The 2nd Asian-European-Institute workshop for BSM & the 10th KIAS workshop on Particle Physics and Cosmology
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RGE studies in the 2HDM

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w/ S.K. Kang, J. Kim, S. Lee, C.T. Lu, K. Cheung, P. Sanyal

[2204.10338] [2205.01701] [2207.05104] [2210.00020]

The 2nd Asian-European-Institute workshop for **BSM** & the 10th KIAS workshop on Particle Physics and Cosmology

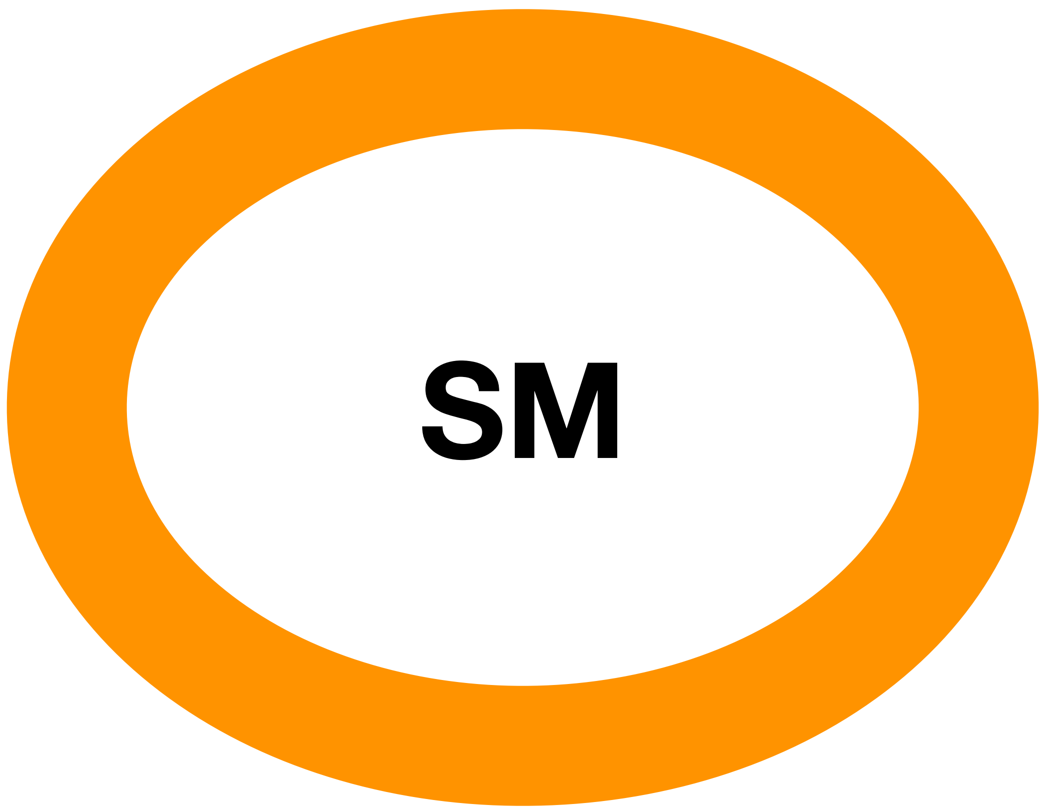
Theoretical structures of "Beyond" the SM

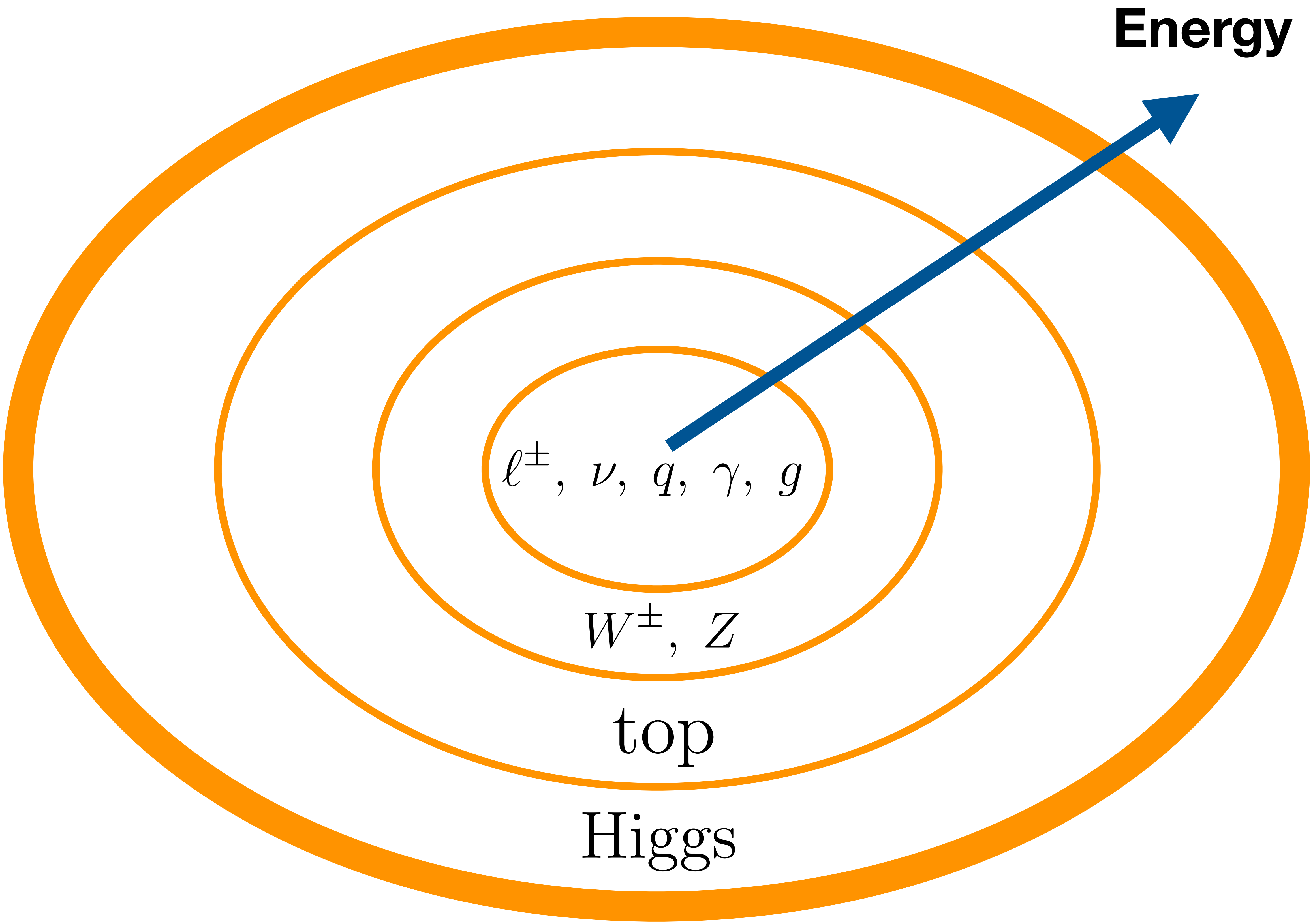

Beyond the …….

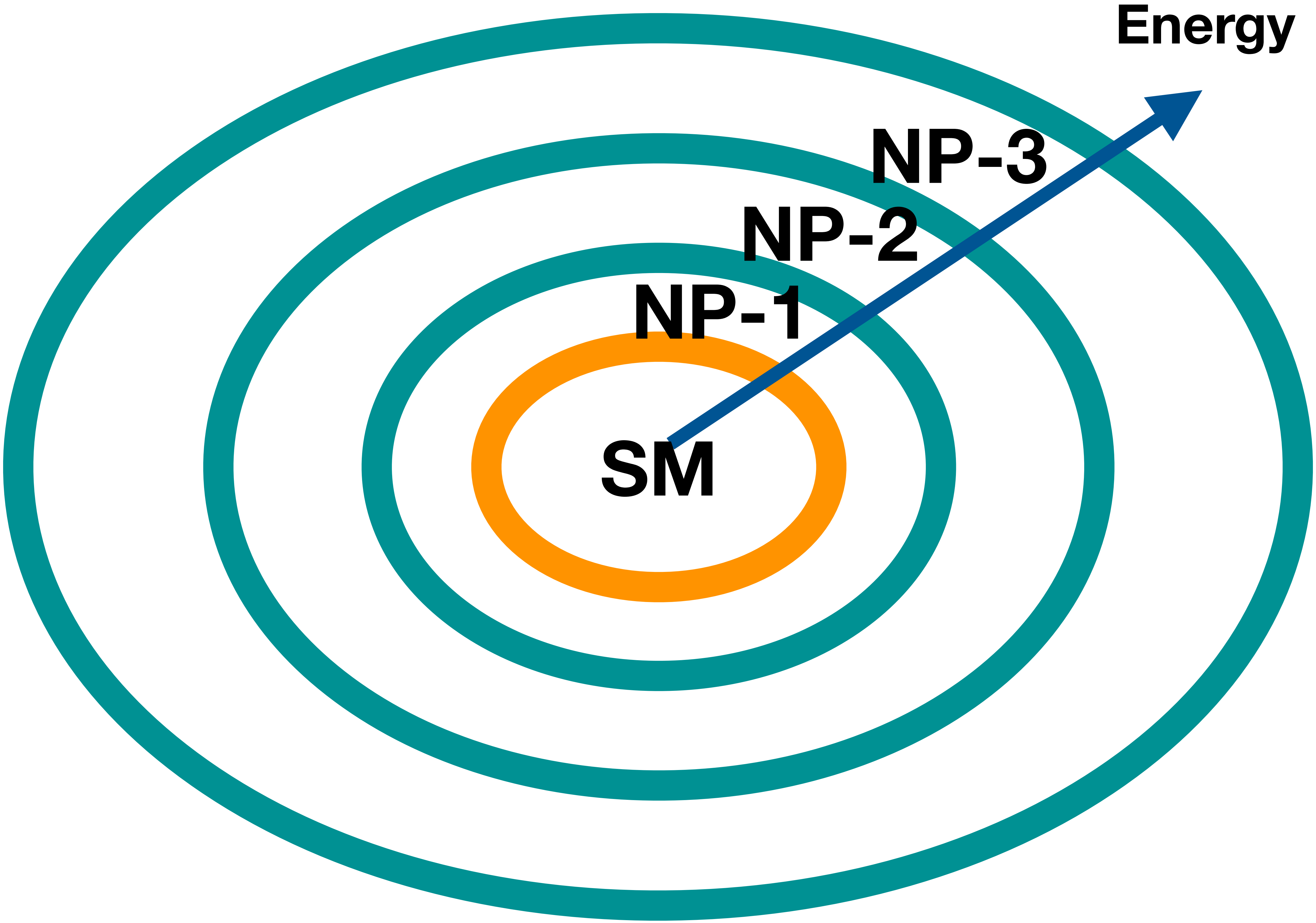
We need to go beyond the wall, looking for NP.

Is there only one wall?

NP




Energy
$\ell^\pm,\ \nu,\ q,\ \gamma,\ g$
$W^\pm,\ Z$
top
Higgs


Energy
NP-3
NP-2
NP-1
SM

If we observe some signals of a NP model, how can we know its credibility?

For the given NP-1 model,

the cutoff scale can tell the next-level BSM model.

1. 2HDM & RGE

2. RGE effects on the CDF W boson mass measurement

3. RGE effects on the muon g-2

4. How to disentangle the high- and low-cutoff scales

5. Conclusions

1. 2HDM & RGE

- Basic theory setup

$$\Phi_i = \begin{pmatrix} w_i^+ \\ \dfrac{v_i + h_i + i\eta_i}{\sqrt{2}} \end{pmatrix}, \quad i = 1, 2,$$

where $v = \sqrt{v_1^2 + v_2^2} = 246\ \text{GeV}.$

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- Discrete Z_2 symmetry to avoid tree-level FCNC

$$\Phi_1 \to \Phi_1, \quad \Phi_2 \to -\Phi_1$$

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- Discrete Z_2 symmetry to avoid tree-level FCNC

$$\Phi_1 \to \Phi_1, \quad \Phi_2 \to -\Phi_1$$

- Scalar potential with CP-invariance

$$\boxed{\begin{aligned} V_\Phi = \; & m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - m_{12}^2 (\Phi_1^\dagger \Phi_2 + \text{H.c.}) \\ & + \frac{1}{2}\lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \frac{1}{2}\lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) \\ & + \frac{1}{2}\lambda_5 \left[(\Phi_1^\dagger \Phi_2)^2 + \text{H.c.} \right], \end{aligned}}$$

- Four types

	Φ_1	Φ_2	u_R	d_R	ℓ_R	Q_L, L_L
Type I	$+$	$-$	$-$	$-$	$-$	$+$
Type II	$+$	$-$	$-$	$+$	$+$	$+$
Type X	$+$	$-$	$-$	$-$	$+$	$+$
Type Y	$+$	$-$	$-$	$+$	$-$	$+$

$$-\mathcal{L}_{\text{Yukawa}} = Y_{u2}\,\overline{Q}_L\,\tilde{\Phi}_2\,u_R + Y_{d2}\,\overline{Q}_L\,\Phi_2\,d_R + Y_{\ell 1}\,\overline{L}_L\,\Phi_1\,e_R + \text{h.c.}$$

- SM Higgs boson

$$\boxed{h_{\rm SM} = s_{\beta-\alpha} h + c_{\beta-\alpha} H.}$$

- 2 scenarios: normal scenario (NS) & inverted scenario (IS)

$$\boxed{\begin{aligned} \text{NS:} \quad & m_h = m_{125}; \\ \text{IS:} \quad & M_H = m_{125}, \end{aligned}}$$

- Higgs alignment limits

$$\boxed{\begin{aligned} \text{NS:} \quad & s_{\beta-\alpha} = 1 \\ \text{IS:} \quad & c_{\beta-\alpha} = 1 \end{aligned}}$$

- theoretical stability ➡ quartic couplings cannot be too large

In the Higgs alignment limit

$$\boxed{M^2 = m_{12}^2/(s_\beta c_\beta)}$$

$$
\begin{aligned}
\lambda_1 &= \frac{1}{v^2}\left[m_{125}^2 + t_\beta^2\left(m_{\varphi^0}^2 - M^2\right)\right], \\
\lambda_2 &= \frac{1}{v^2}\left[m_{125}^2 + \frac{1}{t_\beta^2}\left(m_{\varphi^0}^2 - M^2\right)\right], \\
\lambda_3 &= \frac{1}{v^2}\left[m_{125}^2 - m_{\varphi^0}^2 - M^2 + 2M_{H^\pm}^2\right], \\
\lambda_4 &= \frac{1}{v^2}\left[M^2 + M_A^2 - 2M_{H^\pm}^2\right], \\
\lambda_5 &= \frac{1}{v^2}\left[M^2 - M_A^2\right],
\end{aligned}
$$

Similar masses of BSM Higgs boson for large tan *β*

- theoretical stability ➡ quartic couplings cannot be too large

In the Higgs alignment limit

$$\boxed{M^2 = m_{12}^2/(s_\beta c_\beta)}$$

$$\lambda_1 = \frac{1}{v^2}\left[m_{125}^2 + t_\beta^2\left(\boxed{m_{\varphi^0}^2} - M^2\right)\right],$$

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Similar masses of BSM Higgs boson for large tan β

RGE

- Running of gauge couplings

$$16\pi^2\beta_{g_3} = -7g_3^3,$$

$$16\pi^2\beta_{g_2} = \left(-\frac{10}{3} + \frac{n_d}{6}\right) g_2^3 = -3g_2^3,$$

$$16\pi^2\beta_{g_1} = \left(\frac{20}{3} + \frac{n_d}{6}\right) = 7g_1^3,$$

$$n_g = 2$$

- Running of Yukawa couplings has two contributions.

$$\boxed{\beta_Y = \beta_Y^g + \beta_Y^Y}$$

$$16\pi^2\beta^g_{Y_t} = -\left(\frac{17}{12}g_1^2 + \frac{9}{4}g_2^2 + 8g_3^2\right)Y_t, \qquad 16\pi^2\beta^Y_{Y_t} = \left(\frac{3}{2}Y_b^2 + \frac{9}{2}Y_t^2\right)Y_t$$

$$16\pi^2\beta^g_{Y_\tau} = -\left(\frac{15}{4}g_1^2 + \frac{9}{4}g_2^2\right)Y_\tau, \qquad 16\pi^2\beta^Y_{Y_\tau} = \frac{5}{2}Y_\tau^3$$

- Running of quartic couplings from bosonic contributions and fermonic contributions

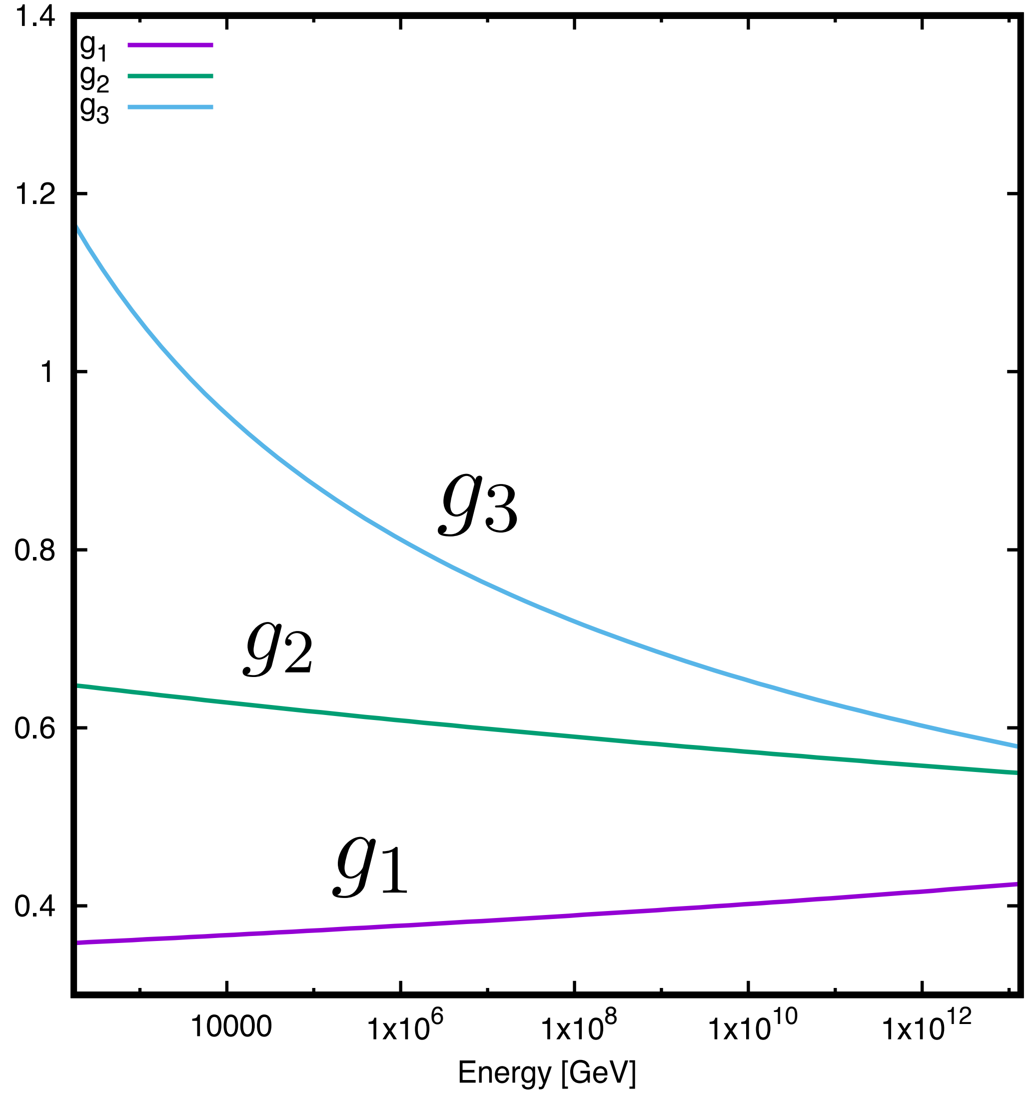
$$\beta_\lambda = \beta^b_\lambda + \beta^Y_\lambda$$

