Continuous spectrum on cosmological collider

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based on 2301.07920

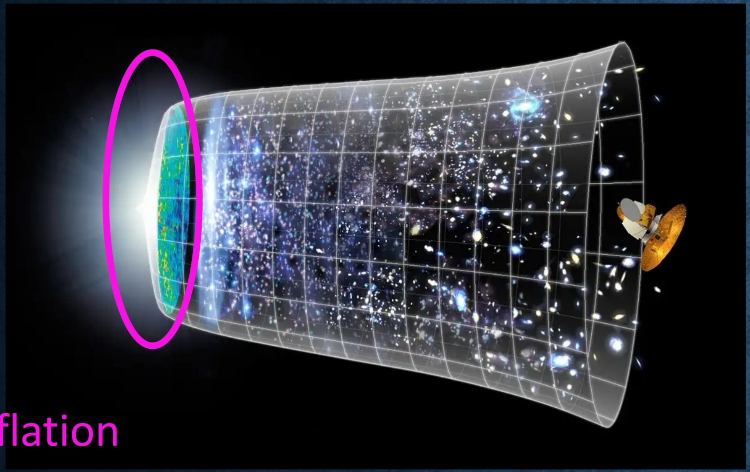
&

ongoing work with

Fumiya Sano, Toshifumi Noumi, Masahide Yamaguchi

KIAS seminar 2023/8/22

Inflation

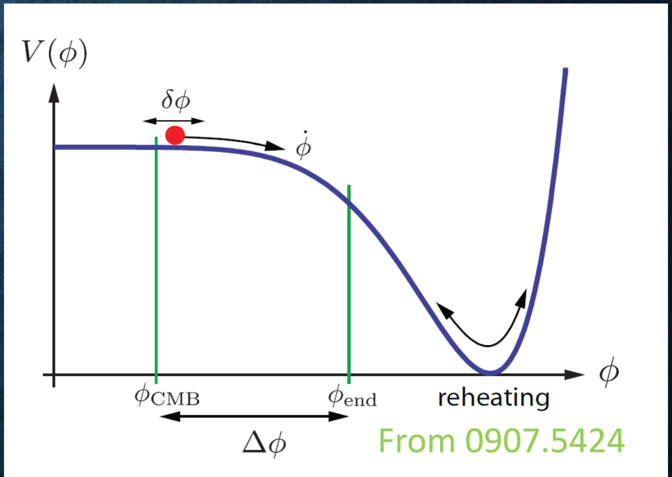


Inflation

·rapid expansion of early universe

·solve initial condition problems (horizon, flatness)

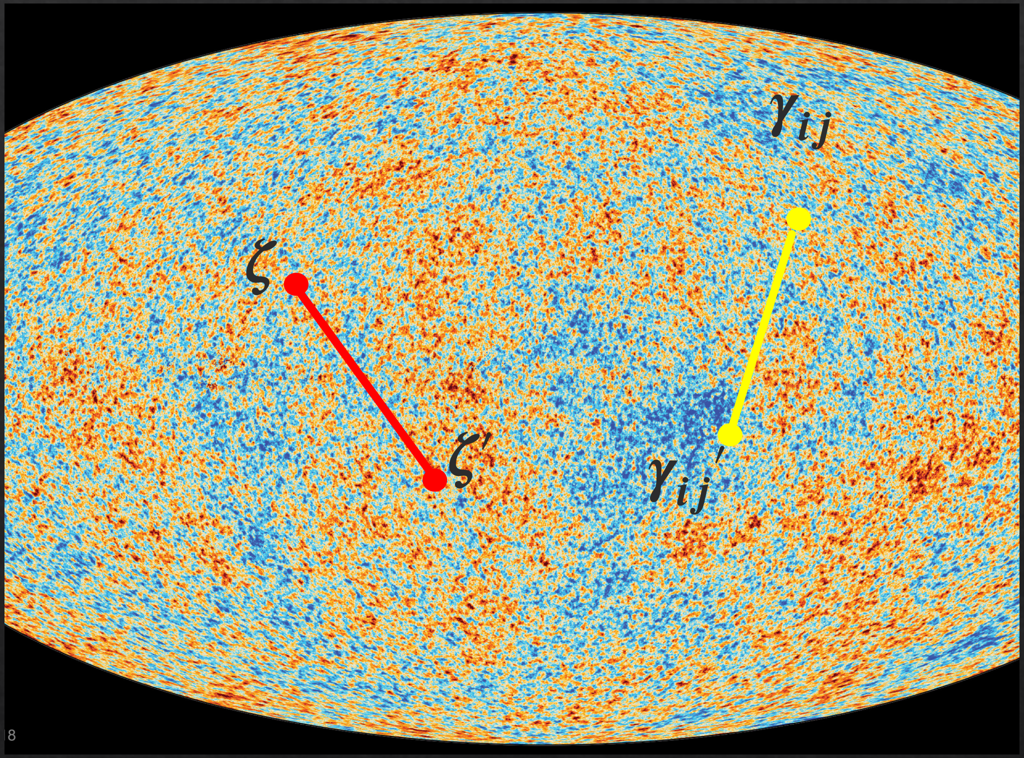
·can be a source for primordial fluctuation



