The Muon g-2 from Lattice QCD

Chulwoo Jung
for RBC/UKQCD collaborations

2nd Asian-European Institutes Workshop for BSM
10th KIAS Workshop on Particle Physics and Cosmology
Nov. 18. 2022

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Muon g-2 Colangelo et. al. arXiv:2203.15810

$$a^{\mu}_{Exp}(\text{BNL E821+ FNAL E989}) = 116592061(41) \times 10^{-11}$$
$$a^{\mu}_{SM} = 116591810(43) \times 10^{-11}$$
$$a^{\mu}_{Exp} - a^{\mu}_{SM} = 251(59) \times 10^{-11},$$

Fermilab E989 expect to achieve $\sim 16 \times 10^{-11}$ accuracy, also J-PARC.
In SM, Largest uncertainty from QCD related quantities : Hadronic Vacuum Polarization (HVP) , Hadronic Light-by-light (Hlbl)
HVP from $e^+e^- \to$ hadrons (R-ratio, Data driven)

$$a^{HVP}_{\mu}|_{e^+e^-} = 6931(40) \times 10^{-11}$$

HVP from Lattice QCD, Muon g-2 theory Initiative WP20 (arXiv:2006.04822) :

$$a^{HVP}_{\mu}|_{e^+e^-} = 7116(184) \times 10^{-11}$$

HVP from Lattice QCD, BMW20:

$$a^{HVP}_{\mu}|_{e^+e^-} = 7075(55) \times 10^{-11}$$

$\sim 2.6\sigma$ tension from R-ratio. Closer to 'no new physics'.

Muon g-2 theory Initiative https://muon-gm2-theory.illinois.edu/

The anomalous magnetic moment of the muon in the Standard Model

T. Aoyama[1,2,3], N. Asmussen[4], M. Benayoun[5], J. Bijnens[6], T. Blum[7,8], M. Bruno[9], I. Caprini[10], C. M. Carloni Calame[11], M. Cè[9,12,13], G. Colangelo[†14], F. Curciarello[15,16], H. Czyż[17], I. Danilkin[12], M. Davier[†18], C. T. H. Davies[19], M. Della Morte[20], S. I. Eidelman[†21,22], A. X. El-Khadra[†23,24], A. Gérardin[25], D. Giusti[26,27], M. Golterman[28], Steven Gottlieb[29], V. Gülpers[30], F. Hagelstein[14], M. Hayakawa[31,2], G. Herdoíza[32], D. W. Hertzog[33], A. Hoecker[34], M. Hoferichter[†14,35], B.-L. Hoid[36], R. J. Hudspith[12,13], F. Ignatov[21], T. Izubuchi[37,8], F. Jegerlehner[38], L. Jin[7,8], A. Keshavarzi[39], T. Kinoshita[40,41], B. Kubis[36], A. Kupich[21], A. Kupść[42,43], L. Laub[14], C. Lehner[†26,37], L. Lellouch[25], I. Logashenko[21], B. Malaescu[5], K. Maltman[44,45], M. K. Marinković[46,47], P. Masjuan[48,49], A. S. Meyer[37], H. B. Meyer[12,13], T. Mibe[†1], K. Miura[12,13,3], S. E. Müller[50], M. Nio[2,51], D. Nomura[52,53], A. Nyffeler[†12], V. Pascalutsa[12], M. Passera[54], E. Perez del Rio[55], S. Peris[48,49], A. Portelli[30], M. Procura[56], C. F. Redmer[12], B. L. Roberts[†57], P. Sánchez-Puertas[49], S. Serednyakov[21], B. Shwartz[21], S. Simula[27], D. Stöckinger[58], H. Stöckinger-Kim[58], P. Stoffer[59], T. Teubner[†60], R. Van de Water[24], M. Vanderhaeghen[12,13], G. Venanzoni[61], G. von Hippel[12], H. Wittig[12,13], Z. Zhang[18],