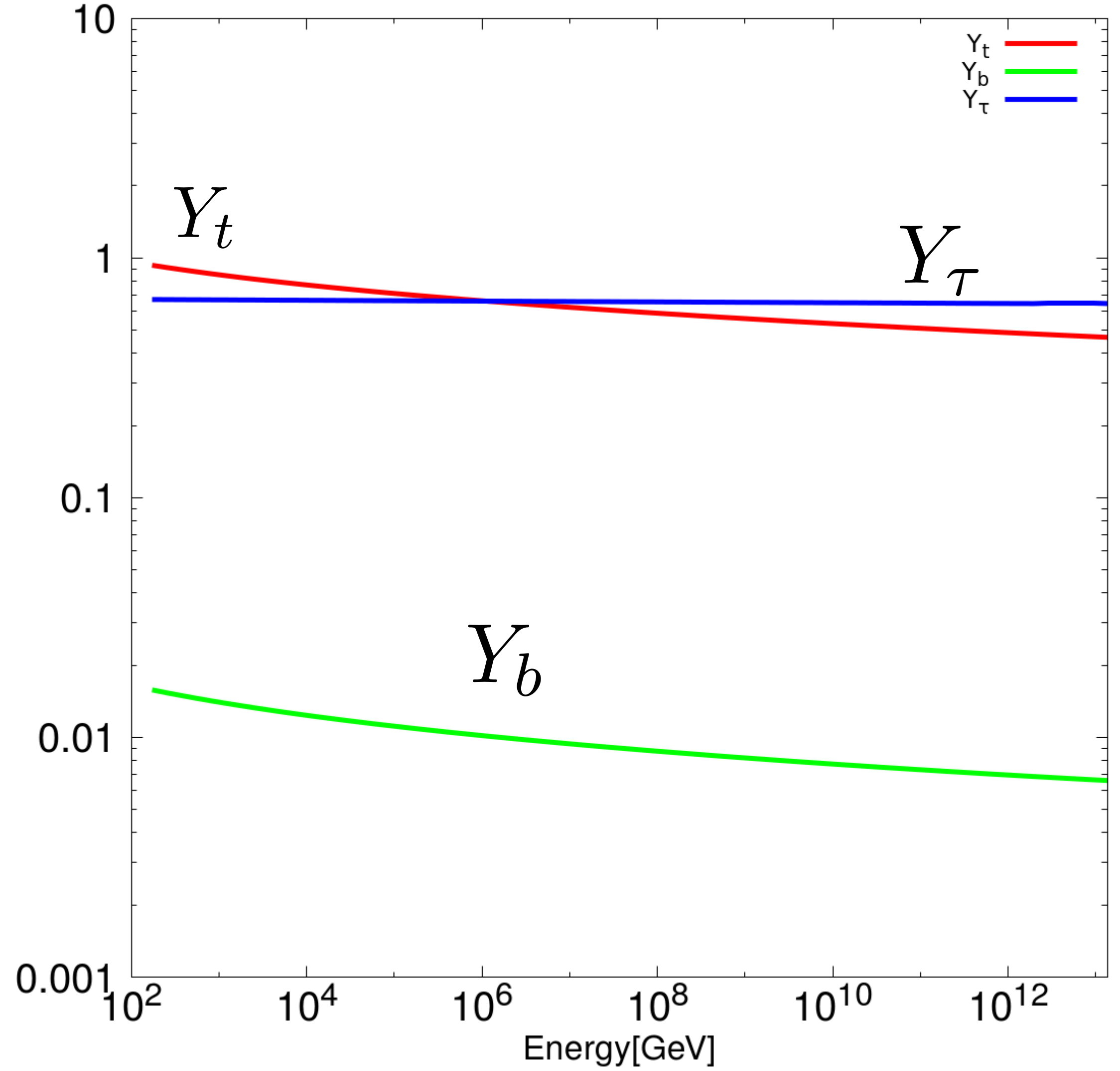
- In type-X, for example,

$$\boxed{\begin{aligned} 16\pi^2\beta^b_{\lambda_1} =& \frac{3}{4}g_1^4 + \frac{3}{2}g_1^2g_2^2 + \frac{9}{4}g_2^4 \\ & - 3g_1^2\lambda_1 - 9g_2^2\lambda_1 + 12\lambda_1^2 + 4\lambda_3^2 + 4\lambda_3\lambda_4 + 2\lambda_4^2 + 2\lambda_5^2, \\ 16\pi^2\beta^Y_{\lambda_1} =& - 4Y_\tau^4 + 4Y_\tau^2\lambda_1. \end{aligned}}$$

	BP-NS	**BP-IS**
M_H in GeV	153.865	93.6073
M_A in GeV	63.0	15.7859
$M_{H^\pm}$ in GeV	176.152	135.00
λ_1	0.52616	1.0251
λ_2	0.25773	0.25767
λ_3	0.52559	0.58636
λ_4	-0.56774	-0.45412
λ_5	0.324993	0.138905
m_{12}^2 in GeV^2	353.226215	393.28757
$\tan\beta$	67.0	22.0
$\sin(\beta-\alpha)$	0.999996	0.00601127
$y_h^\ell \times \sin(\beta-\alpha)$	0.81048833	1.13220955

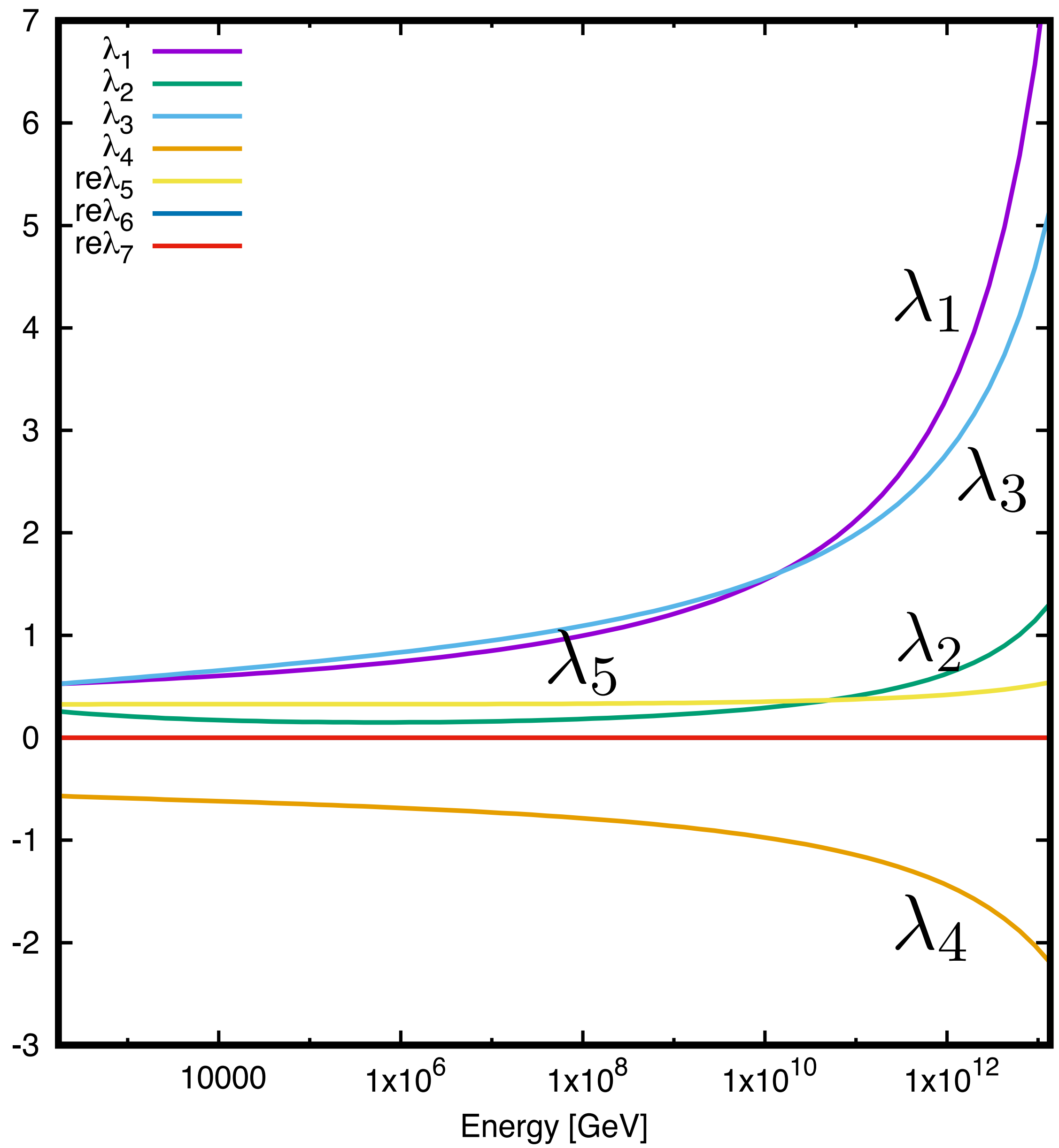
BP-NS



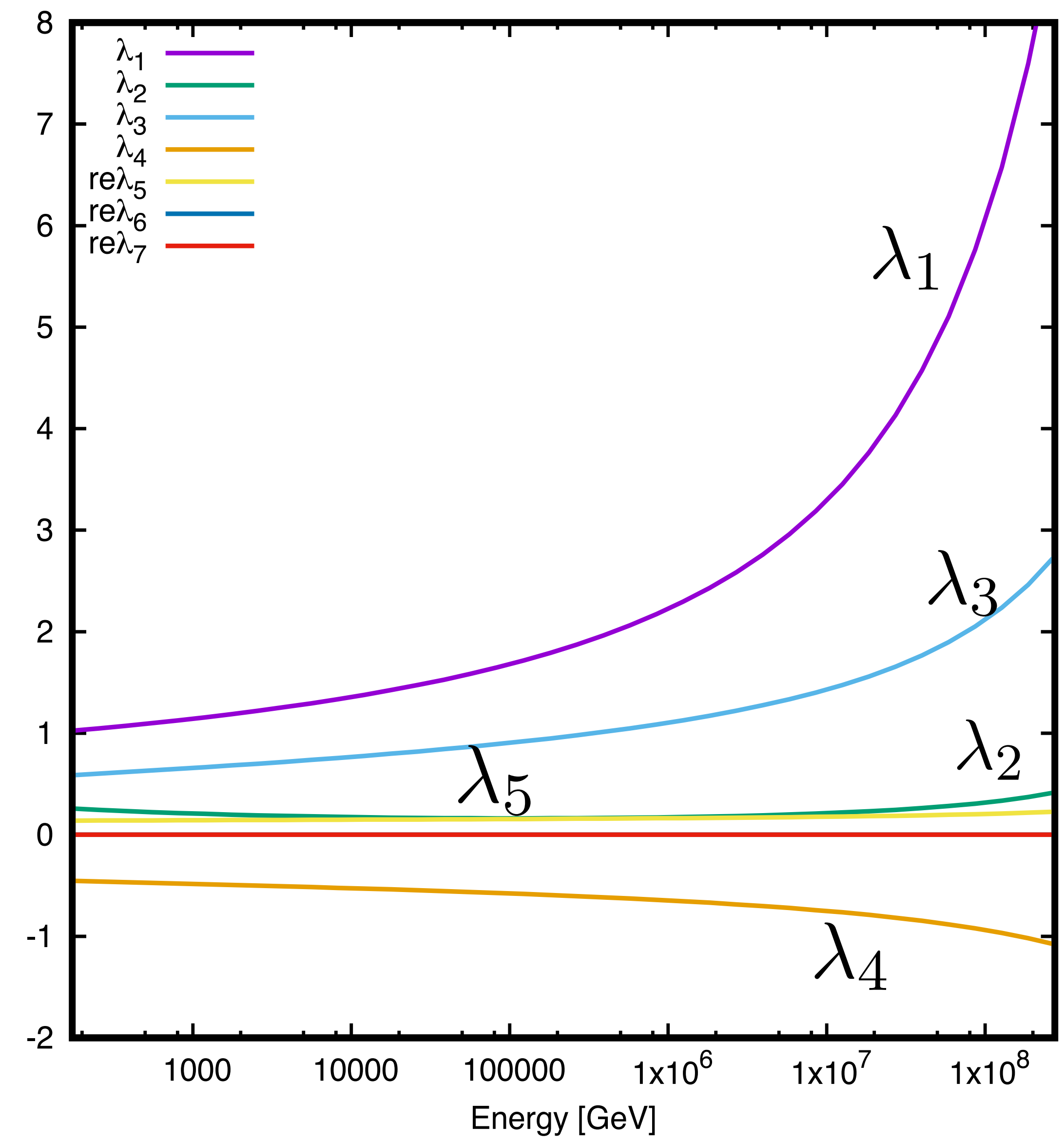


- No dramatic changes

BP-NS



BP-IS



- Quartic couplings can be very large at high energy scale.

- Why is the large quartic coupling a problem?
- It can threaten the theoretical stabiities.

Theoretical stabilities

- Scalar potential bounded from below:

$$\boxed{\begin{aligned}&\lambda_1 > 0, \quad \lambda_2 > 0, \quad \lambda_3 > -\sqrt{\lambda_1\lambda_2}, \\ &\lambda_3 + \lambda_4 - |\lambda_5| > -\sqrt{\lambda_1\lambda_2}.\end{aligned}}$$

- Perturbative unitarity of scalar-scalar scattering at tree level

$$\begin{aligned}
a_\pm &= \frac{3}{2}(\lambda_1+\lambda_2) \pm \sqrt{\frac{9}{4}(\lambda_1-\lambda_2)^2 + (2\lambda_3+\lambda_4)^2}, \\
b_\pm &= \frac{1}{2}\left(\lambda_1+\lambda_2 \pm \sqrt{(\lambda_1-\lambda_2)^2 + 4\lambda_4^2}\right), \\
c_\pm &= \frac{1}{2}\left(\lambda_1+\lambda_2 \pm \sqrt{(\lambda_1-\lambda_2)^2 + 4\lambda_5^2}\right), \\
f_+ &= \lambda_3 + 2\lambda_4 + 3\lambda_5, \quad f_- = \lambda_3 + \lambda_5, \quad f_1 = f_2 = \lambda_3 + \lambda_4, \\
e_1 &= \lambda_3 + 2\lambda_4 - 3\lambda_5, \quad e_2 = \lambda_3 - \lambda_5, \quad p_1 = \lambda_3 - \lambda_4.
\end{aligned}$$

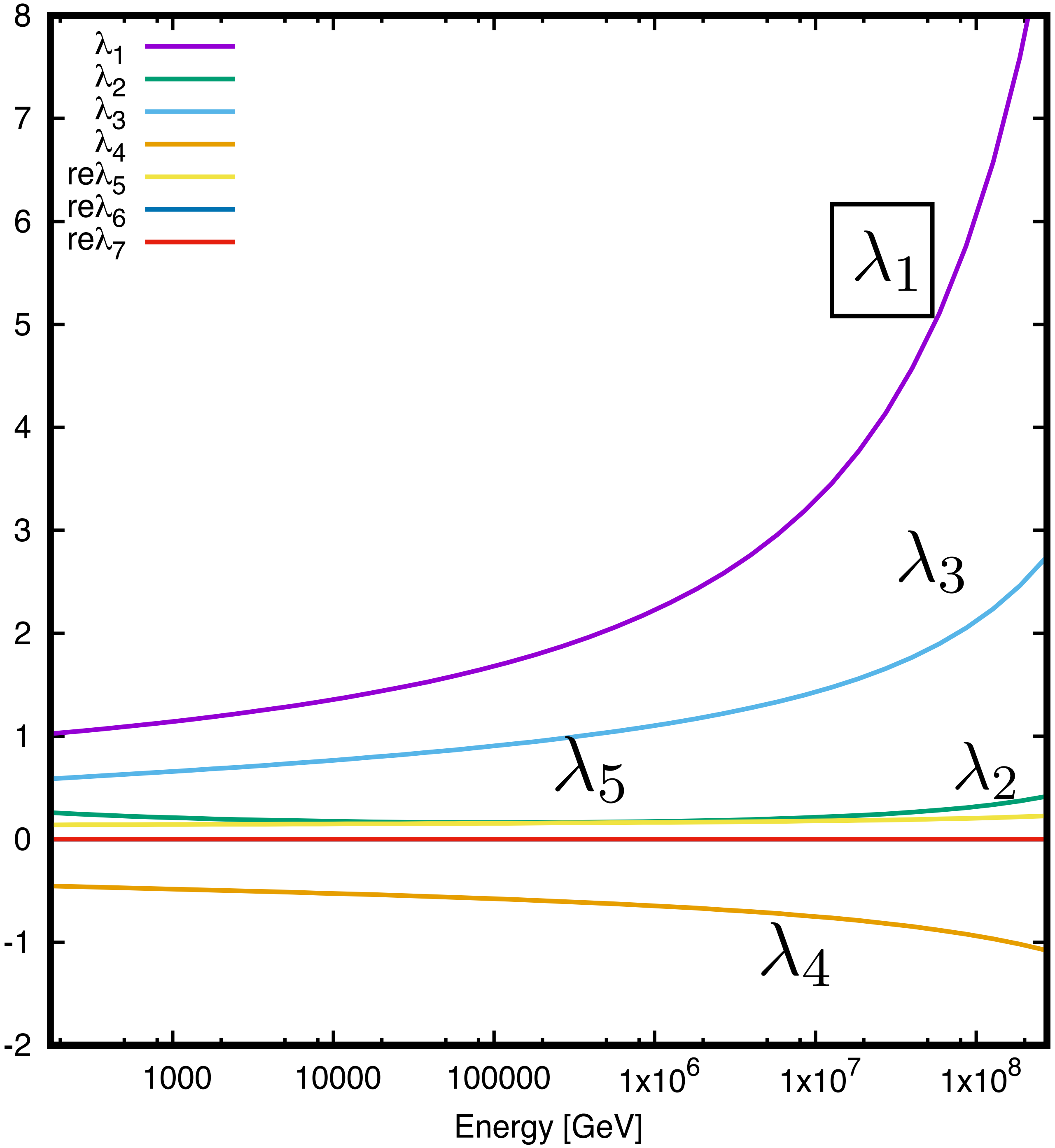
$$< 8\pi$$

- Vacuum stability

$$\boxed{D = m_{12}^2\left(m_{11}^2 - k^2 m_{22}^2\right)(t_\beta - k) > 0,}$$

$$\boxed{k = (\lambda_1/\lambda_2)^{1/4}}$$

BP-IS



Theoretical stability is broken!

