Gravitational Particle Production in the Early Universe

imagine the impossible

KIAS

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STUDY

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Cosmological Gravitational Particle Production (GPP)

- An external field can create particles from the vacuum

    Expanding Universe – Schrödinger 1939
    Electric Fields – Schwinger 1951
    Black Holes – Hawking 1974

- I will concentrate on time-dependent gravitational fields, in particular, the big bang

    Inflation: Quasi deSitter (QdS) phase followed by transition to matter-dominated (MD) then radiation-dominated (RD) phase

- GPP is an example of QFT in classical gravitation background. Many interesting facets, but …

- … my motivation is to explore whether GPP can be the origin of **DARK MATTER** (DM) and whether can provide cosmological constraints on BSM physics

For 40 Years, Leading DM Candidate:
“Weak”-Scale Cold Thermal Relic

- Mass: GeV – TeV
- “Weak-scale” interaction strength with SM
- No self-interactions
- Produced by “freeze-out” from primordial plasma. COLD dark matter.
- “Detectable” by direct detection, indirect detection, decay products, production at colliders
- Just BSM

But not (convincingly) seen

- In Direct detection (but DAMA/LIBRA)
- In Indirect Detection (but galactic-center excess)
- In Decay (but $3.5\ \mathrm{keV}\ \gamma$-ray line)
- In Colliders/Accelerators no BSM signal (but μ_{g-2}, m_W)

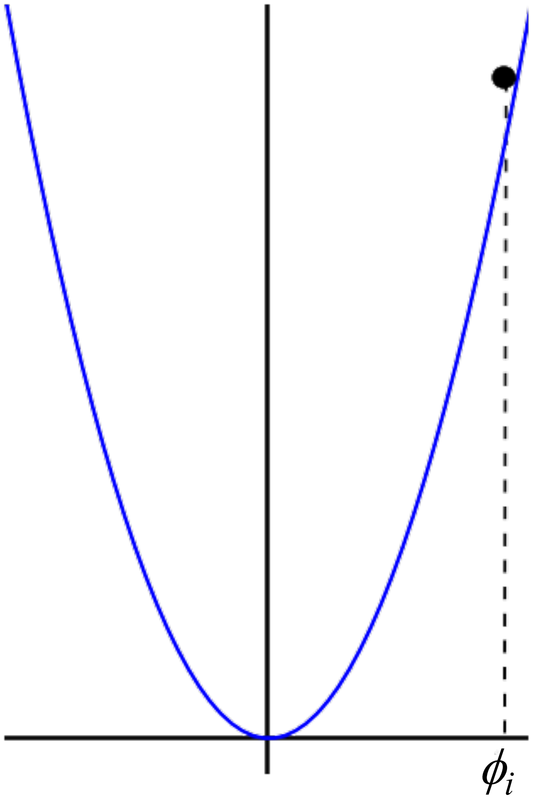
What if DM interacts only gravitationally?

- Gravity must play a prole in its cosmological production
- But gravity weak!

How can GPP produce dark matter?

Ideas for gravitational particle production

Produce particles through misalignment mechanism



- EOM of scalar field

$$\ddot{\phi} + 3H\dot{\phi} + m^2\phi = 0$$

- Scalar field has quantum fluctuations during inflation

$$\Delta\phi = \frac{H}{2\pi}$$

- After inflation field frozen by “Hubble drag” until

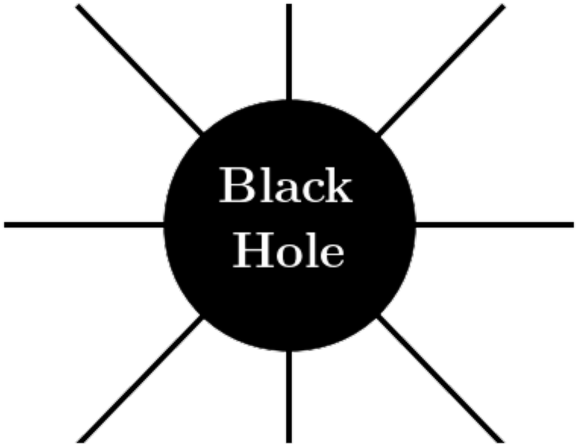
$$H \simeq m_\phi$$

- After which it oscillates with energy density in oscillating field

- E.g., axion

Ideas for gravitational particle production

Produce particles via Hawking radiation from primordial black holes
(Hooper, Krnjaic, & McDermott)



$$\frac{\Omega h^2}{0.12} \approx \left(\frac{10^{11}\mathrm{GeV}}{m}\right)\left(\frac{10^{12}\mathrm{GeV}}{T_i}\right)^3 \left(\frac{\epsilon_{\mathrm{BH}}}{10^{-16}}\right)$$

- PBHs of current interest (after first LIGO event)
- Seeds for PBHs from inflation
- Assumes DM mass about $10^{11}\,\mathrm{GeV}$ (WIMPzilla)

WIMPzilla™ is a very massive* dark-matter candidate

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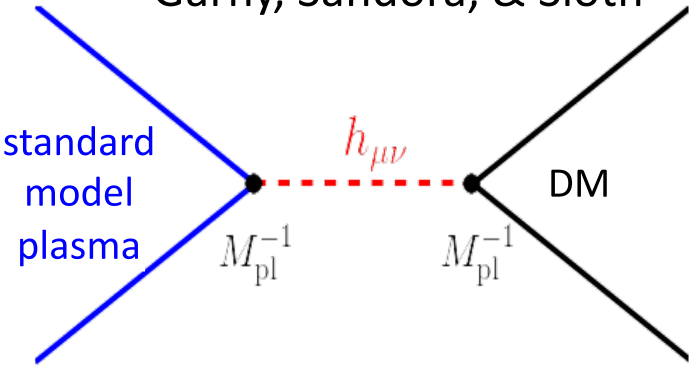
* Very massive ⇒ too massive to be a cold thermal relic ($\gtrsim 200\ \mathrm{TeV}$)

Ideas for gravitational particle production

$$\mathcal{L} = M_{\rm Pl}^{-1} h_{\mu\nu} T^{\mu\nu}$$

Produce particles from SM plasma via
graviton exchange
Garny, Sandora, & Sloth



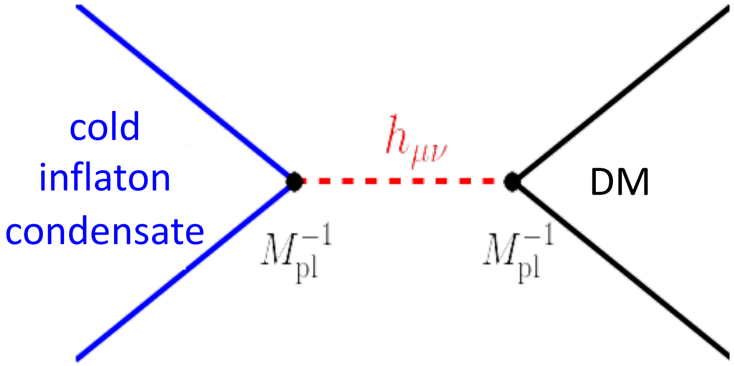
$$\frac{\Omega h^2}{0.12} \approx \left(\frac{\langle \sigma v \rangle}{T^2/M_{\rm Pl}^4}\right)\left(\frac{m}{10^{13}{\rm GeV}}\right)\left(\frac{T_{\rm RH}}{10^{14}{\rm GeV}}\right)^3$$

- Freeze-in
- For DM mass about 10^{13} GeV (WIMPzilla)
- Assumes $m < T_{\rm RH}$

Ideas for gravitational particle production

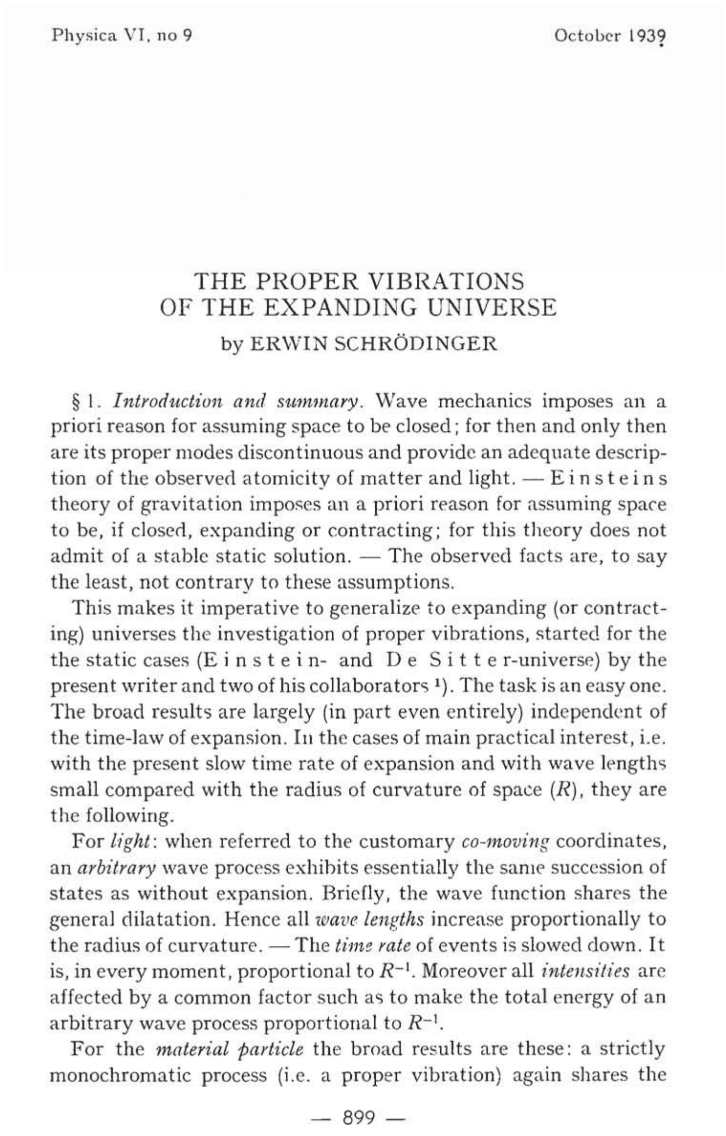
$$\mathcal{L} = M_{\rm Pl}^{-1} h_{\mu\nu} T^{\mu\nu}$$

Produce particles from inflaton field after quasi-de Sitter era via graviton exchange
Ema, Nakayama, Tang; Mambrini & Olive



- Only works for DM mass < inflaton mass
- DM mass for correct Ωh^2 involved function of several parameters
- “Boltzmann” approach not complete treatment (Kaneta, Lee, Oda)

Boltzmann  $\subset$  Schrödinger $+$  Bolgoliubov



Physica VI, no 9 October 1939

THE PROPER VIBRATIONS OF THE EXPANDING UNIVERSE

by ERWIN SCHRÖDINGER

§ 1. *Introduction and summary.* Wave mechanics imposes an a priori reason for assuming space to be closed; for then and only then are its proper modes discontinuous and provide an adequate description of the observed atomicity of matter and light. — Einsteins theory of gravitation imposes an a priori reason for assuming space to be, if closed, expanding or contracting; for this theory does not admit of a stable static solution. — The observed facts are, to say the least, not contrary to these assumptions.

This makes it imperative to generalize to expanding (or contracting) universes the investigation of proper vibrations, started for the the static cases (Einstein- and De Sitter-universe) by the present writer and two of his collaborators [1]). The task is an easy one. The broad results are largely (in part even entirely) independent of the time-law of expansion. In the cases of main practical interest, i.e. with the present slow time rate of expansion and with wave lengths small compared with the radius of curvature of space (R), they are the following.

For *light*: when referred to the customary *co-moving* coordinates, an *arbitrary* wave process exhibits essentially the same succession of states as without expansion. Briefly, the wave function shares the general dilatation. Hence all *wave lengths* increase proportionally to the radius of curvature. — The *time rate* of events is slowed down. It is, in every moment, proportional to R^{-1}. Moreover all *intensities* are affected by a common factor such as to make the total energy of an arbitrary wave process proportional to R^{-1}.

For the *material particle* the broad results are these: a strictly monochromatic process (i.e. a proper vibration) again shares the

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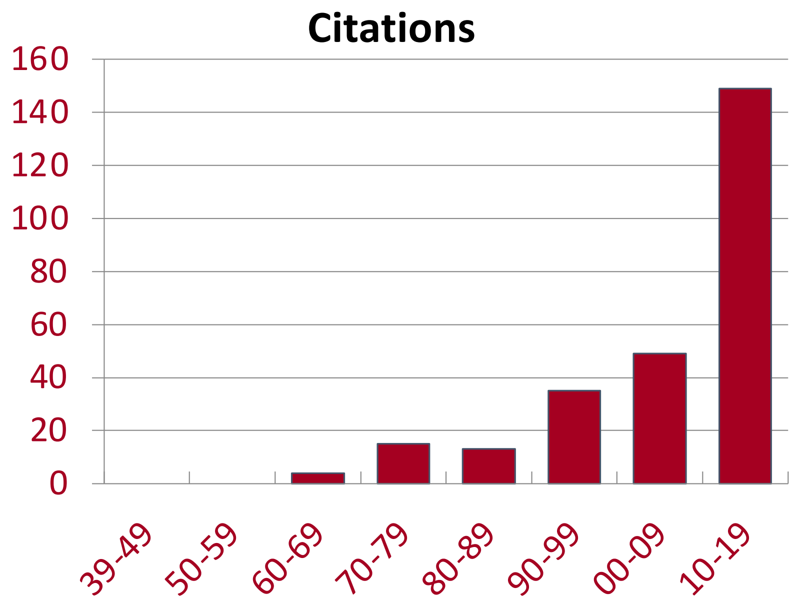
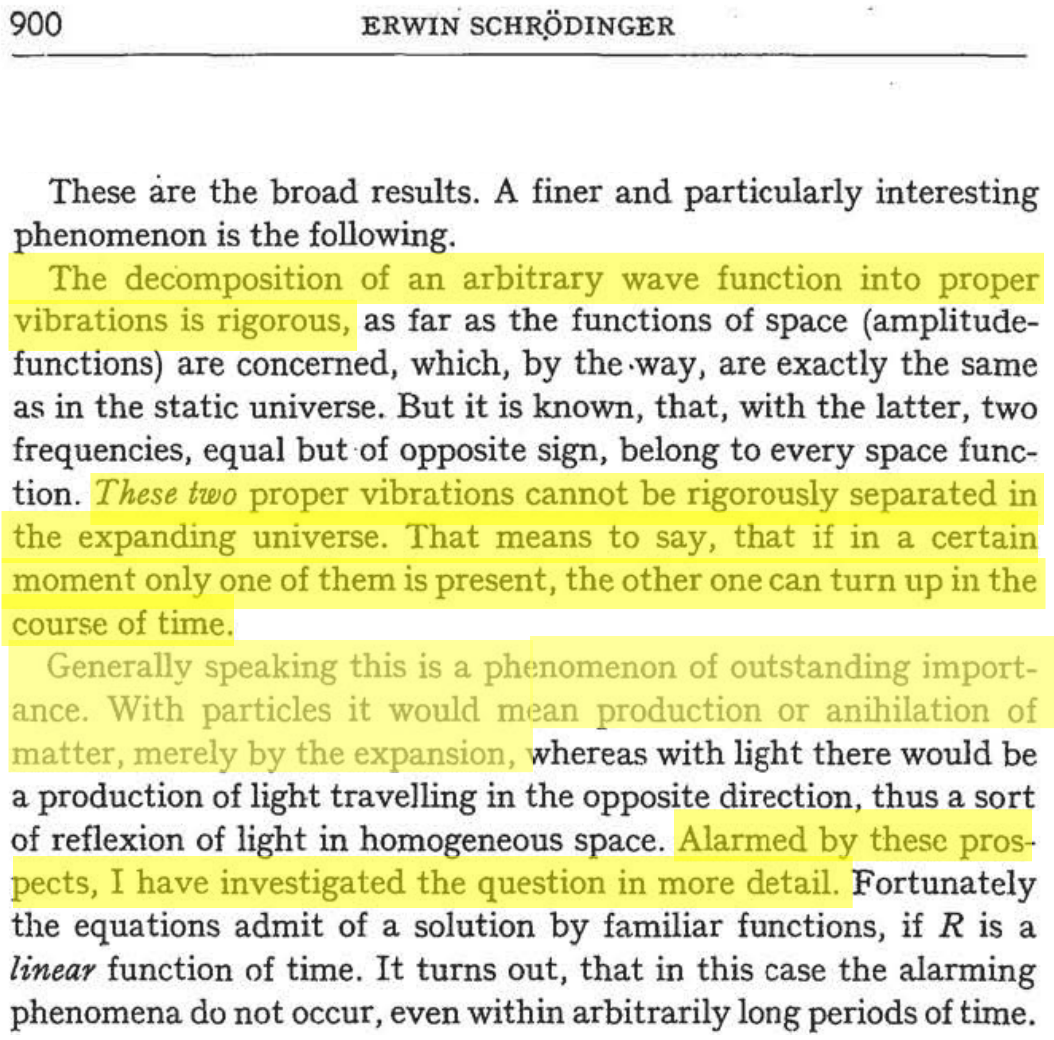




900 ERWIN SCHRÖDINGER

These are the broad results. A finer and particularly interesting phenomenon is the following.

The decomposition of an arbitrary wave function into proper vibrations is rigorous, as far as the functions of space (amplitude-functions) are concerned, which, by the way, are exactly the same as in the static universe. But it is known, that, with the latter, two frequencies, equal but of opposite sign, belong to every space function. *These two* proper vibrations cannot be rigorously separated in the expanding universe. That means to say, that if in a certain moment only one of them is present, the other one can turn up in the course of time.

