Restrictions on the RG flow of effective field theories

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based mostly on *2301.09995*

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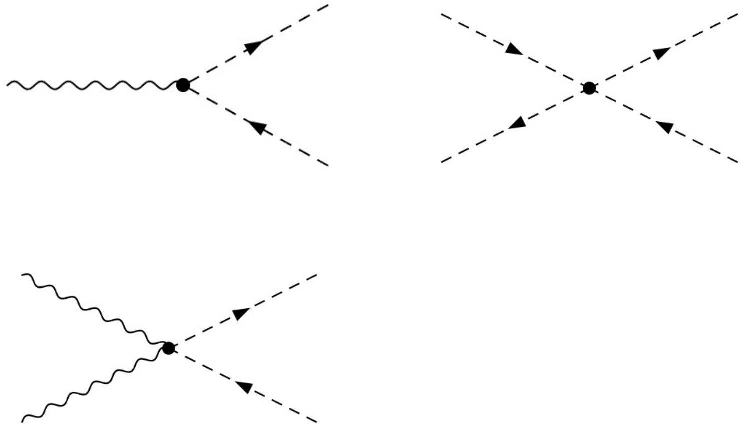
Evolution of scale-dependent parameters in QFT:

(1) Well known for renormalisable theories

(2) Some *aspects* of EFTs well understood

$$\mathcal{L}_{\mathrm{SM}} = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} + D_\mu \phi^\dagger D^\mu \phi - \lambda |\phi|^4 + \cdots$$

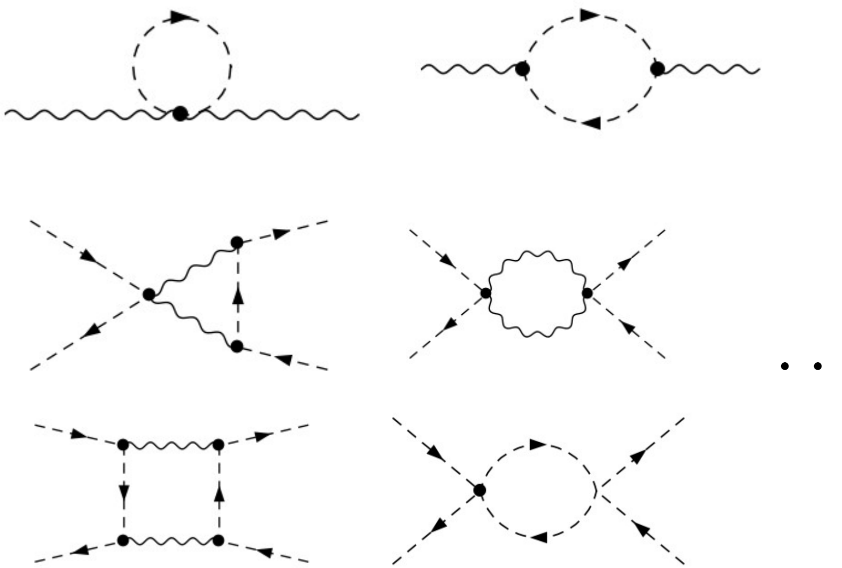


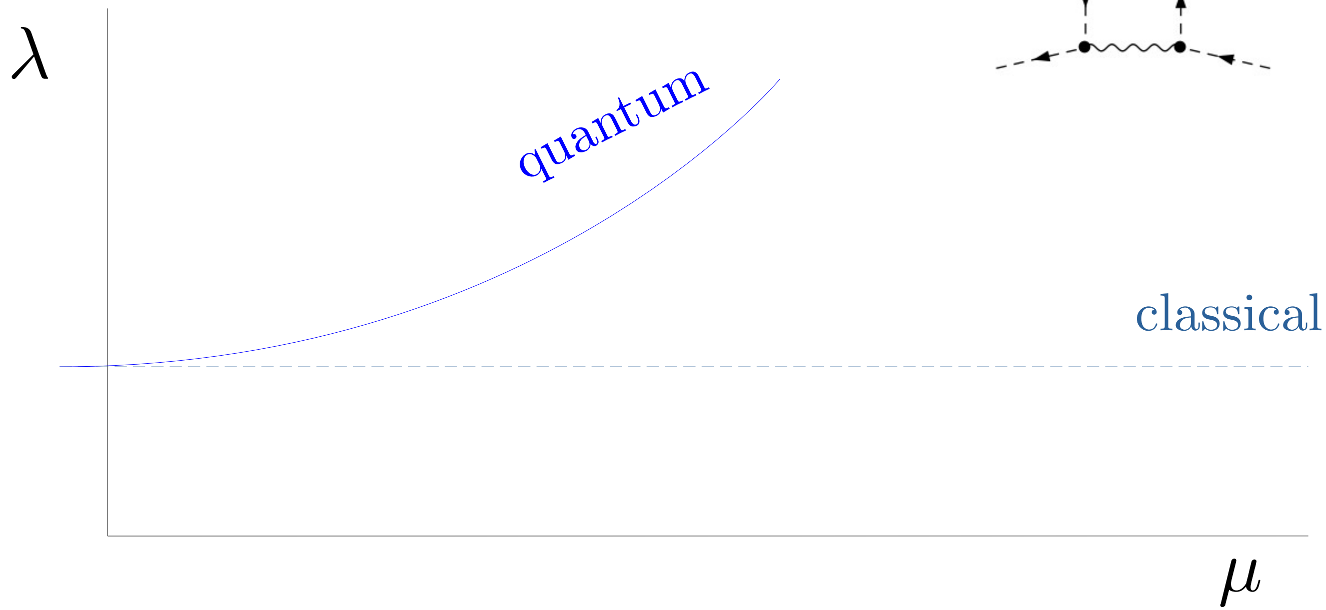
g_1, λ

Evolution of scale-dependent parameters within simplified SM

$$\beta_{g_1} = \frac{41}{6} g_1^3 + 0 \times \lambda$$

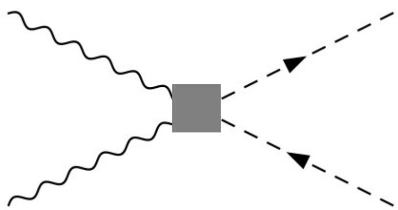
$$\beta_\lambda = \frac{3}{8} g_1^4 + 24\lambda^2$$



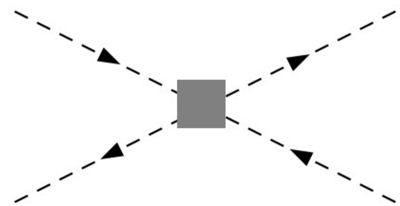


Things change at higher dimensions

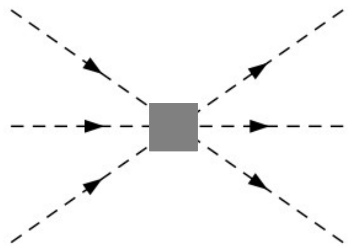
$\mathcal{O}(\frac{1}{\Lambda^2})$



$c_{\phi^2 B^2}$



$c_{\phi^4 D^2}$



c_{ϕ^6}

$$\dot{c}_i \equiv \beta_{c_i} = \gamma_{ij} c_j$$

anomalous dimension matrix
(involves SM couplings only)

Things change at higher dimensions

$$\gamma_{ij} = \begin{array}{cc} & \begin{array}{ccc} c_{\phi^2 B^2} & c_{\phi^4 D^2} & c_{\phi^6} \end{array} \\ \begin{array}{c} c_{\phi^2 B^2} \\ c_{\phi^4 D^2} \\ c_{\phi^6} \end{array} & \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ \times & \times & \times \end{pmatrix} \end{array}$$

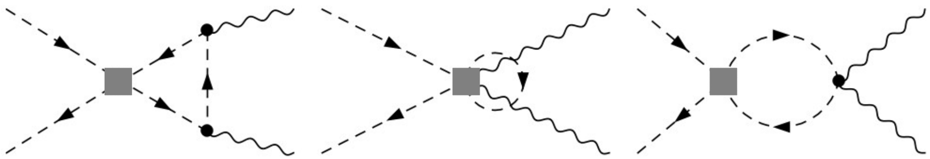
Things change at higher dimensions

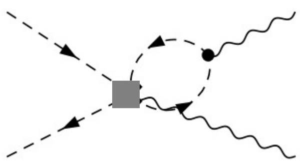
$$\gamma_{ij} = \begin{array}{c c} & \begin{array}{ccc} c_{\phi^2 B^2} & c_{\phi^4 D^2} & c_{\phi^6} \end{array} \\ \begin{array}{c} c_{\phi^2 B^2} \\ c_{\phi^4 D^2} \\ c_{\phi^6} \end{array} & \left(\begin{array}{ccc} \times & \times & 0 \\ \times & \times & 0 \\ \times & \times & \times \end{array} \right) \end{array}$$

Things change at higher dimensions

$$\gamma_{ij} = \begin{array}{cc} & \begin{array}{ccc} c_{\phi^2 B^2} & c_{\phi^4 D^2} & c_{\phi^6} \end{array} \\ \begin{array}{c} c_{\phi^2 B^2} \\ c_{\phi^4 D^2} \\ c_{\phi^6} \end{array} & \begin{pmatrix} \times & \color{red}{0} & 0 \\ \times & \times & 0 \\ \times & \times & \times \end{pmatrix} \end{array}$$

Things change at higher dimensions





$$
\gamma_{ij} = \begin{array}{c} \\ c_{\phi^2 B^2} \\ c_{\phi^4 D^2} \\ c_{\phi^6} \end{array}
\begin{array}{c} \begin{array}{ccc} c_{\phi^2 B^2} & c_{\phi^4 D^2} & c_{\phi^6} \end{array} \\
\begin{pmatrix} \times & 0 & 0 \\ \times & \times & 0 \\ \times & \times & \times \end{pmatrix} \end{array}
$$

On-shell amplitude methods

(+) No fields, no gauge redundancies, no gamma matrices, no polarization vectors, ...

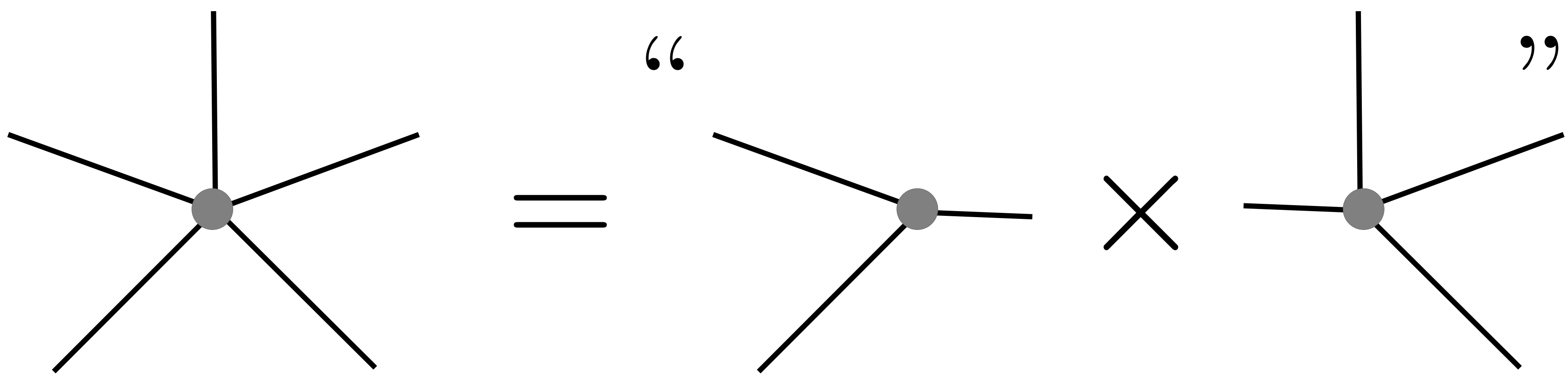
(+) Only observable quantities

(+) Infinitely easier to apply to high-spin particles

(--) Perturbation theory not well understood

The goal: computing amplitudes without using fields/Lagrangians (for **massless** particles)

Strategy:



And 3-point amplitudes are completely determined by quantum mechanics and special relativity

Spinor-helicity variables

$$p = (p^0, \vec{p}) \qquad\qquad P = p_\mu \sigma^\mu = \begin{pmatrix} p_0 + p_3 & p_1 - ip_2 \\ p_1 + ip_2 & p_0 - p_3 \end{pmatrix}$$

$$\det P = 0 \Rightarrow P = \left(\qquad \right) \left(\; \right)$$

$$p\rangle \qquad [p$$

$$\langle p_i p_j \rangle \equiv \langle ij \rangle = \left(\qquad \right) \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \left(\; \right)$$

$$[p_i p_j] \equiv [ij] = \left(\qquad \right) \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \left(\; \right)$$

Spinor-helicity variables

$$p = (p^0, \vec{p}) \qquad P = p_\mu \sigma^\mu = \begin{pmatrix} p_0 + p_3 & p_1 - ip_2 \\ p_1 + ip_2 & p_0 - p_3 \end{pmatrix}$$

$$\langle ij \rangle = -\langle ji \rangle$$

$$[ij] = -[ji]$$

$$[p_i p_j] \equiv [ij] = \left(\qquad \right) \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \left(\right)$$