M. N. Achasov[21], A. Bashir[62], N. Cardoso[47], B. Chakraborty[63], E.-H. Chao[12], J. Charles[25], A. Crivellin[64,65], O. Deineka[12], A. Denig[12,13], C. DeTar[66], C. A. Dominguez[67], A. E. Dorokhov[68], V. P. Druzhinin[21], G. Eichmann[69,47], M. Fael[70], C. S. Fischer[71], E. Gámiz[72], Z. Gelzer[23], J. R. Green[9], S. Guellati-Khelifa[73], D. Hatton[19], N. Hermansson-Truedsson[14], S. Holz[36], B. Hörz[74], M. Knecht[25], J. Koponen[1], A. S. Kronfeld[24], J. Laiho[75], S. Leupold[42], P. B. Mackenzie[24], W. J. Marciano[37], C. McNeile[76], D. Mohler[12,13], J. Monnard[14], E. T. Neil[77], A. V. Nesterenko[68], K. Ottnad[12], V. Pauk[12], A. E. Radzhabov[78], E. de Rafael[25], K. Raya[79], A. Risch[12], A. Rodríguez-Sánchez[6], P. Roig[80], T. San José[12,13], E. P. Solodov[21], R. Sugar[81], K. Yu. Todyshev[21], A. Vainshtein[82], A. Vaquero Avilés-Casco[66], E. Weil[71], J. Wilhelm[12], R. Williams[71], A. S. Zhevlakov[78]

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From arXiv:2203.15810

Contribution	Value $\times 10^{11}$
Experiment (E821 + E989)	116 592 061(41)
HVP LO (e^+e^-)	6931(40)
HVP NLO (e^+e^-)	−98.3(7)
HVP NNLO (e^+e^-)	12.4(1)
HVP LO (lattice, $udsc$)	7116(184)
HLbL (phenomenology)	92(19)
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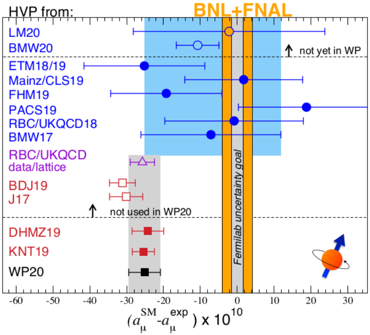
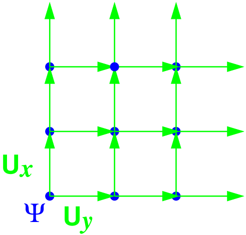
HLbL (lattice, uds)	79(35)
HLbL (phenomenology + lattice)	90(17)
QED	116 584 718.931(104)
Electroweak	153.6(1.0)
HVP (e^+e^-, LO + NLO + NNLO)	6845(40)
HLbL (phenomenology + lattice + NLO)	92(18)
Total SM Value	116 591 810(43)
Difference: $\Delta a_\mu := a_\mu^{\text{exp}} - a_\mu^{\text{SM}}$	251(59)



Urgency for reducing the uncertainty on LQCD calculations of HVP, which has been the focus of several collaborations in the last 2 years. g-2 theory initiative decided to focus on the intermediate distance 'Window' region value of HVP. New results from CLS(2206.06582),ETMC(2206.15084),RBC/UKQCD(in preparation), FHM(2207.04765,$t_0 = 0$fm). Most recent workshop at Higgs Centre. U. of Edinburgh, UK (https://indico.ph.ed.ac.uk/event/112/)

Introduction to lattice QCD

Quantum ChromoDynamics (QCD): Theory of strong interaction which governs interaction between quarks and gluons.
In contrast to Quantum Electrodynamics (QED), The effective coupling of QCD decreases in high energy, hence is calculable by hand, but not in low energy. $\rightarrow$ Nonperturbative techniques such as lattice QCD is needed for *ab initio* calculations. $(\psi(x), A_\mu(x)) \rightarrow (\psi(n), U_\mu(n) = \exp(-iA_\mu))$



$$Z = \int [dU] \det(\not{D} + m) e^{-(S_g)}$$

$$= \int [dU][d\bar{\psi}][d\psi] \ \exp[-(S_g + S_f)]$$

$$S_f = \bar{\psi}(D^\dagger D)^{-1}\psi, \qquad S_{eff} = S_g + S_f$$

$$S_g = \beta \sum \left[(U_\mu(x) U_\nu(x+\hat{\mu}) U^\dagger_\mu(x+\hat{\nu}) U^\dagger_\nu(x) \right]$$

