W boson mass anomaly and grand unification

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W-Boson Anomaly

- Best fit for the M_W (LEP-2, Tevatron, LHC, LHCb data)

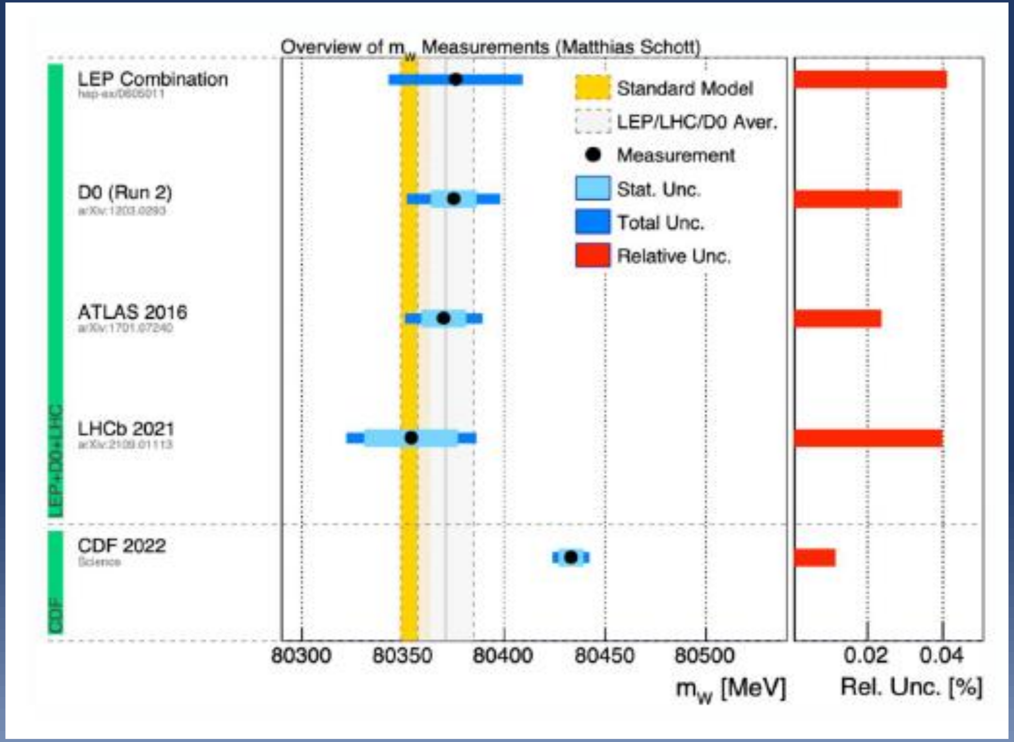
$$M_{W_{fit}} = 80.4133 \pm 0.0080 \mathrm{GeV}$$ 2204.04204

- SM prediction for M_W
  (Fit all data except M_W and predict M_W)

$$M_{W_{pred}} = 80.3499 \pm 0.0056 \mathrm{GeV}$$ (Pull 6.5σ) 2204.04204

- Deviation of the W-boson mass

$$\delta M_W \simeq 63 \text{ MeV}$$



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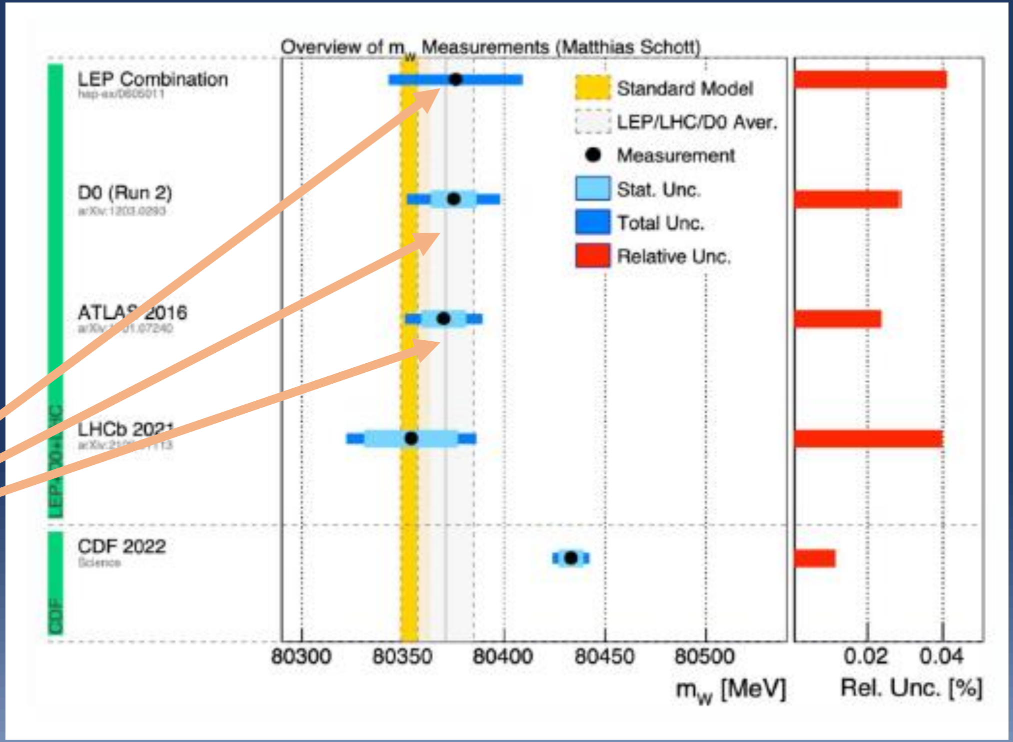
- Even with out CDF, some pull for larger M_W

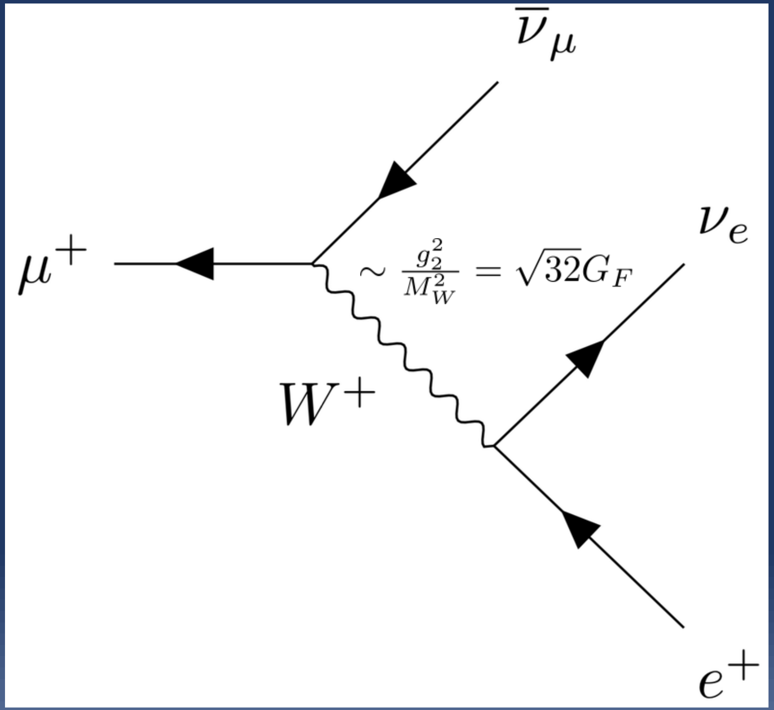


W Boson Mass Shift From New Physics

- Fermi Constant determined from muon decay
  - Very precisely measured

$$\Gamma_\mu = \frac{G_F^2 m_\mu^5}{192\pi^3} \qquad \frac{1}{\Gamma_\mu} = \tau_\mu = 2.1969811(22) \times 10^{-6}\ \mathrm{s}$$

$$G_F = 1.1663787(6) \times 10^{-5}\ \mathrm{GeV}^{-1}$$



W Boson Mass Shift From New Physics

- Fermi Constant determined from muon decay
- Fermi constant input, M_W predicted

$$\frac{G_F}{\sqrt{2}} = \frac{\pi\alpha}{2M_W^2 s_W^2}\left(1+\Delta r\right)$$

  - This defines way to changes M_W

$$\frac{\Delta M_W}{M_W} = \frac{1}{2}\frac{\delta\alpha}{\alpha} - \frac{c_W}{s_W}\delta\theta - \frac{1}{2}\frac{\delta G_F}{G_F}$$