All sensible quantities appearing in an amplitude can be written simply as products of **angles** and **brackets**, e.g.:

$$s_{ij} = (p_i + p_j)^2 = \langle ij \rangle [ij]$$

$$(\overline{u_1} \gamma^\mu P_R u_2)(\overline{u_3} \gamma_\mu P_R u_4) = 2 \langle 13 \rangle [42]$$

Main result

An amplitude can be written simply as a linear combination of products of **angles** and **brackets**:

$$\mathcal{M}(1,2,\cdots,4) = \sum_i \langle 12\rangle^{a_i}\langle 13\rangle^{b_i}\cdots[34]^{c_i}\cdots$$

Little-group scaling:

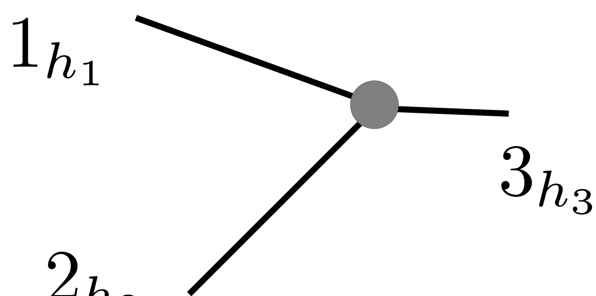
$$\#[i] - \#\langle i\rangle = 2h_i$$

Locality (only single poles!) + unitarity:

$$\text{residue}(\mathcal{M}, \text{pole} = s,t,u) = \mathcal{M}' \times \mathcal{M}''$$

3-point amplitudes are fixed

3-point amplitudes are products of only brackets or only angles, e.g.:



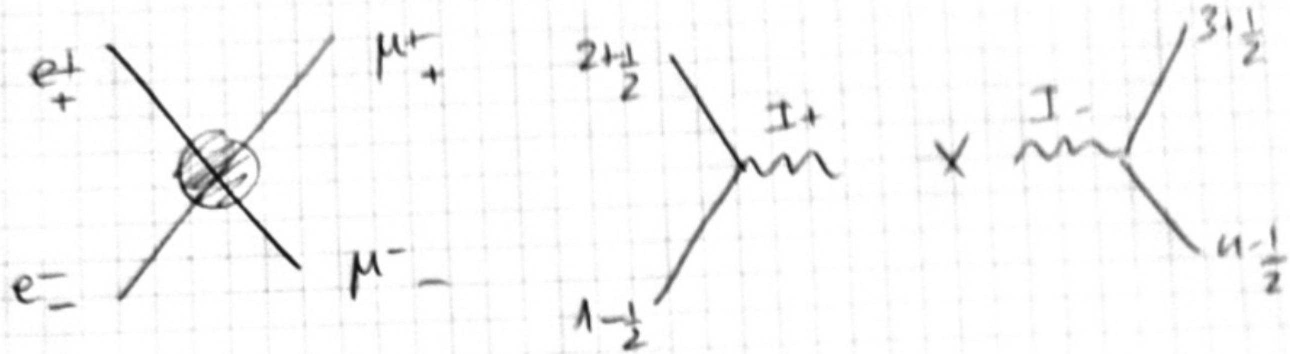
$$\mathcal{M} = [12]^a [23]^b [31]^c$$

$$a = h_1 + h_2 - h_3$$

$$b = h_2 + h_3 - h_1$$

$$c = h_1 + h_3 - h_2$$

e+ e- to mu+ mu-

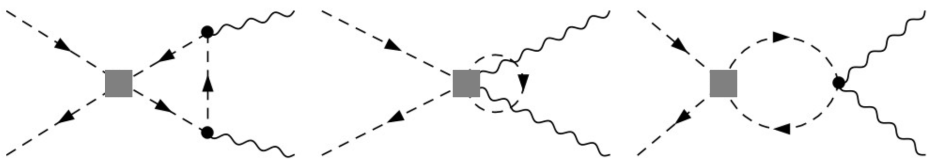


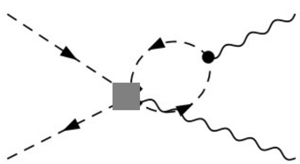
$$\mathcal{M}_s = \frac{[2I]^2}{[12]} \times \frac{\langle 4I\rangle^2}{\langle 34\rangle} = \frac{\langle 41\rangle^2 [12]^2}{\cancel{[12]}\langle 34\rangle}$$

$$= \frac{\langle 41\rangle \cancel{\langle 43\rangle} [32]}{\cancel{\langle 34\rangle}} = \boxed{\langle 14\rangle [23]}$$

$$\Rightarrow \boxed{\mathcal{M} = \frac{\langle 14\rangle [23]}{s}}$$

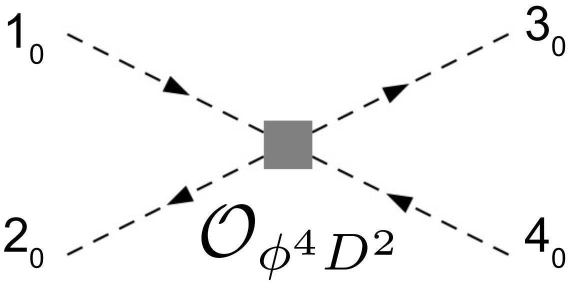
Things change at higher dimensions



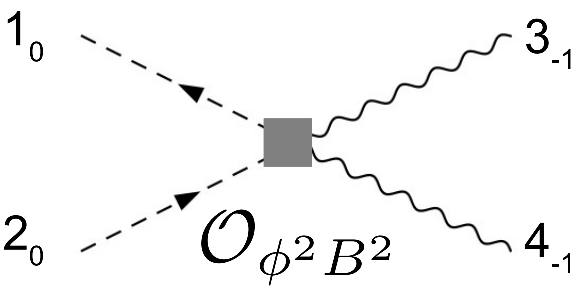


$$\gamma_{ij} = \begin{array}{c} \\ c_{\phi^2 B^2} \\ c_{\phi^4 D^2} \\ c_{\phi^6} \end{array} \begin{array}{c} \begin{array}{ccc} c_{\phi^2 B^2} & c_{\phi^4 D^2} & c_{\phi^6} \end{array} \\ \begin{pmatrix} \times & {\color{red}0} & 0 \\ \times & \times & 0 \\ \times & \times & \times \end{pmatrix} \end{array}$$

On-shell methods in EFT

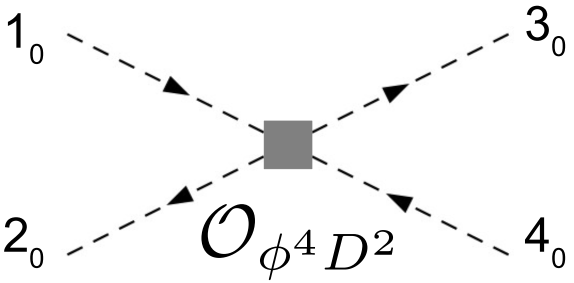


$$= \frac{\langle 13 \rangle [13]}{\Lambda^2}$$

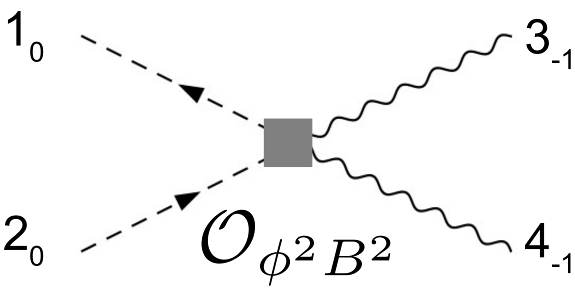


$$= \frac{\langle 34 \rangle^2}{\Lambda^2}$$

On-shell methods in EFT

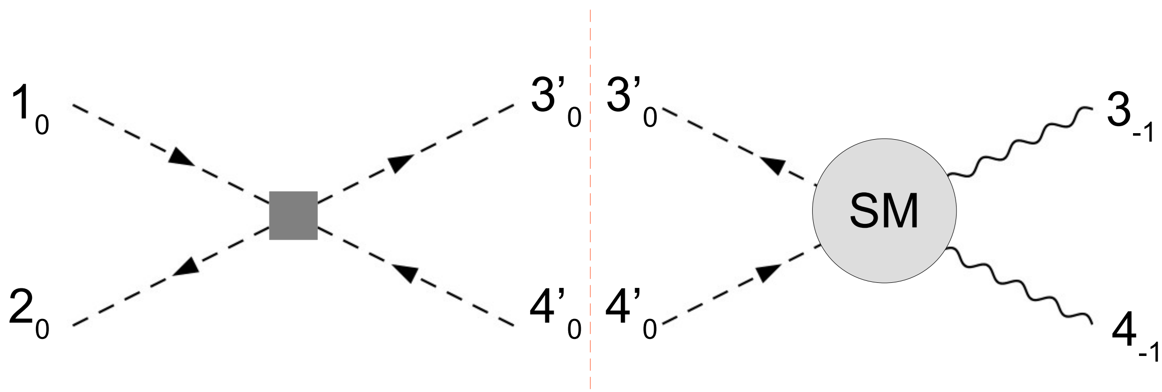


$$= \frac{\langle 13\rangle[13]}{\Lambda^2}$$

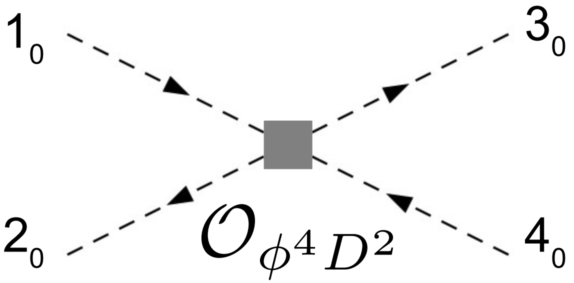


$$= \frac{\langle 34\rangle^2}{\Lambda^2}$$

$$\gamma_{\phi^4 D^2 \to \phi^2 B^2} \propto$$



On-shell methods in EFT



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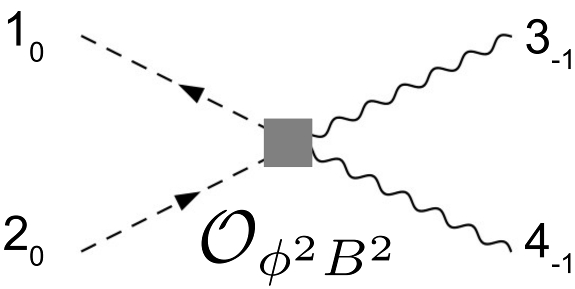

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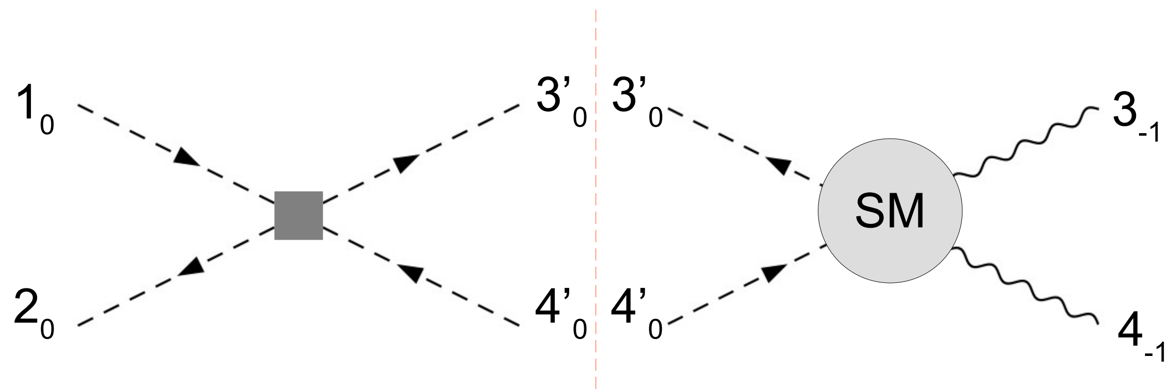
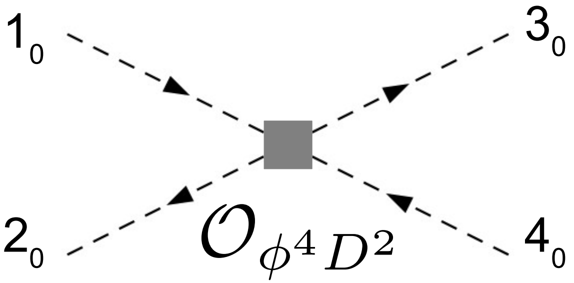
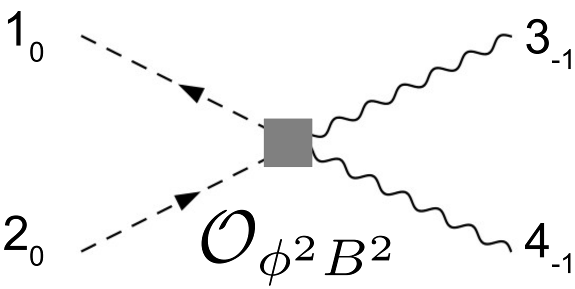


$$= \int d\mathrm{LIPS}\, \frac{\langle 13'\rangle[13']}{\Lambda^2}\, \mathcal{A}_{\mathrm{SM}}$$