· slow-roll inflation by a scalar field: inflaton

· inflaton fluctuation $\delta\phi \Rightarrow$ curvature perturbation ζ

Inflationary observable



scalar power spectrum: $P_\zeta \sim \langle \zeta\zeta' \rangle$

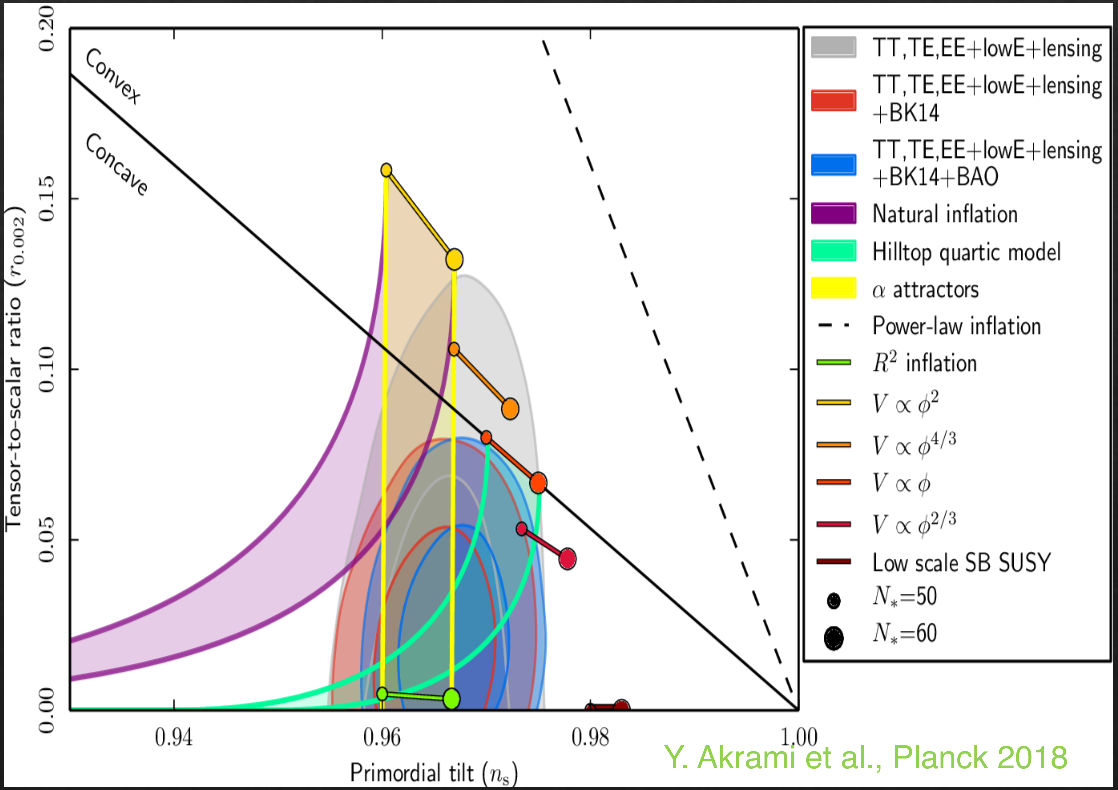
tensor power spectrum: $P_\gamma \sim \langle \gamma\gamma' \rangle$

