Korea Institute for Advanced Study

KIAS Seoul

13 April, 2023

# 1. Status of proton radius puzzle.
# 2. QED radiative corrections for accelerator neutrinos

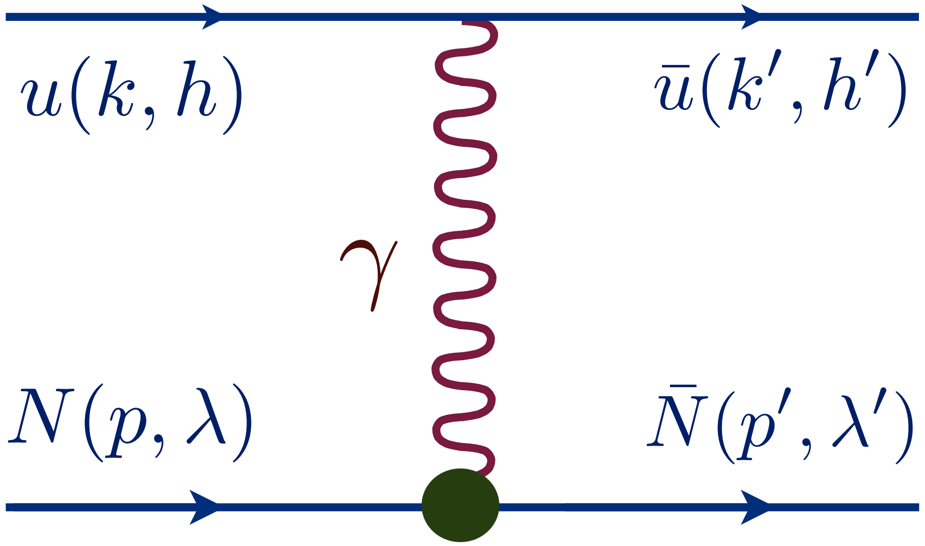
Oleksandr (Sasha) Tomalak

LA-UR-23-22299

# Proton radius puzzle

# 2010-2023

# Tool to explore the proton structure




photon-proton vertex

$$\Gamma^{\mu}(Q^2) = \gamma^{\mu} F_D(Q^2) + \frac{i\sigma^{\mu\nu} q_{\nu}}{2M} F_P(Q^2)$$

Dirac and Pauli form factors

lepton energy $\omega$

momentum transfer $Q^2 = -(k - k')^2$

1ɣ amplitude

$$\mathrm{T} = \frac{e^2}{Q^2} \left(\bar{u}\,(k', h')\,\gamma_{\mu} u\,(k, h)\right) \cdot \left(\bar{N}\,(p', \lambda')\,\Gamma^{\mu}(Q^2) N\,(p, \lambda)\right)$$

# Form factors measurement

Sachs electric and magnetic form factors

$$G_E = F_D - \tau F_P \qquad G_M = F_D + F_P$$

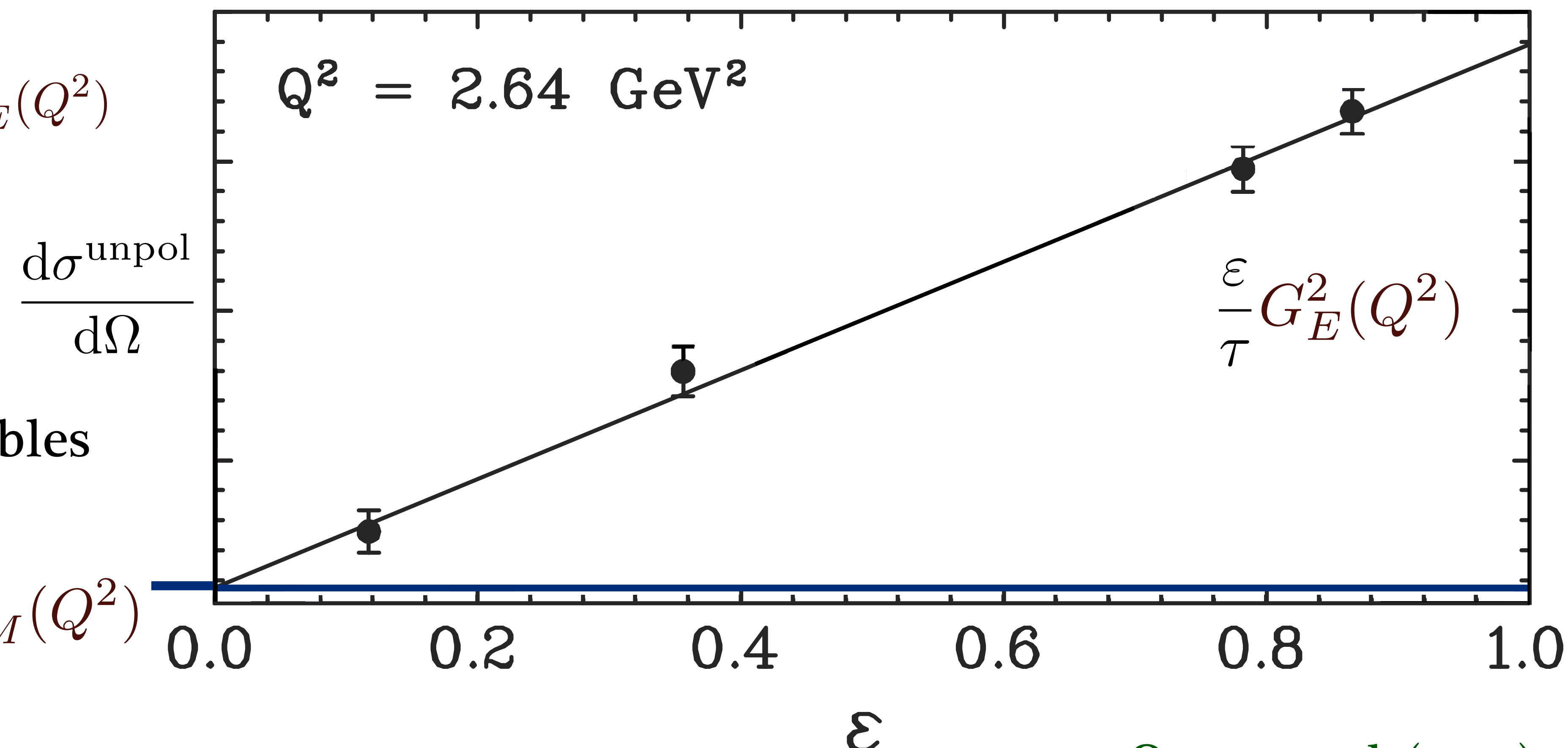
Rosenbluth separation

$$\frac{\mathrm{d}\sigma^{\mathrm{unpol}}}{\mathrm{d}\Omega} \sim G_M^2(Q^2) + \frac{\varepsilon}{\tau} G_E^2(Q^2)$$

$$\tau = \frac{Q^2}{4M^2}$$

$\tau \quad \varepsilon$ kinematical variables

$\varepsilon \leftrightarrow \theta_{\mathrm{lab}}$




Qattan et al. (2005)

- Rosenbluth slope is sensitive to corrections beyond 1γ

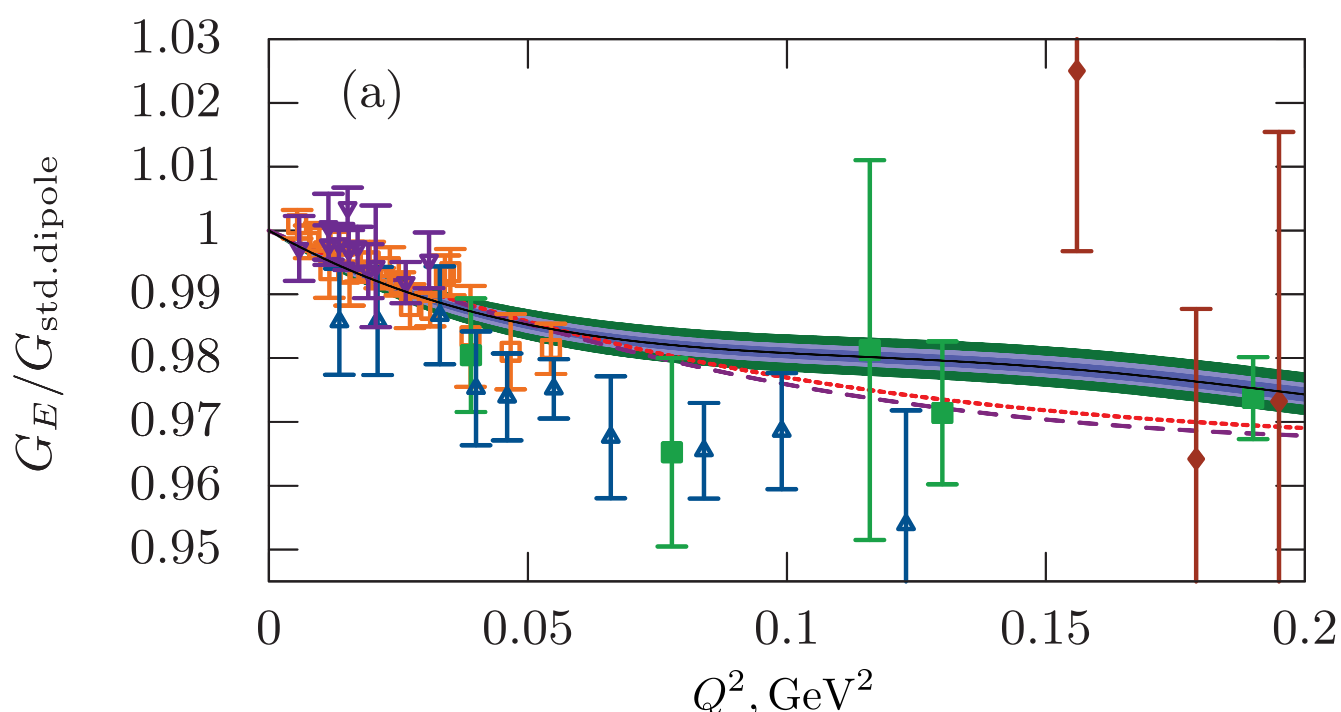
# Proton radius

electric charge radius

$$< r_E^2 > \equiv -6 \left. \frac{\mathrm{d}G_E(Q^2)}{\mathrm{d}Q^2} \right|_{Q^2=0}$$

- ep elastic scattering

$r_E = 0.879 \pm 0.008$ fm




A1@MAMI: J. C. Bernauer et al. (2014)

$0.003 \text{ GeV}^2 < Q^2 < 1 \text{ GeV}^2$

$$G_{std.dipole} = \frac{1}{\left(1 + \frac{Q^2}{\Lambda^2}\right)^2}$$

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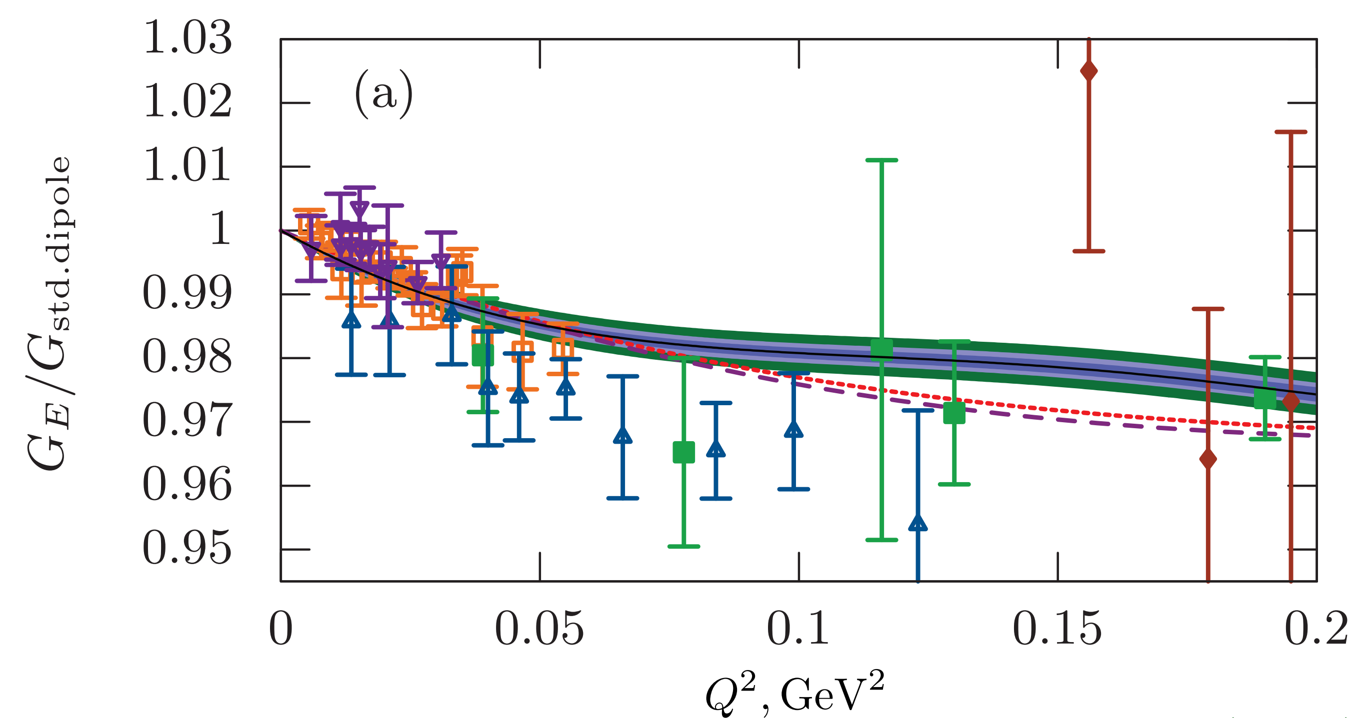


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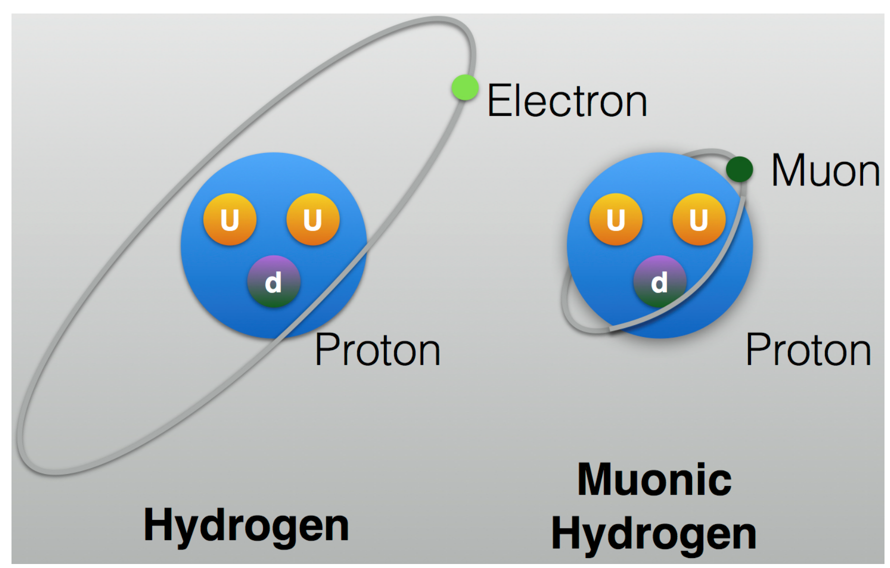
- atomic spectroscopy $\nu_{\mathrm{nS}} = -\frac{R_\infty}{n^2} + \frac{c_r m_r^3 < r_E^2 >}{n^3} + \ldots$

H, D spectroscopy

$$r_E = 0.8758 \pm 0.0077 \text{ fm}$$

CODATA 2010




μH Lamb shift

$$r_E = 0.8409 \pm 0.0004 \text{ fm}$$

CREMA (2010, 2013)

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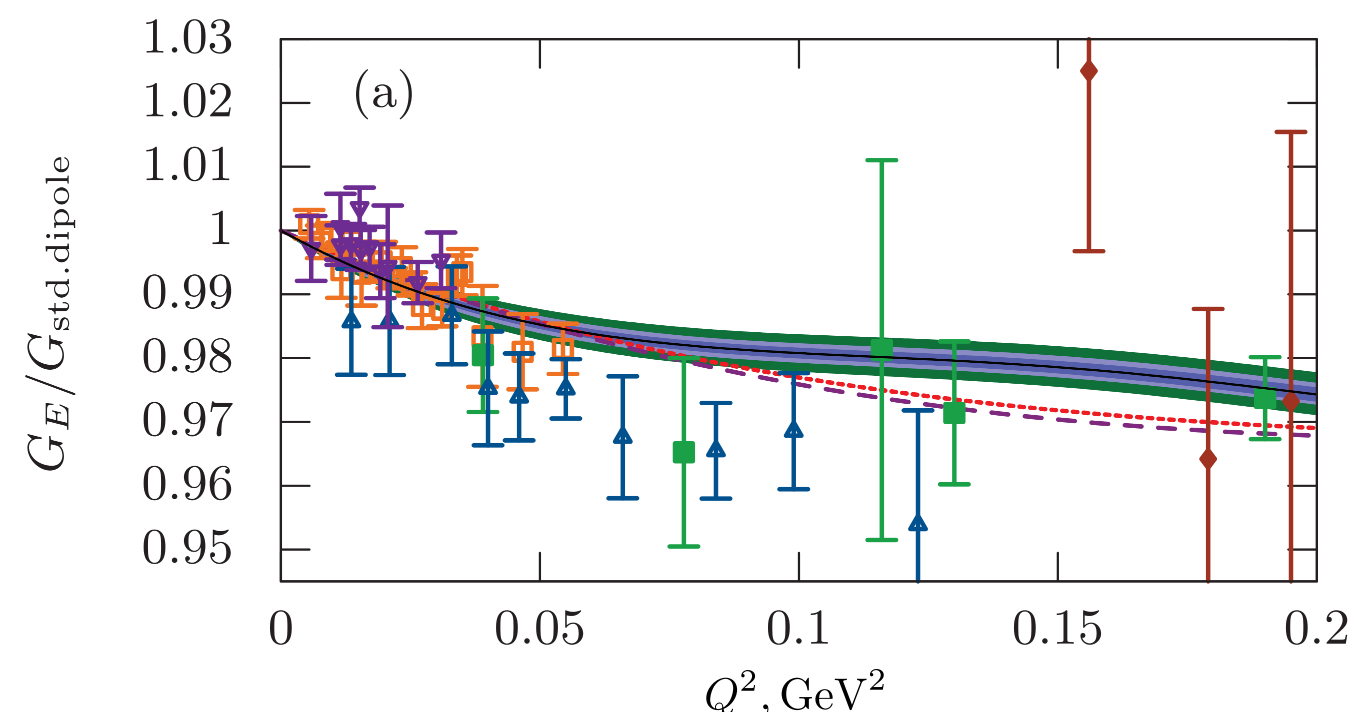


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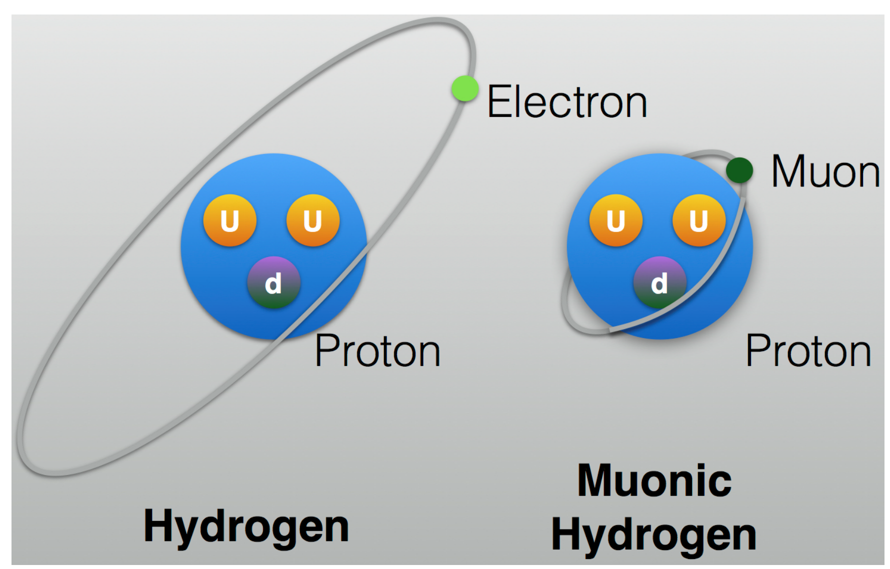


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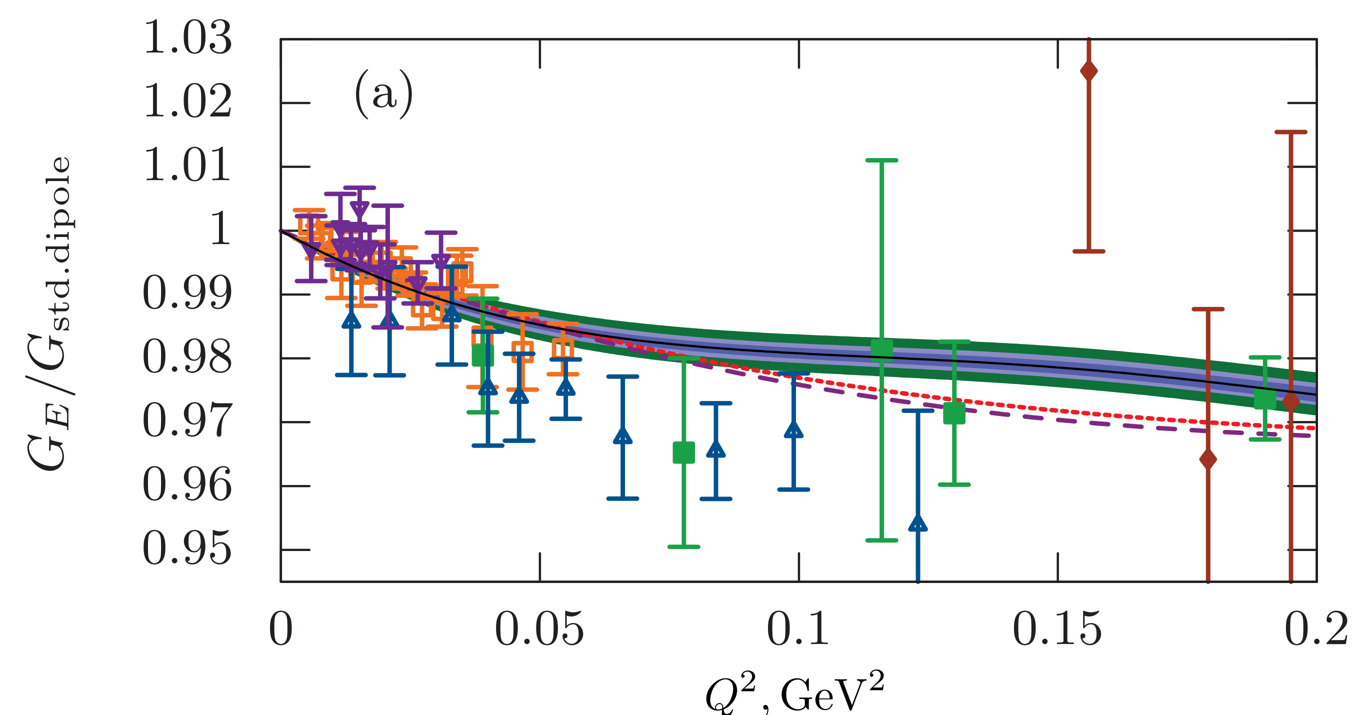


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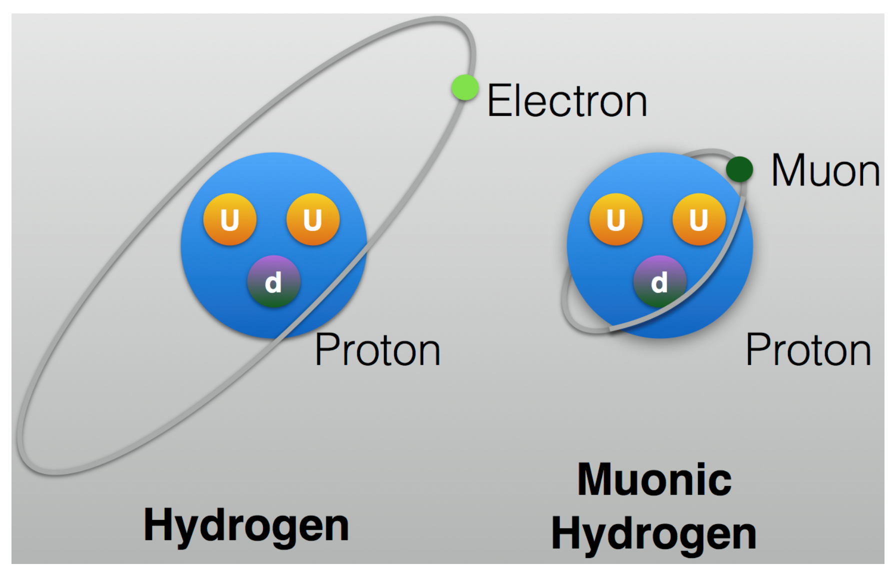


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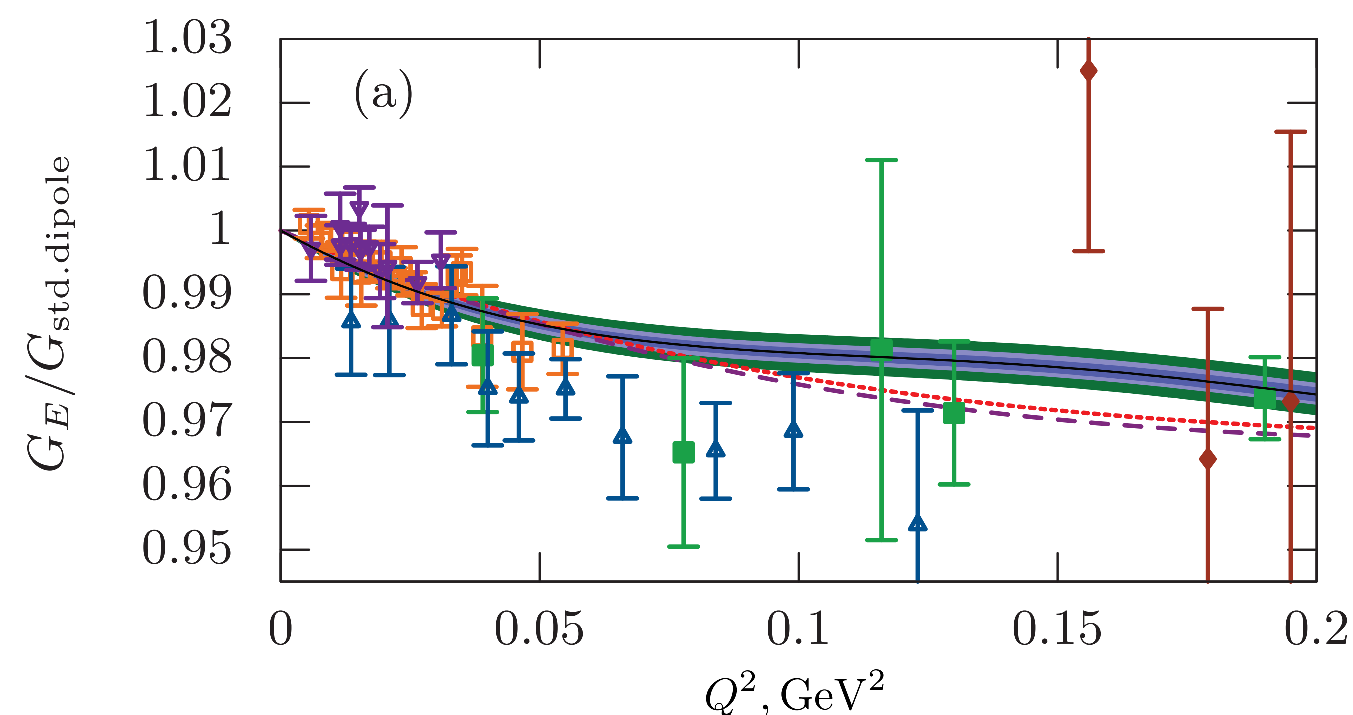