- RGE analysis to calculate the cutoff scale
    1. Run each parameter point to the next high energy scale via the RGEs.
    2. Check three conditions—unitarity, perturbativity, and vacuum stability.
    3. If any condition is broken at a particular energy scale, we stop the evolution and record the energy scale as the cutoff scale.

$$\boxed{g_s, \quad g, \quad g', \quad \lambda_{1,\cdots,5}, \quad \xi_f^{h,H,A}, \quad m_{ij}^2, \quad v_i, \quad (i=1,2).}$$

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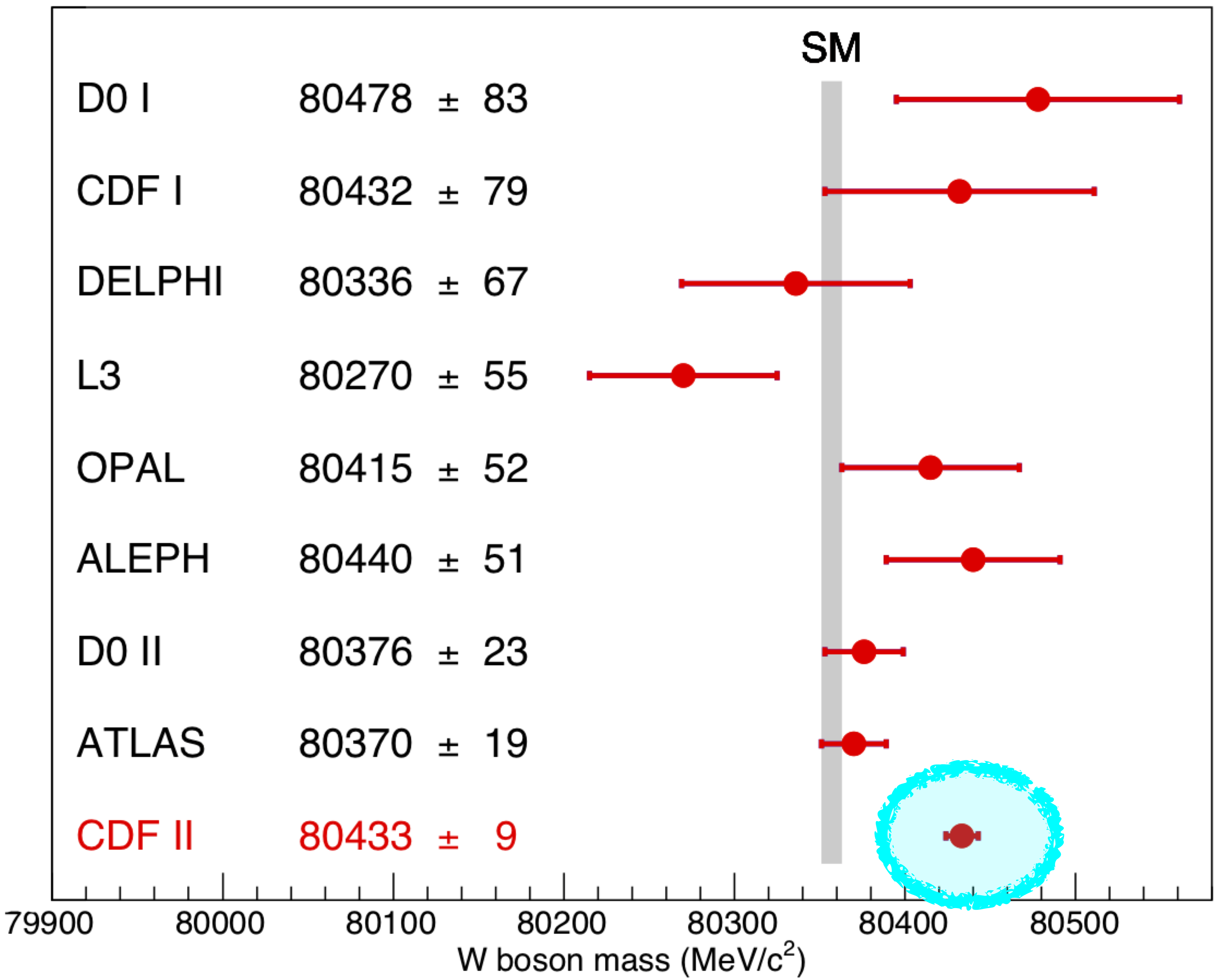
Minimal requirement

Cutoff scale > 1 TeV

4. RGE for the CDF mW

Lee, Cheung, Kim, Lu, JS [2204.10338]

CDF W boson mass



Science 376 (2022)

- Quantify the NP effect by the Peskin-Takeuchi oblique parameters.
- U from dimension-8 operator
- Setting U=0, but S and T as free parameters
- Without and with the CDF mW

$U=0$	PDG 2021			CDF 2022		
	Result	Correlation		Result	Correlation	
14 dof	$\chi^2_{\min} = 15.48$	S	T	$\chi^2_{\min} = 17.82$	S	T
S	0.05 ± 0.08	1.00	0.92	0.15 ± 0.08	1.00	0.93
T	0.09 ± 0.07		1.00	0.27 ± 0.06		1.00

p=0.22

[2204.03796]

Comprehensive study

- Four types & two scenarios = 8 cases
- 8 cases for PDG mW and CDF mW ➨16 cases
- No assumptions on the masses and couplings: 6 parameters

$$\left\{ m_h, \quad M_{H^\pm}, \quad M_H, \quad M_A, \quad m_{12}^2, \quad t_\beta, \quad s_{\beta-\alpha} \right\}.$$

- Scanning ranges

$$\text{NS: } M_H \in [130, 2000] \text{ GeV}, \ M_A \in [15, 2000] \text{ GeV},$$
$$s_{\beta-\alpha} \in [0.8, 1.0], \ m_{12}^2 \in \left[0, 1000^2\right] \text{ GeV}^2,$$
$$\text{IS: } m_h \in [15, 120] \text{ GeV}, \ M_A \in [15, 2000] \text{ GeV},$$
$$c_{\beta-\alpha} \in [0.8, 1.0], \ m_{12}^2 \in \left[0, 1000^2\right] \text{ GeV}^2.$$

$$\text{type-I \& type-X: } \ M_{H^\pm} \in [80, 2000] \text{ GeV}, \quad t_\beta \in [1, 50],$$
$$\text{type-II \& type-Y: } \ M_{H^\pm} \in [580, 2000] \text{ GeV}, \quad t_\beta \in [0.5, 50].$$

- Scanning steps

Step-(i) Theory+FCNC:

Step-(ii) EWPD: **S & T**

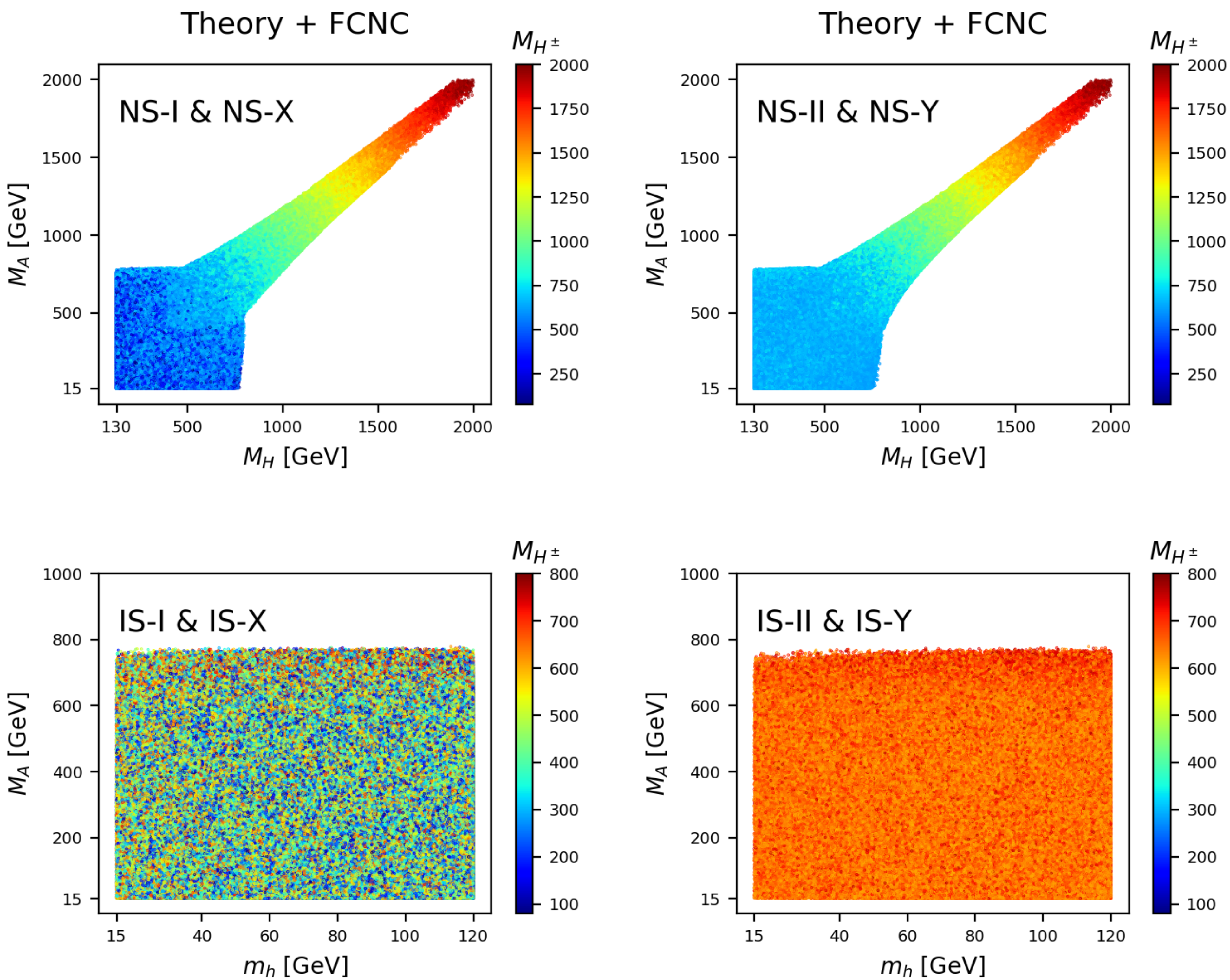
Step-(iii) RGEs for $\Lambda_c > 1$ TeV**:**

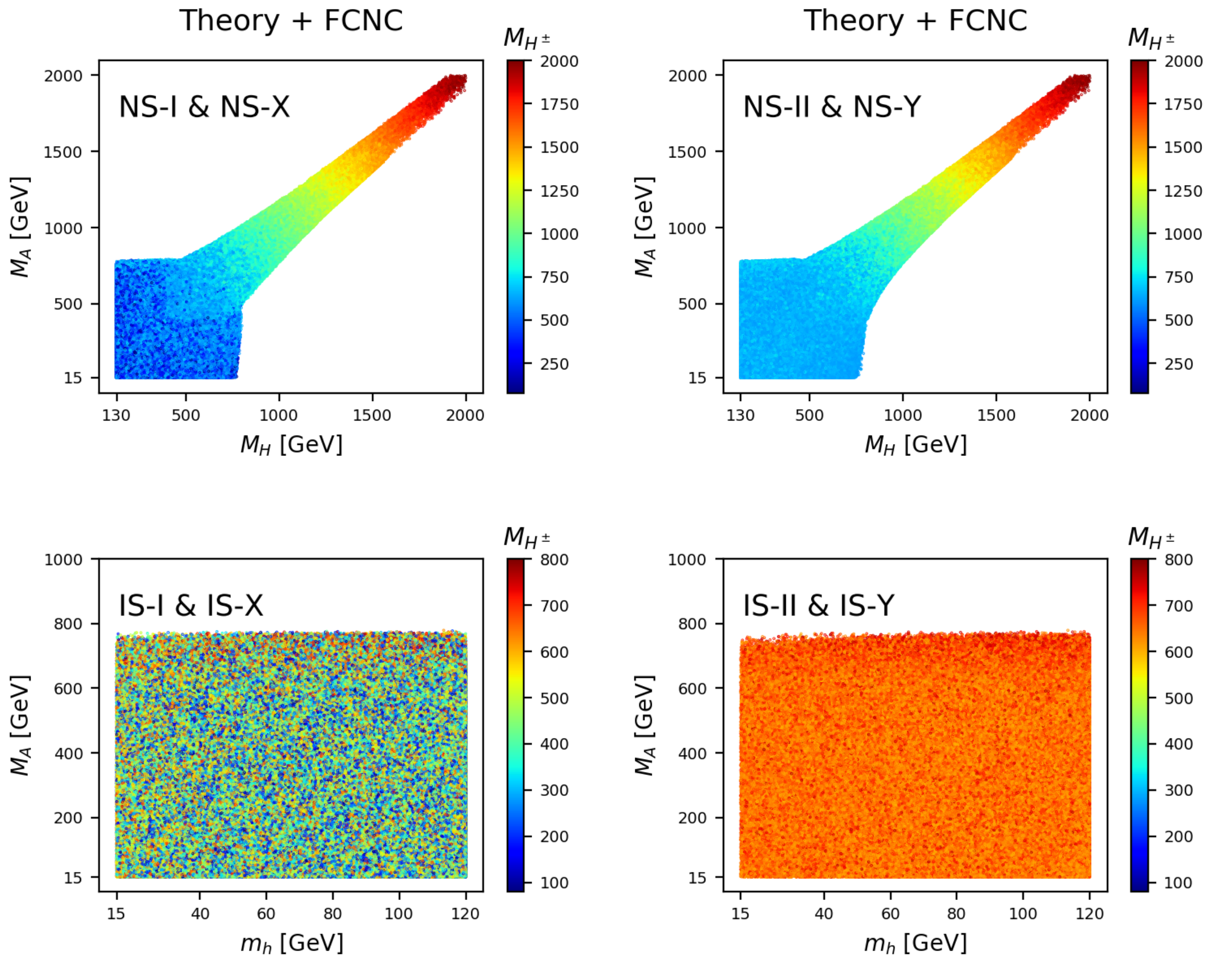
Step-(iv) Collider:

$\boxed{\textbf{Step-(i) Theory+FCNC:}}$

1. Higgs potential being bounded from below;
2. Perturbative unitarity of the scattering amplitudes;
3. Perturbativity of the quartic couplings;
4. Vacuum stability;
5. FCNC observables.

Allowed regions after Step-(i)





- NS: almost degenerate masses for heavy BSM scalars
- IS: light m_h brings down the other new scalars

- Survival percentages w.r.t. ten million points that pass Step-(i)

		EWPD	$\Lambda_c > 1$ TeV	Collider	EWPD	$\Lambda_c > 1$ TeV	Collider
		Normal scenario			Inverted scenario		
type-I	PDG	12.98%	5.13%	0.60%	7.20%	5.08%	0.85%
	CDF	4.42%	1.31%	0.14%	1.30%	0.72%	0.19%
type-II	PDG	10.76%	0.43%	0.20%	2.14%	0	0
	CDF	3.36%	0.03%	0.01%	0.69%	0	0
type-X	PDG	12.98%	5.13%	0.18%	7.20%	5.08%	0.03%
	CDF	4.42%	1.31%	0.03%	1.30%	0.72%	0.01%
type-Y	PDG	10.76%	0.43%	0.20%	2.14%	0	0
	CDF	3.36%	0.03%	0.01%	0.69%	0	0

Step-(iv) Collider:

1. Higgs precision data via HiggsSignals;
2. direct searches at high energy collider via HiggsBounds.

- Survival percentages w.r.t. ten million points that pass Step-(i)