  - In terms of SMEFT this is

$$\mathcal{O}_{HWB} = H^\dagger \tau^I H W^I_{\mu\nu} B^{\mu\nu}$$

$$\mathcal{O}_{HD} = \left(H^\dagger D_\mu H\right)\left(H^\dagger D_\mu H\right)$$

$$\mathcal{O}_{\ell\ell} = \left(\bar\ell_p \gamma_\mu \ell_r\right)\left(\bar\ell_s \gamma^\mu \ell_t\right)$$

$$\mathcal{O}^{(3)}_{H\ell} = \left(H^\dagger \overleftrightarrow{D}^I_\mu H\right)\left(\bar\ell_s \tau^I \gamma^\mu \ell_t\right)$$

$$\frac{\Delta M_W}{M_W} = -\frac{s_{2W}}{c_{2W}}\frac{v^2}{4\Lambda^2}\left(\frac{c_W}{s_W}C_{HD} + \frac{s_W}{c_W}\left(4C^{(3)}_{H\ell} - 2C_{\ell\ell}\right) + 4C_{HWB}\right)$$

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$$\frac{\Delta M_W}{M_W} = -\frac{s_{2W}}{c_{2W}}\frac{v^2}{4\Lambda^2}\left(\frac{c_W}{s_W}\underbrace{C_{HD}}_{\propto T} + \frac{s_W}{c_W}\underbrace{\left(4C^{(3)}_{H\ell} - 2C_{\ell\ell}\right)}_{\text{Give } \delta G_F} + \underbrace{4C_{HWB}}_{\propto S}\right)$$

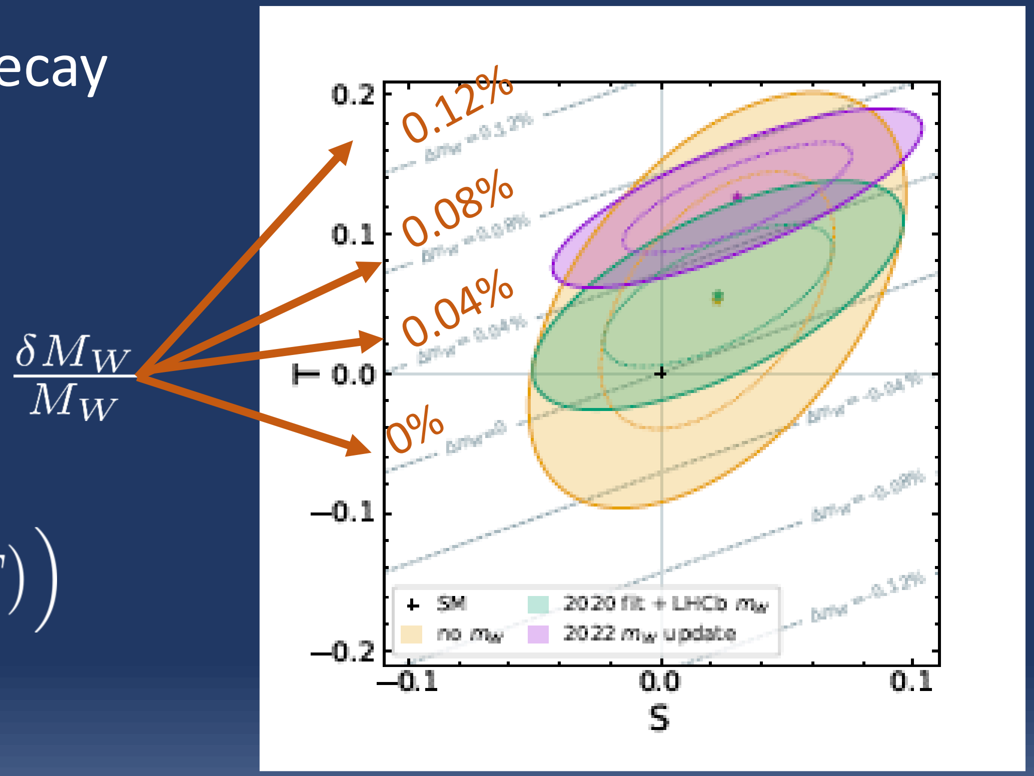
Give $\delta\theta_W$

Give $\delta\theta_W$

W Boson Mass Shift From New Physics

- Fermi Constant determined from muon decay
- Fermi constant generally taken as input
- Example: Supersymmetry
    - Correction now in term so S,T

$$M_W = (M_W)_{SM}\left(1+\frac{1}{2}\frac{c_W^2}{c_W^2-s_W^2}\left(-\frac{1}{2}S+c_W^2T\right)\right)$$



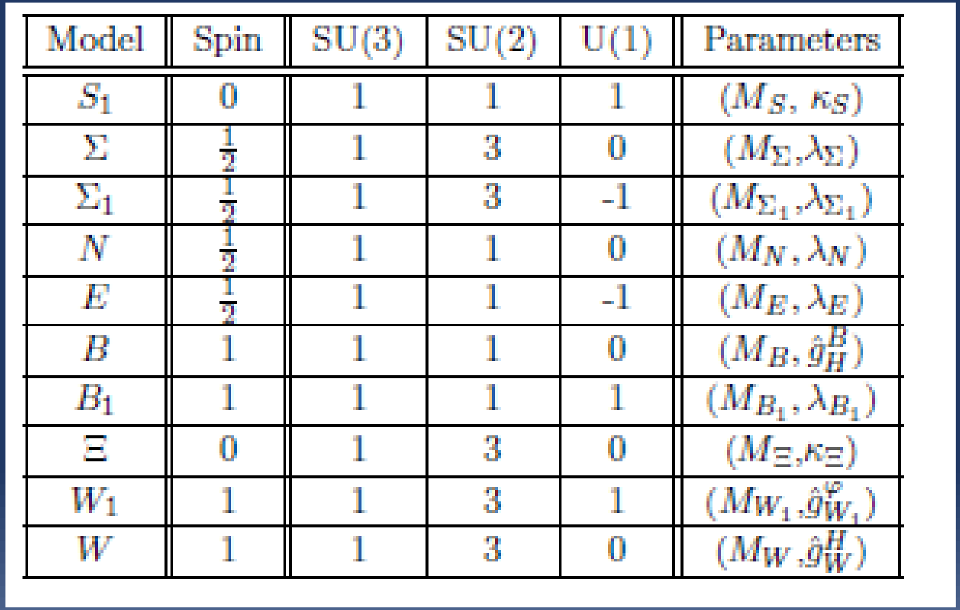
- Light charged Higgsino and Wino

$$T=\frac{3\alpha_2}{16\pi}\frac{M_W^2}{M_2^2}\left(\frac{1-\tan^2\beta}{1+\tan^2\beta}\right)^2 \qquad \frac{\delta M_W}{M_W}=\frac{1}{2}\frac{c_W^2}{c_W^2-s_W^2}T$$