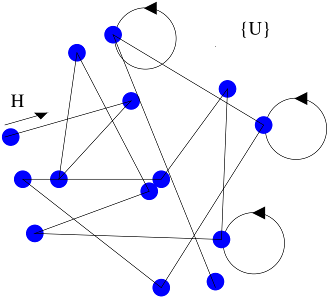
Current "typical" calculation: $V = 64^3 \times 128$, rank(D) $\sim 10^{10}$, nonzero element per row $= \sim 10^2$

Configuration generation, measurements

Importance sampling using $\exp(-(S = (S_g + S_f)))$ as the probability distribution, requires S_f to be real & nonnegative. Introduce fictitious momenta H to evaluate the path integral. $S_f = \det(\not{D} + m)$ is evaluated by Pseudofermions.

$$Z = \int [dU][d\bar{\psi}][d\psi][dH] \ \exp(-(\frac{1}{2}H^2 + S)),$$

$$\frac{dU_\mu(x)}{dt} = iH_\mu(x)U_\mu(x), \frac{d(\frac{1}{2}H^2 + S)}{dt} = 0$$



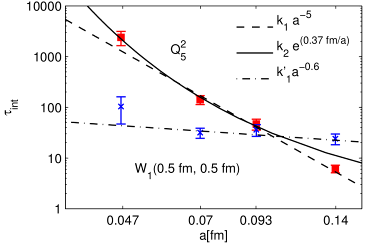
This process achieves $\pi(U) \propto \exp(-S)$ if reversible and
$\pi(U_1)P(U_1 \to U_2) = \pi(U_2)P(U_2 \to U_1)$ (detailed balance).
Accept/reject: Numerical integrators does not preserve $h = (\frac{1}{2}H^2 + S)$ exactly. Calculate h' at each end of trajectory and accept or reject according to $min[1, \exp[-(h' - h)]$.

Systematic errors in LQCD

- Finite lattice spacing errors: $\mathcal{O}(a^2), \mathcal{O}(a^4), \mathcal{O}(a^2 \log a^2) \cdots$
  Simulate with different lattice spacings and extrapolate to $a \to 0$.
  Can add operators in action or observable to eliminate or reduce it (Symanzik improvement).
- Finite Volume errors: $\int p^3 dp \to \sum p$
  Simulate with different lattice volume and extrapolate to $V \to \infty$ using ChPT, etc. Gounaris-Sakurai + Lellouch - Lüscher, Hansen and Patella (HP) $\cdots$
- Chiral symmetry: Crucial for many quantities, helps in controlling operator mixing, renormaliztion...
  different manifestation of breaking for different discretization.

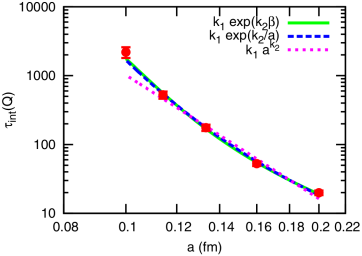
Critical Slowing Down in Hybrid Monte Carlo

Autocorrelation increases rapidly as the lattice spacing(a) decreases



Wilson, Q^2, Wilson loop

(Schaefer et. al., arXiv:1009.5228)



DBW2, Q

(McGlynn & Mawhinney, PhysRevD.90.074502)

$$\tau_{int} \sim a^{-(5\sim6)} \text{ or } exp[a_0/a]$$

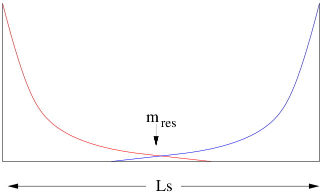
Numerical cost to generate the same number of decorrelated configurations increase at least $\sim (1/a)^9$ for the same physical volume!

