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Primordial Black Holes from Inflation

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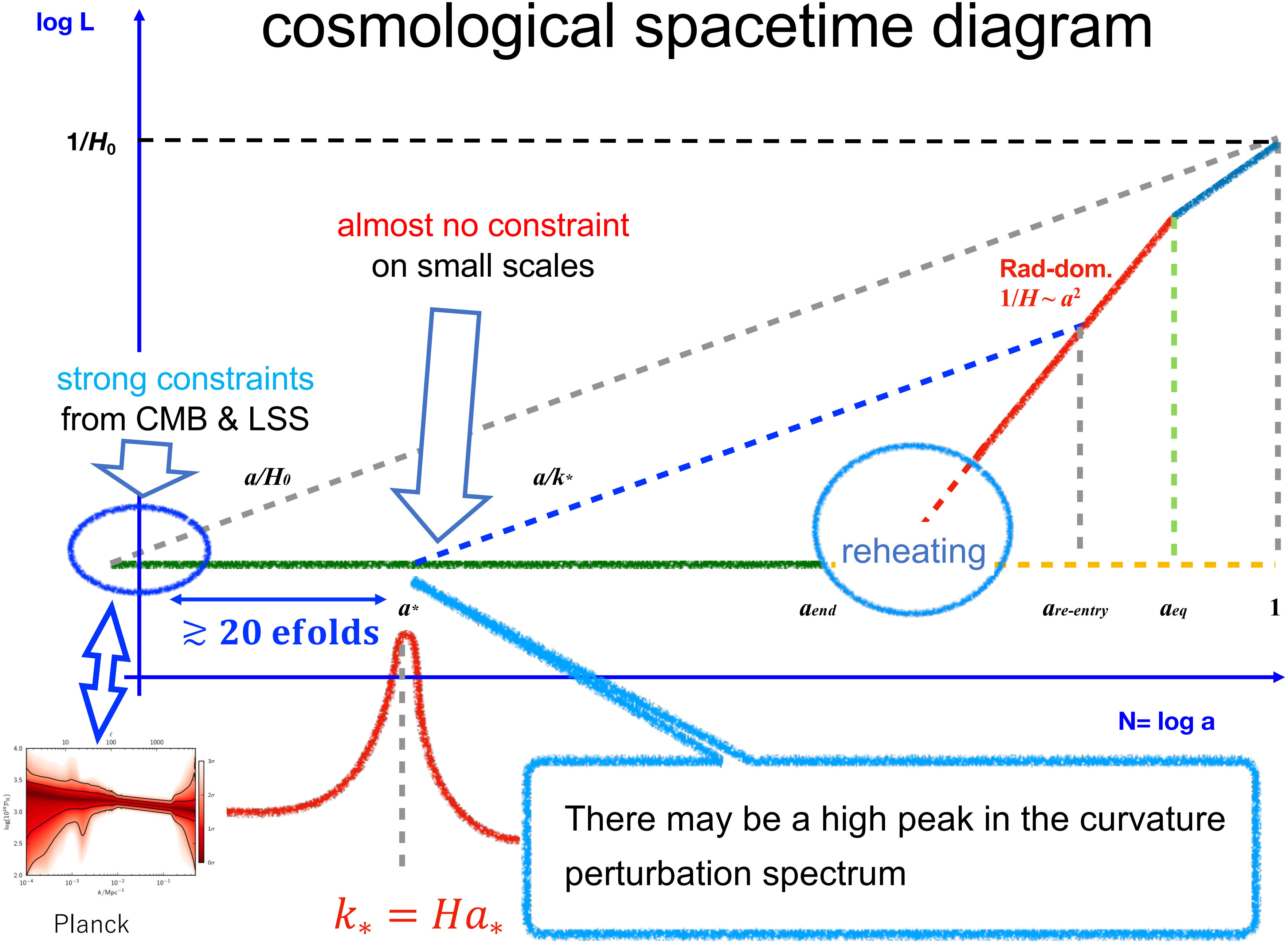
PBH formation

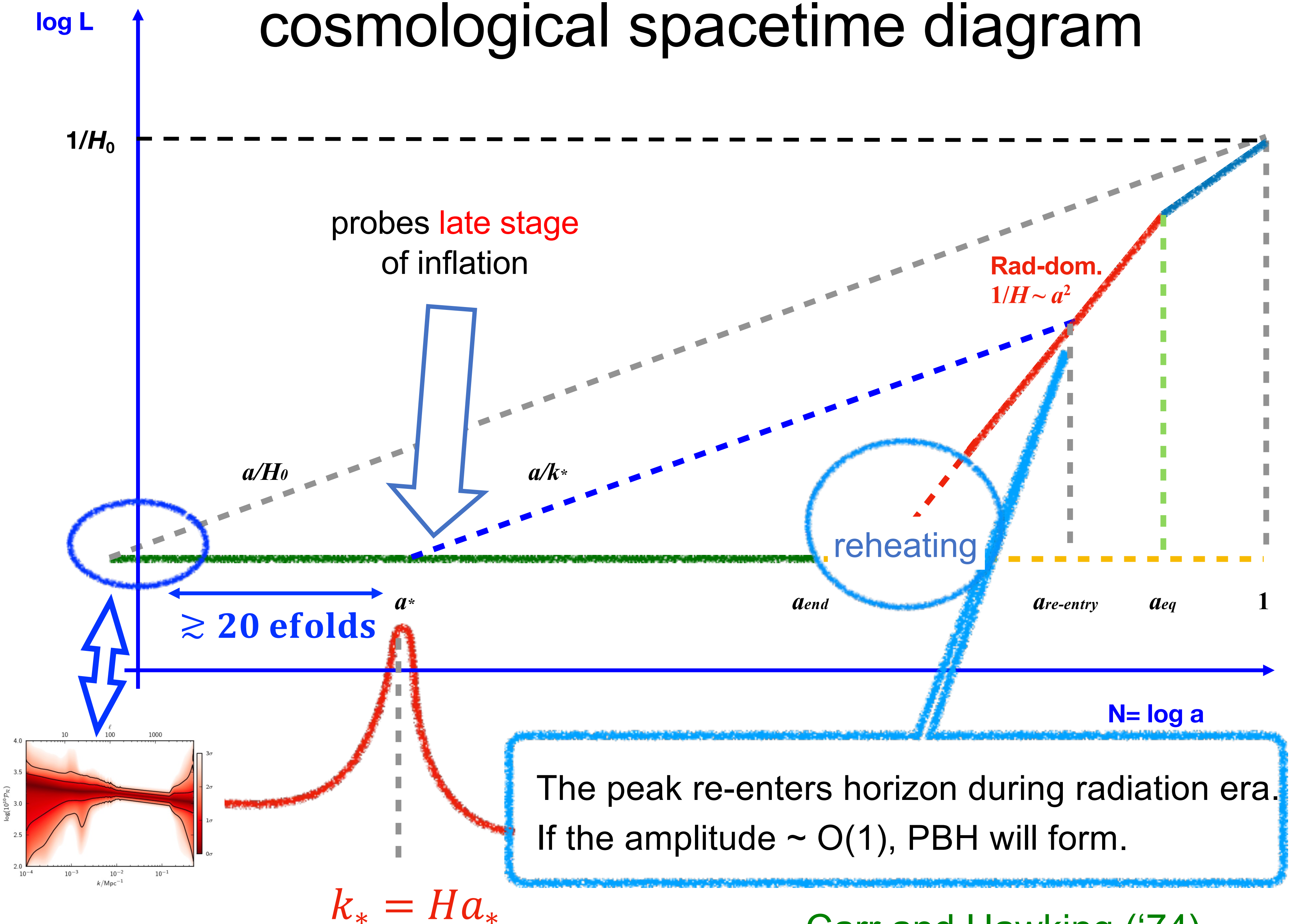
- conventional scenario -

originally proposed by Hawking ('71)