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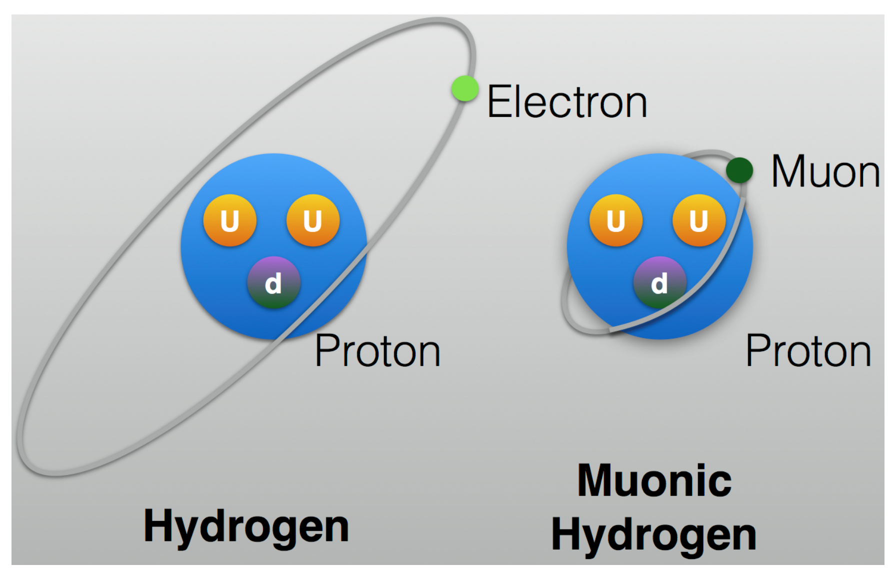


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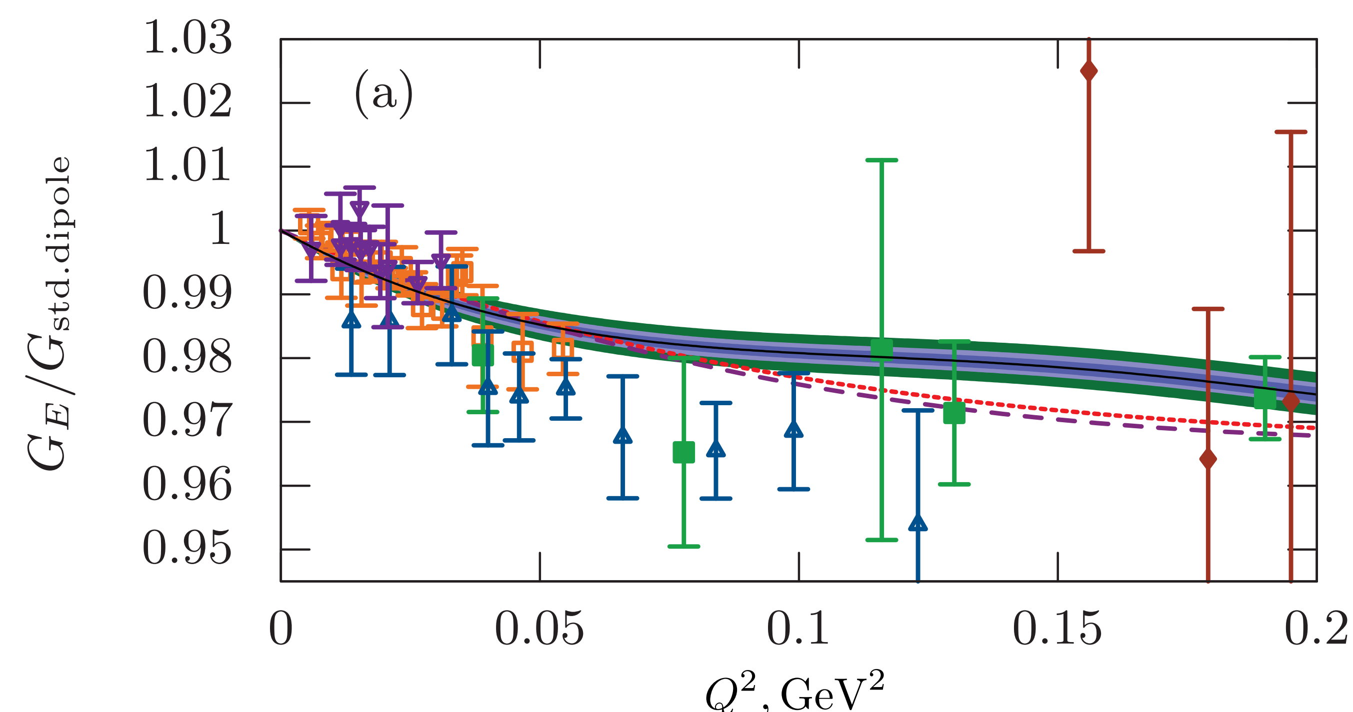


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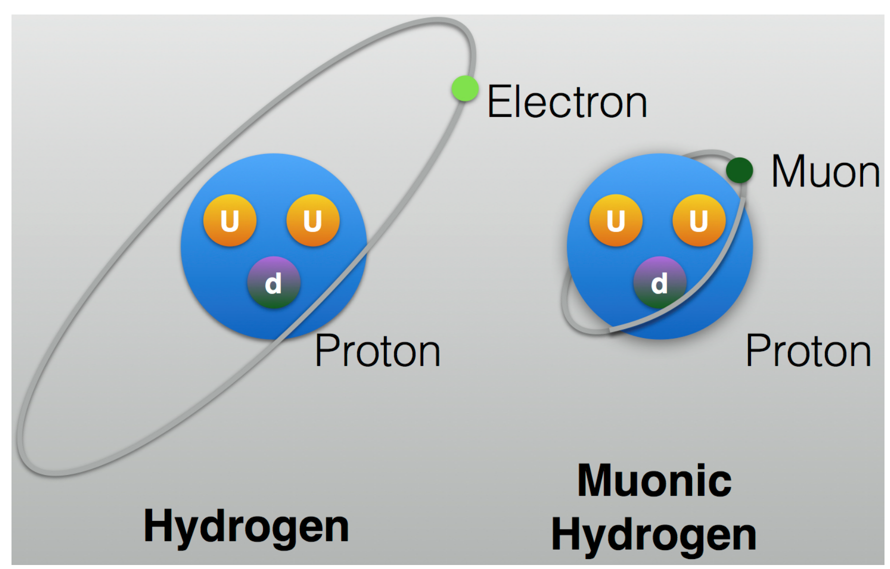


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eH 1S-3S (LKB, Paris, 2018)

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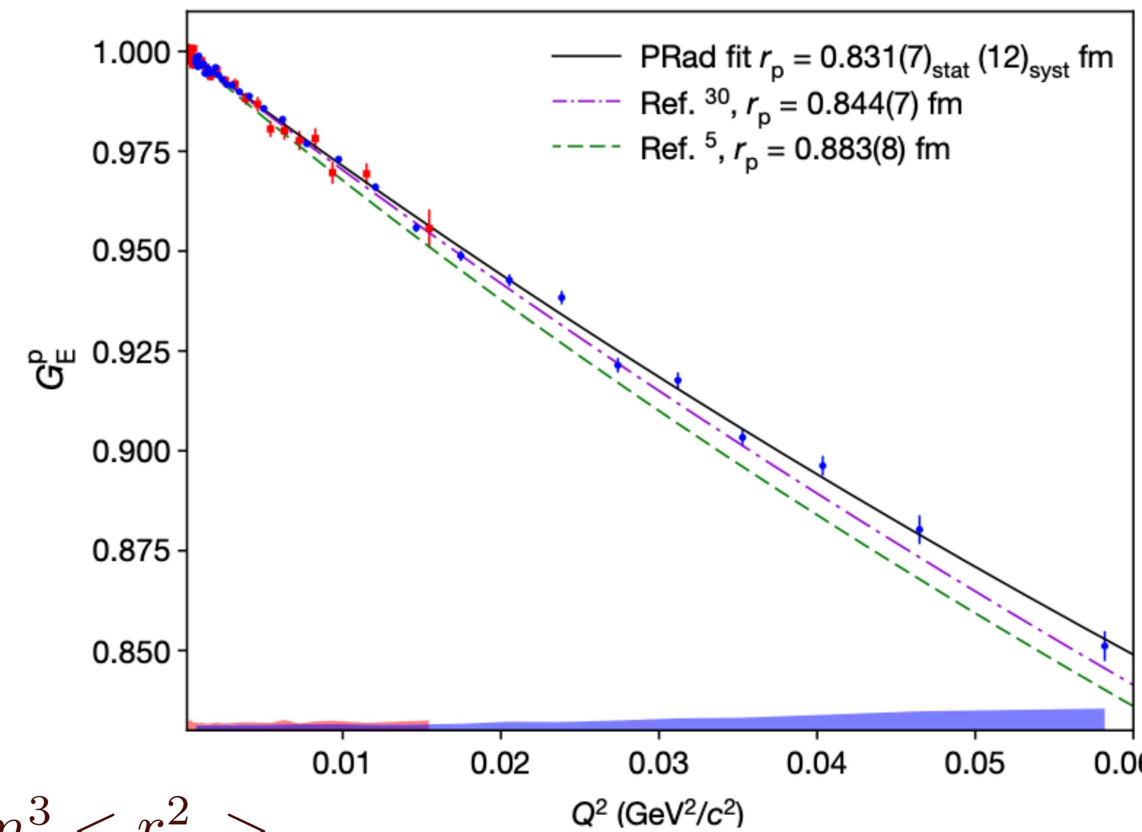


PRad (JLab, 2019)

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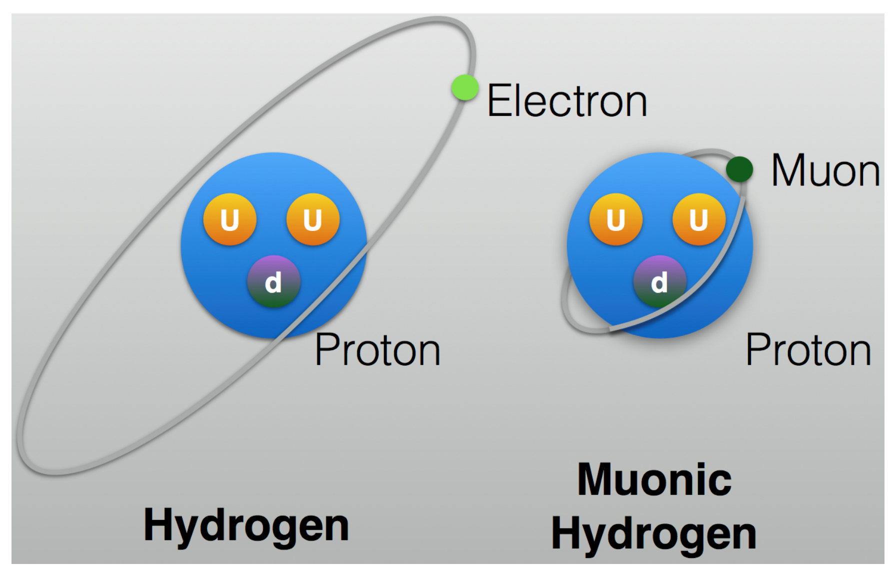
H, D spectroscopy

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CODATA 2010

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CREMA (2010, 2013)

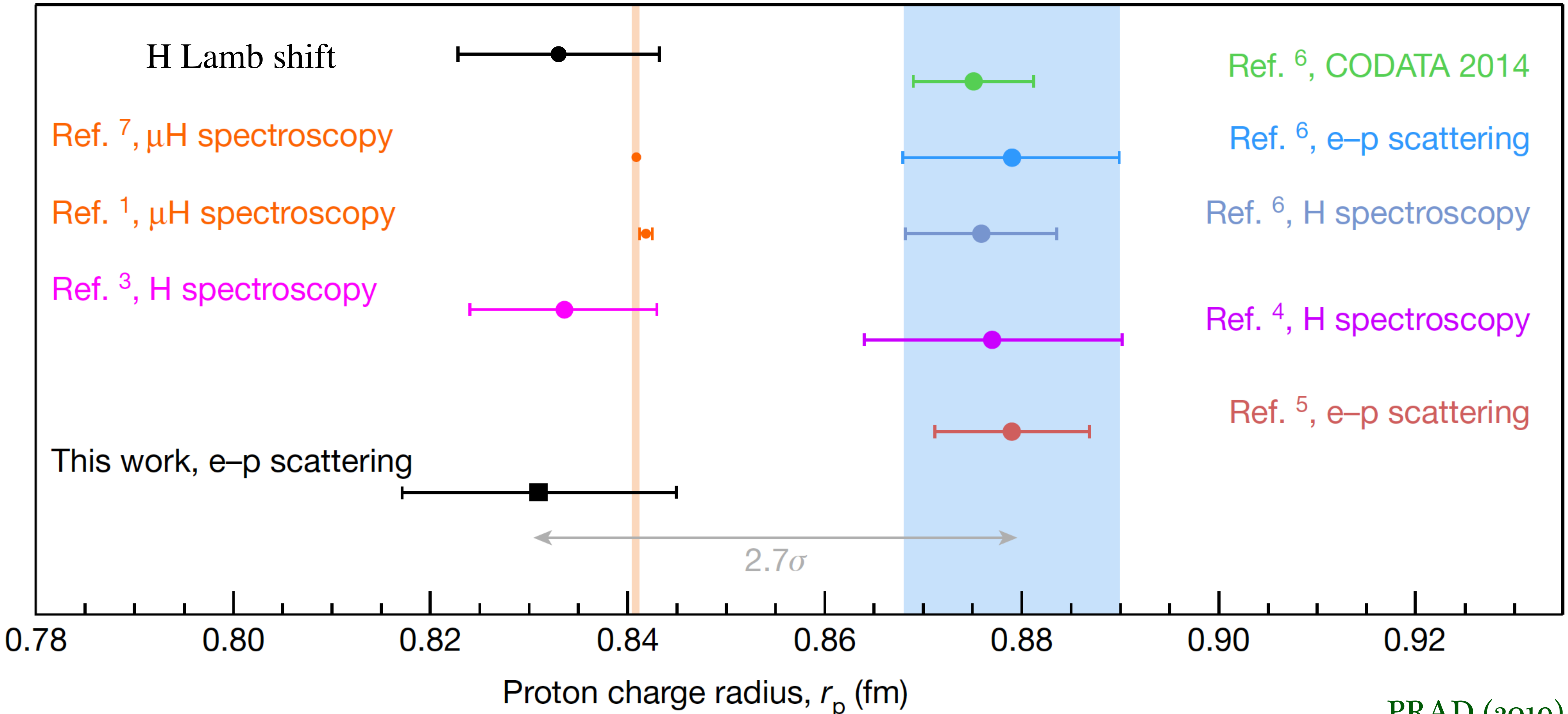
eH 1S-3S (LKB, Paris, 2018)

eH 2S-4P (Garching, 2017)

eH 2S-2P (York U., 2019)

data prefers smaller radius

# Proton radius puzzle




PRAD (2019)

- no puzzle in atomic spectroscopy !!!
- scattering data is not completely understood

# Possible solutions ?

- drawback in experiment: not a single one !

- drawback in theory: many groups reevaluated rad. corrections, $2\gamma$

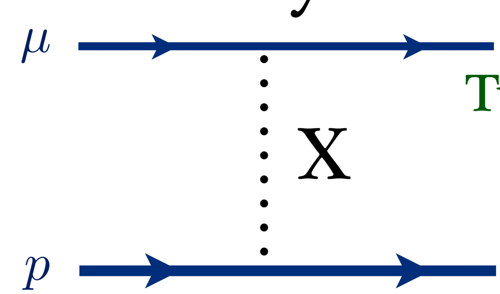
# Possible solutions ?

- drawback in experiment: not a single one !

- drawback in theory: many groups reevaluated rad. corrections, 2ɣ

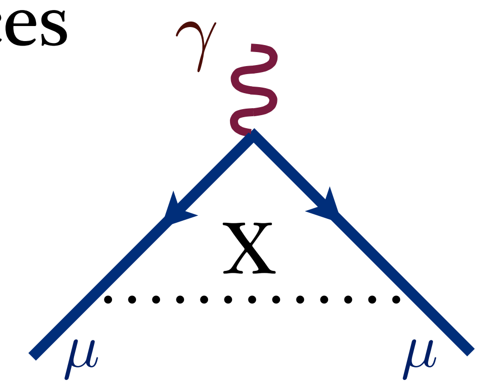
# BSM explanation ?

- lepton universality violating ~MeV muonic forces




Tucker-Smith Yavin (2010)

Barger et al. (2011)




- respect $(g\text{-}2)_\mu$, nuclear and particle physics constraints: fine tuning

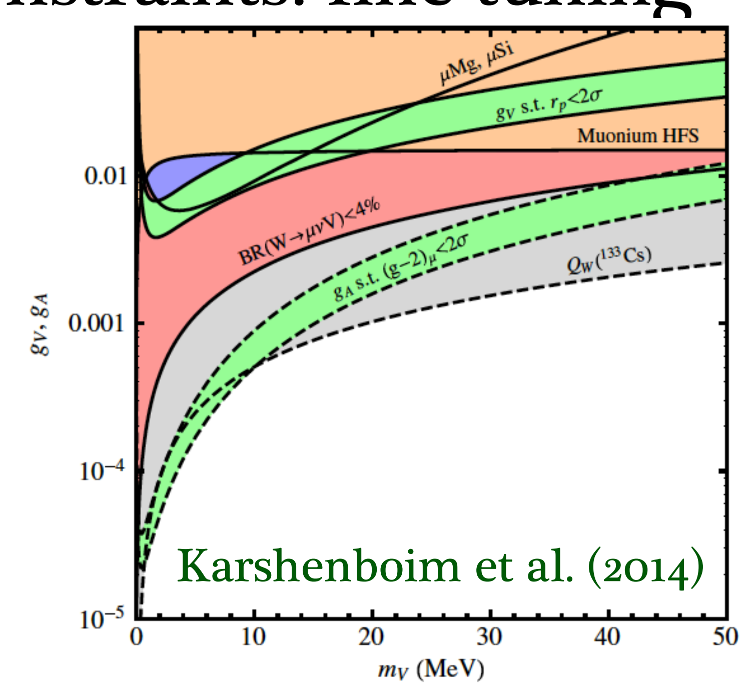
- vector particle: constraints from decay of W

- embedding in renormalizable theory

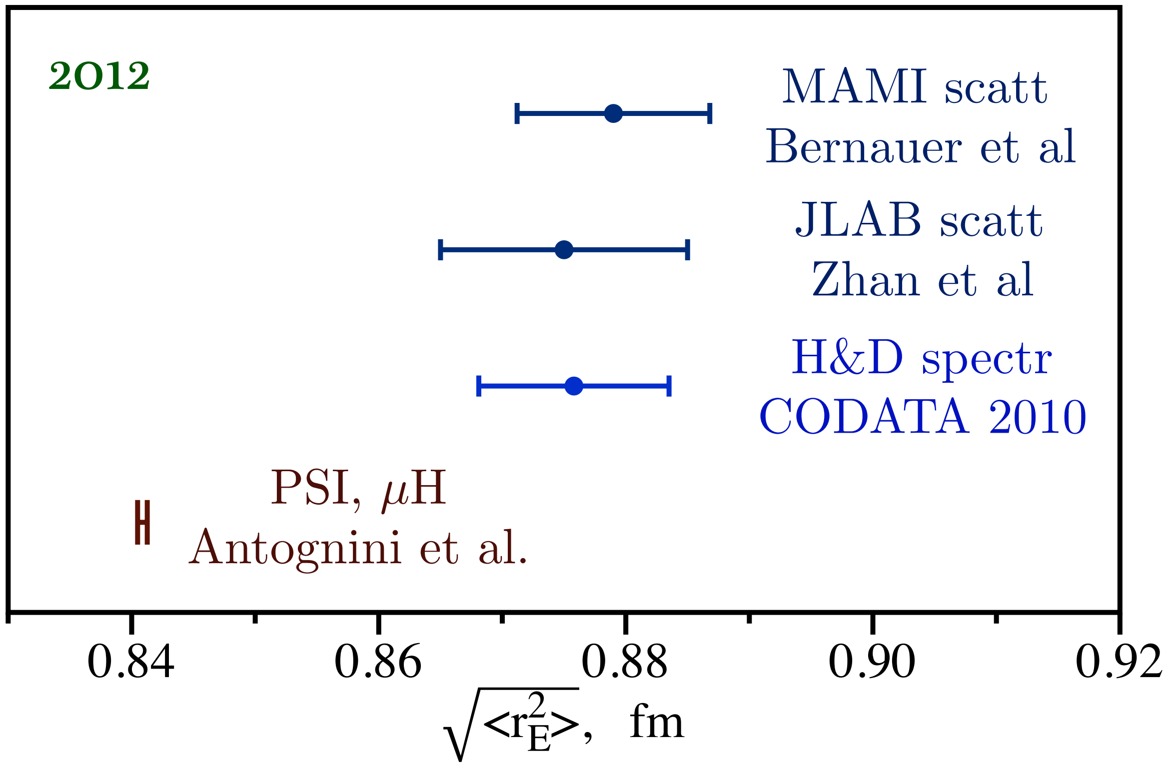
Carlson and Fried (2015)

- scalar particle: 200 keV - 3 MeV

Liu, Cloet and Miller (2018)



# μH Lamb shift and 2γ

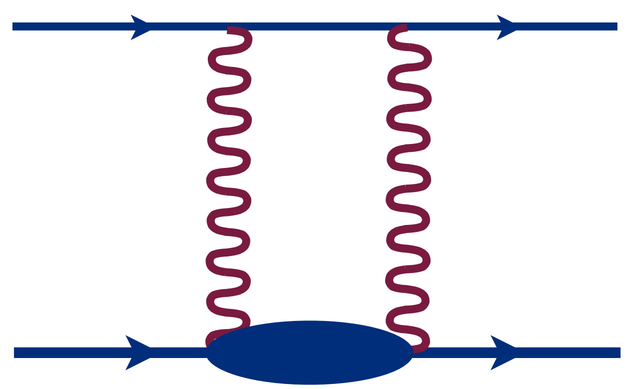



2P-2S transition in μH

- discrepancy: 310 $\mu$eV

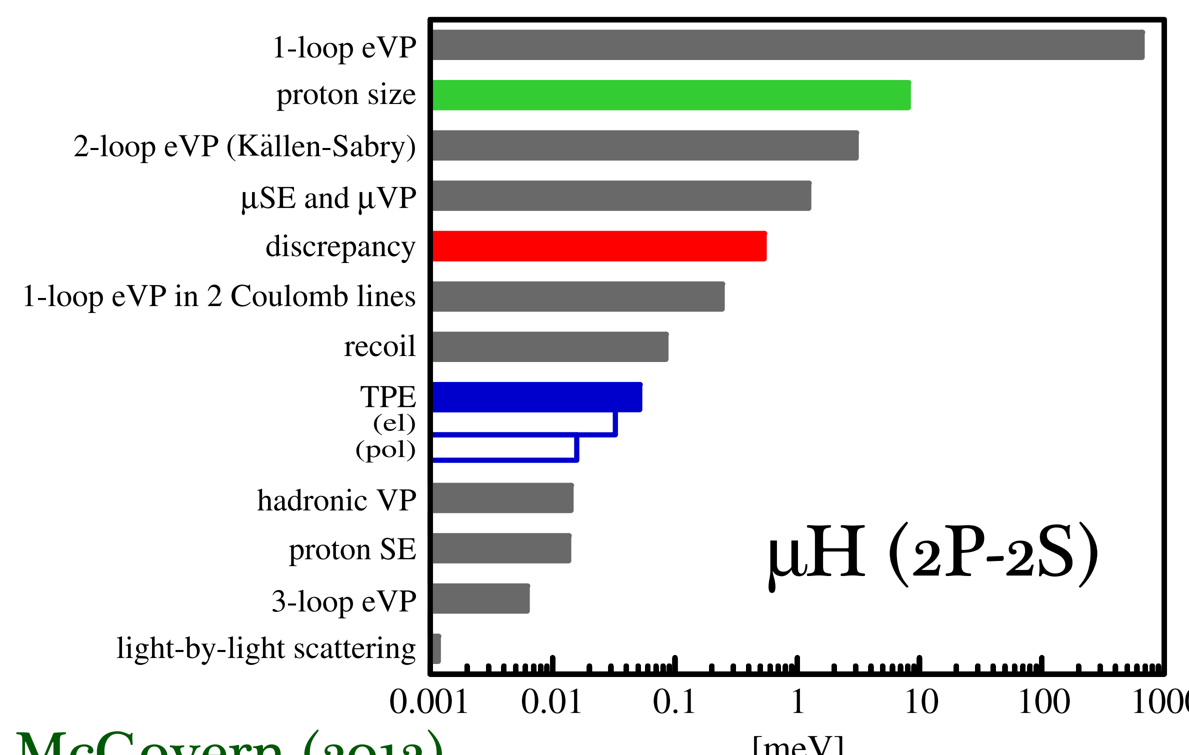
- μH uncertainty: 2.5 $\mu$eV

2γ hadronic correction



$$\Delta E^{2\gamma}_{2P-2S} = 33 \pm 2\ \mu\text{eV}$$

C. Carlson, M. Vanderhaeghen (2011) + M. Birse, J. McGovern (2012)




A. Antognini et al. (2013)

- important to reduce ambiguities of 2γ

# Elastic electron-proton scattering

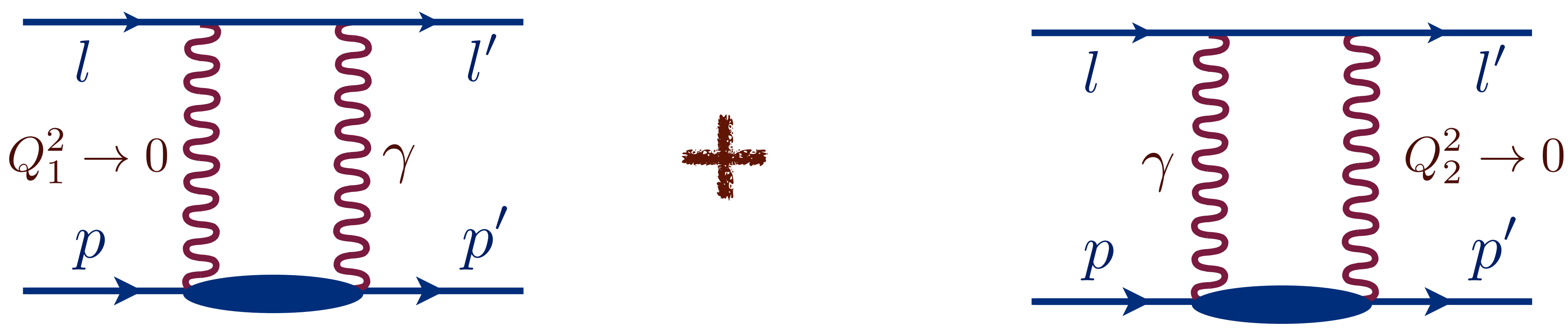
# and two-photon exchange

# Scattering experiments and 2ɣ

- 2ɣ is not among standard radiative corrections

$$\sigma^{\text{exp}} \equiv \sigma_{1\gamma}(1 + \delta_{\text{rad}} + \delta_{\text{soft}} + \delta_{2\gamma})$$

