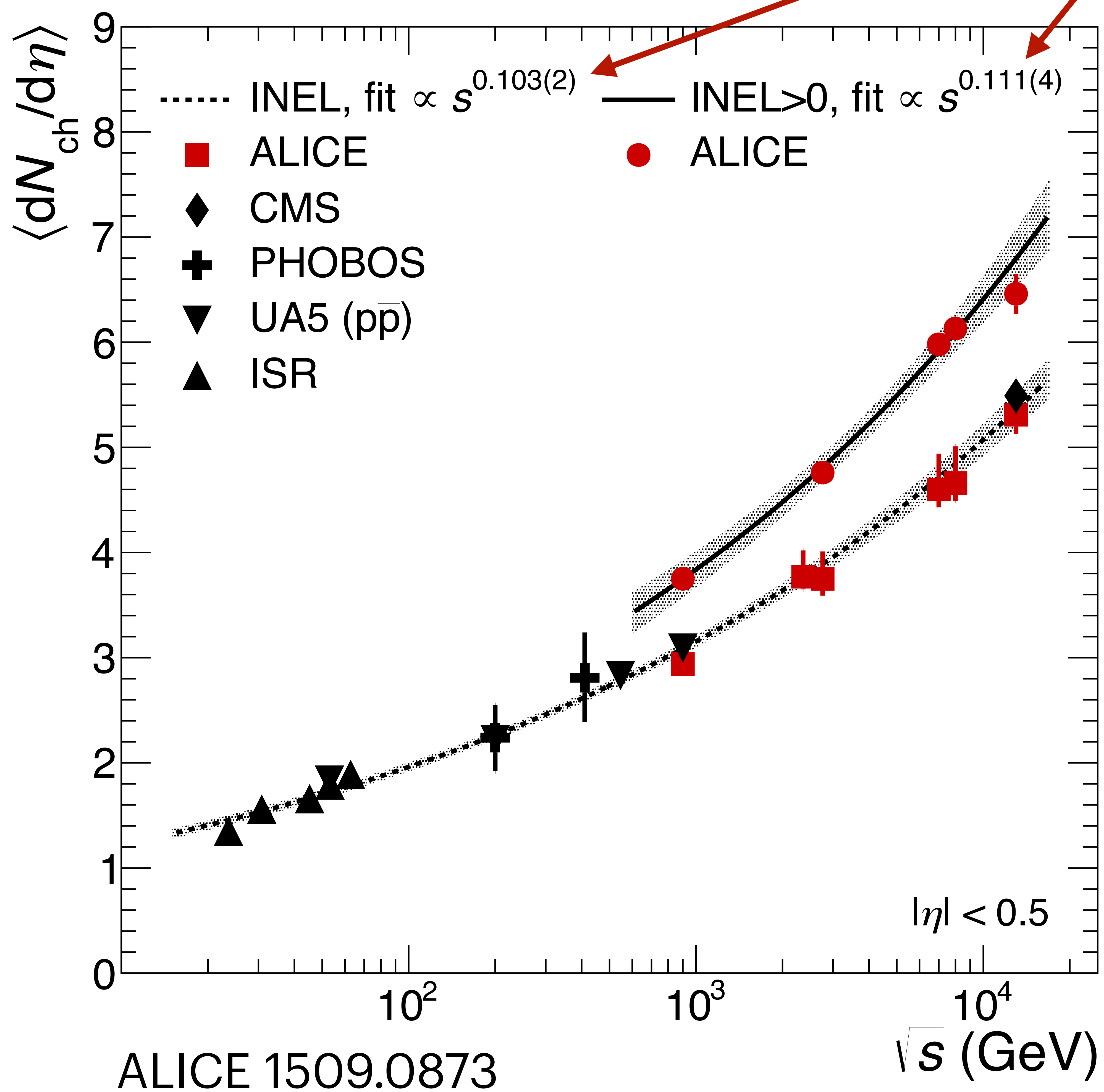

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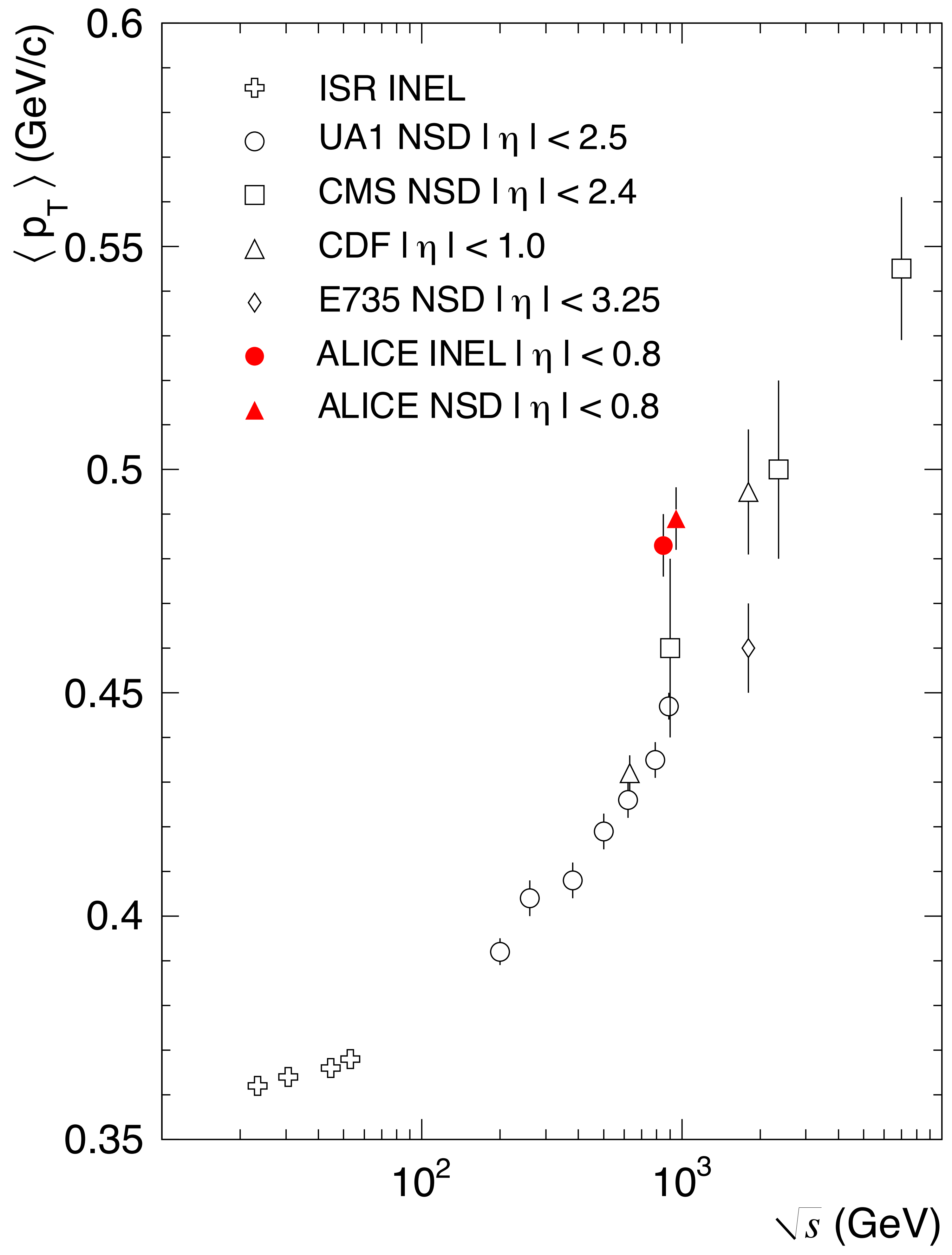
(Very) rough fit