On-shell methods in EFT

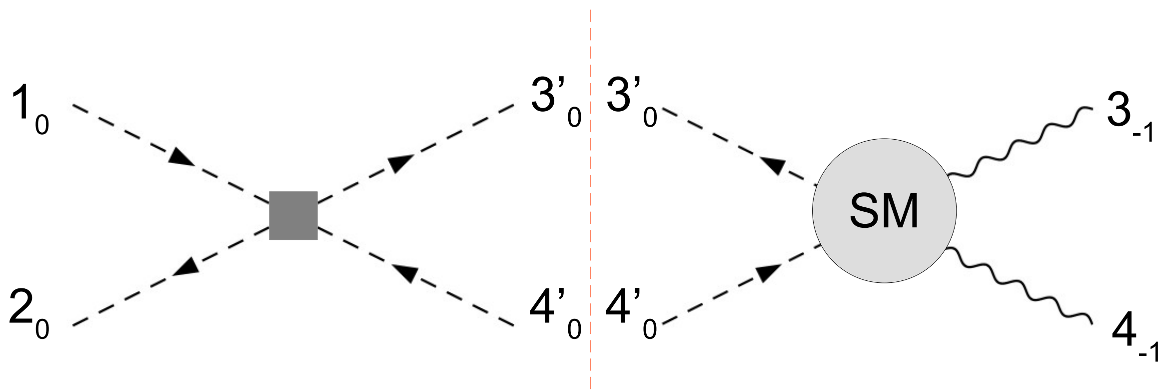


$$= \frac{\langle 13\rangle[13]}{\Lambda^2}$$



$$= \frac{\langle 34\rangle^2}{\Lambda^2}$$

$\gamma_{\phi^4 D^2 \to \phi^2 B^2} \propto$

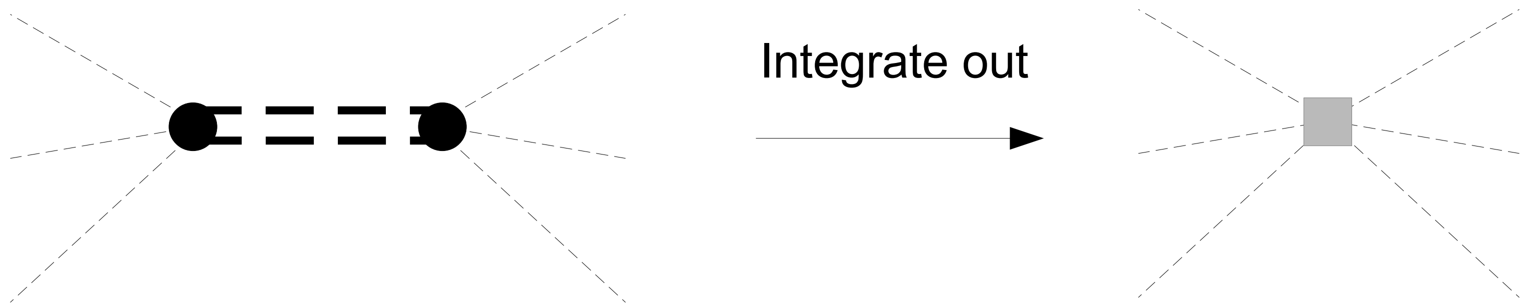


$$= \int d\mathrm{LIPS}\, \frac{\langle 13'\rangle[13']}{\Lambda^2}\, \cancelto{0}{\mathcal{A}_{\rm SM}}$$

Moving to dimension-eight

Besides pure theoretical considerations, anomalous dimensions of dimension-8 operators [Murphy '20; Li, Ren, Shu, Xiao, Yu, Zheng '20] not always phenomenologically irrelevant

Simplest example:



Custodial symmetry violation absent at tree-level dimension-6, one-loop dimension-6 and tree-level dimension-8 [MC, Krause, Nardini '18; Durieux, McCullough, Salvioni '22]

Some partial results:

MC, Guedes, Ramos, Santiago; 2106.05291
Accettulli Huber, Angelis; 2108.03669
Bakshi, MC, Diaz-Carmona, Guedes; 2205.03301
Helset, Jenkins, Manohar; 2212.03253
Asteriadis, Dawson, Fontes; 2212.03258
Bakshi, Diaz-Carmona; 2301.07151

More generally, certain aspects of the full anomalous dimension matrix well understood

Craig, Jiang, Li, Sutherland; 2001.00017

$\overline{w}$					
8	X_L^4	$X_L^3H^2$, $X_L^2\psi^2H$, $X_L\psi^4$	$X_L^2H^4$, $X_L\psi^2H^3$, ψ^4H^2	ψ^2H^5	H^8
6		$X_L^2H^2D^2$, $X_L^2\psi\bar{\psi}D$, $X_L\psi^2HD^2$, ψ^4D^2	$X_LH^4D^2$, $X_L^2\bar{\psi}^2H$, $X_L\psi\bar{\psi}H^2D$, $\psi^2H^3D^2$, $X_L\psi^2\bar{\psi}^2$, $\psi^3\bar{\psi}HD$	H^6D^2, $\psi\bar{\psi}H^4D$, $\psi^2\bar{\psi}^2H^2$	$\bar{\psi}^2H^5$
4			$X_L^2X_R^2$, $X_LX_RH^2D^2$, H^4D^4, $X_LX_R\psi\bar{\psi}D$, $X_R\psi^2HD^2$, $X_L\bar{\psi}^2HD^2$, $\psi\bar{\psi}H^2D^3$, $\psi^2\bar{\psi}^2D^2$	$X_RH^4D^2$, $X_R^2\psi^2H$, $X_R\psi\bar{\psi}H^2D$, $\bar{\psi}^2H^3D^2$, $X_R\psi^2\bar{\psi}^2$, $\psi\bar{\psi}^3HD$	$X_R^2H^4$, $X_R\bar{\psi}^2H^3$, $\bar{\psi}^4H^2$
2				$X_R^2H^2D^2$, $X_R^2\psi\bar{\psi}D$, $X_R\bar{\psi}^2HD^2$, $\bar{\psi}^4D^2$	$X_R^3H^2$, $X_R^2\bar{\psi}^2H$, $X_R\bar{\psi}^4$
0					X_R^4
	0	2	4	6	8

w

Murphy '20;
based on Craig et al '20

$\overline{w}$ \ w	0	2	4	6	8
8	X_L^4	$X_L^3 H^2$, $X_L^2 \psi^2 H$, $X_L \psi^4$	$X_L^2 H^4$, $X_L \psi^2 H^3$, $\psi^4 H^2$	$\psi^2 H^5$	H^8
6		$X_L^2 H^2 D^2$, $X_L^2 \psi\bar{\psi} D$, $X_L \psi^2 H D^2$, $\psi^4 D^2$	$X_L H^4 D^2$, $X_L^2 \bar{\psi}^2 H$, $X_L \psi\bar{\psi} H^2 D$, $\psi^2 H^3 D^2$, $X_L \psi^2 \bar{\psi}^2$, $\psi^3 \bar{\psi} H D$	$H^6 D^2$, $\psi\bar{\psi} H^4 D$, $\psi^2 \bar{\psi}^2 H^2$	$\bar{\psi}^2 H^5$
4			$X_L^2 X_R^2$, $X_L X_R H^2 D^2$, $H^4 D^4$, $X_L X_R \psi\bar{\psi} D$, $X_R \psi^2 H D^2$, $X_L \bar{\psi}^2 H D^2$, $\psi\bar{\psi} H^2 D^3$, $\psi^2 \bar{\psi}^2 D^2$	$X_R H^4 D^2$, $X_R^2 \psi^2 H$, $X_R \psi\bar{\psi} H^2 D$, $\bar{\psi}^2 H^3 D^2$, $X_R \psi^2 \bar{\psi}^2$, $\psi\bar{\psi}^3 H D$	$X_R^2 H^4$, $X_R \bar{\psi}^2 H^3$, $\bar{\psi}^4 H^2$
2				$X_R^2 H^2 D^2$, $X_R^2 \psi\bar{\psi} D$, $X_R \bar{\psi}^2 H D^2$, $\bar{\psi}^4 D^2$	$X_R^3 H^2$, $X_R^2 \bar{\psi}^2 H$, $X_R \bar{\psi}^4$
0					X_R^4

Murphy '20;
based on Craig et al '20

$\overline{w}$ \ w	0	2	4	6	8
8	X_L^4	$X_L^3 H^2$, $X_L^2 \psi^2 H$, $X_L \psi^4$	$X_L^2 H^4$, $X_L \psi^2 H^3$, $\psi^4 H^2$	$\psi^2 H^5$	H^8
6		$X_L^2 H^2 D^2$, $X_L^2 \psi\bar{\psi} D$, $X_L \psi^2 H D^2$, $\psi^4 D^2$	$X_L H^4 D^2$, $X_L^2 \bar{\psi}^2 H$, $X_L \psi\bar{\psi} H^2 D$, $\psi^2 H^3 D^2$, $X_L \psi^2 \bar{\psi}^2$, $\psi^3 \bar{\psi} H D$	$H^6 D^2$, $\psi\bar{\psi} H^4 D$, $\psi^2 \bar{\psi}^2 H^2$	$\bar{\psi}^2 H^5$
4			$X_L^2 X_R^2$, $X_L X_R H^2 D^2$, $H^4 D^4$, $X_L X_R \psi\bar{\psi} D$, $X_R \psi^2 H D^2$, $X_L \bar{\psi}^2 H D^2$, $\psi\bar{\psi} H^2 D^3$, $\psi^2 \bar{\psi}^2 D^2$	$X_R H^4 D^2$, $X_R^2 \psi^2 H$, $X_R \psi\bar{\psi} H^2 D$, $\bar{\psi}^2 H^3 D^2$, $X_R \psi^2 \bar{\psi}^2$, $\psi\bar{\psi}^3 H D$	$X_R^2 H^4$, $X_R \bar{\psi}^2 H^3$, $\bar{\psi}^4 H^2$
2				$X_R^2 H^2 D^2$, $X_R^2 \psi\bar{\psi} D$, $X_R \bar{\psi}^2 H D^2$, $\bar{\psi}^4 D^2$	$X_R^3 H^2$, $X_R^2 \bar{\psi}^2 H$, $X_R \bar{\psi}^4$
0					X_R^4

Murphy '20;
based on Craig et al '20

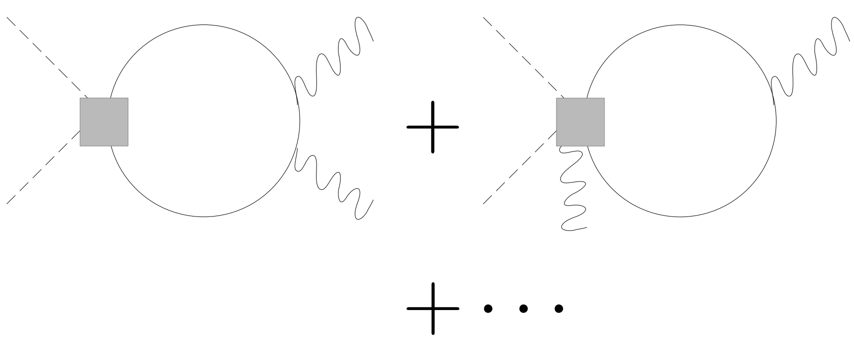
It is obvious that there are zeros in mixing of specific operators of different classes

It is not so clear how to anticipate them, not even with amplitude methods

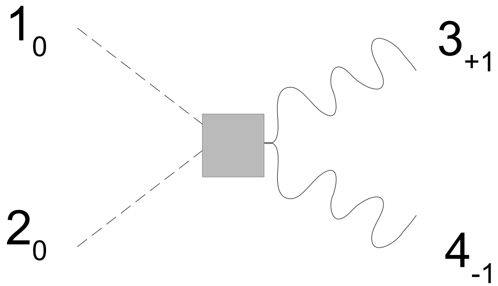
$$\mathcal{O}^{(1)}_{e^2\phi^2 D^3} = i(\bar{e}\gamma^\mu D_\nu e)(D_{(\mu}D_{\nu)}\phi^\dagger\phi) + \text{h.c.}$$

$$\mathcal{O}^{(1)}_{B^2\phi^2 D^2} = (D^\mu\phi^\dagger D^\nu\phi)B_{\mu\rho}B^\rho_\nu$$