- soft-photon contribution is included




L.C. Maximon and J. A. Tjon (2000)

- hard-photon contribution modelled by Feshbach correction

- charge radius insensitive to 2ɣ model

- magnetic radius depends on 2ɣ model

A1@MAMI: J. C. Bernauer et al. (2014)

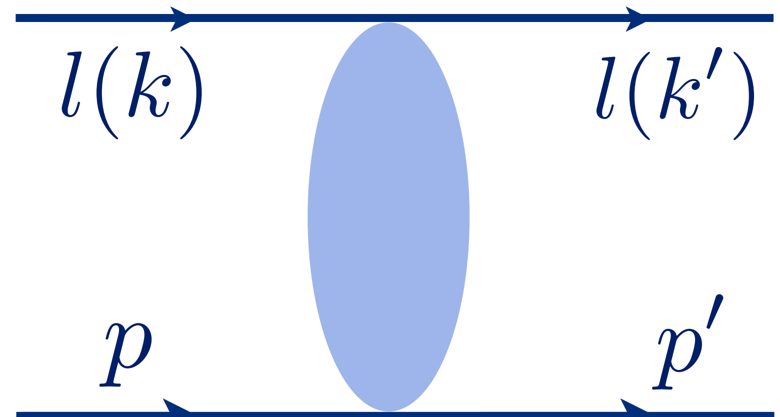
# Elastic lepton-proton scattering and 2ɣ

momentum transfer

$$Q^2 = -(k-k')^2$$

crossing-symmetric variable

$$\nu = \frac{(k, p+p')}{2}$$



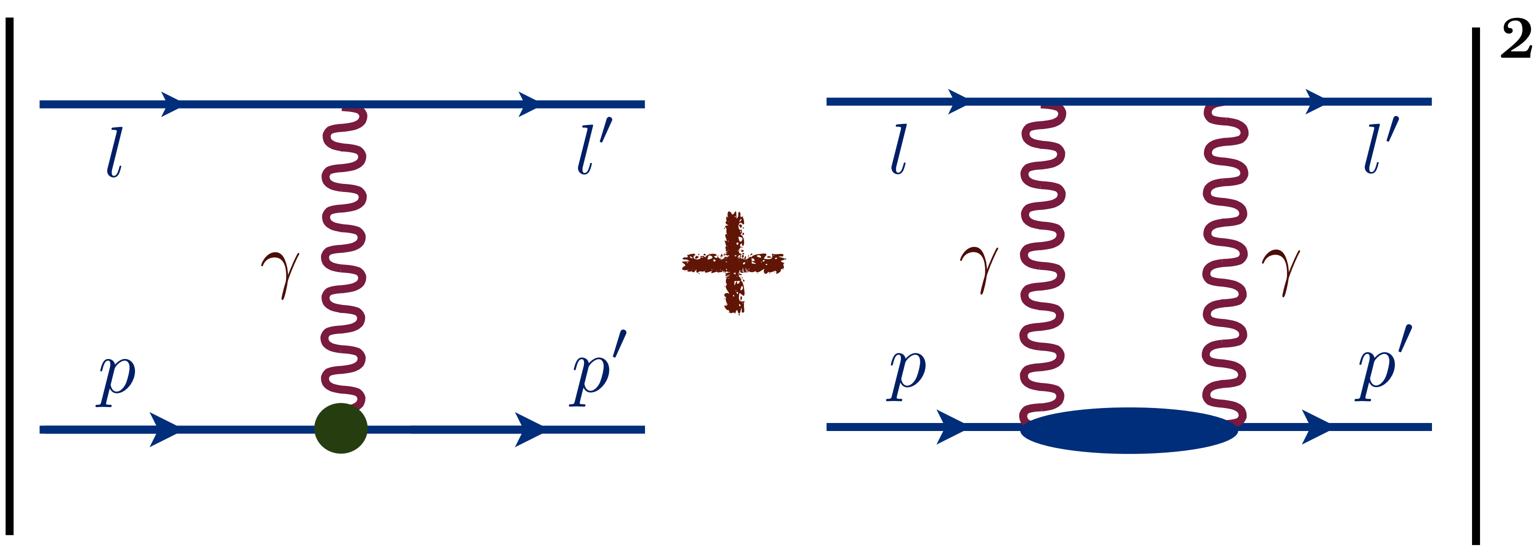

photon polarization parameter

$$\varepsilon$$

forward scattering

$$\varepsilon \to 1$$

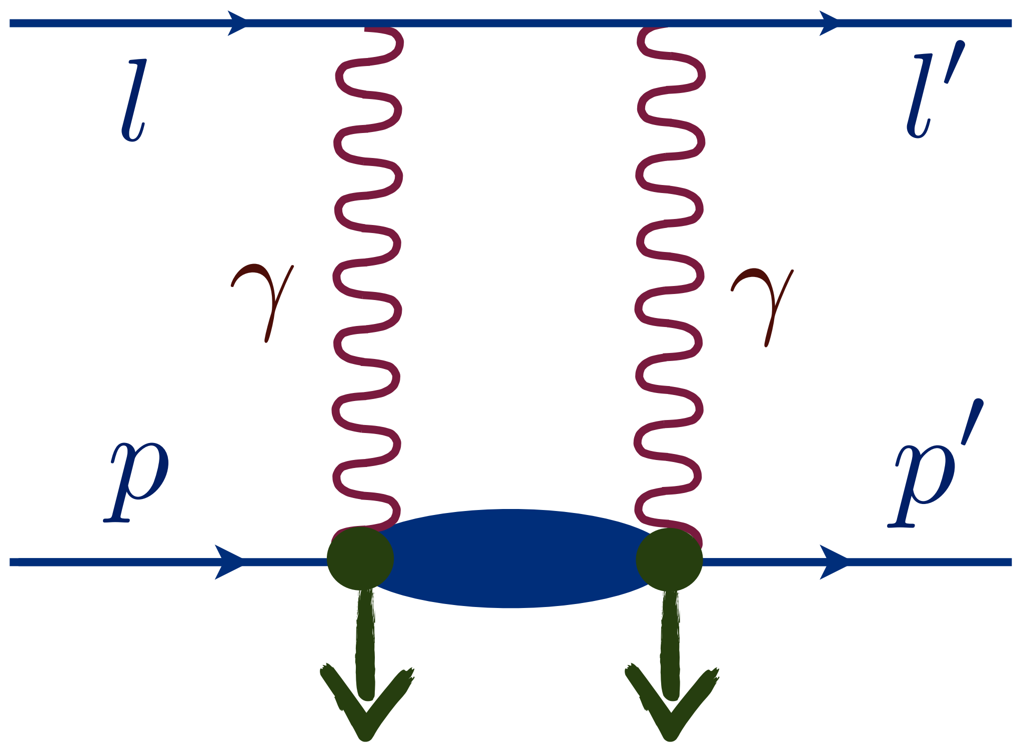
- leading 2ɣ contribution: interference term




$$\delta_{2\gamma} = \frac{2\sum\limits_{\text{spin}} T^{1\gamma}\Re T^{2\gamma}}{\sum\limits_{\text{spin}} |T^{1\gamma}|^2}$$

- 2ɣ correction to cross section is given by amplitudes real parts

non-forward scattering
at low momentum transfer



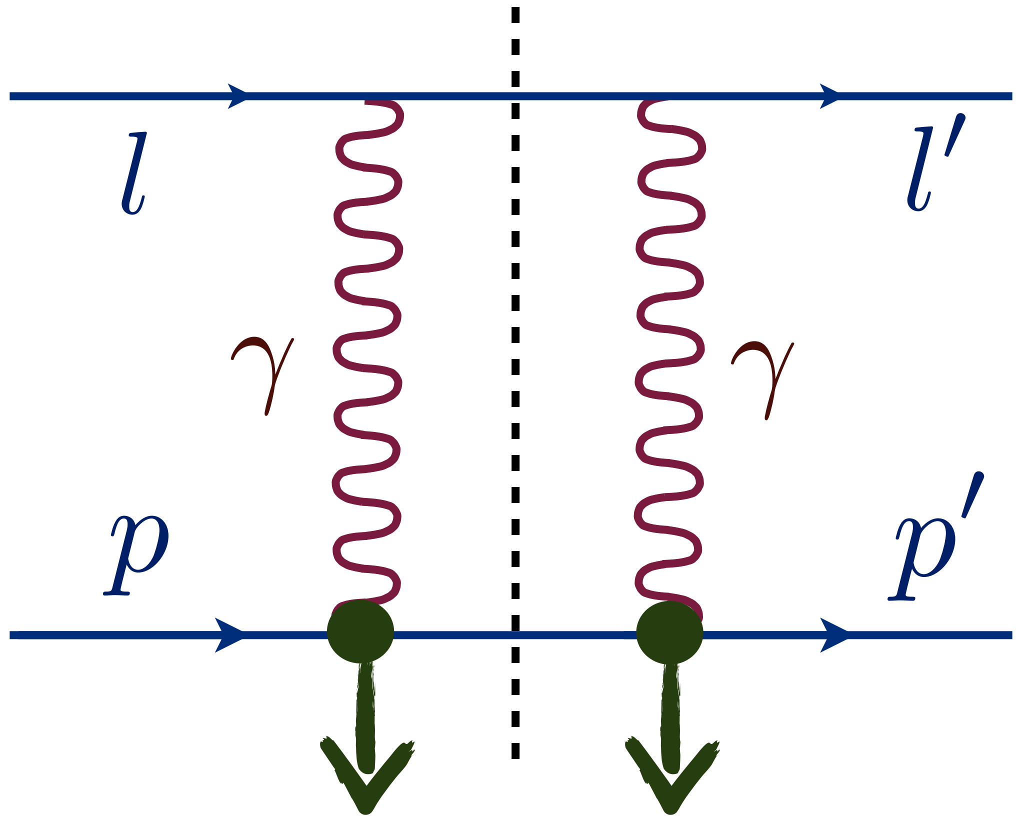

photoproduction vertex or Compton tensor

box diagram

assumption about the vertex

non-forward scattering

proton state




Dirac and Pauli form factors

box diagram

$$\Gamma^\mu(Q^2) = \gamma^\mu F_D(Q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2M} F_P(Q^2)$$

ep scattering: P. G. Blunden, W. Melnitchouk and J. A. Tjon (2003)

assumption about the vertex

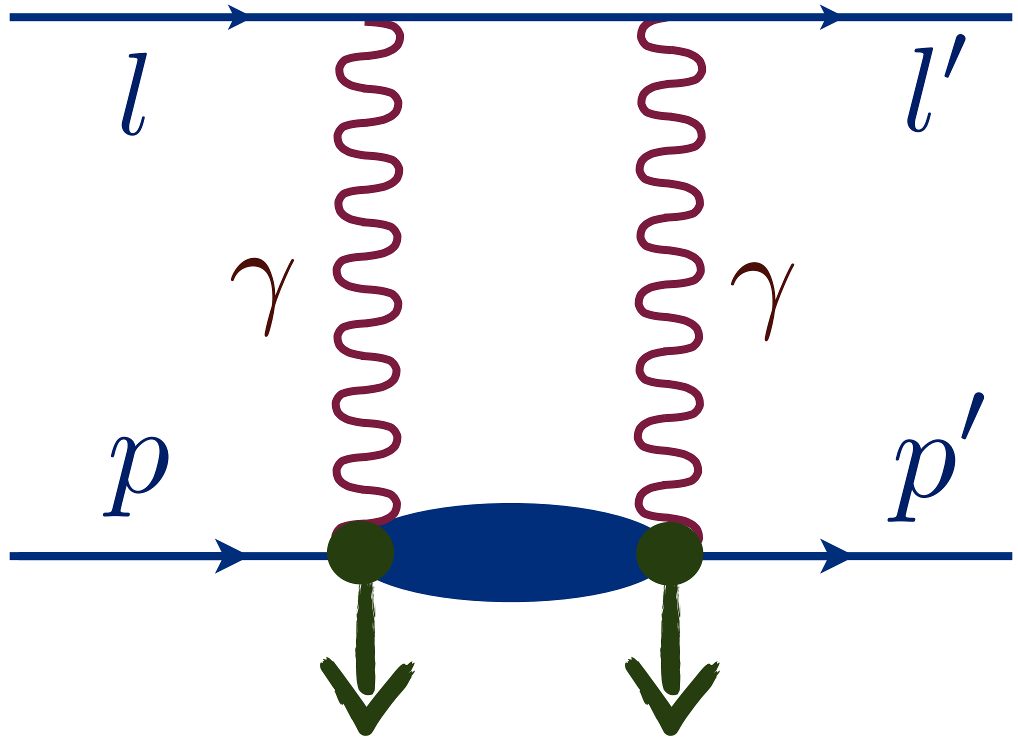
violation of unitarity for resonances !

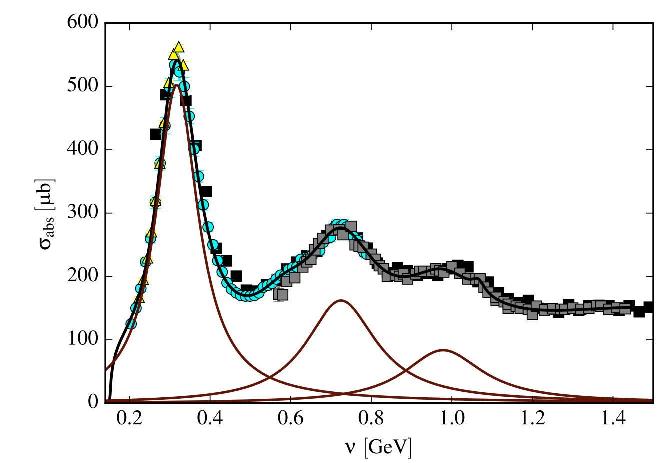
## non-forward scattering

## inelastic states

works at small

scattering angles




forward doubly-virtual Compton tensor

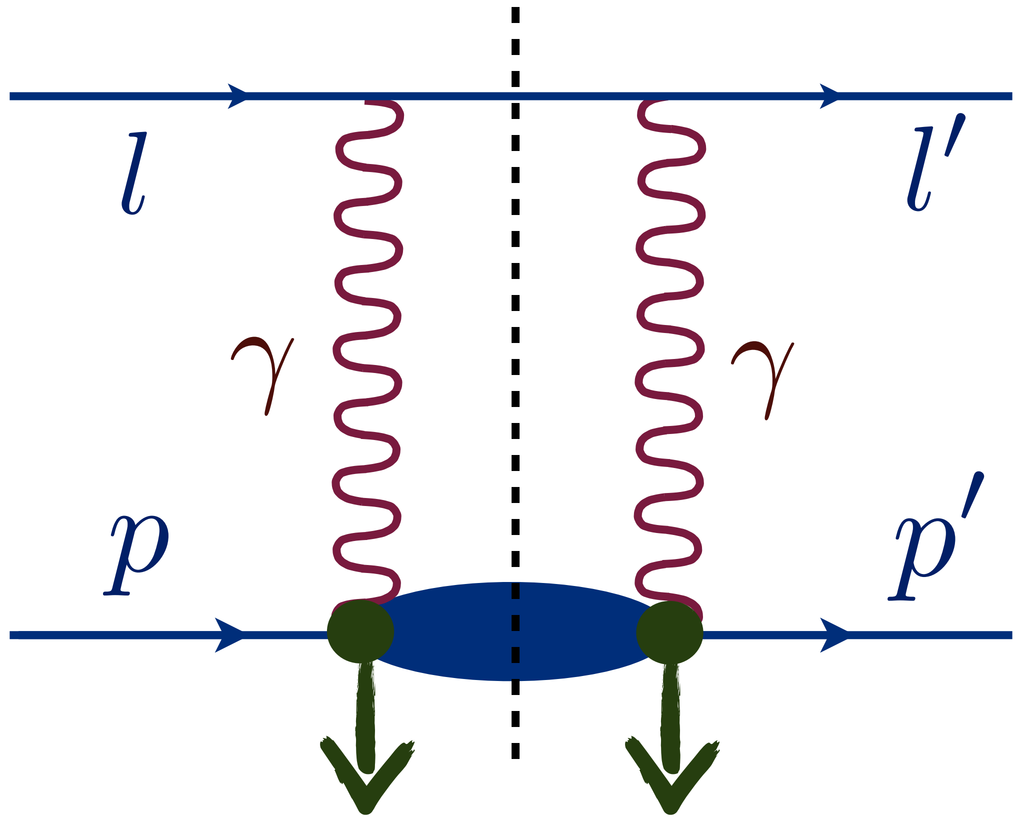
box diagram

unpolarized proton structure

M. E. Christy, P. E. Bosted (2010)

proton + inelastic = total

non-forward scattering
at low momentum transfer




photoproduction vertex or Compton tensor

**box diagram**

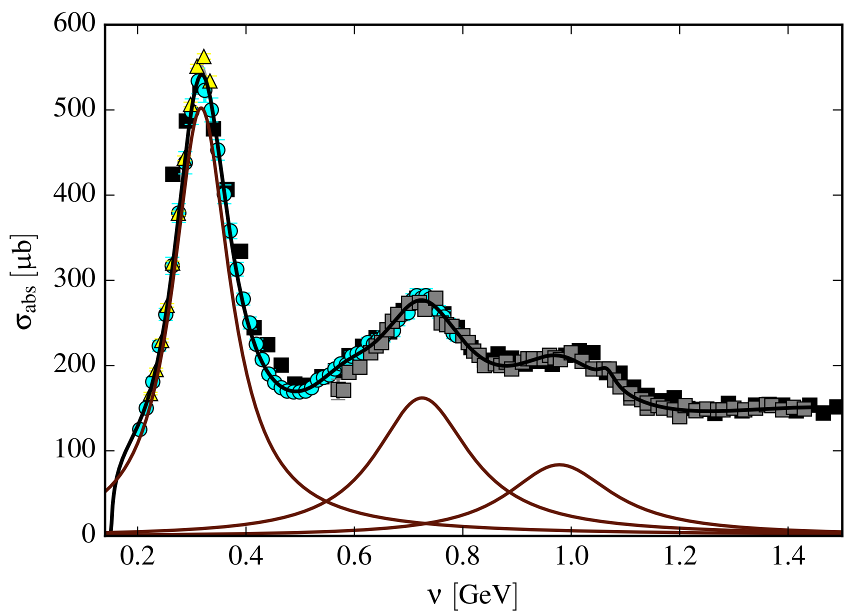
assumption about the vertex

**dispersion relations**

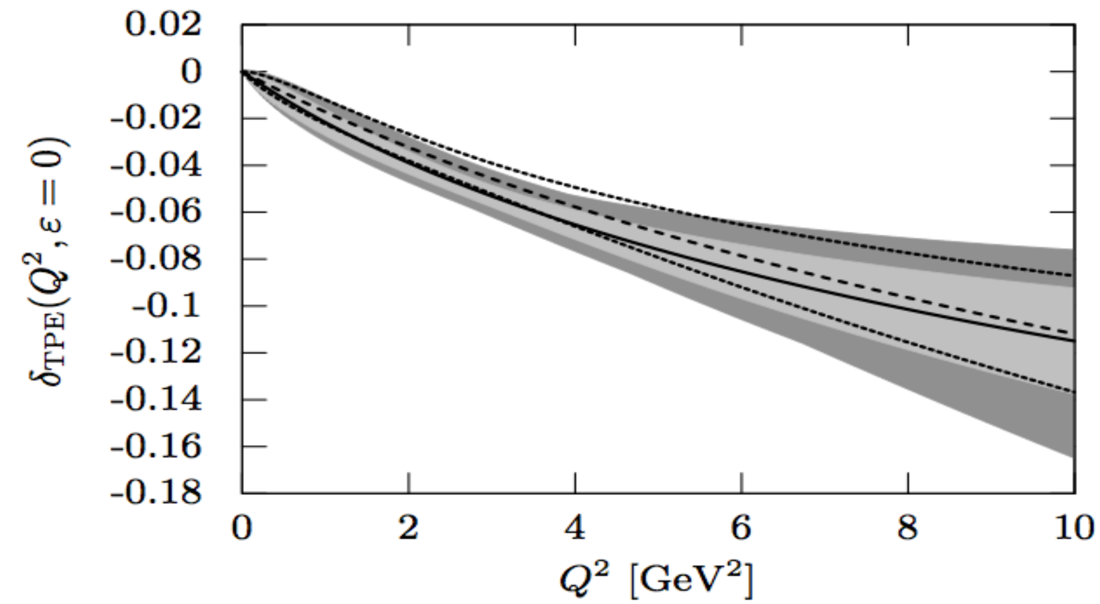
based on on-shell information

# Fixed-$Q^2$ dispersion relation framework

on-shell 1γ amplitudes




2γ prediction




experimental data

elastic and πN

cross section correction

unitarity

$$\Re\mathcal{F}(\nu) = \frac{2\nu}{\pi}\mathcal{P}\int_{\nu_{min}}^{\infty} \frac{\Im\mathcal{F}(\nu' + i0)}{\nu'^2 - \nu^2}\mathrm{d}\nu'$$

disp. rel.

2γ imaginary parts

2γ real parts

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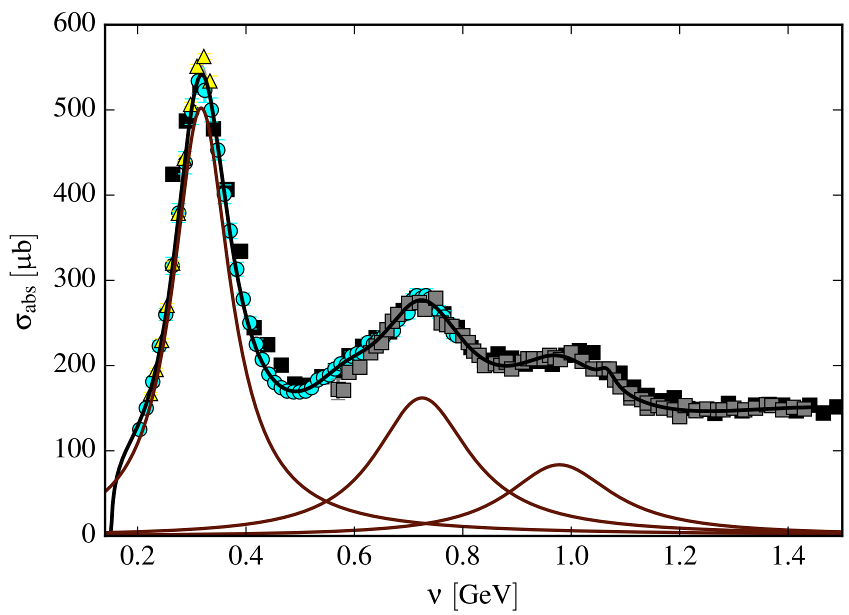


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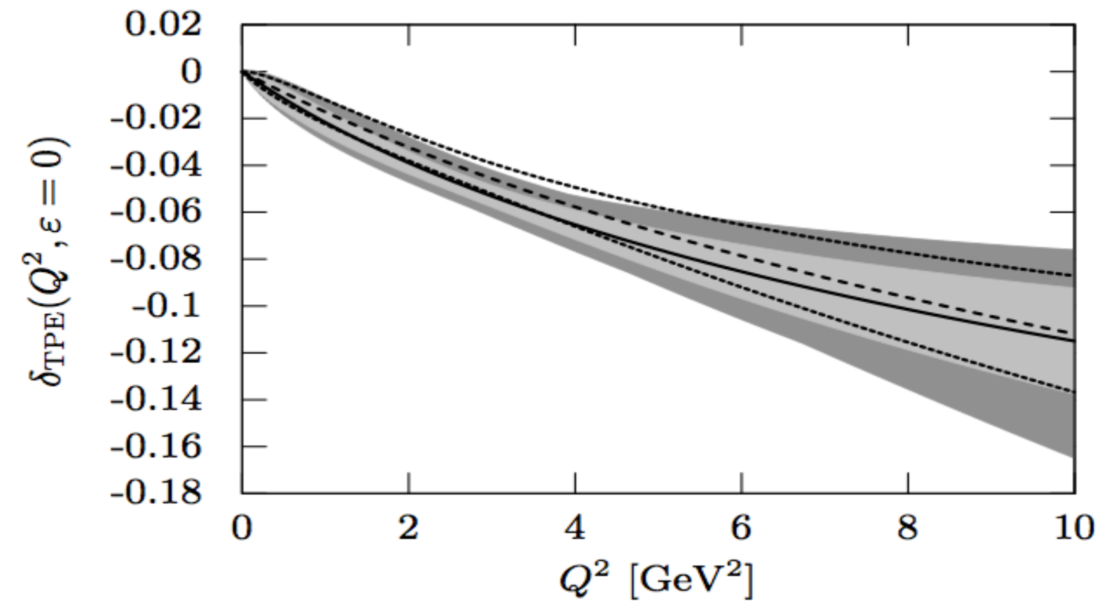


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disp. rel.