$$\langle p_\perp \rangle \propto s^{0.1}$$

And so

$$N \propto s^{0.15}$$



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Azimuthal correlations

Correlations between pairs of particles come from terms in the matrix element of the form

$$|\mathcal{M}|^2 \supset 1 + \sum_{n=1}^{\infty} g_n(k^+k^-, N) \sum_{i \neq j}^{N} \frac{(\vec{p}_{\perp i} \cdot \vec{p}_{\perp j})^n}{Q^{2n}}$$

$$\supset 1 + \sum_{n=1}^{\infty} g_n(k^+k^-, N) \sum_{i \neq j}^{N} \frac{p_{\perp i}^n p_{\perp j}^n}{Q^{2n}} \cos(n(\phi_i - \phi_j))$$

Which, by ergodic assumption and power counting

$$\sim 1 + \sum_{n=1}^{\infty} \frac{g_n(k^+k^-, N)}{N^{2n}} \sum_{i \neq j}^{N} \cos(n(\phi_i - \phi_j))$$

Azimuthal correlations

Now, azimuthal part of flat phase space as N->Infinity

$$\int d\Pi_N \to \int_0^{2\pi} \prod_{i=1}^{N} \frac{d\phi_i}{2\pi}$$

Mean of sum of azimuthal correlations vanishes in this limit

$$\int_0^{2\pi} \prod_{i=1}^{N} \frac{d\phi_i}{2\pi} \sum_{j\neq k}^{N} \cos(n(\phi_j - \phi_k)) = 0$$

The variance, on the other hand

$$\sigma^2 \equiv \int_0^{2\pi} \prod_{i=1}^{N} \frac{d\phi_i}{2\pi} \left[\sum_{j\neq k}^{N} \cos(n(\phi_j - \phi_k))\right]^2 = N^2 \int_0^{2\pi} \prod_{i=1}^{N} \frac{d\phi_i}{2\pi} \cos^2(n(\phi_1 - \phi_2)) = \frac{N^2}{2}$$