cosmological spacetime diagram



cosmological spacetime diagram



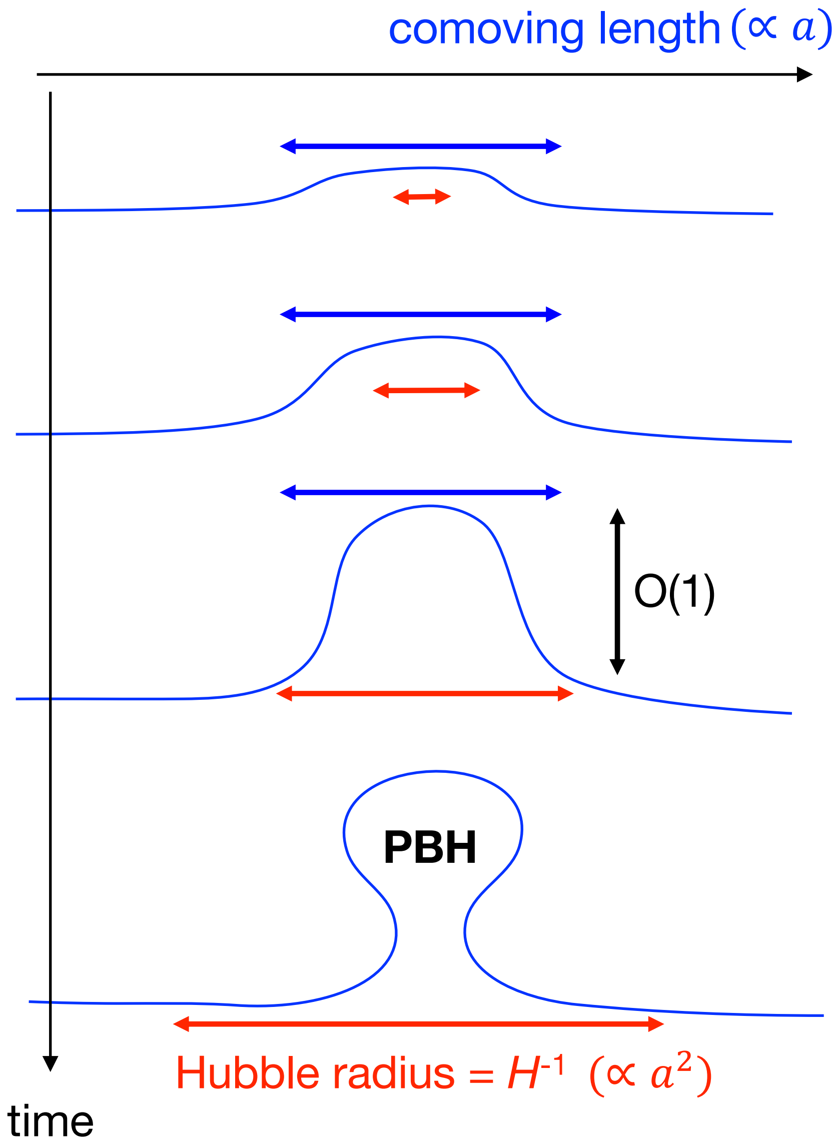
Carr and Hawking ('74)

Conventional PBH formation in a nutshell

- Primordial Black Holes (PBHs) are those formed in the very early universe, conventionally when the universe was radiation-dominated.
- Presumably they originate from a large positive curvature perturbation produced during inflation (which hence should be a rare event).
- For a BH to form during radiation dominance, the perturbation must be O(1) on the Hubble horizon scale.

$$\boldsymbol{M_{\mathrm{PBH}} \sim M_{\mathrm{horizon}}}$$

$$\sim \left(\frac{100\mathrm{MeV}}{T}\right)^2 M_\odot \sim \left(\frac{\ell}{1\mathrm{pc}}\right)^2 M_\odot$$



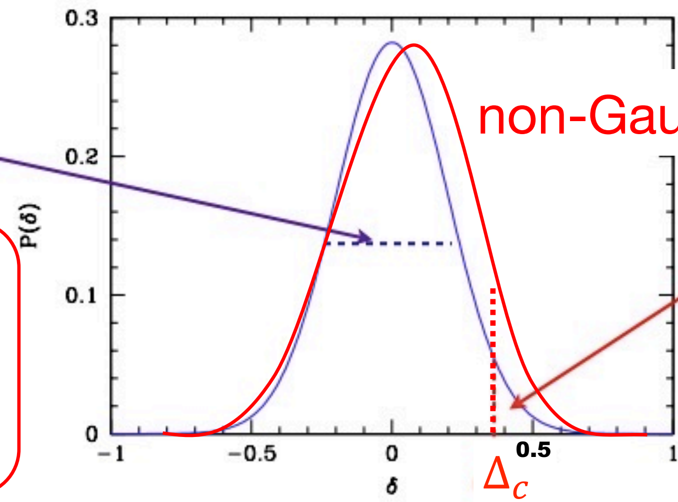
β: fraction of ρ that turns into PBHs

for Gaussian probability distribution

σ(M_H) (mass variance)
typical size of fluctuations

$$^{(3)}R \approx 6H^2 \frac{\delta\rho}{\rho}$$

$$^{(3)}R = O(H^2) \longleftrightarrow \frac{\delta\rho}{\rho} = O(1)$$



- When $\sigma_M \ll \Delta_c$, β can be approximated by exponential:

$$\beta \approx \sqrt{2/\pi}\,\frac{\sigma(M)}{\Delta_c}\exp\left(-\frac{\Delta_c^2}{2\sigma(M)^2}\right) \quad \Delta_c \equiv \left(\frac{\delta\rho_c}{\rho}\right)_{\rm crit} \sim 0.4$$

Carr, ApJ 201, 1 (1975), …

- Recent studies indicates enhanced production: $\Delta_c \sim 0.2$
  using peak theory

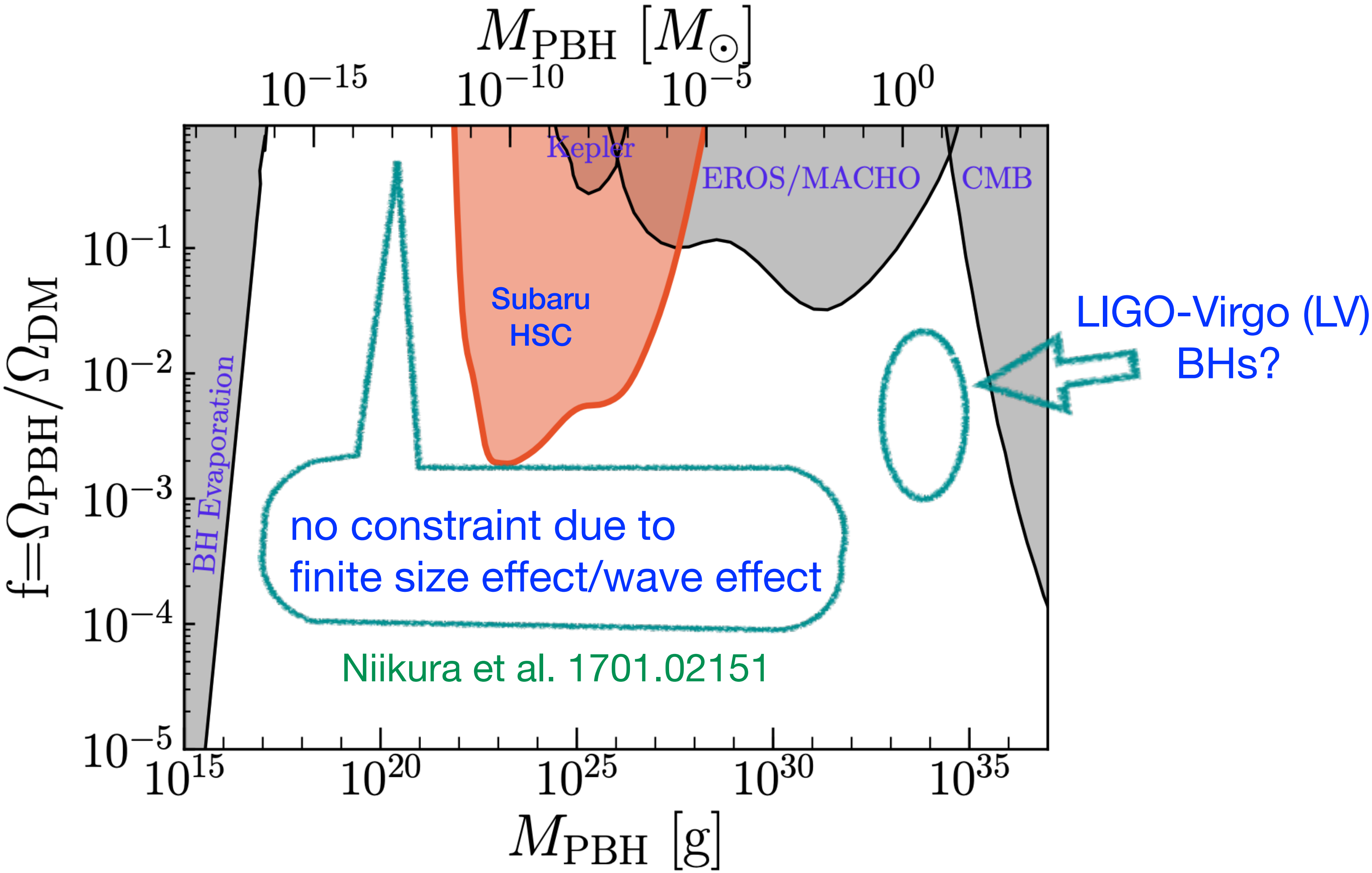
Yoo, Harada, Garriga & Kohri, 1805.03946

- Criterion using compaction function (C~GM/R) may be more relevant

Musco, De Luca, Franciolini & Riotto, 2011.03014

- Non-Gaussianity may significantly affect β

observational constraints



big window at $M_{\rm PBH} \approx 10^{17} - 10^{22}$g ⟷ $T_{\rm re\text{-}entry} \sim 10^4$ -10^8 GeV