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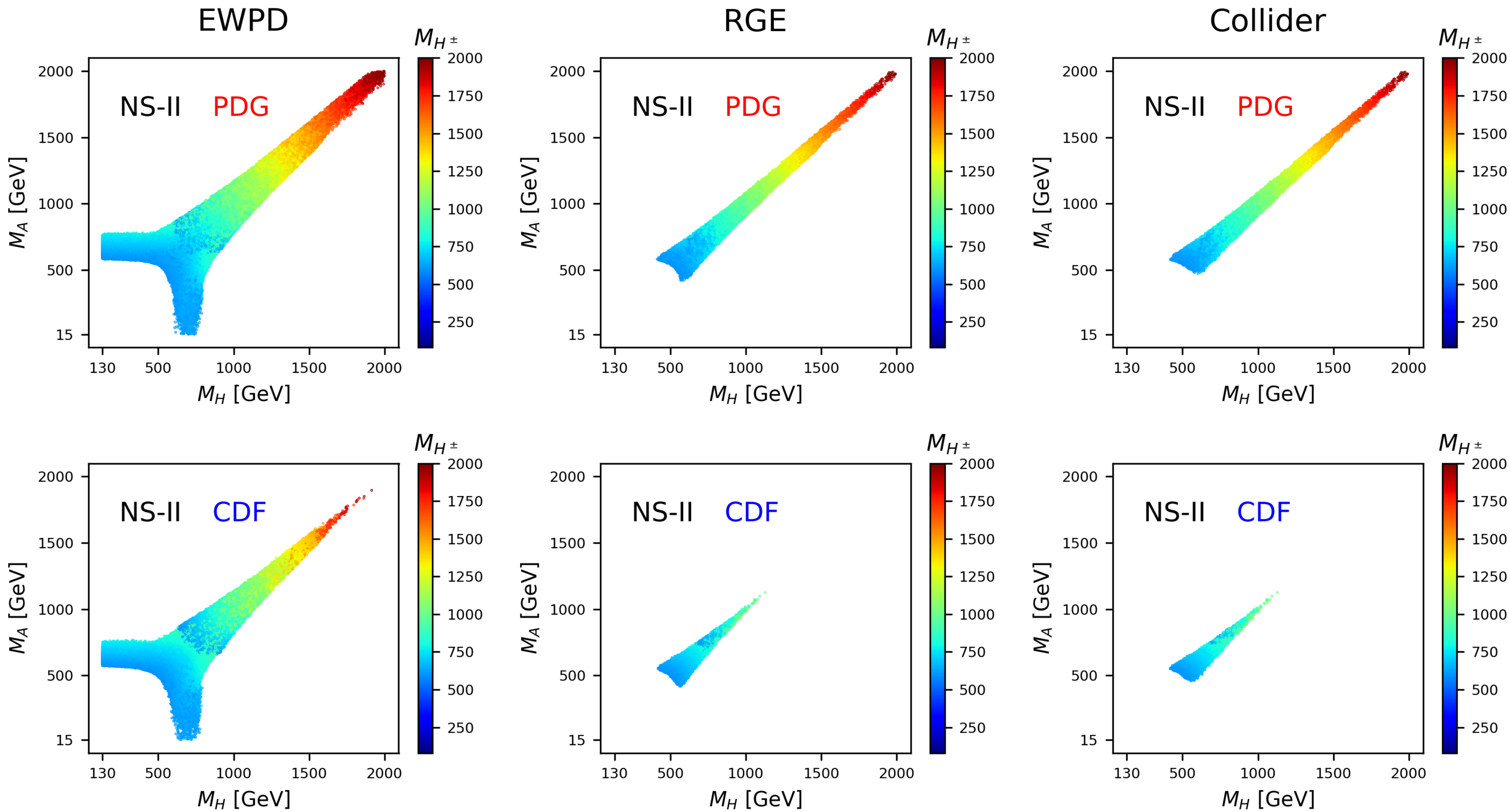
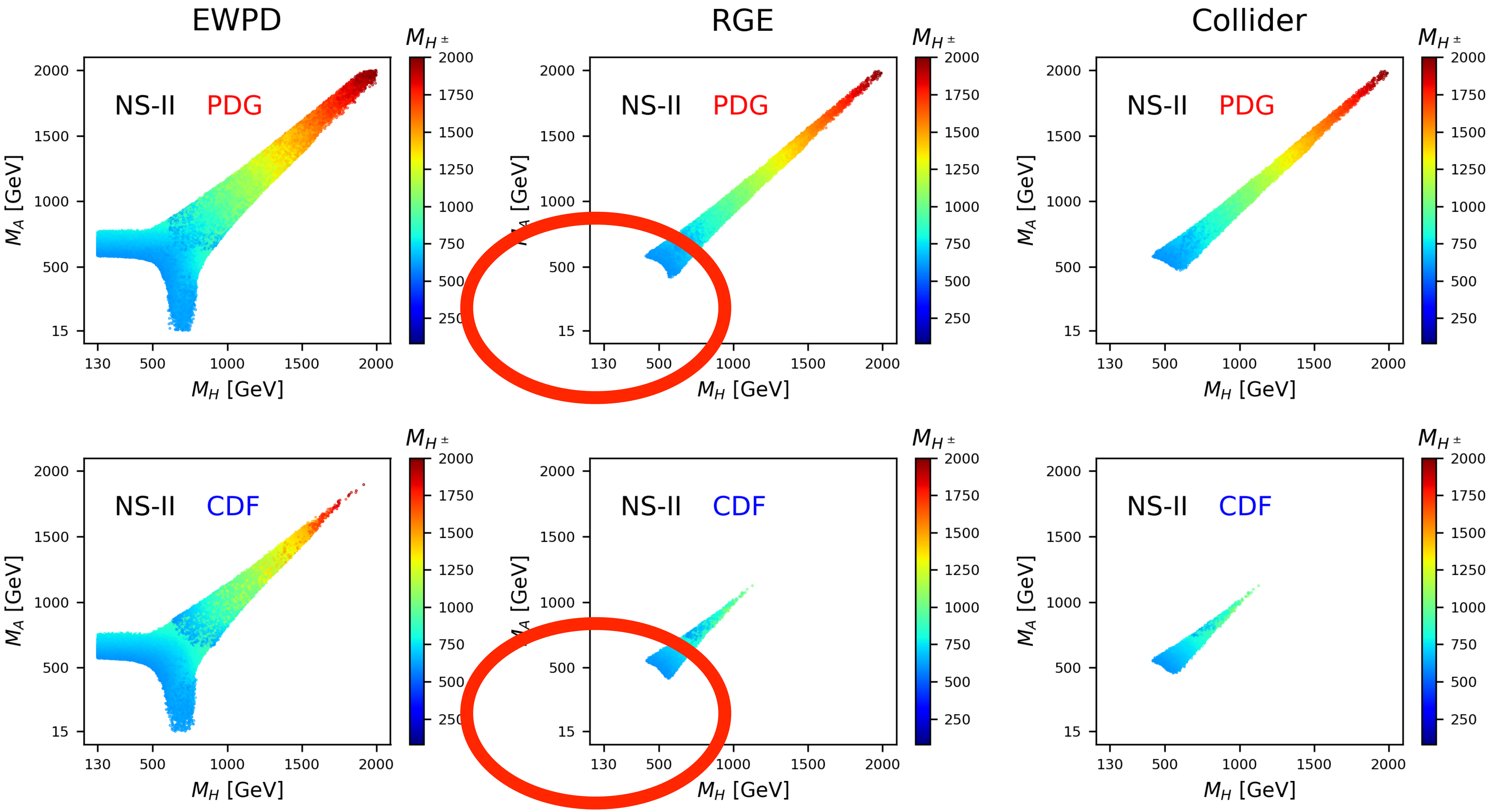
- type-II in NS

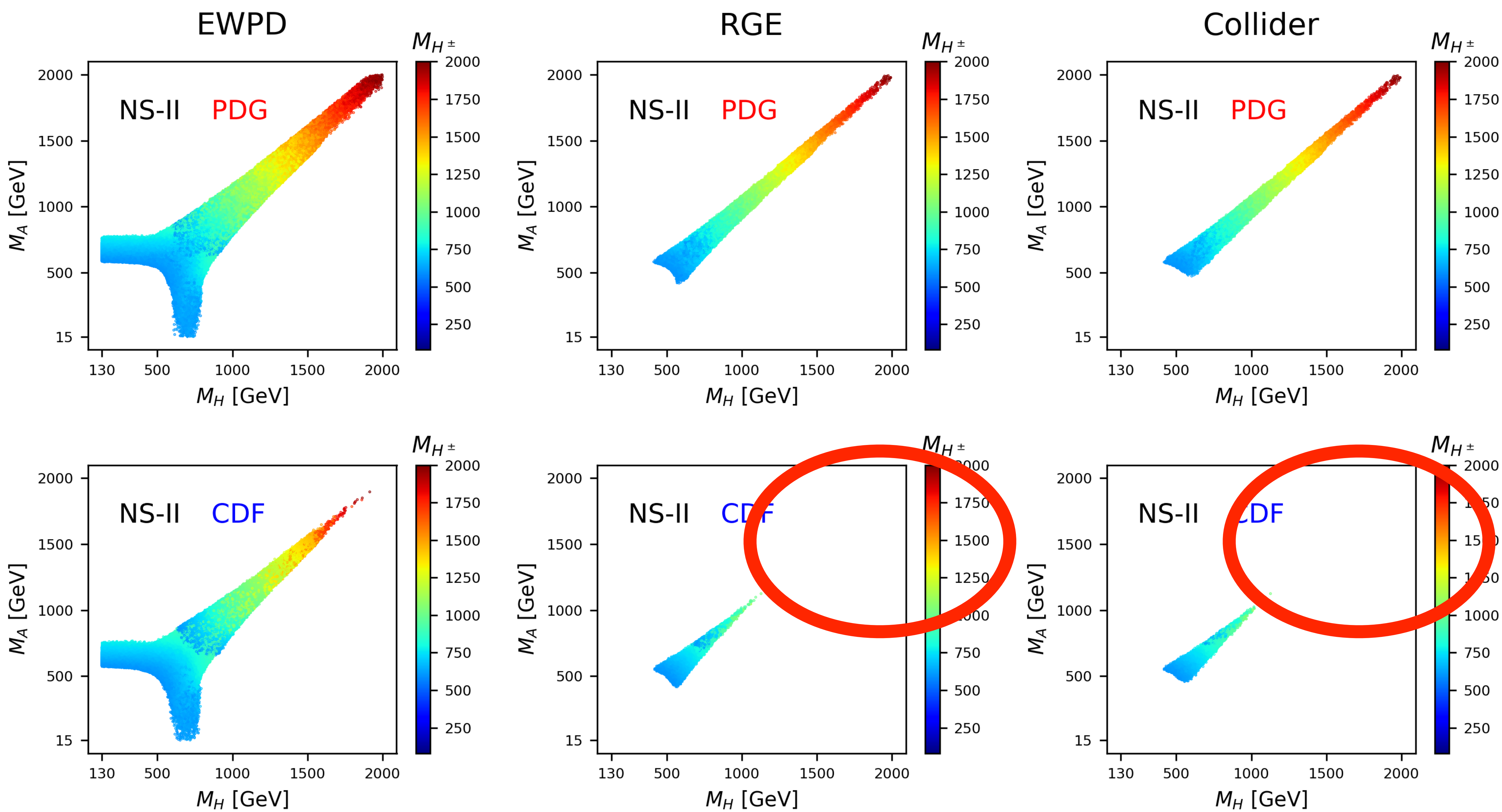

EWPD
RGE
Collider
NS-II PDG
NS-II CDF
M_A [GeV]
M_H [GeV]
$M_{H^\pm}$
2000
1750
1500
1250
1000
750
500
250
15
130

- type-II in NS



- RGE excludes sizable mass gaps among BSM Higgs masses below 500 GeV.

- type-II in NS



- In the CDF, there are upper bounds on BSM Higgs masses.

5. RGE effects on the muon g-2

Kim, Lee, Sanyal, Song [2205.01701]

- Persistent anomaly with 4.2σ which has been around for some time

$$\boxed{\Delta a_\mu = a_\mu^{\rm exp} - a_\mu^{\rm SM} = 251(59) \times 10^{-11}.}$$

- 2HDM: two kinds of contributions

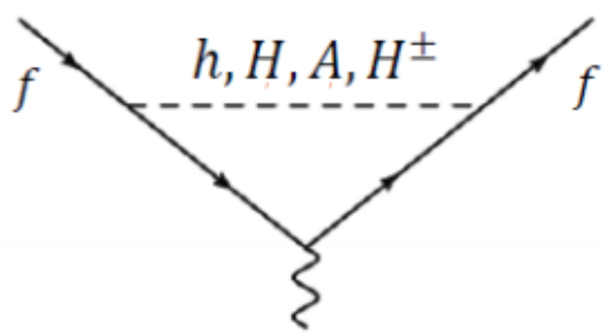
1-loop

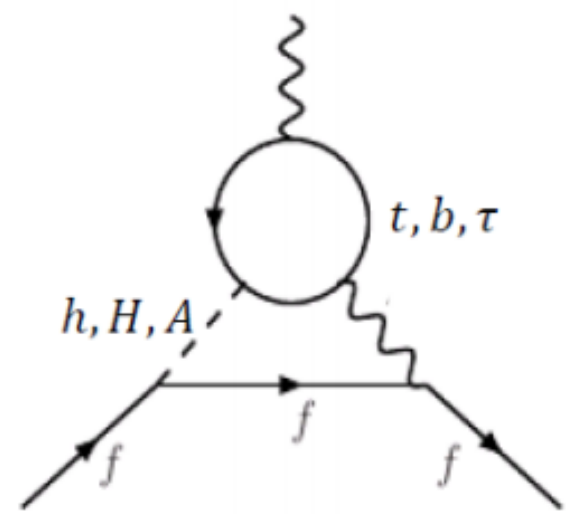
Barr-Zee 2-loop

- Persistent anomaly with 4.2σ which has been around for some time

$$\boxed{\Delta a_\mu = a_\mu^{\rm exp} - a_\mu^{\rm SM} = 251(59)\times 10^{-11}.}$$

- 2HDM: two kinds of contributions





1-loop Barr-Zee 2-loop

- We need
  - Enhance the τ Yukawa coupling
  - Suppress the b quark Yukawa coupling of the BSM Higgs bosons.
  - Light psedoscalar A
- Type-X with large tan$\boldsymbol{\beta}$ and Higgs-phobic A

- Lepton Flavor Universality (LFU) data in danger

1. For the τ decay,

$$\frac{g_\tau}{g_\mu}, \quad \frac{g_\tau}{g_e}, \quad \frac{g_\mu}{g_e}, \quad \left(\frac{g_\tau}{g_\mu}\right)_\pi, \quad \left(\frac{g_\tau}{g_\mu}\right)_K .$$

2. Michel parameters, based on the energy and angular distribution of ℓ^- in the decay of $\tau^- \to \ell^- \nu \nu_\tau$:

$$\rho_e, \quad (\xi\delta)_e, \quad \xi_e, \quad \eta_\mu, \quad \rho_\mu, \quad (\xi\delta)_\mu, \quad \xi_\mu, \quad \xi_\pi, \quad \xi_\rho, \quad \xi_{a_1}.$$

3. Leptonic Z decays:

$$\frac{\Gamma(Z \to \mu^+\mu^-)}{\Gamma(Z \to e^+e^-)}, \quad \frac{\Gamma(Z \to \tau^+\tau^-)}{\Gamma(Z \to e^+e^-)}.$$

2104.10175 [hep-ph]

- Higgs-phobic A in type-X

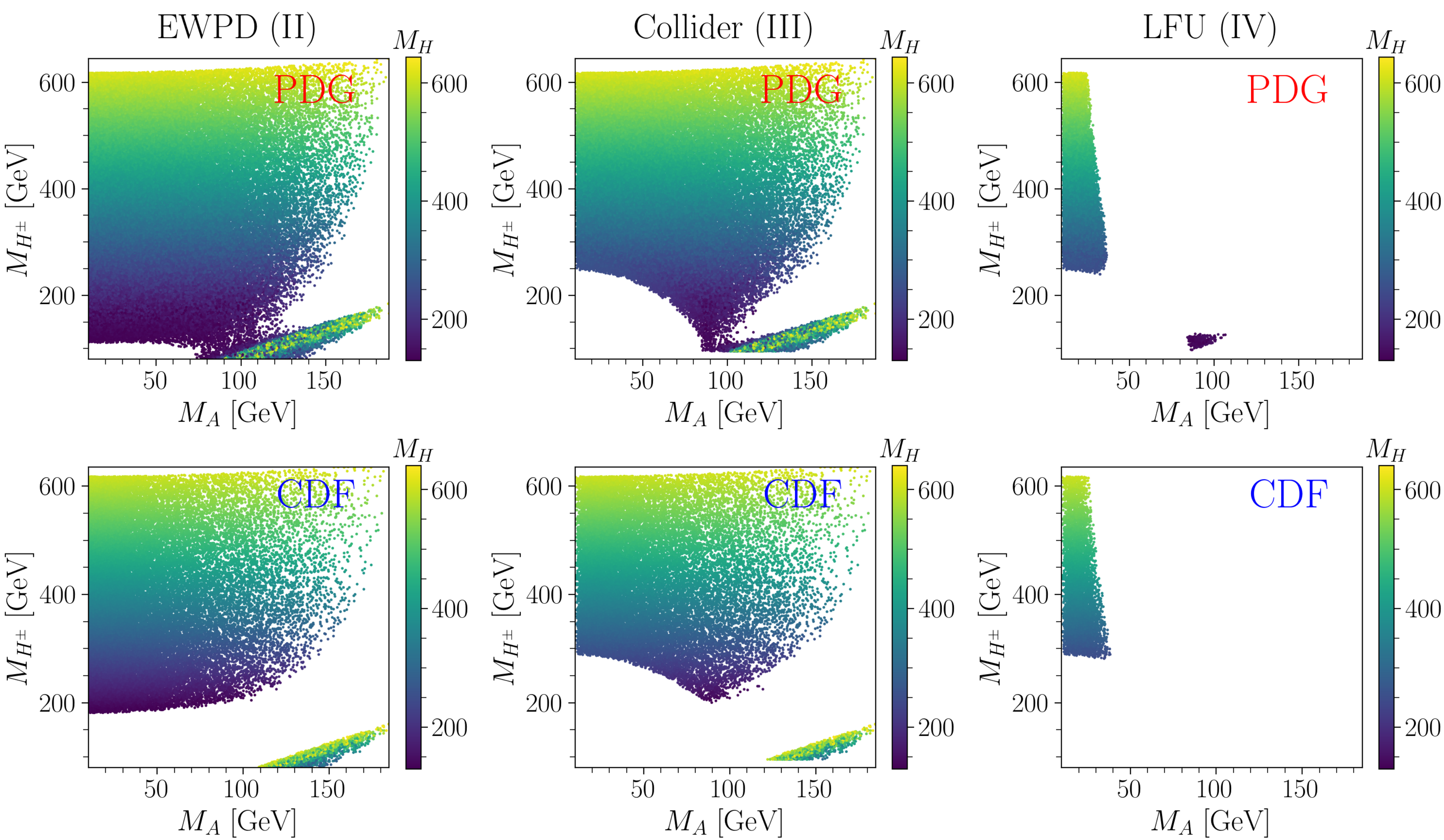
$$\boxed{\hat{\lambda}_{hAA} = \left(2M^2 - 2M_A^2 - m_h^2\right) s_{\beta-\alpha} + \left(m_h^2 - M^2\right)\left(t_\beta - \frac{1}{t_\beta}\right) c_{\beta-\alpha}.}$$

$$\boxed{\text{Higgs-phobic } A: \quad \frac{s_{\beta-\alpha}}{c_{\beta-\alpha}} = -\left(t_\beta - \frac{1}{t_\beta}\right)\frac{m_h^2 - M^2}{2M^2 - 2M_A^2 - m_h^2}.}$$