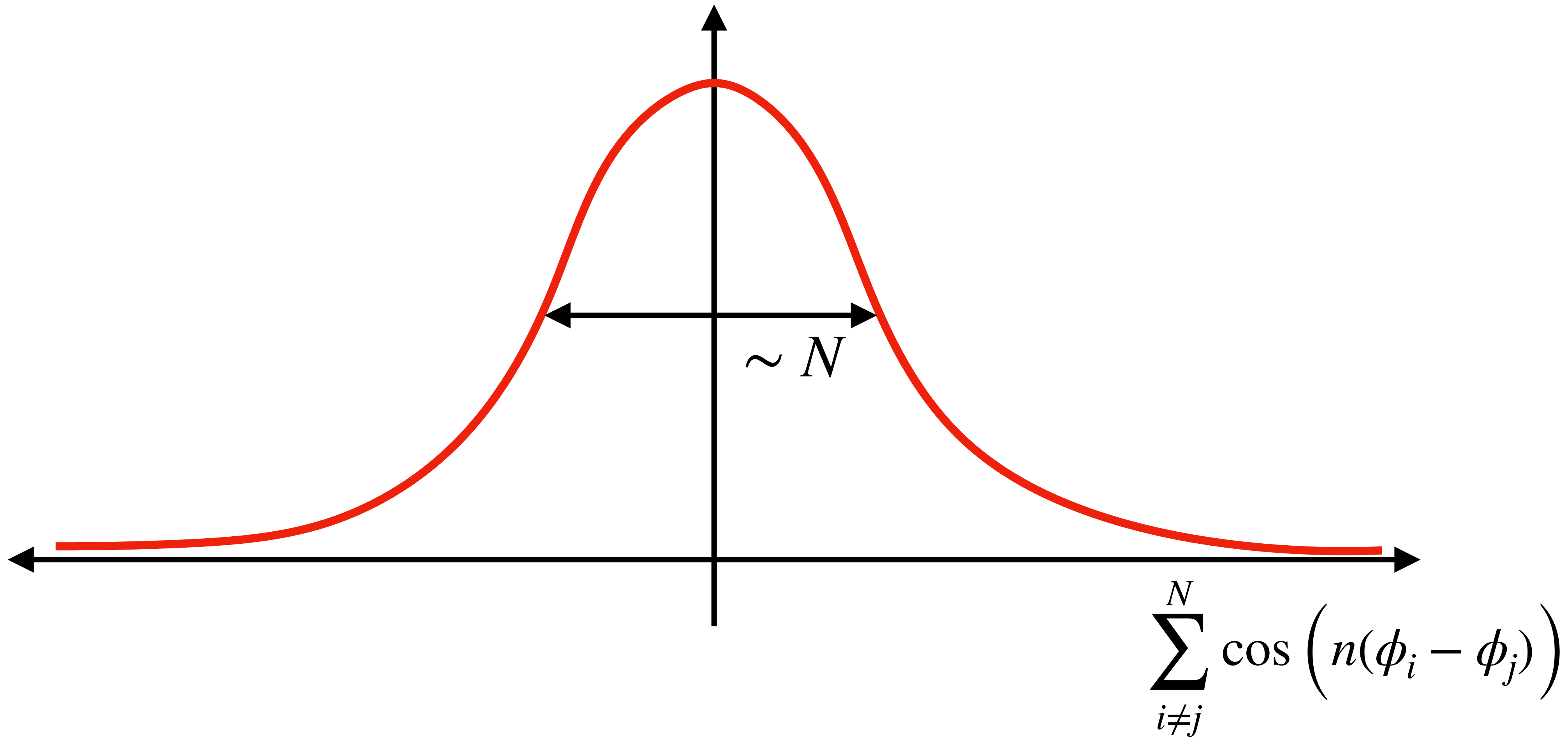
Azimuthal correlations

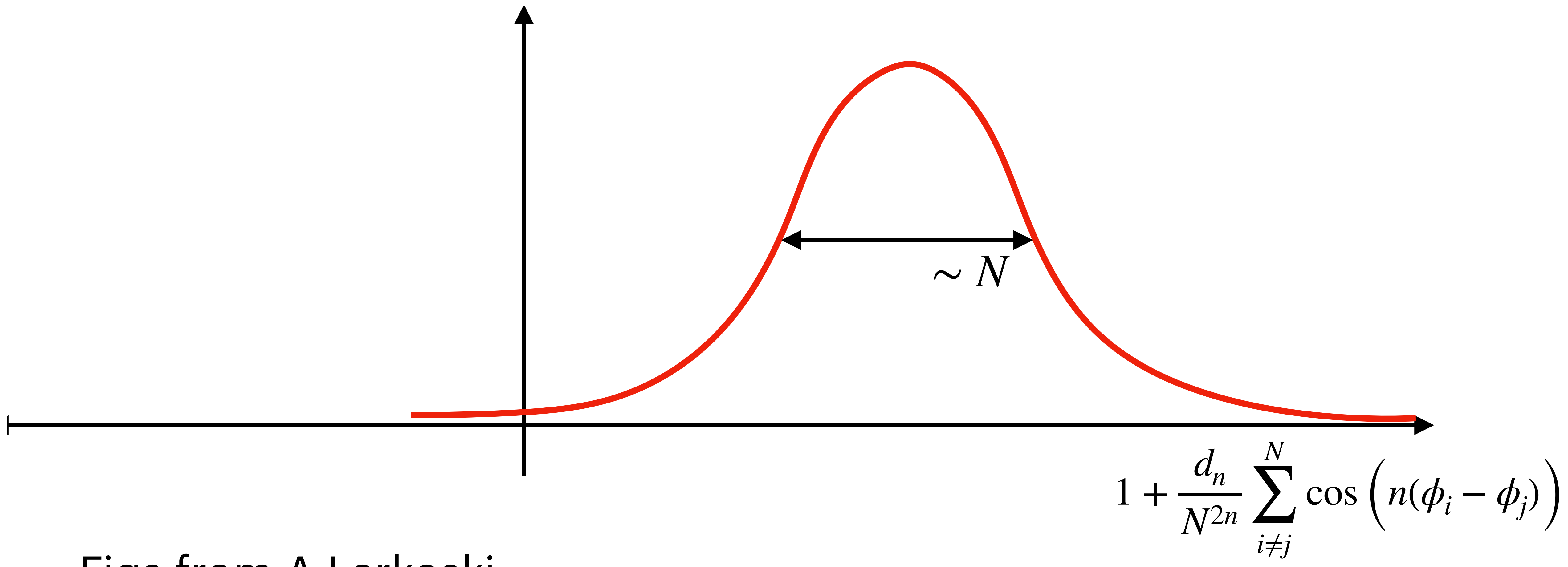
Going back to the matrix element

$$|\mathcal{M}|^2 \sim 1 + \sum_{n=1}^{\infty} \frac{g_n(k^+k^-, N)}{N^{2n}} \sum_{i\neq j}^{N} \cos(n(\phi_i - \phi_j))$$

Positivity at *all* points in phase space requires

$$1 \gtrsim \frac{g_n(k^+k^-, N)}{N^{2n}} \sum_{i\neq j}^{N} \cos(n(\phi_i - \phi_j)) \sim \frac{g_n(k^+k^-, N)}{N^{2n}} \sigma \sim \frac{g_n(k^+k^-, N)}{N^{2n-1}}$$





Figs from A Larkoski

Azimuthal correlations

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And so scaling with N of the coefficients to retain matrix element squared positivity in large N limit

$$g_n(k^+k^-, N) \lesssim N^{2n-1}$$

Azimuthal correlations

Fourier expansion of probability distribution

$$p(\Delta\phi) \;=\frac{1}{2\pi}+\frac{1}{\pi}\sum_{n=1}^{\infty}\frac{d_n(N)}{N^{2n}}\;\cos(n\,\Delta\phi)$$