spectral tilt: $n_s - 1 = \dfrac{d\ln P_\zeta}{d\ln k}, \ldots$

Restrictions on inflaton potential

$$n_s - 1 = \frac{d\ln P_\zeta}{d\ln k} = -3\left(\frac{V'}{V}\right)^2 + 2\frac{V''}{V}$$

$$r = \frac{P_\gamma}{P_\zeta} = 8\left(\frac{V'}{V}\right)^2$$

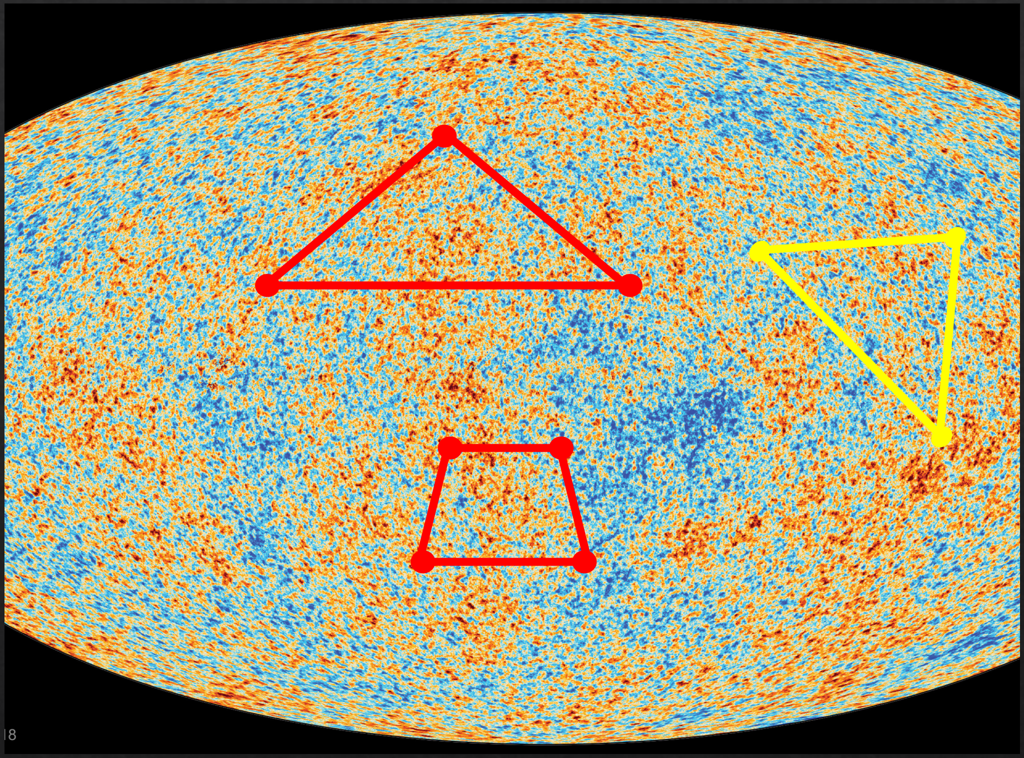