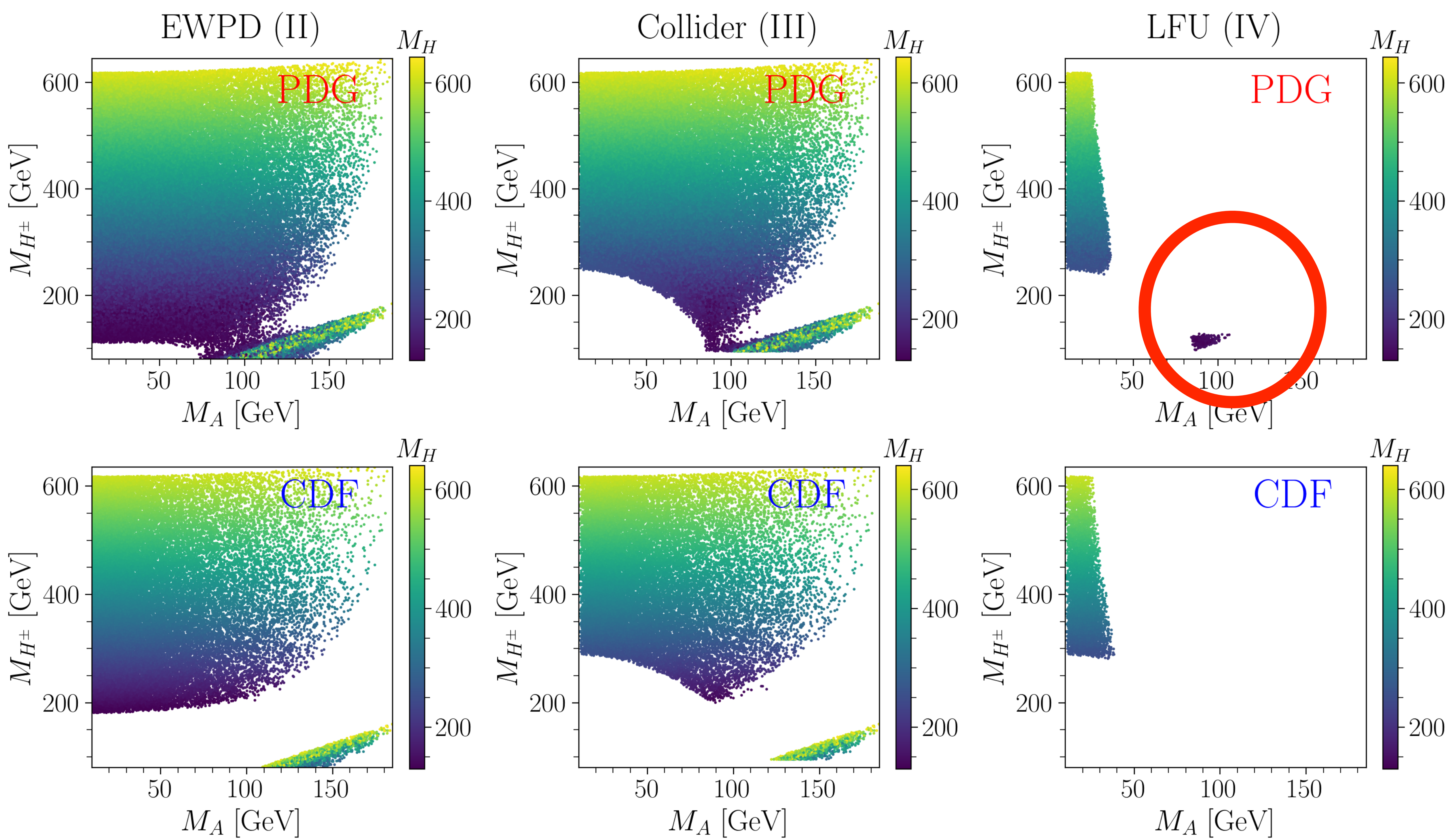
- Higgs-phobic A cannot coexist with 100% alignment. BUT

Chun et al 1507.08067

- Allowed parameter points



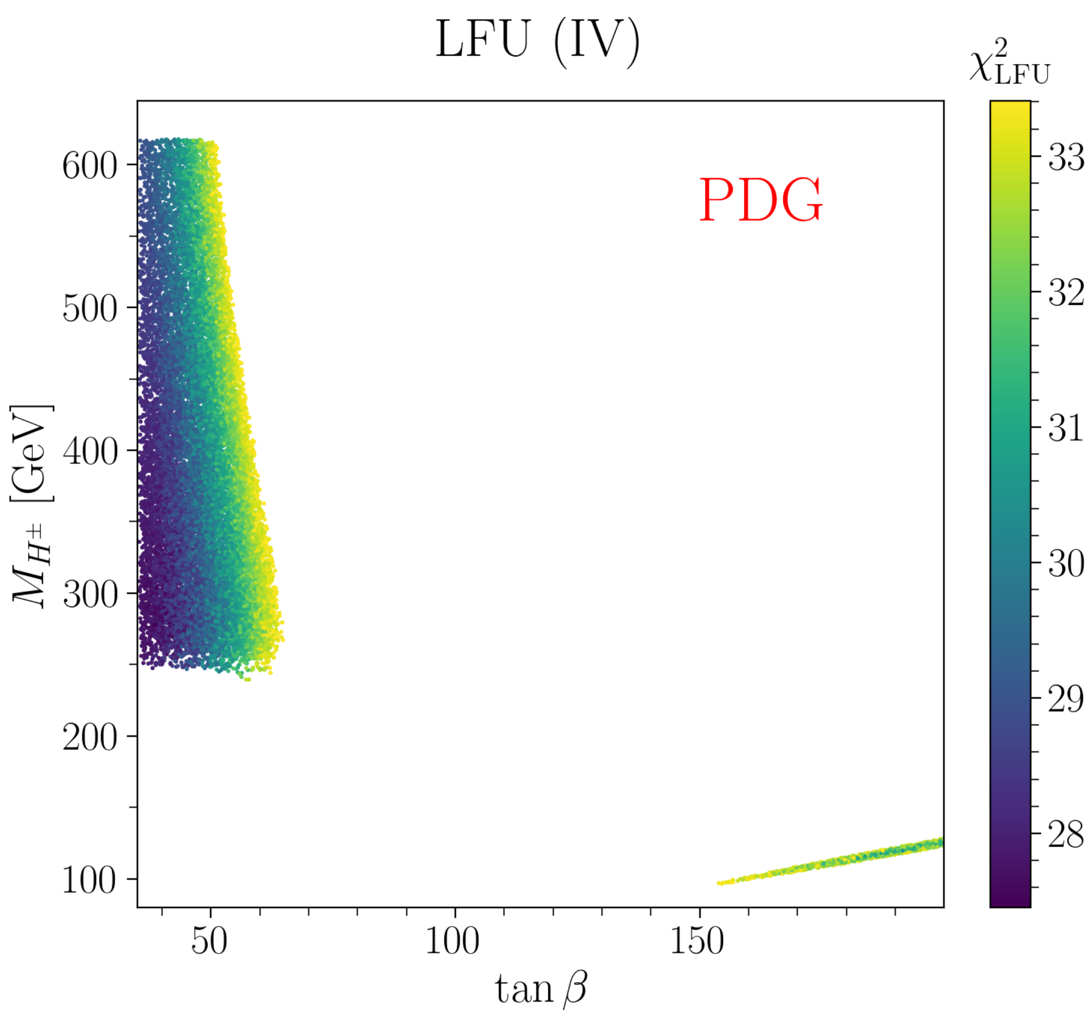
- Difference-1: PDG island



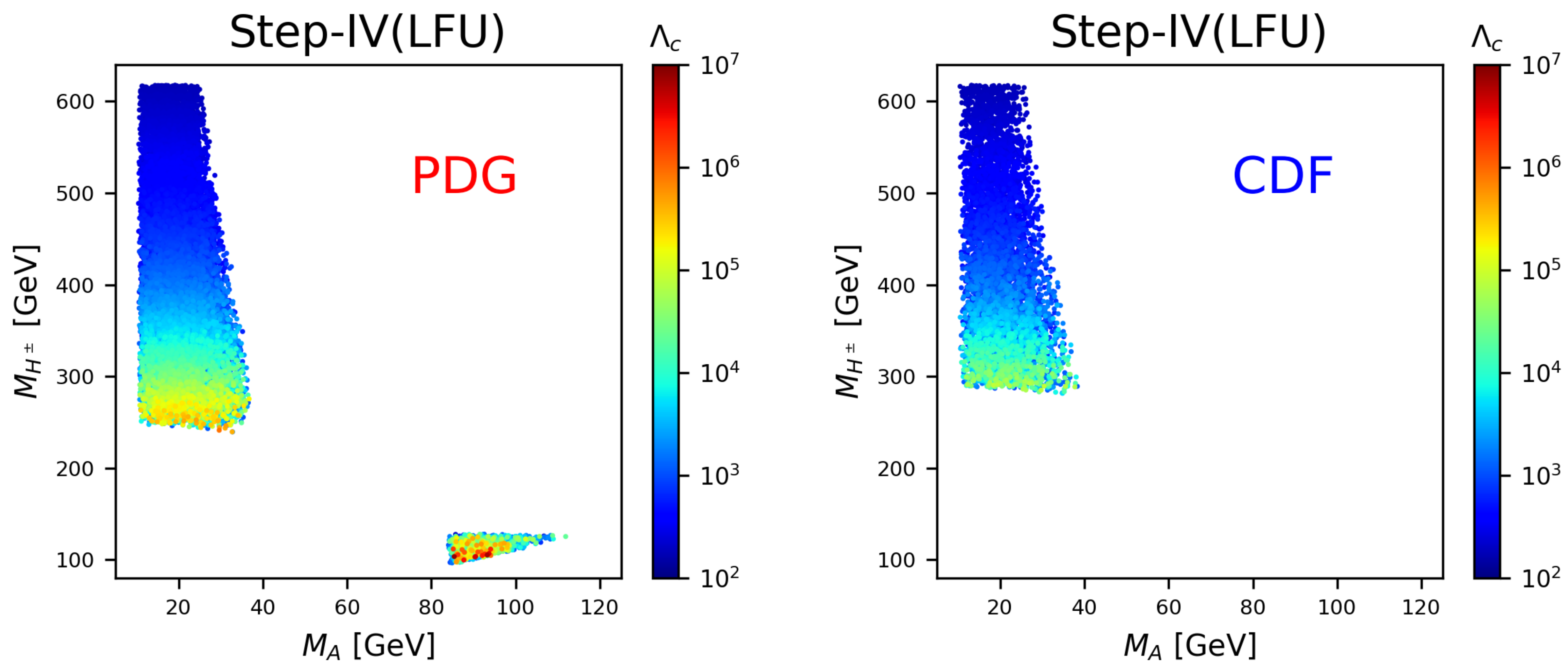
5. Higgs-phobic for muon g-2

- Difference-1: How can the PDG-island evade the LFU?
- Cancellation!

$$\boxed{\epsilon^{\tau}_{\rm tree} = \delta_{\rm tree}\left[\frac{\delta_{\rm tree}}{8} - \frac{m_\mu}{m_\tau}\frac{g\left(\rho^{\mu}_{\tau}\right)}{f\left(\rho^{\mu}_{\tau}\right)}\right]}$$



- PDG vs CDF in the RGE analysis

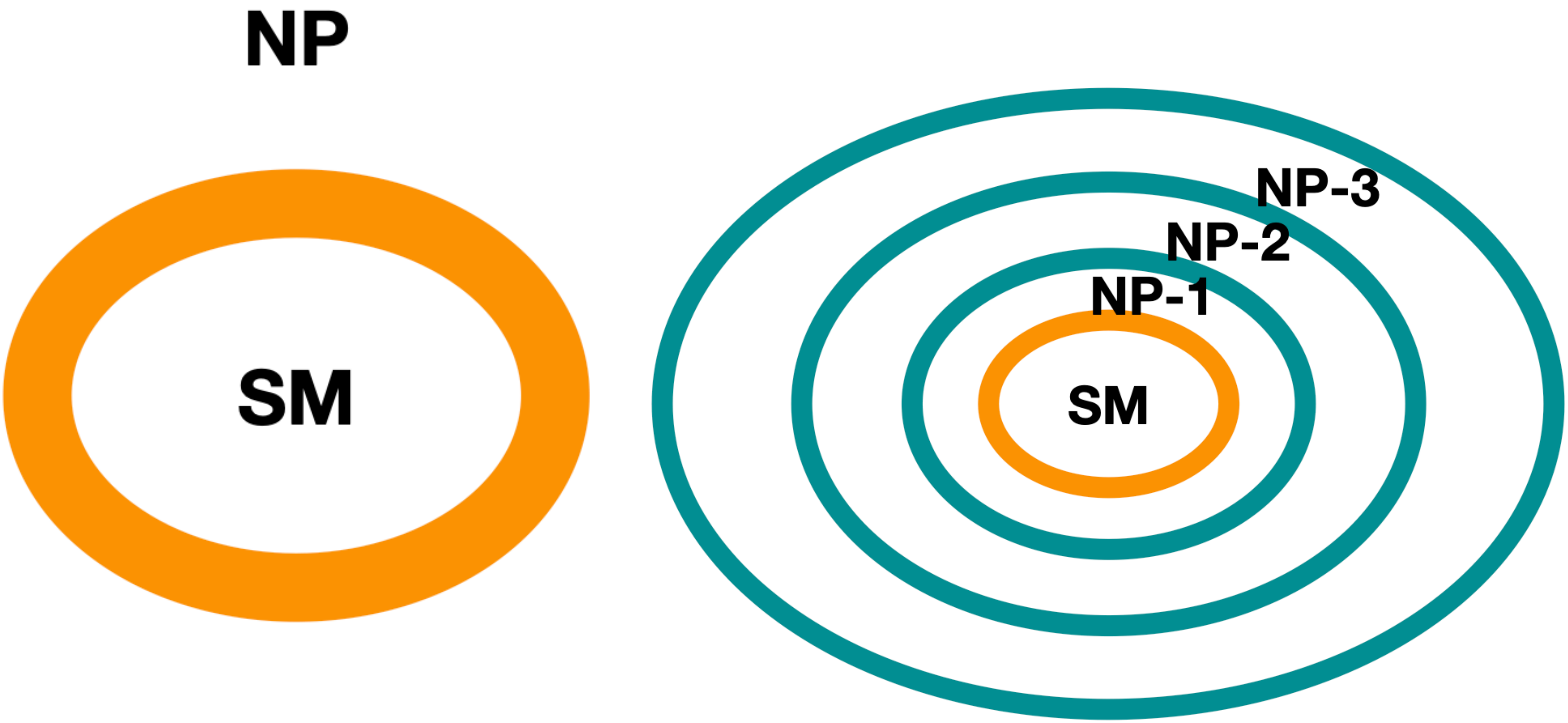


- The maximum cutoff scale is in the PDG-island, which is about 10^7 GeV.

4. How to distinguish the high- and low-cutoff scales?

S.K. Kang, J. Kim, S. Lee, JS [2210.00020]

Can a measurement distinguish?



Can a measurement distinguish?

Yes, if we can measure the cutoff scale of a NP model.

Higgs alignment limit in the inverted scenario

$$M_H = 125 \text{ GeV}, \quad c_{\beta-\alpha} = 1.$$

Yukawa couplings to the SM fermions

$$\xi_f^H = 1, \quad \xi_{t,b,\tau}^h = \frac{1}{t_\beta}, \quad \xi_t^A = -\xi_{b,\tau}^A = \frac{1}{t_\beta}.$$

Yukawa couplings of the BSM Higgs bosons are suppressed by tan β

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Trilinear Higgs couplings

$$\hat{\lambda}_{HHH} = -\frac{3m_{125}^2}{v^2}, \qquad \hat{\lambda}_{hHH} = 0,$$

$$\hat{\lambda}_{hhh} = 3\hat{\lambda}_{hAA} = 3\hat{\lambda}_{hH^+H^-} = -\frac{3(M^2 - m_h^2)(t_\beta^2 - 1)}{t_\beta v^2},$$

$$\hat{\lambda}_{Hhh} = -\frac{m_{125}^2 + 2m_h^2 - 2M^2}{v^2},$$

$$\hat{\lambda}_{HAA} = -\frac{m_{125}^2 + 2M_A^2 - 2M^2}{v^2},$$

$$\hat{\lambda}_{HH^+H^-} = -\frac{m_{125}^2 + 2M_{H^\pm}^2 - 2M^2}{v^2}.$$

Trilinear Higgs couplings

Same as in the SM ← alignment

$$\boxed{\hat{\lambda}_{HHH} = -\frac{3m_{125}^2}{v^2},} \qquad \hat{\lambda}_{hHH} = 0,$$

$$\hat{\lambda}_{hhh} = 3\hat{\lambda}_{hAA} = 3\hat{\lambda}_{hH^+H^-} = -\frac{3(M^2 - m_h^2)(t_\beta^2 - 1)}{t_\beta v^2},$$

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Trilinear Higgs couplings

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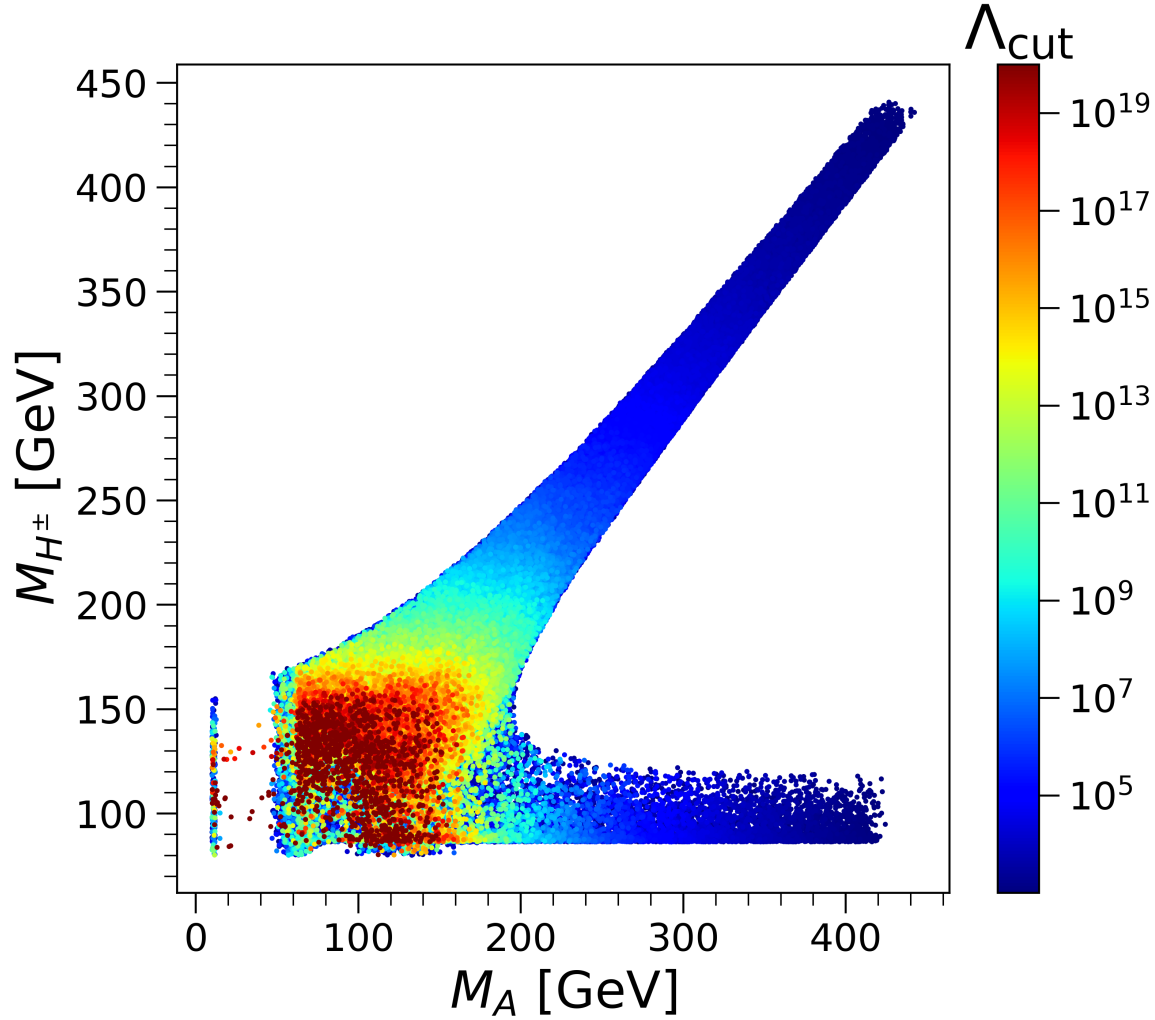
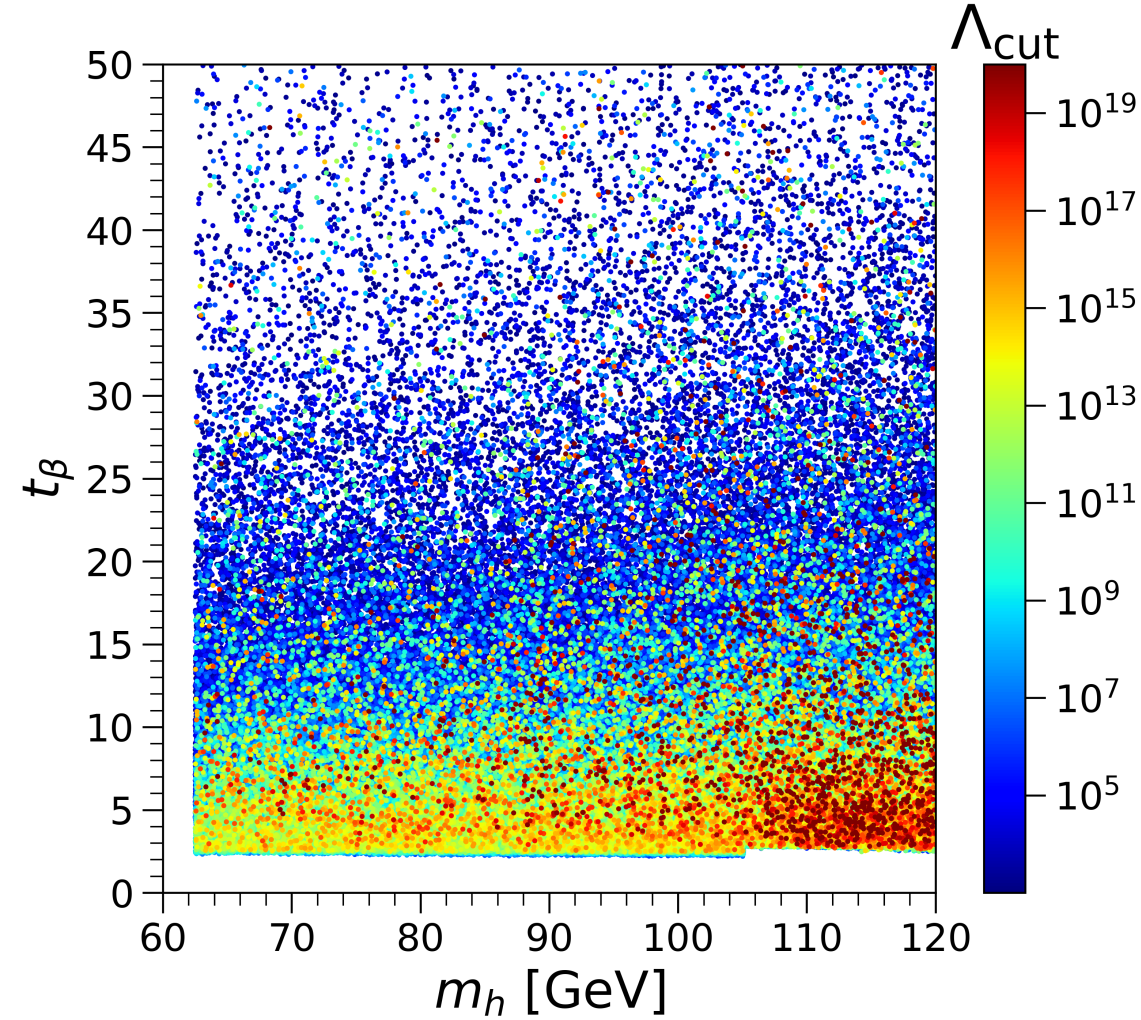
$$\hat{\lambda}_{Hhh} = -\frac{m_{125}^2 + 2m_h^2 - 2M^2}{v^2},$$