In terms of matrix element coefficients

$$d_n(N)=\frac{1}{2Q^2}\int_0^Q dk^+\int_0^Q dk^-\;f(k^+k^-)\,g_n(k^+k^-,N)$$

$$g_n(k^+k^-,N)\lesssim N^{2n-1} \quad\Rightarrow\quad d_n(N)\lesssim N^{2n-1}$$

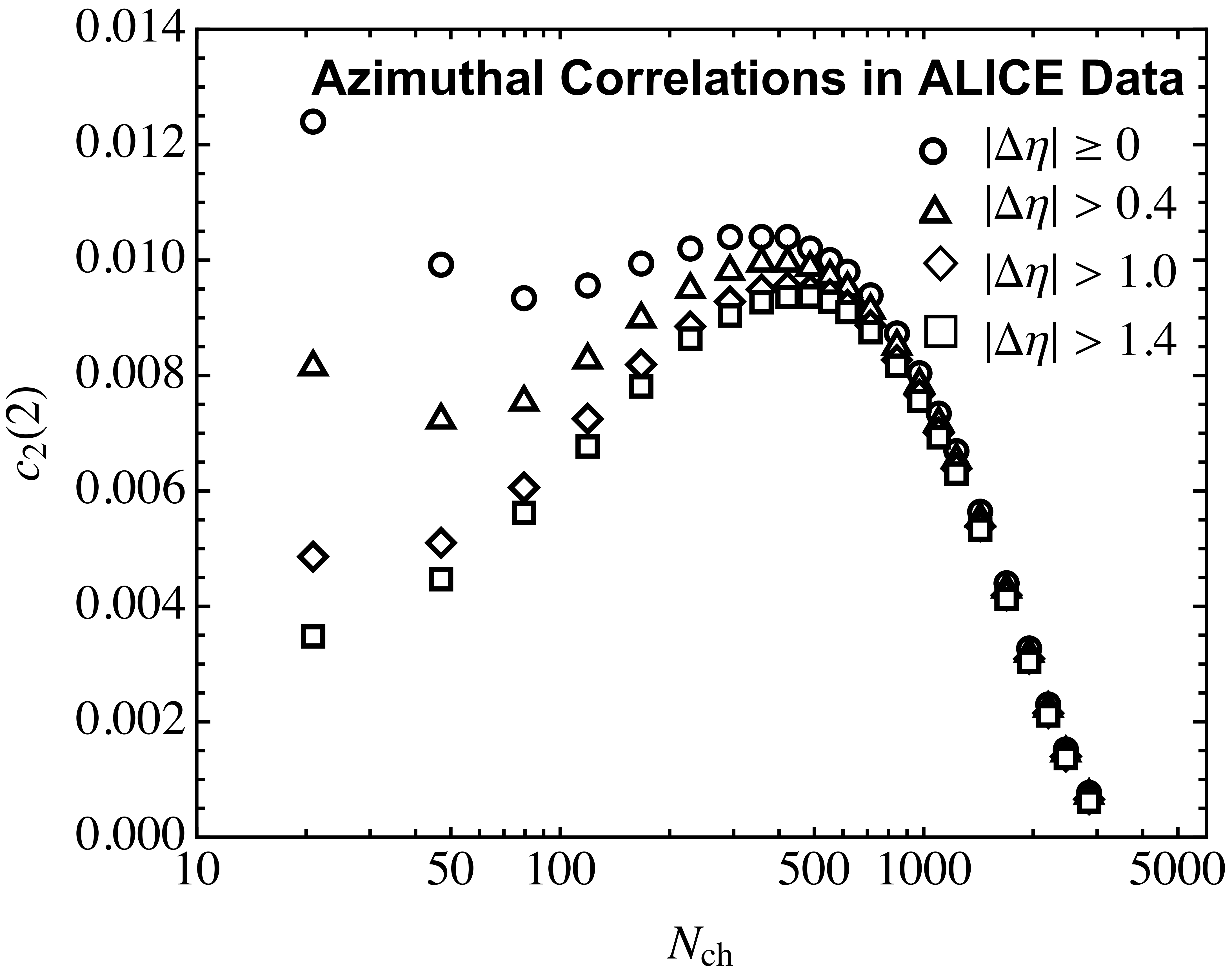
i.e. Azimuthal correlations vanish at large N

$$\lim_{N\to\infty}\frac{d_n(N)}{N^{2n}}=0$$

Ellipticity

First non-trivial azimuthal correlation used as evidence for collective flow/ QGP

Proxy to flow in reaction plane often used is pairwise azimuthal correlation moment