Generally speaking this is a phenomenon of outstanding importance. With particles it would mean production or anihilation of matter, merely by the expansion, whereas with light there would be a production of light travelling in the opposite direction, thus a sort of reflexion of light in homogeneous space. Alarmed by these prospects, I have investigated the question in more detail. Fortunately the equations admit of a solution by familiar functions, if R is a *linear* function of time. It turns out, that in this case the alarming phenomena do not occur, even within arbitrarily long periods of time.

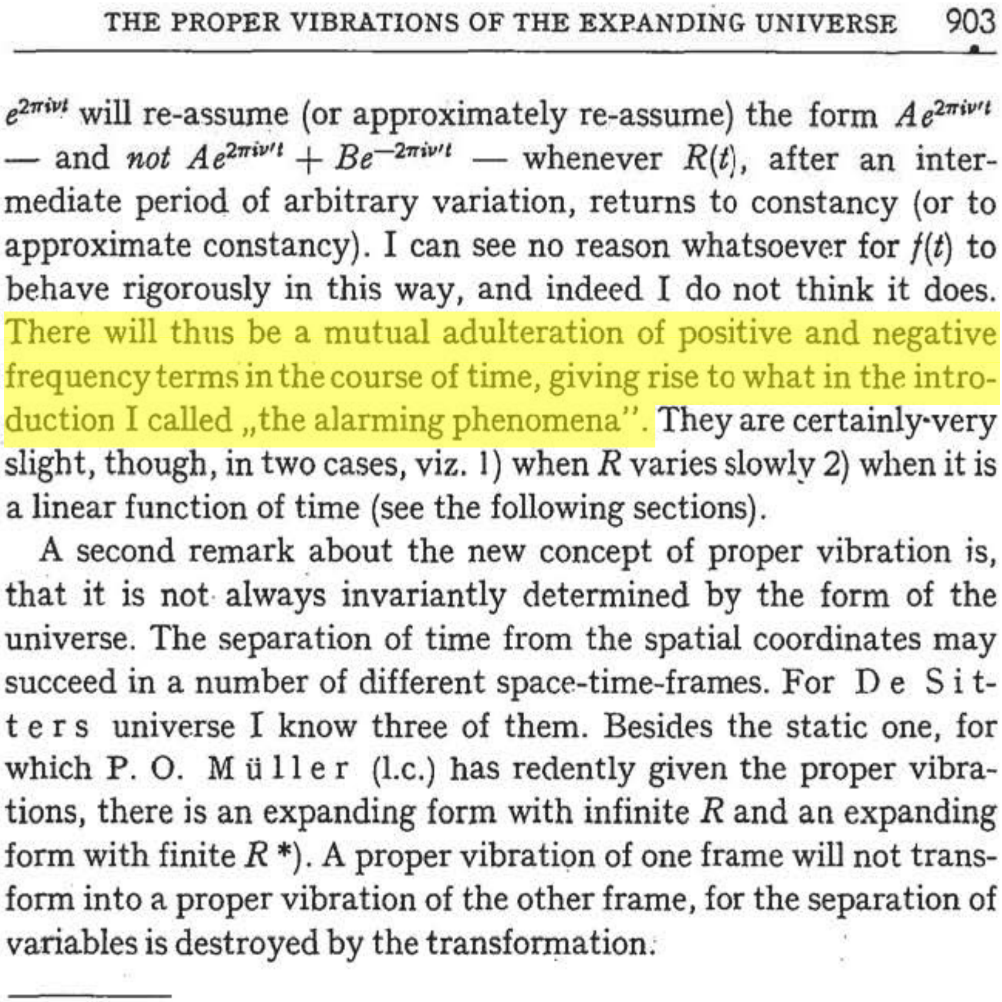
Even in an expanding universe, a particle's wavefunction can be decomposed into "proper vibrations" (positive & negative energy modes):

$$\Psi(t) = \alpha\, e^{-i\omega t} + \beta\, e^{+i\omega t}$$

If start with pure incoming or outgoing waves, in and out may become mixed.

Phenomenon of "outstanding importance." The expansion of the universe creates particles!

This alarms me [ed. why?] so I wrote a paper.



$e^{2\pi i\nu t}$ will re-assume (or approximately re-assume) the form $Ae^{2\pi i\nu' t}$ — and *not* $Ae^{2\pi i\nu' t} + Be^{-2\pi i\nu' t}$ — whenever $R(t)$, after an intermediate period of arbitrary variation, returns to constancy (or to approximate constancy). I can see no reason whatsoever for $f(t)$ to behave rigorously in this way, and indeed I do not think it does. There will thus be a mutual adulteration of positive and negative frequency terms in the course of time, giving rise to what in the introduction I called „the alarming phenomena". They are certainly very slight, though, in two cases, viz. 1) when R varies slowly 2) when it is a linear function of time (see the following sections).

A second remark about the new concept of proper vibration is, that it is not always invariantly determined by the form of the universe. The separation of time from the spatial coordinates may succeed in a number of different space-time-frames. For De Sitters universe I know three of them. Besides the static one, for which P. O. Müller (l.c.) has redently given the proper vibrations, there is an expanding form with infinite R and an expanding form with finite R *). A proper vibration of one frame will not transform into a proper vibration of the other frame, for the separation of variables is destroyed by the transformation.

Schrödinger's two favorite phrases:

1. alarming phenomenon
2. adulteration

Schrödinger was alarmed by creation of a *single* particle

per Hubble time $(H_0^{-1} \sim 10^{10}\ \text{yr})$

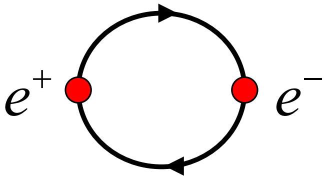
per Hubble volume $(H_0^{-3} \sim 10^{57}\ \text{km}^3)$

with Hubble energy $(H_0 \sim 10^{-33}\ \text{eV})$

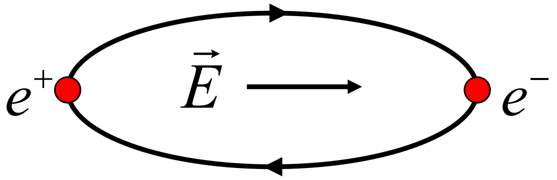
Of all the circumstances faced by Schrödinger in 1939, why did this alarm him?

Disturbing the Quantum Vacuum

Electric Field → Particle creation



Particle creation if energy gained in acceleration from E-field over a Compton wavelength exceeds the particle’s rest mass.

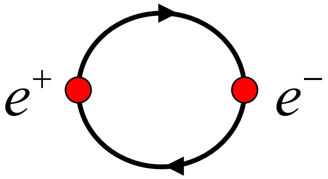


$$\left|\vec{E}_{\text{crit}}\right| = \frac{m_e^2 c^3}{e\hbar} \approx 10^{16}\ \text{V cm}^{-1} \qquad \Gamma \propto e^{-\pi E_{\text{crit}}/\left|\vec{E}\right|}$$

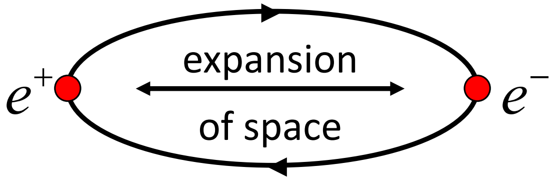
Sauter (1931); Heisenberg & Euler (1935); Weisskopf (1936); Schwinger (1951)

Disturbing the Quantum Vacuum

Expanding universe → Particle creation



Particle creation if energy gained in acceleration from expansion over a Compton wavelength exceeds the particle’s rest mass.



$v = c$ at Hubble radius

$$H_{\text{crit}} = m$$

$$\Gamma \propto e^{-\pi H_{\text{crit}}/H}$$

Schrödinger’s Alarming Phenomenon (1939)

Schrödinger’s Alarming Phenomenon

“Outstanding” importance?

> Schrödinger 1939: “Generally speaking this is a phenomenon of outstanding importance. With particles it would mean the production or annihilation of matter, merely by the expansion.” [why would that be of outstanding importance?]

Forgotten in 40s, 50s, 60s (by Schrödinger also).

Great Cosmological Significance?

> Leonard Parker Thesis 1966. In 1968 paper: “…for the early stages of a Friedmann expansion it [particle creation] may well be of great cosmological significance, especially since it seems inescapable if one accepts quantum field theory and general relativity.” [no speculation as to the “great cosmological significance”]

First attempt:

> Zel’dovich 1970s proposed an application: explaining why the universe is homogeneous and isotropic.

Schrödinger’s Alarming Phenomenon

Other interest in GGP in the 1970s (mostly regarded as a curiosity).

US: Parker, Ford, Fulling, Allen, Friedman, Wald, …
Soviet Union: Zel’dovich, Starobinski, Grishchuk, Grib, Mostepanenko, Lukash, … (CGGP in the CCCP)
UK: Bunch, Davies, Birrell, Hawking, …

Great cosmological significance in the 1980s (inflation):

Sasaki, Kodama, **Mukhanov & Chibisov**, Vilenkin, Linde, Abbott, Wise, Lyth, Salopek, Bond, …

Could there be more?

- Gravitational Particle Production universal*
- Gravitational Particle Production not a large effect (cf. curvature perturbations ≈ 10^{-5})
- What else could be observable?
  - **Dark matter (DM)**
  - CMB Isocurvature perturbations
  - CMB Nongaussianities

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* So long as Weyl conformal symmetry violated.

DM From Schrödinger's Alarming Phenomenon

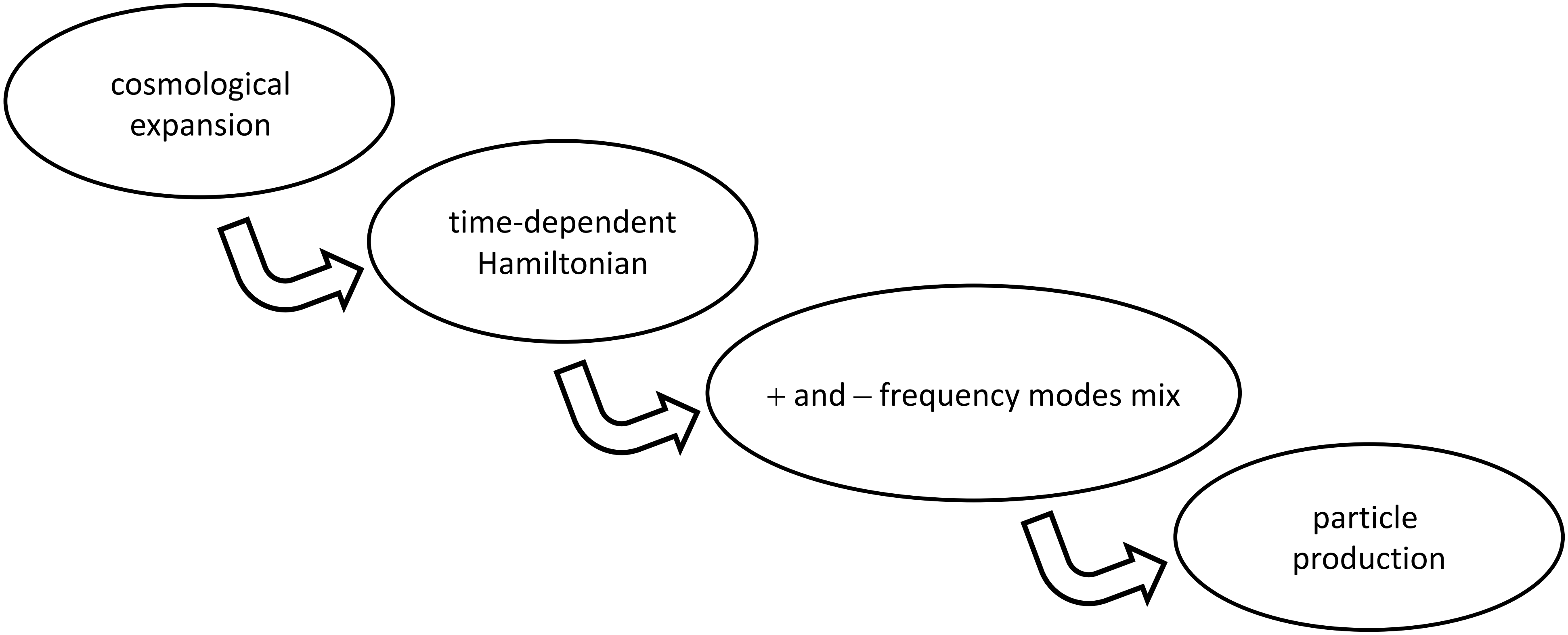
Dark matter?

- Chung, Kolb, Riotto (1998); Kuzmin & Tkachev (1999)
  - My collaborators:
    - Ivone Albuquerque
    - Edward Basso
    - Daniel Chung
    - Patrick Crotty
    - Michael Fedderke
    - Gian Giudice
    - Lam Hui
    - Siyang Lin
    - Andrew Long
    - Evan McDonough
    - Toni Riotto
    - Rachel Rosen
    - Leo Senatore
    - Alexi Starobinski
    - Igor Tkachev
    - Mark Wyman

Gravitational Particle Production (GPP)

- In Minkowskian QFT, a particle is an IR of the Poincaré group.
- But, expanding universe not Poincaré invariant.
- Notion of a “particle” is approximate.

Schrodinger (1939); Parker (1965, 68); Fulling, Ford, & Hu; Zel’dovish; Starobinski; Grib, Frolov, Mamaev, & Mostepanenko; Mukhanov & Sasaki, Birrell & Davies…



cosmological expansion

time-dependent Hamiltonian

+ and – frequency modes mix

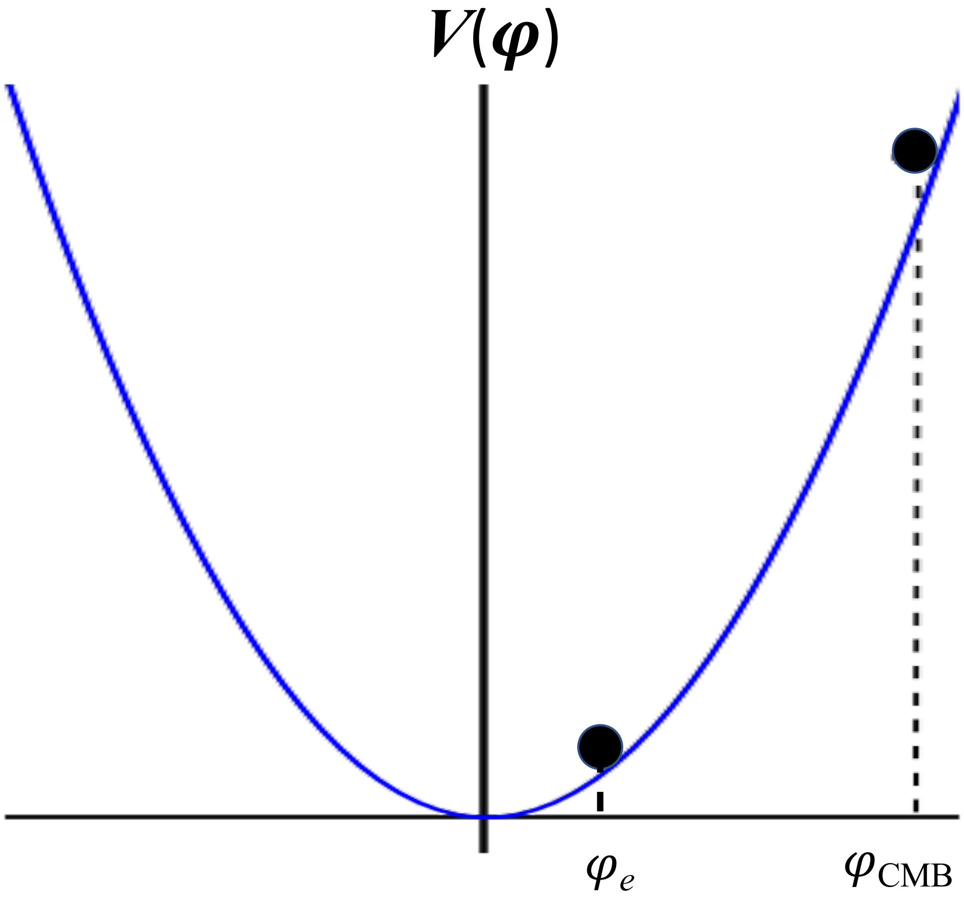
particle production

$\exists$ Standard Inflationary Picture, but not Standard Inflationary Model

But there is a “simple” inflationary model: single-field with quadratic inflaton potential:

$$V(\varphi) = \frac{1}{2}\mu^2\varphi^2$$

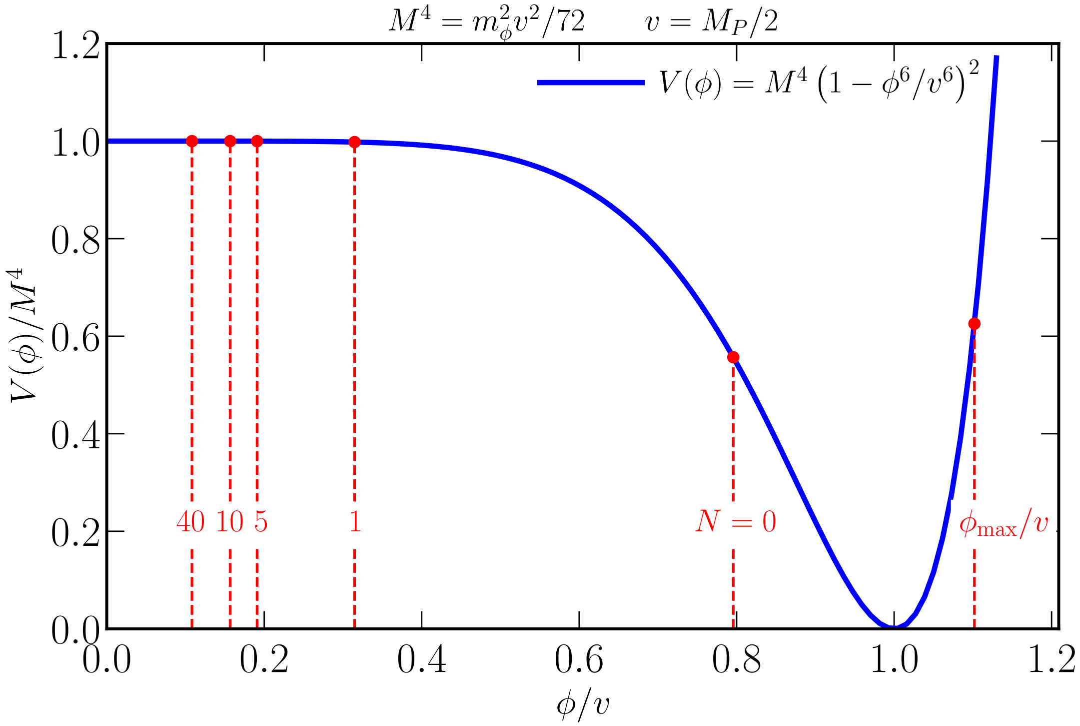
Simple model ruled out by CMB measurements. *But* CMB measurements probe inflaton potential 60 or so e-folds before the end of inflation. We will often be interested in inflaton potential near the end or after inflation ends when φ is close to the minimum of its potential and quadratic description may be a good approximation.