Proportional to tan β

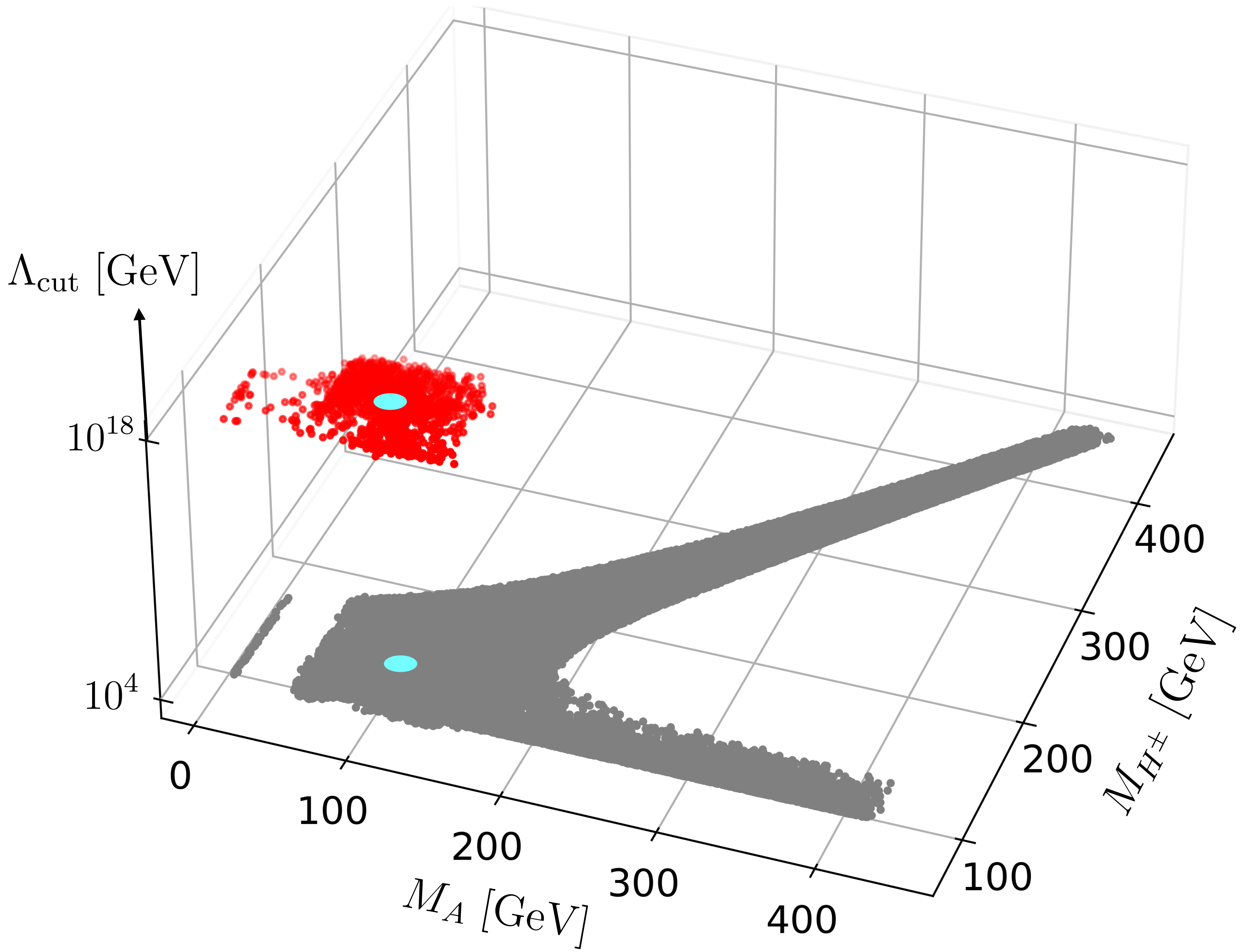
$$\hat{\lambda}_{HAA} = -\frac{m_{125}^2 + 2M_A^2 - 2M^2}{v^2},$$

$$\hat{\lambda}_{HH^+H^-} = -\frac{m_{125}^2 + 2M_{H^\pm}^2 - 2M^2}{v^2}.$$





Happy that the model can retain the stability all the way up to the Planck-cutoff scale?



Light masses cannot tell whether Λ is high or low.

Measuring tanβ can help?

In type-I, large tan β cannot be measured

$$\xi_f^H = 1, \quad \xi_{t,b,\tau}^h = \frac{1}{t_\beta}, \quad \xi_t^A = -\xi_{b,\tau}^A = \frac{1}{t_\beta}.$$

In type-I, large tan β cannot be measured.

- The tan β-dependent production, the gluon fusion, is suppressed if tan β is large.
- The decay branching ratios are insensitive to tan β in type I.

BP-1: $m_h = 70$ GeV, $M_A = 110$ GeV, $M_{H^\pm} = 110$ GeV,

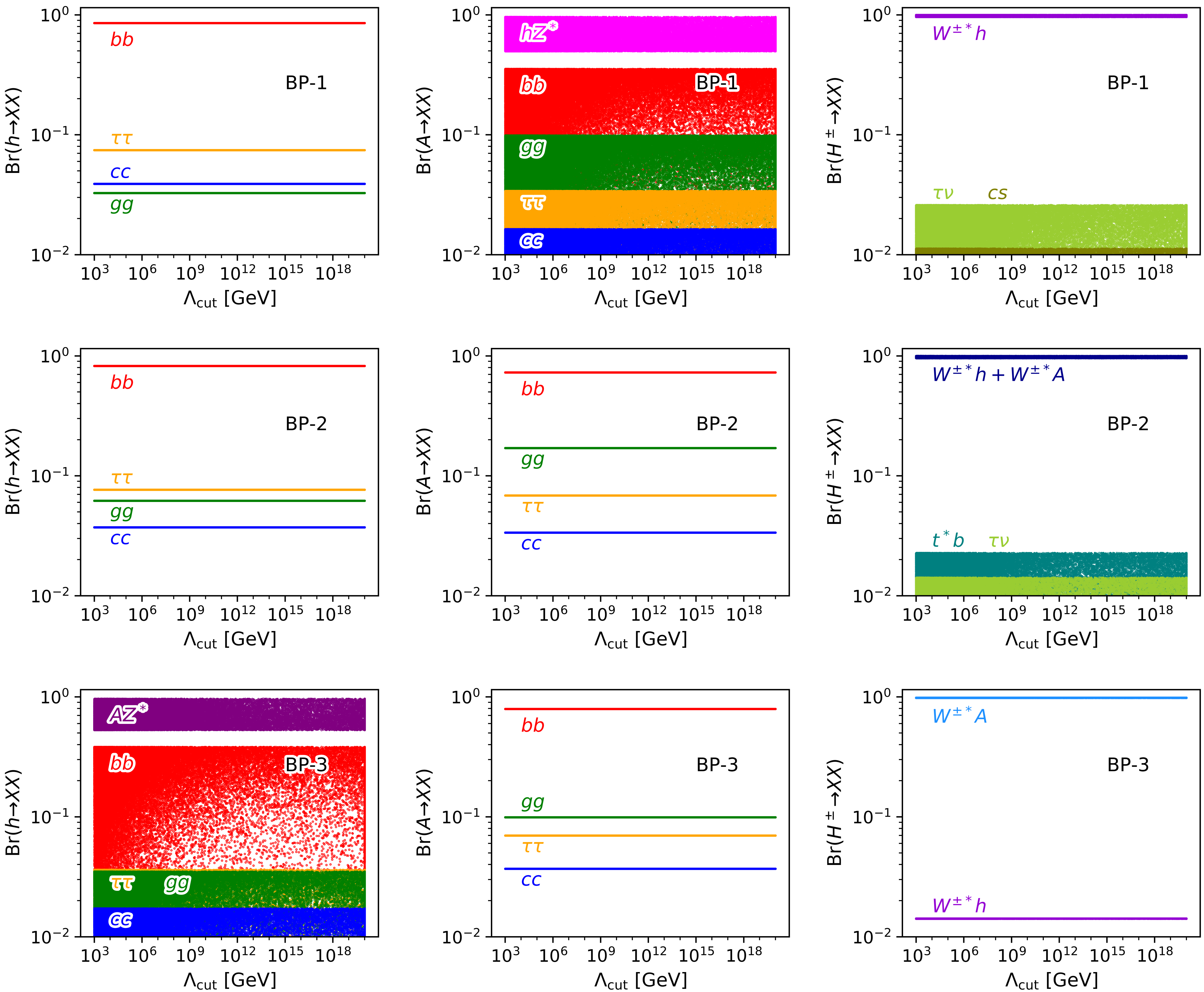
BP-2: $m_h = 100$ GeV, $M_A = 100$ GeV, $M_{H^\pm} = 140$ GeV,

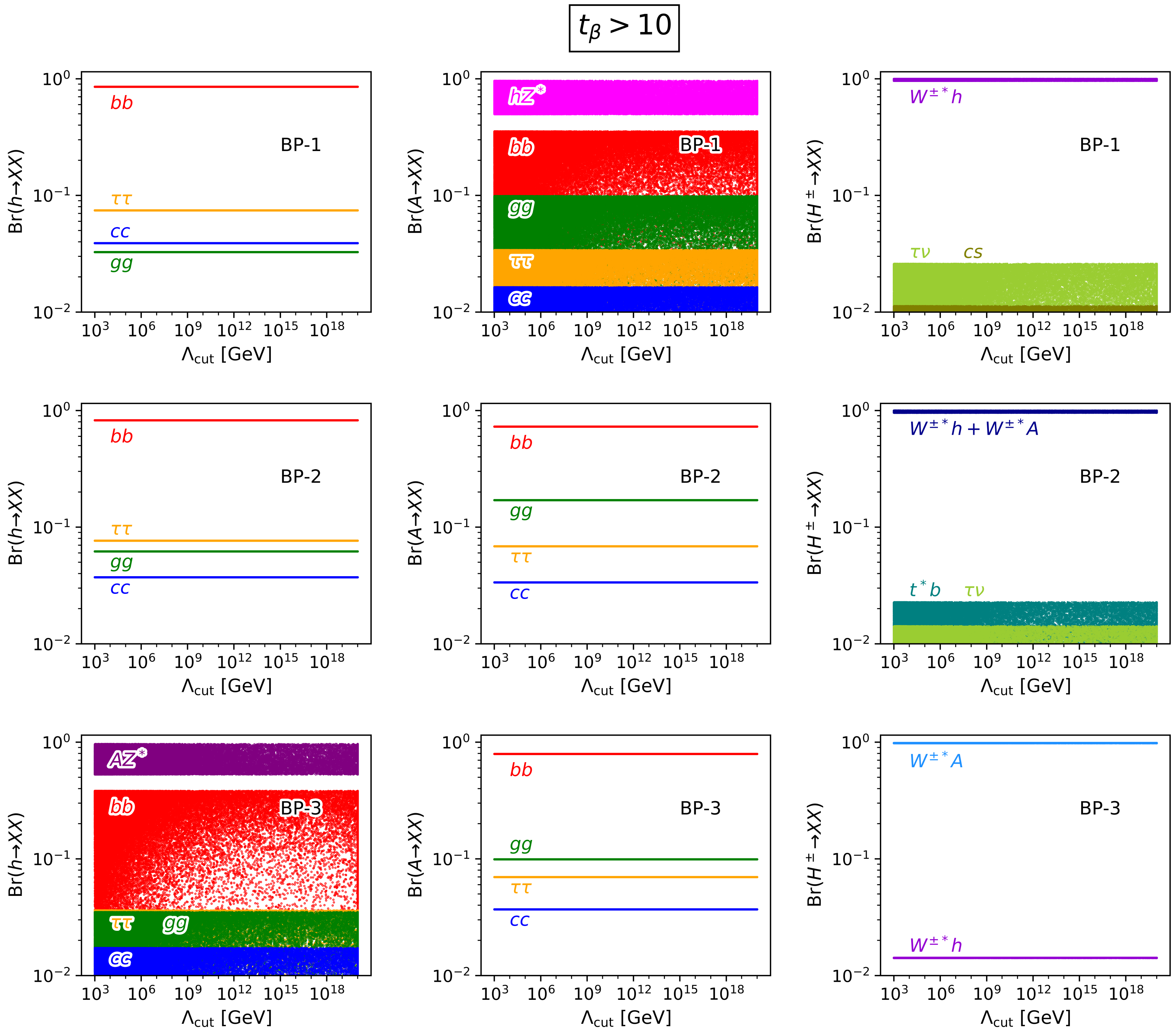
BP-3: $m_h = 110$ GeV, $M_A = 70$ GeV, $M_{H^\pm} = 140$ GeV.

All accommodate Λ=1 TeV and Λ=10^{18} GeV.