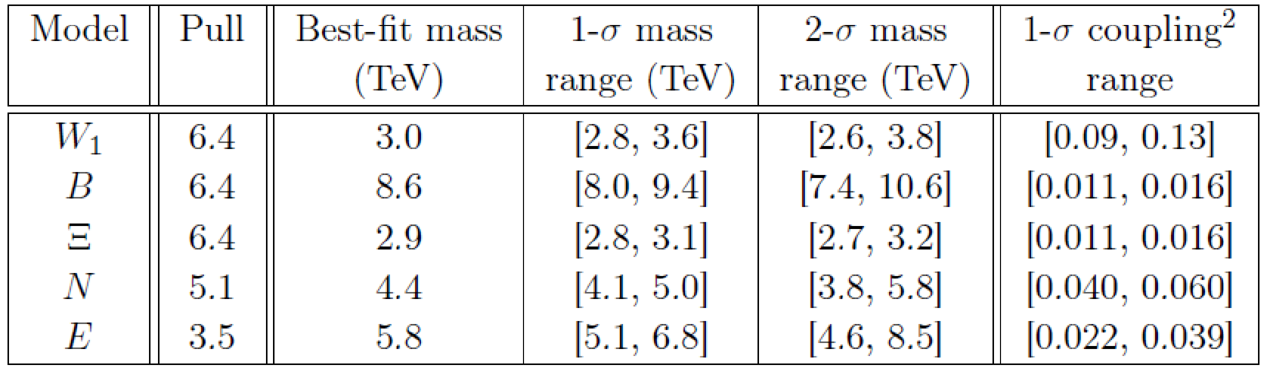
M_2 GeV	100	150	200	250
$\delta M_W/M_W$	0.09%	0.04%	0.02%	0.01%

Possible Tree-Leve Solutions

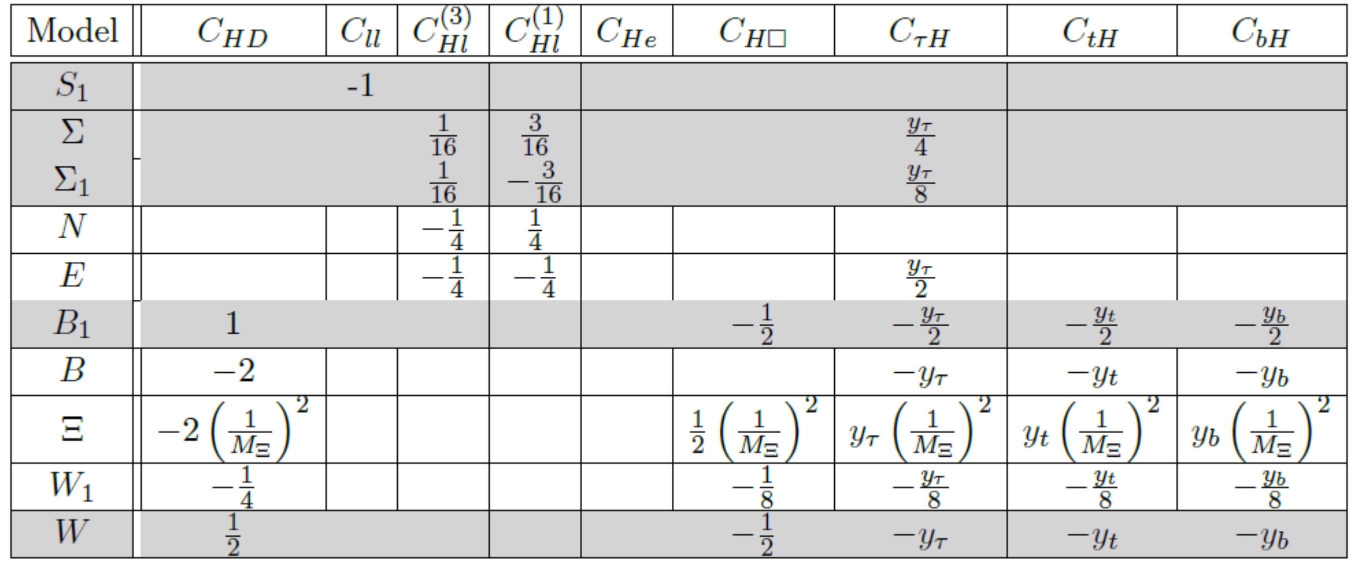
- Possible single field extensions which explain deviation



Model	Spin	SU(3)	SU(2)	U(1)	Parameters
S_1	0	1	1	1	$(M_S,\ \kappa_S)$
Σ	$\frac{1}{2}$	1	3	0	$(M_\Sigma,\lambda_\Sigma)$
Σ_1	$\frac{1}{2}$	1	3	-1	$(M_{\Sigma_1},\lambda_{\Sigma_1})$
N	$\frac{1}{2}$	1	1	0	(M_N,λ_N)
E	$\frac{1}{2}$	1	1	-1	(M_E,λ_E)
B	1	1	1	0	$(M_B,\hat{g}_H^B)$
B_1	1	1	1	1	(M_{B_1},λ_{B_1})
Ξ	0	1	3	0	(M_Ξ,κ_Ξ)
W_1	1	1	3	1	$(M_{W_1},\hat{g}_{W_1}^\varphi)$
W	1	1	3	0	$(M_W,\hat{g}_W^H)$



Model	Pull	Best-fit mass (TeV)	1-σ mass range (TeV)	2-σ mass range (TeV)	1-σ coupling^2 range
W_1	6.4	3.0	[2.8, 3.6]	[2.6, 3.8]	[0.09, 0.13]
B	6.4	8.6	[8.0, 9.4]	[7.4, 10.6]	[0.011, 0.016]
Ξ	6.4	2.9	[2.8, 3.1]	[2.7, 3.2]	[0.011, 0.016]
N	5.1	4.4	[4.1, 5.0]	[3.8, 5.8]	[0.040, 0.060]
E	3.5	5.8	[5.1, 6.8]	[4.6, 8.5]	[0.022, 0.039]



Model	C_{HD}	C_{ll}	$C_{Hl}^{(3)}$	$C_{Hl}^{(1)}$	C_{He}	$C_{H\Box}$	$C_{\tau H}$	C_{tH}	C_{bH}
S_1		-1							
Σ			$\frac{1}{16}$	$\frac{3}{16}$			$\frac{y_\tau}{4}$		
Σ_1			$\frac{1}{16}$	$-\frac{3}{16}$			$\frac{y_\tau}{8}$		
N			$-\frac{1}{4}$	$\frac{1}{4}$					
E			$-\frac{1}{4}$	$-\frac{1}{4}$			$\frac{y_\tau}{2}$		
B_1	1					$-\frac{1}{2}$	$-\frac{y_\tau}{2}$	$-\frac{y_t}{2}$	$-\frac{y_b}{2}$
B	-2						$-y_\tau$	$-y_t$	$-y_b$
Ξ	$-2\left(\frac{1}{M_\Xi}\right)^2$					$\frac{1}{2}\left(\frac{1}{M_\Xi}\right)^2$	$y_\tau\left(\frac{1}{M_\Xi}\right)^2$	$y_t\left(\frac{1}{M_\Xi}\right)^2$	$y_b\left(\frac{1}{M_\Xi}\right)^2$
W_1	$-\frac{1}{4}$					$-\frac{1}{8}$	$-\frac{y_\tau}{8}$	$-\frac{y_t}{8}$	$-\frac{y_b}{8}$
W	$\frac{1}{2}$					$-\frac{1}{2}$	$-y_\tau$	$-y_t$	$-y_b$

Bagnaschi, Ellis, Madigan, Mimasu, Sanz, You

$$\mathcal{O}_{HD} = \left(H^\dagger D_\mu H\right)\left(H^\dagger D_\mu H\right)$$

Electroweak Symmetry Breaking with Triplet

- Triplet can contribute to Electroweak symmetry breaking

Generates Σ_3 vev

$$V(H,\Sigma_3) \supset -\mu_H^2|H|^2 + \lambda_H|H|^4 + \boxed{A_{3H}H^\dagger\Sigma_3 H} + h.c. + 2\boxed{\mu_3^2}\mathrm{Tr}(\Sigma_3^\dagger\Sigma_3)$$

$\sim$ TeV

- If $Y=0$, then only contributes to W mass

$$\langle H\rangle = (0,v)^T, \quad \langle\Sigma_3\rangle = \frac{1}{2}\begin{pmatrix} v_T & 0 \\ 0 & -v_T \end{pmatrix}$$

$$[W_\mu,\langle\Sigma_3\rangle] = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & -v_T W_\mu^+ \\ v_T W_\mu^+ & 0 \end{pmatrix} \quad \Longrightarrow \quad \delta M_W^2 = 2g_2^2 v_T^2$$

- Only contributes to the W boson
  - If we take hypercharge non-zero would also contribute to Z