r

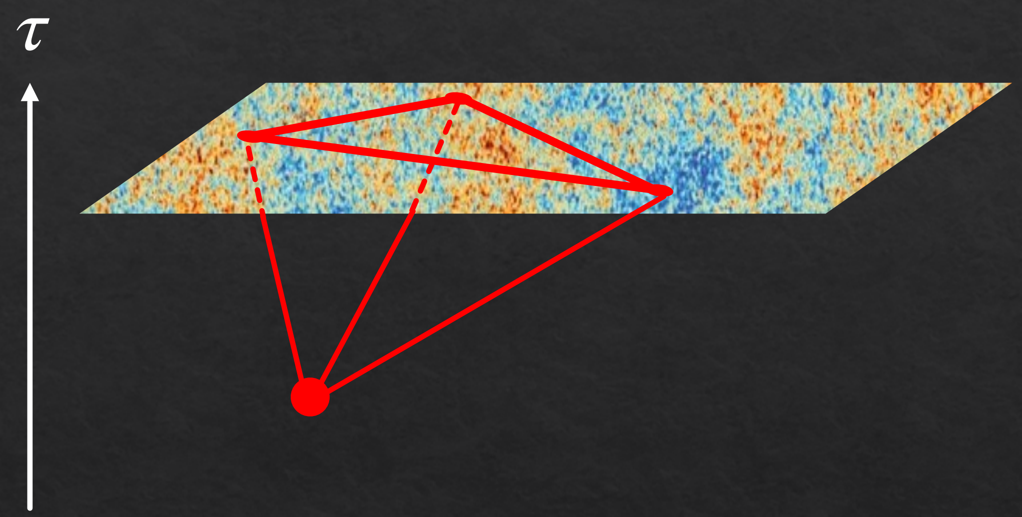
n_s

Y. Akrami et al., Planck 2018

Beyond 2-pt. function: non-Gaussianity (NG)



NG from self-interaction
(=gravitational floor)



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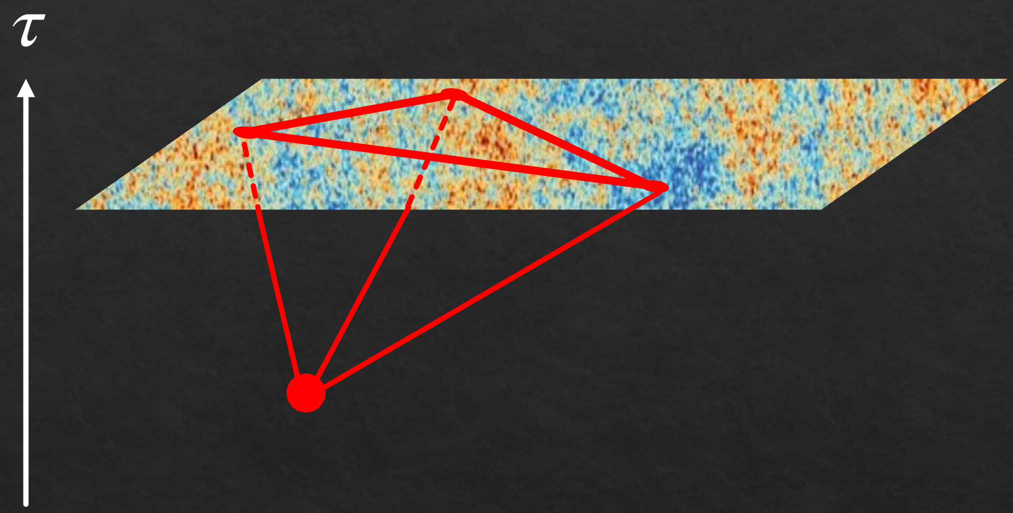