$\exists$ Standard Inflationary Picture, but not Standard Inflationary Model

Also, recent studies employing Hilltop Potential (Basso, Chung, Kolb, Long)

$$V(\varphi) = M^4 \left(1 - \varphi^6/v^6\right)^2$$



and rapid-turn inflation models (hyperbolic inflation, angular inflation, racetrack inflation, orbital inflation,…) with two fields (Kolb, Long, McDonough, Payeur)

$\exists$ Standard Inflationary Picture, but not Standard Inflationary Model

But there is a “simple” inflationary model:
single-field with quadratic inflaton potential:

$$V(\varphi) = \frac{1}{2}\mu^2\varphi^2$$

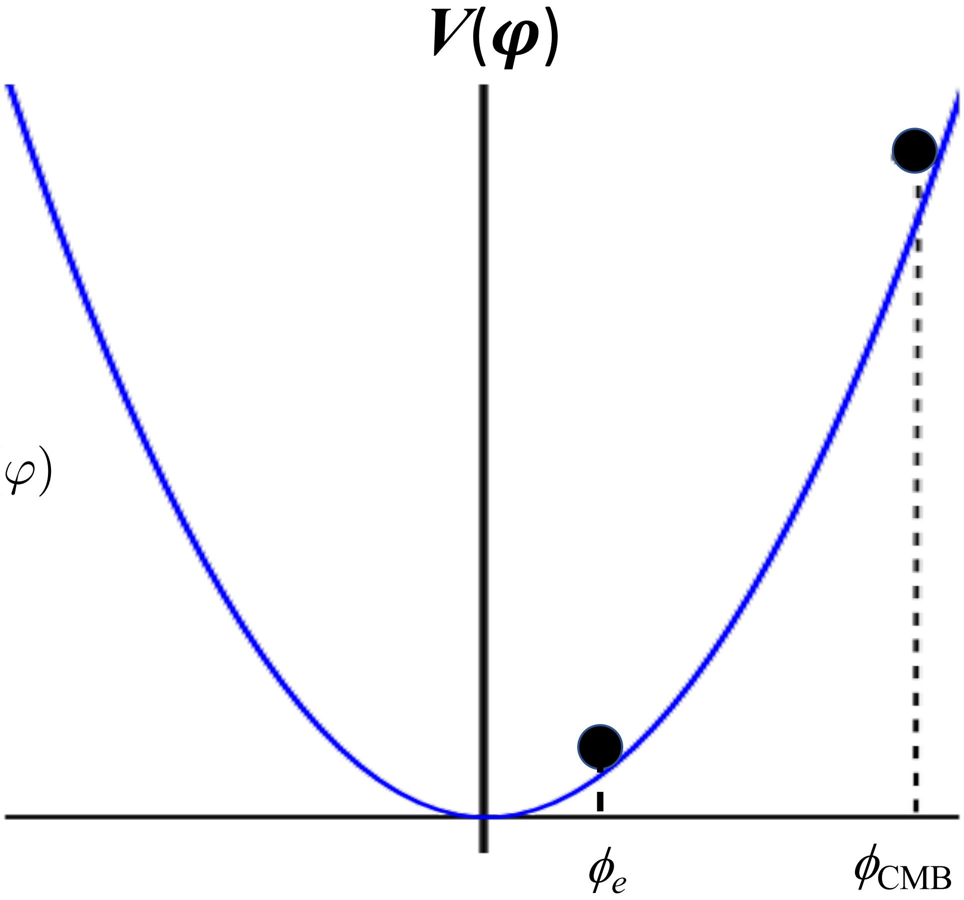
EOM: $\ddot{\varphi} + 3H\dot{\varphi} + \partial_\varphi V(\varphi) = 0$

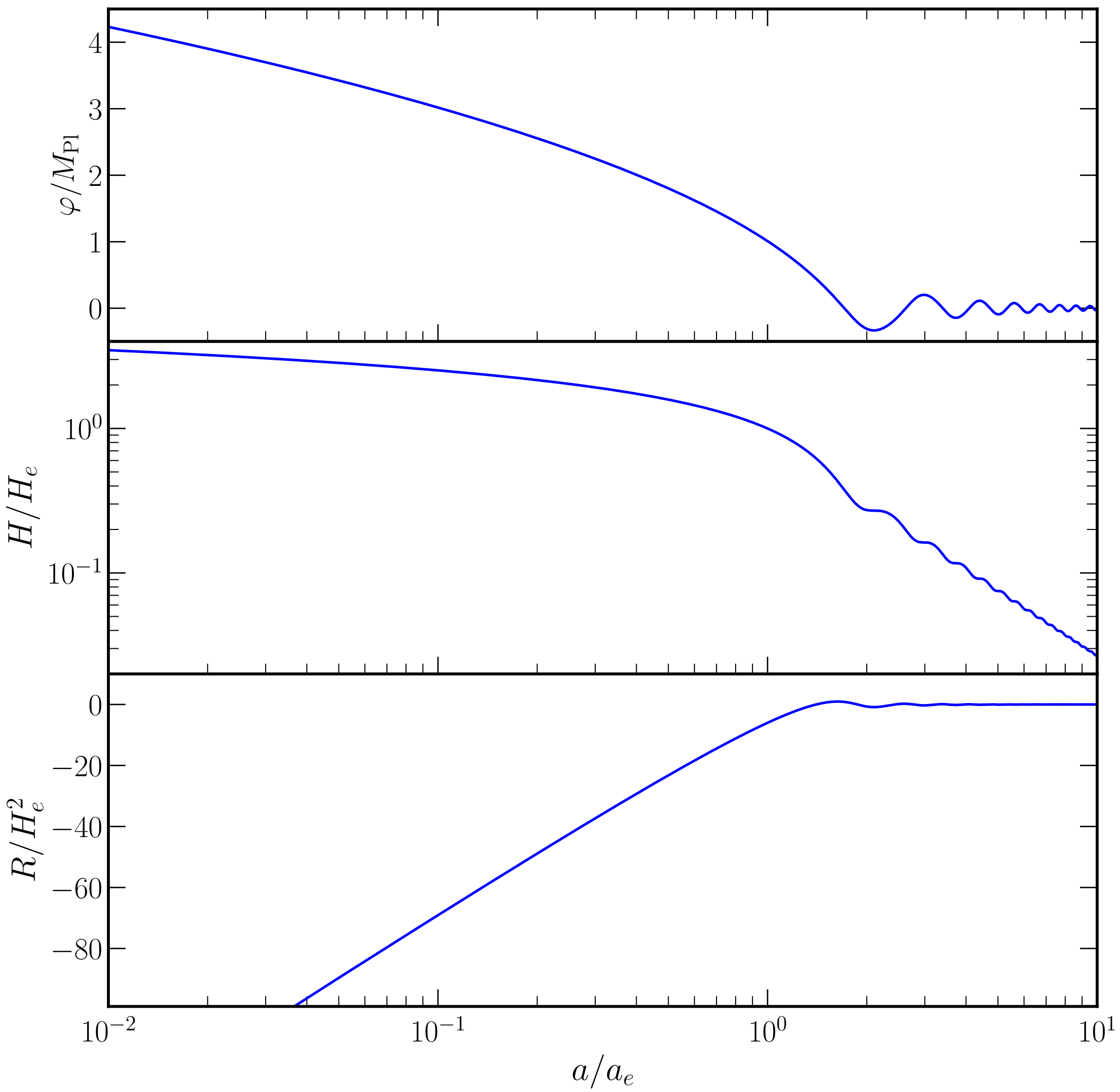
Slow roll during inflation ($\ddot{\varphi} = 0$): $3H\dot{\varphi} = -\partial_\varphi V(\varphi)$

Inflation is accelerated expansion: $\ddot{a} \propto -(\rho + 3p)$

$$\rho = \tfrac{1}{2}\dot{\varphi}^2 + V(\varphi)$$
$$p = \tfrac{1}{2}\dot{\varphi}^2 - V(\varphi)$$

$$\ddot{a} \propto -(\rho + 3p) \propto V(\varphi) - \dot{\varphi}^2$$





Quadratic Inflation

Potential

$$V(\varphi) = \frac{1}{2}m_\varphi^2\varphi^2$$

Expansion rate H

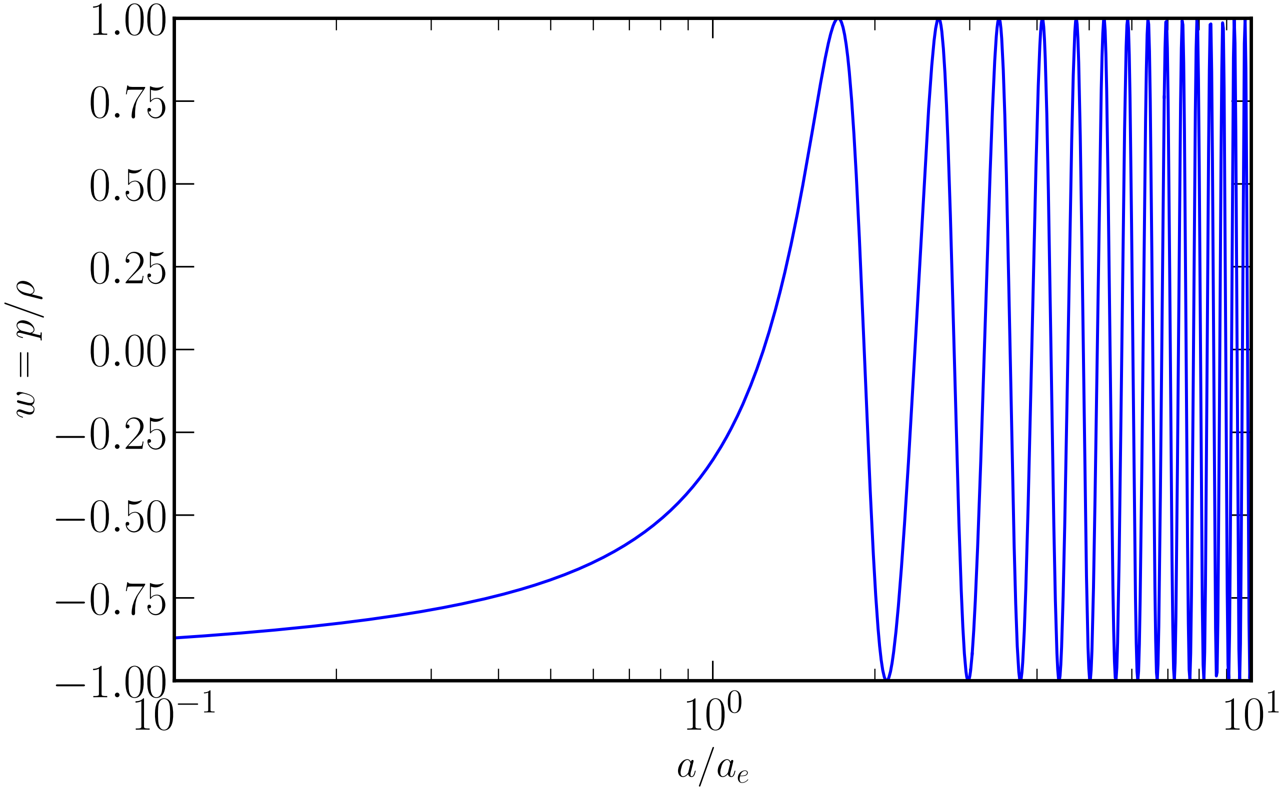
$$3M_{\rm Pl}^2H^2 = \rho_\varphi = \frac{1}{2}\dot{\varphi}^2 + V(\varphi)$$

$$p_\varphi = \frac{1}{2}\dot{\varphi}^2 - V(\varphi)$$

Ricci Scalar Curvature R

$$M_{\rm Pl}^2R = -(\rho - 3p)$$

Quadratic Inflation



Potential

$$V(\varphi) = \frac{1}{2} m_\varphi \phi^2$$

Equation of state parameter

$w = p/\rho$

Scalar field ϕ in FRW background

covariant action for spectator scalar field (not the inflaton)

Gravity enters the picture

$$S[\varphi(x), g_{\mu\nu}(x)] = \int d^4x\sqrt{-g}\left[\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - \frac{1}{2}m^2\varphi^2 + \frac{1}{2}\xi R\varphi^2\right]$$

in spatially flat FRW background : $ds^2 = a^2(\eta)[d\eta^2 - d\mathbf{x}^2]$ (η is conformal time)

$$S[\varphi(\eta, \boldsymbol{x})] = \int_{-\infty}^{\infty} d\eta \int d^3\mathbf{x}\left[\frac{1}{2}a^2(\partial_\eta\varphi)^2 - \frac{1}{2}a^2(\nabla\varphi)^2 - \frac{1}{2}a^4m^2\varphi^2 + \frac{1}{2}a^4\xi R\varphi^2\right]$$

field rescaling

$$\phi(\eta, \boldsymbol{x}) = a(\eta)\varphi(\eta, \boldsymbol{x})$$

action for canonically-normalized field

$aH \to 0$ to zero at $\eta = \pm\infty$

$$S[\phi(\eta, \boldsymbol{x})] = \int_{-\infty}^{\infty} d\eta \int d^3\mathbf{x}\left[\frac{1}{2}(\partial_\eta\phi)^2 - \frac{1}{2}(\nabla\phi)^2 - \frac{1}{2}m_{\text{eff}}^2\phi^2 - \frac{1}{2}\partial_\eta(aH\phi^2)\right]$$

time-dependent effective mass

$$m_{\text{eff}}^2(\eta) = a^2(\eta)\left[m^2 + \left(\frac{1}{6} - \xi\right)R(\eta)\right]$$

cosmological expansion ⇒
time-dependent background ⇒
time-dependent Hamiltonian for spectator fields

Scalar field ϕ in FRW background

Solutions to wave equation include both + and – frequency terms

$$\chi_k(\eta) = \frac{\alpha_k(\eta)}{\sqrt{2\omega_k(\eta)}}\, e^{-i\int \omega_k(\eta) d\eta} + \frac{\beta_k(\eta)}{\sqrt{2\omega_k(\eta)}}\, e^{+i\int \omega_k(\eta) d\eta}$$

Assume start with only + frequency term: $\chi_k(\eta) = \frac{1}{\sqrt{2\omega_k(\eta)}}\, e^{-i\int \omega_k(\eta) d\eta}$

$$\partial_\eta^2 \chi_k(\eta) + \omega_k^2 \left[1 + 3 \left(\frac{\partial_\eta \omega_k}{2\omega_k^2} \right)^2 - \frac{\partial_\eta^2 \omega_k}{2\omega_k^3} \right] \chi_k(\eta) = 0$$

Mixing of + and – frequency terms depends on "Adiabaticity parameter" A_k:

$$A_k \equiv \frac{\partial_\eta \omega_k}{\omega_k^2}$$

$A_k \ll 1$, + frequency solution remains good solution

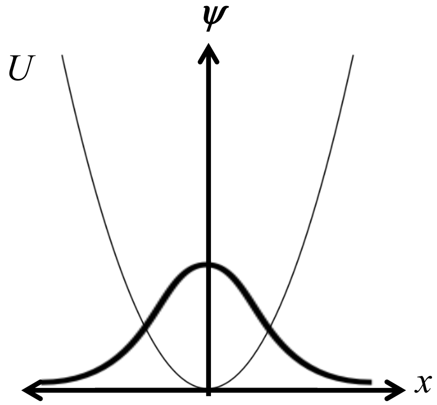
$A_k \gg 1$, + and – frequency terms mix

Schrödinger’s Alarming Phenomenon

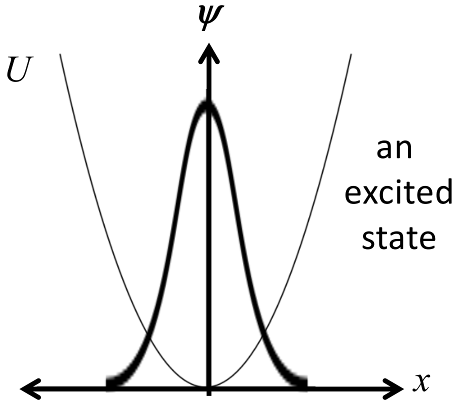
Expansion of the universe causes explicit time dependence in action for “spectator” fields.
Initial ~ QdS (early-time) vacuum may not evolve to final ~ Minkowski (late-time) vacuum, but to an excited state populated by particles.

$$\ddot{x}(t)+\omega^2(t)\;x(t)=0$$

Spring constant varied slowly (adiabatically)



Spring constant varied abruptly (nonadiabatically)



Adiabticity Parameter A_k

Mixing of + and – frequency terms depends on "Adiabaticity parameter" A_k:

$$A_k \equiv \frac{\partial_\eta \omega_k}{\omega_k^2}$$

$A_k \ll 1,$ + frequency solution remains good

$A_k \gg 1,$ + and – frequency terms mix

$$\omega_k^2 = k^2 + a^2(\eta)\left[m^2 + \left(\frac{1}{6} - \xi\right) R(\eta)\right]$$

Define some dimensionless parameters

$$\alpha \equiv a/a_e$$

$$\mu \equiv m/H_e$$

$$h \equiv H/H_e$$

$$A_k = \frac{\alpha^3\mu^2 h + \alpha^3 h(R/H_e^2)(1/6-\xi) - \frac{1}{2}\alpha^2(R'/H_e^2)(1/6-\xi)}{[k^2 + \alpha^2\mu^2 + \alpha^2(R/H_e^2)(1/6-\xi)]^{3/2}}$$

Scalar field ϕ in FRW background

$$m_{\text{eff}}^2(\eta) = a^2(\eta)\left[m^2 + \left(\frac{1}{6} - \xi\right) R(\eta)\right]$$

Abrupt changes in $a(\eta)$ leads to nonadiabatic changes in $\omega_k(\eta)$, which *adulterates* positive and negative frequency modes, leading to of particle creation in the expanding universe.