Triple Higgs Boson and the Fermi Constant

- This mass correction alters weak mixing angle

$$s_W^2 = \overbrace{1 - \frac{M_{W_0}^2}{M_Z^2}}^{\tilde{s}_W^2} + \frac{2e^2 v_T^2}{\tilde{s}_W^2 M_Z^2}$$

$\tilde{s}_W^2$ → Diagonalizes A,Z

$$\frac{G_F}{\sqrt{2}} \simeq \frac{\alpha\pi}{M_W^2}\left(s_W^2 + \overbrace{\sqrt{s_W^2 + \frac{4e^2 v_T^2}{M_Z^2}}}^{\tilde{s}_W^2}\right)^{-1}$$

- This leads to a correction to the measured W boson mass

$$M_W \simeq (M_W)_{SM}\left(1 + \frac{1}{2}\frac{c_W^2}{c_W^2 - s_W^2}\frac{4v_T^2}{v^2}\right)$$

$$v_T \sim 3\ \mathrm{GeV} \rightarrow \delta M_W \sim 60\mathrm{GeV}$$

$$v_T = \frac{A_{3H} v^2}{2\mu_3^2} \qquad v^2 = \left(\mu_H^2 + A_{3H} v_T\right)/(2\lambda_H)$$

- Is there motivation for this new field?

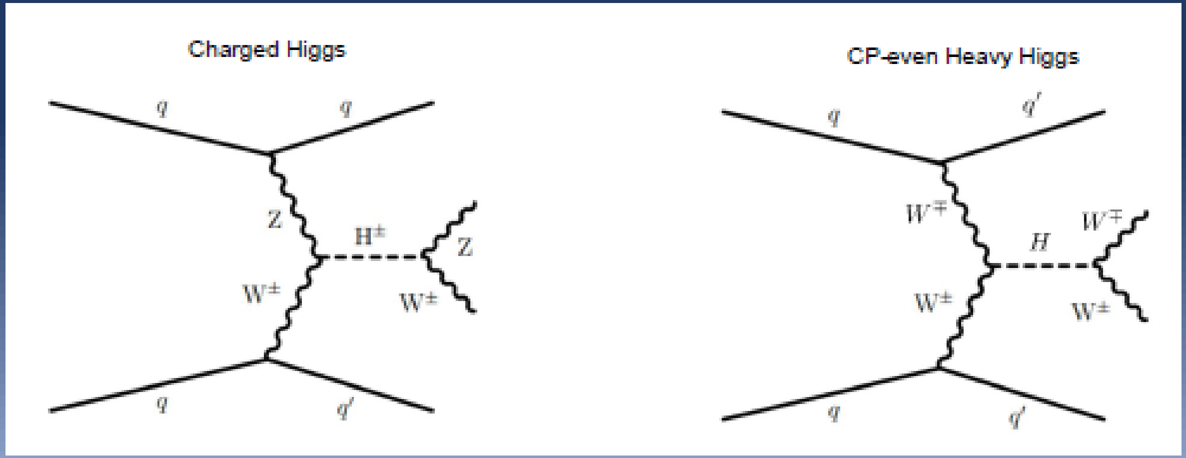
$$\mu_3 \sim \ \mathrm{TeV}, \quad \mathrm{A_{3H}} \sim 200\ \mathrm{GeV}$$

Triple Higgs Boson Signatures

- The neutral Higgs bosons small mix
- Couplings to SM fermions quite suppressed

$$\theta_{H,H^{\pm}} \simeq \frac{A_{3H}v}{\mu_3^2} \simeq 0.03$$

- Dominant signature from decays to SM massive bosons



Couplings $\propto v_T$
so suppressed

“Who Order That”(Issac Rabi)

- Like the discovery of the muon, why would nature have a triplet?
  - At face value it does nothing but give mass to $W^{\pm}$

"Who Order That"(Issac Rabi)

- Like the discovery of the muon, Who order a triplet?

- SU(5) Grand unification?

$$\Sigma_{24} \supset \Sigma_3 = (1,3,0) \ \ [SU(3), SU(2), U(1)]$$

  - Gauge coupling unification

Dotted: SM

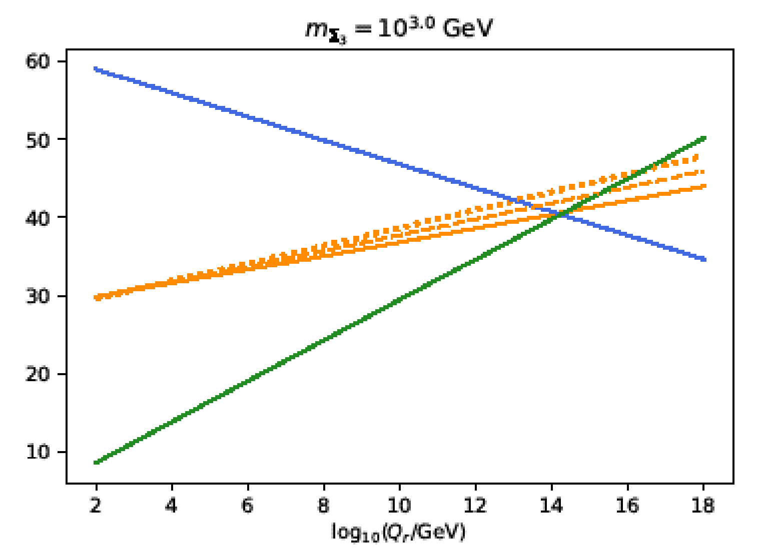
$$\Delta b_2 = 0$$

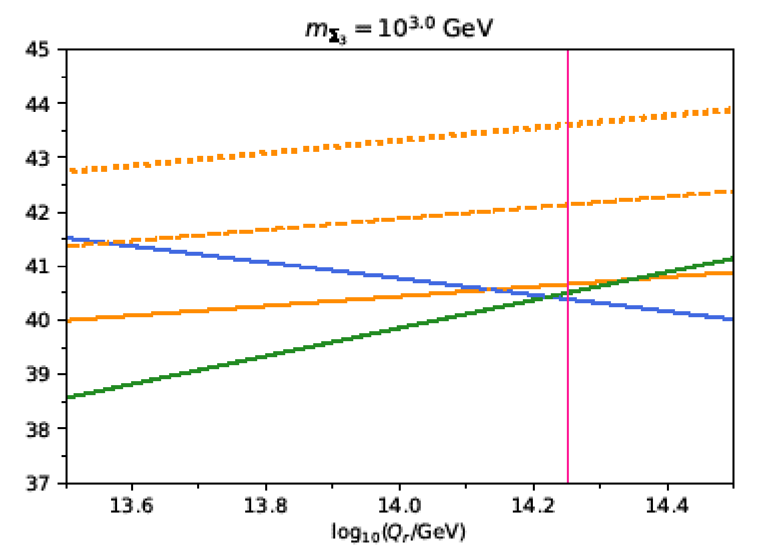
Dashed: Real triplet

$$\Delta b_2 = \frac{1}{3}$$

Solid: Complex Triplet

$$\Delta b_2 = \frac{2}{3}$$



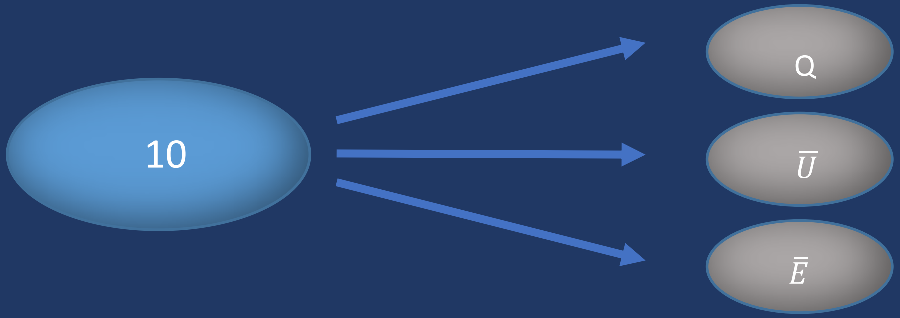


$$M_{GUT} \sim 10^{14} \text{ GeV}$$

Proton Decay?