- $f_{\rm NL}$~$O(\epsilon)$ for simple model Maldacena '03

simple : single scalar + Einstein gravity + canonical kinetic term + slow-roll

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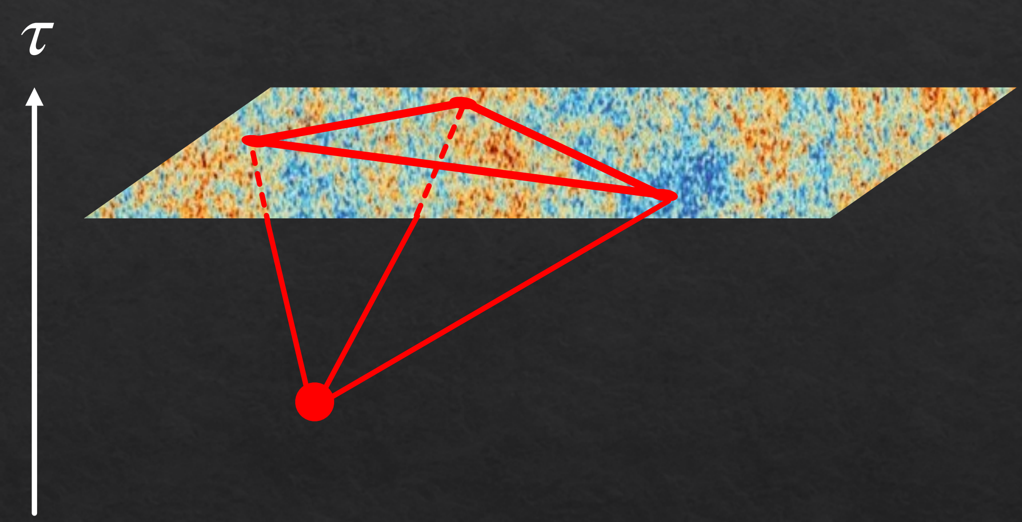

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- $f_{\rm NL} \gg 1$ for more general class of inflation models

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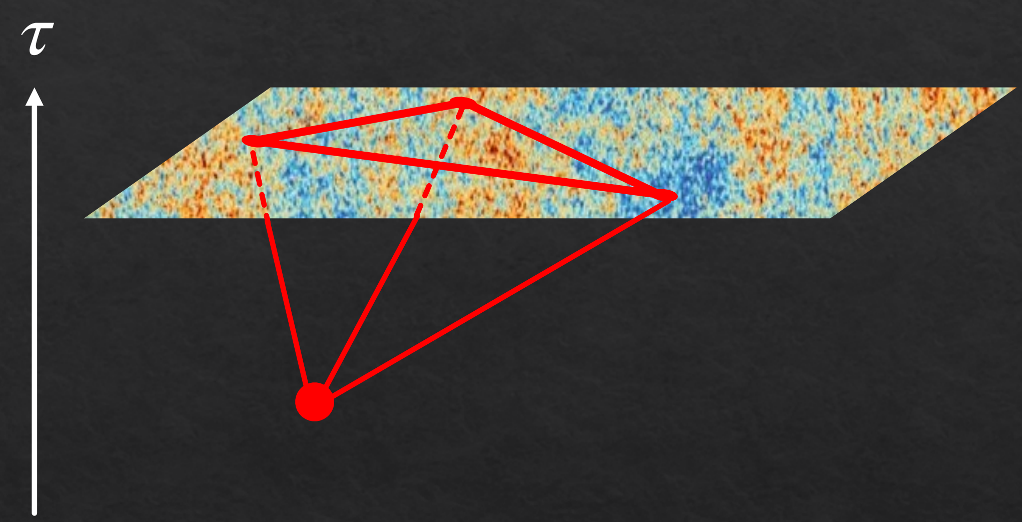