Nonadiabaticity proportional to $\dfrac{\partial_\eta \omega_k}{\omega_k^2}$

Adiabatic deep in quasi-de Sitter phase
Adiabatic at late time after inflation

$$\text{Nonadiabatic:} \begin{cases} \xi = 0: \begin{cases} k/a \ll H \text{ when mode exits horizon during inflation. Super-Hubble.} \\ k/a \gg H \text{ at end of inflation. Sub-Hubble radius.} \end{cases} \\ \xi = 1/6: \text{ at end of inflation. Sub-Hubble radius.} \end{cases}$$

Initial Conditions

Initial Conditions: as $a \longrightarrow 0$, frequency $\omega_k^2 = k^2 + a^2(\eta)\left[m^2 + \left(\frac{1}{6} - \xi\right) R(\eta)\right] \longrightarrow k^{\,2}$ motivates "Bunch-Davies" (Minkowski) initial conditions for χ_k and $\partial_\eta \chi_k$:

$$\chi_k(\eta) \xrightarrow{\eta\to-\infty} \chi_k^{\mathrm{BD}}(\eta) \equiv \frac{1}{\sqrt{2k}}\, e^{-ik\eta}$$

$$\partial_\eta \chi_k(\eta) \xrightarrow{\eta\to-\infty} -i\sqrt{\frac{k}{2}}e^{-ik\eta}$$

as $\eta \longrightarrow \infty$ the physical momentum is much larger than H and the field should not "feel" the curvature of spacetime.

NO APOLOGIES!

Scalar field ϕ in FRW background

Solutions to wave equation include both + and – frequency terms

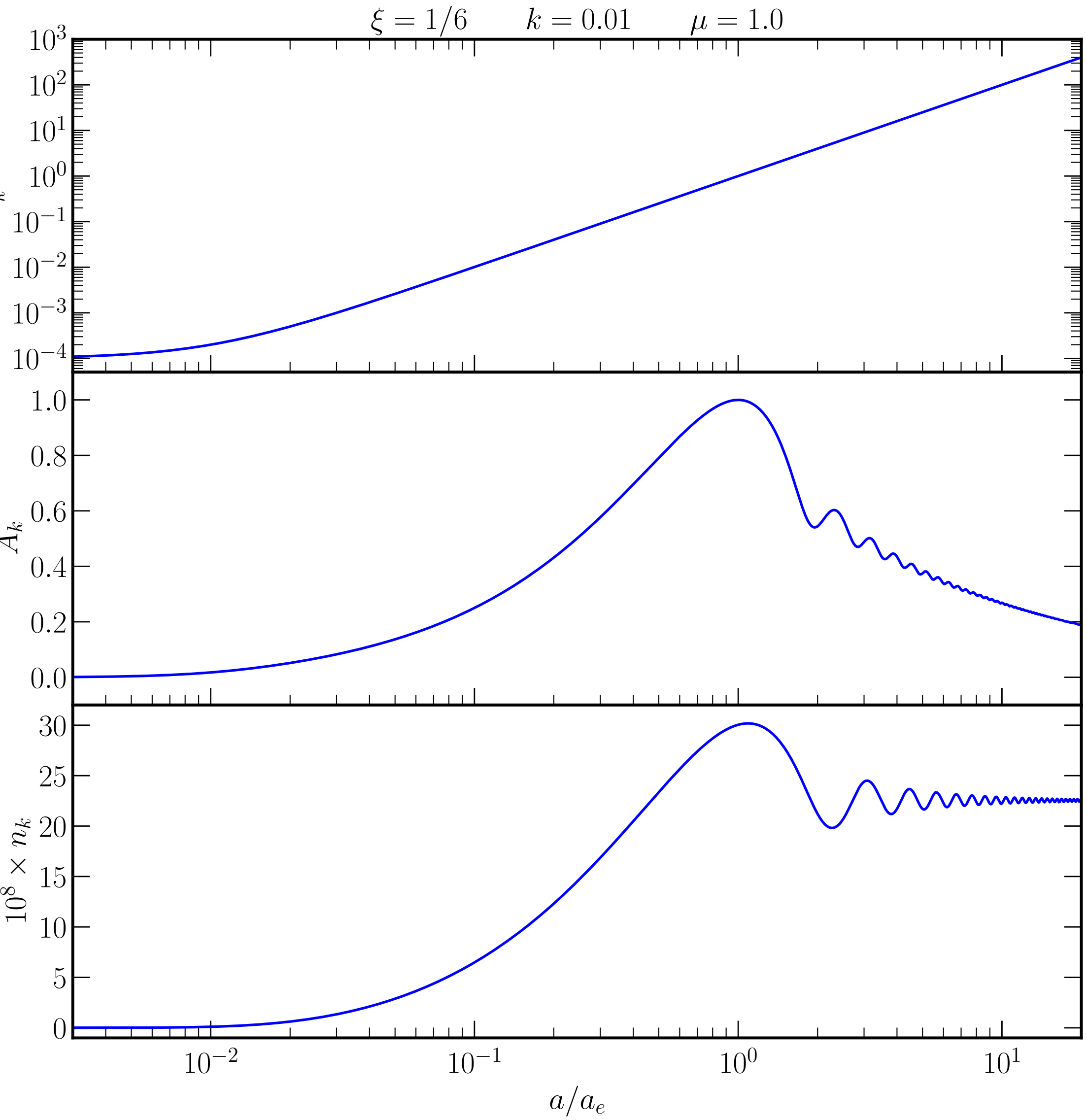
$$\chi_k(\eta) = \frac{\alpha_k(\eta)}{\sqrt{2\omega_k(\eta)}}\, e^{-i\int \omega_k(\eta)d\eta} + \frac{\beta_k(\eta)}{\sqrt{2\omega_k(\eta)}}\, e^{+i\int \omega_k(\eta)d\eta}$$

If start with only outgoing waves, $\beta_k(\eta) = 0$,
will generate incoming waves, $\beta_k(\eta) \neq 0$.

Comoving number density of particles at late time is

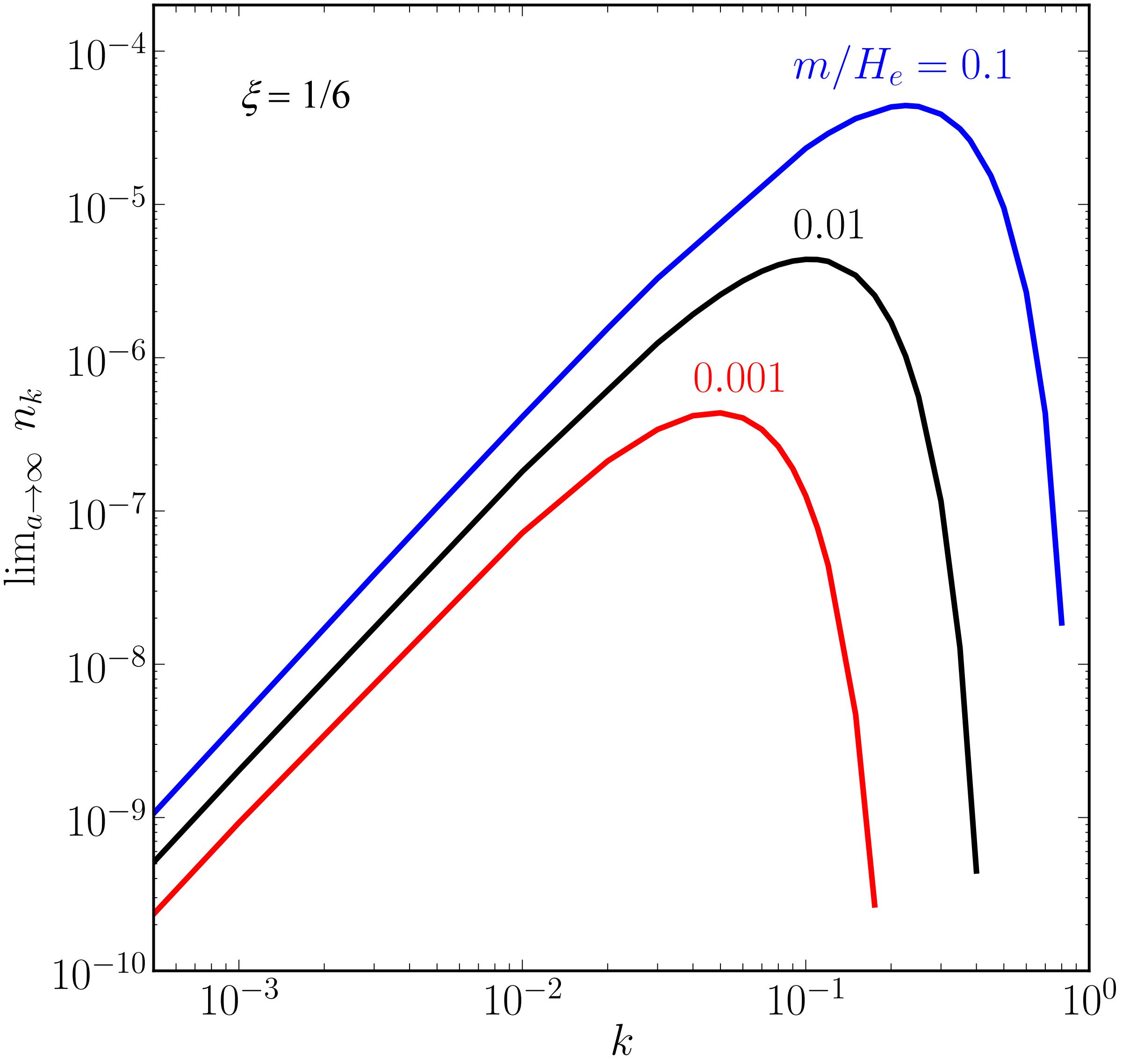
$$n\,a^3 = \frac{1}{(2\pi)^3}\int d^3k\,|\beta_k(\eta)|^2 = \int \frac{dk}{k}\;\frac{1}{2\pi^2}\;k^3|\beta_k(\eta)|^2$$

$$n_k \equiv \frac{1}{2\pi^2}k^3|\beta_k(\eta)|^2$$ Spectral density

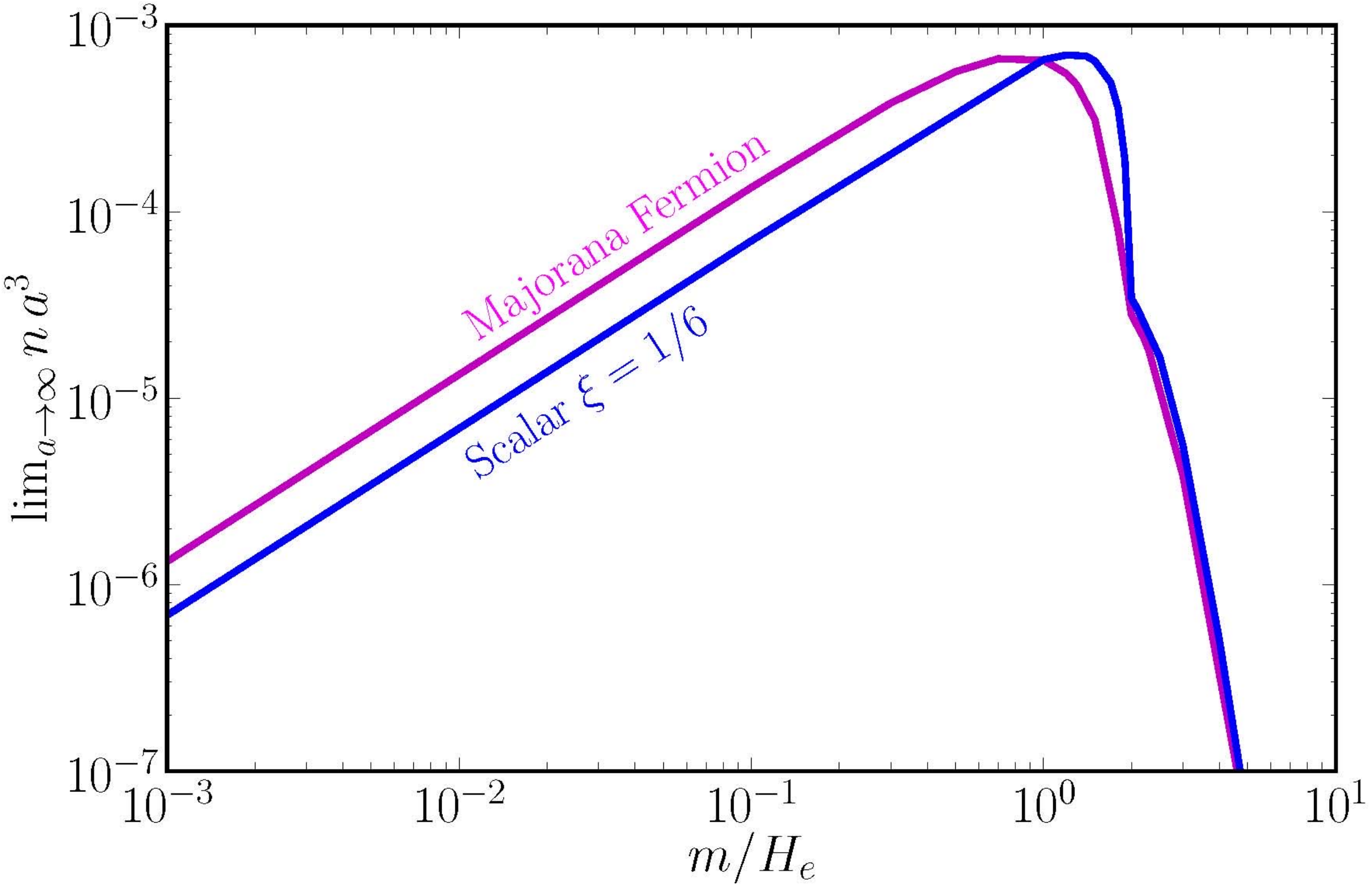


Conformally-Coupled Scalar

$$\mu = m_\phi / H_e$$



Notice spectrum is BLUE, by which I mean spectrum vanishes as $k \longrightarrow 0$

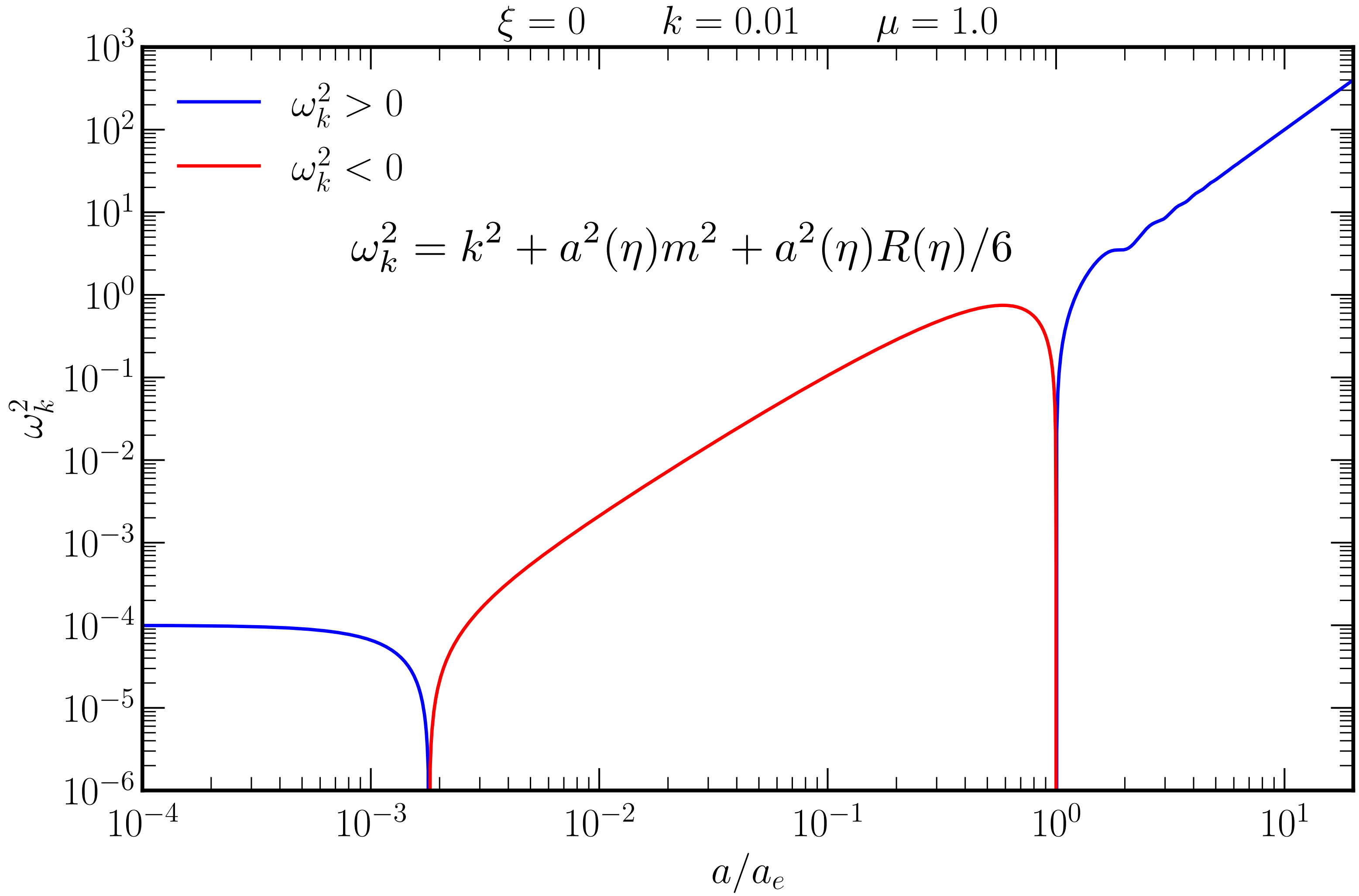


$na^3 \rightarrow 0$ as $m \rightarrow 0$

For conformally-coupled scalar, conformal symmetry only broken by mass term.