Different discretization in Lattice QCD

Naive discretization $(\partial_\mu + iA_\mu)\psi(x) \rightarrow \frac{(U_\mu(x)\psi(x+\mu) - U^\dagger_\mu(x-\mu)\psi(x-\mu))}{2a}$ turns p into $\sin(p)$. $2^4 = 16$ particles instead of 1 (doublers)!
It is impossible to have a chirally invariant, doubler-free, local, translationally invariant, real bilinear fermion action on the lattice (Nielsen-Ninomiya no-go theorem).
Various solutions:

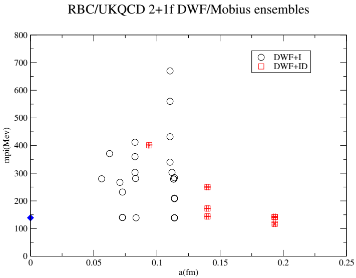
- Wilson Fermion(CLS): Add Laplacian-like term $-\frac{a}{2}\Delta\psi(x) = -\frac{a}{2}\sum_\mu[\psi(x + a\hat{\mu}) + \psi(x - a\hat{\mu}) - 2\psi(x)]$ Additive mass renormaliztion $\rightarrow$ mass tuning needed. Spurious zero mode in the Dirac operator (exceptional configurations)
- Twisted Wilson Fermion(ETMC): massless 2-flavor Wilson fermion + $m_l + i\mu_l\tau^3\gamma^5$
- Staggered (Kogut-Susskind) fermion (BMW,FHM,LM): $\psi \rightarrow \psi\Pi_{i=1\dots4}\gamma_i^{x_i}$ turns the action into 4 degenerate "particles" with 4 poles each. Keep only 1 spinor per site, interpret remaining 4 poles as 4 degenerate fermions. Chiral symmetry only partially preserved. 1 of 15 "pions" is a Goldstone pion. Special ChPT(Staggered ChPT, SChPT) to deal with taste breaking better. Still 4 flavor. "Rooting" needed to match nature.

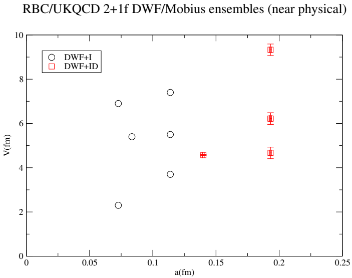
Domain Wall fermion / Overlap fermions (RBC/UKQCD): Dirac operator in 5D with repeating gauge field in 4D. Bulk contribution eliminated by introducing 'Pauli-Villars' with antiperiodic boundary condition in 5th dim. Residual symmetry breaking term well represented by a mass term for low enenegy quantities.



- Good chrial symmetry: Remnant symmetry breaking (residual mass) can be controlled separately from lattice spacing by increasing the extent of the 5th dimension (L_s) and the coupling between 4d slices
- In contrast to Wilson fermions where the discretized Dirac operator can have poles near the valence mass (exceptional configurations), DWF formalism guarantees safety as long as valence mass is positive. Allowing simulation at physical point for moderate lattice spacing without the need for chrial extrapolation or link smearing for fermions, etc. $\rightarrow$ Focus on physical point. Avoid relying on ChPT.
- More numerical cost compared to other discretizations

RBC/UKQCD 2+1f Ensembles





DWF+I: Iwasaki gauge action
DWF+ID: Iwasaki + Dislocation Suppressing Determinant Ratio (DSDR):
Suppresses the chiral symmetry breaking on larger lattice spacing.