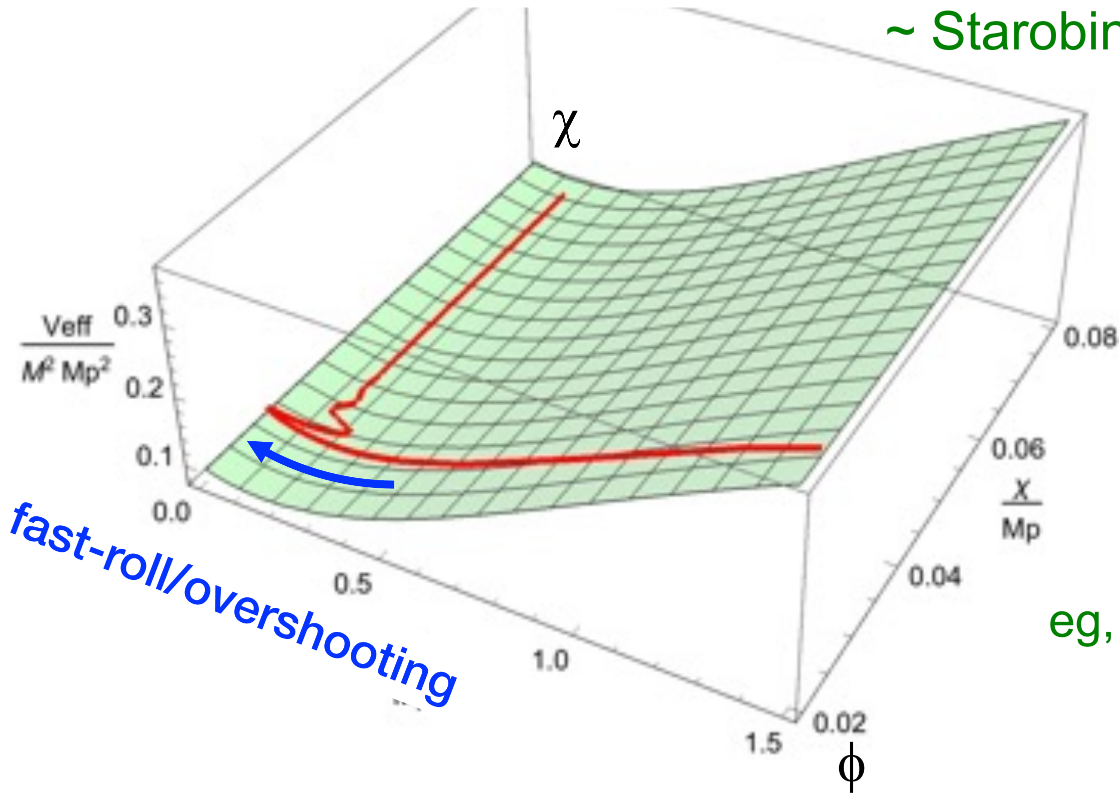
Inflation models

Model 1a: two-field inflation

Pi, Zhang, Huang & MS, 1712.09896

$$S_J = \int d^4x\sqrt{-g}\left\{\frac{M_{\rm Pl}^2}{2}\left(R+\frac{R^2}{6M^2}\right)-\frac{1}{2}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi - V(\chi) - \frac{1}{2}\xi R\chi^2\right\}.$$

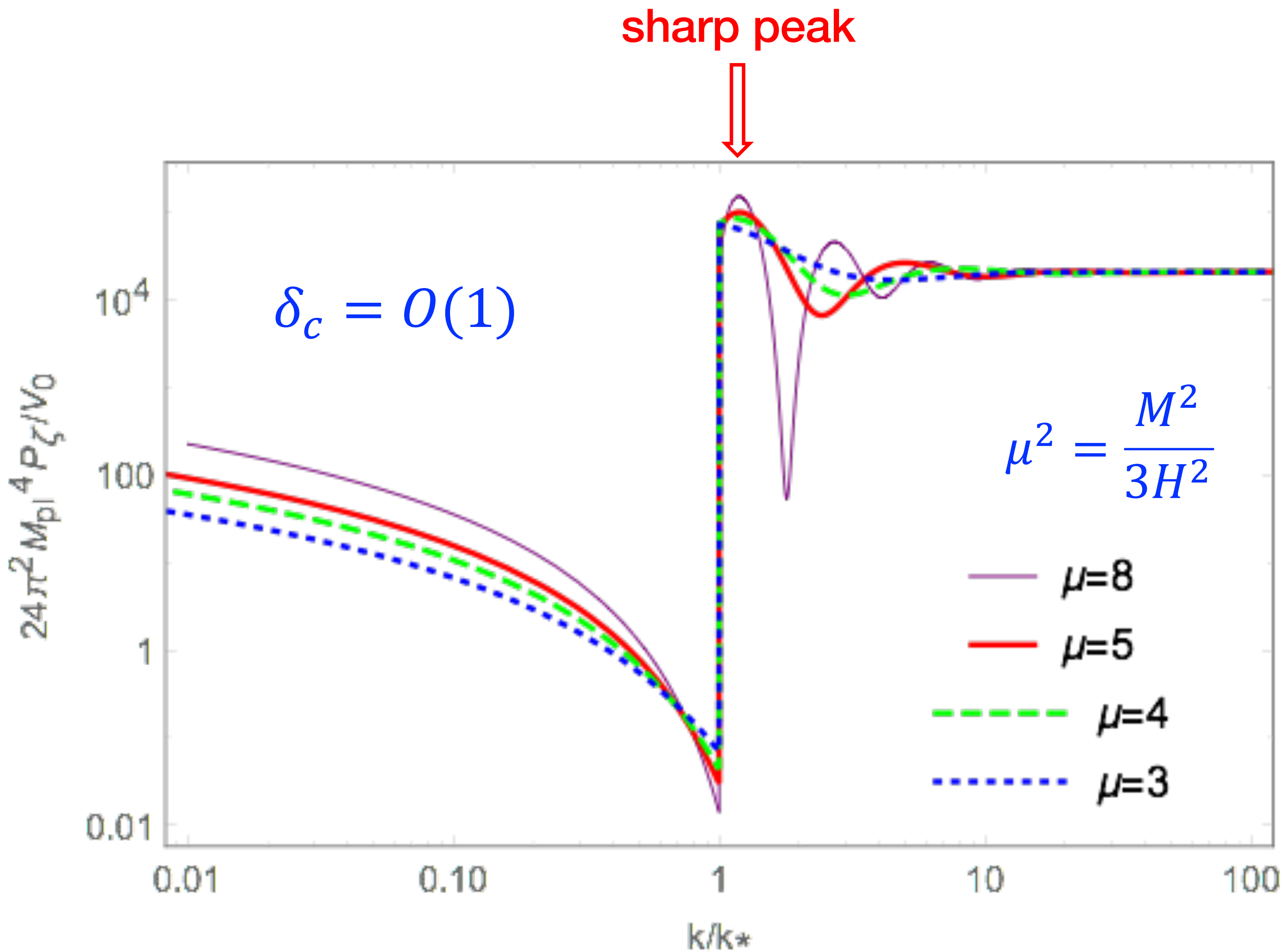
~ Starobinsky (scalaron) + curvaton



many 2-stage models can account for PBH formation

eg, Kawasaki et al., 1606.07631

- Scalaron ϕ becomes massive at the end of the 1st stage.
- Field χ plays the role of inflaton at the 2nd stage.



