Xiao-Gang He

Seminar at KIAS, Oct. 14, 2022

1. Muon g-2 Anomaly in SM

2. The Minimal Gauge Sector Extension: $U(1)_{L\mu-L\tau}$

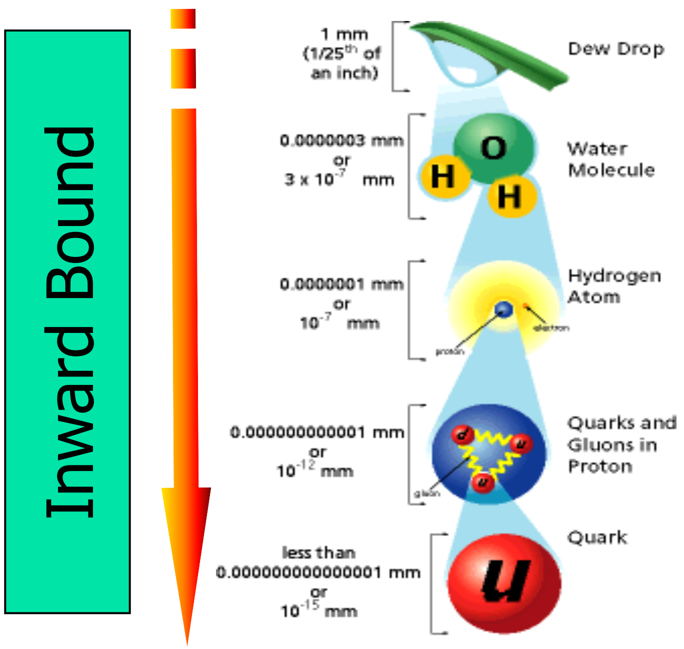
3. Small and Large Z′ Mass Solutions for Muon g-2

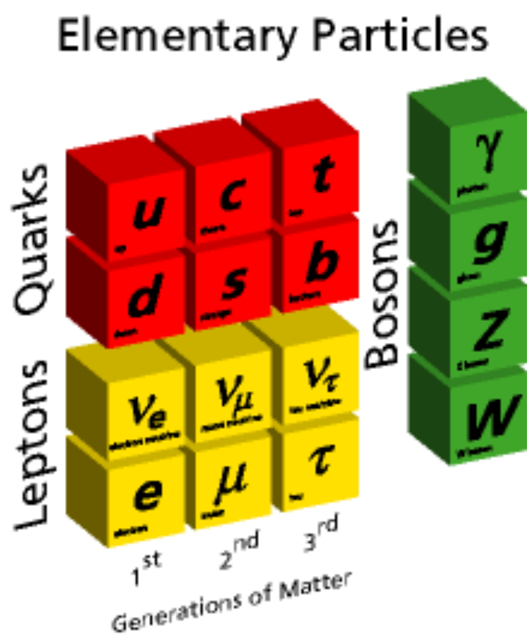
4. Conclusions

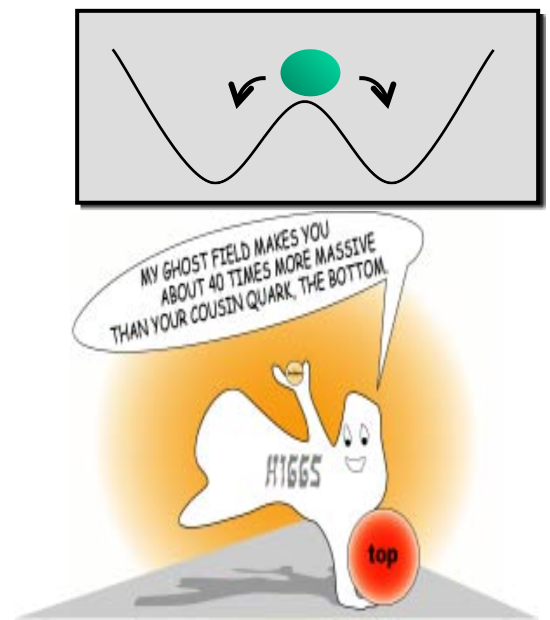
1. Muon g-2 Anomaly in SM

The standard model of strong and electroweak interactions

SU(3) x SU(2) x U(1) gauge theory for strong and electroweak interactions

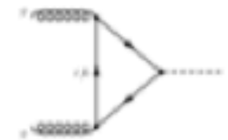




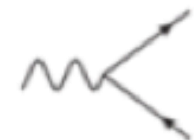


In the SM, only 3 generations of quarks and leptons are allowed.