HVP computation in LQCD Bernecker & Meyer, Eur. Phys. J. A 47, 148 (2011),

$$a_\mu = 4\alpha^2 \int_0^\infty dq^2 f(q^2)[\Pi(q^2) - \Pi(q^2 = 0)]\,,$$

$$J_\mu(x) = i \sum_f Q_f \overline{\Psi}_f(x)\gamma_\mu \Psi_f(x)$$

$$\sum_x e^{iqx} \langle J_\mu(x) J_\nu(0)\rangle = (\delta_{\mu\nu} q^2 - q_\mu q_\nu)\Pi(q^2)$$

$$\Pi(q^2) - \Pi(q^2 = 0) = \sum_t \left(\frac{\cos(qt) - 1}{q^2} + \frac{1}{2} t^2 \right) C(t)$$

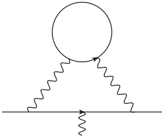
$$C(t) = \frac{1}{3} \sum_{\vec{x}} \sum_{j=0,1,2} \langle J_j(\vec{x}, t) J_j(0)\rangle$$

$$a_\mu = \sum_t w(t) C(t)\,.$$

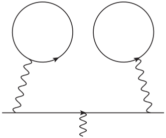
$$C(t) = C^{(0)}(t) + \alpha C^{(1)}_{\mathrm{QED}}(t) + \sum_f \Delta m_f C^{(1)}_{\Delta \mathrm{m_f}}(t) \quad + \mathcal{O}(\alpha^2, \alpha\Delta m, \Delta m^2)\,,$$

HVP from DWF 2+1f arXiv:1801.07224

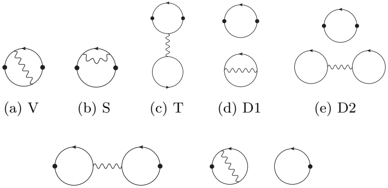
Typically QCD configurations isospin symmetric, expand on QED and SIB. Some diagrams are estimated