- 2-stage model can produce a sharp peak from the transition

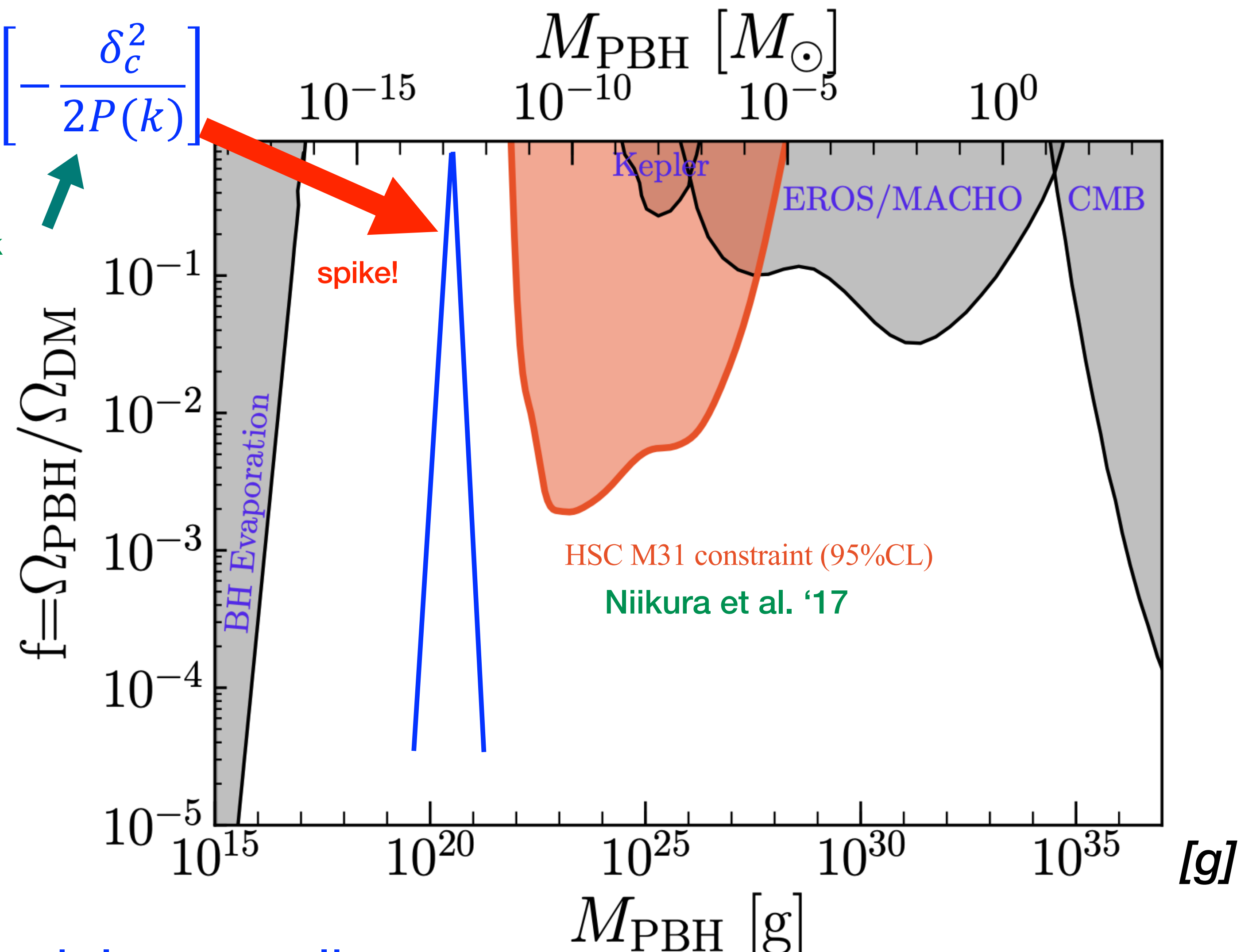
- non-Gaussianity is small in most 2-stage models.

PBH mass function

$\delta_c = O(1)$

$$f(M_{\rm PBH}) \propto \exp\left[-\frac{\delta_c^2}{2P(k)}\right]$$

sharp peak



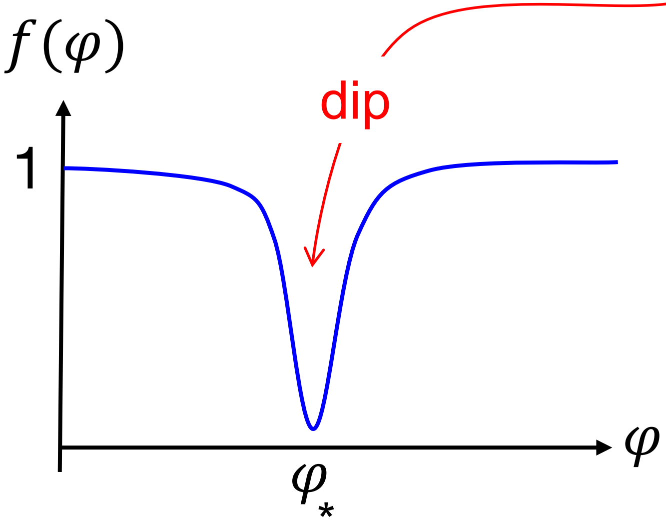
2-stage model can realize

PBH=CDM scenario with a monochromatic PBH mass

Model 1b: non-minimal curvaton

Pi & MS, 2112.12680

$$\mathscr{L} = -\frac{1}{2}(\partial\varphi)^2 - V(\varphi) - \frac{1}{2}f(\varphi)^2(\partial\chi)^2 - \frac{1}{2}m_\chi^2\chi^2 \quad (m_\chi^2 \ll H^2)$$

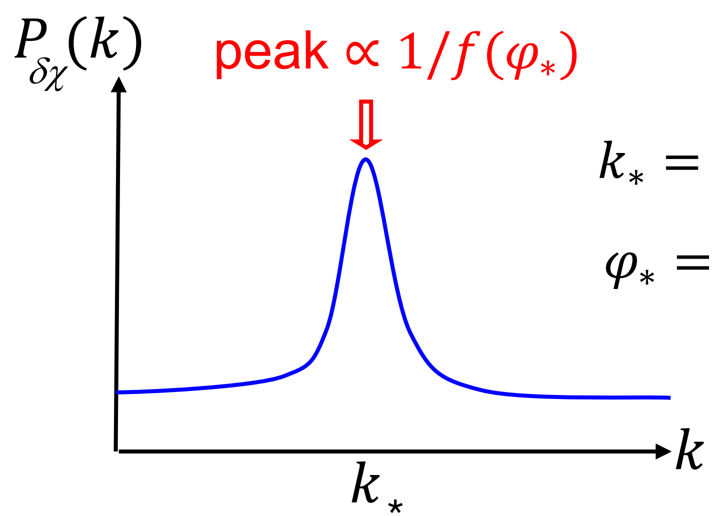


- assume f <<1 when $\varphi \sim \varphi_*$
- vacuum fluctuation: $f(\varphi)\delta x = \frac{H}{2\pi}$

$\Rightarrow$ $\delta x = \frac{H}{2\pi f}$

- $\delta\chi$ is enhanced at $\varphi = \varphi_*$

$\Rightarrow$ leads to PBH formation



$k_* = Ha(t_*)$

$\varphi_* = \varphi(t_*)$

Fully non-Gaussian curvature perturbation

$$e^{4\zeta} - \left[\frac{4r}{3+r}\left(1+\frac{\delta\chi}{\chi}\right)^2\right] e^{\zeta} + \left[\frac{3r-3}{3+r}\right] = 0$$

MS, Valiviita & Wands, astro-ph/0607627

ζ = curvature perturbation on uniform density slices

$r = \rho_\chi/\rho_{\rm tot}$ at epoch of curvaton decay

- PBH formation is fully non-perturbatively non-Gaussian:

  criterion $\zeta > \zeta_{\rm cr}$~0.5 gives a highly nonlinear expression in $\delta \equiv \delta\chi/\chi$

$\Delta(\mathrm{R})$: $\delta\rho/\rho$ smoothed over comoving scale R

$$\Delta(\mathrm{R}) \approx \frac{4k_*^2 R^2}{9}\zeta(R)$$

for spectrum peaked at $k=k_*$

- Power spectrum is well approximated by that of $\delta\chi^2$ for $r \ll 1$.

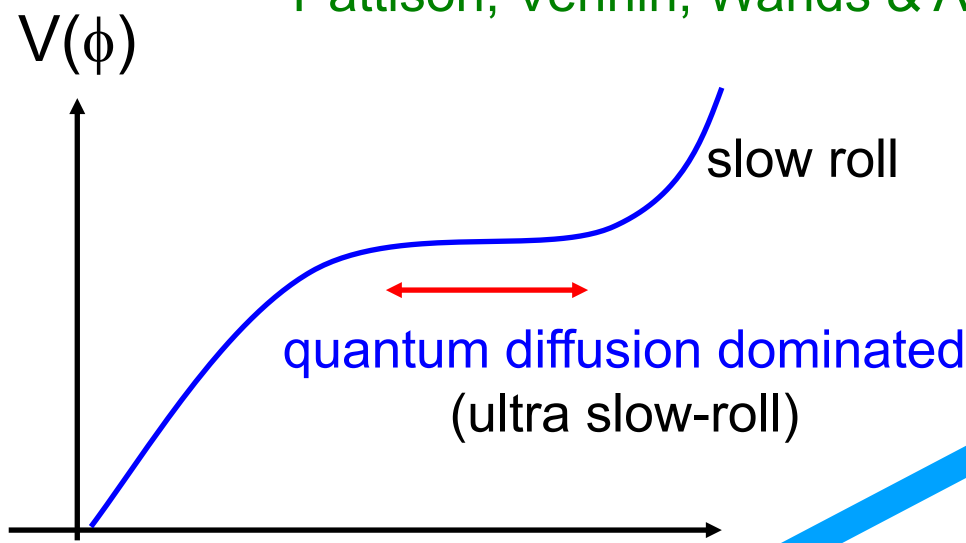
Pi & MS, 2112.12680

$$P_{\delta\chi}(k_*) \longrightarrow \zeta = \frac{\delta\chi}{\chi} + \frac{3}{4r}\left(\frac{\delta\chi}{\chi}\right)^2 \quad \text{for } r \ll 1 \longrightarrow P_\zeta(k_*)$$

perturbative approximation is valid

Model 2a: potential with inflection point

Pattison, Vennin, Wands & Assadullahi, 2101.05741, …



$$\delta\phi \sim \frac{\dot\phi}{H}$$

vacuum fluctuations $\frac{H}{2\pi}$

classical background motion

$$\mathcal{R} = \delta N = -\frac{H}{\dot\phi}\,\delta\phi \sim O(1)$$

tail of the PDF can be exponentially enhanced: $e^{-c\mathcal{R}}$ instead of $\exp(-c\mathcal{R}^2)$