SU(5) Grand Unification

- Matter fields embedded into larger representation
  - Leads to charge quantization
  - Simplest group which fits the SM, SU(5)

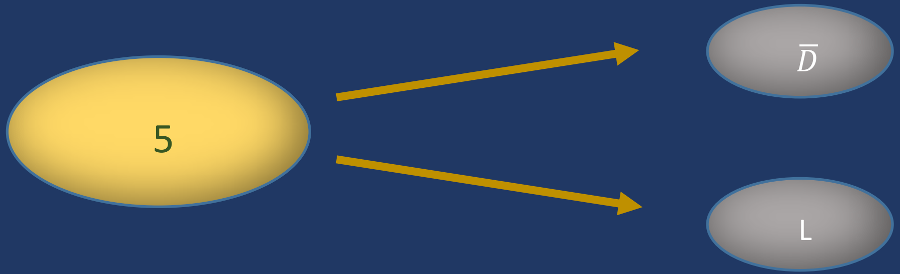


10

Q

$\bar{U}$

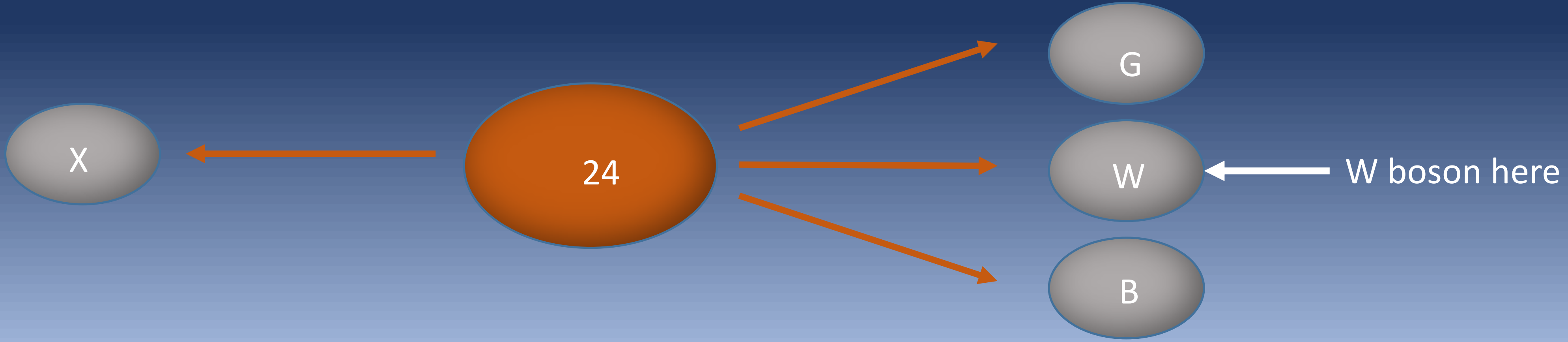
$\bar{E}$



5

$\bar{D}$

L



X

24

G

W

B

W boson here

Grand Unification

- Matter fields embedded into larger representation

- SU(5) Breaking with single real 24 Rep

Can we use this Y=0 Triplet?

$$24_H = \begin{pmatrix} \boxed{\Sigma_3} & X/\sqrt{2} \\ X^\dagger/\sqrt{2} & \Sigma_8 \end{pmatrix} \quad + \quad \text{Singlet}$$

- For a generic renormalizable interactions

$$\frac{m^2_{\Sigma_3}}{m^2_{\Sigma_8}} = 4$$

- So light triplet alone from SU(5) breaking is impossible

Grand Unification

- Matter fields embedded into larger representation

- SU(5) Breaking with single real 24 Rep

- If we include an additional 24, can get a light triplet
    - Lots of freedom from many couplings

$$\begin{aligned} V &\ni 2\mu_{24}^2 \mathrm{Tr}(\Sigma_{24}^\dagger \Sigma_{24}) + 2A_1 \mathrm{Tr}(\Sigma_{24H}\Sigma_{24}^\dagger \Sigma_{24}) + 2A_2 \mathrm{Tr}(\Sigma_{24}^\dagger \Sigma_{24H}\Sigma_{24}) \\ &+ \lambda_1 \mathrm{Tr}(\Sigma_{24H}^2)\mathrm{Tr}(\Sigma_{24}^\dagger \Sigma_{24}) + 2\lambda_2 \mathrm{Tr}(\Sigma_{24H}^2 \Sigma_{24}^\dagger \Sigma_{24}) + 2\lambda_3 \mathrm{Tr}(\Sigma_{24H}\Sigma_{24}^\dagger \Sigma_{24H}\Sigma_{24}) \end{aligned}$$

- Relevant masses

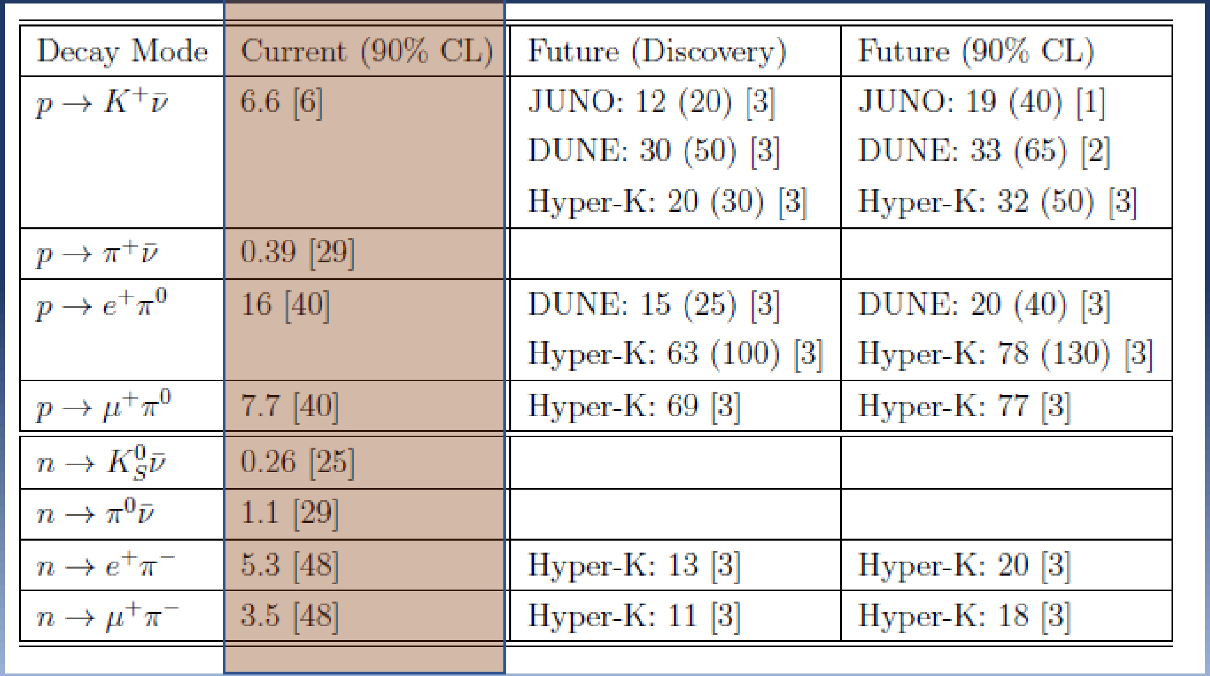
$$\left.\begin{aligned} m_{\Sigma_8}^2 &= \mu_{24}^2 + 2A_1 v_{\rm GUT} + 4(\lambda_2+\lambda_3)v_{\rm GUT}^2 \simeq 5A_1 v_{\rm GUt}, \\ m_{\Sigma_3}^2 &= \mu_{24}^2 - 3A_1 v_{\rm GUT} + 9(\lambda_2+\lambda_3)v_{\rm GUT}^2 \simeq 0 \end{aligned}\right\} \quad \begin{aligned} &\lambda_3 \simeq \lambda_2 \\ &A_1 v_{\rm GUT} \simeq \mu_{24}^2 \end{aligned}$$