$\sqrt{s_{NN}} = 2.76$ TeV

$|\eta| < 1$

$0.2 < p_{\perp} < 3.0$ GeV

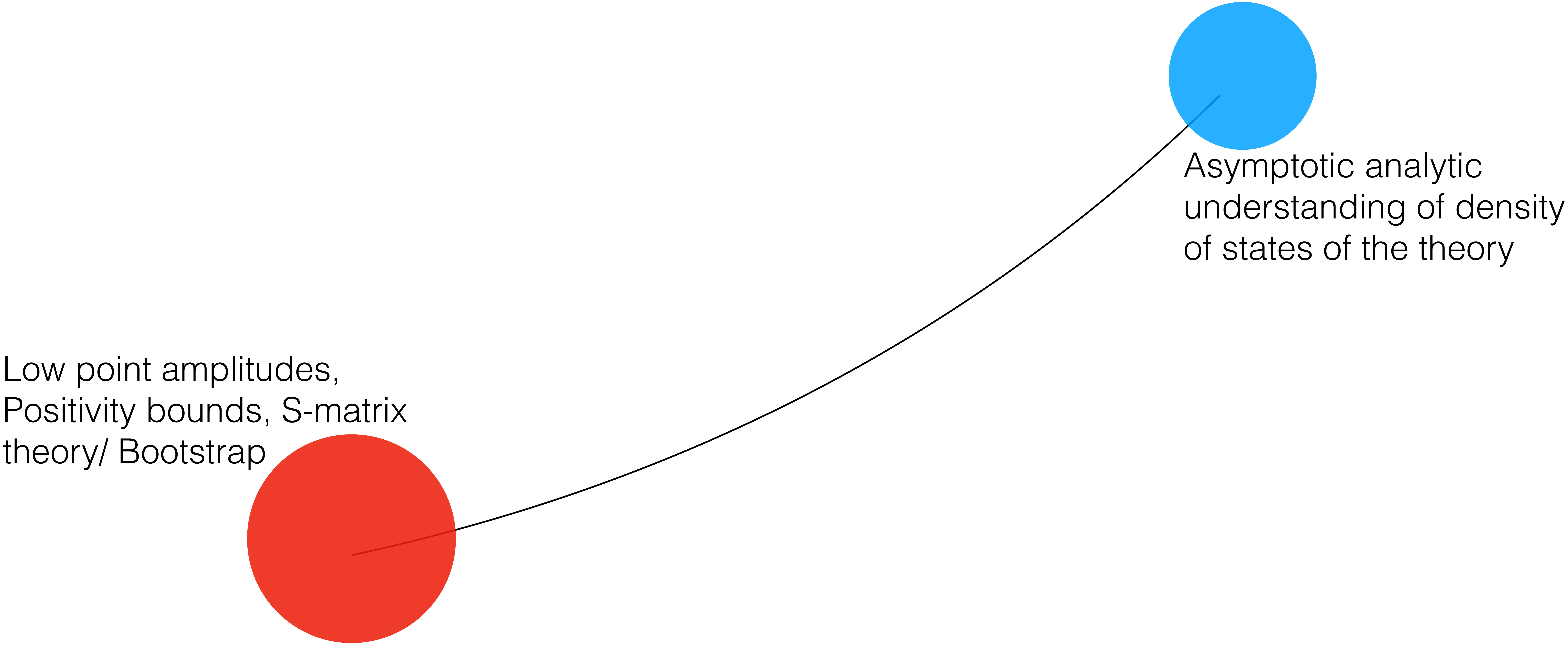
arXiv:1406.2474

The vanishing at large N predicted by the above analysis is borne out in data. May be interpreted in models with collision parameter (centrality) -> 0

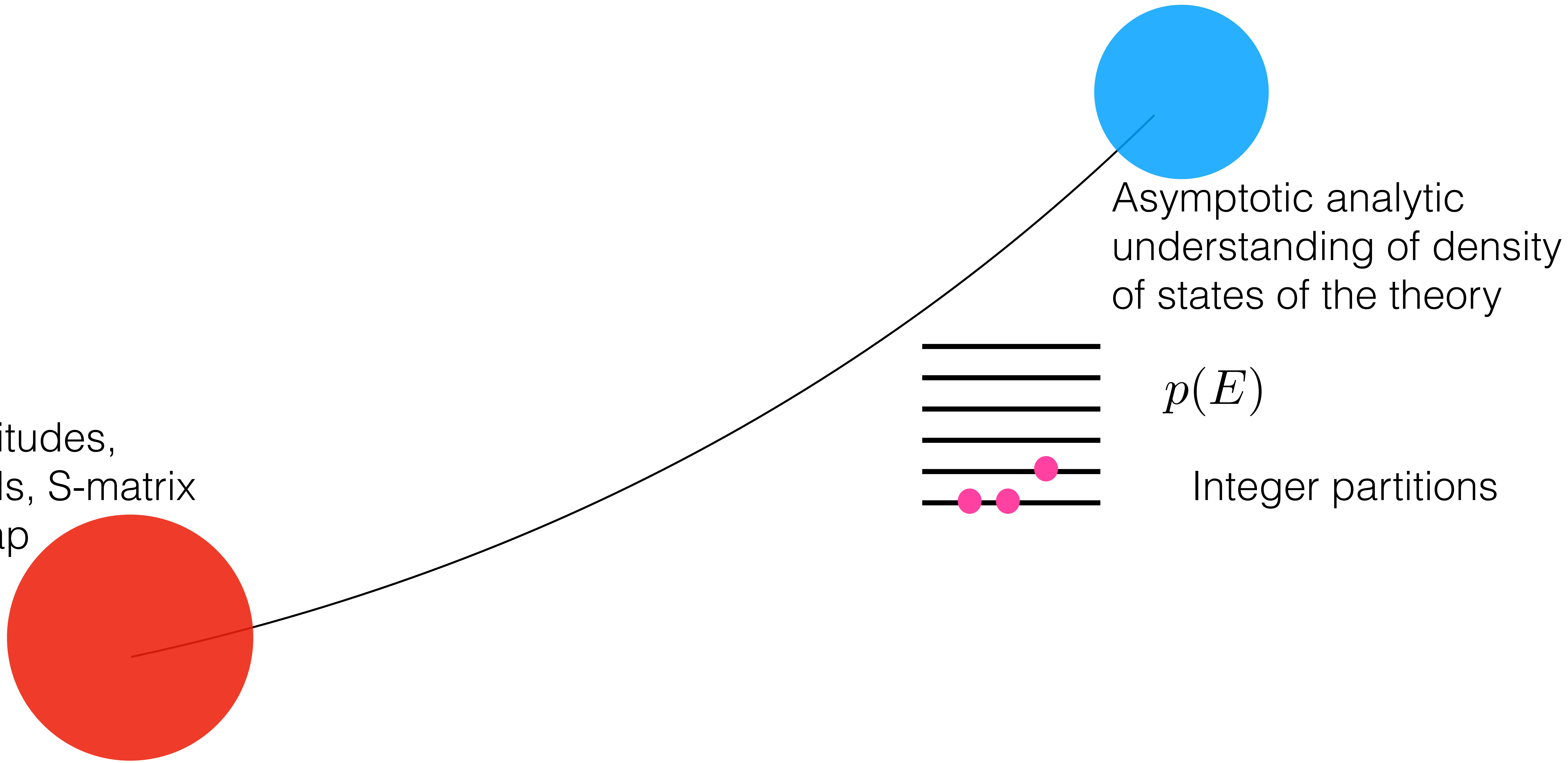
$$c_2(2) = \frac{1}{N} + \int_0^{2\pi} d\Delta\phi\, p(\Delta\phi)\, \cos(2\Delta\phi) = \frac{1}{N} + \frac{d_2(N)}{N^4}$$

What does large N buy us?