➡ fully non-Gaussian PBH formation

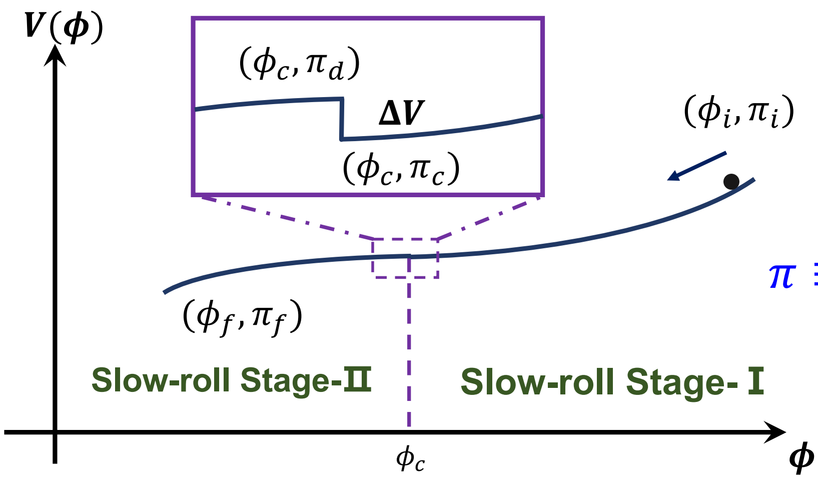
exponential tail is actually quite common in featured potential

Pi & MS, 2211.13932

Model 2b: Amplification by Upward Step

- One Small Step for an Inflaton, One Giant Leap for Inflation -

Cai, Ma, MS, Wang & Zhou, 2112.13836



$$\pi \equiv \frac{d\phi}{Hdt}$$

- energy conservation at the step:

$$\pi_d = -\sqrt{\pi_c^2 - 6\Delta V/V}$$

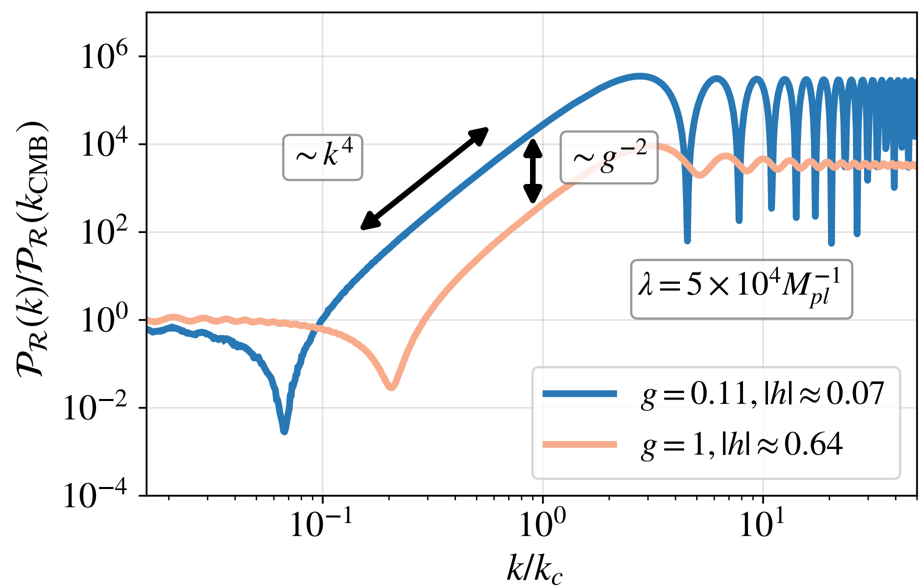
(in $M_{\text{Planck}} = 1$ units)

$$\pi_c = -\sqrt{2\varepsilon_c}$$

ε_c: SR parameter at $\phi = \phi_c$

even for a tiny step, $\Delta V << V$,
$P_R(k)$ is enhanced by $1/g^2$ if

$$g \equiv \frac{\pi_d}{\pi_c} \ll 1$$



non-perturbative non-Gaussianity at tail of distribution

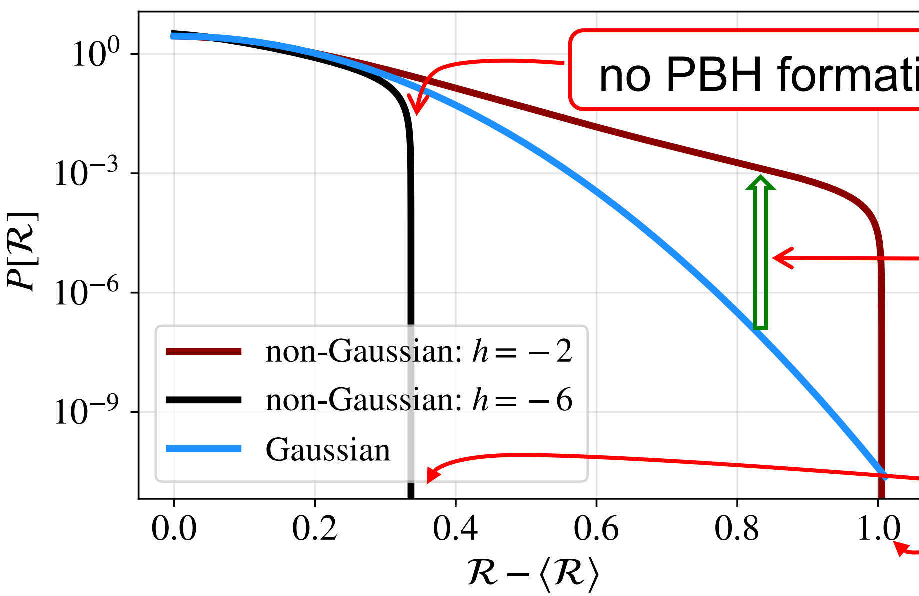
- perturbative non-Gaussianity is small if $-h \equiv \frac{6\sqrt{2\varepsilon_V}}{|\pi_d|} \ll 1$

$$\mathcal{R} = \mathcal{R}_G + \frac{|h|}{4}\mathcal{R}_G^2 + \frac{|h|^2}{8}\mathcal{R}_G^3 + \cdots \Longrightarrow \mathcal{P}(k) \approx \mathcal{P}_G(k)$$

power spectrum is given by Gaussian part

- tail of distribution is extremely non-Gaussian

$$\left|\frac{d\mathcal{R}_G}{d\mathcal{R}}\right| \qquad P[\mathcal{R}] = \frac{2-|h|\mathcal{R}}{\Omega}\exp\left[-\frac{\mathcal{R}^2(4-|h|\mathcal{R})^2}{32\sigma_\mathcal{R}^2}\right]$$