Since metric is conformally Minkowski, massless, conformally-coupled scalar field does not feel expansion.

Adiabaticity Parameter A_k



Conformal and Minimal Couplings Very Different

Mode equation:

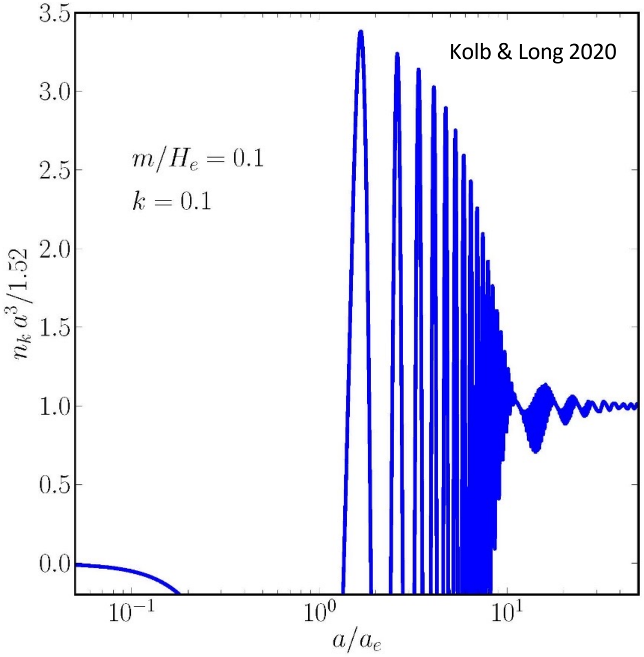
$$\partial_\eta^2 \chi_k(\eta) + \omega_k^2(\eta)\chi_k(\eta) = 0$$

$\omega_k^2 < 0$ Possible for minimal coupling—expect growth

Why should ξ be constant? There should be an RG flow for ξ.

Might set ξ to 0 or $1/6$ at some scale (say $M_{\rm Pl}$?) but at other scales there should be log corrections.

Minimal: $\xi = 0$



Evolution complicated by 2 frequency scales: m and R

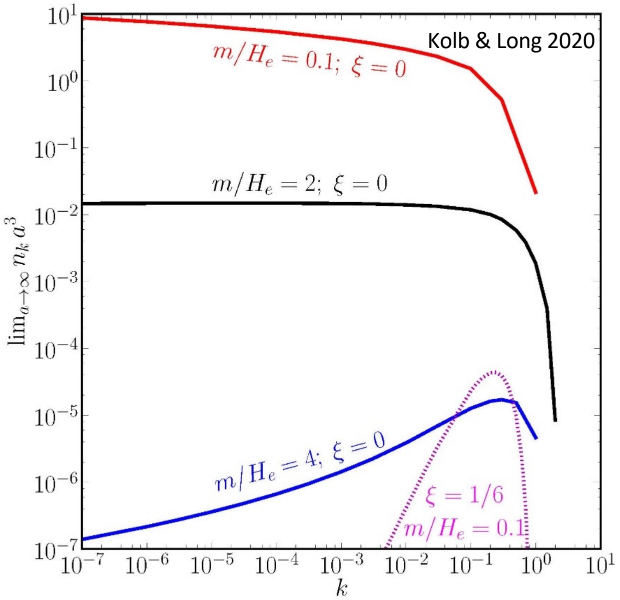
$$\omega_k^2 = k^2 + a^2(\eta)\left(m^2 + \frac{1}{6}R(\eta)\right)$$

For small m, nonadiabatic deep in inflation as mode becomes tachyonic*

Irruption when tachyonic: $k = aH$

Suppression in spectrum at $k > 1$

Suppression in spectrum for $m/H_e > 1$



Spectrum diverges In IR for $m\,/\,H_e < 2$

Isocurvature issues

---

*In inflation $R \sim -H^2$

Conversion of *comoving density* to Ωh^2

After inflation universe dominated by coherent oscillations of inflaton. Energy density decreases as a matter-dominated universe. Eventually inflaton decays, “reheating” the universe to some “reheat” temperature T_{RH}, after which the universe evolves as a radiation-dominated universe, eventually becoming matter dominated around $z = 30{,}000$, then dark-energy dominated at a redshift ≈ 1.

All the while na^3 remaining constant.

$$\frac{\Omega h^2}{0.12} = \frac{m}{H_e}\left(\frac{H_e}{10^{12}\,\mathrm{GeV}}\right)^2\left(\frac{T_{\mathrm{RH}}}{10^9\,\mathrm{GeV}}\right)\frac{\lim_{a\to\infty} n\,a^3}{10^{-5}}$$

We don’t know H_e or T_{RH}, but the above values are “representative” choices.

So $na^3 \approx 10^{-5}$ seems desirable.

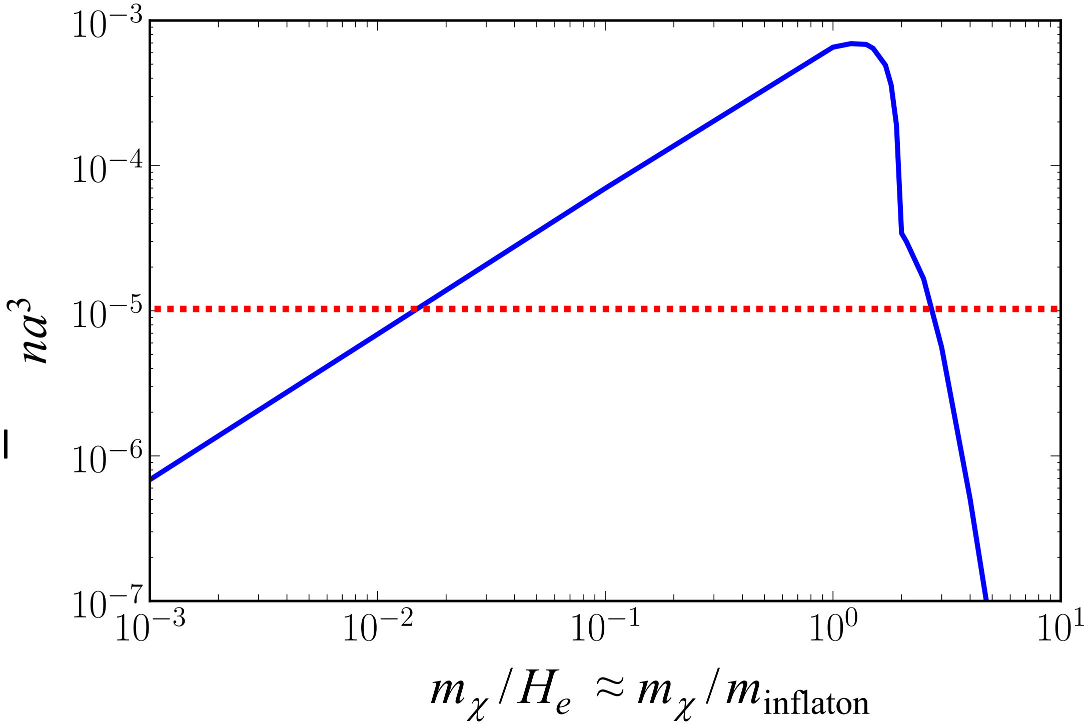
Scalar field ϕ in FRW background

Conformally coupled scalar

$\xi = 1/6$

$$\frac{\Omega_\chi h^2}{0.12} = \frac{m_\chi}{H_e}\left(\frac{H_e}{10^{12}\mathrm{GeV}}\right)^2\left(\frac{T_{\mathrm{RH}}}{10^9\mathrm{GeV}}\right)\frac{[na^3]}{10^{-5}}$$

- Calculation assumes particular inflationary model (chaotic, which is ruled out).
- But general picture holds in other models since action occurs around end of inflation.
- We don't know, but $H_e \approx 10^{12}$ GeV and $T_{\mathrm{RH}} \approx 10^9$ GeV are "common."
- If stable and dark matter, $\Omega_\chi h^2 = 0.12 \implies m \approx H_e$. Could have been anything!
- Perhaps inflation scale represents new physics scale, stable particle at that mass scale natural DM candidate.
- WIMPZILLA miracle!

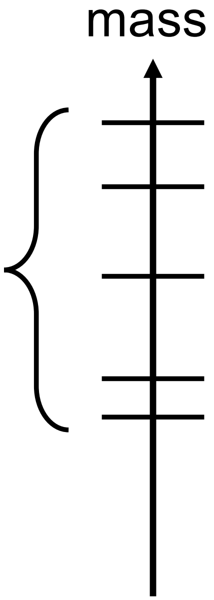


Conformally-coupled scalar WIMPZILLA DM candidate

$m_\chi = \mathcal{O}(m_{\mathrm{inflaton}})$

GPP & Dark Matter

- Inflation indicates a new mass scale
- In most models, $m_{\text{inflaton}} \approx H_{\text{inflation}} \approx 10^{12} - 10^{14}$ GeV?
- $H_{\text{inflation}}$ detectable via primordial gravitational waves in CMB
- (I, at least) expect other particles with mass $\approx m_{\text{inflaton}}$



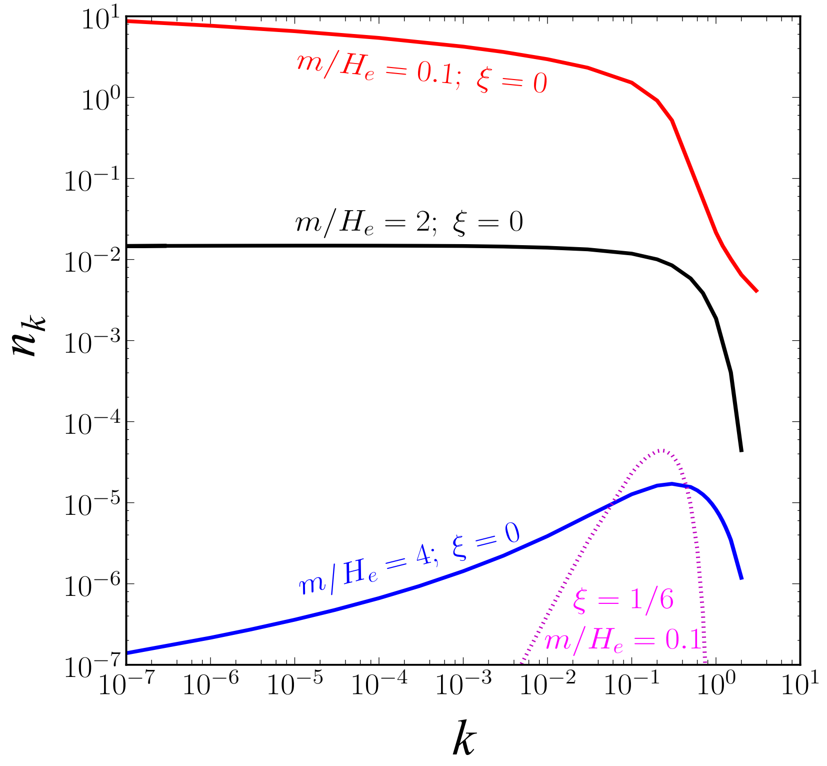
all produced à la Schrödinger's alarming phenomenon

m_{inflaton}

lightest stable? Dark Matter "WIMPzilla"

Scalar field ϕ in FRW background

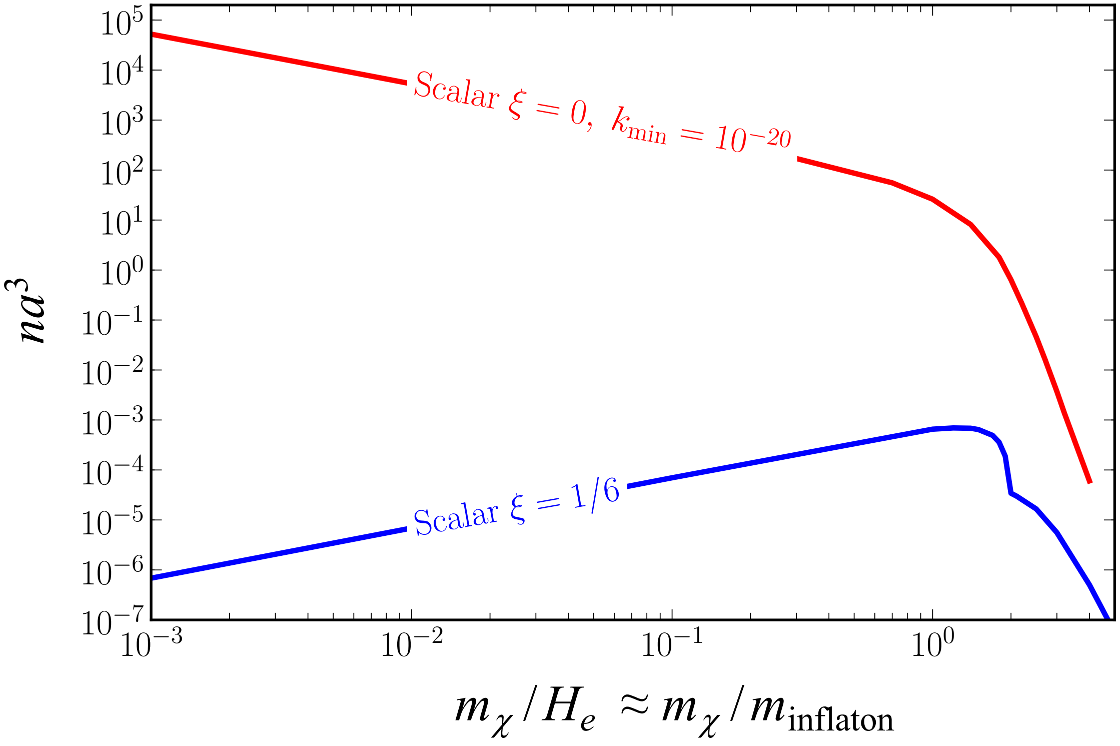
$$[na^3] = \frac{1}{2\pi^2}\int \frac{dk}{k} n_k$$



$\xi = 0$ red for $m/H_e < 2$
blue for $m/H_e > 2$

$\xi = 1/6$ blue for all m/H_e

$$\frac{\Omega_\chi h^2}{0.12} = \frac{m_\chi}{H_e}\left(\frac{H_e}{10^{12}\mathrm{GeV}}\right)^2\left(\frac{T_{\mathrm{RH}}}{10^9\mathrm{GeV}}\right)\frac{[na^3]}{10^{-5}}$$



Minimally-coupled scalar WIMPZILLA not DM candidate unless $m_\chi / H_e \approx m_\chi / m_{\mathrm{inflaton}} \gtrsim$ a few

Scalar field ϕ in FRW background

Red Spectrum Leads to dangerous Isocurvature Fluctuations

If WIMPZILLAS contribute to matter density, two sources of density fluctuations:

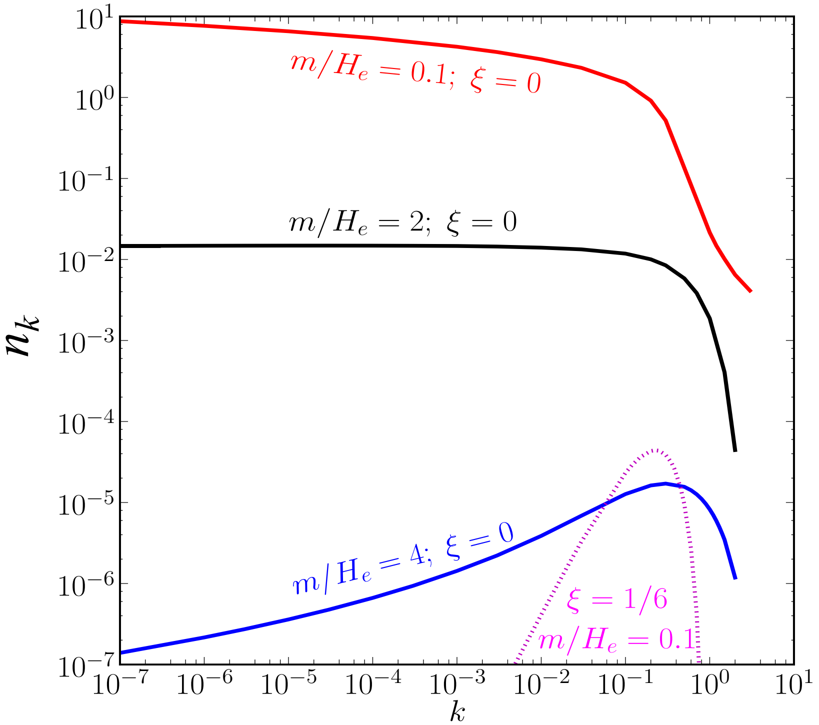
- Curvature fluctuations from inflation
- Fluctuations in χ field

They are uncorrelated.