Connected diagram



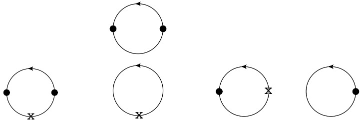
Disconnected diagram



(a) V (b) S (c) T (d) D1 (e) D2

(f) F (g) D3

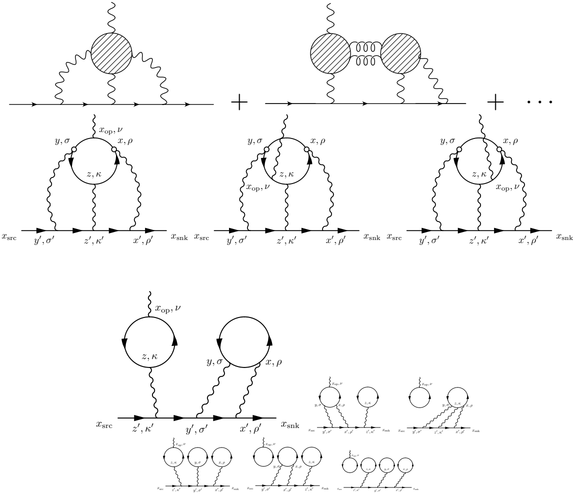
QED correction



(a) M (b) R (c) O

IB correction

Hadronic light by light (Hlbl) Blum et al., arXiv: 1911.08123



from arXiv: 1801.07224

$a_\mu^{\text{ud, conn, isospin}}$	$202.9(1.4)_S(0.2)_C(0.1)_V(0.2)_A(0.2)_Z$	$649.7(14.2)_S(2.8)_C(3.7)_V(1.5)_A(0.4)_Z(0.1)_{E48}(0.1)_{E64}$
$a_\mu^{\text{s, conn, isospin}}$	$27.0(0.2)_S(0.0)_C(0.1)_A(0.0)_Z$	$53.2(0.4)_S(0.0)_C(0.3)_A(0.0)_Z$
$a_\mu^{\text{c, conn, isospin}}$	$3.0(0.0)_S(0.1)_C(0.0)_Z(0.0)_M$	$14.3(0.0)_S(0.7)_C(0.1)_Z(0.0)_M$
$a_\mu^{\text{uds, disc, isospin}}$	$-1.0(0.1)_S(0.0)_C(0.0)_V(0.0)_A(0.0)_Z$	$-11.2(3.3)_S(0.4)_V(2.3)_L$
$a_\mu^{\text{QED, conn}}$	$0.2(0.2)_S(0.0)_C(0.0)_V(0.0)_A(0.0)_Z(0.0)_E$	$5.9(5.7)_S(0.3)_C(1.2)_V(0.0)_A(0.0)_Z(1.1)_E$
$a_\mu^{\text{QED, disc}}$	$-0.2(0.1)_S(0.0)_C(0.0)_V(0.0)_A(0.0)_Z(0.0)_E$	$-6.9(2.1)_S(0.4)_C(1.4)_V(0.0)_A(0.0)_Z(1.3)_E$
a_μ^{SIB}	$0.1(0.2)_S(0.0)_C(0.2)_V(0.0)_A(0.0)_Z(0.0)_{E48}$	$10.6(4.3)_S(0.6)_C(6.6)_V(0.1)_A(0.0)_Z(1.3)_{E48}$
$a_\mu^{\text{udsc, isospin}}$	$231.9(1.4)_S(0.2)_C(0.1)_V(0.3)_A(0.2)_Z(0.0)_M$	$705.9(14.6)_S(2.9)_C(3.7)_V(1.8)_A(0.4)_Z(2.3)_L(0.1)_{E48}$ $(0.1)_{E64}(0.0)_M$
$a_\mu^{\text{QED, SIB}}$	$0.1(0.3)_S(0.0)_C(0.2)_V(0.0)_A(0.0)_Z(0.0)_E(0.0)_{E48}$	$9.5(7.4)_S(0.7)_C(6.9)_V(0.1)_A(0.0)_Z(1.7)_E(1.3)_{E48}$
$a_\mu^{\text{R-ratio}}$	$460.4(0.7)_{RST}(2.1)_{RSY}$	
a_μ	$692.5(1.4)_S(0.2)_C(0.2)_V(0.3)_A(0.2)_Z(0.0)_E(0.0)_{E48}$ $(0.0)_b(0.1)_c(0.0)_{\overline{S}}(0.0)_{\overline{Q}}(0.0)_M(0.7)_{RST}(2.1)_{RSY}$	$715.4(16.3)_S(3.0)_C(7.8)_V(1.9)_A(0.4)_Z(1.7)_E(2.3)_L$ $(1.5)_{E48}(0.1)_{E64}(0.3)_b(0.2)_c(1.1)_{\overline{S}}(0.3)_{\overline{Q}}(0.0)_M$

TABLE I. Individual and summed contributions to a_μ multiplied by 10^{10}. The left column lists results for the window method with $t_0 = 0.4$ fm and $t_1 = 1$ fm. The right column shows results for the pure first-principles lattice calculation. The respective uncertainties are defined in the main text.

Contribution	Value $\times 10^{11}$
Experiment (E821 + E989)	116 592 061(41)
HVP LO (e^+e^-)	6931(40)
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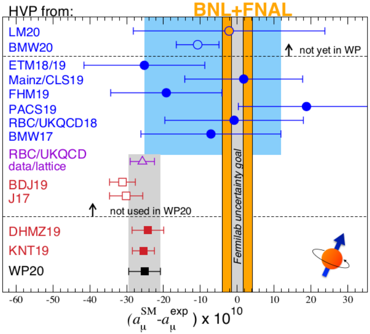
Difference: $\Delta a_\mu := a_\mu^{\text{exp}} - a_\mu^{\text{SM}}$	251(59)



Window decomposition of HVP

LQCD and $e^+e^- \to$ hadrons has different systematics:

- Short distance : LQCD finite lattice spacing error
- Immediate distance: LQCD has smaller errors
- Long distance : LQCD Finite Volume effect + exponential signal decay

RBC proposed Window method, which allows splitting and combining contributions from $e^+e^- \to$hadron,
To facilitate the comparison with BMW result, focus on the intermediate window from both LQCD and R-ratio