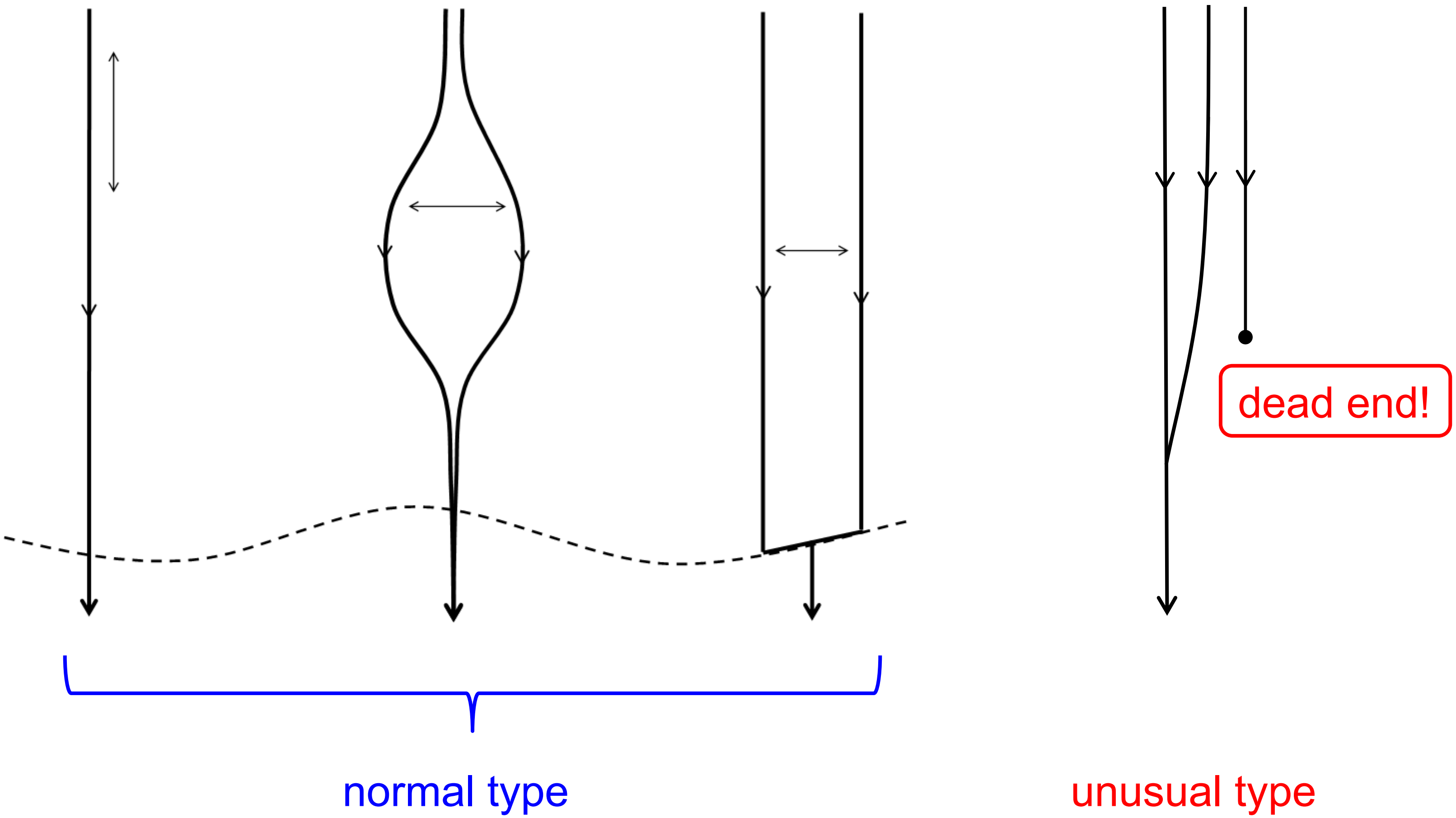


no PBH formation if $\mathcal{R}_{\rm cut} < \mathcal{R}_{\rm crit}$~0.5

exponential enhancement for $|h|$<O(1)

$\mathcal{R}_{\rm cut} \equiv \frac{2}{|h|}$ $\cdots$ ∃cutoff

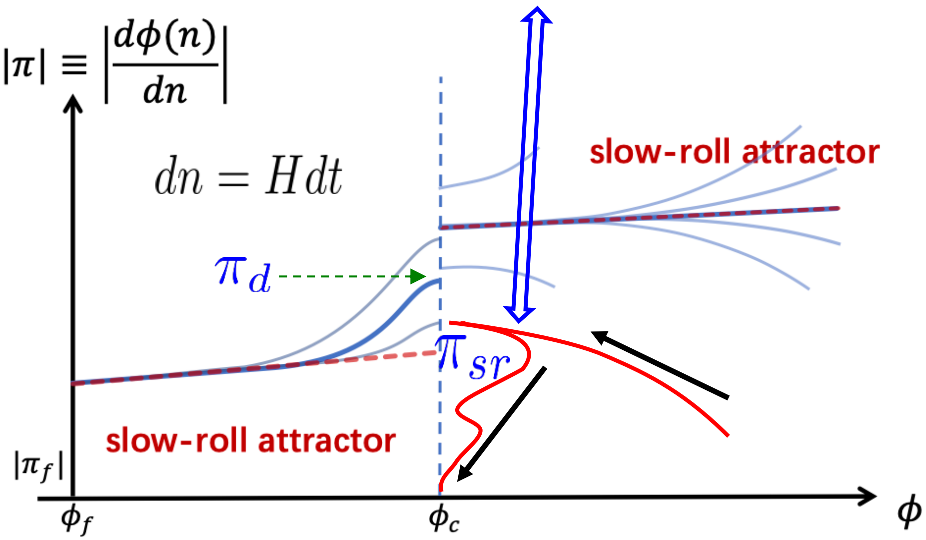
unusual type of field space trajectories

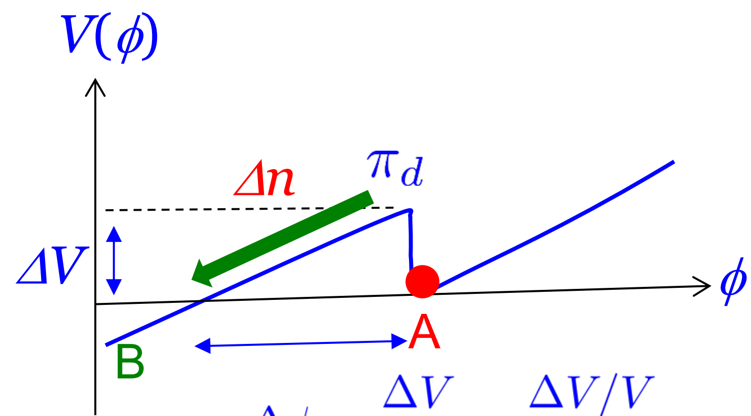


PBH formation during inflation

$$\mathcal{R} \simeq \frac{2}{|h|}\left(1-\sqrt{1-|h|\mathcal{R}_G}\right)$$ PDF cutoff at $\mathcal{R} = \mathcal{R}_{\rm cut} \equiv \frac{2}{|h|}$

⇨ trajectories that can't climb the step





$$\Delta\phi = \frac{\Delta V}{V'} = \frac{\Delta V/V}{\sqrt{2\epsilon}}$$

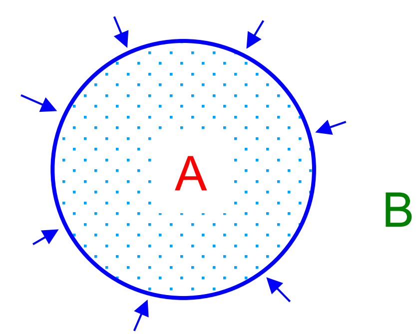
$$\Delta n \simeq \frac{\Delta\phi}{|\pi_{sr}|} = \frac{\Delta V/V}{\pi_{sr}^2}$$

⇧ # of e-folds region A expands

region stuck at $\phi=\phi_c$ will become PBH!

region A expands until $V(\phi)$ surrounding it becomes smaller than $V(\phi_A)=V_0$

$$M_{\rm BH} \simeq H^{-1} e^{3\Delta n}$$



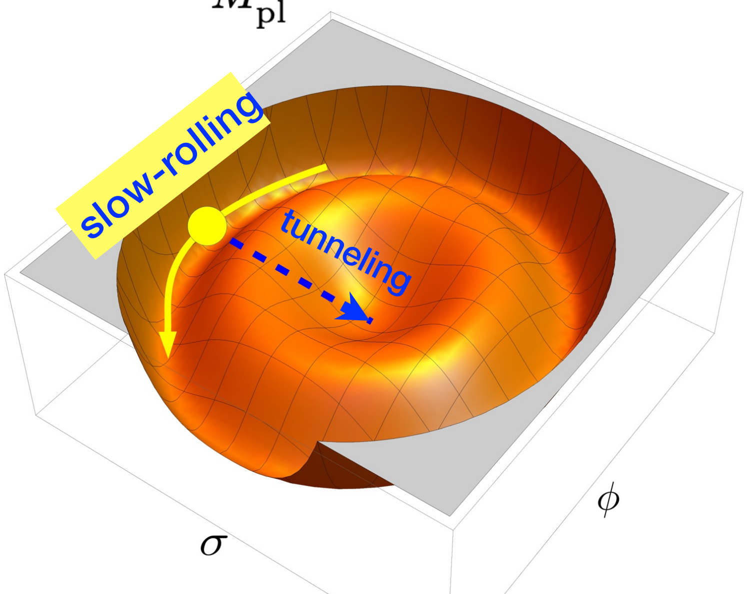
Model 4: PBH-as-MVP scenario

PBH formation during inflation due to vacuum tunneling
(not from curvature perturbation)