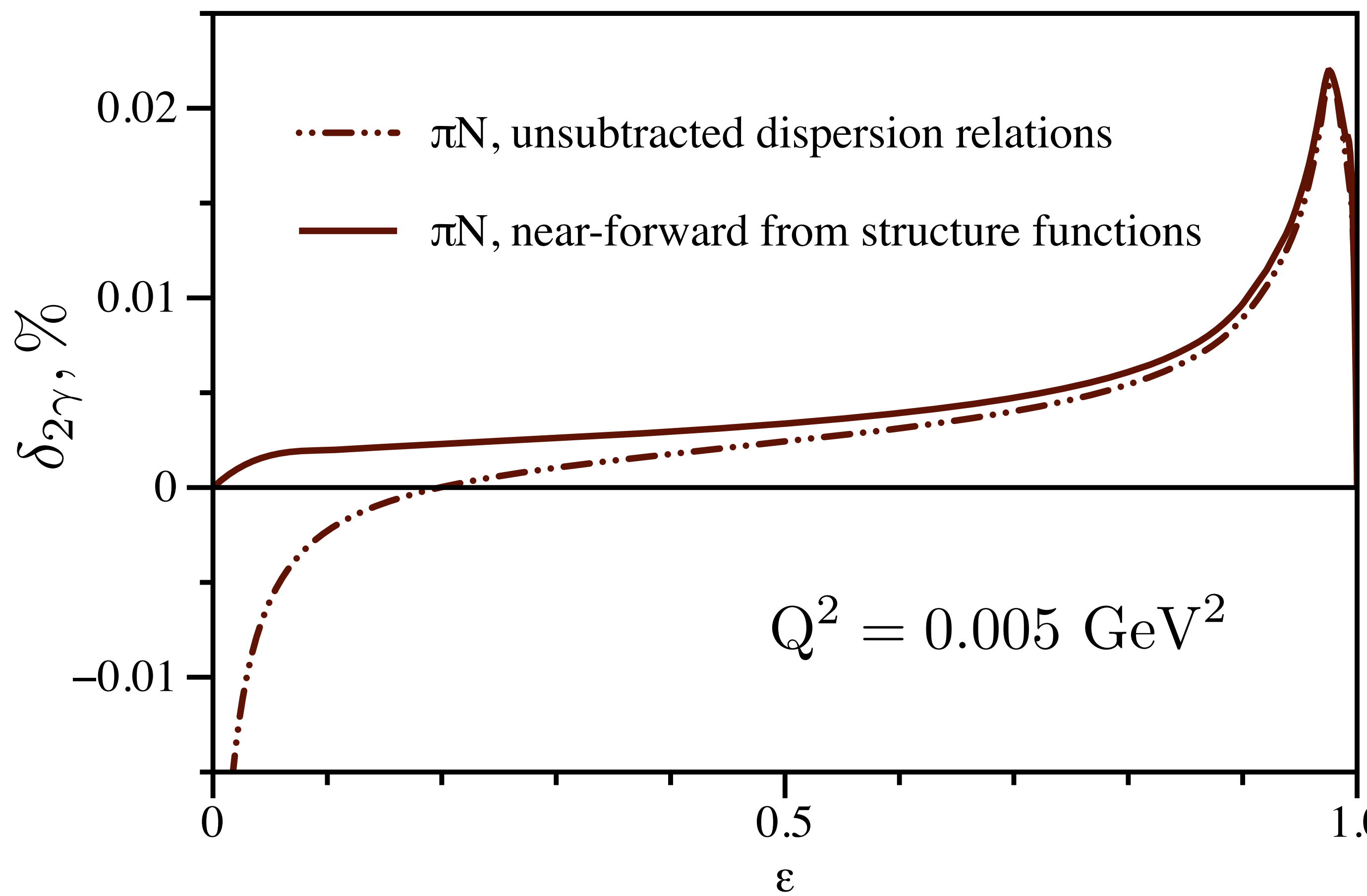
2γ imaginary parts

2γ real parts

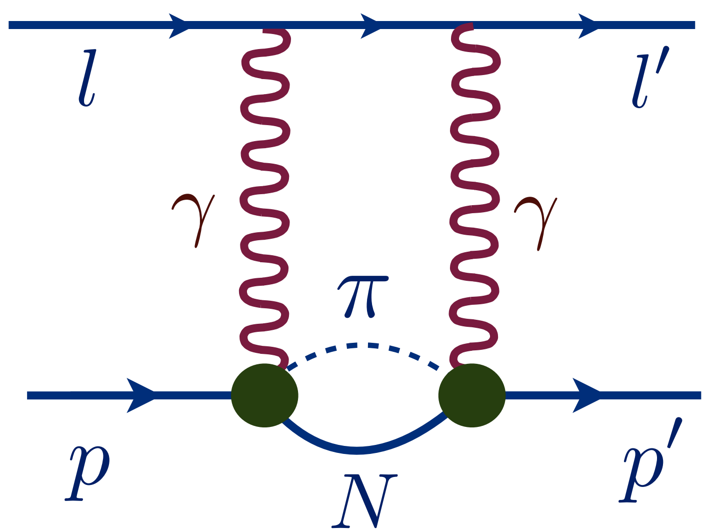
Dm. Borisyuk and A. Kobushkin (2008-15)

O. T. and M. Vanderhaeghen (2014-17)

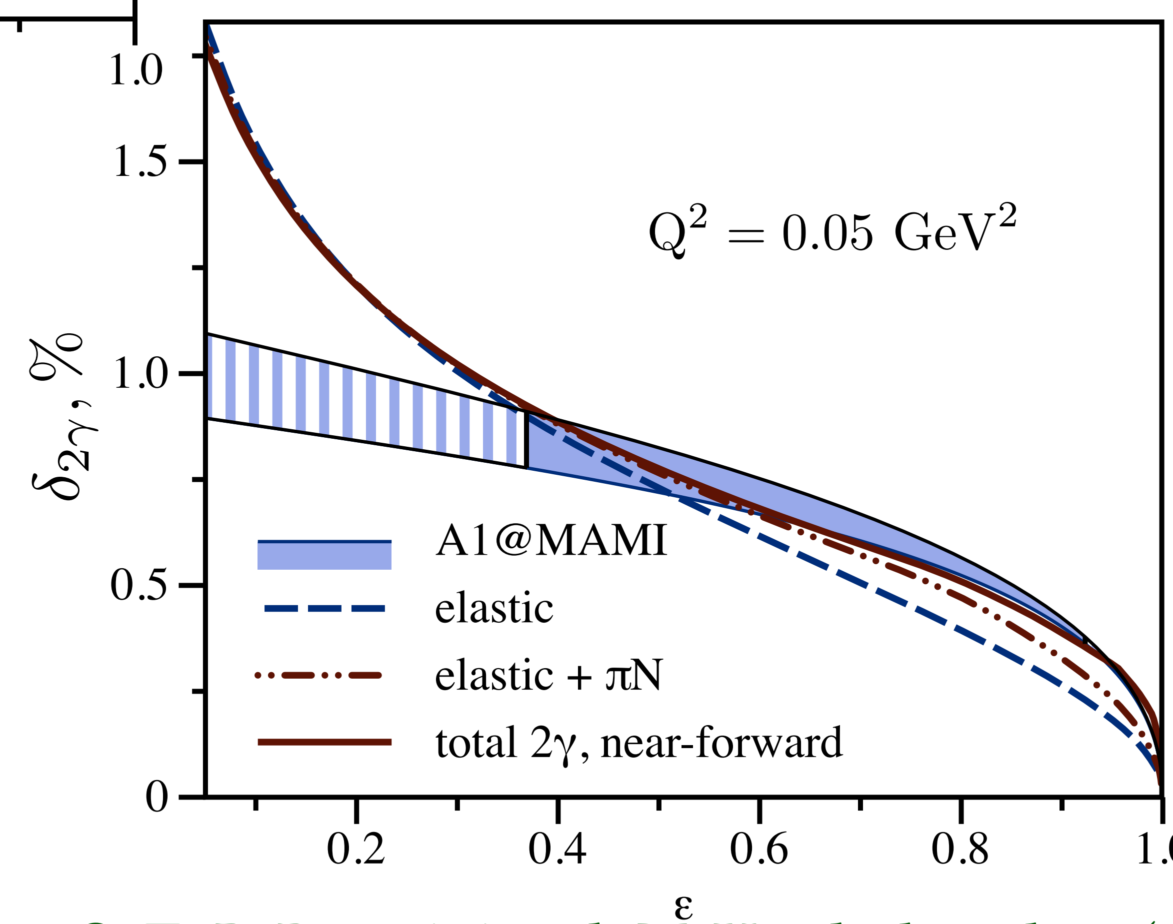
# πN in dispersive framework (e-p)




- dispersion relations agree with near-forward at large $\varepsilon$

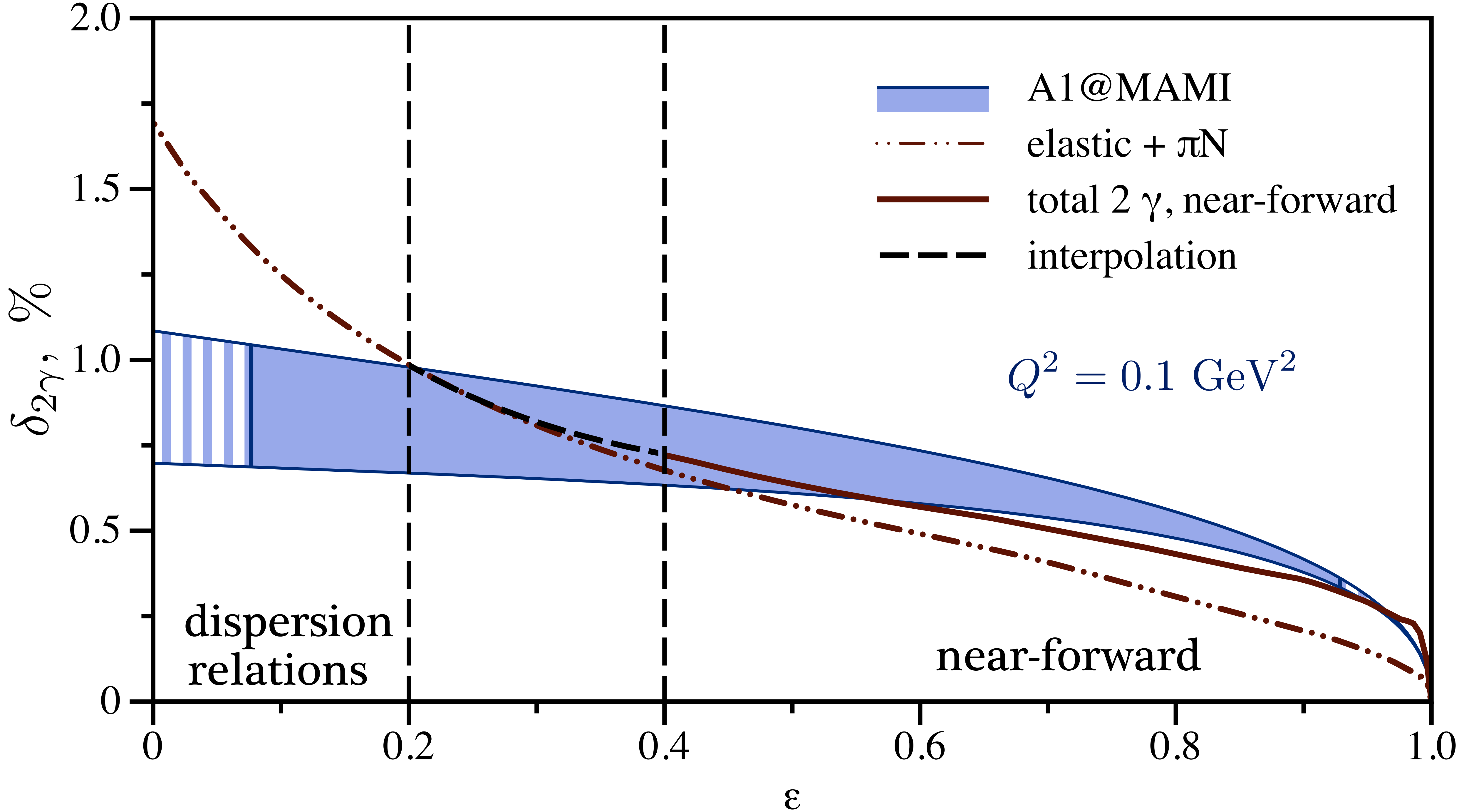



- πN is dominant inelastic 2γ



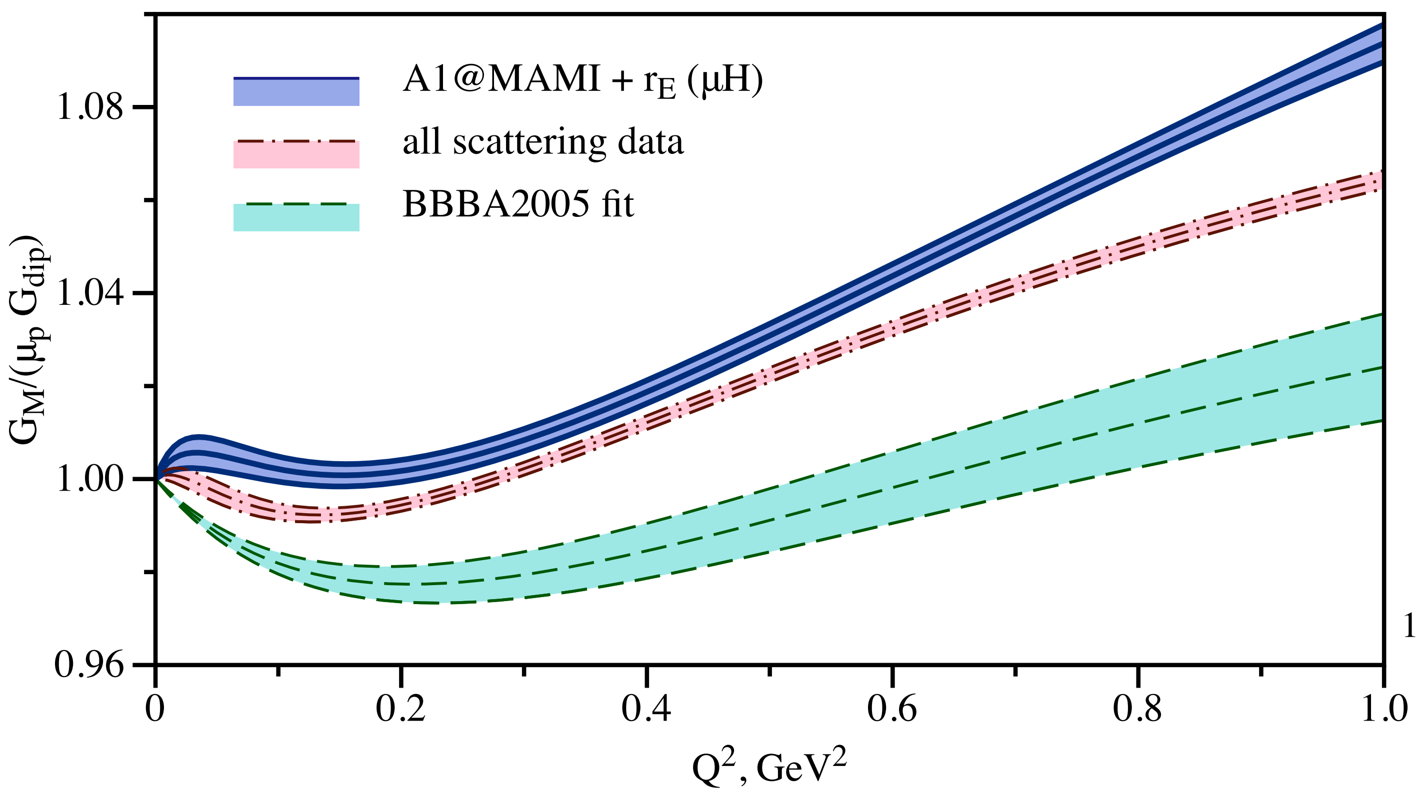

O. T., B. Pasquini and M. Vanderhaeghen (2017)

# Our best 2γ knowledge




- small $Q^2$: near-forward at large $\varepsilon$, all inelastic states
- $Q^2 \lesssim 1$ GeV$^2$: elastic+πN within dispersion relations
- intermediate range: interpolation

# Applications to nucleon form factors




z expansion fit

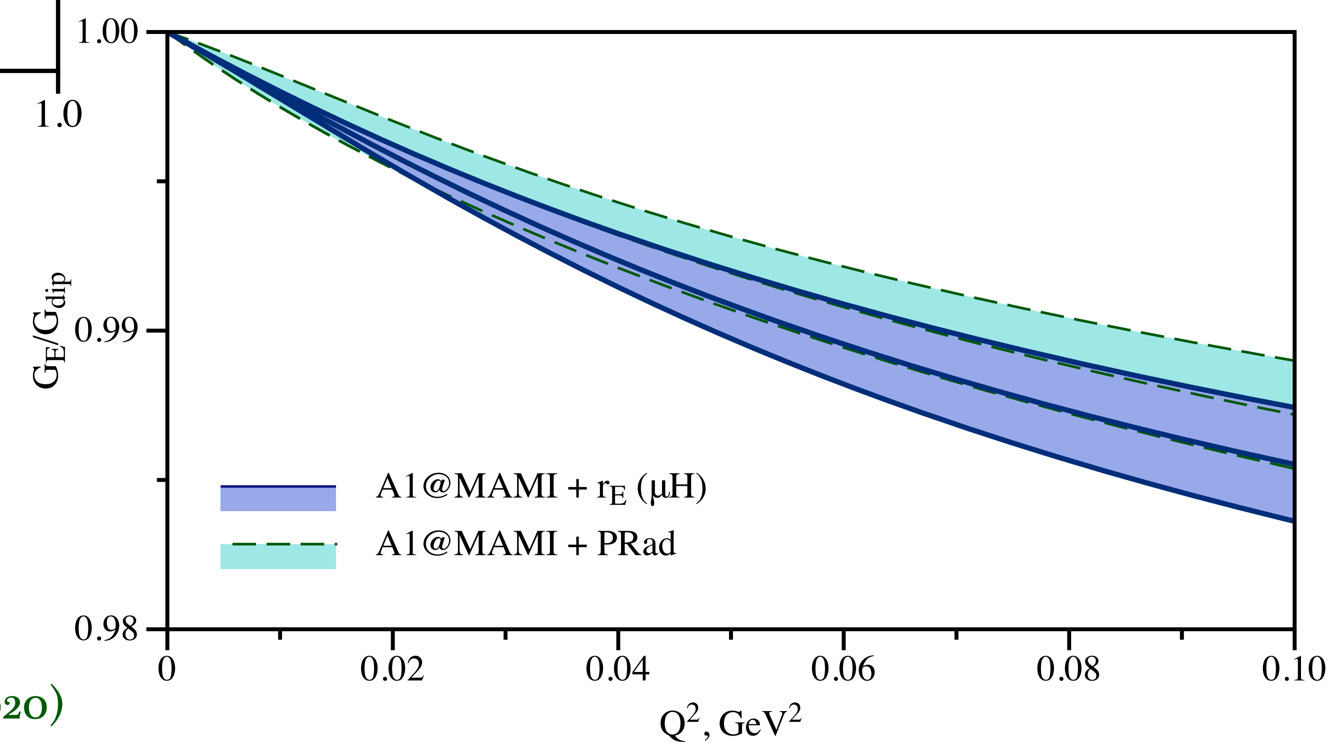
$$z(Q^2) = \frac{\sqrt{t_{cut} + Q^2} - \sqrt{t_{cut} - t_0}}{\sqrt{t_{cut} + Q^2} + \sqrt{t_{cut} - t_0}}$$

with 4-5 independent coefficients

$$G(Q^2) = \sum_{k=0}^{k_{max}} a_k z(Q^2)^k$$

BBBA: fit used in neutrino physics

PRad data or μH charge radius ?




K. Borah, G. Lee, R. J. Hill and O. T. (2020)

- first model-independent fits presenting covariance matrix
- 2γ provides nontrivial hadronic radiative correction
- proton charge radius as a constraint

# Elastic muon-proton scattering

# and two-photon exchange

# Elastic muon-proton scattering

- charge radius extractions:

| | |
|---|---|
| eH, eD spectroscopy | ep scattering |
| μH, μD spectroscopy | μp scattering ???? |

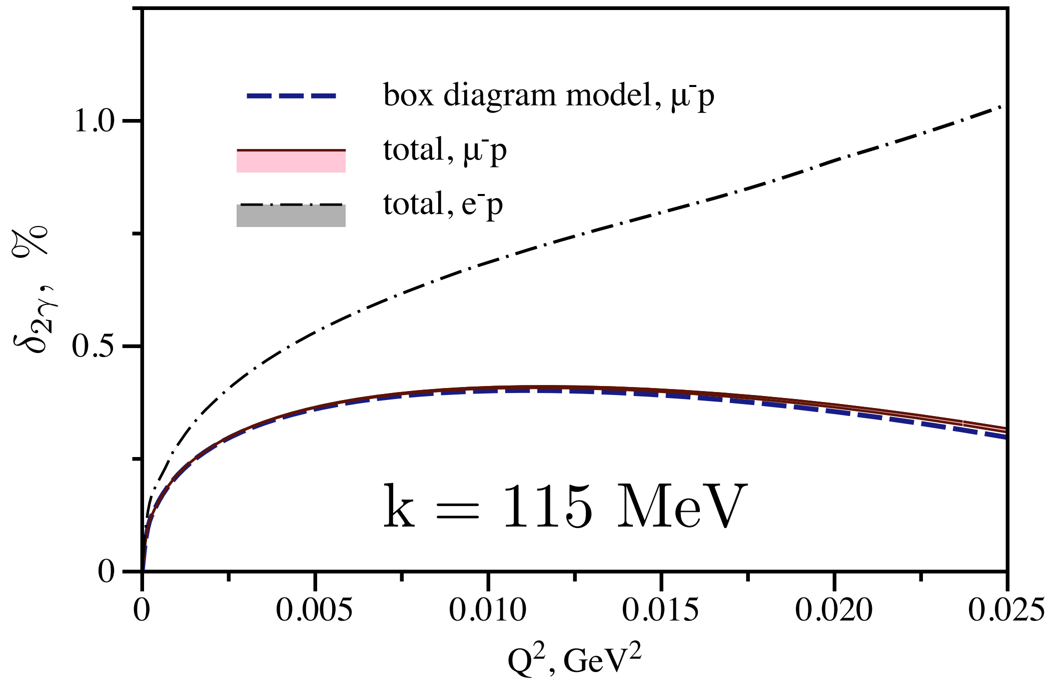
- μp elastic scattering is planned by MUSE@PSI(2018-19)
  measure with both electron/muon charges

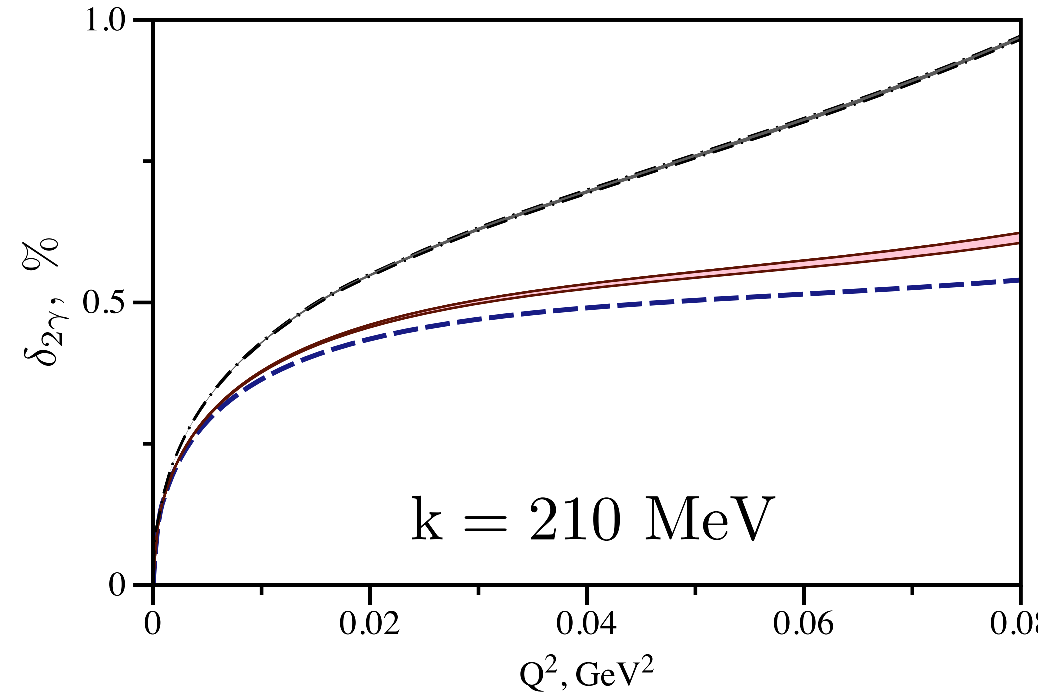
- three nominal beam energies: 115, 153, 210 MeV, $Q^2 < 0.1$ GeV$^2$

- 2γ correction in MUSE ?