Proton Decay : Dimension-6

- Dimension-6 proton decay
  - Expected lifetime over 5 orders of magnitude too small

$$\tau_p \simeq 2.4 \times 10^{34}\ \text{yrs} \left(\frac{M_X}{5.3\times10^{15}\ \text{GeV}}\right)^4$$

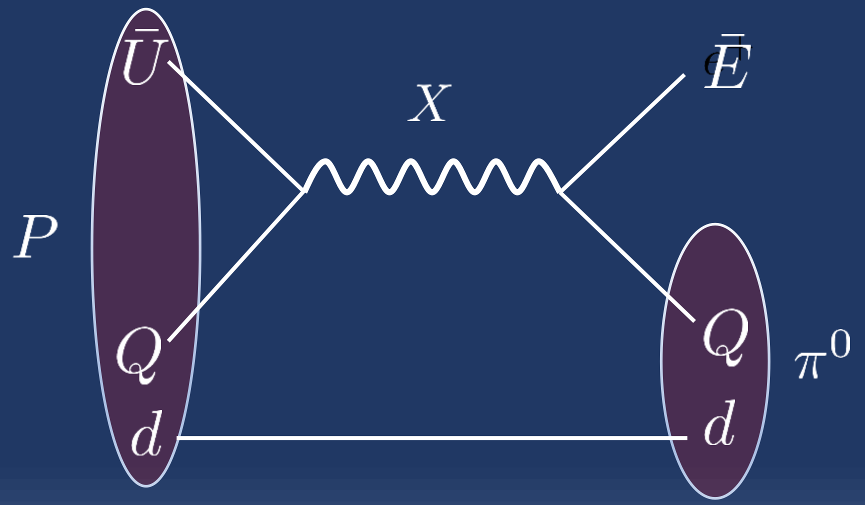
10^{33} yrs

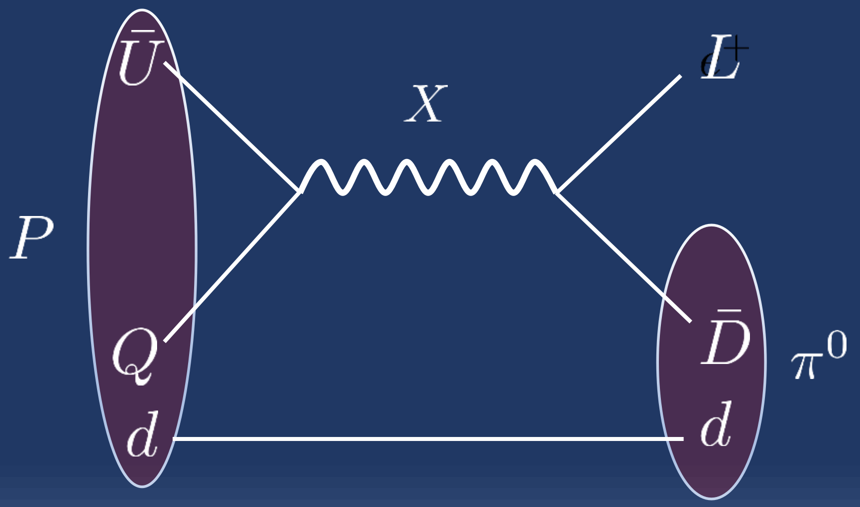


Decay Mode	Current (90% CL)	Future (Discovery)	Future (90% CL)
$p \to K^+\bar{\nu}$	6.6 [6]	JUNO: 12 (20) [3] DUNE: 30 (50) [3] Hyper-K: 20 (30) [3]	JUNO: 19 (40) [1] DUNE: 33 (65) [2] Hyper-K: 32 (50) [3]
$p \to \pi^+\bar{\nu}$	0.39 [29]		
$p \to e^+\pi^0$	16 [40]	DUNE: 15 (25) [3] Hyper-K: 63 (100) [3]	DUNE: 20 (40) [3] Hyper-K: 78 (130) [3]
$p \to \mu^+\pi^0$	7.7 [40]	Hyper-K: 69 [3]	Hyper-K: 77 [3]
$n \to K^0_S\bar{\nu}$	0.26 [25]		
$n \to \pi^0\bar{\nu}$	1.1 [29]		
$n \to e^+\pi^-$	5.3 [48]	Hyper-K: 13 [3]	Hyper-K: 20 [3]
$n \to \mu^+\pi^-$	3.5 [48]	Hyper-K: 11 [3]	Hyper-K: 18 [3]

Proton Decay : Dimension-6

- Dimension-6 proton decay
- The decay proceeds via the heavy gauge boson





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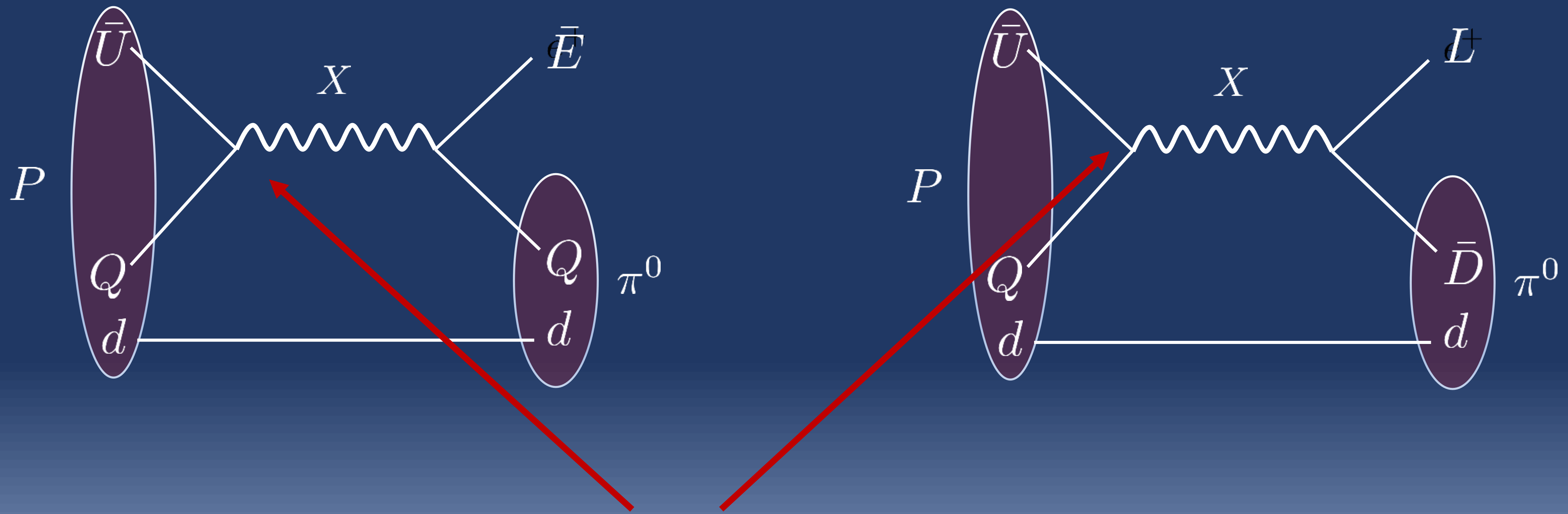