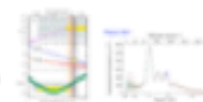
gg -> Higgs ~ (number of heavy quarks)^2, if fourth generation exist, their mass should be large, 9 times bigger production of Higgs. LHC data ruled out more than 3 generations of quarks.



LEP already ruled out more than 3 neutrinos with mass less than $m_Z/2$.



Cosmology and astrophysics, number of light neutrinos also less than 4.



SM, triangle anomaly cancellation: equal number of quarks and leptons!

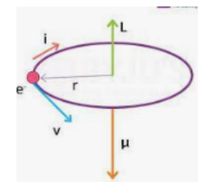
There are only three generations of sequential quarks and leptons!

SM works well. But there are some anomalies at some high statistic level, among them, the muon g-2 anomaly is an serious one with 4.2 s from SM prediction!

Muon magnetic dipole momentum in SM

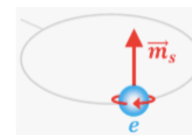
The energy of a particle with magnetic dipole μ interact with a magnetic field **B** is given by: $H = -\ \mu \cdot \mathbf{B}$

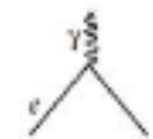
Classically, a particle of charge q moving in circle in magnetic field with angular momentum L, the magnetic dipole: $\mu_L = q\mathbf{L}/2m$.



Quantum mechanically, a Dirac particle has an intrinsic magnetic dipole moment: $\mu = q\mathbf{S}/m$ which can be written as $\mu = g\, q\mathbf{S}/2m$, ------- g=2.

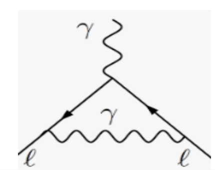
g is called the g-factor. (Dirac)



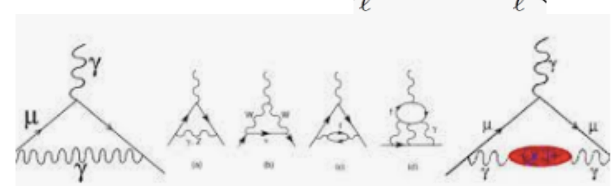


In quantum field theory, there is correction at loop level making a = (g-2)/2 not zero. This is the anomalous dipole moment of a particle. At one loop

for charge leptons, i = e, μ, τ, $a_i = \alpha/2\pi$ (Schwinger)



In the SM, including QEC, Strong and electroweak contributions, a_μ has been calculated to very high precision.



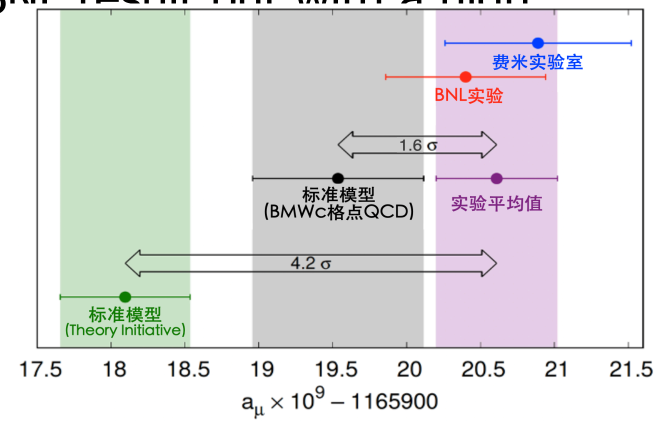
Experimental measurements

Muon a_μ has also been measured to high precision.

BNL experiment (1997 – 2001) final result

for $\Delta a_\mu = a_\mu(\text{exp}) - a_\mu(\text{SM})$ at 2.7σ larger than zero.

FNL experiment first result announce in April, 2021, confirm BNL result but with a high confidence level at 3.3σ.