# MUSE@PSI (2018-19) estimates ($\mu^-p$)

- proton box diagram model + inelastic 2γ




O. T. and M. Vanderhaeghen (2014, 2016)

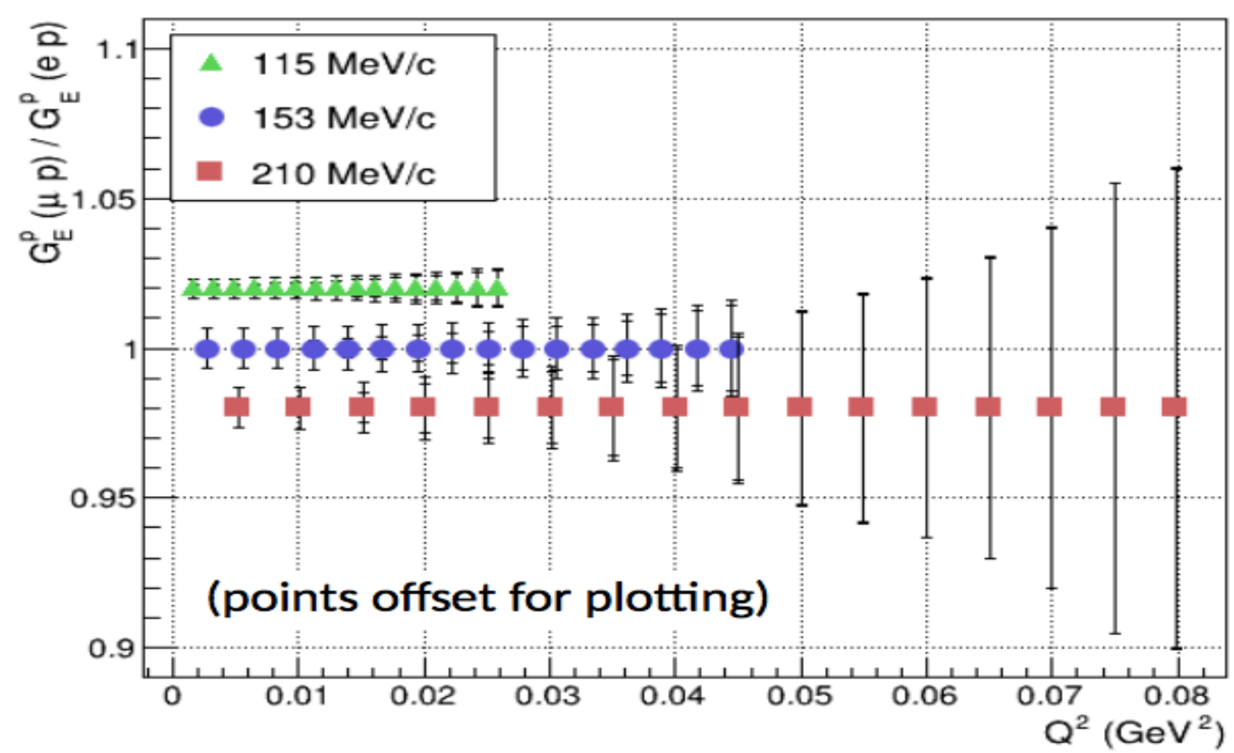
- expected muon over electron ratio

small inelastic 2γ

↓

small 2γ uncertainty

- MUSE can test $r_E$ in one charge channel




K. Mesick talk (PAVI 2014), MUSE TDR (2016)

# COMPASS proton radius experiment

- elastic $\mu$p scattering at SPS with 100 GeV beam

- measure $\mathrm{G}_\mathrm{E}^2 + \tau \mathrm{G}_\mathrm{M}^2$ at forward angles

# 2γ corrections?

- Feshbach correction (+ recoil)

$$\delta_{2\gamma} = \frac{\alpha \pi Q}{2\omega}\left(1 + \frac{m}{M}\right)$$

→ 2-3 orders below MUSE

- inelastic states: kinematically enhanced

- sub per mille level of 2γ in COMPASS kinematics

# 1S-2S transition in hydrogen and 2γ

- measurements of 1S-2S transition in eH with $4 \times 10^{-15}$ accuracy:

$$\nu_{1S-2S}(H) = 2466061413187018(11)\ \text{Hz}$$ 2010th

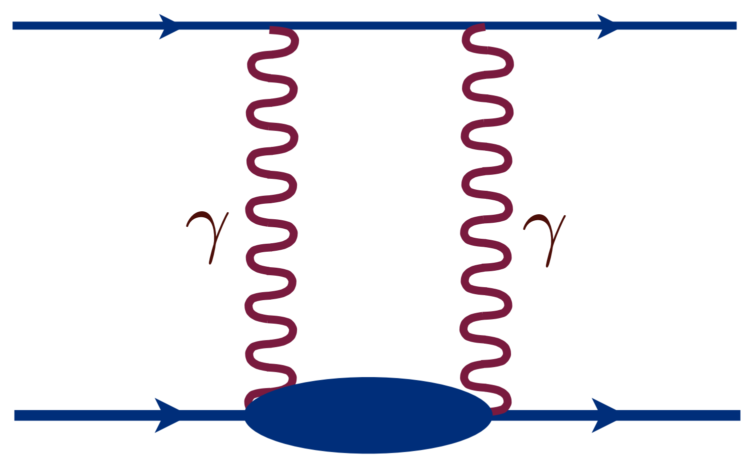
A. Matveev et al. (2013)

- more precise than recent Lamb shift measurement (error: 3.2 kHz)

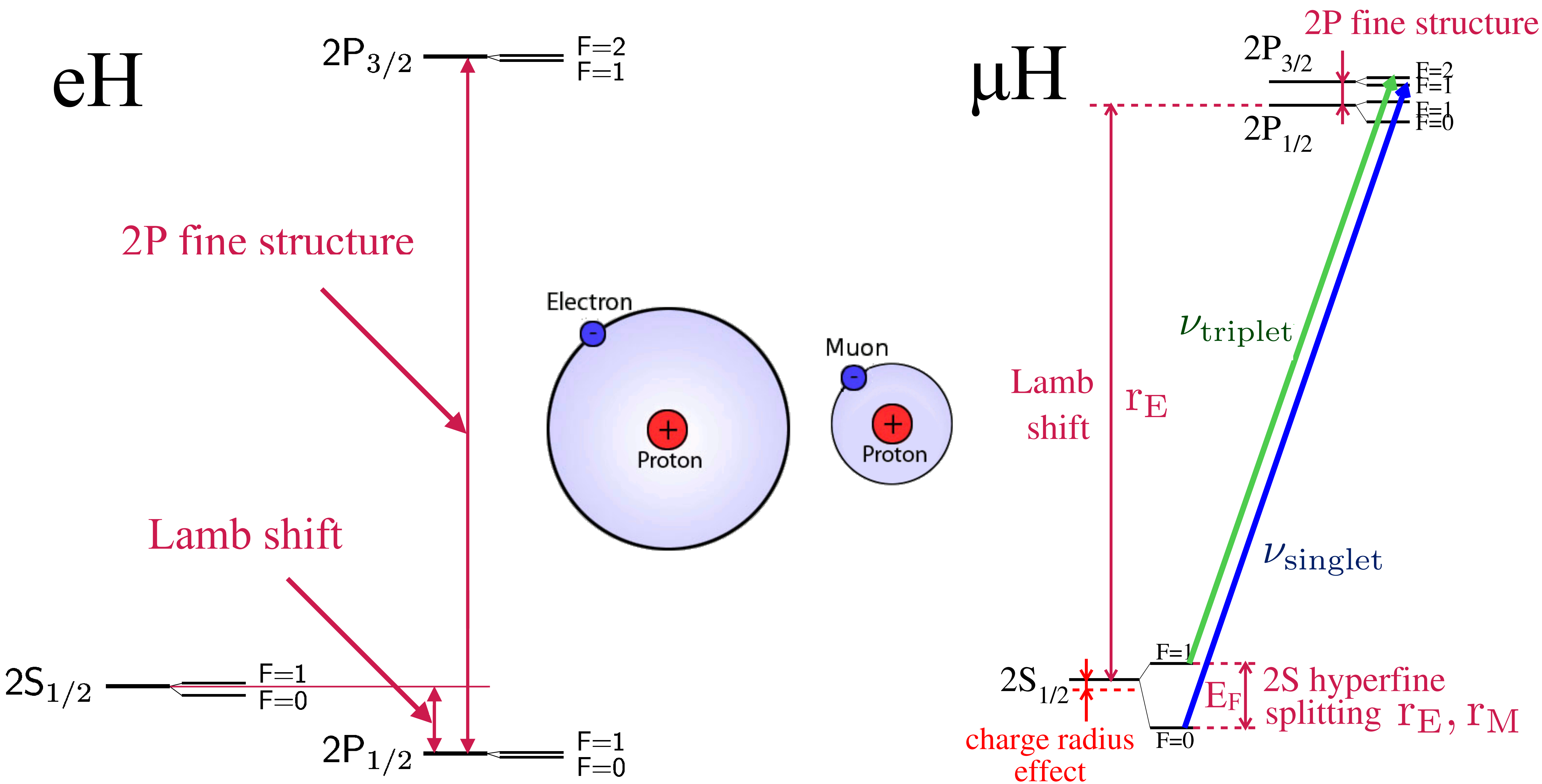
N. Bezginov et al. (2019)

$$\nu_{nS} = -\frac{R_\infty}{n^2} + \frac{L_{1S}(r_E)}{n^3}$$

- main input to determine Rydberg constant



# Lamb shift and hyperfine splitting in H



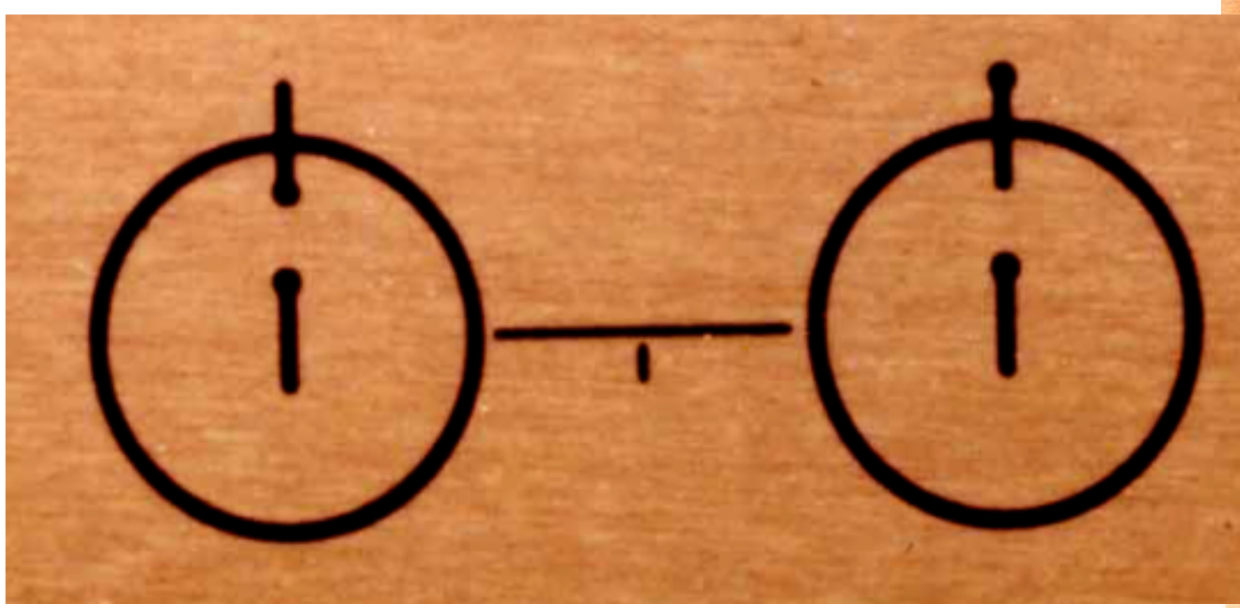

- 1S HFS in μH with 1 ppm accuracy at PSI, J-PARC, RIKEN-RAL

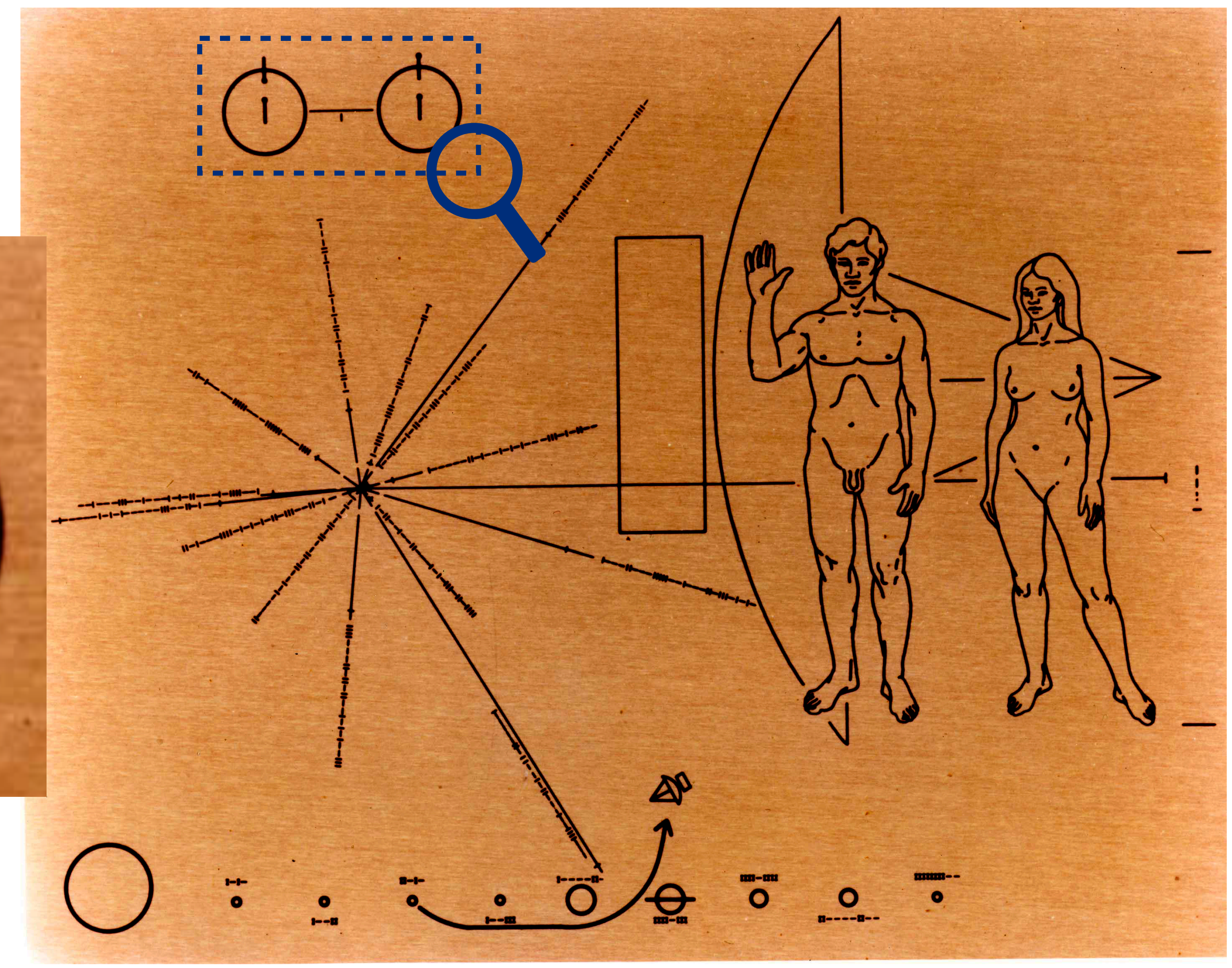
R. Pohl et al. (2016)

# μH 1S HFS from eH 1S HFS

- measurements of 1S HFS in eH (21 cm line):

$$\nu_{\mathrm{HFS}}(\mathrm{H}) = 1420.4057517667(9)\ \mathrm{MHz}$$ 1970th





Pioneer plaque, NASA Ames

- relation between eH and $\mu$H through g1 and g2 in 2$\gamma$

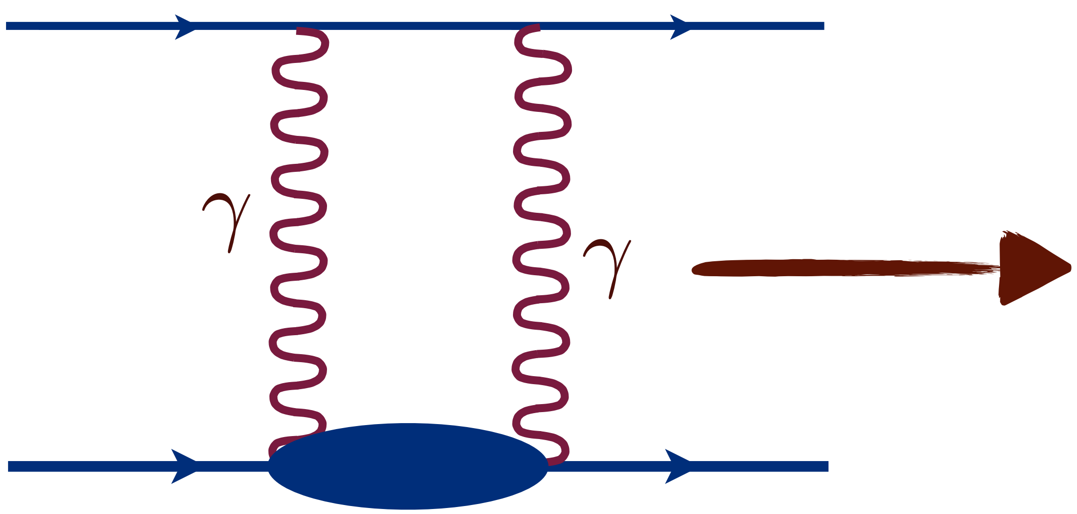
O. T., Eur.Phys.J.A 55 (2019) 5, 64

# Conclusions

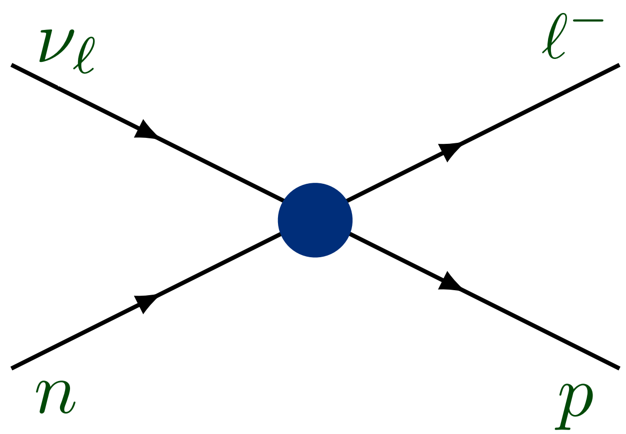
- proton charge radius puzzle dissolves with new measurements

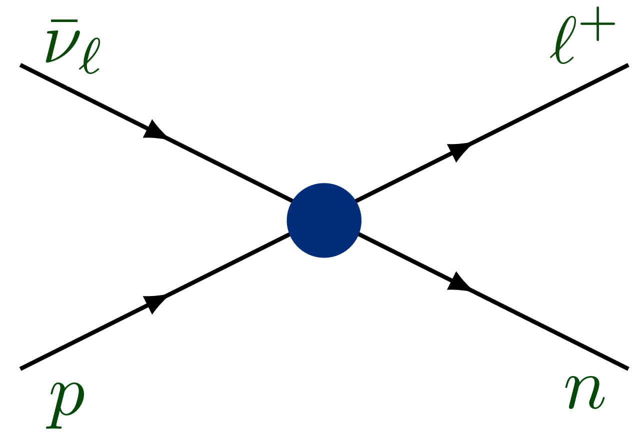
- tensions in scattering data are not resolved

- forthcoming muon scattering data will shed new light




largest theoretical uncertainty in low-energy proton structure

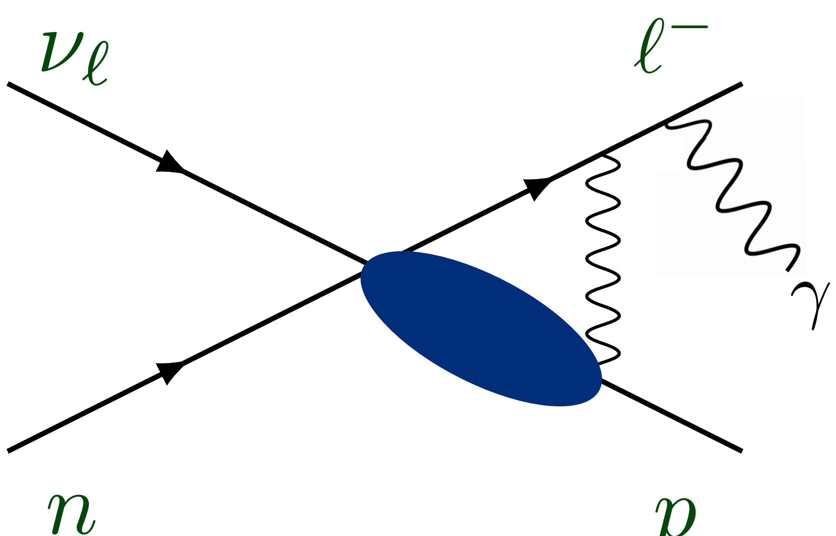



O. T., Qing Chen, Richard J. Hill and Kevin S. McFarland, Nature Commun. 13 (2022), 1, 5286

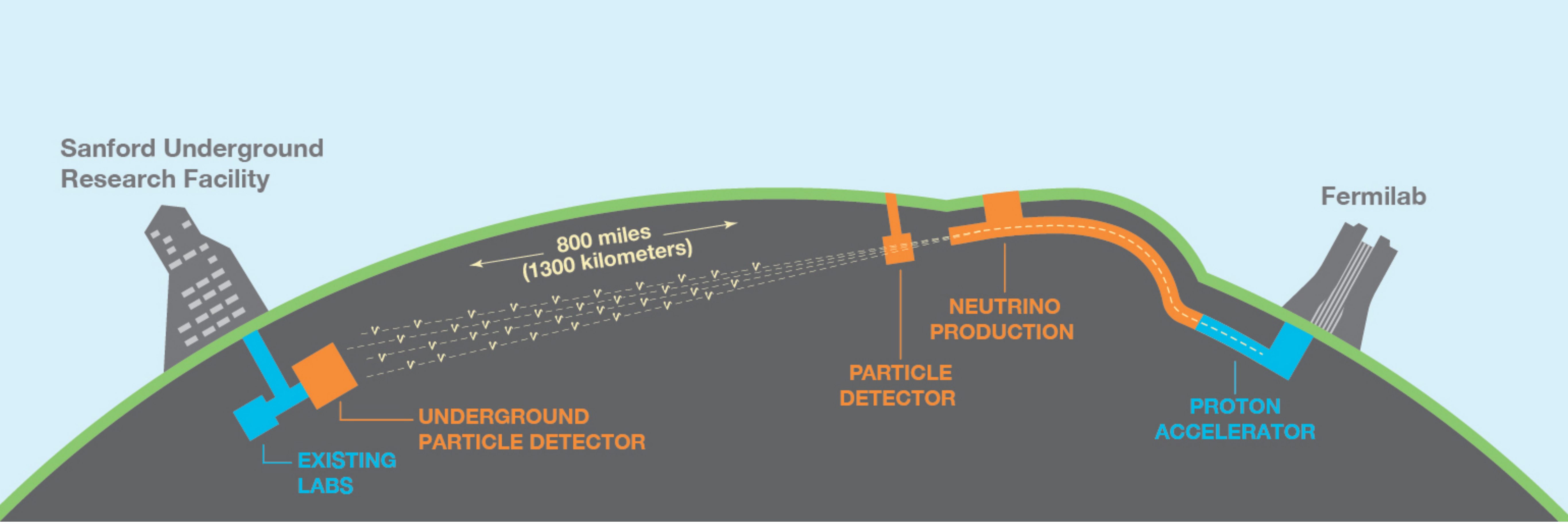
# Radiative corrections in charged-current elastic scattering on free nucleons

O. T., Qing Chen, Richard J. Hill, Kevin S. McFarland and Clarence Wret
editors suggestion in Phys. Rev. D (2022)



# Neutrino experiments

- DUNE and Hyper-K: leading-edge $\nu$ science experiments

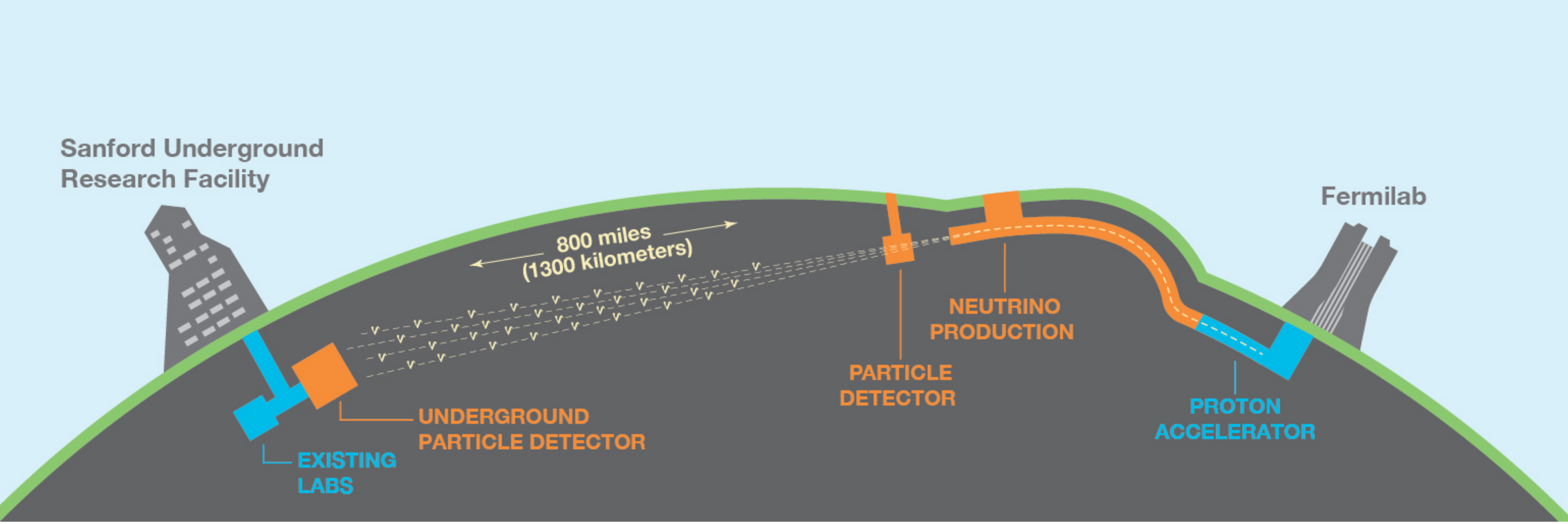



- origin of matter-antimatter asymmetry — $\delta_{CP}$
- mass hierarchy and oscillation parameters — PMNS matrix, $\Delta m_{31}^2$
- Grand Unified Theories — proton decay
- dynamics of supernova explosion — wait for one;)

 DUNE, CDR (2016), TDR (2020)

# Neutrino experiments

- DUNE and Hyper-K: leading-edge $\nu$ science experiments

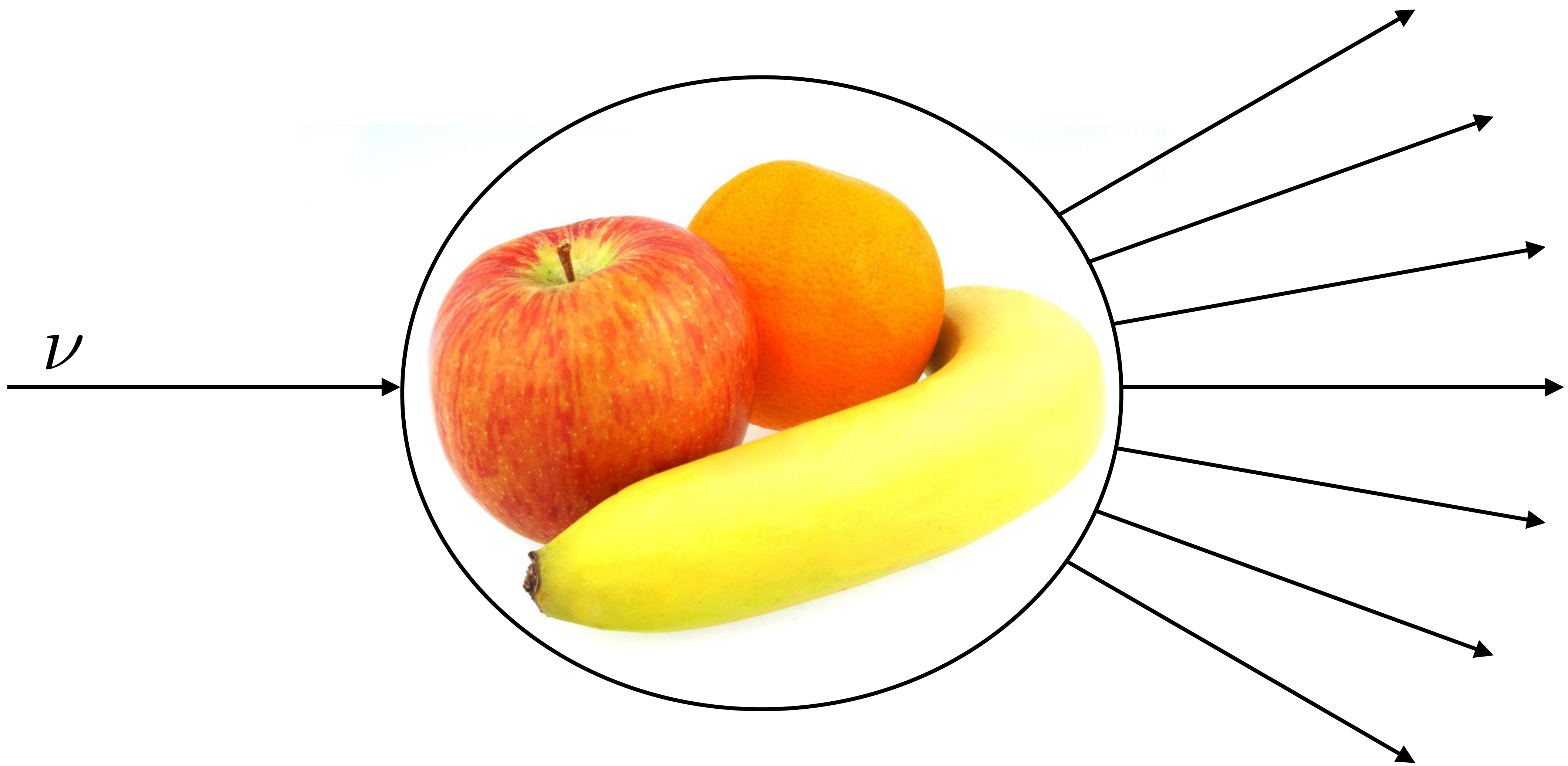



- measurement of $\nu_\mu(\overline{\nu}_\mu)$ disappearance and $\nu_e(\overline{\nu}_e)$ appearance