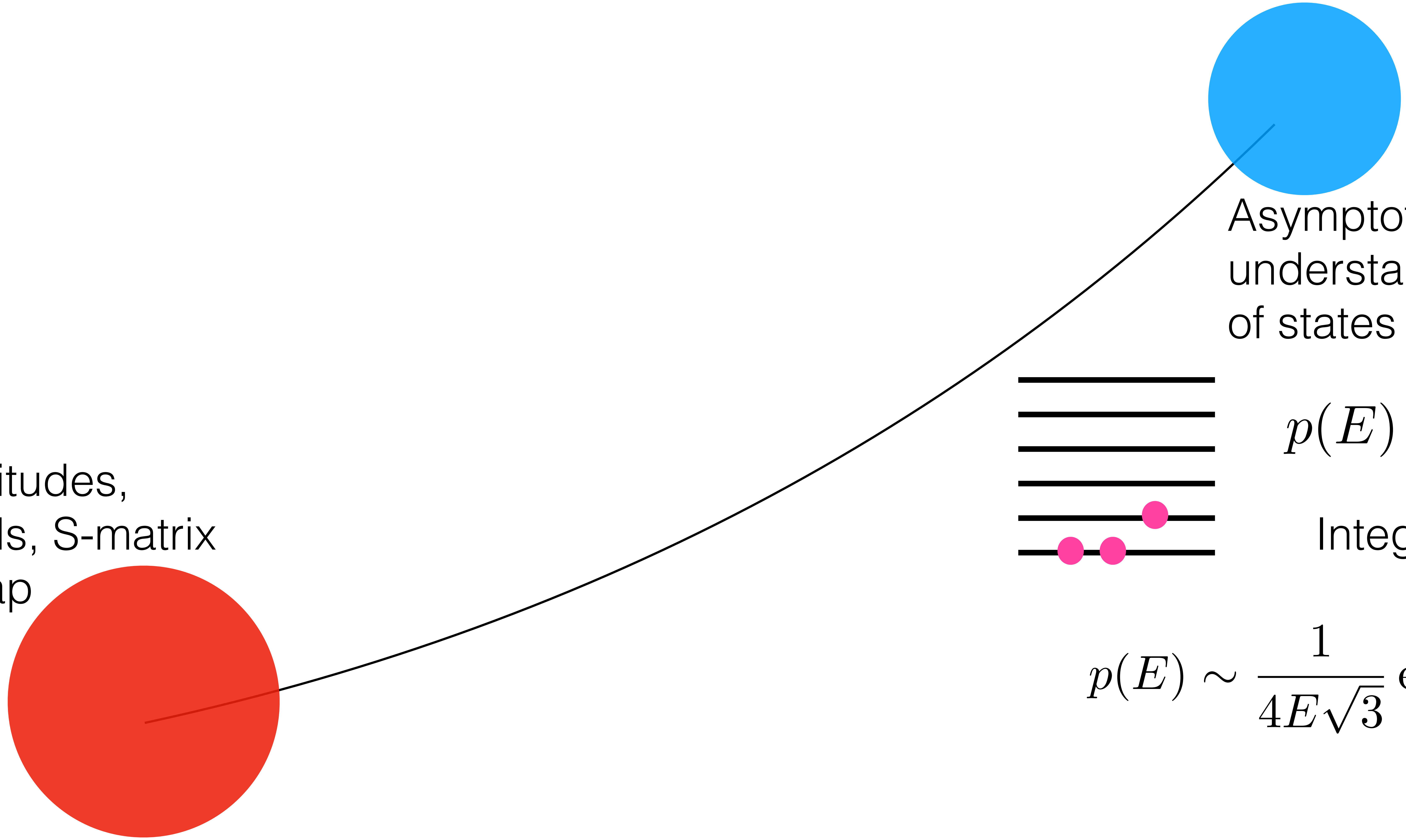
Analytic probes of S-matrix



Analytic probes of S-matrix



Analytic probes of S-matrix



Low point amplitudes, Positivity bounds, S-matrix theory/ Bootstrap

Asymptotic analytic understanding of density of states of the theory

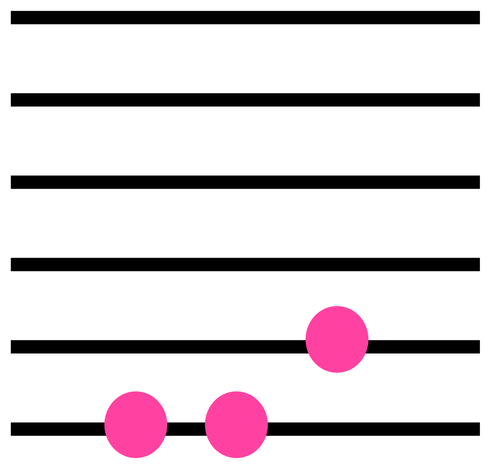
$p(E)$

Integer partitions

$$p(E) \sim \frac{1}{4E\sqrt{3}} \exp\left(\pi\sqrt{\frac{2E}{3}}\right)$$

Analytic probes of S-matrix

Asymptotic analytic understanding of density of states of the theory



$p(E)$

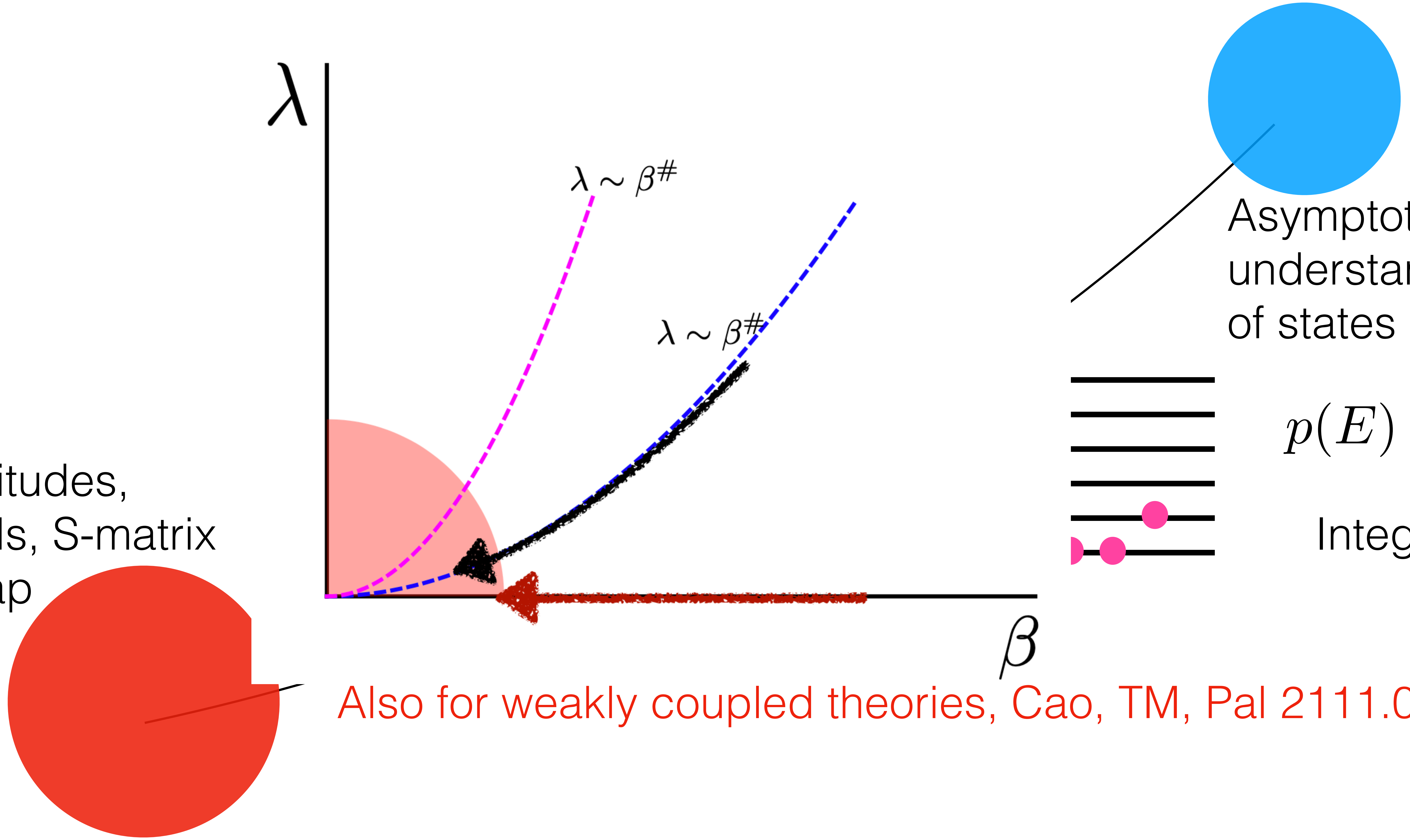
Integer partitions

Low point amplitudes, Positivity bounds, S-matrix theory/ Bootstrap

For the standard model, TM, Pal 2010.08560