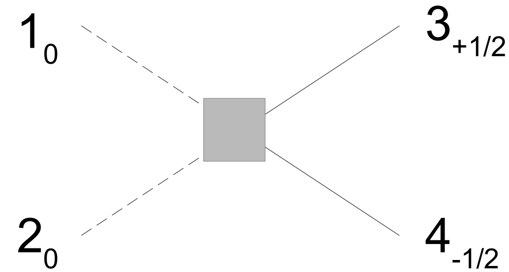
$$\mathcal{O}^{(2)}_{e^2\phi^2 D^3} = i(\bar{e}\gamma^\mu D_\nu e)(\phi^\dagger D_{(\mu}D_{\nu)}\phi) + \text{h.c.}$$



$$\underbrace{\mathcal{O}^{(1)}_{e^2\phi^2 D^3} - \mathcal{O}^{(2)}_{e^2\phi^2 D^3}}_{\widetilde{\mathcal{O}}^{(1)}_{e^2\phi^2 D^3}} \not\longrightarrow \mathcal{O}^{(1)}_{B^2\phi^2 D^2}$$

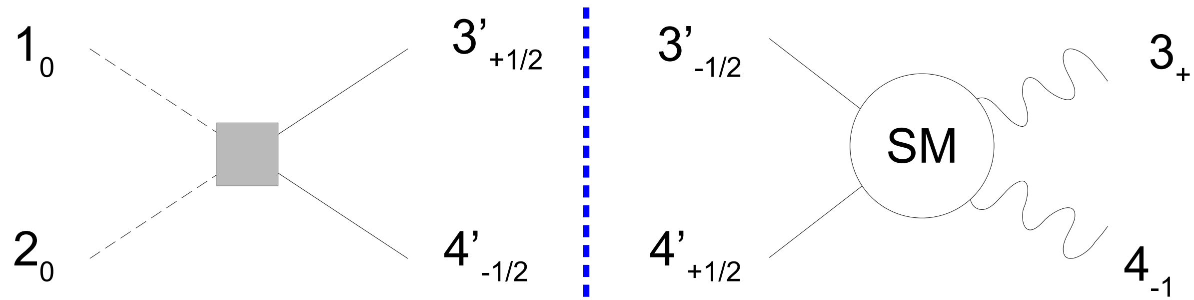


$$= \langle 41\rangle^2[31]^2$$



$$= \langle 43\rangle\langle 41\rangle[43][31]$$

$$\gamma_{\tilde{c}^{(1)}_{e^2\phi^2D^3}\to c^{(1)}_{B^2\phi^2D^2}} \propto$$



$$= \int d\mathrm{LIPS}\langle 4'3'\rangle\langle 4'1\rangle[4'3'][3'1]\frac{\langle 3'4\rangle^2}{\langle 3'3\rangle\langle 34'\rangle}$$

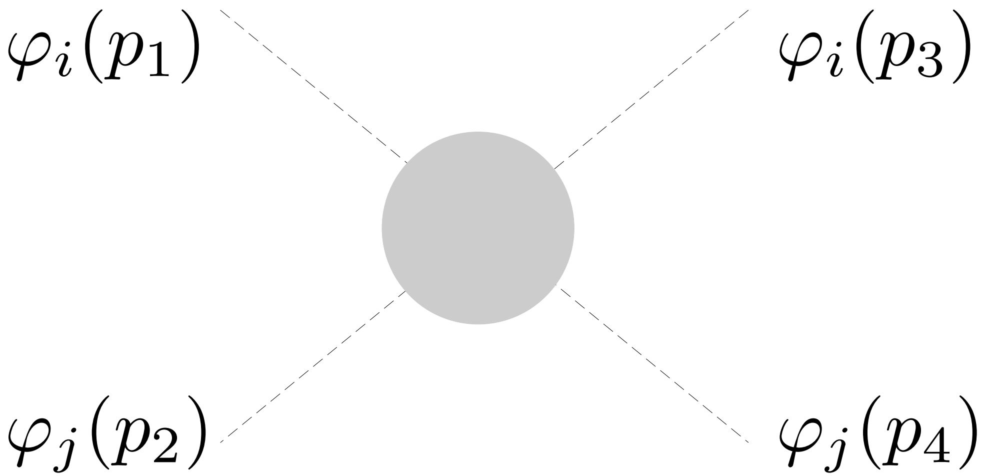
$$= \int_0^{2\pi} d\phi \int_0^{\pi/2} d\theta s_\theta c_\theta \left[\#_1 e^{i\phi} + \cancelto{0}{\#_2 e^{2i\phi}} + \cdots\right]$$

A different perspective: certain operators are constrained by positivity, from unitarity+locality $\left[\text{Adams, Arkani-Hamed, Nicolis, Rattazzi '06}\right]$

$$c^{(1)}_{B^2\phi^2D^2} \leq 0$$

But some others are not:

Positivity bounds



$$s = (p_1 + p_2)^2$$

$$t = (p_1 + p_3)^2$$

$$u = (p_2 + p_3)^2$$

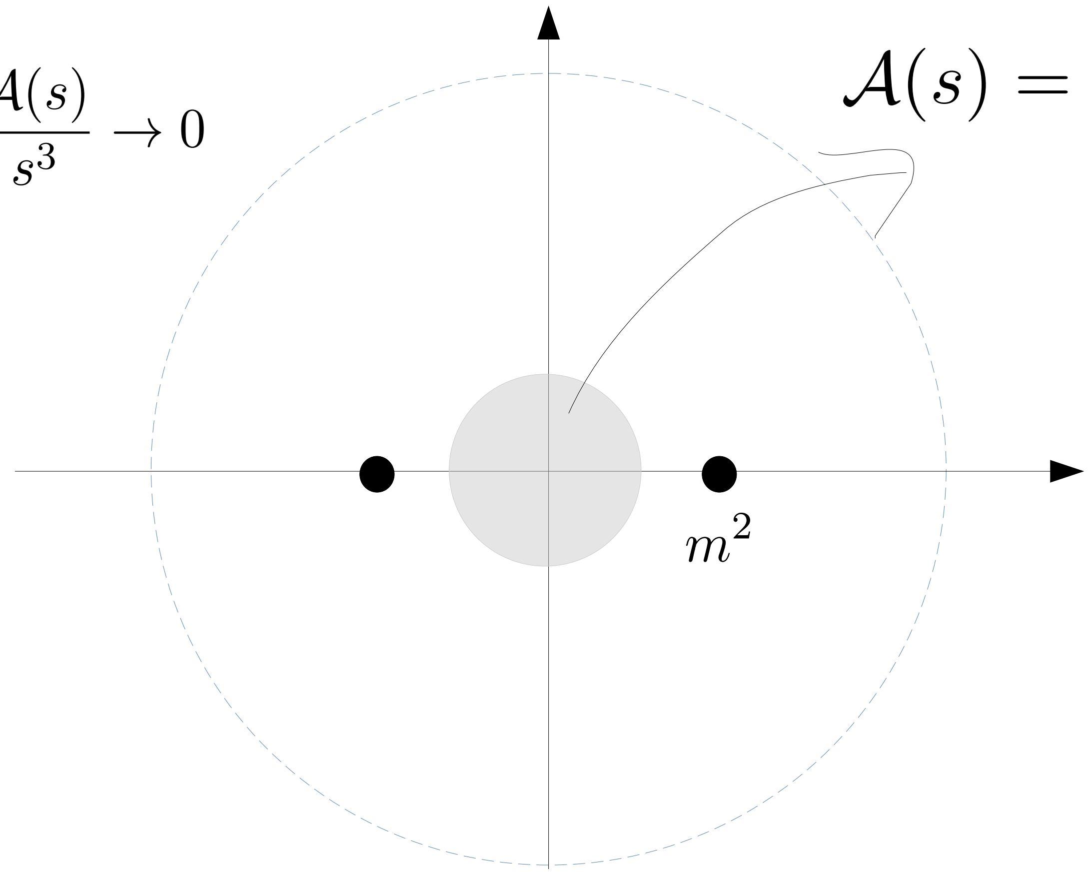
$$s + t + u = 0$$

$$\mathcal{A}(s) = \mathcal{A}(-s)$$

$s \in \mathbb{C}$

$\frac{\mathcal{A}(s)}{s^3} \to 0$

$\mathcal{A}(s) = a_0 + a_2 s^2 + \cdots$



$$0 = \mathrm{Res}[\frac{\mathcal{A}(s)}{s^3}, s = 0] + 2\,\mathrm{Res}[\frac{\mathcal{A}(s)}{s^3}, s = m^2]$$

$$= a_2 - \frac{1}{\pi}\int s\,\frac{\sigma(s)}{(m^2)^3} \Rightarrow a_2 > 0$$

A different perspective: certain operators are constrained by positivity, from unitarity+locality [Adams, Arkani-Hamed, Nicolis, Rattazzi '06]

$$c^{(1)}_{B^2\phi^2D^2} \leq 0$$

But some others are not:

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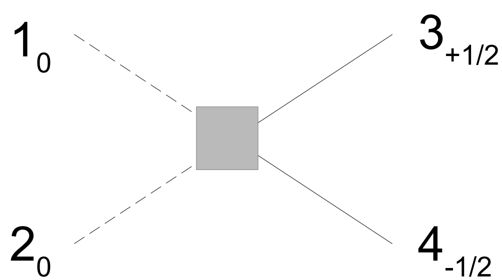

$$= \langle 43\rangle\langle 41\rangle[43][31]$$

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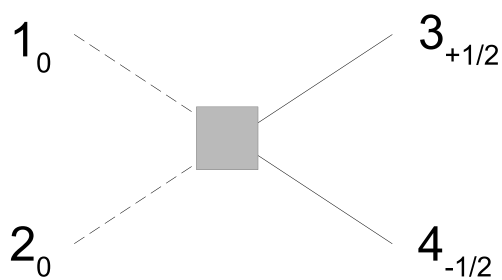
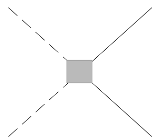
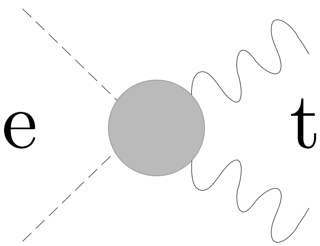


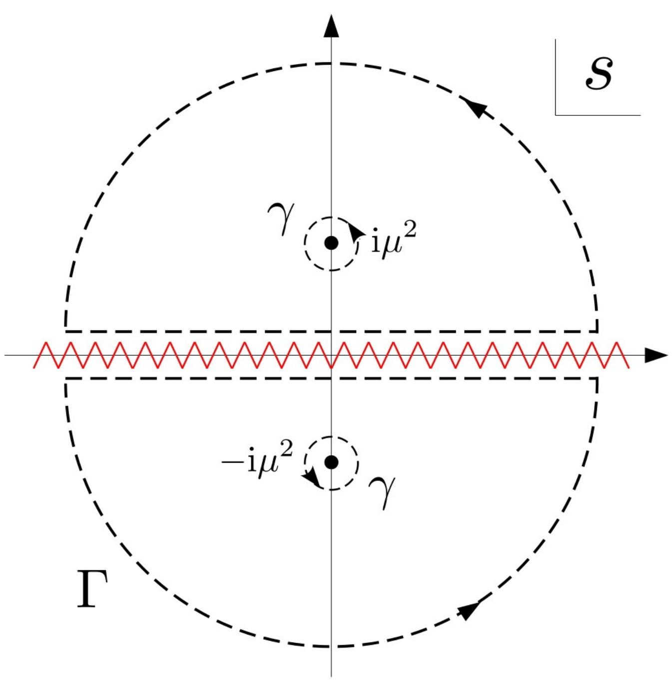
$$= \langle 43\rangle\langle 41\rangle[43][31]$$

"Therefore",

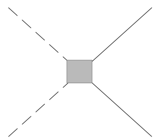
$$\dot{c}^{(1)}_{B^2\phi^2 D^2} = \#_1 \tilde{c}^{(1)}_{e^2\phi^2 D^3} + \cdots \Rightarrow \#_1 = 0$$

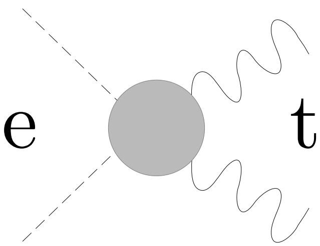
1. For any $(c^{(1)}_{e^2\phi^2 D^3}, c^{(2)}_{e^2\phi^2 D^3})$ compatible with the positivity bounds ($c^{(1)}_{e^2\phi^2 D^3} + c^{(2)}_{e^2\phi^2 D^3} \leq 0$) , there exists UV such that only $(c^{(1)}_{e^2\phi^2 D^3}, c^{(2)}_{e^2\phi^2 D^3})$ (and lower-dimensional  ones) at tree level.