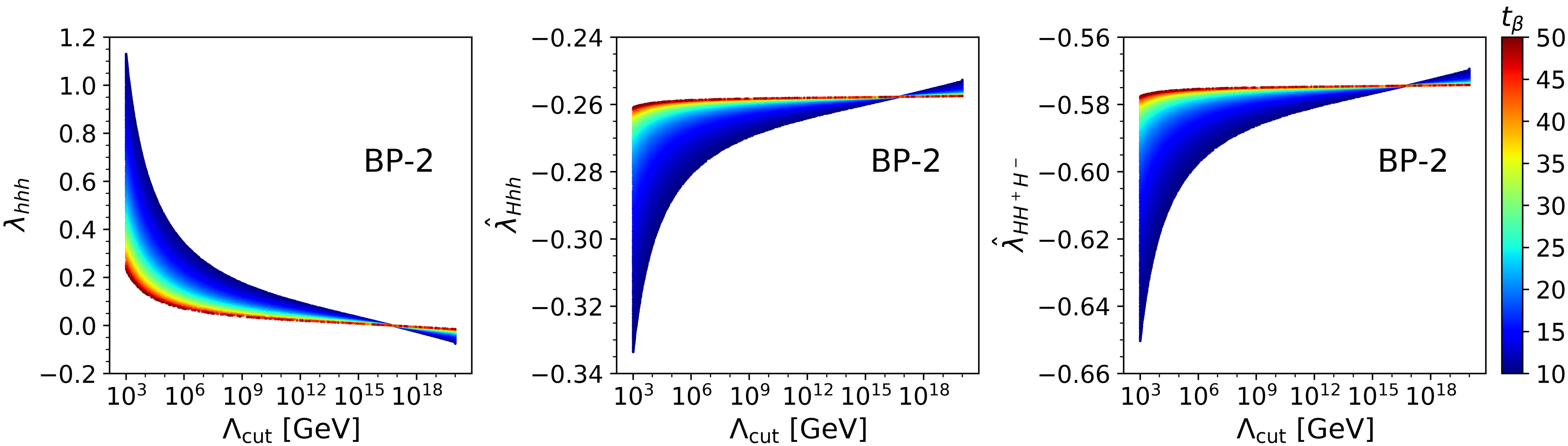
$$\boxed{\text{Large } t_\beta \text{ case: } t_\beta > 10.}$$

$t_\beta > 10$


Br(h→XX)
Br(A→XX)
Br(H±→XX)
Λcut [GeV]
BP-1
BP-2
BP-3
bb
ττ
cc
gg
hZ*
W±* h
τν
cs
W±* h + W±* A
t* b
W±* A
AZ*

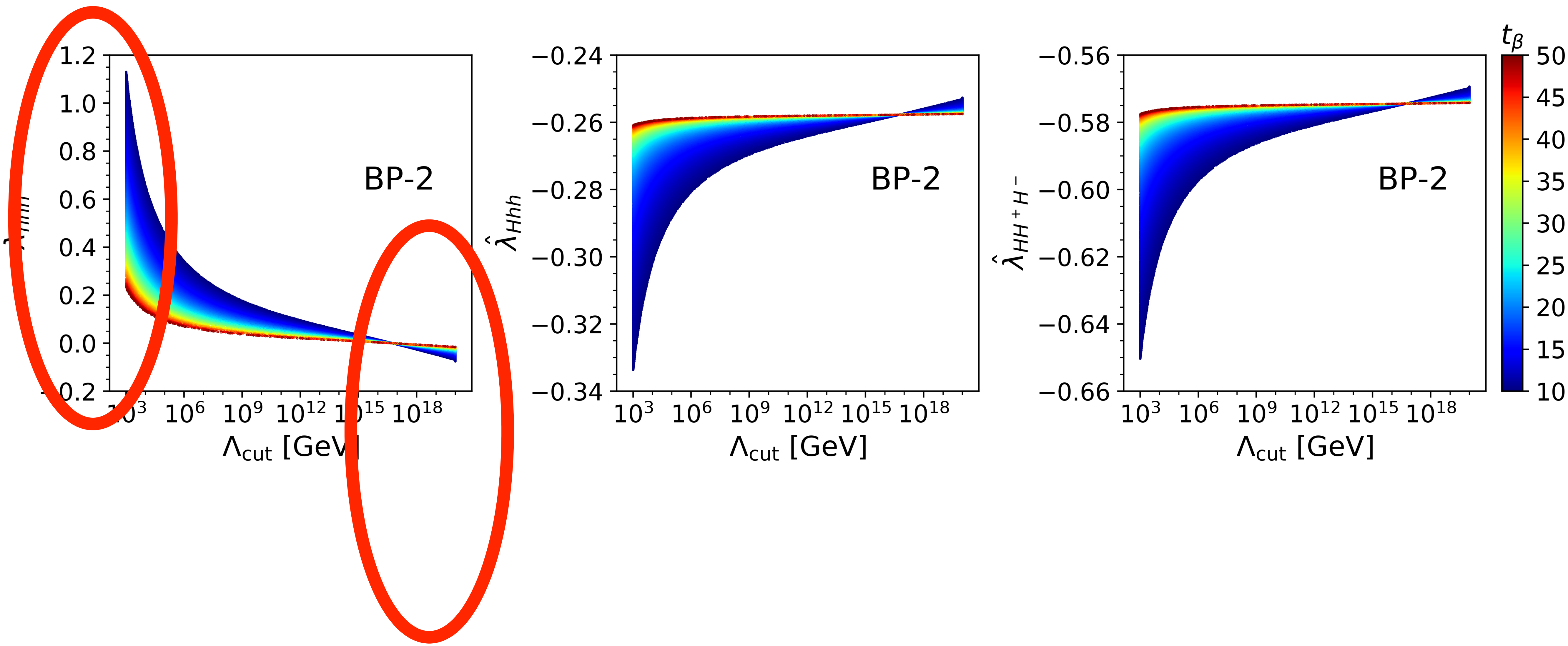


Dominant decay modes of the BSM Higgs bosons are insensitive to Λ.

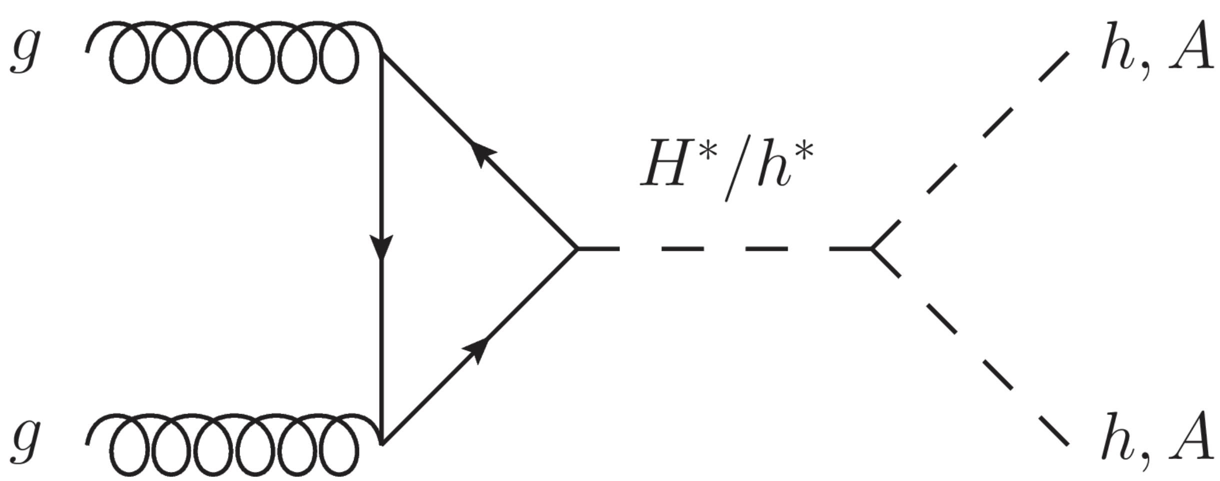


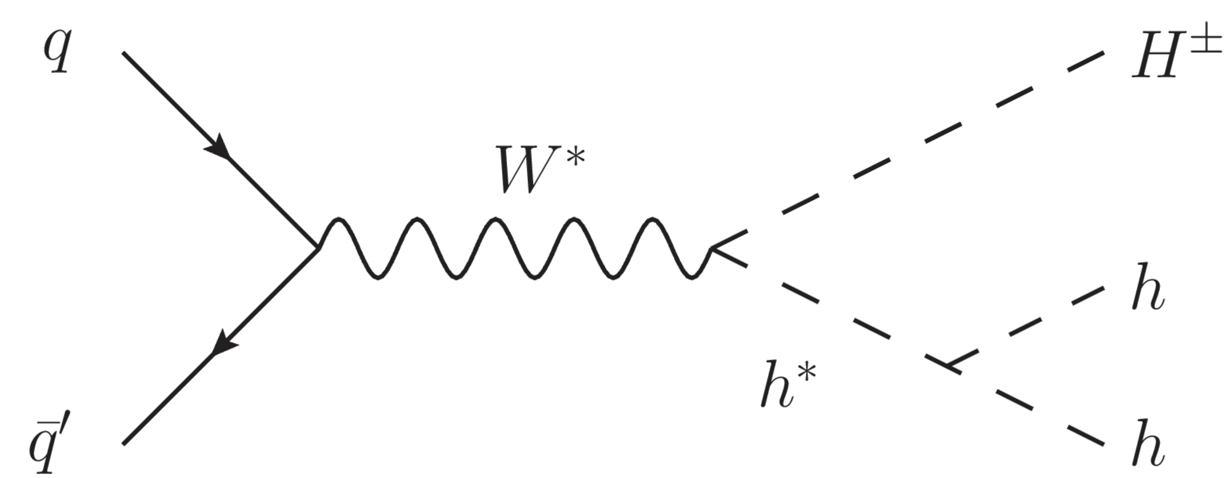
BP-2
BP-2
BP-2
$\hat{\lambda}_{hhh}$
$\hat{\lambda}_{Hhh}$
$\hat{\lambda}_{HH^+H^-}$
$\Lambda_{\rm cut}$ [GeV]
$\Lambda_{\rm cut}$ [GeV]
$\Lambda_{\rm cut}$ [GeV]
t_β

Not overlapped, at least, for Λ=1 TeV and Λ=10^{18} GeV.



Di-Higgs and Tri-Higgs productions to probe the trilinear Higgs couplings





Various channels for the tri-Higgs productions

$$q\bar{q} \to Z^* \to Ah^* \to Ahh, \quad q\bar{q} \to Z^* \to A^*h \to Ahh,$$

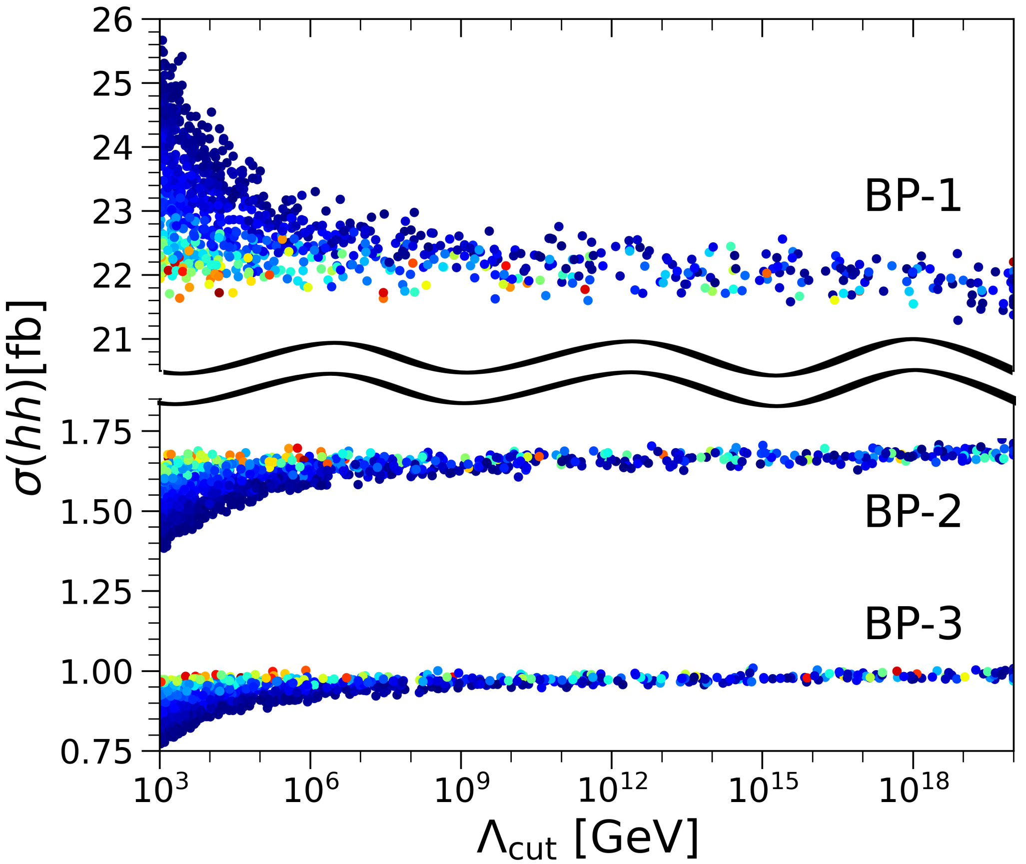
$$q\bar{q} \to Z^* \to Ah^* \to AAA,$$

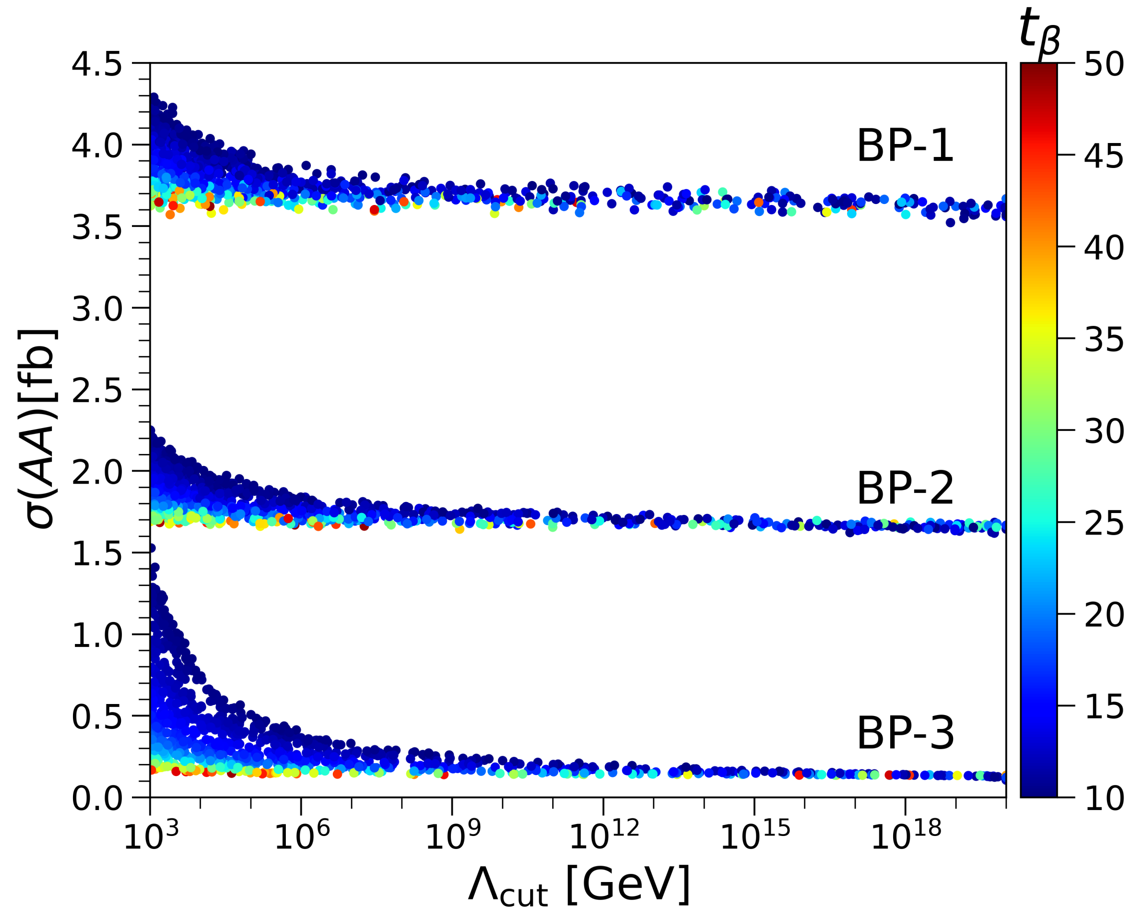
$$q\bar{q}' \to W^* \to H^\pm h^* \to H^\pm hh,$$

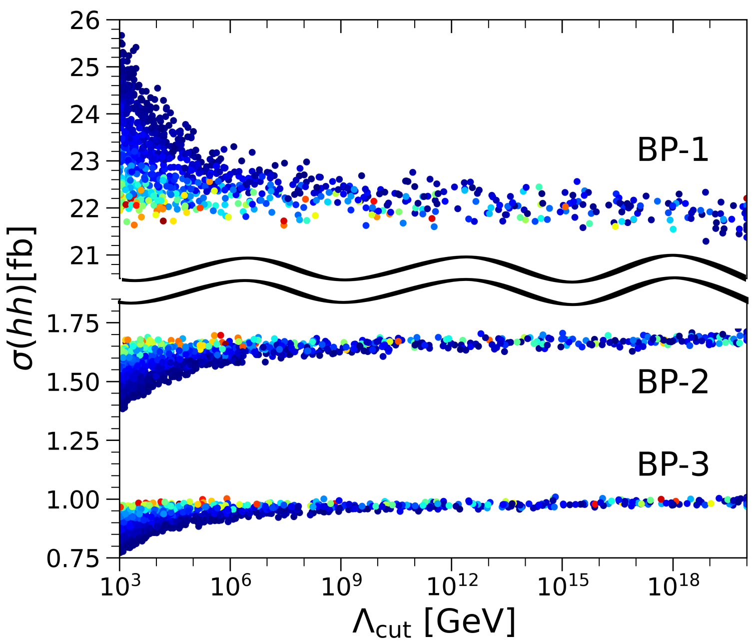
$$q\bar{q}' \to W^* \to H^\pm h^* \to H^\pm AA,$$

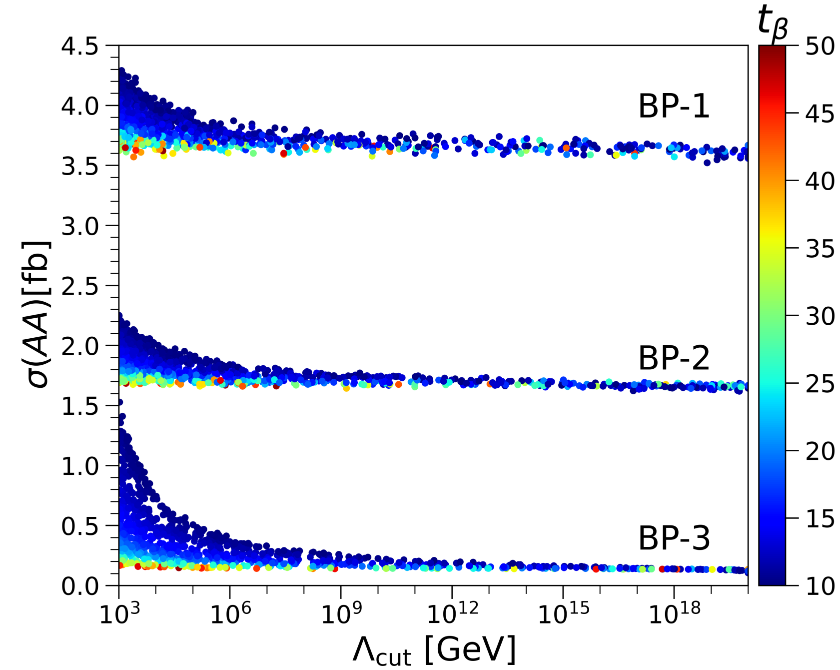
$$q\bar{q}' \to W^* \to H^\pm A^* \to H^\pm Ah.$$

Di-Higgs processes are NOT efficient as a discriminator.