Combining BNL and FNL results, $\Delta a_\mu = 251(59) \times 10^{-11}$.

The deviation away from SM is at 4.2σ level!

Recent Lattice calculation indicates the deviation is only at one σ level. More accurate theory calculations and Experimental measurement needed to confirm this anomaly.

Even the anomaly itself still needs to be confirmed, a lot of efforts have been made to explore the anomaly through beyond SM physics to match data.

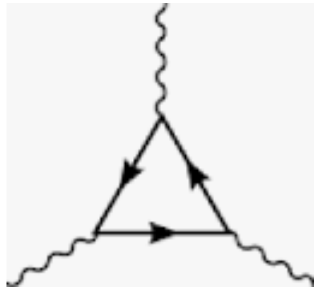
This talk is about $U(1)_{L\mu - L\tau}$ model explanation.

2. The Minimal Gauge Sector Extension: $U(1)_{L\mu-L\tau}$

The simplest model which has an additional gauge boson than that in SM:

The smallest gauge group: U(1) whose gauge boson usually called a Z'
No additional matter and Higgs fields compared with SM particle contents,

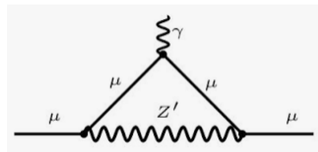
The only triangle gauge anomaly free models are:
$U(1)_{\mathbf{Li-Lj}}$ with i, j = e, μ or τ



(PRD 43 (1991) R22; PRD 44(1991) 2118)

SM is triangle anomaly free, in $U(1)_{\mathbf{Li-Lj}}$ model, the anomaly produce by L_i is cancelled by L_j. The Z' only couple to lepton current differences:

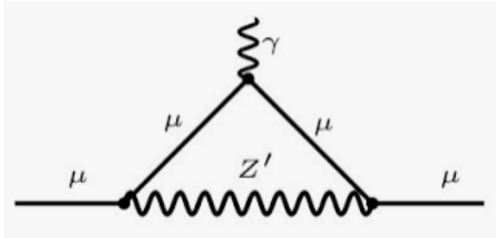
L = $-\tilde{g}(\bar{\mu}\gamma^{\mu}\mu - \bar{\tau}\gamma^{\mu}\tau + \bar{\nu}_{\mu}\gamma^{\mu}L\nu_{\mu} - \bar{\nu}_{\tau}\gamma^{\mu}L\nu_{\tau})Z'_{\mu}$



Many interesting consequences, new gauge boson for collider search, g-2 for charged leptons. If Z' couples to dark matter it is a leptophilic dark matter interaction…. i or j to be e, has very stringent constraints, i=μ, j = τ less constrained. Our emphasis!

Z′ contribution to muon g-2

The simplest version has Z′ coupling to μ and τ in diagonal form. An additional anomalous Δa_μ will be generated at one loop level
(Baek, Desh, He and Ko, PRD64(2001)055006)



$$\Delta a_\mu^{Z'} = \frac{\tilde{g}^2}{8\pi^2}\frac{m_\mu^2}{m_{Z'}^2}\int_0^1 \frac{2x^2(1-x)dx}{1-x+(m_\mu^2/m_{Z'}^2)x^2} .$$

In the large $m_{Z'} >> m_\mu$ limit,

$$\Delta a_\mu^{Z'} = (\tilde{g}^2/12\pi^2)(m_\mu^2/m_{Z'}^2).$$

To explain the muon g-2 anomaly:

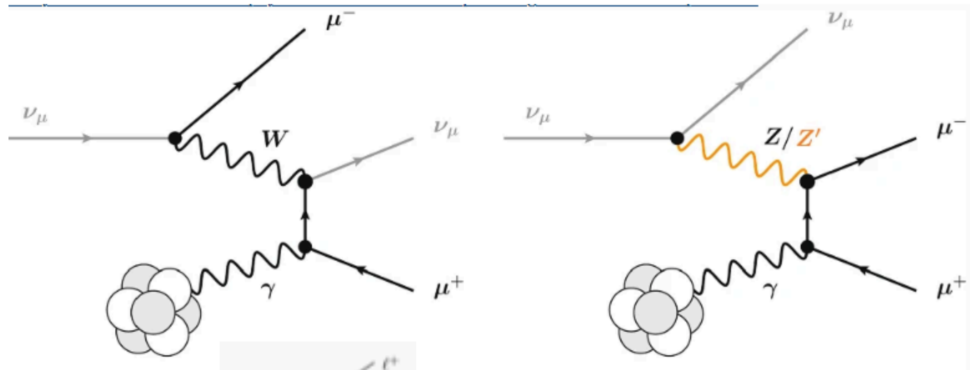
$$\tilde{g}^2/m_{Z'}^2 = (2.66 \pm 0.63) \times 10^{-5}\text{GeV}^{-2}.$$