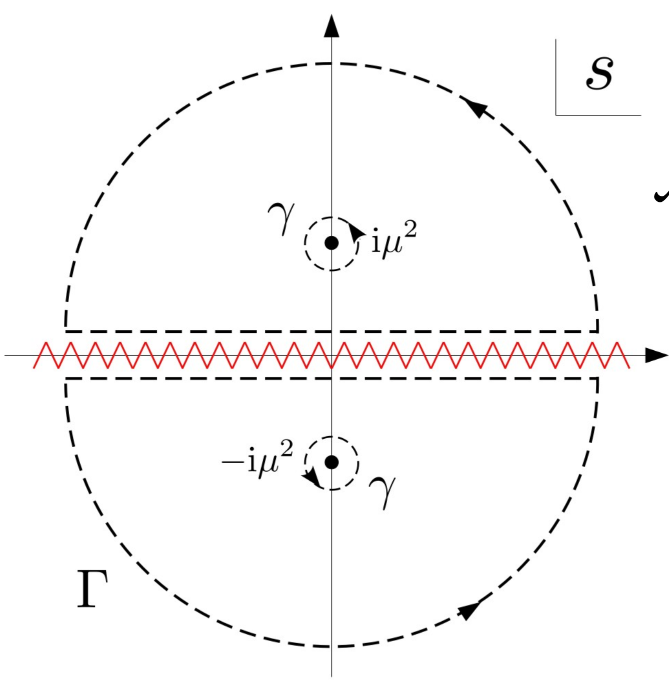
2. Within any such UV, compute  to order $O(g^2)$



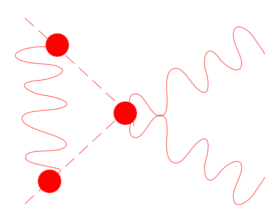
$$\Sigma(\mu) \equiv \frac{1}{2\pi \mathrm{i}} \int_\gamma \frac{\mathcal{A}(s) s^3}{(s^2+\mu^4)^3} \geq 0$$

1. For any $(c^{(1)}_{e^2\phi^2 D^3}, c^{(2)}_{e^2\phi^2 D^3})$ compatible with the positivity bounds ($c^{(1)}_{e^2\phi^2 D^3} + c^{(2)}_{e^2\phi^2 D^3} \leq 0$) , there exists UV such that only $(c^{(1)}_{e^2\phi^2 D^3}, c^{(2)}_{e^2\phi^2 D^3})$ (and lower-dimensional  ones) at tree level.

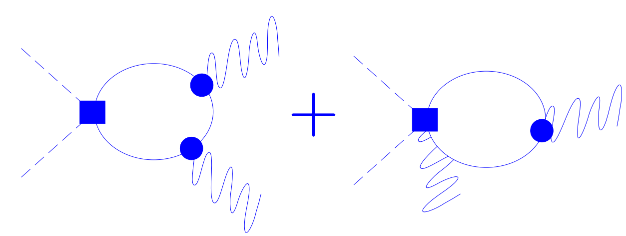
2. Within any such UV, compute  to order $O(g^2)$



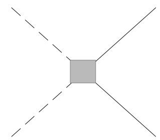
$$\mathcal{A}(s) \sim -(\beta_4 + \beta_8 s^2 + \beta_{12} s^4 + \cdots) \log \frac{s}{\Lambda^2}$$

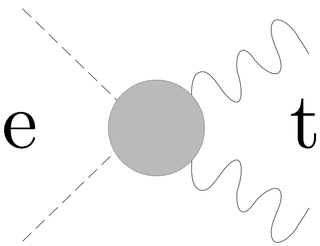


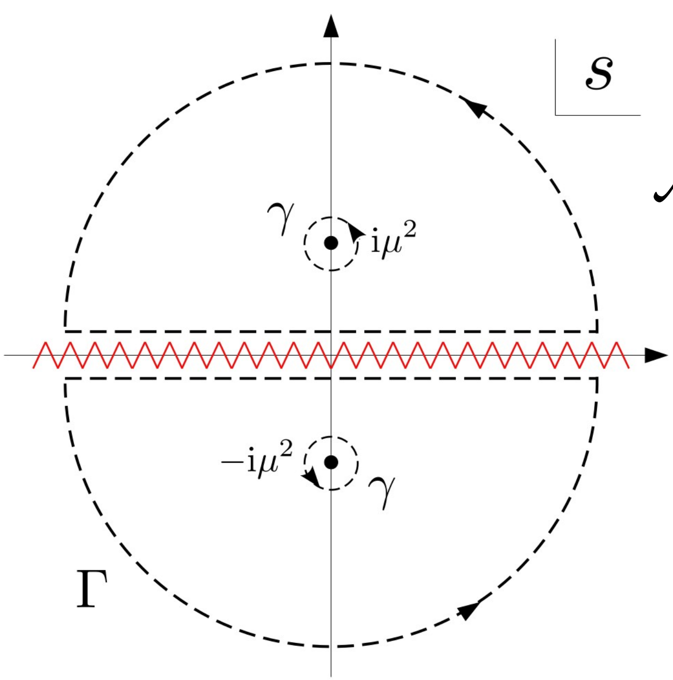
$+\cdots$



$+\cdots$

1. For any $(c^{(1)}_{e^2\phi^2 D^3}, c^{(2)}_{e^2\phi^2 D^3})$ compatible with the positivity bounds ($c^{(1)}_{e^2\phi^2 D^3} + c^{(2)}_{e^2\phi^2 D^3} \leq 0$) , there exists UV such that only $(c^{(1)}_{e^2\phi^2 D^3}, c^{(2)}_{e^2\phi^2 D^3})$ (and lower-dimensional  ones) at tree level.

2. Within any such UV, compute  to order $\mathrm{O}(g^2)$



$$\mathcal{A}(s) \sim -(\beta_4 + \beta_8 s^2 + \beta_{12} s^4 + \cdots) \log \frac{s}{\Lambda^2}$$

$$\Sigma(\mu) = -\beta_8 + \beta_{12}\mu^4 + \cdots$$

$$\Rightarrow \lim_{\mu \to 0} \Sigma(\mu) = -\beta_8 \geq 0$$

So $\beta_8 \leq 0$ in any of the aforementioned UV, and therefore for all values of $(c^{(1)}_{e^2\phi^2D^3}, c^{(2)}_{e^2\phi^2D^3})$ compatible with $c^{(1)}_{e^2\phi^2D^3} + c^{(2)}_{e^2\phi^2D^3} \leq 0$

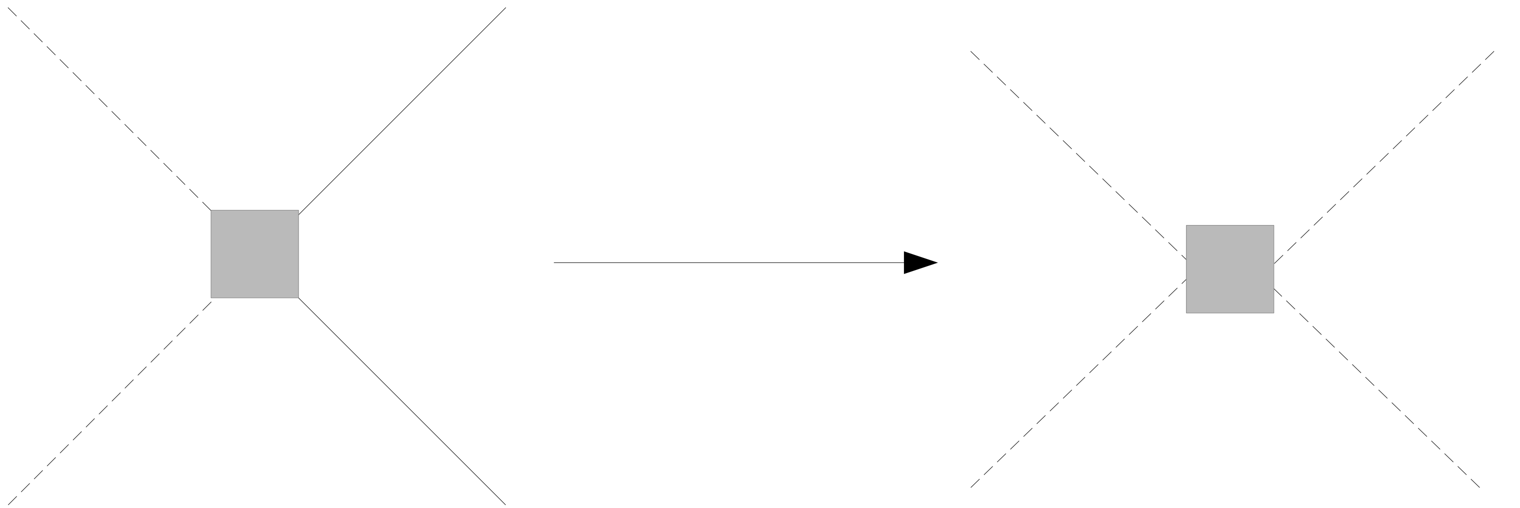
3. The beta function is linear in the Wilson coefficients:

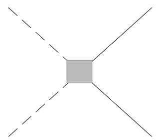
$$\beta_8 = \alpha(c^{(1)}_{e^2\phi^2D^3} + c^{(2)}_{e^2\phi^2D^3})\,, \quad \alpha \geq 0$$

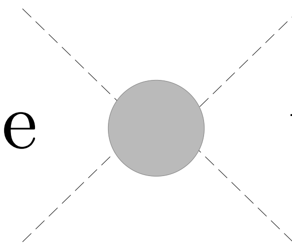
Therefore,

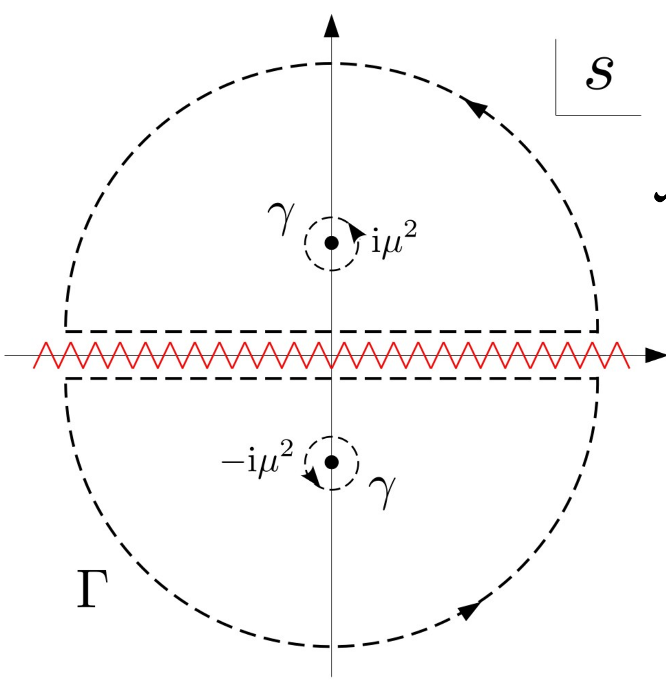
$$\underbrace{\mathcal{O}^{(1)}_{e^2\phi^2D^3} - \mathcal{O}^{(2)}_{e^2\phi^2D^3}}_{\widetilde{\mathcal{O}}^{(1)}_{e^2\phi^2D^3}} \not\longrightarrow \mathcal{O}^{(1)}_{B^2\phi^2D^2}$$

How do things change if we consider instead...?

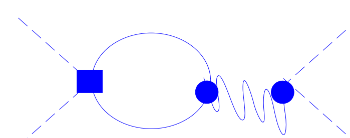


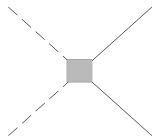
1. For any $(c^{(1)}_{e^2\phi^2 D^3}, c^{(2)}_{e^2\phi^2 D^3})$ compatible with the positivity bounds ($c^{(1)}_{e^2\phi^2 D^3} + c^{(2)}_{e^2\phi^2 D^3} \leq 0$) , there exists UV such that only $(c^{(1)}_{e^2\phi^2 D^3}, c^{(2)}_{e^2\phi^2 D^3})$ (and lower-dimensional  ones) at tree level.

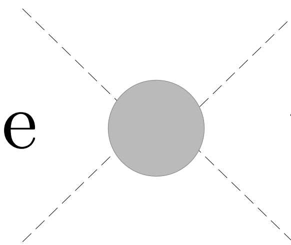
2. Within any such UV, compute  to order $O(g^2)$

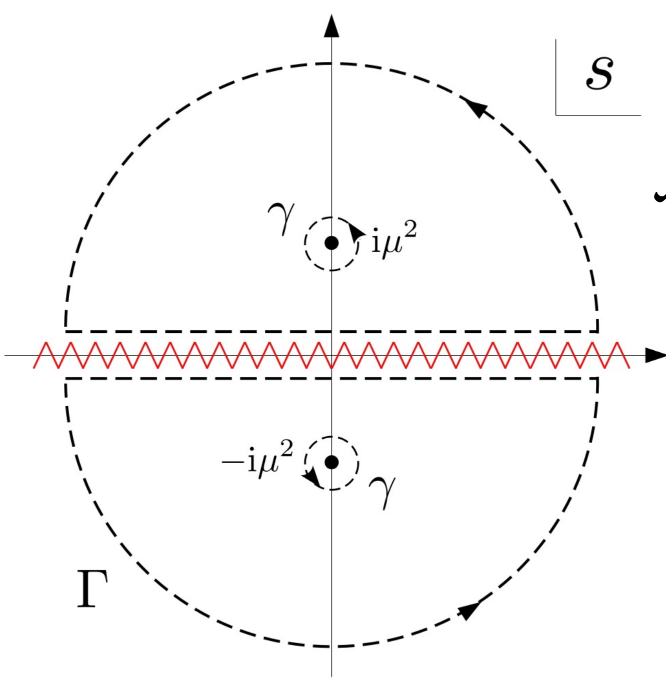


$$\mathcal{A}(s) \sim -(\beta_4 + \beta_8 s^2 + \beta_{12} s^4 + \cdots)\log\frac{s}{\Lambda^2}$$

$\cdots$  $+\cdots$

1. For any $(c^{(1)}_{e^2\phi^2 D^3}, c^{(2)}_{e^2\phi^2 D^3})$ compatible with the positivity bounds ($c^{(1)}_{e^2\phi^2 D^3} + c^{(2)}_{e^2\phi^2 D^3} \leq 0$) , there exists UV such that only $(c^{(1)}_{e^2\phi^2 D^3}, c^{(2)}_{e^2\phi^2 D^3})$ (and lower-dimensional  ones) at tree level.