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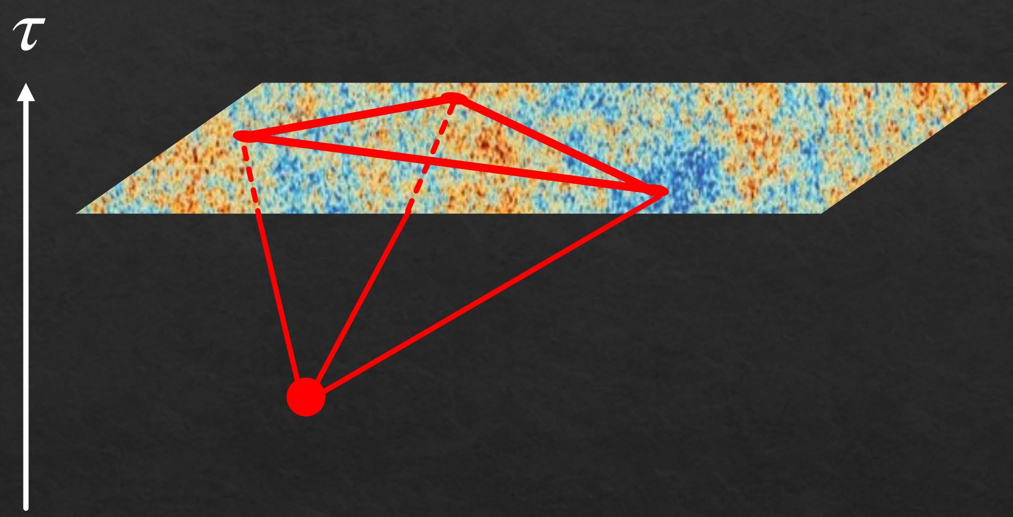
    simple : single scalar + Einstein gravity + canonical kinetic term + slow-roll

- $f_{\mathrm{NL}} \gg 1$ for more general class of inflation models

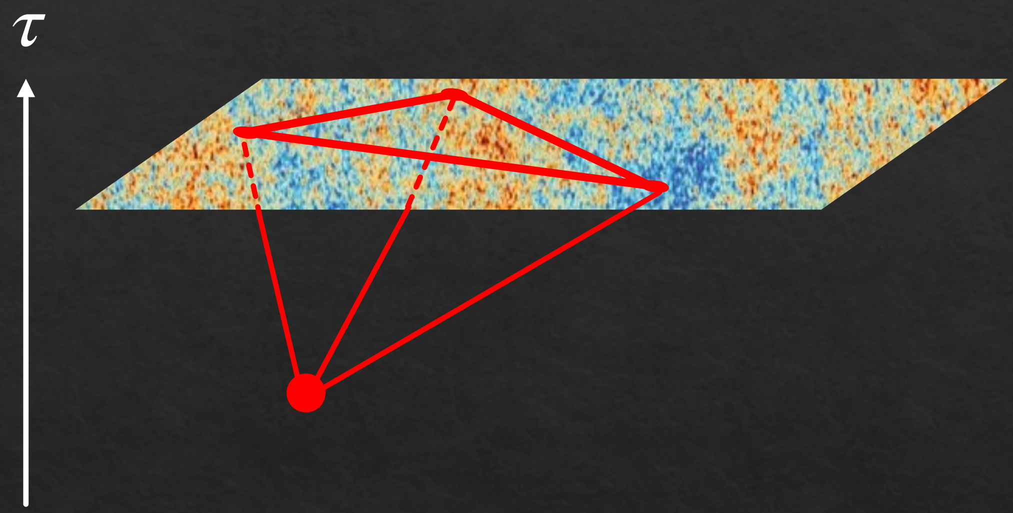
- Maldacen's consistency relation, Suyama-Yamaguchi bound, …

NG from self-interaction (=gravitational floor)



NG $\rightarrow$ strong restrictions on inflation models

NG from self-interaction (=gravitational floor)



NG → strong restrictions on inflation models

There's more: Cosmological collider (CC)