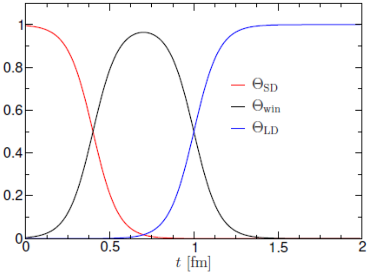
$$a_{\mu,SD} = \sum_t w(t)\,[1 - \Theta(t, t_0, \Delta)]\, C(t)\,.$$

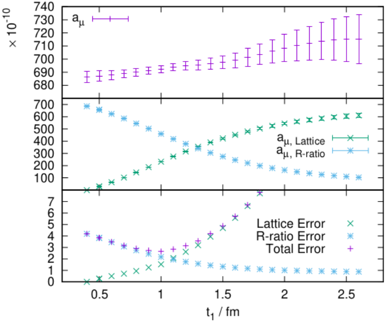
$$a_{\mu,W} = \sum_t w(t)\,[\Theta(t, t_0, \Delta) - \Theta(t, t_1, \Delta)]\, C(t)\,.$$

$$a_{\mu,LD} = \sum_t w(t)\,[\Theta(t, t_1, \Delta)]\, C(t)\,.$$

$$\Theta(t, t', \Delta) = \frac{1}{2}\left(1 + tanh\frac{t - t'}{\Delta}\right)$$

$$t_0 = 0.4fm, t_1 = 1fm, \Delta = 0.15fm$$





Weight function in Euclidean time and error comparison

Recent results on $a_{\mu,W}$

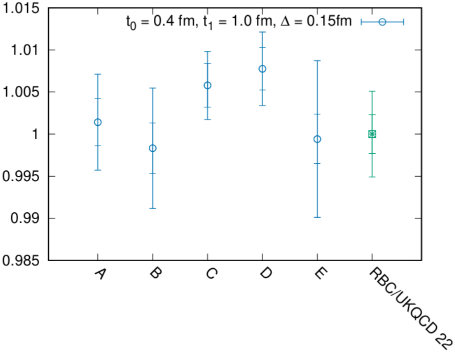
RBC/UKQCD:

- $4\times$ statistics on 2 physical ensembles measured previously
- 3rd physical ensemble at lattice spacing a ~0.07fm , 4th in progress
- Use both continuum and lattice p for HVP kernel.
- Additional measurement with 'conserved' (preserves lattice action) current operator
- Additional normalization constant for the vecrtor current Z_V.
- Many additional ensembles around existing physical point ensembles for better control of systematic errors
- 2 different definitions for the 'physical' point:
  RBC($m_\pi = 0.135$Gev, $m_K = 0.4957$Gev, $m_\Omega = 1.67225$Gev)
  BMW($m_\pi = 0.13497$Gev, $m_{s\bar{s}} = 0.6898$Gev, w_0=0.17236 fm).
- Blinding: Multiply $(b_0 + b_1 a^2 + b_2 a^4)$ to $C(t)$ for different groups. Eliminated after unblinding

New Mobius ensembles tuned to precision HVP (including $N_f = 2+1+1$ ensembles):

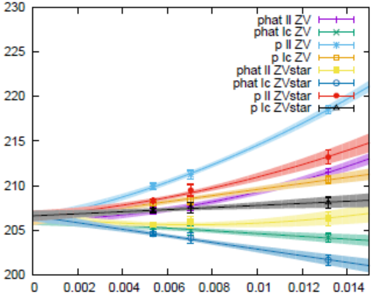
id	a^{-1} / GeV	m_π / GeV	m_K / GeV	m_{D_s} / GeV	$m_\pi L$	L_s
1	1.73	0.210	0.530	–	3.8	24
3	1.73	0.210	0.600	–	3.8	24
4	1.73	0.280	0.530	–	3.8	24
2	1.73	0.280	0.530	–	3.8	32
A	1.73	0.280	0.530	–	3.8	8
5	1.73	0.280	0.530	1.9	3.8	24
7	1.73	0.280	0.530	1.3	3.8	24
8	2.359	0.280	0.530	1.9	3.8	12
B	1.73	0.140	0.500	–	2.5	24
C	1.73	0.140	0.500	–	5.0	24
D	1.73	0.280	0.500	–	5.0	24
E	3.5	0.280	0.530	–	3.8	12
48I	1.73	0.140	0.500	–	3.8	24
64I	2.359	0.140	0.500	–	3.8	12
96I	2.7	0.135	0.500	–	4.8	12