2. Within any such UV, compute  to order $\mathrm{O}(g^2)$



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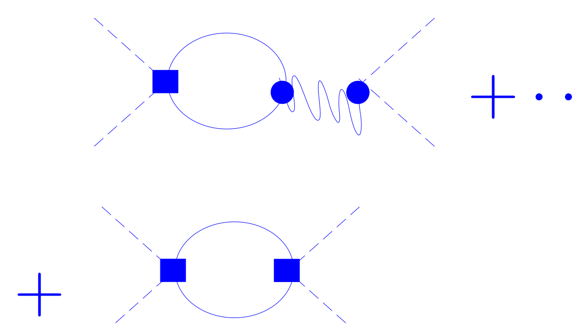
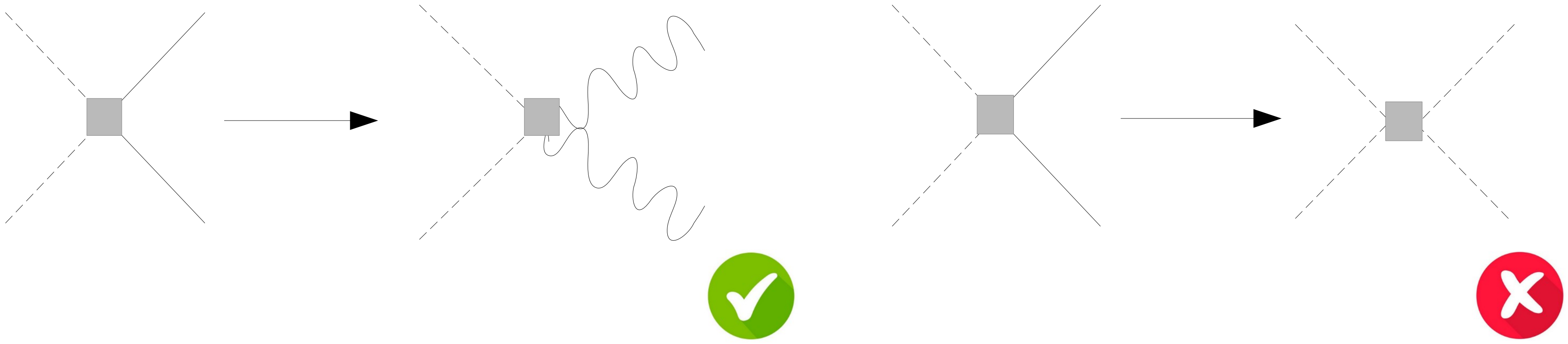
$\cdots$



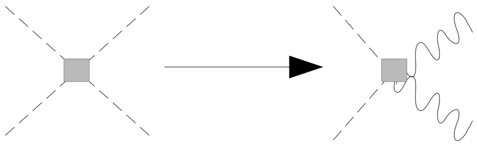
Resorting to the UV to understand the IR is only a trick . In general:

(1) Some tree-level O_i obey $c_i \geq 0$

(2) If O_i involves fields not present in O_j and c_j not constrained by positivity, then $\gamma_{ij} = 0$



Other aspects of anomalous dimensions: signs and inequalities

Let us consider the mixing 

Positivity bounds:

$$c^{(2)}_{\phi^4} \geq 0\,, \quad c^{(1)}_{\phi^4} + c^{(2)}_{\phi^4} \geq 0\,, \quad c^{(1)}_{\phi^4} + c^{(2)}_{\phi^4} + c^{(3)}_{\phi^4} \geq 0$$

$$\dot{c}^{(1)}_{B^2\phi^2D^2} \geq 0$$

From where we obtain:

$$\begin{aligned}\dot{c}^{(1)}_{B^2\phi^2D^2} &= \alpha(c^{(1)}_{\phi^4} + c^{(2)}_{\phi^4} + c^{(3)}_{\phi^4}) + \beta(c^{(1)}_{\phi^4} + c^{(2)}_{\phi^4}) + \gamma c^{(2)}_{\phi^4} + \cdots \\ &= (\alpha+\beta)c^{(1)}_{\phi^4} + (\alpha+\beta+\gamma)c^{(2)}_{\phi^4} + \alpha c^{(3)}_{\phi^4} + \cdots\,,\end{aligned}$$

Other aspects of anomalous dimensions: signs and inequalities

$$\begin{aligned}\dot{c}^{(1)}_{B^2\phi^2D^2} &= \alpha(c^{(1)}_{\phi^4} + c^{(2)}_{\phi^4} + c^{(3)}_{\phi^4}) + \beta(c^{(1)}_{\phi^4} + c^{(2)}_{\phi^4}) + \gamma c^{(2)}_{\phi^4} + \cdots \\ &= (\alpha+\beta)c^{(1)}_{\phi^4} + (\alpha+\beta+\gamma)c^{(2)}_{\phi^4} + \alpha c^{(3)}_{\phi^4} + \cdots\,,\end{aligned}$$

1. The anomalous dimensions are positive

	$c^{(1)}_{\phi^4D^4}$	$c^{(2)}_{\phi^4D^4}$	$c^{(3)}_{\phi^4D^4}$
$c^{(1)}_{B^2\phi^2D^2}$	+	+	+

2. They fulfill

$$\gamma_{c^{(1)}_{B^2\phi^2D^2},\,c^{(2)}_{\phi^4D^4}} \geq \gamma_{c^{(1)}_{B^2\phi^2D^2},\,c^{(1)}_{\phi^4D^4}} \geq \gamma_{c^{(1)}_{B^2\phi^2D^2},\,c^{(3)}_{\phi^4D^4}}$$

Full electroweak SMEFT (with no flavour)

	$c^{(1)}_{\phi^4 D^4}$	$c^{(2)}_{\phi^4 D^4}$	$c^{(3)}_{\phi^4 D^4}$	$\tilde{c}^{(1)}_{e^2\phi^2 D^3}$	$\tilde{c}^{(2)}_{e^2\phi^2 D^3}$	$\tilde{c}^{(1)}_{l^2\phi^2 D^3}$	$\tilde{c}^{(2)}_{l^2\phi^2 D^3}$	$\tilde{c}^{(3)}_{l^2\phi^2 D^3}$	$\tilde{c}^{(4)}_{l^2\phi^2 D^3}$	$c_{e^4 D^2}$	$c^{(1)}_{l^4 D^2}$	$c^{(2)}_{l^4 D^2}$	$c^{(1)}_{l^2 e^2 D^2}$	$c^{(2)}_{l^2 e^2 D^2}$
$c^{(1)}_{B^2\phi^2 D^2}$	+	+	+	$\boxed{0}$	−	$\boxed{0}$	−	$\boxed{0}$	−	0	0	0	0	0
$c^{(1)}_{W^2\phi^2 D^2}$	+	+	+	0	0	$\boxed{0}$	−	$\boxed{0}$	−	0	0	0	0	0
$\tilde{c}^{(2)}_{e^2\phi^2 D^3}$	+	+	+	×	×	$\boxed{0}$	−	$\boxed{0}$	−	−	0	0	$\boxed{0}$	−
$\tilde{c}^{(2)}_{l^2\phi^2 D^3}$	+	+	+	$\boxed{0}$	−	×	×	×	×	0	−	−	$\boxed{0}$	−
$\tilde{c}^{(4)}_{l^2\phi^2 D^3}$	+	+	+	$\boxed{0}$	−	×	×	×	×	0	−	−	$\boxed{0}$	−
$c_{e^2 B^2 D}$	0	0	0	$\boxed{0}$	−	0	0	0	0	−	0	0	$\boxed{0}$	−
$c_{l^2 B^2 D}$	0	0	0	0	0	$\boxed{0}$	−	$\boxed{0}$	−	0	−	−	$\boxed{0}$	−
$c_{e^2 W^2 D}$	0	0	0	$\boxed{0}$	−	0	0	0	0	0	0	0	$\boxed{0}$	−
$c^{(1)}_{l^2 W^2 D}$	0	0	0	0	0	$\boxed{0}$	−	$\boxed{0}$	−	0	−	−	0	0
$c^{(2)}_{l^2 e^2 D^2}$	0	0	0	$\boxed{0}$	−	$\boxed{0}$	−	$\boxed{0}$	−	−	−	−	×	×

Full electroweak SMEFT (with no flavour)

	$c^{(1)}_{\phi^4 D^4}$	$c^{(2)}_{\phi^4 D^4}$	$c^{(3)}_{\phi^4 D^4}$	$\tilde{c}^{(1)}_{e^2\phi^2 D^3}$	$\tilde{c}^{(2)}_{e^2\phi^2 D^3}$	$\tilde{c}^{(1)}_{l^2\phi^2 D^3}$	$\tilde{c}^{(2)}_{l^2\phi^2 D^3}$	$\tilde{c}^{(3)}_{l^2\phi^2 D^3}$	$\tilde{c}^{(4)}_{l^2\phi^2 D^3}$	$c_{e^4 D^2}$	$c^{(1)}_{l^4 D^2}$	$c^{(2)}_{l^4 D^2}$	$c^{(1)}_{l^2 e^2 D^2}$	$c^{(2)}_{l^2 e^2 D^2}$
$c^{(1)}_{B^2\phi^2 D^2}$	$\boxed{\frac{g^2}{3}}$	$\boxed{\frac{g^2}{2}}$	$\boxed{\frac{g^2}{6}}$	$\boxed{0}$	$-$	$\boxed{0}$	$-$	$\boxed{0}$	$-$	0	0	0	0	0
$c^{(1)}_{W^2\phi^2 D^2}$	$+$	$+$	$+$	0	0	$\boxed{0}$	$-$	$\boxed{0}$	$-$	0	0	0	0	0
$\tilde{c}^{(2)}_{e^2\phi^2 D^3}$	$+$	$+$	$+$	$\times$	$\times$	$\boxed{0}$	$-$	$\boxed{0}$	$-$	$-$	0	0	$\boxed{0}$	$-$
$\tilde{c}^{(2)}_{l^2\phi^2 D^3}$	$+$	$+$	$+$	$\boxed{0}$	$-$	$\times$	$\times$	$\times$	$\times$	0	$-$	$-$	$\boxed{0}$	$-$
$\tilde{c}^{(4)}_{l^2\phi^2 D^3}$	$+$	$+$	$+$	$\boxed{0}$	$-$	$\times$	$\times$	$\times$	$\times$	0	$-$	$-$	$\boxed{0}$	$-$
$c_{e^2 B^2 D}$	0	0	0	$\boxed{0}$	$-$	0	0	0	0	$-$	0	0	$\boxed{0}$	$-$
$c_{l^2 B^2 D}$	0	0	0	0	0	$\boxed{0}$	$-$	$\boxed{0}$	$-$	0	$-$	$-$	$\boxed{0}$	$-$
$c_{e^2 W^2 D}$	0	0	0	$\boxed{0}$	$-$	0	0	0	0	0	0	0	$\boxed{0}$	$-$
$c^{(1)}_{l^2 W^2 D}$	0	0	0	0	0	$\boxed{0}$	$-$	$\boxed{0}$	$-$	0	$-$	$-$	0	0
$c^{(2)}_{l^2 e^2 D^2}$	0	0	0	$\boxed{0}$	$-$	$\boxed{0}$	$-$	$\boxed{0}$	$-$	$-$	$-$	$-$	$\times$	$\times$

Full electroweak SMEFT (with no flavour)