$$p(\Delta) \sim \frac{50674491\; 3^{5/8} \left(\frac{31}{5}\right)^{3/8} 7^{7/8} \pi^{10}}{131072000 \sqrt[4]{2}\sqrt{13}\,\Delta^{55/8}} \exp\left(\frac{2}{3}\sqrt{2}\sqrt[4]{\frac{217}{15}}\pi\Delta^{3/4} - \frac{37\sqrt[4]{\frac{15}{217}}\pi}{4\sqrt{2}}\Delta^{1/4} + 28\zeta'(-2)\right)$$

Analytic probes of S-matrix



Asymptotic analytic understanding of density of states of the theory

Integer partitions

Low point amplitudes, Positivity bounds, S-matrix theory/ Bootstrap

Also for weakly coupled theories, Cao, TM, Pal 2111.07472

Analytic probes of S-matrix

Bootstrap approach?

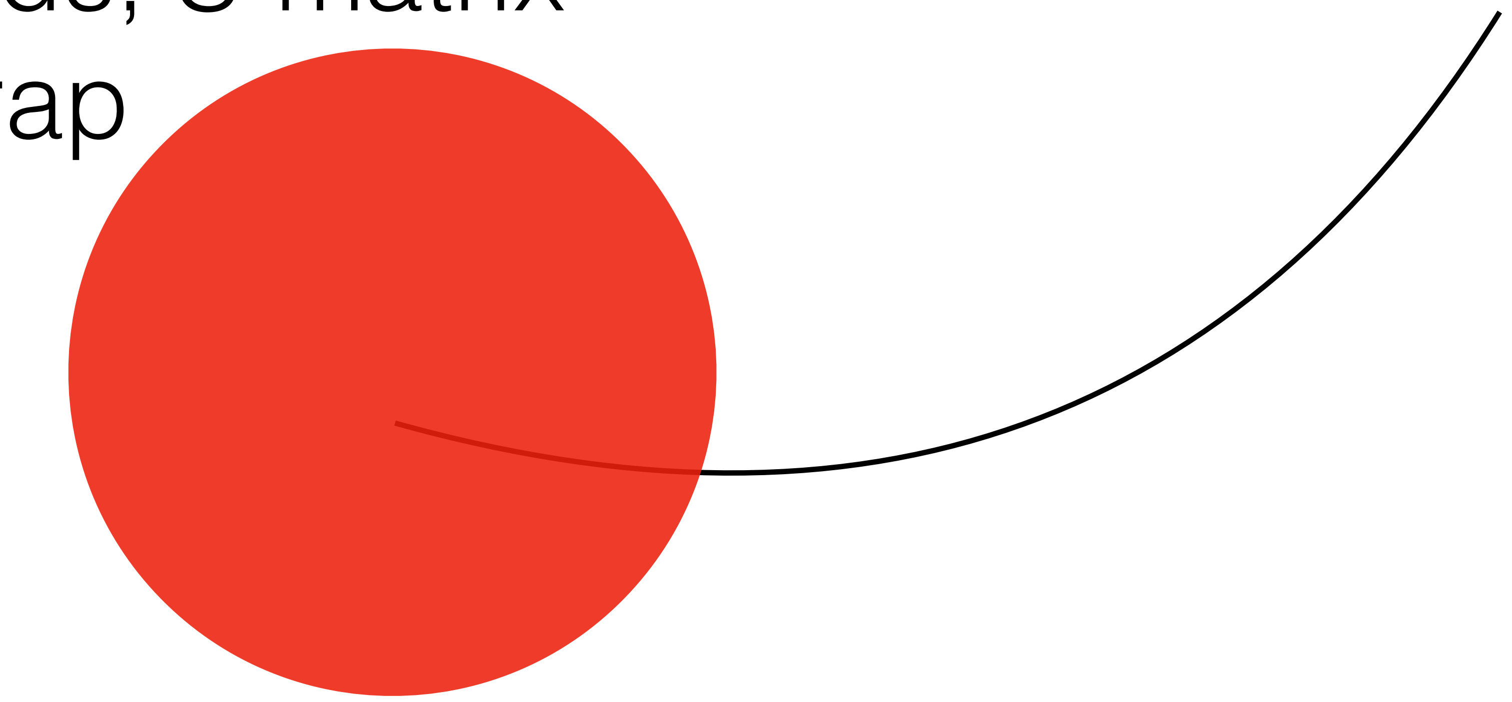
pp or AA to N hadrons has *some* S-matrix element, that has to obey certain symmetries