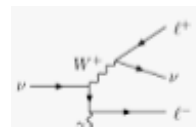
One must check if the above region is ruled out by other processes.

The trident neutrino scattering data come in and rule out the above solution
For large Z′ mass indicated above!

3. Small and Large Z′ Mass Solutions for Muon g-2

The trident neutrino constraint





Normalize SM contribution to experimental measurement, σ_{exp}/σ_{SM}.

If data agree with SM, $\sigma/\sigma(SM) = 1$.

Experimental data: $\sigma_{exp}/\sigma_{SM}|_{trident}:\ 1.58 \pm 0.57,\ 0.82 \pm 0.28$ and $0.72^{+1.73}_{-0.72}$
from CHARM-II, CCFR and NuTeV.

With Z′ contribution:

$$\frac{\sigma_{Z'}}{\sigma_{SM}}\Big|_{trident} = \frac{(1+4s_W^2+8\tilde{g}^2 m_W^2/g^2 m_{Z'}^2)^2 + 1}{1+(1+4s_W^2)^2}$$

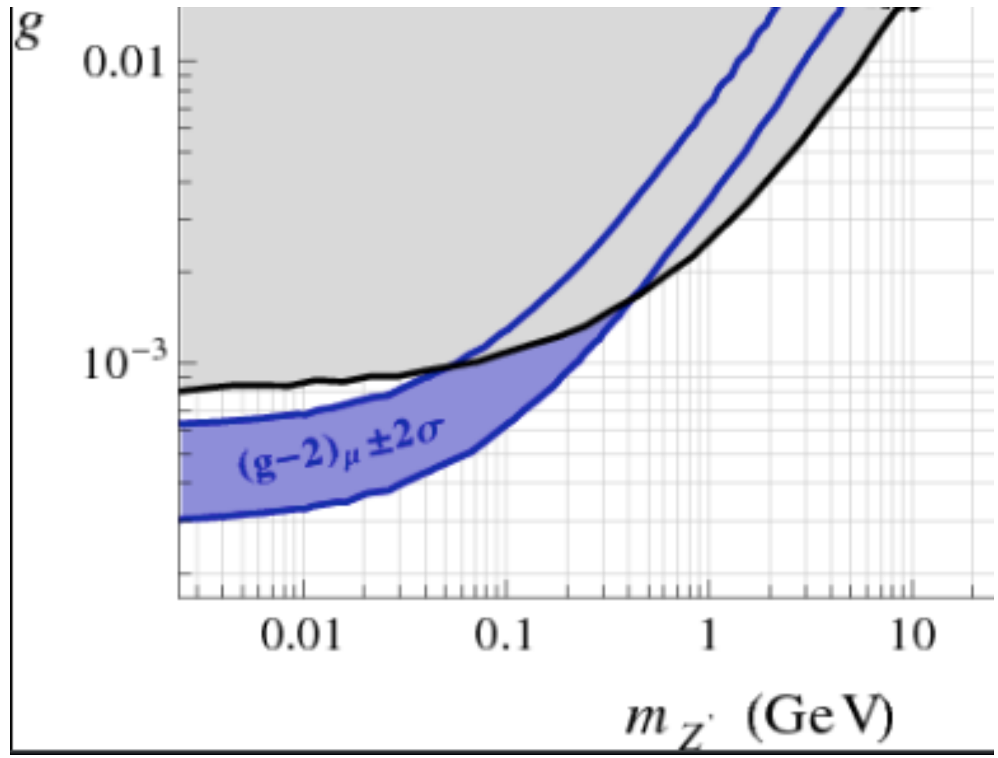
Using central value for 2.66×10^{-5} GeV^{-2} for $\tilde{g}^2/m_{Z'}^2$ one obtains $\sigma_{Z'}/\sigma_{SM} = 5.86$

The model is ruled out as a solution for muon g-2 anomaly for large Z′ mass!
(much larger than muon mass)

Small Z′ solution

What about small mass $m_{Z'} << m_\tau$?