	$c^{(1)}_{\phi^4D^4}$	$c^{(2)}_{\phi^4D^4}$	$c^{(3)}_{\phi^4D^4}$	$\tilde{c}^{(1)}_{e^2\phi^2D^3}$	$\tilde{c}^{(2)}_{e^2\phi^2D^3}$	$\tilde{c}^{(1)}_{l^2\phi^2D^3}$	$\tilde{c}^{(2)}_{l^2\phi^2D^3}$	$\tilde{c}^{(3)}_{l^2\phi^2D^3}$	$\tilde{c}^{(4)}_{l^2\phi^2D^3}$	$c_{e^4D^2}$	$c^{(1)}_{l^4D^2}$	$c^{(2)}_{l^4D^2}$	$c^{(1)}_{l^2e^2D^2}$	$c^{(2)}_{l^2e^2D^2}$
$c^{(1)}_{B^2\phi^2D^2}$	$\boxed{\frac{g^2}{3}}$	$\boxed{\frac{g^2}{2}}$	$\boxed{\frac{g^2}{6}}$	$\boxed{0}$	$-$	$\boxed{0}$	$-$	$\boxed{0}$	$-$	0	0	0	0	0
$c^{(1)}_{W^2\phi^2D^2}$	$+$	$+$	$+$	0	0	$\boxed{0}$	$-$	$\boxed{0}$	$-$	0	0	0	0	0
$\tilde{c}^{(2)}_{e^2\phi^2D^3}$	$+$	$+$	$+$	$\times$	$\times$	$\boxed{0}$	$\boxed{-\frac{4\|Y\|^2}{3}}$	$\boxed{0}$	$-$	$-$	0	0	$\boxed{0}$	$-$
$\tilde{c}^{(2)}_{l^2\phi^2D^3}$	$+$	$+$	$+$	$\boxed{0}$	$-$	$\times$	$\times$	$\times$	$\times$	0	$-$	$-$	$\boxed{0}$	$-$
$\tilde{c}^{(4)}_{l^2\phi^2D^3}$	$+$	$+$	$+$	$\boxed{0}$	$-$	$\times$	$\times$	$\times$	$\times$	0	$-$	$-$	$\boxed{0}$	$-$
$c_{e^2B^2D}$	0	0	0	$\boxed{0}$	$-$	0	0	0	0	$-$	0	0	$\boxed{0}$	$-$
$c_{l^2B^2D}$	0	0	0	0	0	$\boxed{0}$	$-$	$\boxed{0}$	$-$	0	$-$	$-$	$\boxed{0}$	$-$
$c_{e^2W^2D}$	0	0	0	$\boxed{0}$	$-$	0	0	0	0	0	0	0	$\boxed{0}$	$-$
$c^{(1)}_{l^2W^2D}$	0	0	0	0	0	$\boxed{0}$	$-$	$\boxed{0}$	$-$	0	$-$	$-$	0	0
$c^{(2)}_{l^2e^2D^2}$	0	0	0	$\boxed{0}$	$-$	$\boxed{0}$	$-$	$\boxed{0}$	$-$	$-$	$-$	$-$	$\times$	$\times$

Full electroweak SMEFT (with no flavour)

	$c^{(1)}_{\phi^4D^4}$	$c^{(2)}_{\phi^4D^4}$	$c^{(3)}_{\phi^4D^4}$	$\tilde{c}^{(1)}_{e^2\phi^2D^3}$	$\tilde{c}^{(2)}_{e^2\phi^2D^3}$	$\tilde{c}^{(1)}_{l^2\phi^2D^3}$	$\tilde{c}^{(2)}_{l^2\phi^2D^3}$	$\tilde{c}^{(3)}_{l^2\phi^2D^3}$	$\tilde{c}^{(4)}_{l^2\phi^2D^3}$	$c_{e^4D^2}$	$c^{(1)}_{l^4D^2}$	$c^{(2)}_{l^4D^2}$	$c^{(1)}_{l^2e^2D^2}$	$c^{(2)}_{l^2e^2D^2}$
$c^{(1)}_{B^2\phi^2D^2}$	$\boxed{\frac{g^2}{3}}$	$\boxed{\frac{g^2}{2}}$	$\boxed{\frac{g^2}{6}}$	$\boxed{0}$	$-$	$\boxed{0}$	$-$	$\boxed{0}$	$-$	0	0	0	0	0
$c^{(1)}_{W^2\phi^2D^2}$	$+$	$+$	$+$	0	0	$\boxed{0}$	$-$	$\boxed{0}$	$-$	0	0	0	0	0
$\tilde{c}^{(2)}_{e^2\phi^2D^3}$	$+$	$+$	$+$	$\boxed{g^2-\lvert Y\rvert^2}$	$\times$	$\boxed{0}$	$\boxed{-\frac{4\lvert Y\rvert^2}{3}}$	$\boxed{0}$	$-$	$-$	0	0	$\boxed{0}$	$-$
$\tilde{c}^{(2)}_{l^2\phi^2D^3}$	$+$	$+$	$+$	$\boxed{0}$	$-$	$\times$	$\times$	$\times$	$\times$	0	$-$	$-$	$\boxed{0}$	$-$
$\tilde{c}^{(4)}_{l^2\phi^2D^3}$	$+$	$+$	$+$	$\boxed{0}$	$-$	$\times$	$\times$	$\times$	$\times$	0	$-$	$-$	$\boxed{0}$	$-$
$c_{e^2B^2D}$	0	0	0	$\boxed{0}$	$-$	0	0	0	0	$-$	0	0	$\boxed{0}$	$-$
$c_{l^2B^2D}$	0	0	0	0	0	$\boxed{0}$	$-$	$\boxed{0}$	$-$	0	$-$	$-$	$\boxed{0}$	$-$
$c_{e^2W^2D}$	0	0	0	$\boxed{0}$	$-$	0	0	0	0	0	0	0	$\boxed{0}$	$-$
$c^{(1)}_{l^2W^2D}$	0	0	0	0	0	$\boxed{0}$	$-$	$\boxed{0}$	$-$	0	$-$	$-$	0	0
$c^{(2)}_{l^2e^2D^2}$	0	0	0	$\boxed{0}$	$-$	$\boxed{0}$	$-$	$\boxed{0}$	$-$	$-$	$-$	$-$	$\times$	$\times$

Outlook

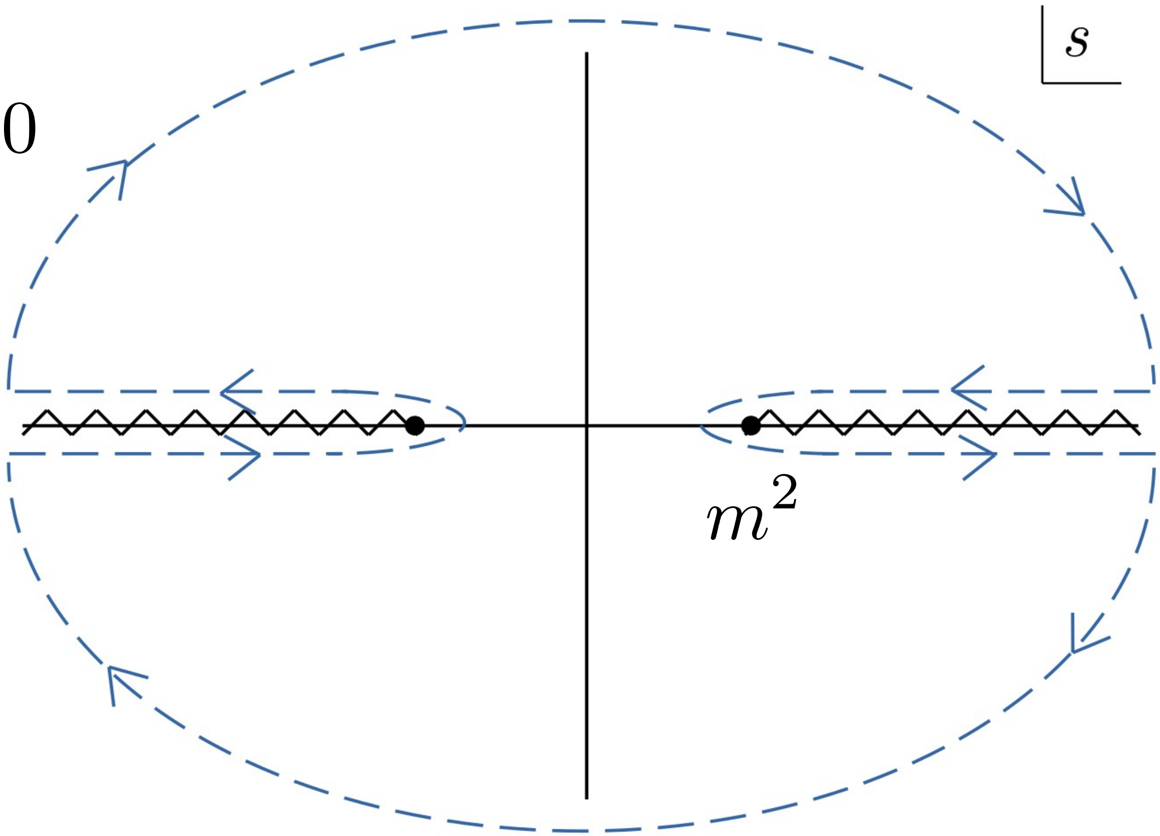
Positivity bounds on dimension-8 interactions (which are important/interesting by themselves) restrict different aspects of (certain) anomalous dimensions (zeros, signs, inequalities)

Further applications include full SMEFT [MC, Li 'ongoing work], LEFT and other EFTs.

Phenomenological relevance of dimension-8 quantum corrections still to be fully understood

Thank you!

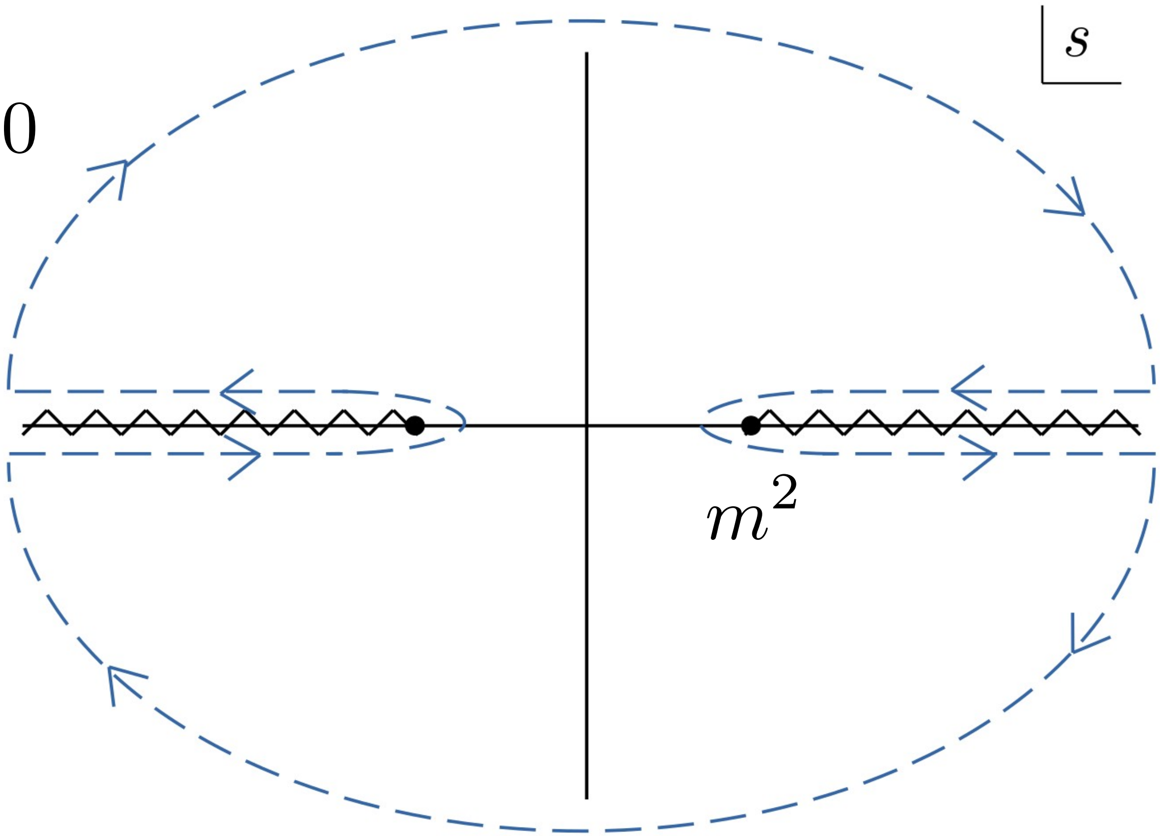
$$\frac{\mathcal{A}(s)}{s^3} \to 0$$



$$\int \frac{\mathcal{A}(s)}{s^3} = 2\int_{m^2}^{\infty} \frac{1}{s^3} \lim_{\epsilon\to 0}[\mathcal{A}(s+i\epsilon) - \mathcal{A}(s-i\epsilon)]$$

$$= 2\int_{m^2}^{\infty} \frac{1}{s^3} \lim_{\epsilon\to 0}[\mathcal{A}(s+i\epsilon) - \mathcal{A}(s+i\epsilon)^*] = 2i\int_{m^2}^{\infty} \frac{\sigma(s)}{s^2}$$