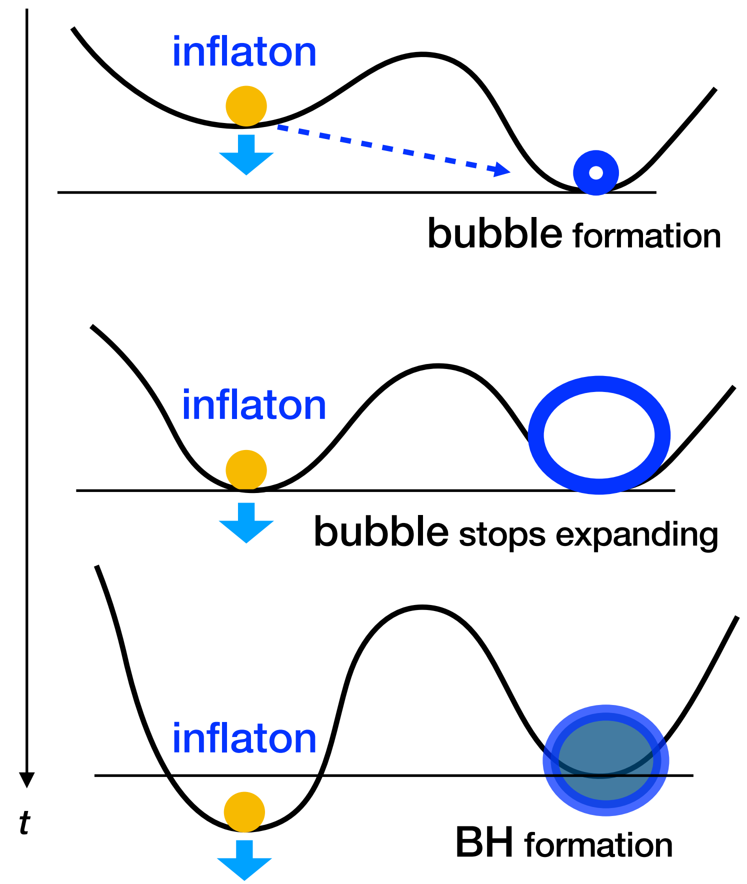
Garriga, Vilenkin & Zhang, 1512.01819, Deng & Vilenkin, 1710.02865,…

example:

$$V(\phi,\sigma) = m^2(\phi^2+\sigma^2) - a(\phi^2+\sigma^2)^2 + \frac{c}{M_{\rm pl}^2}(\phi^2+\sigma^2)^3 + gM_{\rm pl}^4 \sin\left(\frac{\phi}{fM_{\rm pl}}\right)$$

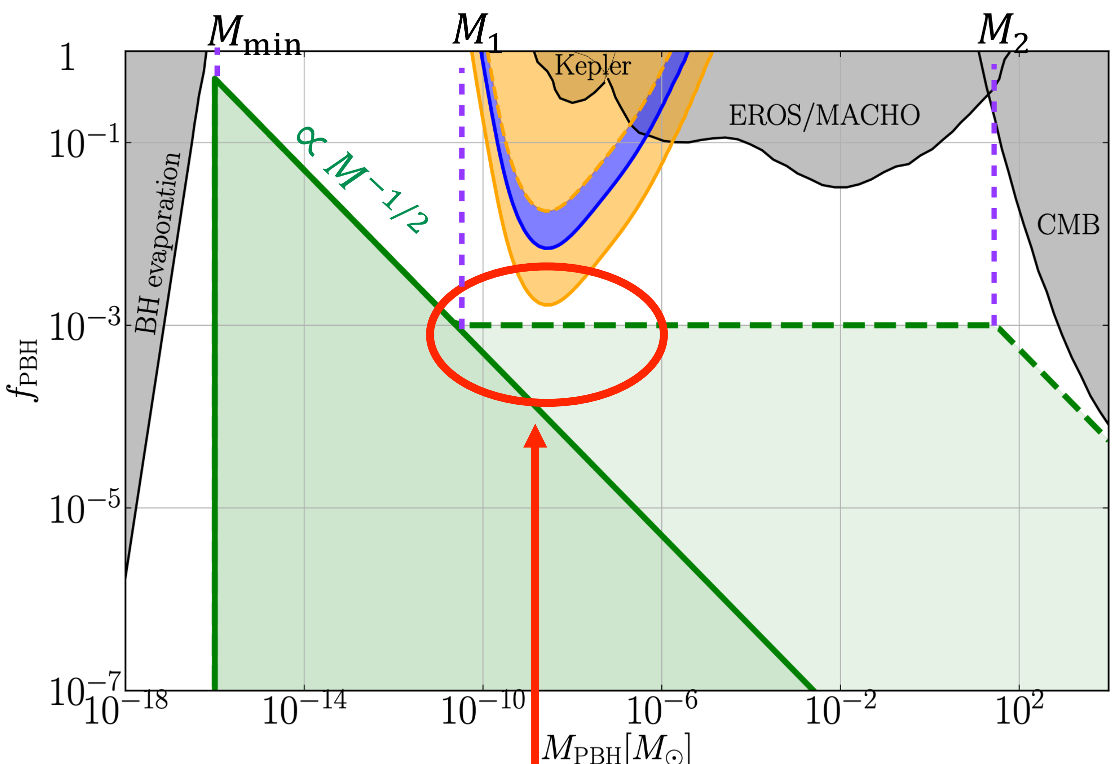


can probe multiverse!



Mass function

Kusenko, MS, Sugiyama, Takada, Takhistov & Vitagliano, 2001.09160



may be tested by Subaru HSC!

analysis is in progress

- for scale M re-entering horizon during radiation-dom stage

$$f(M) = \lambda \left(\frac{M}{M_{\rm min}}\right)^{-1/2} \quad : M_{\rm min} < M$$

$M \simeq M_{\rm min} \cdots$ CDM

- if there is an intermediate matter-dom stage

$$f(M) = \lambda \left(\frac{M_1}{M_{\rm min}}\right)^{-1/2} \quad : M_1 < M < M_2$$

$M \simeq M_2 \cdots$ LIGO BHs

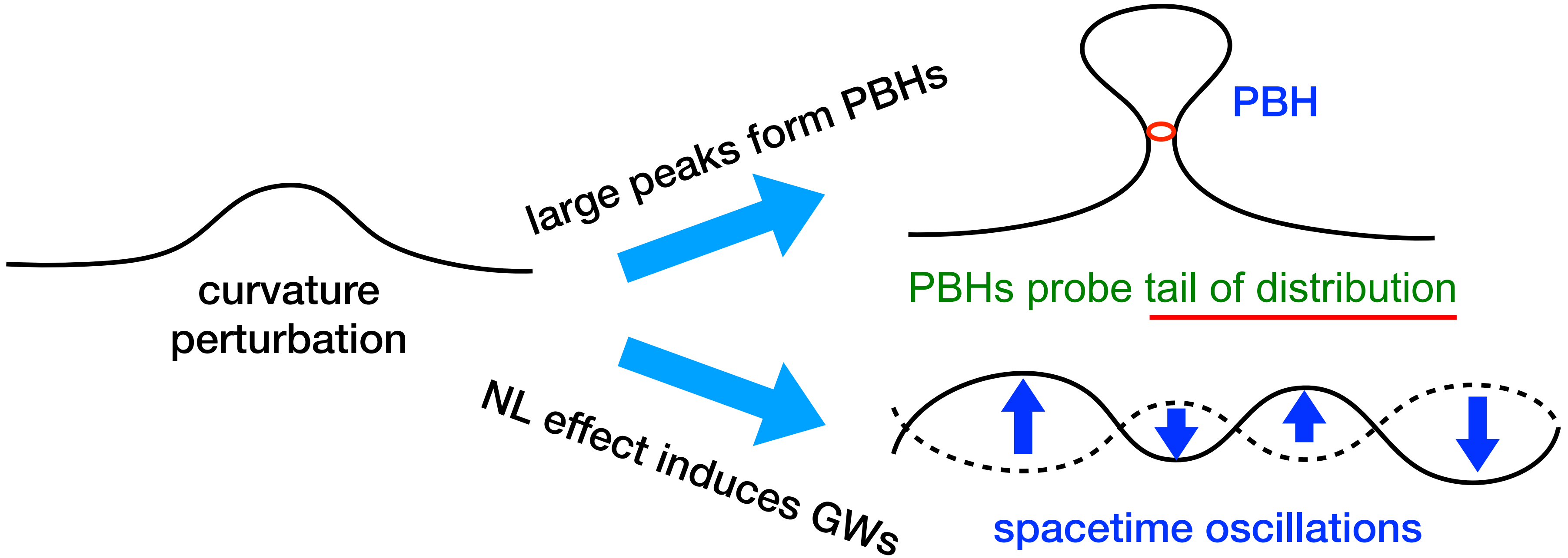
$$f(M) = \lambda \left(\frac{M_2}{M_1}\right)^{1/2} \left(\frac{M}{M_{\rm min}}\right)^{-1/2} : M_2 < M$$

$M \gg M_2 \cdots$ SMBHs

Implications to GW cosmology?

induced GWs

GWs can capture PBHs!