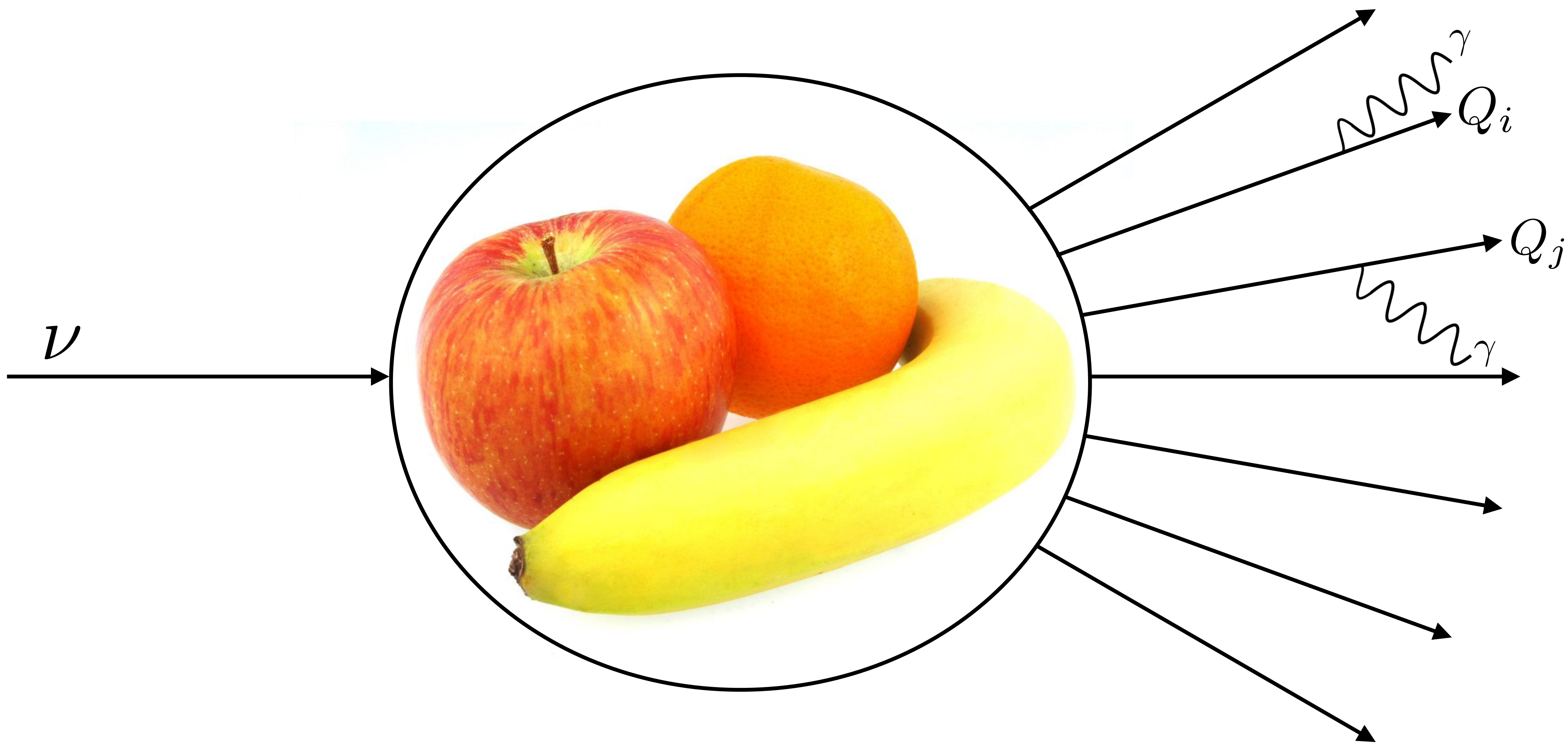
$$N_\nu \sim \int \mathrm{d}E_\nu \Phi_\nu\left(E_\nu\right) \times \sigma\left(E_\nu\right) \times R\left(E_\nu, E_\nu^{\mathrm{rec}}\right)$$

- near detector: determine flux and cross sections

# Neutrino interactions

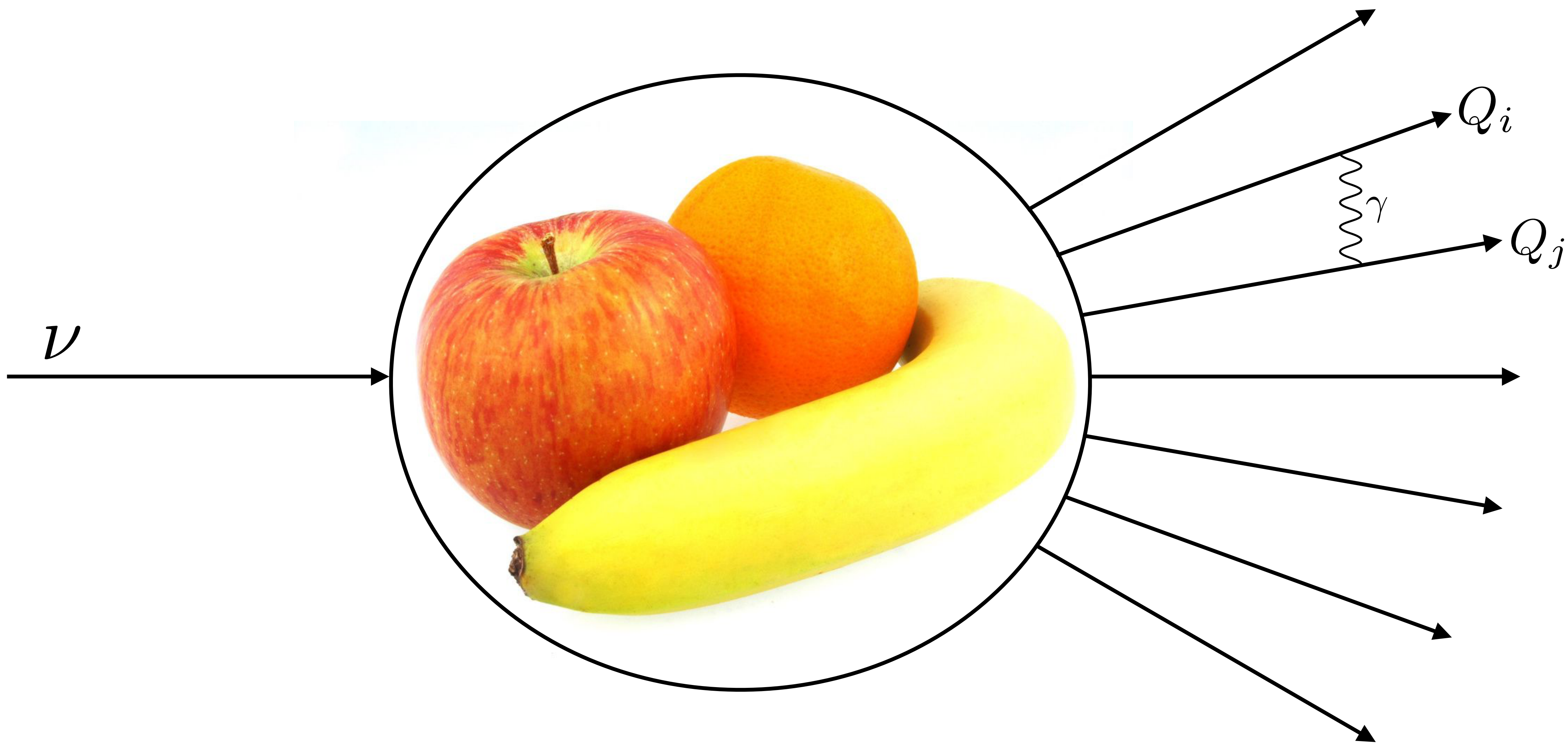


# QED corrections



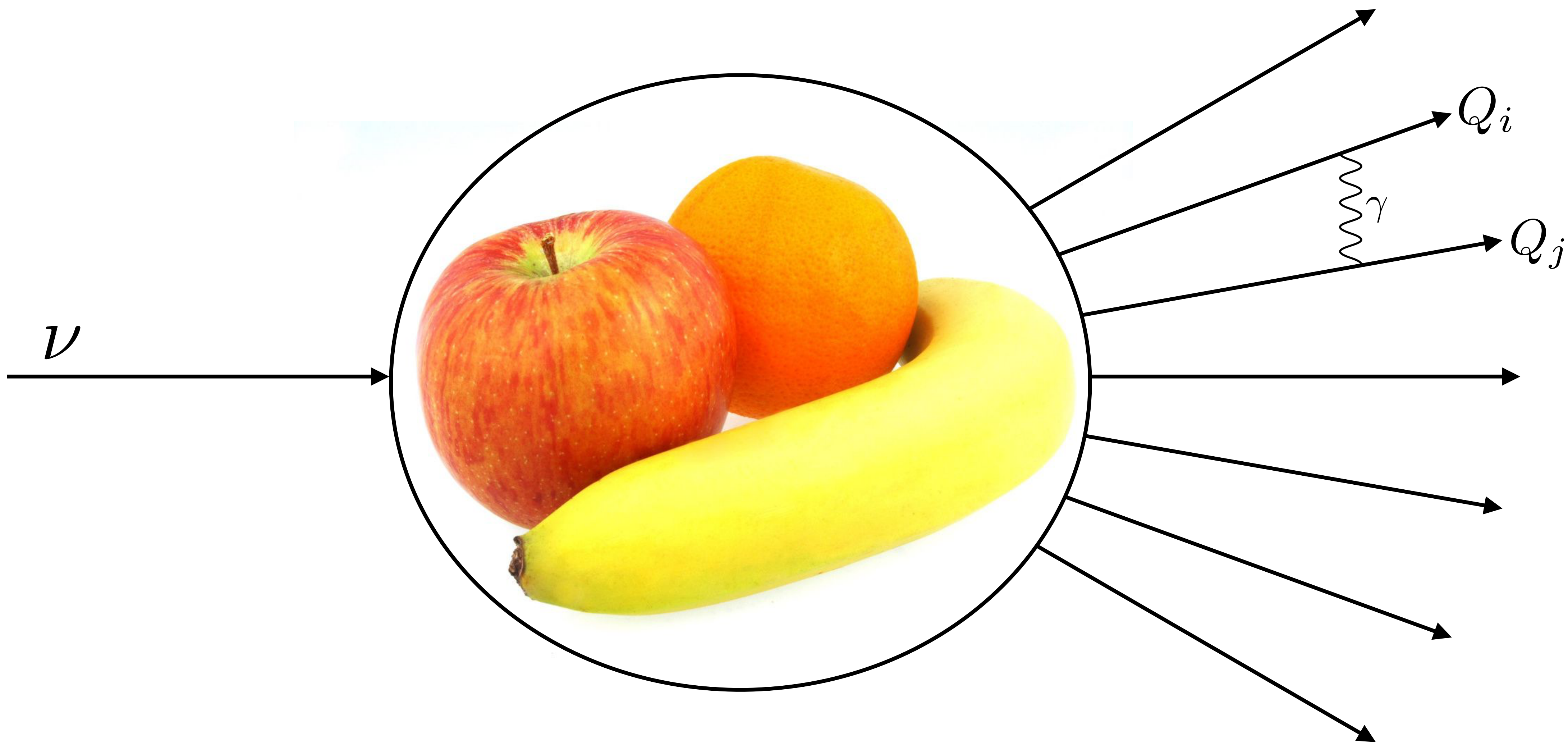

- all charged particles couple to real and virtual photons

# QED corrections




- all charged particles couple to real and virtual photons

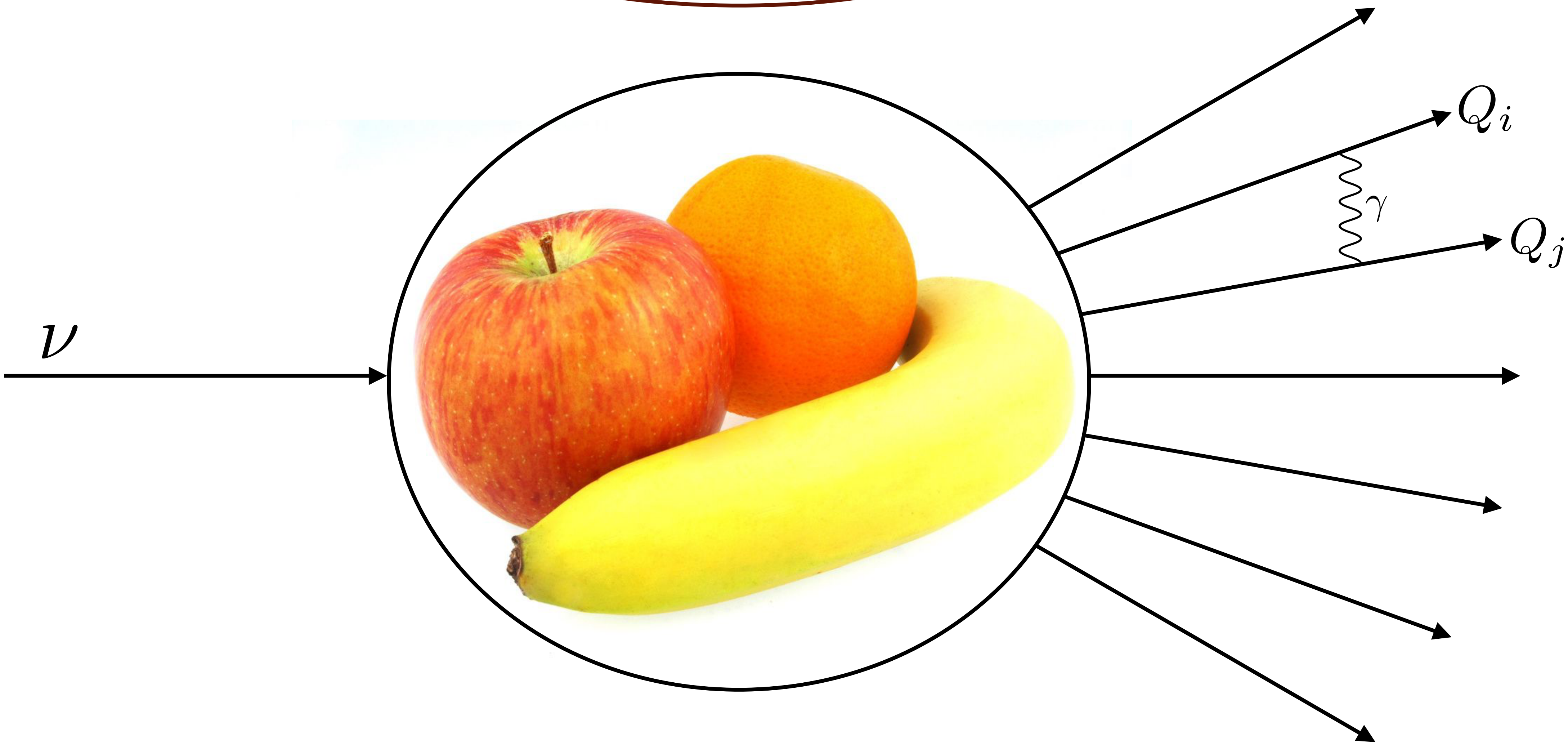
# QED corrections




- $\frac{\alpha}{\pi} \sim 0.2~\%$ suppression by electromagnetic coupling constant

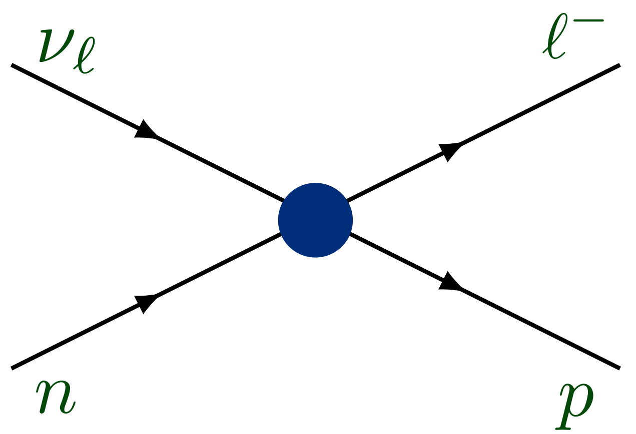
# QED corrections

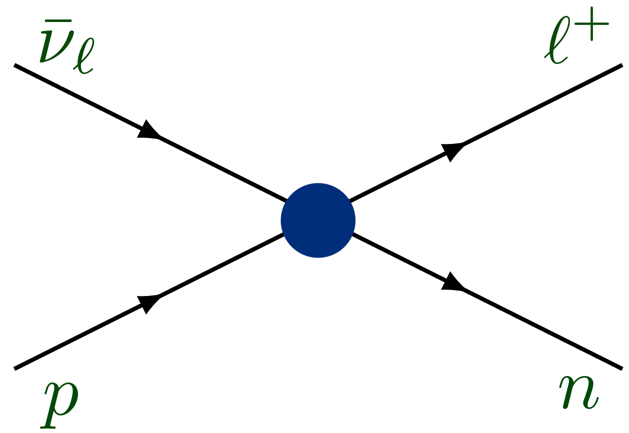
$m_e \ll m_\mu \ll E_\nu$




$\frac{\alpha}{\pi} \sim 0.2\ \%$ multiplied by $\ln \frac{E_\nu}{m_e} \sim 6 - 10$ or $\ln^2 \frac{E_\nu}{m_e} \sim 36 - 100$

- scale separation introduces large flavor-dependent QED logarithms



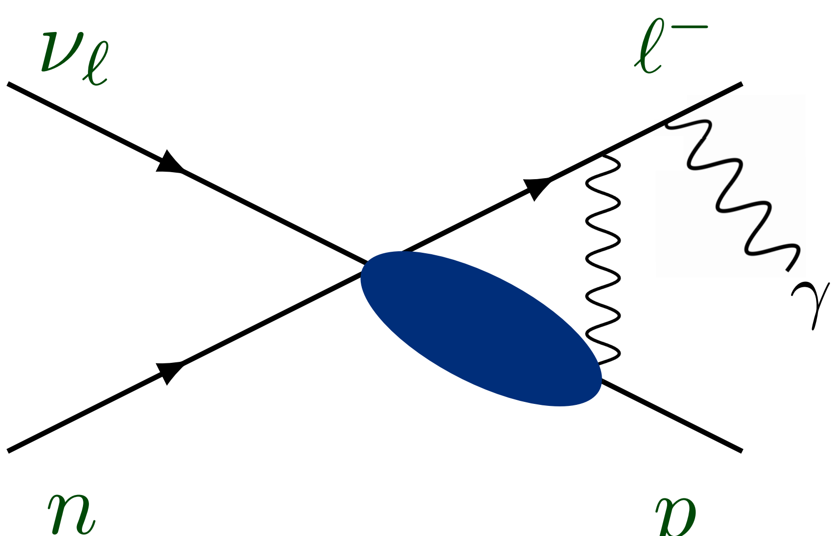




O. T., Qing Chen, Richard J. Hill and Kevin S. McFarland, Nature Commun. 13 (2022), 1, 5286

# Radiative corrections in charged-current elastic scattering on free nucleons

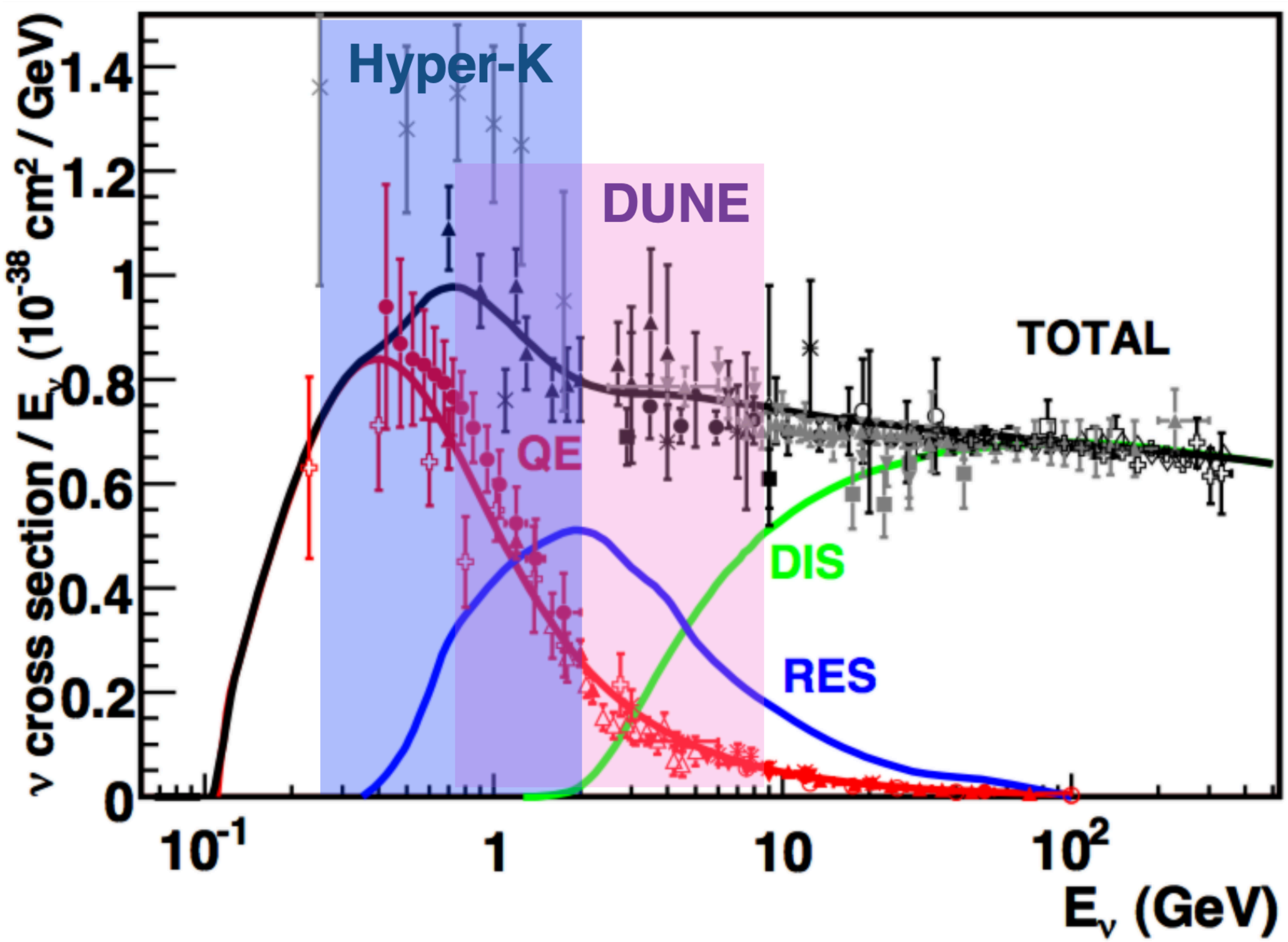
O. T., Qing Chen, Richard J. Hill, Kevin S. McFarland and Clarence Wret

editors suggestion in Phys. Rev. D (2022)



# CCQE. Why should we care?

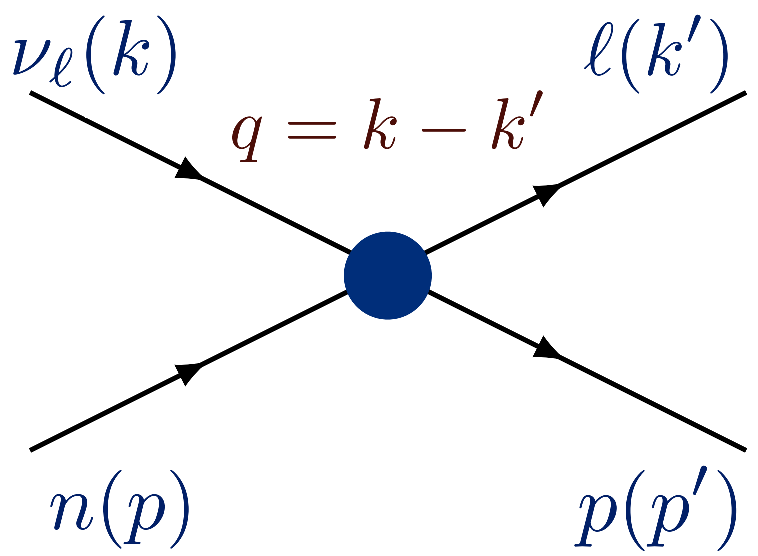
- neutrino-nucleus cross sections and future accelerator-based fluxes




Formaggio and Zeller (2013)

- basic process: bulk of events at Hyper-K and DUNE
- channel for reconstruction of neutrino energy

# CCQE scattering on free nucleon




neutrino energy $E_\nu$

momentum transfer $Q^2 = -q^2$

contact interaction at GeV energies

- assuming isospin symmetry, nucleon current:

$$\Gamma^\mu\left(Q^2\right) = < p|\bar{u}\left(\gamma^\mu - \gamma^\mu\gamma_5\right)d|n >$$

$$\Gamma^\mu(Q^2) = \gamma^\mu F_D^V(Q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2M}F_P^V(Q^2) + \gamma^\mu\gamma_5 F_A(Q^2) + \frac{q^\mu}{M}\gamma_5 F_P(Q^2)$$

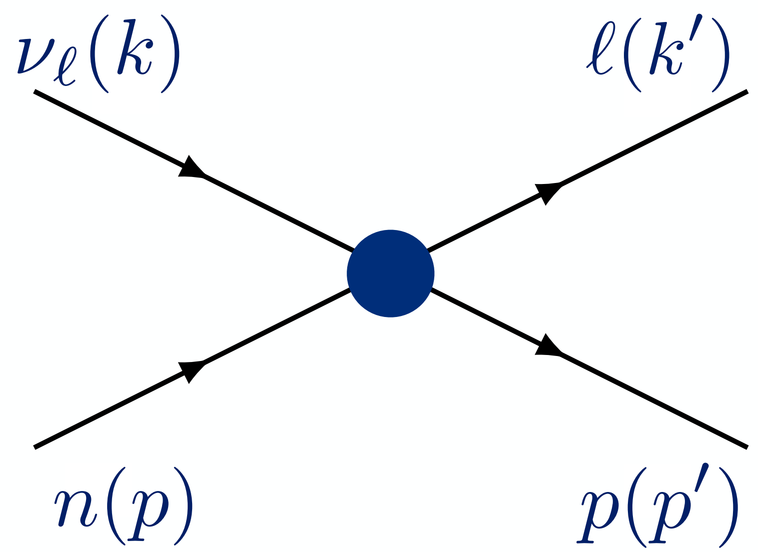
form factors: isovector Dirac and Pauli axial and pseudoscalar

$$F_{D,P}^V = F_{D,P}^p - F_{D,P}^n$$

tree-level amplitude

$$\mathrm{T} = \frac{\mathrm{G_F}V_{ud}}{\sqrt{2}}\left(\bar{\ell}(k')\gamma_\mu\left(1-\gamma_5\right)\nu_\ell(k)\right)\left(\bar{p}(p')\Gamma^\mu(Q^2)n(p)\right)$$

# CCQE scattering on free nucleon




$$\nu = E_\nu/M - \tau - r^2$$

$$r = \frac{m_\ell}{2M} \qquad \tau = \frac{Q^2}{4M^2}$$

unpolarized cross section

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2} \sim \frac{M^2}{E_\nu^2}\left(\left(\tau + r^2\right) A(Q^2) - \nu B(Q^2) + \frac{\nu^2}{1+\tau} C(Q^2)\right)$$

Llewellyn Smith (1972)

- structure-dependent functions

$$A = \tau\left(G_M^V\right)^2 - \left(G_E^V\right)^2 + (1+\tau)F_A^2 - r^2\left(\left(G_M^V\right)^2 + F_A^2 - 4\tau F_P^2 + 4F_A F_P\right)$$

$$B = \pm 4\tau F_A G_M^V \qquad C = \tau\left(G_M^V\right)^2 + \left(G_E^V\right)^2 + (1+\tau)F_A^2$$

- pseudoscalar form factor contribution is suppressed by lepton mass
- cross section is sensitive to both vector and axial contributions

# Elastic scattering on free nucleon

- only 3 experiments performed with deuterium bubble chamber

  direct access to form-factor shape

ANL 1982: 1737 events

BNL 1981: 1138 events

FNAL 1983: 362 events

world data: ~3200 events

Fermilab bubble chamber, Richard Drew

- axial form factor extracted based on electromagnetic structure

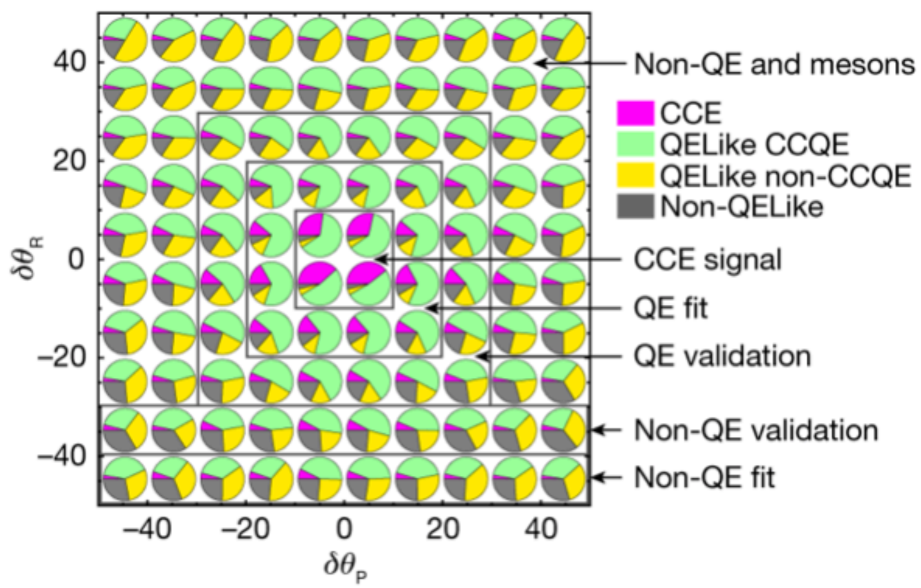
A.S. Meyer, M. Betancourt, R. Gran and R.J. Hill (2016)

# MINERvA result with free protons

- idea of scattering on molecular hydrogen realized !!!