DM density perturbations uncorrelated with baryon and photon perturbations.

For red spectrum, strict CMB limits

Chung, Kolb, Riotto, Senatore for minimally-coupled scalars



Assumes <u>before</u> reheating mode becomes

nonrelativistic $k/a < m$

and sub-Hubble $k/a < H$

Red spectrum survives "early" reheating.

Scalar Fields Not the Only Game in Town

spin-0 (real scalar)

Chung, Kolb, & Riotto (1998); Kuzmin & Tkachev (1998)

$$\mathcal{L} = \frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - \frac{1}{2}m^2\varphi^2 + \frac{1}{2}\xi R\varphi^2$$

spin-$1/2$ (Dirac)

Chung, Kolb, & Riotto (1998); Kuzmin & Tkachev (1998); Chung, Everett, Yoo, & Zhou (2011)

$$\mathcal{L} = \frac{i}{2}\bar{\Psi}\underline{\gamma}^\mu(\nabla_\mu\Psi) - \frac{1}{2}m\bar{\Psi}\Psi + \mathrm{h.c.}$$

spin-1 (de Broglie-Proca)

Dimopoulos (2006) – not for DM; Graham, Mardon, & Rajendran (2016);
Ahmed, Grzadkowski, & Socha (2020); Kolb & Long (2020)

$$\mathcal{L} = -\frac{1}{4}g^{\mu\alpha}g^{\nu\beta}F_{\mu\nu}F_{\alpha\beta} + \frac{1}{2}m^2g^{\mu\nu}A_\mu A_\nu - \frac{1}{2}\xi_1 Rg^{\mu\nu}A_\mu A_\nu - \frac{1}{2}\xi_2 R^{\mu\nu}A_\mu A_\nu$$

spin-$3/2$ (Rarita-Schwinger)

Kallosh, Kofman, Linde, & Van Proeyen (1999)
Giudice, Riotto, & Tkachev (1999); Lemoine (1999); Kolb & Long (2020)

$$\mathcal{L} = \frac{i}{4}\bar{\Psi}_\mu\left(\underline{\gamma}^\mu\underline{\gamma}^\rho\underline{\gamma}^\sigma - \underline{\gamma}^\sigma\underline{\gamma}^\rho\underline{\gamma}^\mu\right)(\nabla_\rho\Psi_\sigma) - \frac{1}{2}m\bar{\Psi}_\mu\underline{\gamma}^\mu\underline{\gamma}^\sigma\Psi_\sigma + \mathrm{h.c.}$$

spin-2 (Fierz-Pauli)

DM: Kolb, Liang, Long Rosen (2022); [see also Babichev, et al (2016)
Bernard, Deffayet, & von Strauss (2015); Mazuet & Volkov (2018)]

$$\mathcal{L} = \frac{1}{2}M_g^2 h_{\mu\nu}\left(\tilde{\mathcal{E}}^{\mu\nu\rho\sigma} + m^2\mathcal{M}^{\mu\nu\rho\sigma}\right)h_{\rho\sigma}$$

Dirac field ψ in FRW background

Dirac Equation in FRW:

$$i\partial_\eta \begin{pmatrix} u_A(\eta) \\ u_B(\eta) \end{pmatrix} = \begin{pmatrix} a(\eta)m & k \\ k & -a(\eta)m \end{pmatrix} \begin{pmatrix} u_A(\eta) \\ u_B(\eta) \end{pmatrix}$$

Dispersion relation same as conformally-coupled scalar

$$\omega_k^2(\eta) = k^2 + m^2 a^2(\eta)$$

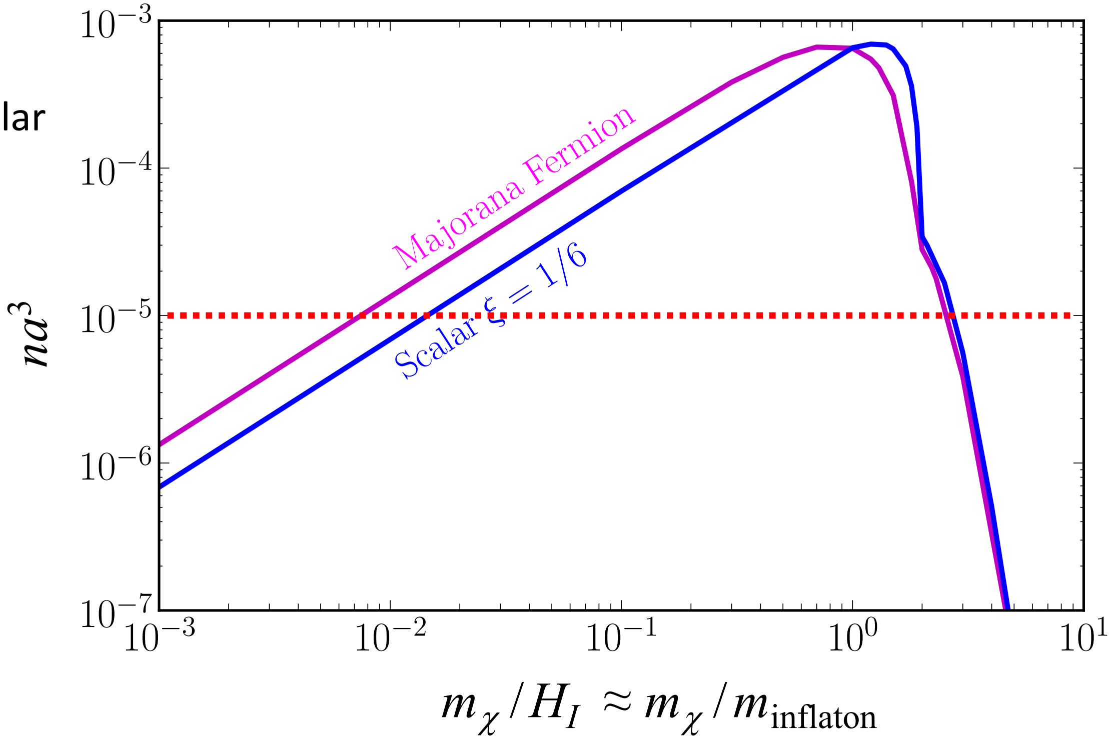
Adiabaticity parameter k/m × conformal scalar

$$A_k = \frac{a^2 H m k}{\left(k^2 + a^2 m^2\right)^{3/2}}$$

Blue spectrum: no isocurvature issues

Dirac WIMPZILLA DM candidate for $m_\chi = \mathcal{O}(m_{\text{inflaton}})$

$$\frac{\Omega_\chi h^2}{0.12} = \frac{m_\chi}{H_e} \left(\frac{H_e}{10^{12}\text{GeV}}\right)^2 \left(\frac{T_{\text{RH}}}{10^9\text{GeV}}\right) \frac{\left[na^3\right]}{10^{-5}}$$



Fields with Spin $>$ 1/2

For bosons, $\omega_k(\eta)$ tells all:

$$\omega_k^2(\eta) = \begin{cases} k^2 + a^2(\eta)m^2 + (\frac{1}{6} - \xi)a^2(\eta)R(\eta) & s=0 \\ k^2 + a^2(\eta)m^2 \quad \text{Like conformally-coupled scalar: in massless limit no production} & s=1 \quad \lambda=\pm 1 \\ k^2 + a^2(\eta)m^2 + \dfrac{1}{6}\dfrac{k^2a^2(\eta)R(\eta)}{k^2+a^2(\eta)m^2} + 3\dfrac{k^2a^4(\eta)H^2(\eta)m^2}{(k^2+a^2(\eta)m^2)^2} \quad \text{Interesting (i.e., complicated)} & s=1 \quad \lambda=0 \\ k^2 + a^2(\eta)m^2 + \frac{1}{6}a^2(\eta)R(\eta) \quad \text{Like minimally-coupled scalar; graviton in massless limit} & s=2 \quad \lambda=\pm 2 \\ k^2 + a^2(\eta)m^2 + \frac{1}{6}\dfrac{a^2(\eta)(2k^2+a^2(\eta)m^2)R(\eta)}{k^2+a^2(\eta)m^2} - \dfrac{a^2(\eta)k^2(2k^2-a^2(\eta)m^2)H^2(\eta)}{(k^2+a^2(\eta)m^2)^2} & s=2 \quad \lambda=\pm 1 \\ \text{way, way too long to show} & s=2 \quad \lambda=0 \end{cases}$$

de Broglie—Proca field A_μ in FRW background

Graham, Mardon, & Rajendran (2016); Ahmed, Grzadkowski, & Socha (2020); Kolb & Long (2020)

Dispersion relation for transverse modes, $\omega_k^2 = k^2 + a^2(\eta)m^2$, same as conformally-coupled scalar.

Dispersion relation for longitudinal modes, $\omega_k^2 = k^2 + a^2(\eta)m^2 + \frac{1}{6}\frac{k^2a^2(\eta)R(\eta)}{k^2+a^2(\eta)m^2} + 3\frac{k^2a^4(\eta)H^2(\eta)m^2}{(k^2+a^2(\eta)m^2)^2}$, “interesting.”

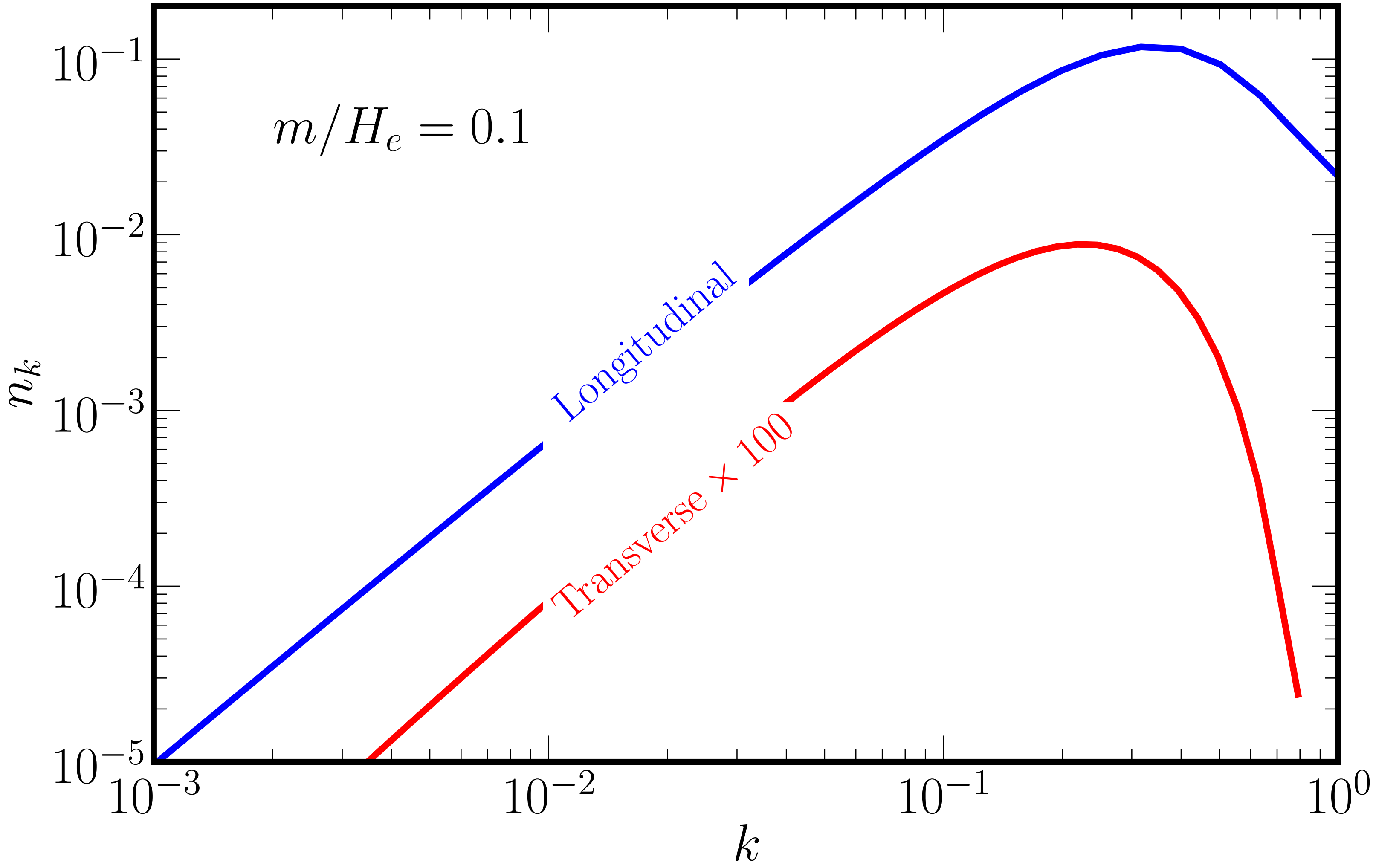
- power spectrum is blue-tilted at low k
- negligible power at CMB scales
- no problem with isocurvature even for m << H_I

Late Reheating $$\frac{\Omega h^2}{0.12} = \left(\frac{H_e}{10^{11}\mathrm{GeV}}\right)^2 \left(\frac{T_{\mathrm{RH}}}{5\times 10^7\mathrm{GeV}}\right) \qquad \left(T_{\mathrm{RH}} < 8.4\times 10^8 \left(\frac{m}{\mathrm{GeV}}\right)^{1/2} \mathrm{GeV}\right)$$

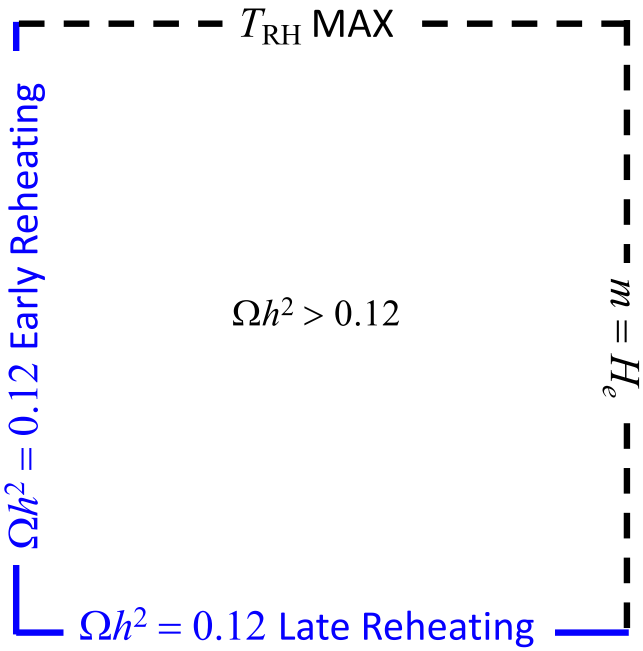
Early Reheating $$\frac{\Omega h^2}{0.12} = \left(\frac{m}{10^{-6}\mathrm{eV}}\right)^{1/2} \left(\frac{H_e}{10^{14}\mathrm{GeV}}\right)^2 \qquad \left(T_{\mathrm{RH}} > 8.4\times 10^8 \left(\frac{m}{\mathrm{GeV}}\right)^{1/2} \mathrm{GeV}\right)$$