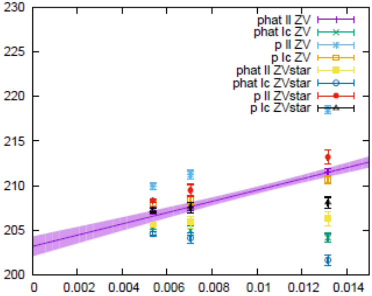
Blinding



Multiply $(b_0 + b_1 a^2 + b_2 a^4)$ to $C(t)$ for different groups.

$$a_W(ud) = 206.36(44)(43), a_{SD}(ud) = 48.7(5)(16) \times 10^{-10}$$

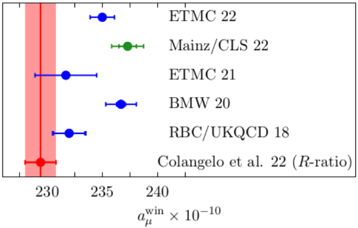




More recent results

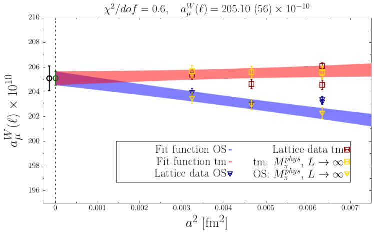
CLS (arXiv: 2206.06582)

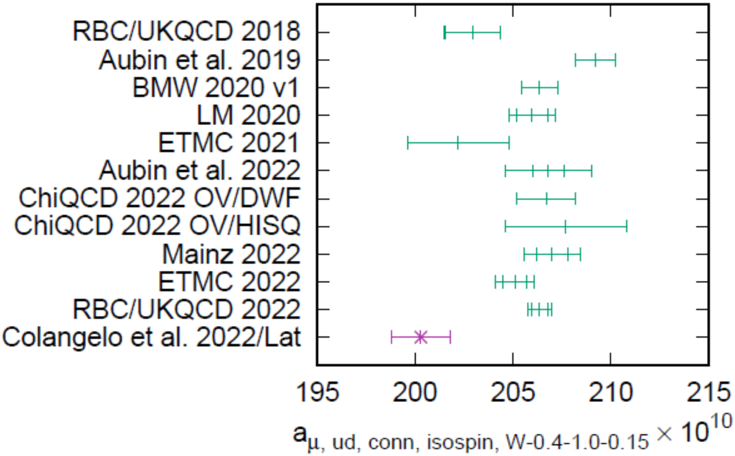
- Wilson fermion with $a = 0.04 - 0.1$fm
- $a_W = (237.30 \pm 0.79_{stat} \pm 1.22_{syst}) \times 10^{-10}$
  $a_W(ud) = (207.00 \pm 0.83_{stat} \pm 1.20_{syst}) \times 10^{-10}$



ETMC (arXiv: 2206.15084)

- Twisted Wilson fermion, 2+1+1f, $a = 0.057$-0.08 fm
- $a_{SD}, a_W(all), a_W(ud) = 69.33(29), 235.0(1.1), 205.10(56) \times 10^{-10}$





From C. Lehner.

Summary

- Most recent lattice results on the 'Window' (0.4-1fm) region of HVP moves away from ($e^+e^- \rightarrow$ hadrons). Improved results in short distance and other parts already reported or coming shortly.
- Concerted effort from multiple LQCD collaborations with different discretizations is critical in reliable estimate of the systematic errors.
- There are still significant portion of the tension between experiment and data driven approach not yet unexplained.
- Improvement LQCD results in the short- and long-distance region of HVP crucial in getting more accurate picture. A lot of improvement being made. Stay tuned!

Thank you!