In this region, the previous result for trident neutrino scattering is not valid because the q^2 exchange by Z′ is comparable, the heavy Z′ mass limit Cannot be applied!



More involved numerical calculations obtain the results shown in the figure On the right.

It is a folklore that in $U(1)_{L\mu-L\tau}$ in order to explain muon g-2 anomaly, $m_{Z'}$ must be less than 300 MeV!

In this rest of the talk, I will show that this needs not to be the case!

The model can admit solution without affecting to trident neutrino scattering!

Physics Letters B 827 (2022) 136989

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

ELSEVIER

arXiv: 2112.0992

Widening the $U(1)_{L_\mu - L_\tau}$ Z' mass range for resolving the muon $g-2$ anomaly

Yu Cheng[a,b], Xiao-Gang He[a,b,c], Jin Sun[a,b,*]

It has long been shown that the $U(1)_{L\mu\text{-}L\tau}$ can be modified to have Z' coupling to leptons in the following form

$$-\tilde{g}(\bar{\mu}\gamma^\mu \tau + \bar{\tau}\gamma^\mu \mu + \bar{\nu}_\mu \gamma^\mu L \nu_\tau + \bar{\nu}_\tau \gamma^\mu L \nu_\mu) Z'_\mu \; .$$

(PRD 50 (1994) 4571)

Achieved by the discrete symmetry: Z' -> - Z', mu to - tau and tau -> - mu

In this case, no $\nu_\mu N \rightarrow \nu_i \mu \bar{\mu} N$ ocess in the model and can naturally avoid trident neutrino data and may survive! In this case

Easy to explain muon g-2 anomaly!

$$\Delta a_\mu^{Z'} = \frac{\tilde{g}^2 m_\mu^2}{12\pi^2 m_{Z'}^2}\left(\frac{3m_\tau}{m_\mu} - 2\right) \; .$$

However, the above interaction will generate $\tau \rightarrow \mu \bar{\nu}_\mu \nu_\tau + \mu \bar{\nu}_\tau \nu_\mu$

$$R_{\tau\mu} = \Gamma(\tau \rightarrow \mu\nu\bar{\nu})/\Gamma_{SM}(\tau \rightarrow \mu\nu\bar{\nu}) = 1.0066 \pm 0.0041$$

The data rules out the above simple at more than 5σ level!

Need to work harder to find solution!

Solutions to problems facing

Needs to introduce new ingredients to cancel the large contribution to obtain the wanted interactions. Triplet Higgs Δ_i: (1,3,1)for type-II seesaw model will help!

The mode: $H_{1,2,3}\ :\ (1,2,1/2)\ (< H_i >= v_i/\sqrt{2})$ $)_{L\mu-L\tau}$ charges (2,0,-2)

Impose an exchange symmetry Z_2 : $Z' \to -Z'$, $H_1 \leftrightarrow H_1$ and $H_2 \leftrightarrow H_3$

Also v2 = v3 (no spontaneous Z_2 symmetry breaking).

$$
\begin{aligned}
L_H =& -\tilde{g}(\bar{l}_2\gamma^\mu L l_2 - \bar{l}_3\gamma^\mu L l_3 + \bar{e}_2\gamma^\mu R e_2 - \bar{e}_3\gamma^\mu R e_3)Z'_\mu \\
& -[Y^l_{11}\bar{l}_1 R e_1 + Y^l_{22}(\bar{l}_2 R e_2 + \bar{l}_3 R e_3)]H_1 \\
& -Y^l_{23}(\bar{l}_2 R e_3 H_2 + \bar{l}_3 R e_2 H_3) + H.C.
\end{aligned}
$$