Same Vertex: Can we suppress it

Proton Decay : Dimension-6

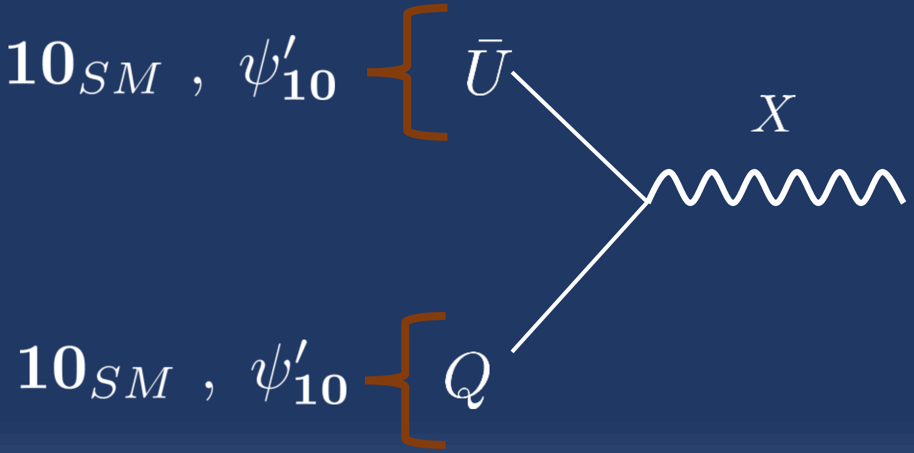
- Dimension-6 proton decay
- The decay proceeds via the heavy gauge boson
- Mixing and suppression of proton decay

$$\mathcal{L} \supset \underbrace{\left(M_{10}\mathbf{10} + M_\psi \psi_{\mathbf{10}}\right)} \bar{\psi}_{\mathbf{10}}$$

This linear combination massless : SM field

Proton Decay : Dimension-6

- Dimension-6 proton decay
- The decay proceeds via the heavy gauge boson
- Mixing SU(5) reps and suppression of proton decay



$$\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \mathbf{10} \\ \psi_{\mathbf{10}} \end{pmatrix} = \begin{pmatrix} \mathbf{10}_{\rm SM} \\ \psi'_{\mathbf{10}} \end{pmatrix}$$

SM field interaction combination of these

Proton Decay : Dimension-6

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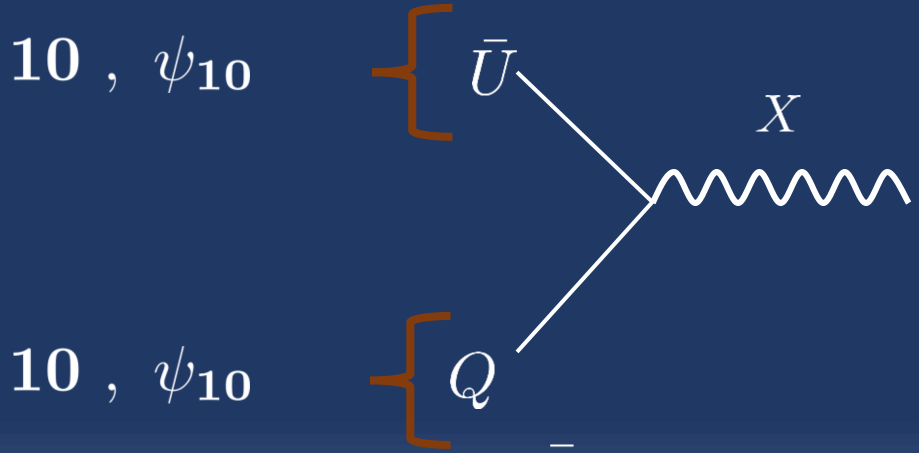
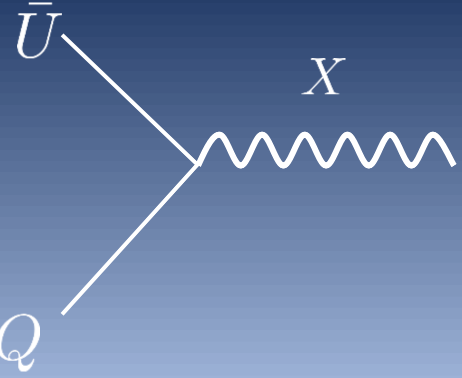
SM field interaction combination of these



$$\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \mathbf{10} \\ \psi_{\mathbf{10}} \end{pmatrix} = \begin{pmatrix} \mathbf{10}_{\mathrm{SM}} \\ \psi'_{\mathbf{10}} \end{pmatrix}$$



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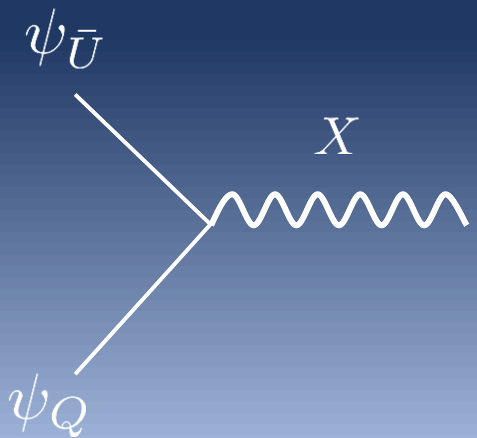


$$\propto \sin^2\theta + \cos^2\theta{=}1$$

Proton Decay : Dimension-6

- Dimension-6 proton decay
- The decay proceeds via the heavy gauge boson
- Mixing SU(5) reps and suppression of proton decay
- Mixing incomplete SU(5) reps to suppress proton decay

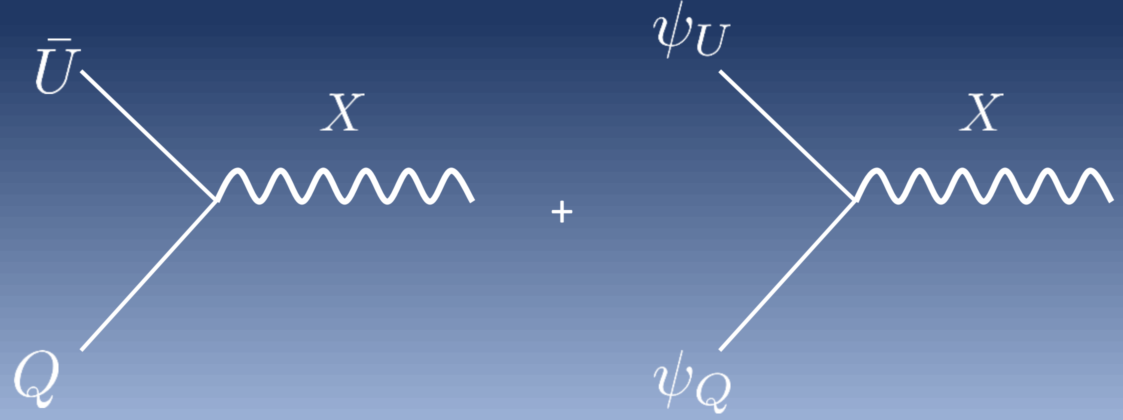
$$\mathcal{L} \supset \bar{\psi}_{\mathbf{10}}\mathbf{10}\,(M_{10} + \lambda_\psi\langle\Sigma_{24H}\rangle) + \bar{\psi}_{\mathbf{10}}\mathbf{10}\,(M + \lambda\langle\Sigma_{24H}\rangle)$$

Creates independent mass
matrices for EACH SM rep

Proton Decay : Dimension-6

- Dimension-6 proton decay
- The decay proceeds via the heavy gauge boson
- Mixing SU(5) reps and suppression of proton decay
- Mixing incomplete SU(5) reps to suppress proton decay
  - Each SM rep has a different mixing angle

$$\begin{pmatrix}\cos\theta_Q & \sin\theta_Q \\ -\sin\theta_Q & \cos\theta_Q\end{pmatrix}\begin{pmatrix}Q \\ \psi_Q\end{pmatrix}=\begin{pmatrix}Q' \\ \psi'_Q\end{pmatrix} \qquad \begin{pmatrix}\cos\theta_U & \sin\theta_U \\ -\sin\theta_U & \cos\theta_U\end{pmatrix}\begin{pmatrix}\bar{U} \\ \psi_{\bar{U}}\end{pmatrix}=\begin{pmatrix}\bar{U}' \\ \psi'_{\bar{U}}\end{pmatrix}$$



$$\propto \cos\theta_Q \cos\theta_U + \sin\theta_Q \sin\theta_U$$

Proton Decay: Dimension-6

- Dimension-6 proton decay
- The decay proceeds via the heavy gauge boson
- Mixing SU(5) reps and suppression of proton decay
- Mixing incomplete SU(5) reps to suppress proton decay
  - Each SM rep has a different mixing angle
  - With a proton lifetime of

$$\tau(p \to e^+\pi^0) \approx 3.3 \times 10^{27}\mathrm{yrs}\, A_{\mathrm{mix}}^{-1}(i=1)\, \left(\frac{M_X}{10^{14}\mathrm{GeV}}\right)^4 \left(\frac{g_5}{0.55}\right)^{-4},$$

$$A_{\mathrm{mix}}(i) \quad \simeq \quad (\cos\theta_Q \cos\theta_U + \sin\theta_Q \sin\theta_U)^2$$