Understanding of strongly coupled theories from a bootstrap approach, recently been applied to QFT, i.e. to the S-matrix

Recent: M. F. Paulos, J. Penedones, J. Toledo, B. C. van Rees, and P. Vieira, ’16, ’17. More recently e.g. L. Cordova and P. Vieira, ’18; D. Mazac and M. Paulos ’18,’19; Cordova, He, Kruczenski, Vieira, ’19; Karateev, Kuhn, Penedones ’19; Correia, Sever, Zhiboedov, ’20; Homrich, Penedones, Toledo, van Rees, Vieira, ’20 …

Those are 2 to 2. This is 2 to N>>1

Low point amplitudes, Positivity bounds, S-matrix theory/ Bootstrap



Addendum: manipulating flat phase space in the Large N limit

$$\int d\Pi_N = (2\pi)^{4-3N} Q^{2N-4} \frac{2\pi^{N-1}}{(N-1)!(N-2)!}$$

Pseudorapidity

$$Q^2 = k^+ k^-$$

$$\begin{aligned}
p_{\text{flat}}(\eta) &\sim \lim_{N\to\infty} \int \prod_{i=1}^{N} \left[p_{\perp i}\, dp_{\perp i}\, d\eta_i\, \frac{d\phi_i}{2\pi} \right] \delta(\eta - \eta_1) \\
&\quad \times \delta\left(k^- - \sum_i p_{i\perp} e^{\eta_i}\right) \delta\left(k^+ - \sum_i p_{i\perp} e^{-\eta_i}\right) \delta^{(2)}\left(\sum_i \vec{p}_{i\perp}\right) \\
&\propto \lim_{N\to\infty} \int dp_{\perp 1}\, p_{\perp 1}\, d\eta_1\, \delta(\eta - \eta_1) \left[(k^+ - p_{\perp 1} e^{-\eta_1})(k^- - p_{\perp 1} e^{\eta_1}) - p_{\perp 1}^2 \right]^N \\
&\propto \lim_{N\to\infty} \int dp_\perp\, p_\perp \left(1 - \frac{k^+ e^{\eta} + k^- e^{-\eta}}{k^+ k^-} p_\perp \right)^N \\
&= \int_0^\infty dp_\perp\, p_\perp\, e^{-\frac{k^+ e^{\eta} + k^- e^{-\eta}}{k^+ k^-} N p_\perp} \\
&= \frac{(k^+ k^-)^2}{N^2} \left(k^+ e^{\eta} + k^- e^{-\eta} \right)^{-2} .
\end{aligned}$$

Transverse mom

$$p_{\text{flat}}(p_\perp) \propto p_\perp \int_{-\infty}^{\infty} d\eta\, e^{-\frac{k^+ e^{\eta} + k^- e^{-\eta}}{k^+ k^-} N p_\perp} = p_\perp\, K_0\left(\frac{2 N p_\perp}{\sqrt{k^+ k^-}} \right)$$

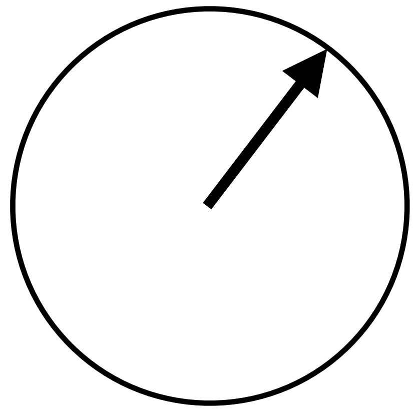
Lorentz invariant phase space is a Stiefel Manifold

Henning, TM arxiv:1902.06747

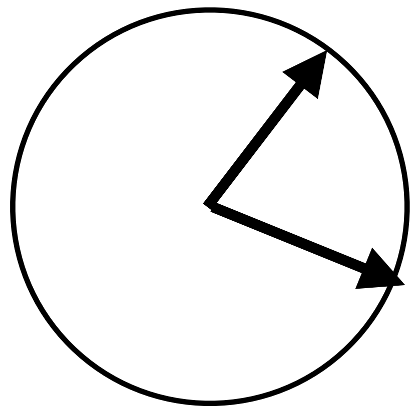
$$\delta^{(4)}(p_a + p_b - p_1 - \ldots - p_N) \prod_{i=1}^{N} \delta(p_i^2 - m_i^2)$$

Momentum conservation On-shell

Sphere $\dfrac{O(N)}{O(N-1)}$



Stiefel $\dfrac{O(N)}{O(N-2)}$



(& then Complexified, O -> U)