…ass matrix is

$$
M_{\mu,\tau} = \begin{pmatrix} Y_{22}v_2/\sqrt{2} & Y_{23}v_2/\sqrt{2} \\ Y_{23}v_2/\sqrt{2} & Y_{22}v_2\sqrt{2} \end{pmatrix},
$$

Which is diagonalized by

$$
\begin{pmatrix} \mu \\ \tau \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} e_2 \\ e_3 \end{pmatrix}
$$

In the mass eigenstate basis, Z′ interaction with leptons is

$$
-\tilde{g}(\bar{\mu}\gamma^\mu\tau + \bar{\tau}\gamma^\mu\mu + \bar{\nu}_\mu\gamma^\mu L\nu_\tau + \bar{\nu}_\tau\gamma^\mu L\nu_\mu)Z'_\mu
$$

need to fi…

$$
\tau \to \mu\bar{\nu}_\mu\nu_\tau + \mu\bar{\nu}_\tau\nu_\mu
$$

Introduce $\Delta_{1,2,3}$ $(<\Delta_i>=\tilde{v}_i/\sqrt{2})$ with $U(1)_{L_\mu - L_\tau}$ charges $(0,-2,2)$

Under Z_2 symmetry: $\Delta_1 \leftrightarrow \Delta_1,\ \Delta_2 \leftrightarrow \Delta_3$

vev constrained to be less than 1 Gev from electroweak parameter ρ

$$-\left[\bar{l}^c_\mu L l_\mu (Y^\nu_{22}(\Delta_2+\Delta_3) - 2Y^\nu_{23}\Delta_1) + \bar{l}^c_\tau L l_\tau (Y^\nu_{22}(\Delta_2+\Delta_3) + 2Y^\nu_{23}\Delta_1) + 2\bar{l}^c_\mu L l_\tau (Y^\nu_{22}(\Delta_2-\Delta_3))\right]/2 + H.C.$$

$$\Delta = \begin{pmatrix} \Delta^+/\sqrt{2} & \Delta^{++} \\ \Delta^0 & -\Delta^+/\sqrt{2} \end{pmatrix}$$

$$\bar{\nu}^c_i L \nu_j \Delta^0 - \bar{e}^c_i L e_j \Delta^{++} - \frac{1}{\sqrt{2}}(\bar{\nu}^c_i L e_j + \bar{e}^c_i L \nu_j)\Delta^+$$

In the charged lepton eigenstate basis:

$$\left(\sqrt{2}(\bar{\nu}^c_e, \bar{\nu}^c_\mu, \bar{\nu}^c_\tau) M(\Delta^+) + (\bar{e}^c, \bar{\mu}^c, \bar{\tau}^c) M(\Delta^{++})\right) L(e,\mu,\tau)^T$$
$$-(\bar{\nu}^c_e, \bar{\nu}^c_\mu, \bar{\nu}^c_\tau) M(\Delta^0) L(\nu_e,\nu_\mu,\nu_\tau)^T \ , \qquad (13)$$

The non-zero elements in M(D) are

$$M_{11} = Y^\nu_{11}\Delta_1,\ M_{23} = M_{32} = \frac{1}{2}Y^\nu_{22}(\Delta_2 - \Delta_3),$$
$$M_{22,33} = \frac{1}{2}Y^\nu_{22}(\Delta_2+\Delta_3) \mp Y^\nu_{23}\Delta_1 \ .$$

Exchange $\Delta^{0,+,++}$ will generate needed interactions!

Solution to τ -> μ ν ν

Exchange Δ^+_i

$$\rho_{\mu\mu}(\bar\nu_\mu\gamma^\mu L\nu_\mu + \bar\nu_\tau\gamma^\mu L\nu_\tau)\bar\mu\gamma_\mu L\tau$$
$$+(\rho_{\tau\mu}\bar\nu_\tau\gamma^\mu L\nu_\mu + \rho_{\mu\tau}\bar\nu_\mu\gamma^\mu L\nu_\tau)\bar\mu\gamma_\mu L\tau$$

$$\rho_{\mu\mu,\tau\mu} = \left(|Y^\nu_{22}|^2/m^2_{\Delta_2} \mp |Y^\nu_{22}|^2/m^2_{\Delta_3}\right)/4 \text{ and } \rho_{\mu\tau} = \rho_{\tau\mu} - |Y^\nu_{23}|^2/m^2_{\Delta_1}$$