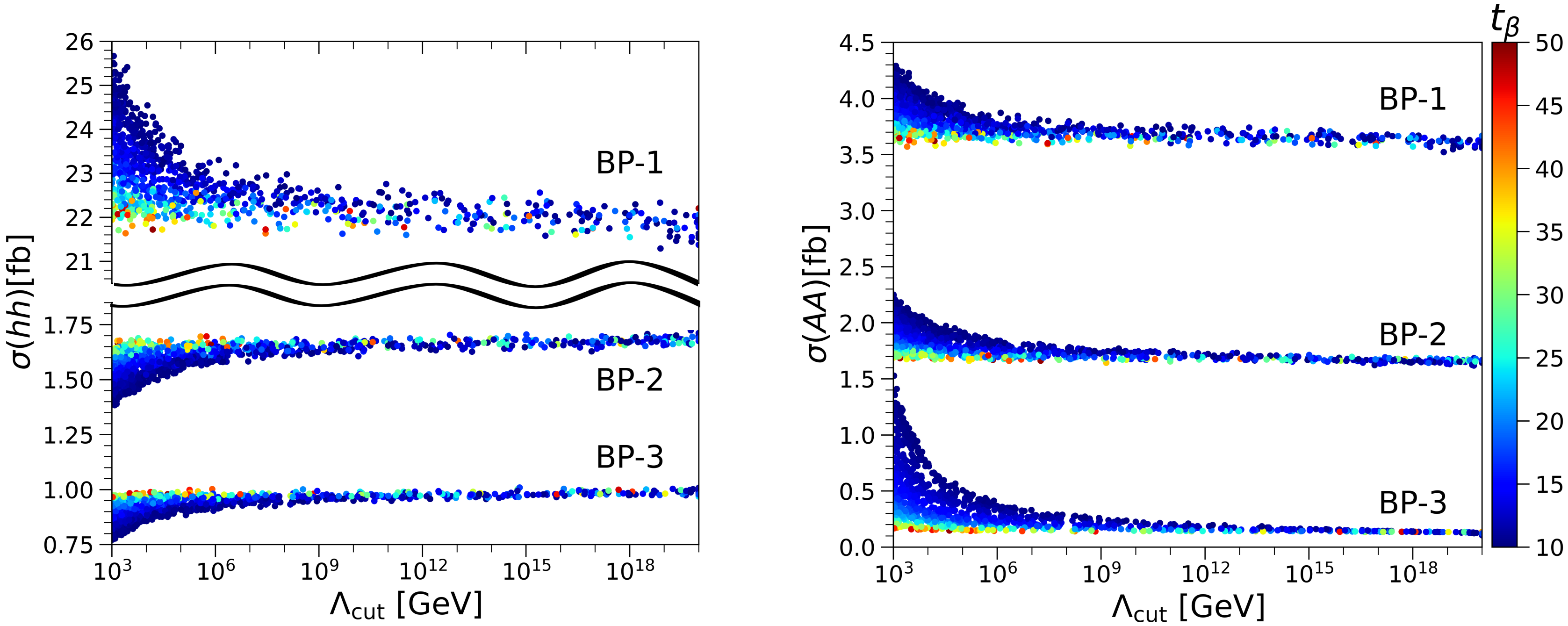




Because Hhh and hhh contributions are mixed.

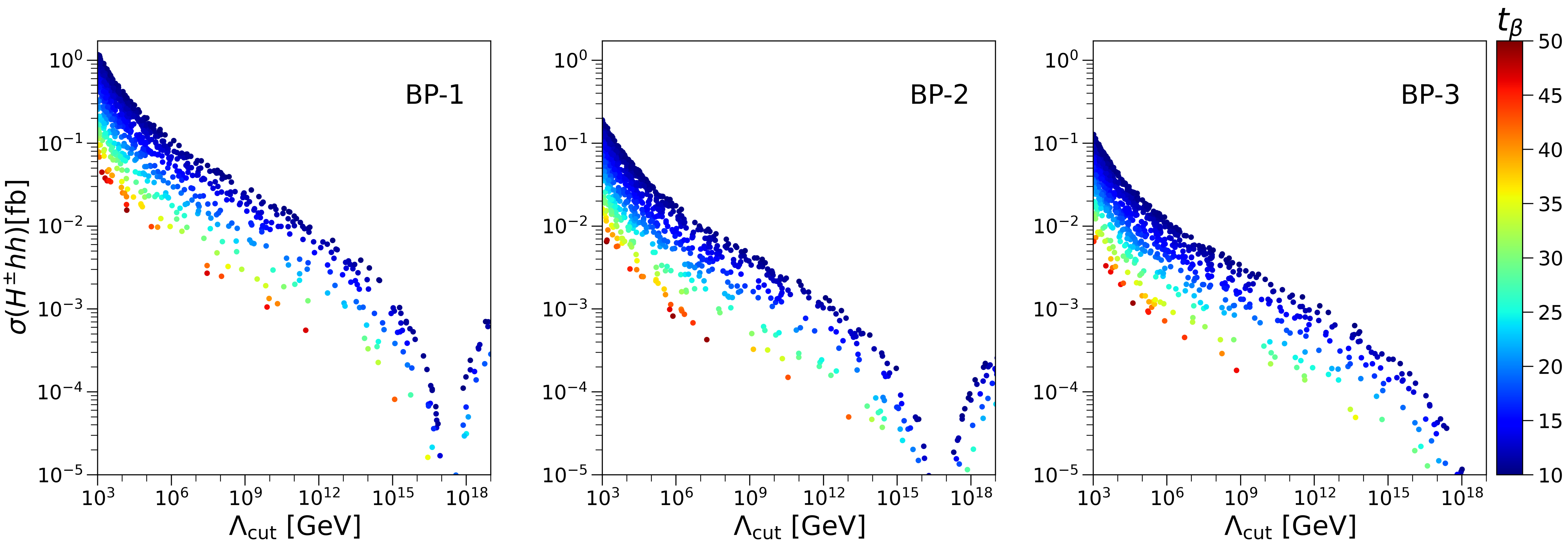
$$\text{BP-1 with } t_\beta = 10 \text{ and } \Lambda_{\rm cut} = 1 \text{ TeV}$$

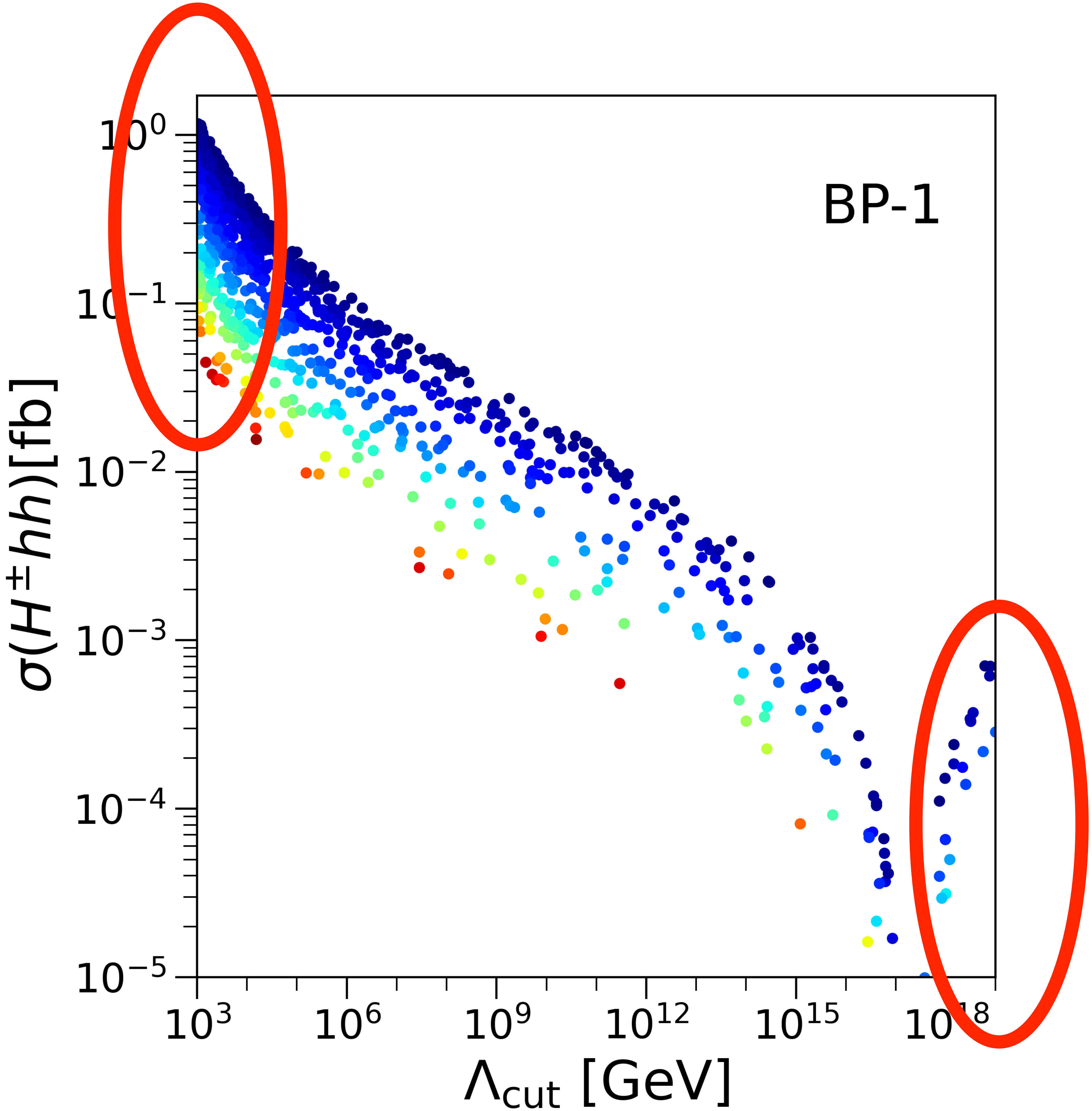
$$\sigma(gg \to H \to hh) \simeq 36.5 \text{ fb},$$
$$\sigma(gg \to h \to hh) \simeq 1.1 \text{ fb},$$
$$\sigma(gg \to hh)_{\rm intf} \simeq -12.1 \text{ fb}$$



Discriminating the high and low cutoff scales requires the precision measurement on the cross section within 1 fb.

Tri-Higgs processes are sensitive to Λ.

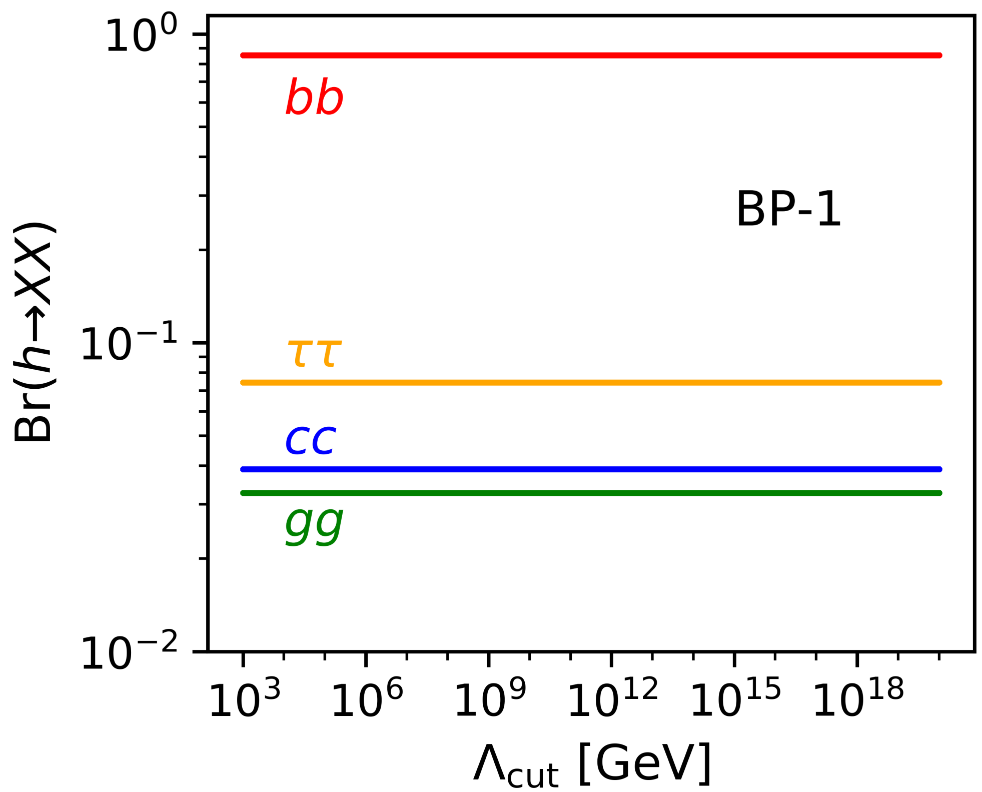


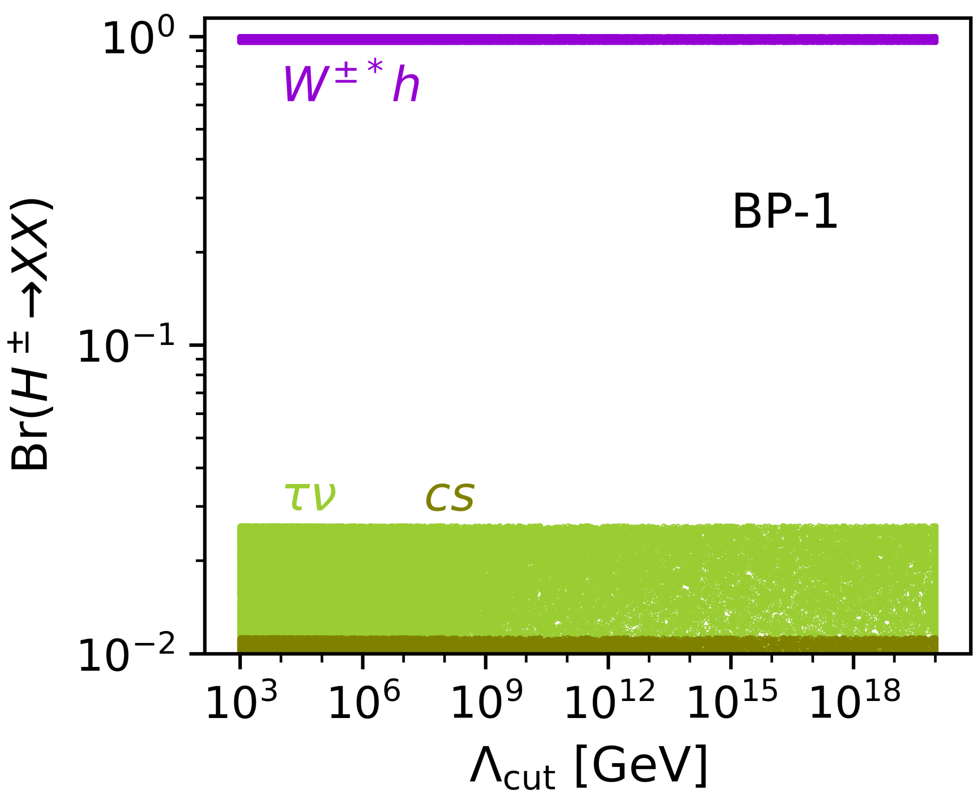

BP-1
$\sigma(H^{\pm}hh)$[fb]
$\Lambda_{\rm cut}$ [GeV]
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^3
10^6
10^9
10^{12}
10^{15}
10^{18}

Backgrounds?

NOT studied yet!!!

- Final states of H+ h h?   6b + lepton + MET

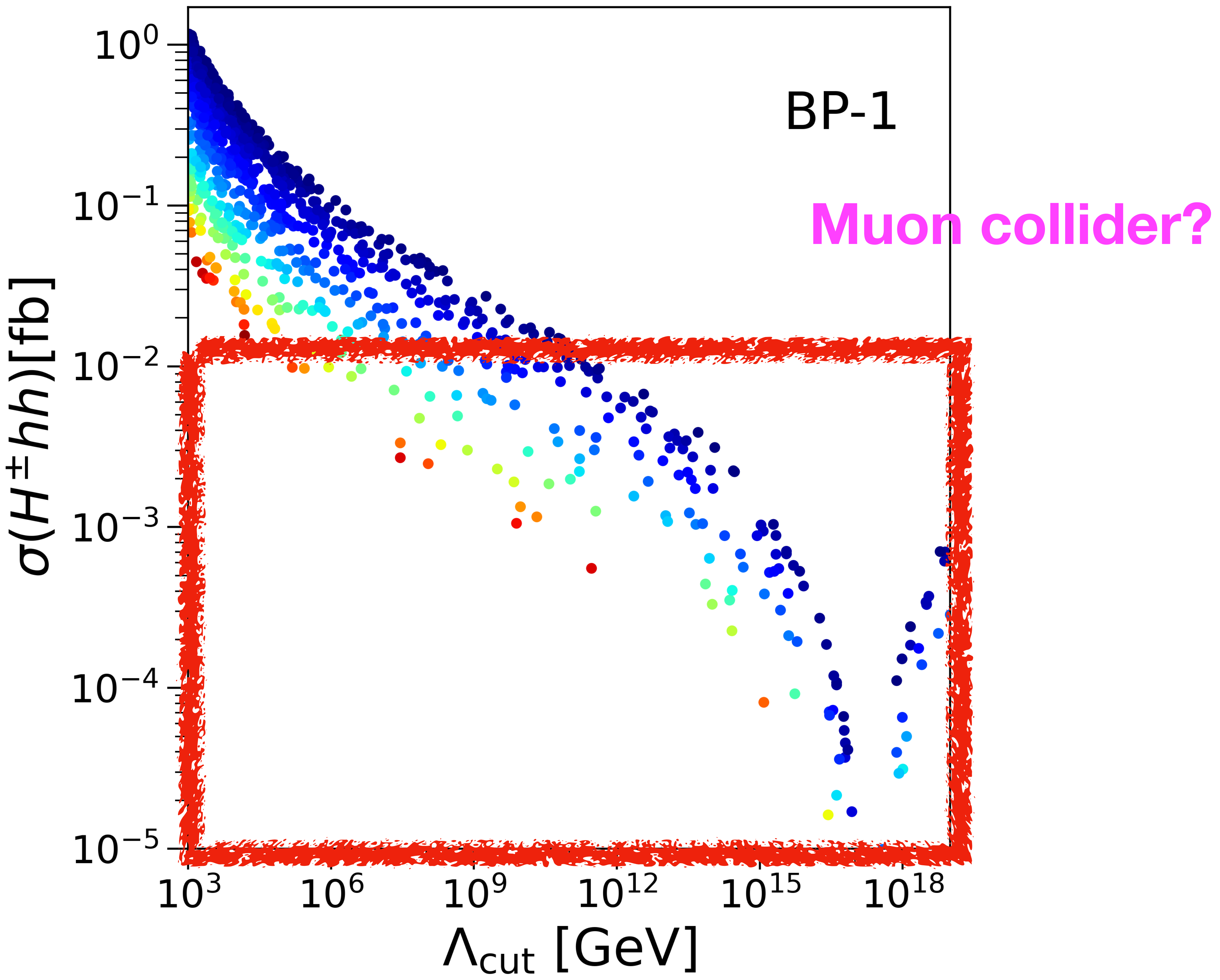




Backgrounds

$$t + \bar{t} + \ell\nu \to b j_b^{\rm mis} j_b^{\rm mis} + b j_b^{\rm mis} j_b^{\rm mis} + \ell\nu,$$
$$t + \bar{t} + jj \to b\ell\nu + b j_b^{\rm mis} j_b^{\rm mis} + j_b^{\rm mis} j_b^{\rm mis},$$

After basic selection, the background cross sections are below 1 ab.



BP-1
Muon collider?
$\sigma(H^{\pm}hh)$[fb]
$\Lambda_{\rm cut}$ [GeV]
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^3
10^6
10^9
10^{12}
10^{15}
10^{18}

5. Conclusions

- As one of the simplest extension of the SM, 2HDM is a good candidate for the first-stage NP model.
- Distinguishing the high- and low-cutoff scales is feasible through the measurement of the trilinear Higgs couplings.
- Tri-Higgs production is sensitive to the cutoff scale of the type-I 2HDM in the inverted scenario.