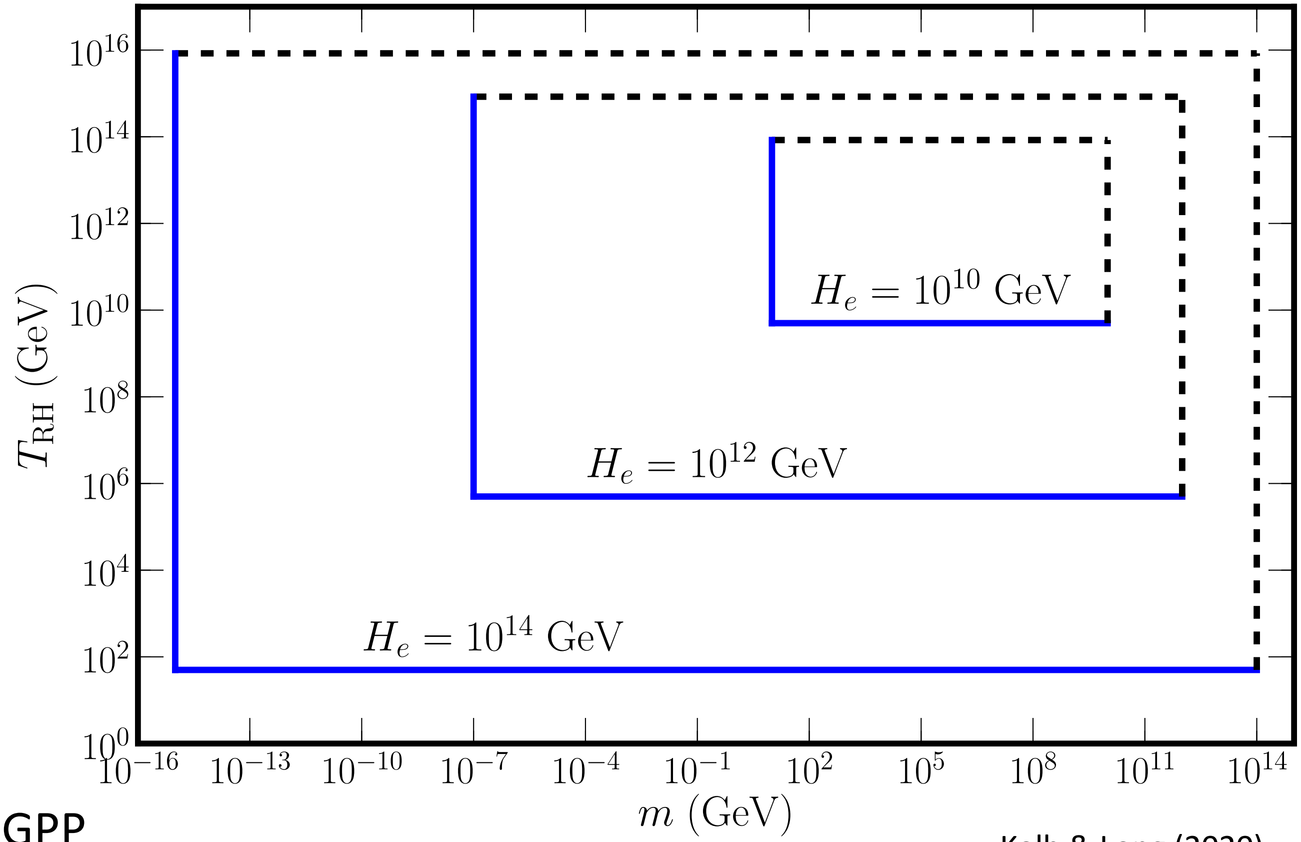
Ωh^2 depends on m, H_e , and T_{RH}

de Broglie—Proca field A_μ in FRW background



de Broglie—Proca field A_μ in FRW background





Kolb & Long (2020)

Very light (μeV) DM from GPP

or

Very massive (10^{14} GeV) DM from GPP

Fierz-Pauli field $f_{\mu\nu}$ in FRW background

- One thought that massive gravity theories had ghostly 6th degree of freedom at the nonlinear level until de Rahm, Gabadadze & Trolly (dRGT 2011)

- Hassan & Rosen (2012) showed how to construct ghost-free “bimetric” theories with correct number of propagating d.o.f.

- We (with Ling, Long, and Rosen) are examining two massive spin-2 theories à la Hassan & Rosen

mirror matter

$$S[g,f,\Phi] = \int d^4x \left[m_g^2 \sqrt{-g}\, R(g) + m_f^2 \sqrt{-f}\, R(f) - 2m^4 \sqrt{-g}\, V(\mathbb{X};\beta_n) + \sqrt{-g}\, \mathcal{L}_m(g,\Phi) + \sqrt{-f}\, \mathcal{L}_m(f,\Phi) \right]$$

$$\mathbb{X}^{\mu}{}_{\nu} = \sqrt{g^{\mu\lambda} f_{\lambda\nu}}$$

doubly-coupled

$$S[g,f,\Phi] = \int d^4x \left[m_g^2 \sqrt{-g}\, R(g) + m_f^2 \sqrt{-f}\, R(f) - 2m^4 \sqrt{-g}\, V(\mathbb{X};\beta_n) + \sqrt{-g^{\rm eff}}\, \mathcal{L}_m(g^{\rm eff},\Phi) \right]$$

$$g^{\rm eff}_{\mu\nu} = a^2 g_{\mu\nu} + 2ab(g\mathbb{X}) + b^2 f_{\mu\nu}$$

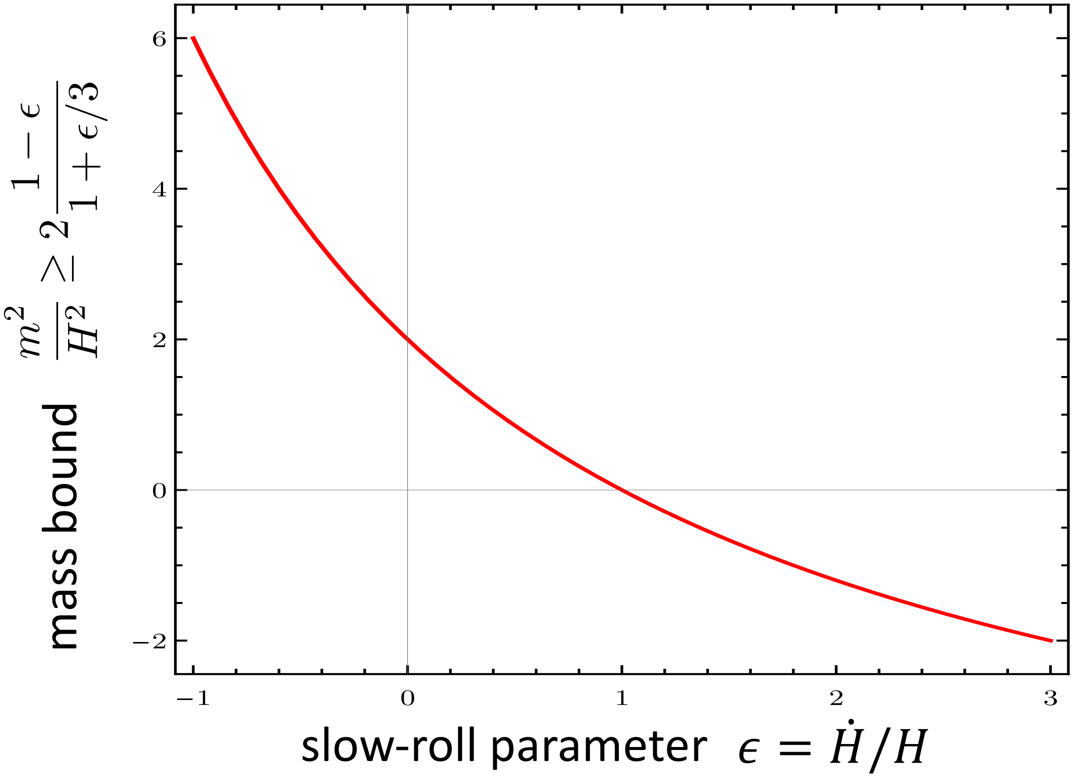
- Looking for WIMPZILLAS, but interesting things along the way

Fierz—Pauli field $f_{\mu\nu}$ in FRW background

- In 1987 Higuchi demonstrated that for fields of spin two or greater in de Sitter space, there are ghosts unless

$$m^2 > 2H^2$$

- We (with Ling, Long, and Rosen) generalize the Higuchi bound to FRW:



$$m^2 \geq 2H^2 \ \frac{H^2 + \dot{H}}{H^2 - \dot{H}/3} = 2H^2 \frac{1-\epsilon}{1+\epsilon/3}$$

$$\epsilon = \begin{cases} 0 & \text{dS} \\ -3/2 & \text{MD} \\ -2 & \text{RD} \end{cases}$$

Cosmological Limit on the mass of massive spin-2 field!

Rarita—Schwinger field ψ_μ in FRW background

"Dirac" Equation in FRW:

$$i\partial_\eta \begin{pmatrix} u_A(\eta) \\ u_B(\eta) \end{pmatrix} = \begin{pmatrix} a(\eta)m & k \\ k & -a(\eta)m \end{pmatrix} \begin{pmatrix} u_A(\eta) \\ u_B(\eta) \end{pmatrix}$$

$s = 3/2$; $\lambda = \pm\, 3/2$ (same as $s = 1/2$)

$$i\partial_\eta \begin{pmatrix} u_A(\eta) \\ u_B(\eta) \end{pmatrix} = \begin{pmatrix} a(\eta)m & (C_A + iC_B)k \\ (C_A - iC_B)k & -a(\eta)m \end{pmatrix} \begin{pmatrix} u_A(\eta) \\ u_B(\eta) \end{pmatrix}$$

nonzero for gravitino

$s = 3/2$; $\lambda = \pm\, 1/2$

C_A & C_B function of (H, m, R, $\partial_\eta m$)

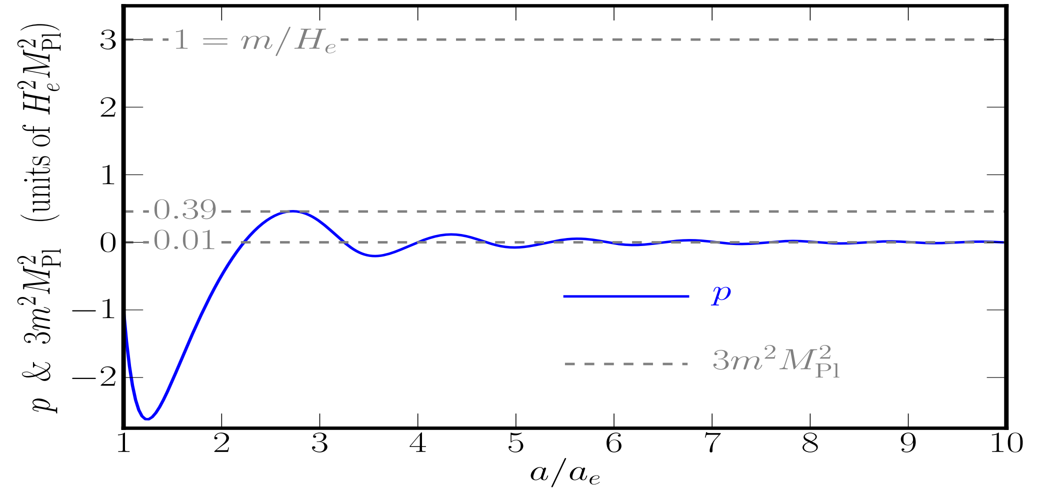
$C_A{}^2 + C_B{}^2 = c_s{}^2 =$ sound speed

New feature: $c_s = \dfrac{\left|p(\eta) - 3m^2 M_{\rm Pl}^2\right|}{\rho(\eta) + 3m^2 M_{\rm Pl}^2}$ time-dependent effective sound speed!

Can vanish when $p = 3m^2 M_{\rm Pl}^2$!!

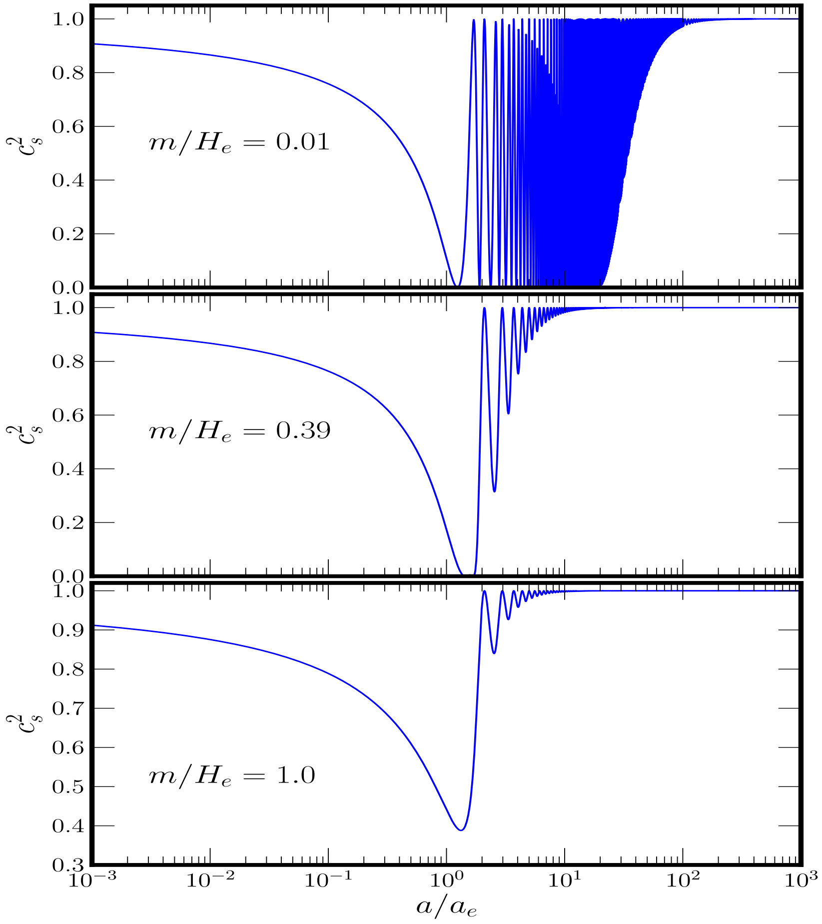
Rarita—Schwinger field ψ_μ in FRW background

$$c_s = \frac{\left|p(\eta) - 3m^2 M_{\rm Pl}^2\right|}{\rho(\eta) + 3m^2 M_{\rm Pl}^2}$$



Sound speed will vanish (perhaps many times) if $m < 0.39\ H_e$
(assumes harmonic potential after inflation)

vanishing sound speed



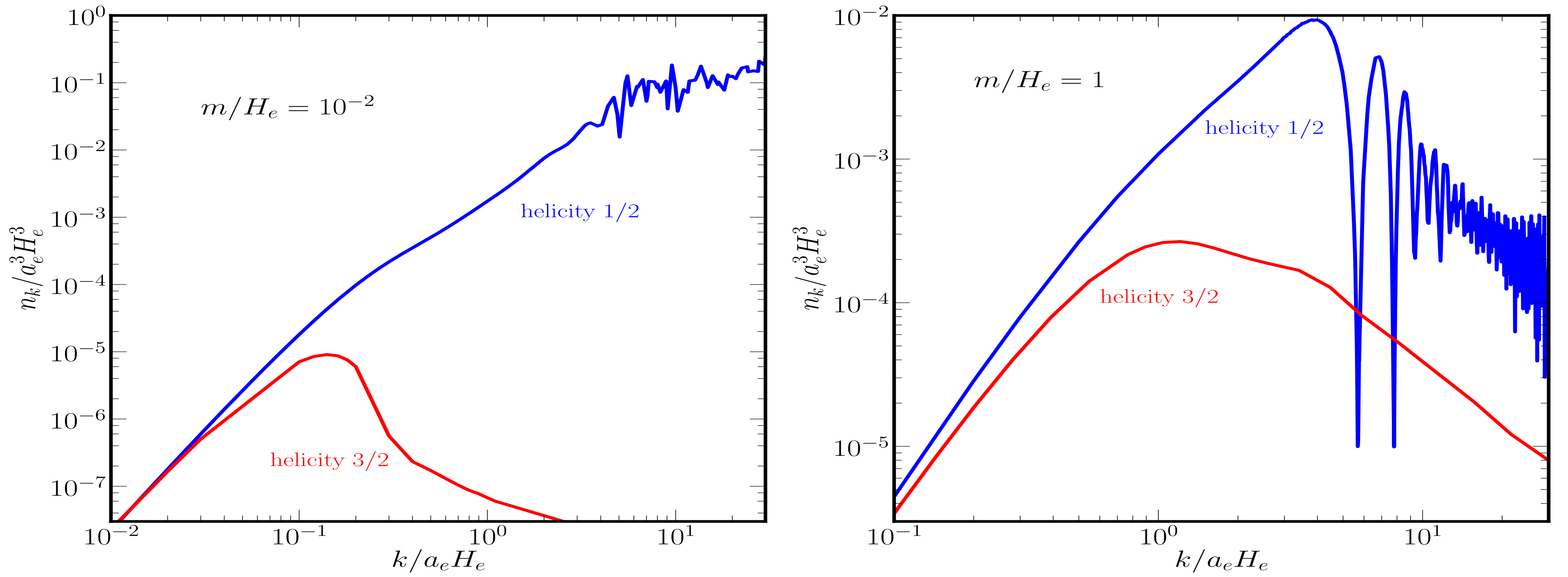
Rarita—Schwinger field ψ_μ in FRW background

Dispersion relation is $\omega_k^2(\eta) = c_s^2 k^2 + a^2(\eta) m^2$

Usual case: $c_s^2 = 1 \Rightarrow \omega_k(\eta) = k$ and constant for $k \Rightarrow \infty$

GPP depends on changing $\omega_k(\eta)$, so no production of high-k modes!

If $c_s^2 = 0$: as $k \Rightarrow \infty$, $\omega_k(\eta)$ is independent of k, production of high-k modes unsuppressed!



Rarita—Schwinger field ψ_μ in FRW background

Supergravity employs spin-3/2 field (gravitino, inflation, …), the superpartner to graviton.

Catastropic production of gravitinos dependent on model.

For models with a single chiral superfield gravitino mass is time dependent ($\partial_\eta m \neq 0$).

$c_s = 1$ at all times ⟹ no catastrophic production

For models with multiple chiral superfields (most modern models)

c_s depends on relative orientation of inflaton direction & susy breaking

$c_s = 0$ in models with a nilpotent superfield and orthogonal constraint KKLT

mixing between the goldstino & inflatino may avoid the catastrophe (explicit calculation needed)

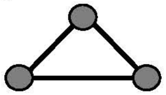
Dudas, Garcia, Mambrini, Olive, Peloso, & Verner (2021); Antoniadis, Benakli, & Ke (2021)

Models with $c_s = 0$ are in a SWAMPLAND! Kolb, Long, & McDonough (2021)

GGP may provide constraints on SUGRA model building.