  - Lifetime constraints requirements

$$(a)\, \sin\theta_U \sim 10^{-4} \quad \text{and} \quad \cos\theta_Q \sim 10^{-4}$$
$$(b)\, \sin\theta_Q \sim 10^{-4} \quad \text{and} \quad \cos\theta_U \sim 10^{-4}$$

Proton Decay: Colored Higgs

- SM Yukawa Couplings

$$-\mathcal{L} \quad \ni \quad \frac{1}{4} Y_{10,i}\delta_{ij}\mathbf{10}_i\mathbf{10}_j H_5 + h.c.$$

$$\Rightarrow \quad \begin{aligned} &(Y_{10})_1 \sin\theta_Q \sin\theta_U Q_i \cdot H\bar{U}_i = y_u Q_i \cdot H\bar{U}_i \\ &(Y_{10})_1 = \frac{y_u}{\sin\theta_U \sin\theta_Q} \end{aligned}$$

- Colored Higgs Yukawa couplings

$$-\mathcal{L} \quad \ni \quad \frac{1}{2}(Y_{10})_1 H_C \left(Q_1 \cdot Q_1\right) = \frac{1}{2}\frac{y_u}{\sin\theta_U \sin\theta_Q} H_C \left(Q_1 \cdot Q_1\right)$$

  - Proton Decay greatly enhanced since

$\sin\theta_Q \sim 1, \;\; \sin\theta_U \sim 10^{-4}$ Or $\sin\theta_Q \sim 10^{-4}, \;\; \sin\theta_U \sim 1$

Proton Decay: Colored Higgs

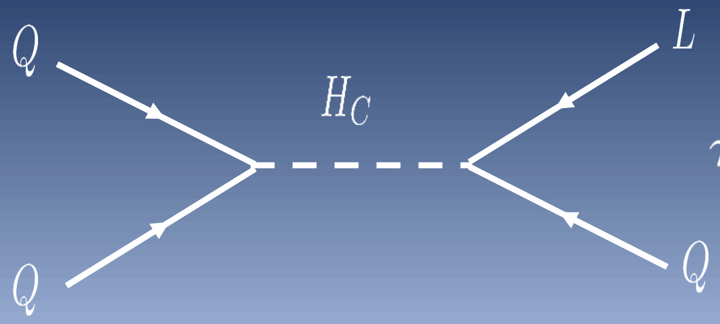
- SM Yukawa Couplings

$$(Y_{10})_1 = \frac{y_u}{\sin\theta_U}$$

- Colored Higgs Yukawa couplings

$$-\mathcal{L} \quad \ni \quad \frac{1}{2}(Y_{10})_1 H_C\,(Q_1 \cdot Q_1) = \frac{1}{2}\frac{y_u}{\sin\theta_u} H_C\,(Q_1 \cdot Q_1)$$

- Proton Decay greatly enhanced since $\sin\theta_Q \sim 1, \quad \sin\theta_U \sim 10^{-4}$



$$\tau_p = \frac{1}{\Gamma_{p\to K^+\overline{\nu}_\tau}} = 9.4\times 10^{33}\text{yrs}\left(\frac{m_{H_C}}{2\times 10^{13}\text{GeV}}\right)^4\left(\frac{10^{-4}}{\sin\theta_U\sin\theta_Q}\right)^2$$

Threshold Corrections: Colored Higgs Mass

- The breaking of SU(5) splits some masses

$$\Sigma_{3H},\ \Sigma_{8H},\ H_C,\ X_{1,2},\ \Sigma_8$$

- These masses are constrained by gauge coupling unification
  - Three question, 6 unknown leads to continuum of solutions

$$\boxed{M_X \sim 10^{14}\ \text{GeV}}$$

$$\begin{aligned}
\tilde{\alpha}_1^{-1}(M_X) &= \alpha_1^{-1}(M_X) - \frac{b_{1,H_C}}{2\pi}\ln\frac{M_X}{m_{H_C}} - \frac{b_{1,X_1}}{2\pi}\ln\frac{M_X}{m_{X_1}} - \frac{b_{1,X_2}}{2\pi}\ln\frac{M_X}{m_{X_2}},\\
\tilde{\alpha}_2^{-1}(M_X) &= \alpha_2^{-1}(M_X) - \frac{b_{2,X_1}}{2\pi}\ln\frac{M_X}{m_{X_1}} - \frac{b_{2,X_2}}{2\pi}\ln\frac{M_X}{m_{X_2}} - \frac{b_{2,\Sigma_{3H}}}{2\pi}\ln\frac{M_X}{2m_{\Sigma_{8H}}},\\
\tilde{\alpha}_3^{-1}(M_X) &= \alpha_3^{-1}(M_X) - \frac{b_{3,H_C}}{2\pi}\ln\frac{M_X}{m_{H_C}} - \frac{b_{3,X_1}}{2\pi}\ln\frac{M_X}{m_{X_1}} - \frac{b_{3,X_2}}{2\pi}\ln\frac{M_X}{m_{X_2}},\\
&- \frac{b_{3,\Sigma_{8H}}}{2\pi}\ln\frac{M_X}{m_{\Sigma_{8H}}} - \frac{b_{3,\Sigma_8}}{2\pi}\ln\frac{M_X}{m_{\Sigma_8}}
\end{aligned}$$

Threshold Corrections: Colored Higgs Mass

- The breaking of SU(5) splits some masses

$$\Sigma_{3H},\ \Sigma_{8H},\ H_C,\ X_{1,2},\ \Sigma_8$$

- These masses are determined by gauge coupling unification
    - Three question, 6 unknown leads to continuum of solutions

- Equations afford a solution

$$m_{H_C} \sim 10 M_X \qquad\qquad \text{others} \sim \pm 10 M_X$$

- Lifetime well beyond experimental limit (maybe short not an exhaustive study)

$$\tau_p = \frac{1}{\Gamma_{p\to K^+\bar{\nu}_\tau}} = 9.4\times 10^{41}\text{yrs}\left(\frac{m_{H_C}}{2\times 10^{15}\text{GeV}}\right)^4\left(\frac{10^{-4}}{\sin\theta_U \sin\theta_Q}\right)^2$$

SM Yukawa Couplings: The Low Scale

- Minimal SU(5): Yukawa couplings explained by M_P suppressed operator

$$-\mathcal{L} \ni \sqrt{2}\frac{c_{ij}}{M_*}\bar{\mathbf{5}}_i\Sigma_{24H}\mathbf{10}'_j H_5^*$$

- Because GUT scale suppressed contribution too small

$$y_b(M_X) - y_\tau(M_X) \sim \frac{M_X}{M_P} \sim 10^{-4}$$ Theory

$$y_b(M_X) - y_\tau(M_X) = \frac{m_b(M_X) - m_\tau(M_X)}{v} \simeq 4 \times 10^{-3}$$ Experiment

- Yukawa can be accommodated by vector $5 + \bar{5}$
    - Mixing can split the Yukawa Couplings

Conclusions

- W boson mass anomaly dominated by CDF measurement
    - Put is present in a lesser degree in other experiments
- W boson constrained by Fermi Constant
    - Shifts in $\alpha,\ \theta_W,\ G_F$ can lead to deviation in M_W
- Loop level explanations are tightly constrained by experiment
    - Requires $\sim 100\ \mathrm{GeV}$ electroweak interacting particle
- Fits to SMEFT show only a few possibilities
    - Triplet Higgs being one of them
    - Some others seem difficult to realize
- Triplet Higgs can be motivated by Grand unification
    - Unification scale suppressed
    - Dimension-6 proton decay suppressed by mixing
    - Colored Higgs mediated proton decay enhanced, but still ok
    - Yukawa couplings also explained by mixing