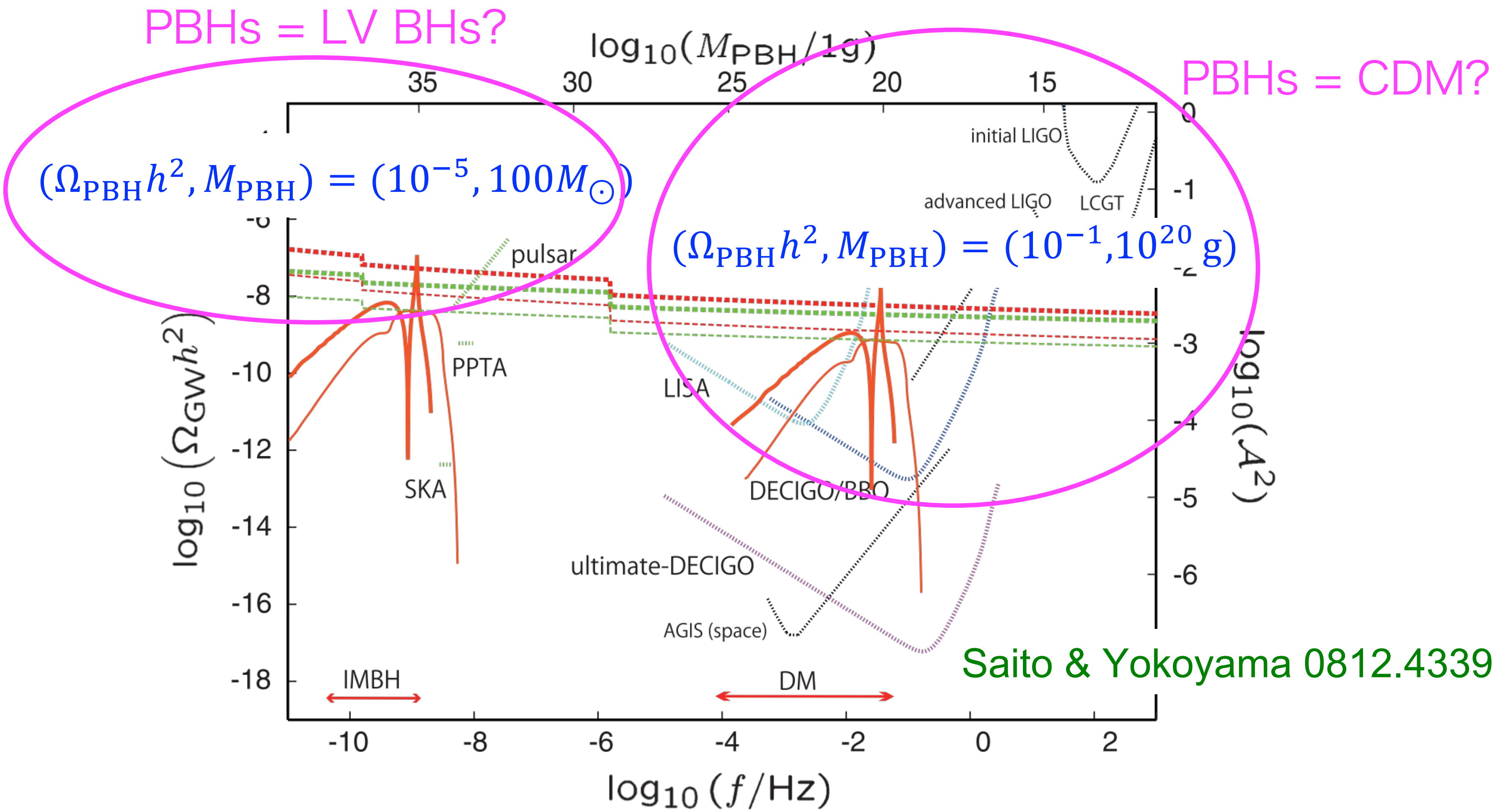
PBHs probe tail of distribution

iGWs probe power spectrum

PBHs = CDM with $M_{PBH} \sim 10^{21}$g generates GWs with $f \sim 10^{-3}$ Hz ⇨ Background GWs at LISA band

PBHs =LV BHs with $M_{PBH} \sim 10 M_{\odot}$ generates GWs with $f \sim 10^{-8}$ Hz ⇨ Background GWs at PTA band

GWs can test PBH scenario!

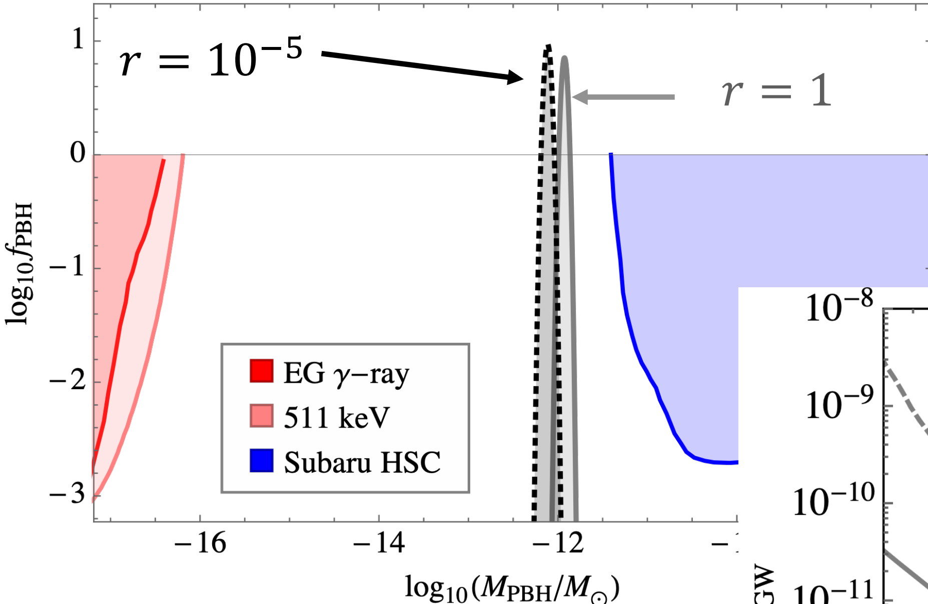


- PBHs = LV BHs scenario is already constrained by NANOGrav(PTA)

Cai, Pi, Wang & Yang 1907.06372

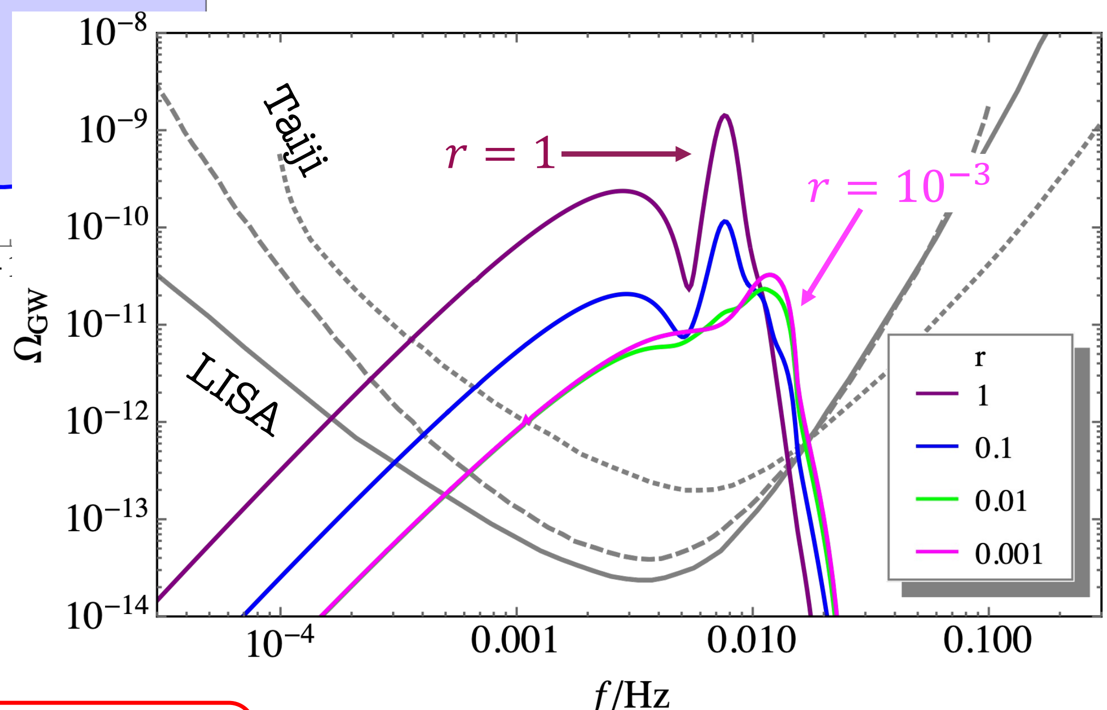
iGWs in non-minimal curvaton model

Pi & MS, 2112.12680



PBH mass function

width: $\frac{\Delta M}{M} \approx 0.1$



iGW spectrum

PBH=CDM scenario can be clearly verified or falsified by LISA!

summary

- various inflation models can lead to PBH formation
- late stage of inflation can be probed by PBHs and the associated secondary/induced GWs
- PBHs may be formed during inflation
- PBHs may play central roles in GW cosmology