$M_{\Delta i}$ are the Δ masses which are larger than m_W . $M_{total}(\tau \to \mu\nu\bar\nu)$

$$\left(\frac{\rho_{\tau\mu}}{2} - \frac{g^2}{4m_W^2} - \frac{\tilde g^2}{m^2_{Z'}}\right)\bar\mu\gamma_\mu\tau\bar\nu_\tau\gamma^\mu L\nu_\mu$$
$$-\left(\frac{\rho_{\tau\mu}}{2} - \frac{g^2}{4m_W^2}\right)\bar\mu\gamma_\mu\gamma_5\tau\bar\nu_\tau\gamma^\mu L\nu_\mu$$
$$+\left[\left(\frac{\rho_{\mu\tau}}{2} - \frac{\tilde g^2}{m^2_{Z'}}\right)\bar\mu\gamma_\mu\tau - \left(\frac{\rho_{\mu\tau}}{2}\right)\bar\mu\gamma_\mu\gamma_5\tau\right]\bar\nu_\mu\gamma^\mu L\nu_\tau$$
$$+(\rho_{\mu\mu})\bar\mu\gamma_\mu L\tau(\bar\nu_\mu\gamma^\mu L\nu_\mu + \bar\nu_\tau\gamma^\mu L\nu_\tau)\ . \qquad (17)$$

$$\rho_{\tau\mu}/2 - \tilde g^2/m^2_{Z'} = 0.$$

straints

$$\tau \to \mu\nu\bar\nu$$

Additional effective interactions

Exchange Δ^{++} at tree level will give

$$Br(\tau \to 3\mu) = \frac{m_\tau^5 \tau_\tau}{3(16\pi)^3}\left(\frac{|Y^\nu_{22}|^2}{m^2_{\Delta_2}} - \frac{|Y^\nu_{22}|^2}{m^2_{\Delta_3}}\right)^2 .$$

Exchange $\Delta^{+,\,++}$ at one loop level will genetrate a non-zero Δa_μ

$$-\frac{m_\mu^2}{16\pi^2}\left[\left(\frac{|Y^\nu_{22}|^2}{m^2_{\Delta_2}} + \frac{|Y^\nu_{22}|^2}{m^2_{\Delta_3}}\right) + \frac{|Y^\nu_{23}|^2}{m^2_{\Delta_1}}\right]$$

Also at tr

$$[(1+4s_W^2-(m_W^2/g^2)\delta_{\nu_\mu\mu})^2+(1-(m_W^2/g^2)\delta_{\nu_\mu\mu})^2$$
$$+(-(m_W^2/g^2)\delta_{\nu_\tau\mu})^2]\,/[(1+4s_W^2)^2+1], \qquad (19)$$

$\delta_{\nu_\mu\mu} = |Y^\nu_{22}|^2/m^2_{\Delta_2} + |Y^\nu_{22}|^2/m^2_{\Delta_3} + 4|Y^\nu_{23}|^2/m^2_{\Delta_1}$ $\quad$ $\delta_{\nu_\tau\mu} = |Y^\nu_{22}|^2/m^2_{\Delta_2} - |Y^\nu_{22}|^2/m^2_{\Delta_3}$

Numerical Analysis

For illustration taking Δ_2 and Δ_3 mass to be the same.