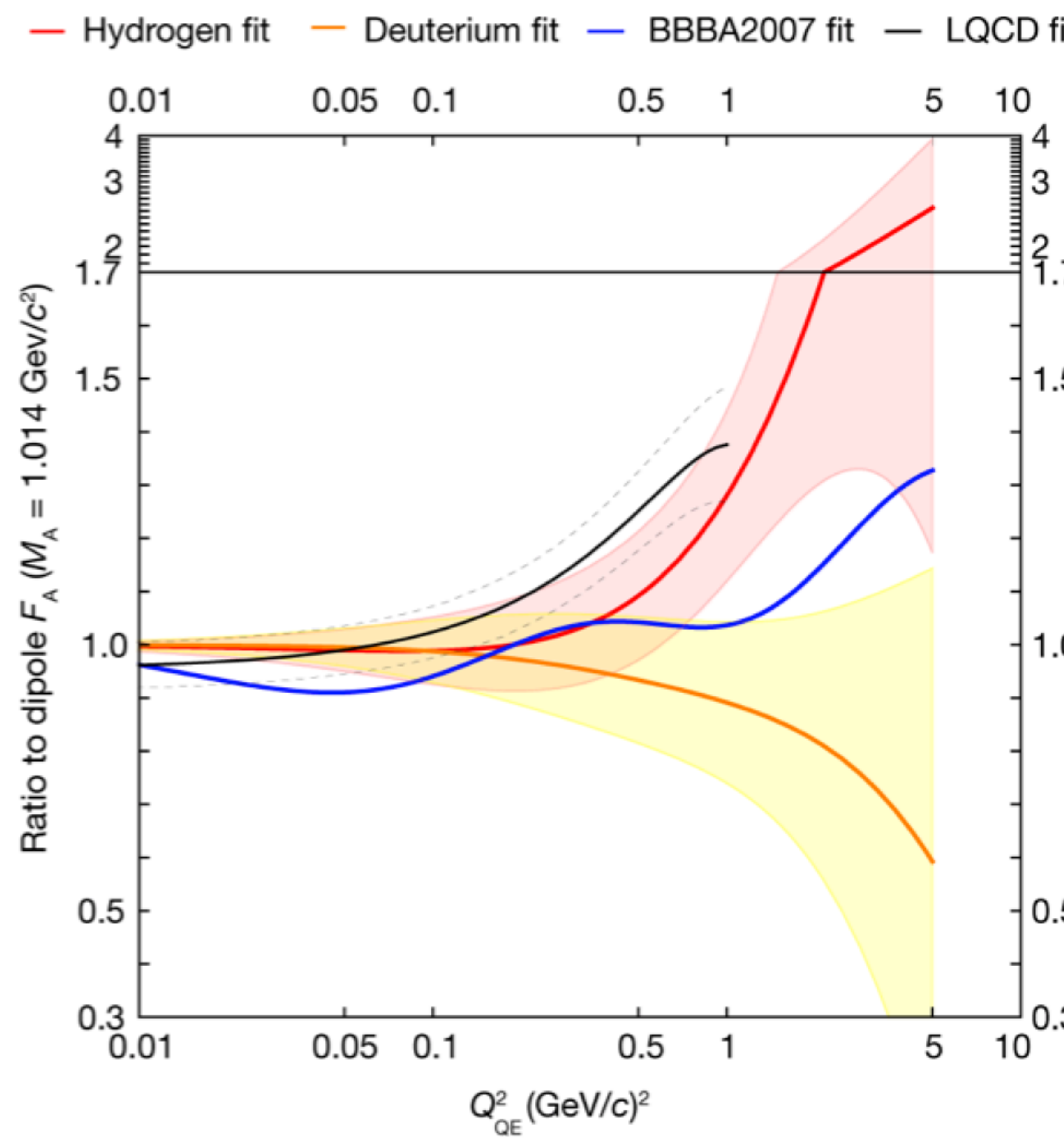
$$\overline{\nu}_\mu p \to \mu^+ n$$

muon kinematic selection




5580 events over
12500 background

background nuclear events constrained by scattering of $\nu$

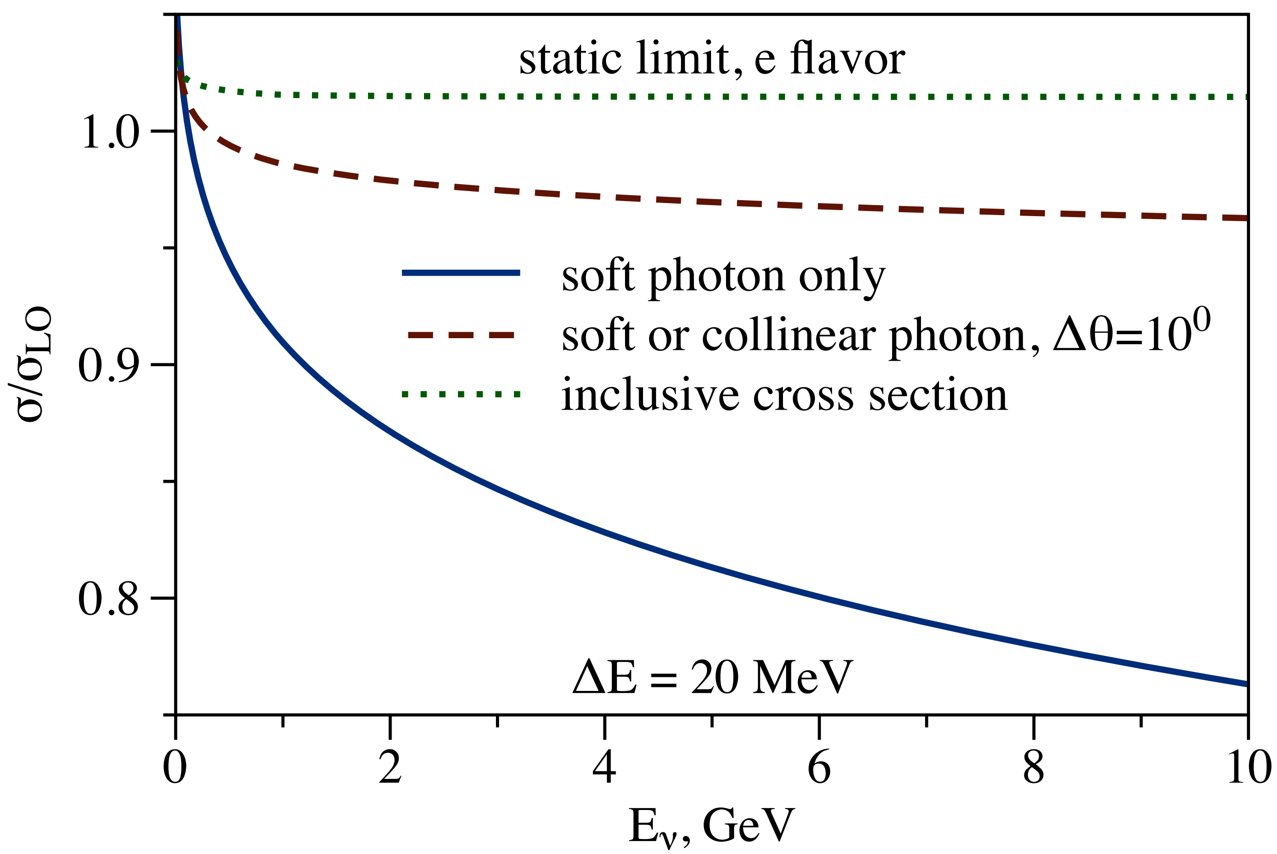


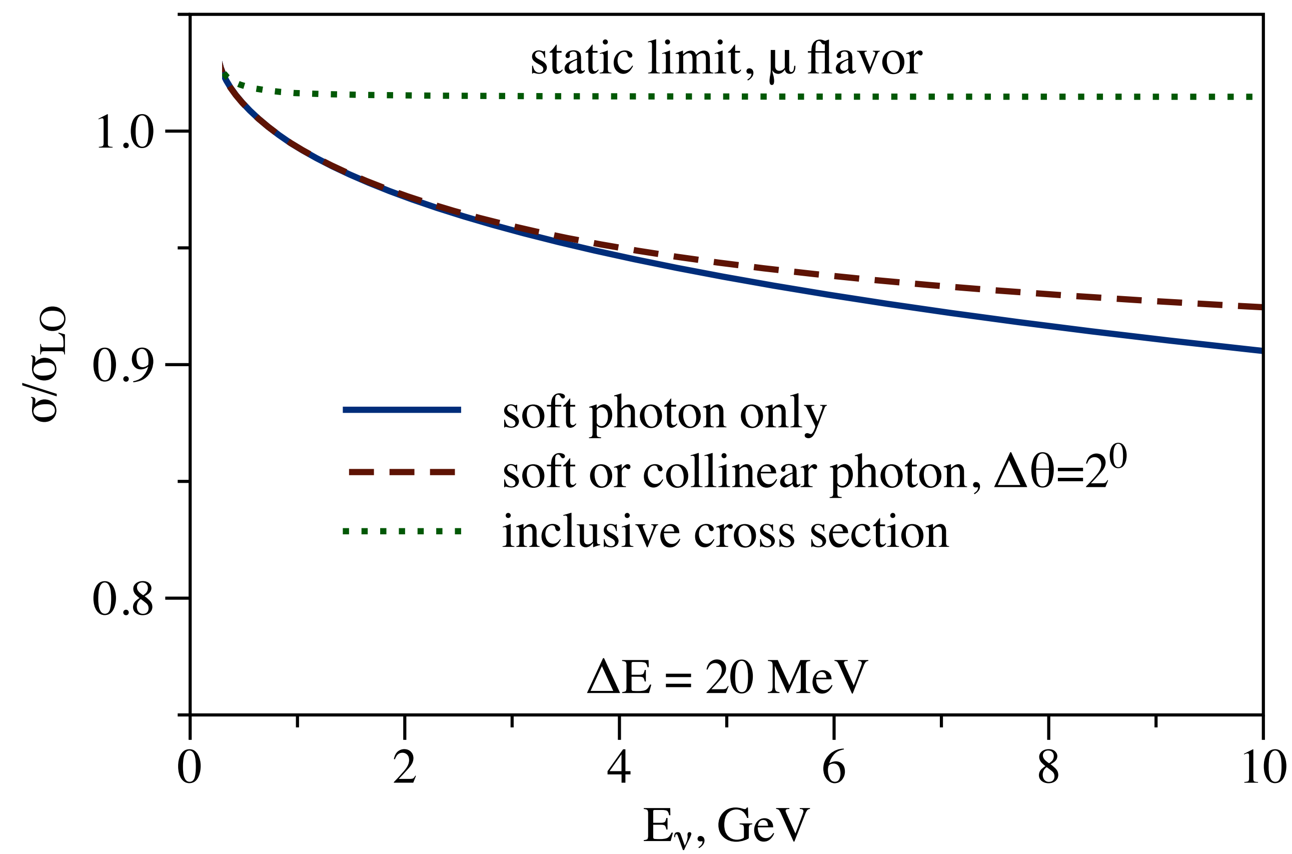

- 1st measurement of axial form factor on "free" protons $\overline{\nu}_\mu p \to \mu^+ n$

T. Cai et al., MINERvA Collaboration, Nature (2023), 614, 48-53

# Static nucleon limit

- formal limit of infinitely heavy nucleus $m_\ell \ll E_\ell \ll M$
- provides correct soft and collinear logarithms
- soft-photon energy < 20 MeV, jet size: 10° for electron and 2° for muon

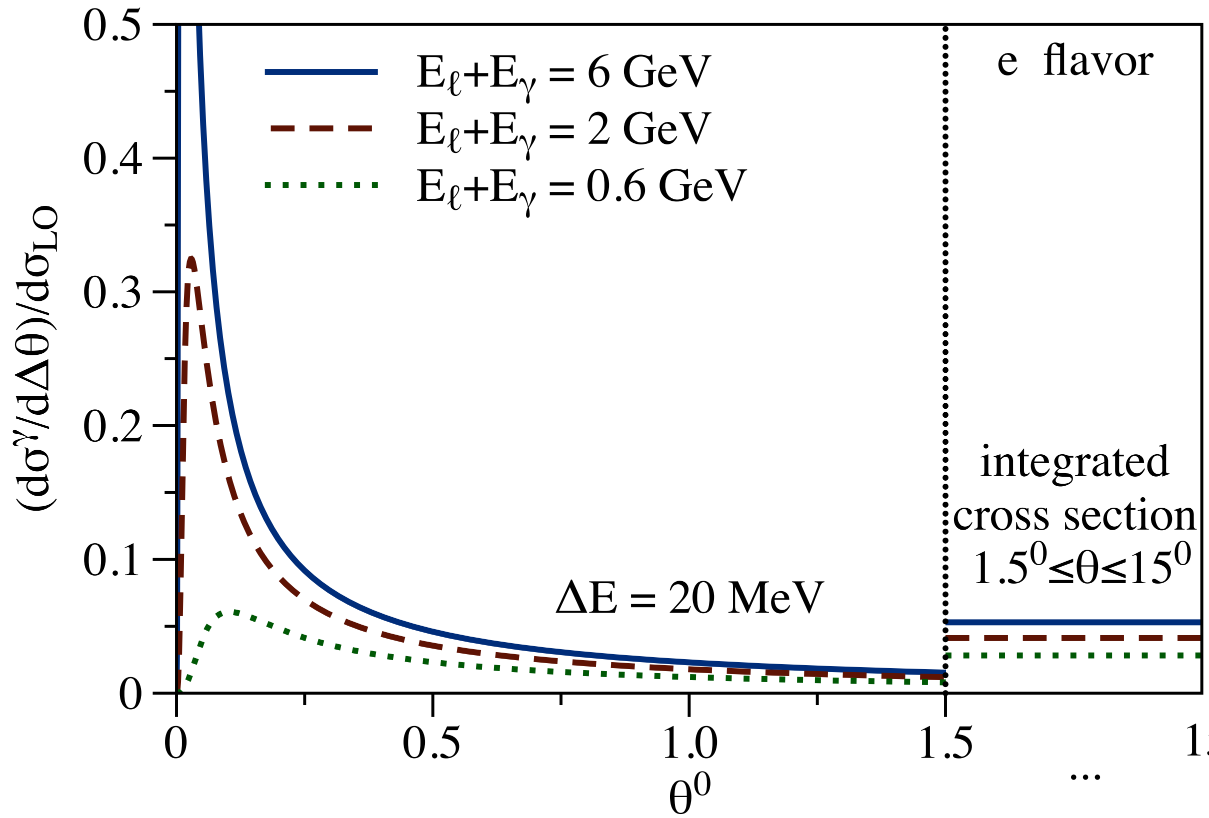


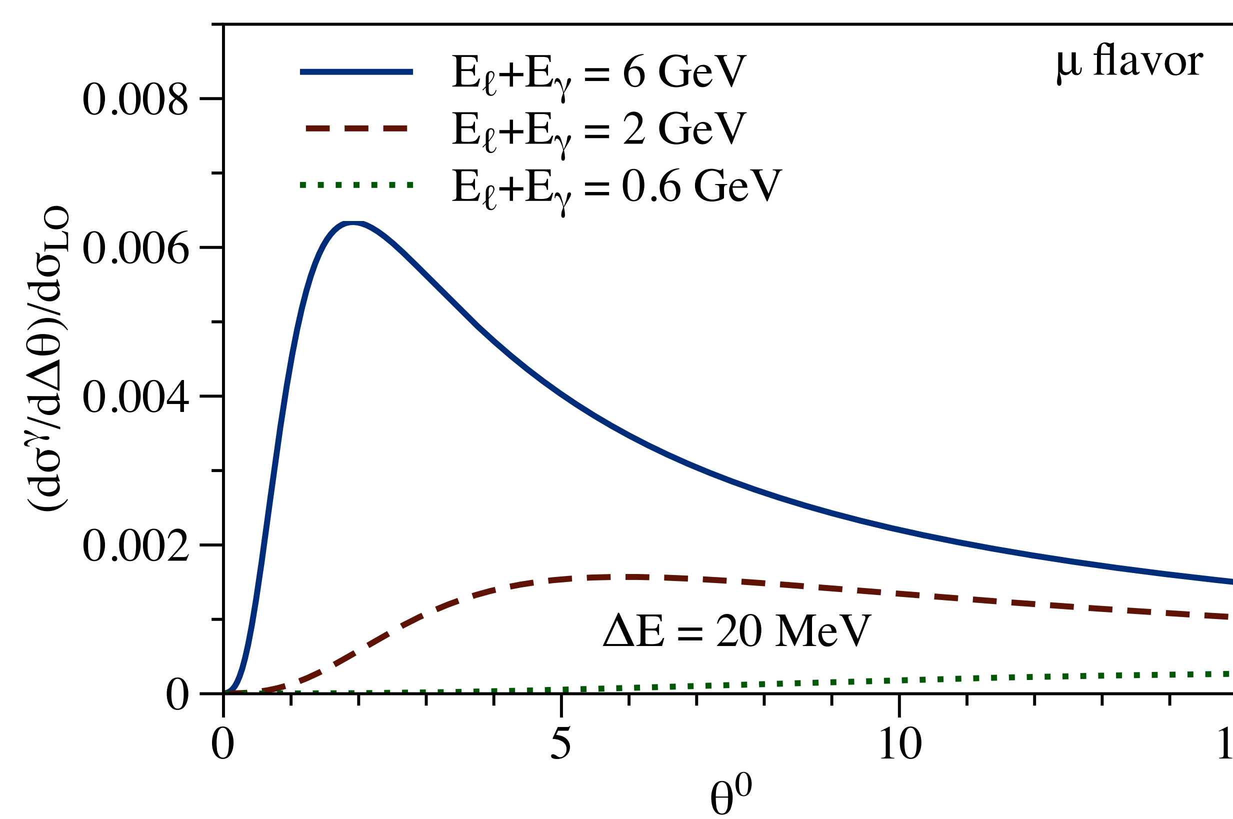




- flavor-dependent effect, same for $\nu_\ell n \to \ell^- p$ vs $\bar{\nu}_\ell p \to \ell^+ n$
- collinear observable: cancellation of virtual vs real logs
- inclusive observables (+$\gamma$): few % level, flavor independent

# Electron vs muon jets

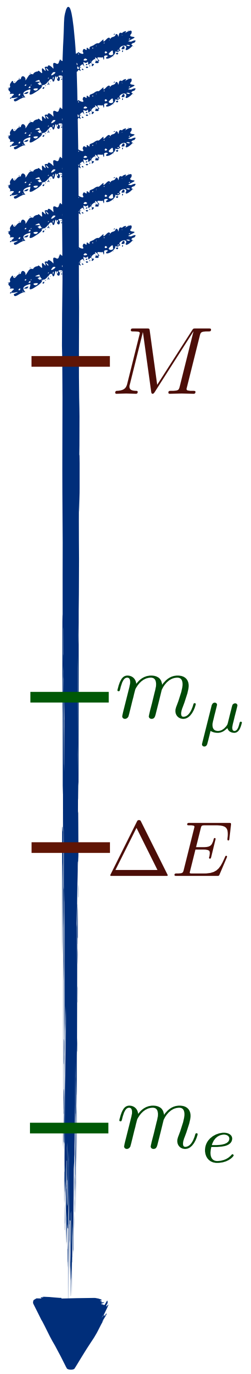
- factorization for radiation of collinear photons
- cone angle is defined to lepton direction
- photons of energy > 20 MeV, fixed energy in the cone




- flavor-dependent effect, same for $\nu_\ell n \to \ell^- p$ vs $\bar{\nu}_\ell p \to \ell^+ n$
- forward-peaked radiation for electron flavor
- negligible radiation for muons with shifted peak position

# Factorization approach



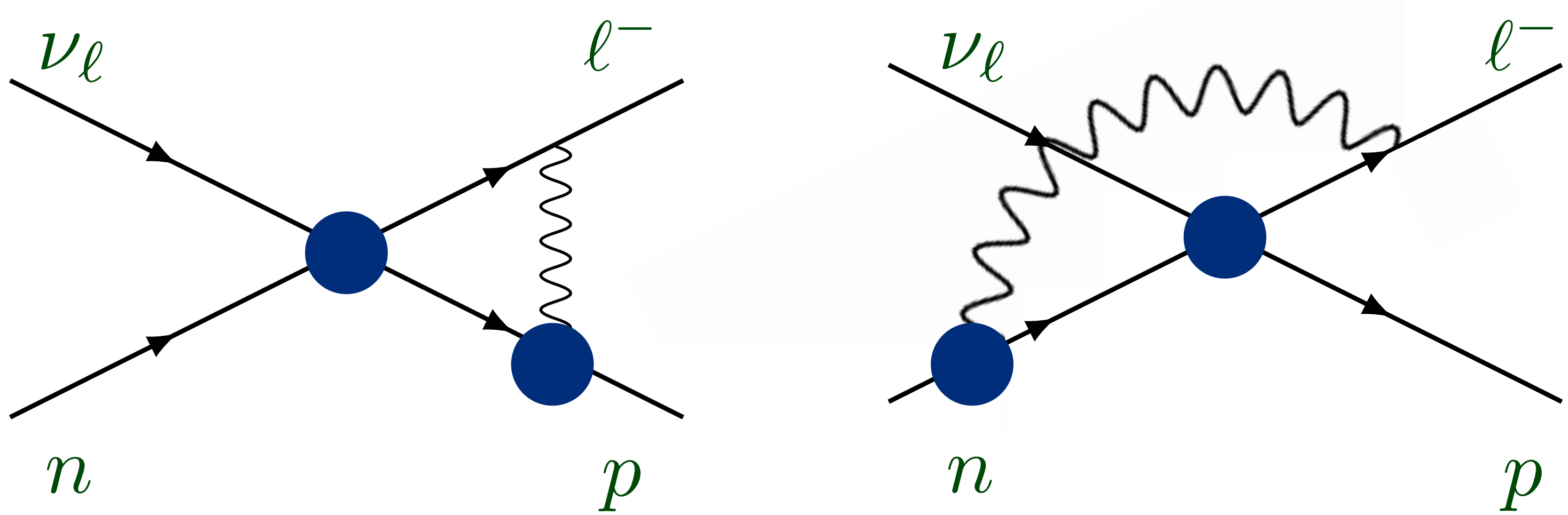

- cross section is given by factorization formula

$$d\sigma \sim S\left(\frac{\Delta E}{\mu}\right) J\left(\frac{m_\ell}{\mu}\right) H\left(\frac{M}{\mu}\right)$$

- determine hard function at hard scale by matching experiment or hadronic model to the theory with heavy nucleon

- soft and collinear functions are evaluated perturbatively

# Hadronic model at GeV scale




- exchange of photon between the charged lepton and nucleons

- assume onshell form for each interaction with dipole form factors

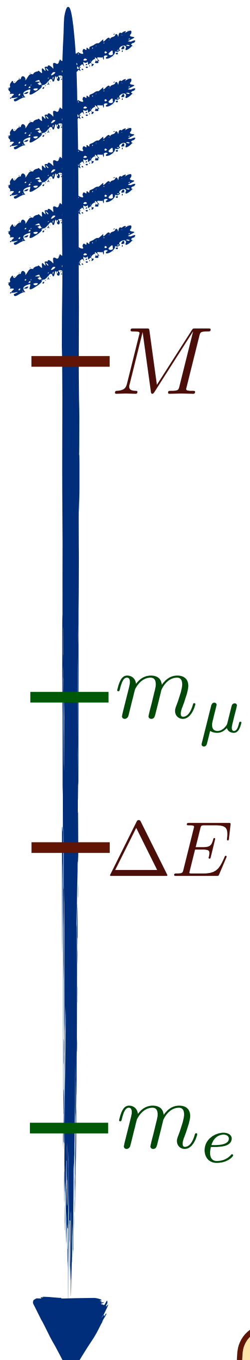
discussed for neutrino-nucleon scattering: Graczyk (2013)