Long-term Program

Complexity

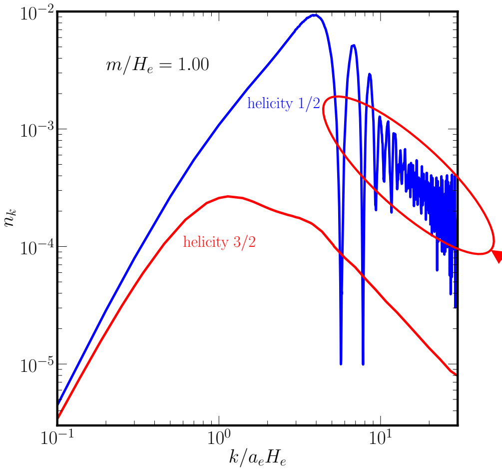
* related to but not cosmological collider program	1-pt function 	2-pt function 	3-pt function* 
Observable	**Dark Matter**	**Isocurvature Fluctuations**	**CMB Non-Gaussianities**
Massive scalar field (conformal)	Kuzmin & Tkachev (99)	Expected to be very small	Chung & Yoo
Massive scalar field (minimal)	Kuzmin & Tkachev (99)	Chung, Kolb, Riotto & Senatore	
Massive Dirac field	Chung, Kolb & Riotto (98, 99)	Similar to conformal scalar	Similar to conformal scalar?
Proca-de Broglie field	Massive: Kolb & Long Light: Graham, Mardon & Rajendran		
Massive Rarita-Schwinger field	Kolb, McDonough, Long		
Massive Fierz-Pauli field	Kolb, Ling, Long & Rosen (in progress)		

Complexity

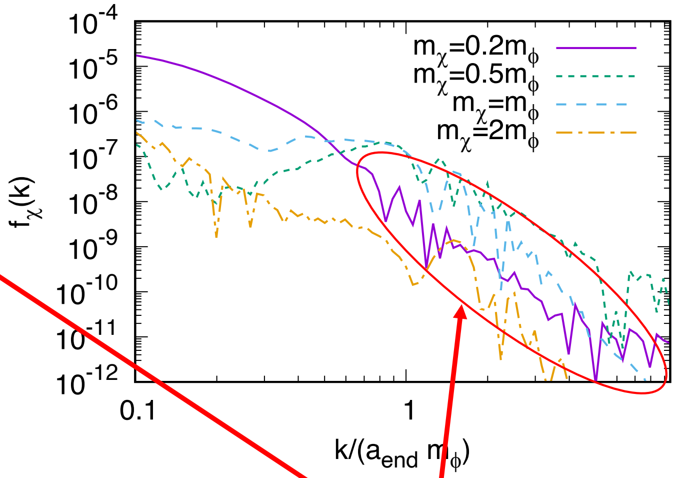
Quantum interference in gravitational particle production

(Basso, Chung, Kolb, Long)

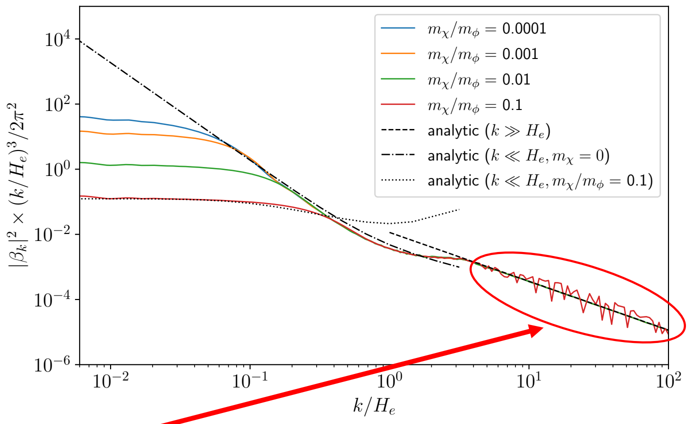
Catastrophic Production of Slow Gravitinos
Kolb, Long, Mcdonough



Production of Purely Gravitational Dark Matter
Ema, Nakayama, Tang

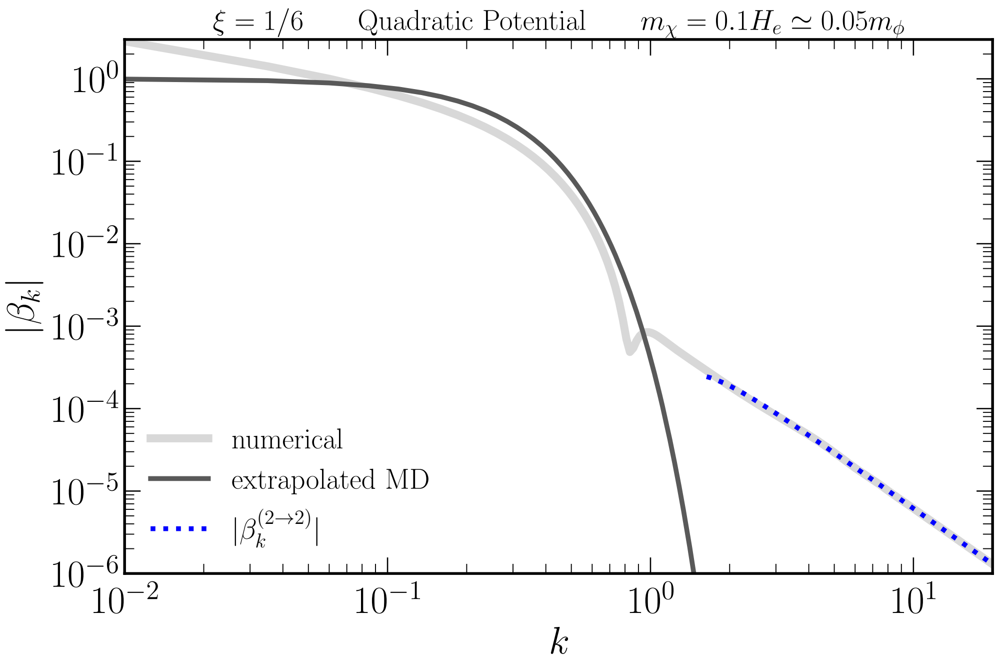


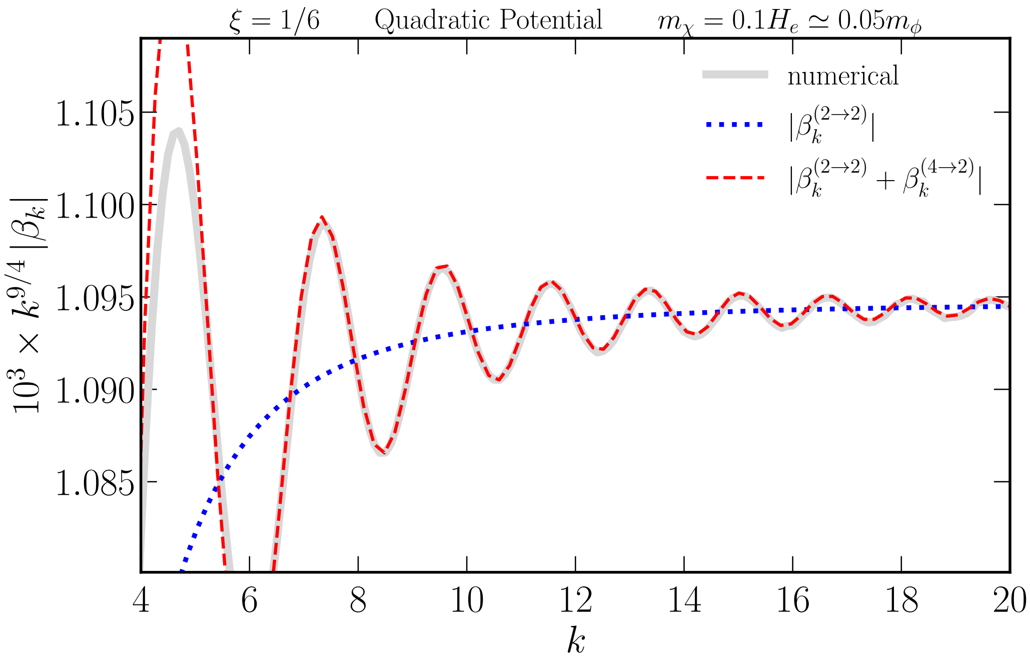
Boltzmann or Bogoliubov?
Kaneta, Mook, Oda



WTF? (Why These Features?) also, power-law decrease instead of exponential

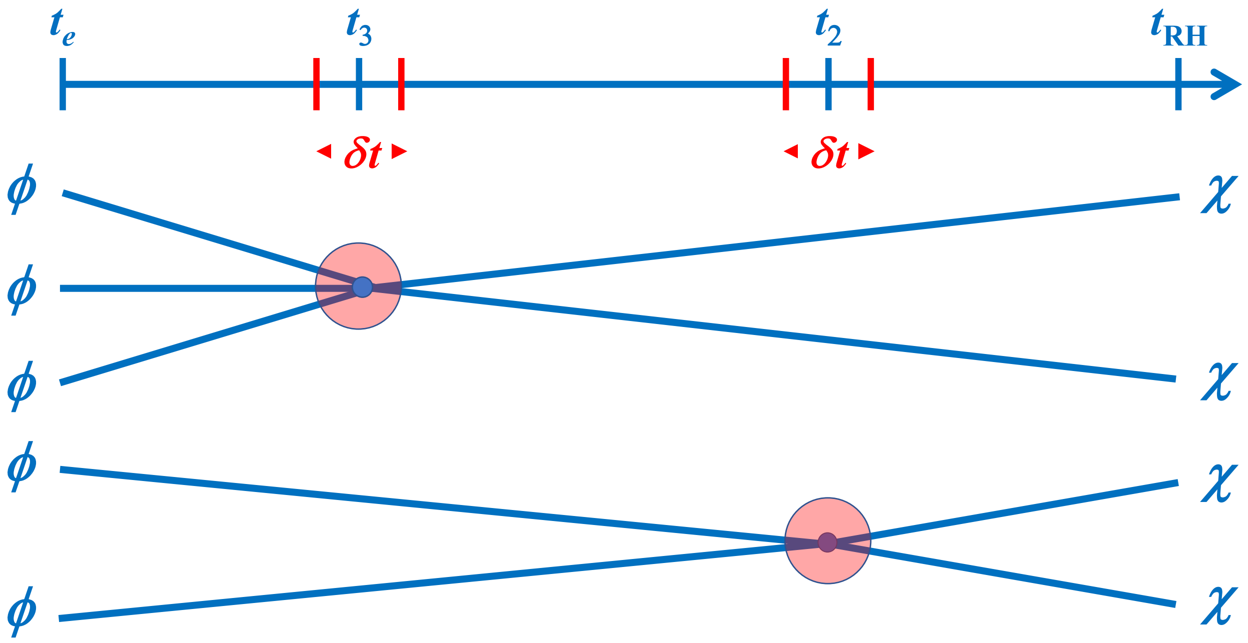
We argue that these features are due to the quantum interference of coherent scattering reactions. We find analytic formulae for the particle production amplitude for a conformally-coupled scalar field, including an interference effect in the kinematic region where the production can be interpreted as inflaton scattering into scalar final states via graviton exchange.



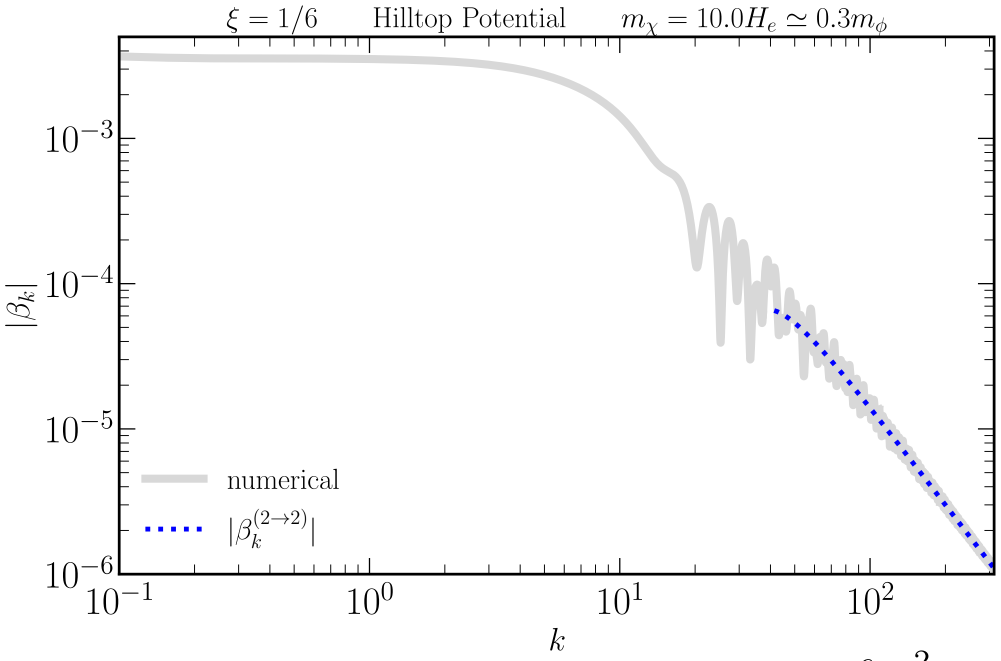


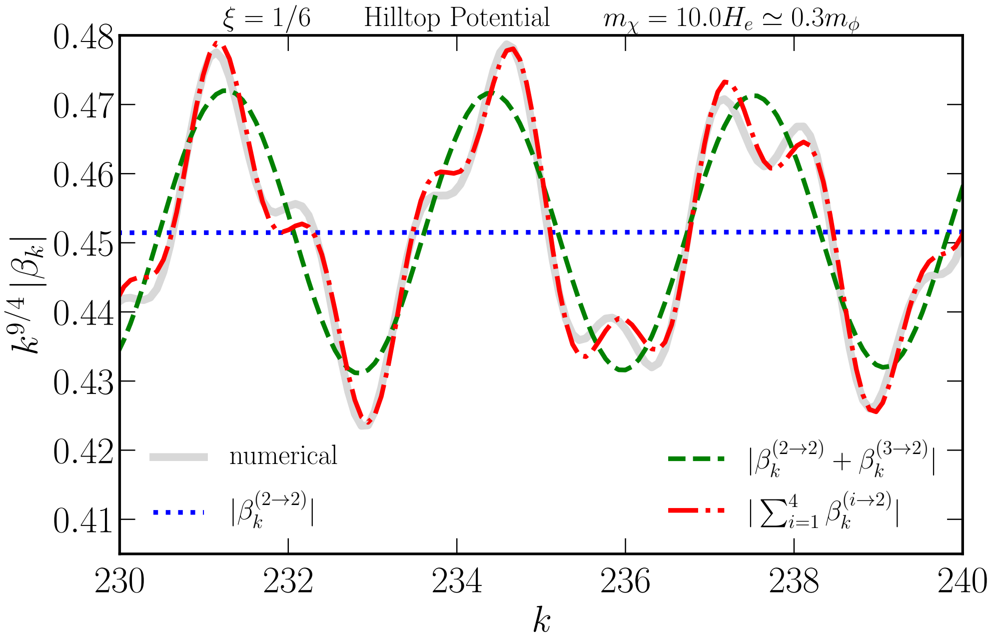
- Extrapolated MD (matter dominated-no inflaton oscillations) $|\beta_k| \propto \exp(-k^{3/2})$ for $k > 1$
- Numerical (quadratic inflaton potential with inflaton oscillations) $|\beta_k| \propto k^{-9/4}$ for $k > 1$
- Power-law behavior can be understood as $\phi + \phi \to \chi + \chi$ via a classical Boltzmann approach
- But, $\phi + \phi \to \chi + \chi$ via a classical Boltzmann approach cannot explain oscillations
- Oscillations due to quantum interference $|\, c_1\langle\chi\chi|U|\phi\phi\rangle + c_2\langle\chi\chi|U|\phi\phi\phi\phi\rangle \,|^2$

Quantum interference in gravitational particle production



- Initial macroscopic inflaton scattering state can be viewed as cold coherent superposition of $n\phi$ states
- Bogoliubov treatment allows processes that can be interpreted as $|\ c_1\langle\chi\chi|U|\phi\phi\rangle + c_2\langle\chi\chi|U|\phi\phi\phi\phi\rangle\ |^2$





- Hilltop inflaton potential $V \sim \left(1 - \frac{\phi^6}{v^6}\right)^2$ effective cubic term
- Quantum interference much more pronounced

Finally, Summary: GPP can make DM & constrain BSM physics!

Dark matter might have only gravitational interactions (that's all we really "know")

If so, dark matter must have a gravitational origin.

Cosmological Gravitational Particle Production through Schrödinger's alarming phenomenon promising.

Scalars:

Conformally-coupled: promising DM candidate if $m \approx H_e$ (WIMPZILLA miracle).

Minimally-coupled: not promising DM candidate.

Late reheating: Ωh^2 much too large unless $m \gtrsim$ few H_e. Early reheating: Ωh^2 much too large unless $m \gtrsim$ few H_e.

Isocurvature constraints unless $m \gtrsim$ few H_e.

Dirac fermions:

Similar to conformally-coupled scalars: promising DM candidate if $m \approx H_e$ (WIMPZILLA miracle).

de Broglie—Proca vectors:

DM candidate could be very light ($\mu\mathrm{eV}$) or very massive (H_e)

Rarita-Schwinger fermions:

Catastrophic production if c_s vanishes. Implications for models of supergravity.

Fierz-Pauli tensors:

FRW-generalization of the Higuchi bound; DM relic abundance in progress.

Spin greater than 2: Alexander, Jenks, McDonough

A Question

What to make of a QFT that is “perfectly” reasonable in Minkowski,

but

Pathological in FLRW?
(examples: Fierz-Pauli, spin-2 with small mass)

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