This choice eliminate potential $\tau \to \mu\gamma$ constraint,

also force vanishing $\tau \to \mu\mu\bar{\mu}$

Choose $\rho_{\tau\mu}/2 - \tilde{g}^2/m^2_{Z'} = 0.$

to reduce contribution to $\tau \to \mu\nu\bar{\nu}$

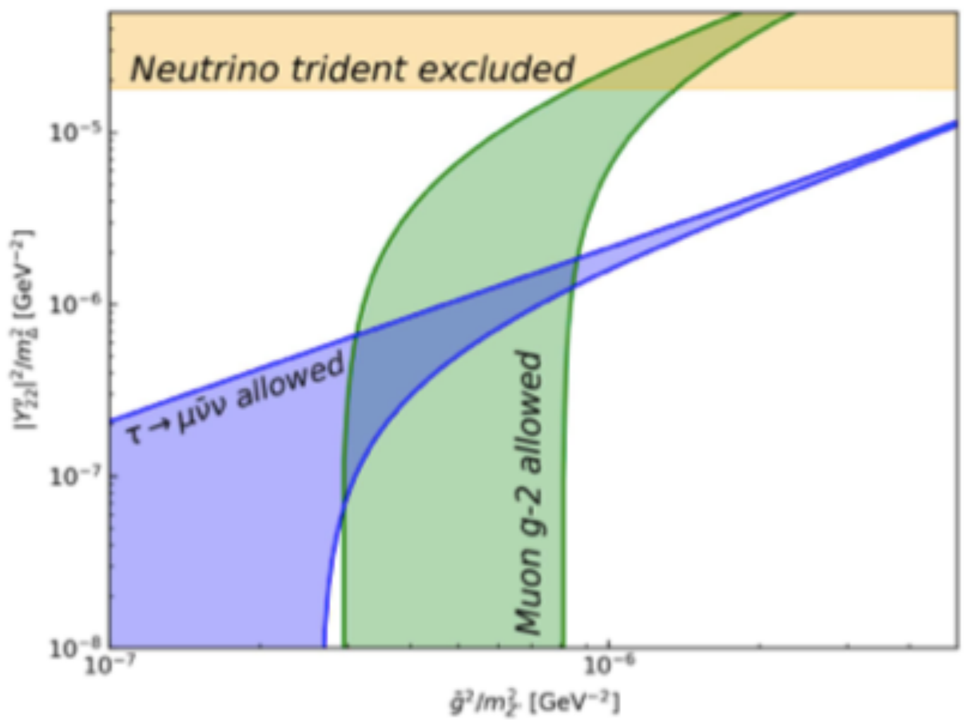
Also set $Y^{\nu}_{11,23} << Y^{\nu}_{22}$ avoid

potential danger from $\tau \to e\nu\bar{\nu}$

Constraints from

$$e^+e^- \to Z \to \mu^+\mu^-/\tau^+\tau^-/\nu\bar{\nu}$$

Are also iso satisfied.



There are large parameter space to allow the model to satisfy all known Constraints! Z' mass can be large!

4. Conclusions

In the simplest model $U(1)_{L\mu-L\tau}$, Z' interaction with charged leptons are diagonal.

If required to explain muon g-2 anomaly, trident neutrino scattering data constrain the Z' mass to be less than 300 MeV or so.

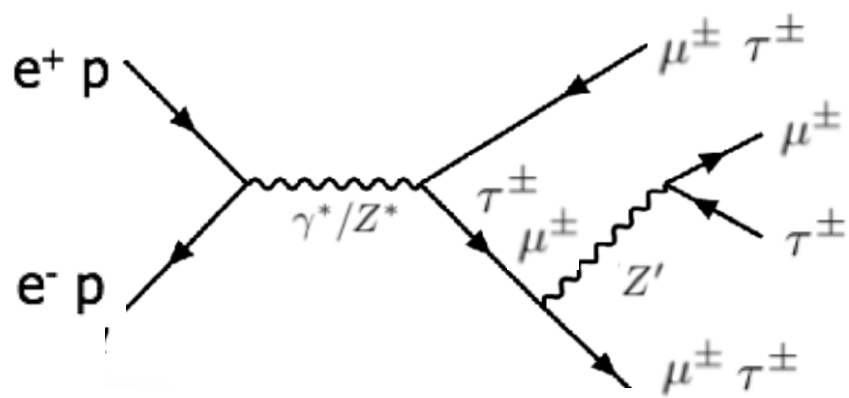
It is possible to evade trident neutrino constrain with modified model. There are large parameter space to allow the model to satisfy all known constraints!

Z' mass can be large!

Interesting smoking gun signature for modified model:

$$\gamma^*/Z^* \rightarrow \mu^\pm\tau^\mp + (Z' \rightarrow \mu^-\tau^+, \mu^+\tau^-).$$

e+ e- or pp ->



$\mu^\pm\mu^\pm\tau^\mp\tau^\mp$ states

Thank you for your attentions!