- add self energy for charged particles

- reproduce soft and collinear regions of SCET

- best determination of hard function

# Factorization approach




- cross section is given by factorization formula

$$d\sigma \sim S\left(\frac{\Delta E}{\mu}\right) J\left(\frac{m_\ell}{\mu}\right) H\left(\frac{M}{\mu}\right)$$

- determine hard function at hard scale by matching experiment or hadronic model to the theory with heavy nucleon

- RGE evolution of the hard function to scales $\Delta E, m_\ell$

- soft and collinear functions are evaluated perturbatively

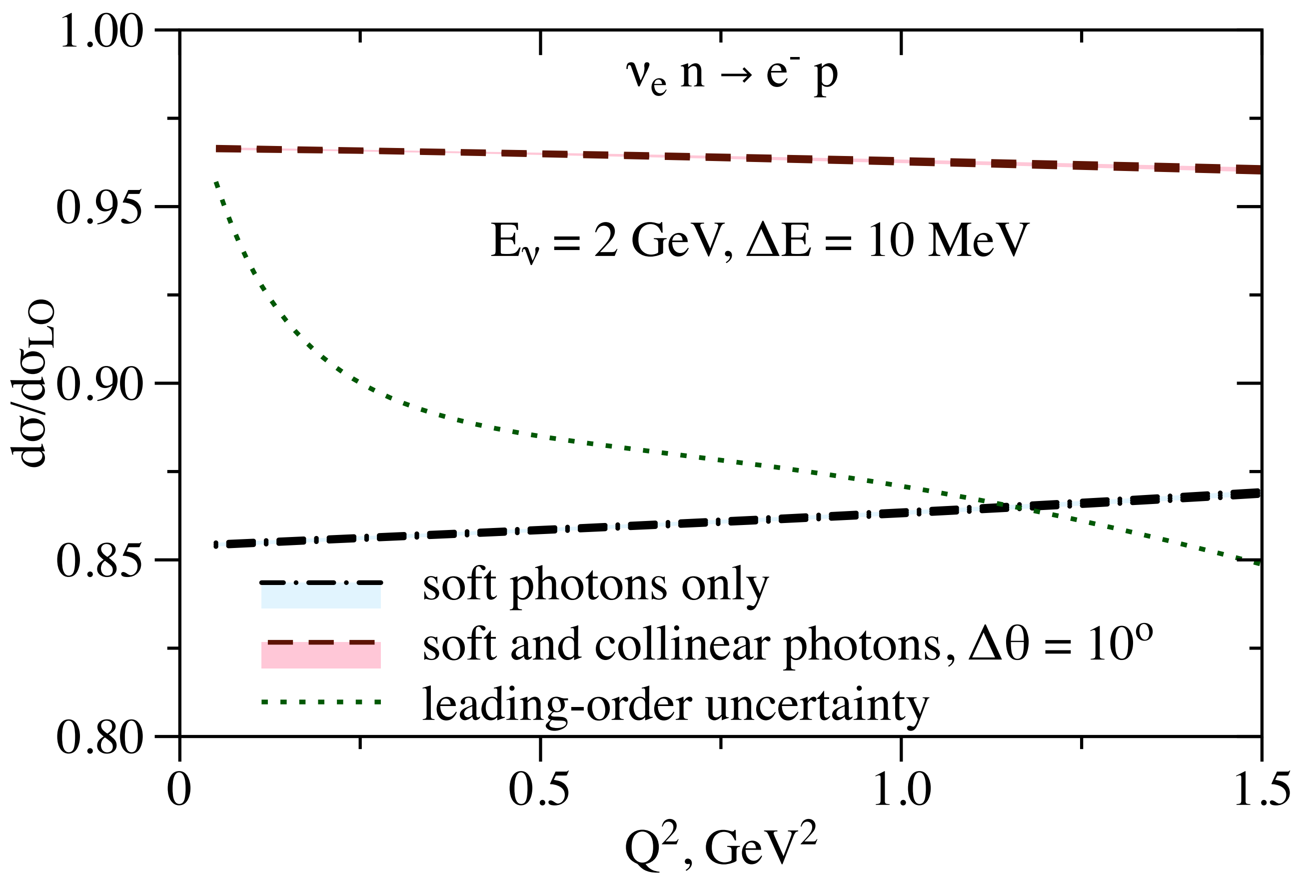
- calculate cross section at low energies accounting for all large logs

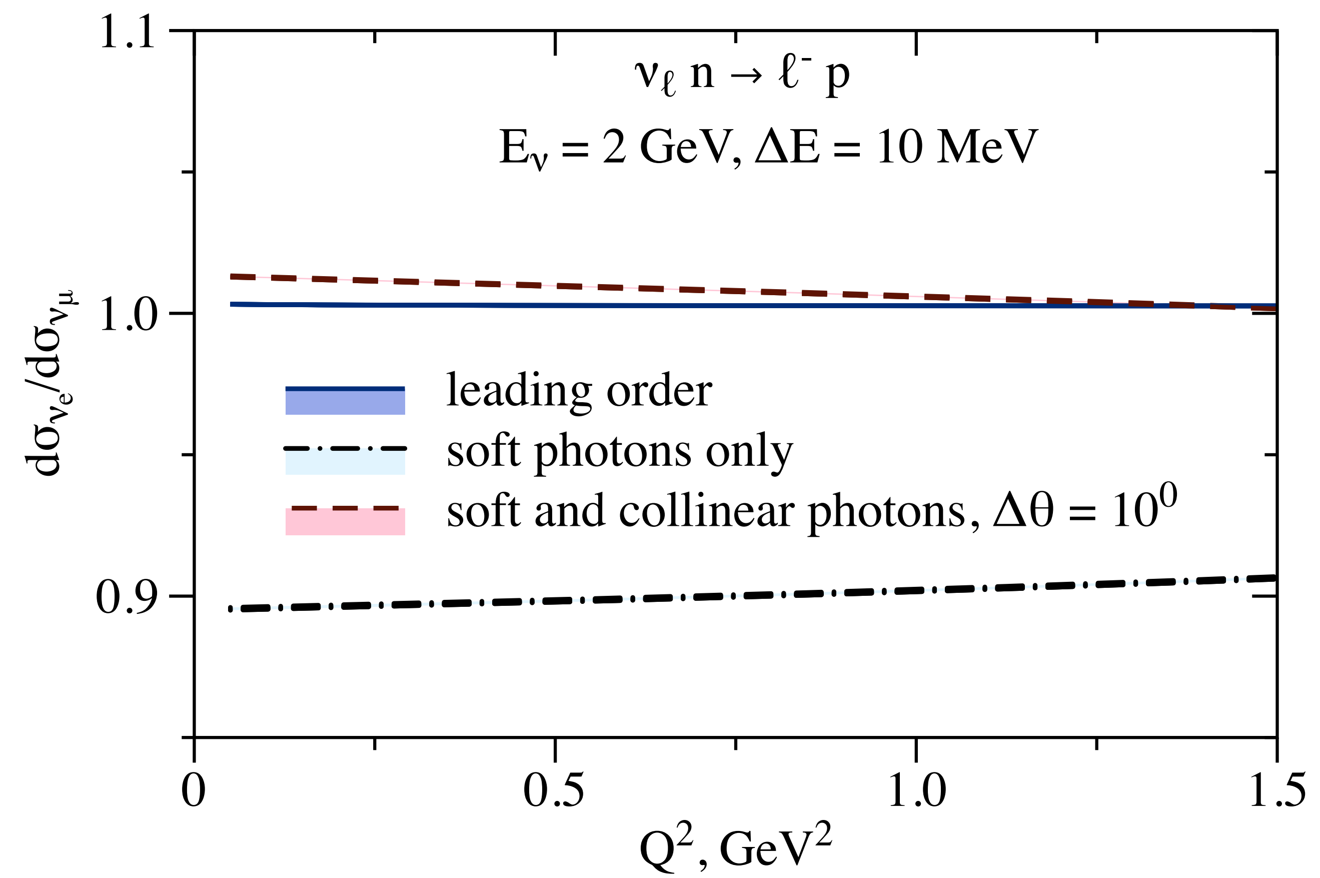
ep scattering with soft radiation only: Richard J. Hill (2016)

- soft and collinear functions determined analytically
- hard function describes physics at GeV energies

# Exclusive observables

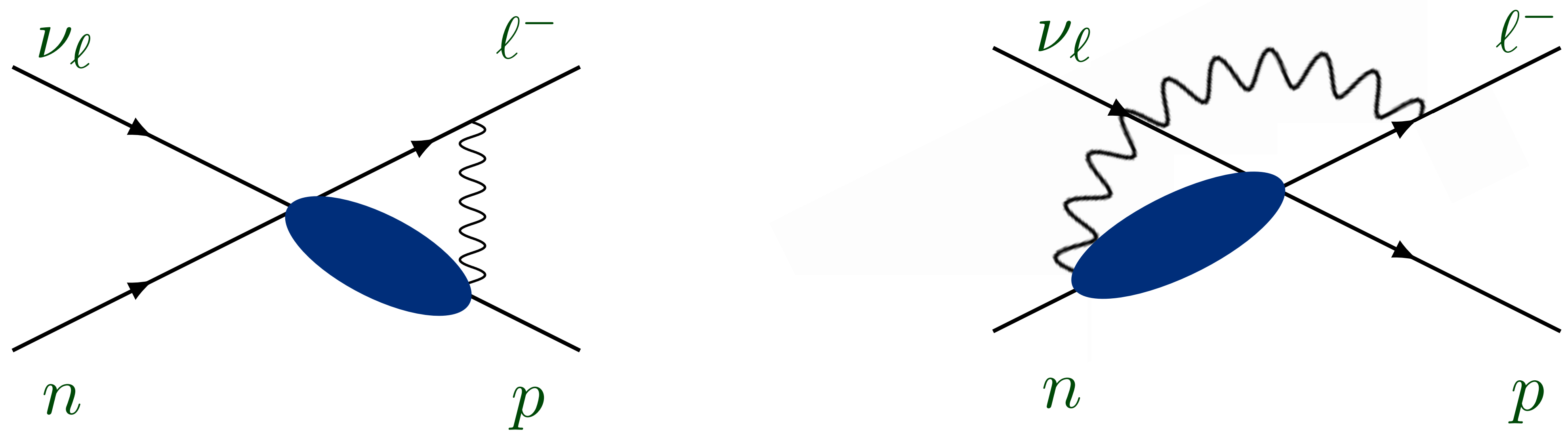
- cancellation of uncertainties from hard function for e/$\mu$ and ratio to LO



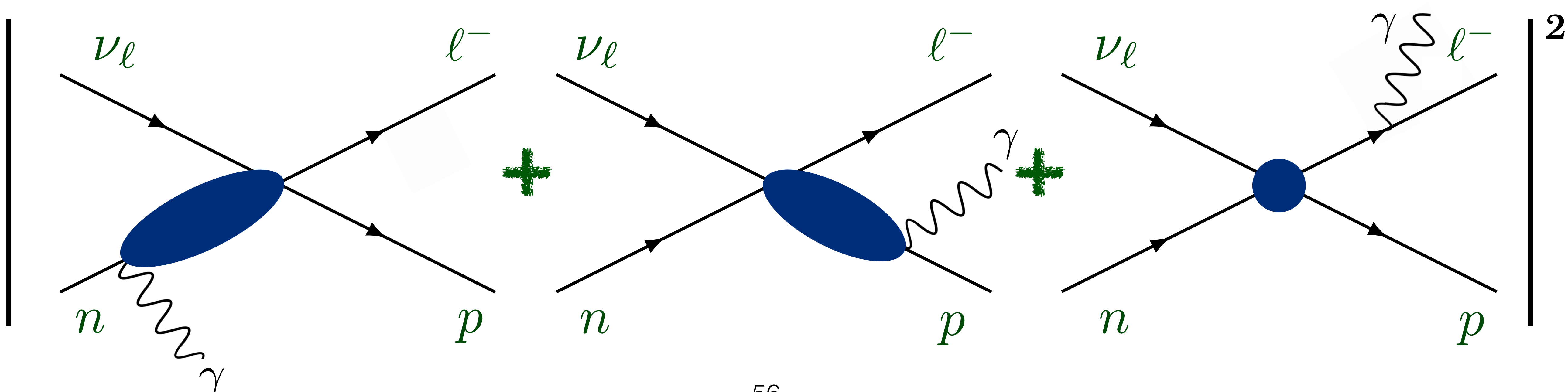

- ratios: cancellation of uncertainty from hard function

# Inclusive observables



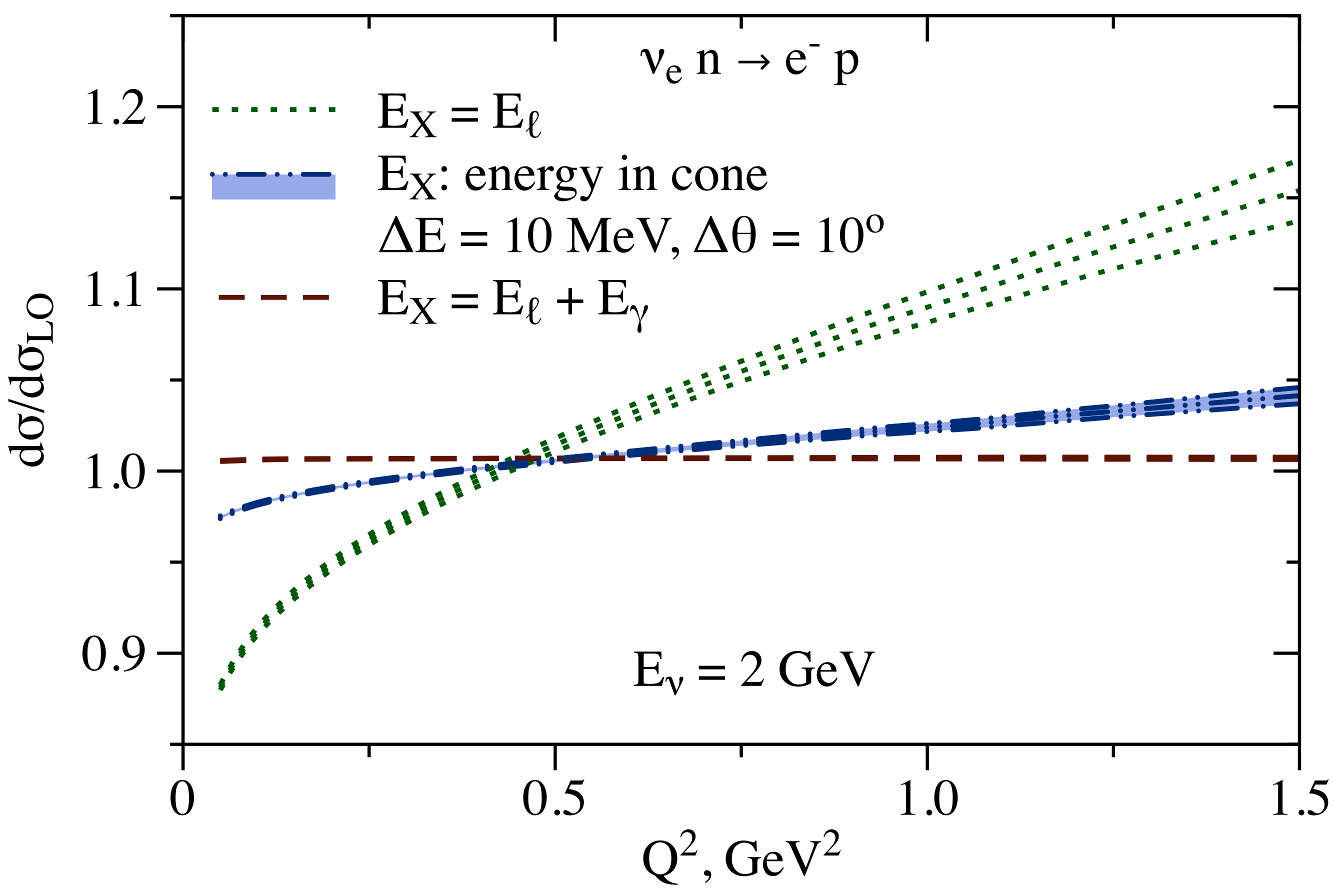

- the same gauge-invariant model for the real radiation

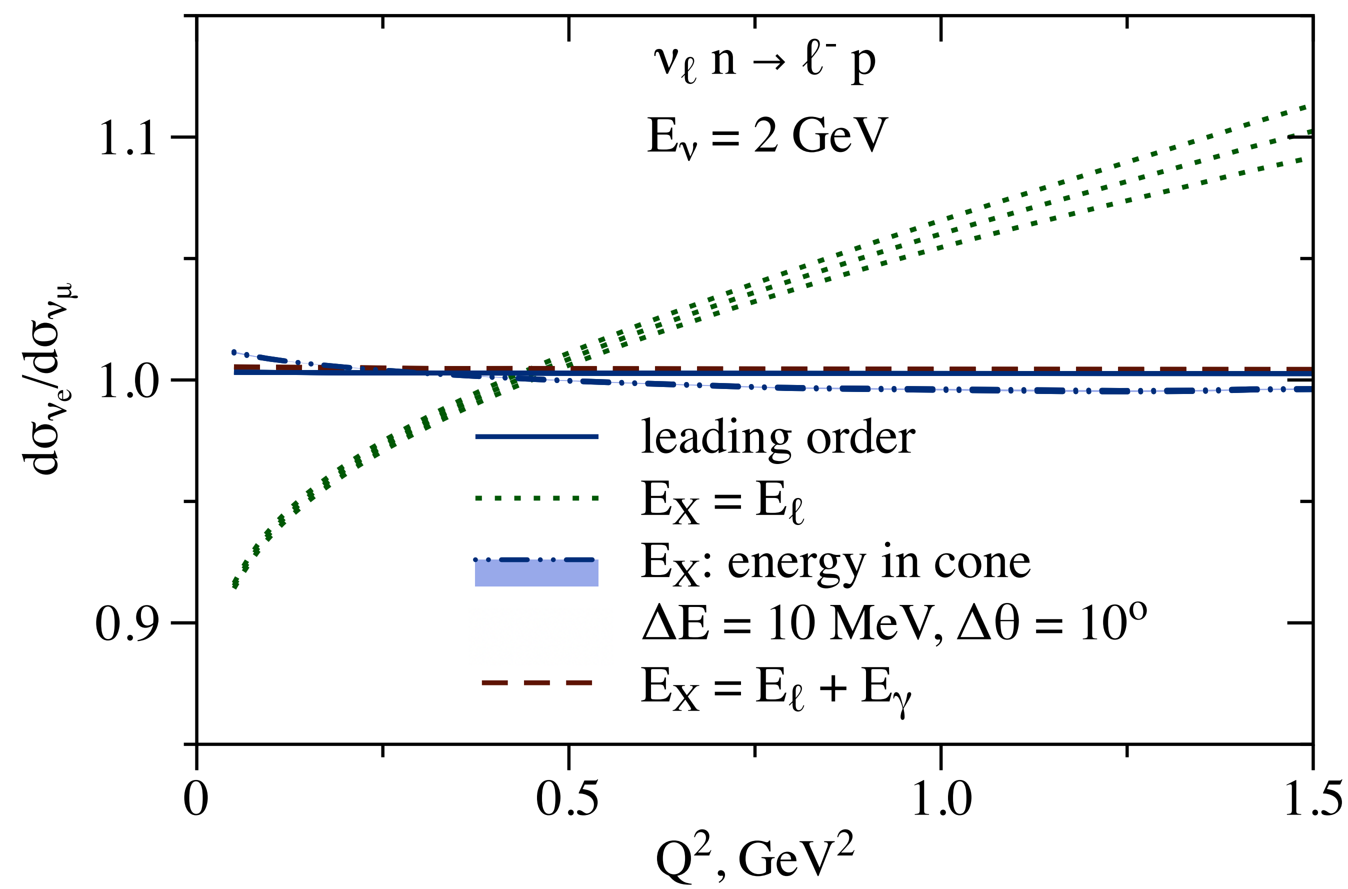
- arbitrary hard photons are part of the observable



# Inclusive observables

- kinematics $Q^2 = 2M\,(E_\nu - E_X)$ is reconstructed with 3 different $E_X$




- dependence on reconstruction of kinematics and cuts
- predict $\sigma_{\nu_e}$ from $\sigma_{\nu_\mu}$ measurements with neutrino beam ✓

# Electron/muon ratio

| | $E_\nu$, GeV | | $\left(\frac{\sigma_e}{\sigma_\mu}-1\right)_{\mathrm{LO}}$, % | $\frac{\sigma_e}{\sigma_\mu}-1$, % |
|---|---|---|---|---|
| T2K/HyperK | 0.6 | $\nu$ | $2.47 \pm 0.06$ | $2.84 \pm 0.06 \pm 0.37$ |
| | | $\bar{\nu}$ | $2.04 \pm 0.08$ | $1.84 \pm 0.08 \pm 0.20$ |
| NOvA/DUNE | 2.0 | $\nu$ | $0.322 \pm 0.006$ | $0.54 \pm 0.01 \pm 0.22$ |
| | | $\bar{\nu}$ | $0.394 \pm 0.003$ | $0.20 \pm 0.01 \pm 0.19$ |

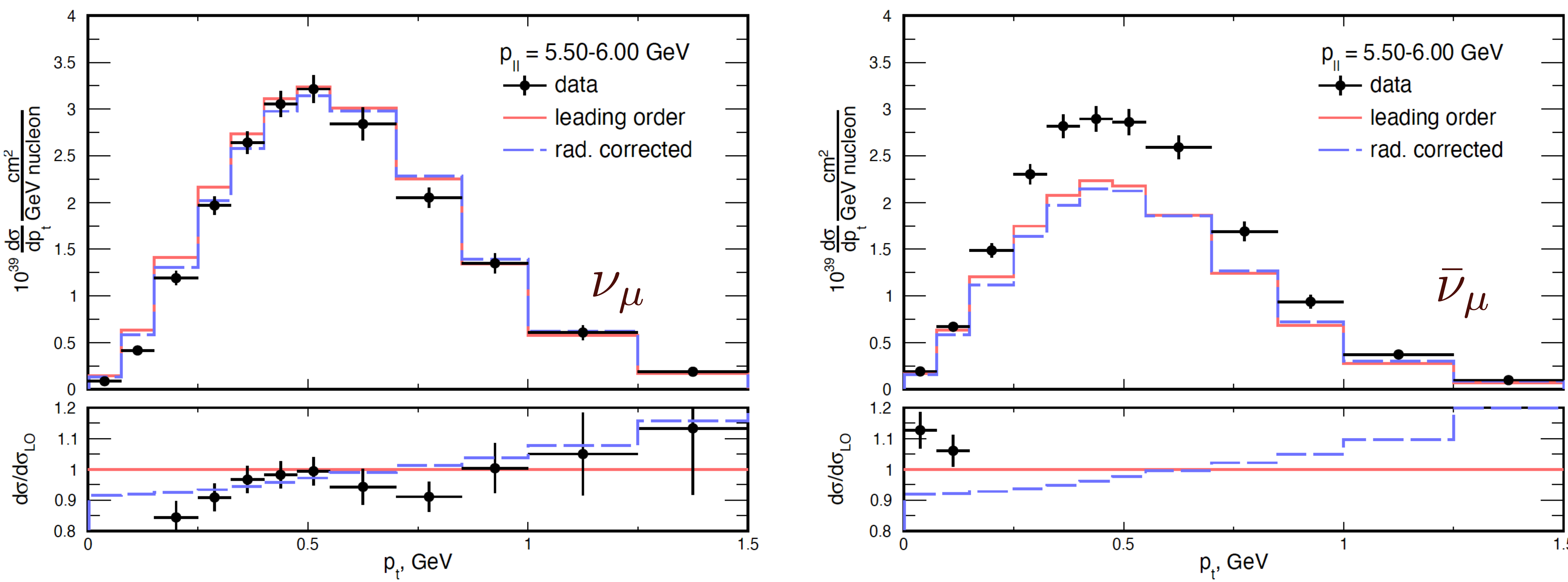
TABLE II: Inclusive electron-to-muon cross-section ratios for neutrinos and antineutrinos without kinematic cuts. Uncertainties at leading order are from vector and axial nucleon form factors. For the final result, we include an additional hadronic uncertainty from the one-loop correction to the first uncertainty, and provide a second uncertainty as the magnitude of the radiative correction.

$$\frac{\sigma\left(m_\ell \to 0\right)}{\sigma\left(m_\ell = 0\right)} \approx 1 + A m_\ell^2 + \alpha B m_\ell^2 \ln m_\ell$$

- inclusive cross sections and flavor ratios determined by KLN
- nuclear effects: suppressed by expansion parameters squared

# Comparison to data

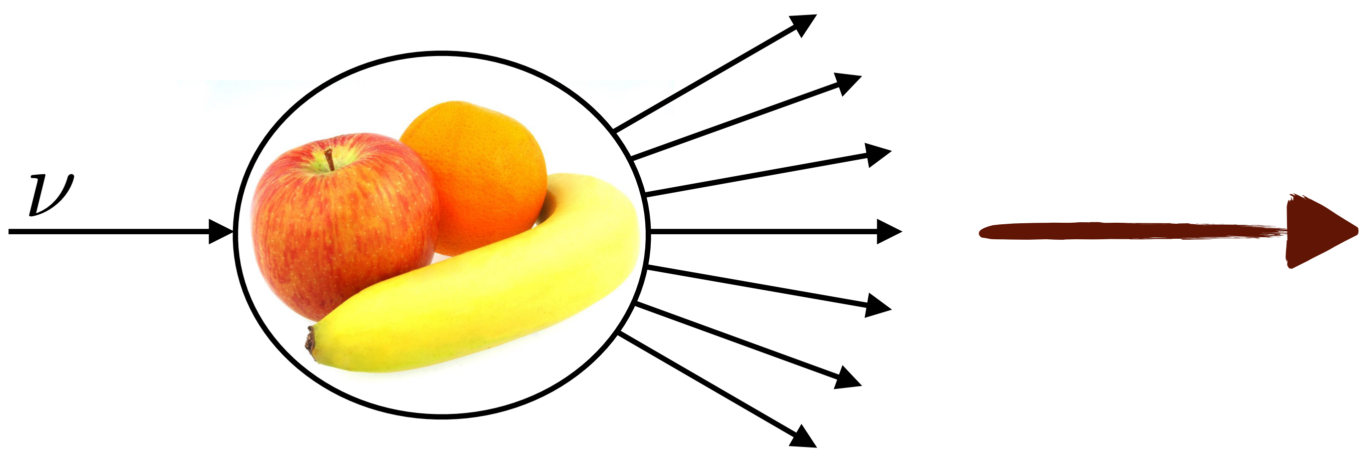
- medium-energy flux data from MINERvA@FERMILAB




O. T., Qing Chen, Richard J. Hill, Kevin S. McFarland and Clarence Wret
editors suggestion, Phys. Rev. D 106, 093006 (2022)

- electron flavor: measurements are uncertain
- muon flavor: comparable to experimental precision

# Conclusions




radiative corrections in EFT framework

- radiative corrections to neutrino-nucleon cross sections formulated in factorization framework

- charged-current elastic electron vs muon cross-section ratios evaluated from theory with sub-percent uncertainty

- ongoing work on applications

Thanks for your attention !!!