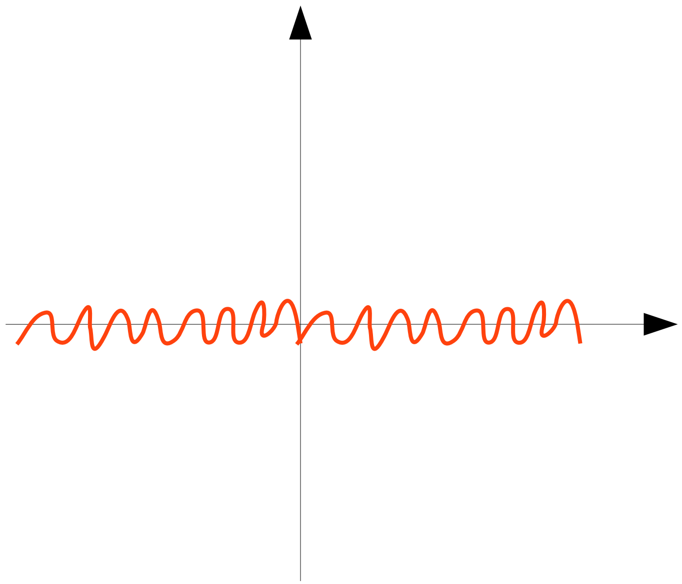
$$\frac{\mathcal{A}(s)}{s^3} \to 0$$



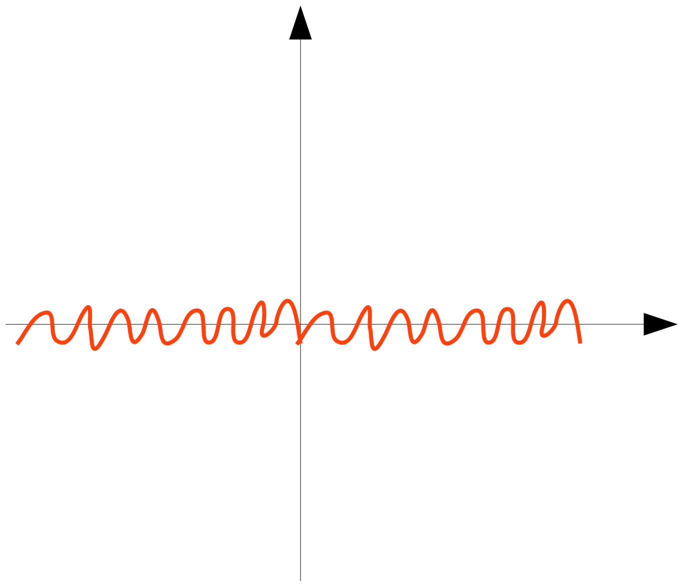
$$2i \int_{m^2}^{\infty} \frac{\sigma(s)}{s^2} = 2\pi i \, \mathrm{Res}[\frac{\mathcal{A}(s)}{s^3}, s = 0] = 2\pi i a_2$$

$$\Rightarrow a_2 \geq 0$$

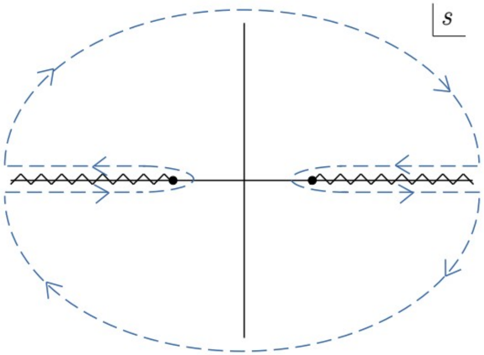
Positivity in the presence of light loops



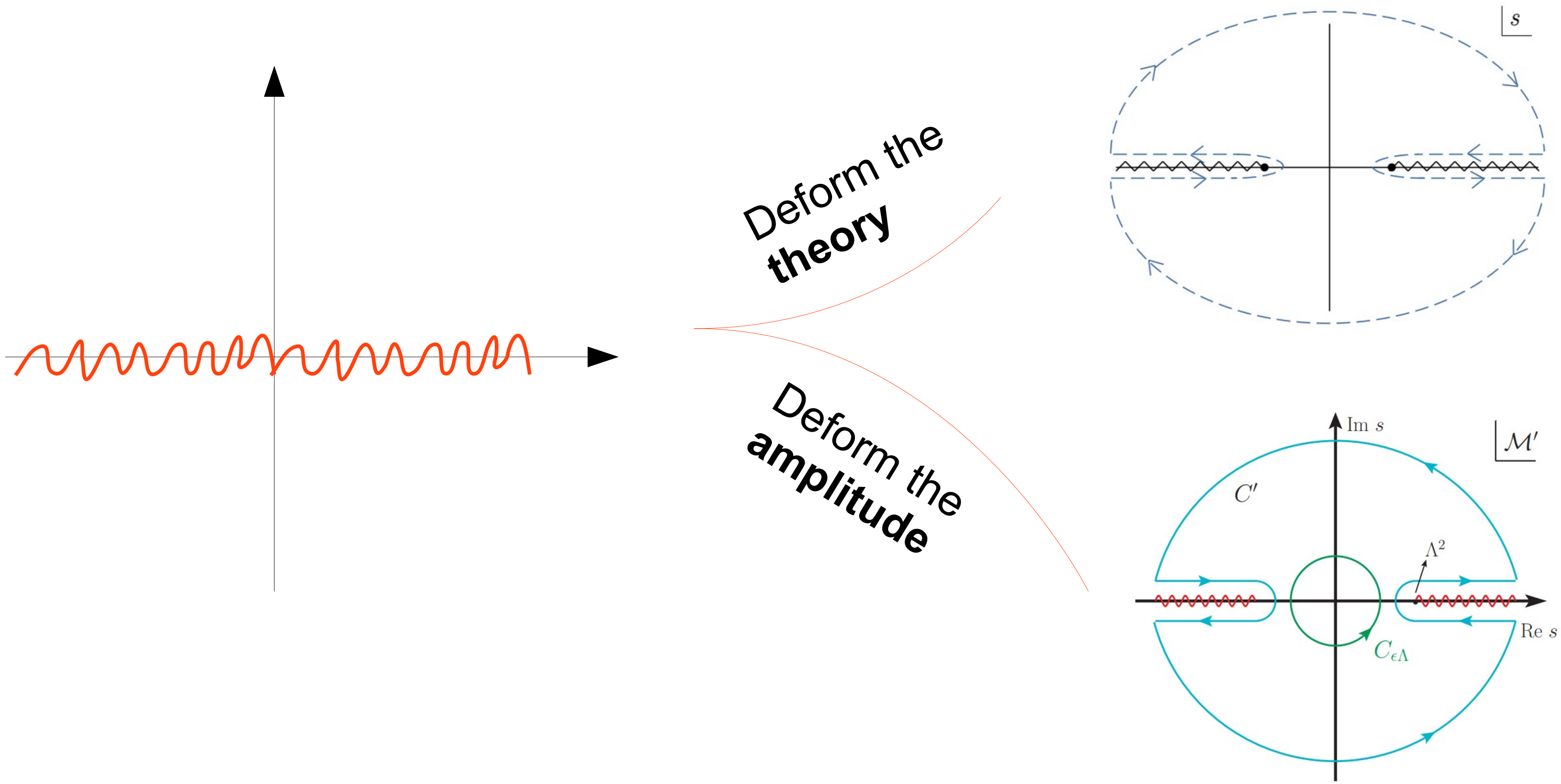
Positivity in the presence of light loops



Deform the **theory**

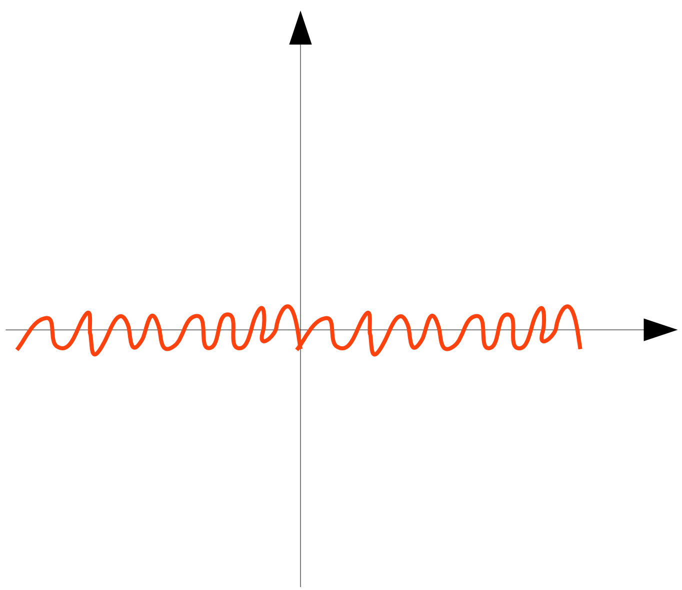


Positivity in the presence of light loops

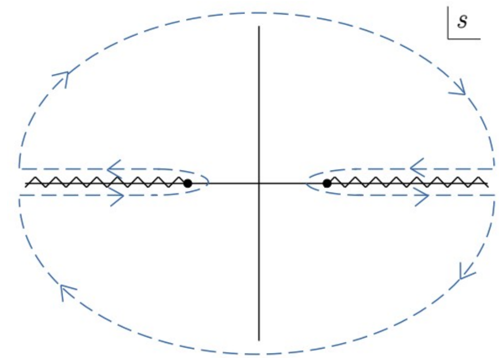


(b) Subtracted amplitude

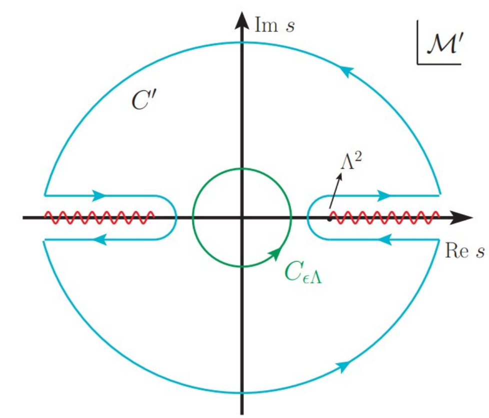
Positivity in the presence of light loops



Deform the **theory**



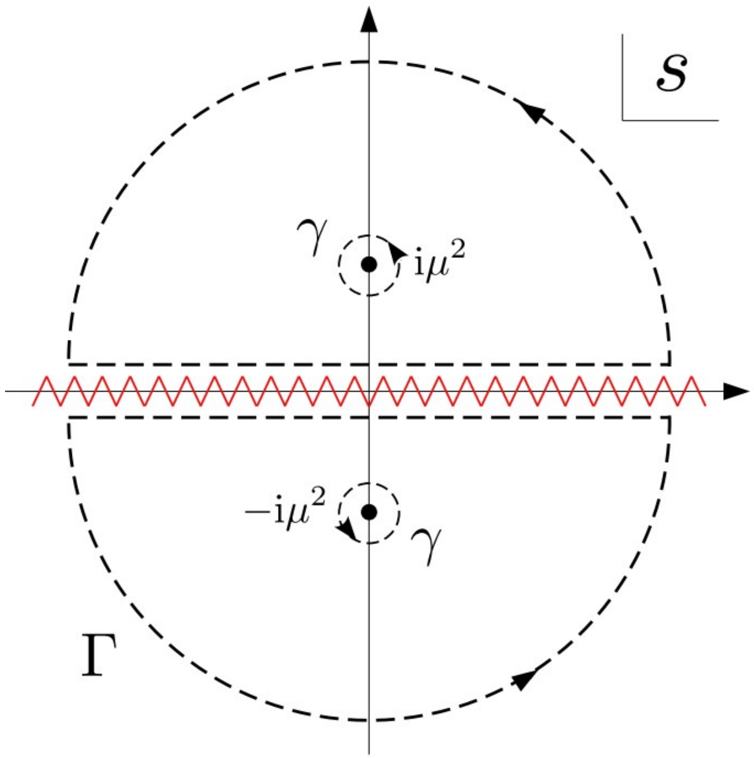
Deform the **amplitude**



(b) Subtracted amplitude

Deform the **integration path**

Positivity in the presence of light loops



$$\Sigma(\mu) \equiv \frac{1}{2\pi \mathrm{i}} \int_\gamma \frac{\mathcal{A}(s)s^3}{(s^2+\mu^4)^3} = \frac{1}{2\pi \mathrm{i}} \int_\Gamma \frac{\mathcal{A}(s)s^3}{(s^2+\mu^4)^3}$$

$$\begin{aligned}\Sigma(\mu) &= \frac{1}{\pi \mathrm{i}} \int_0^\infty \frac{s^3}{(s^2+\mu^4)^3} \lim_{\epsilon\to 0} \left[\mathcal{A}(s+\mathrm{i}\epsilon) - \mathcal{A}(s-\mathrm{i}\epsilon)\right] \\ &= \frac{1}{\pi \mathrm{i}} \int_0^\infty \frac{s^3}{(s^2+\mu^4)^3} \lim_{\epsilon\to 0} \left[\mathcal{A}(s+\mathrm{i}\epsilon) - \mathcal{A}(s+\mathrm{i}\epsilon)^*\right] = \frac{2}{\pi} \int_0^\infty \frac{\mathrm{Im}\mathcal{A}(s)s^3}{(s^2+\mu^4)^3} \geq 0\end{aligned}$$