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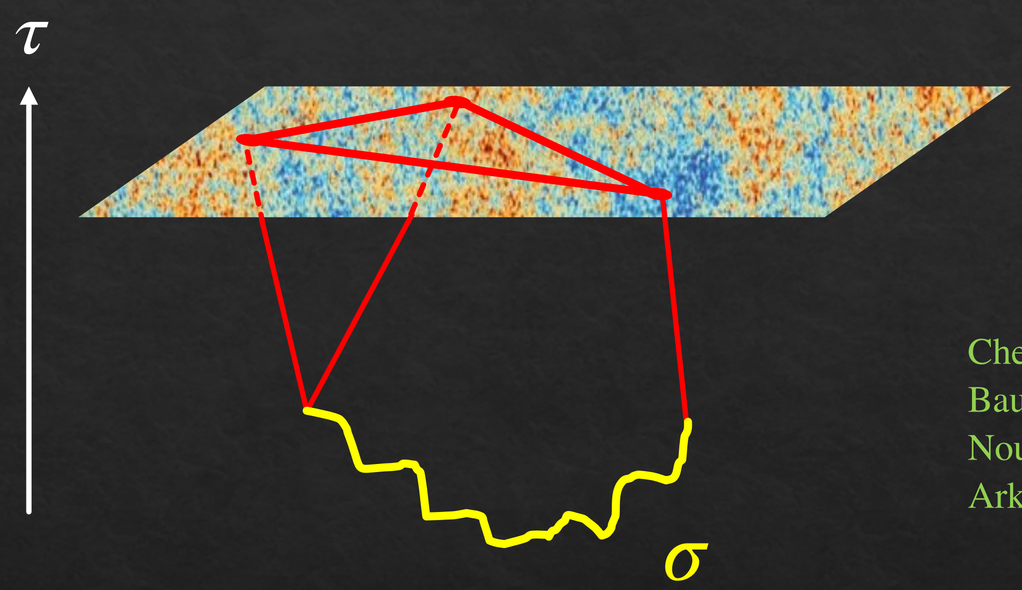
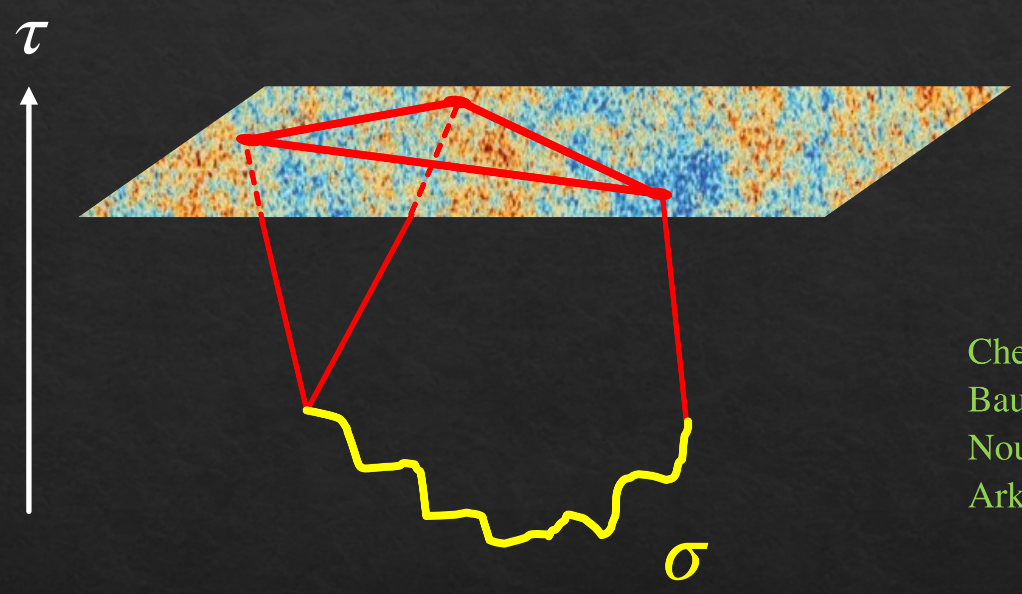


Chen, Wang, '10
Baumann, Green, '12
Noumi, Yamaguchi, Yokoyama, '13
Arkani-Hamed, Maldacena, '15

Cosmological collider (CC)



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- NG can contain information of heavy (new) particle σ with specific oscillation signal (frequency = m_σ)

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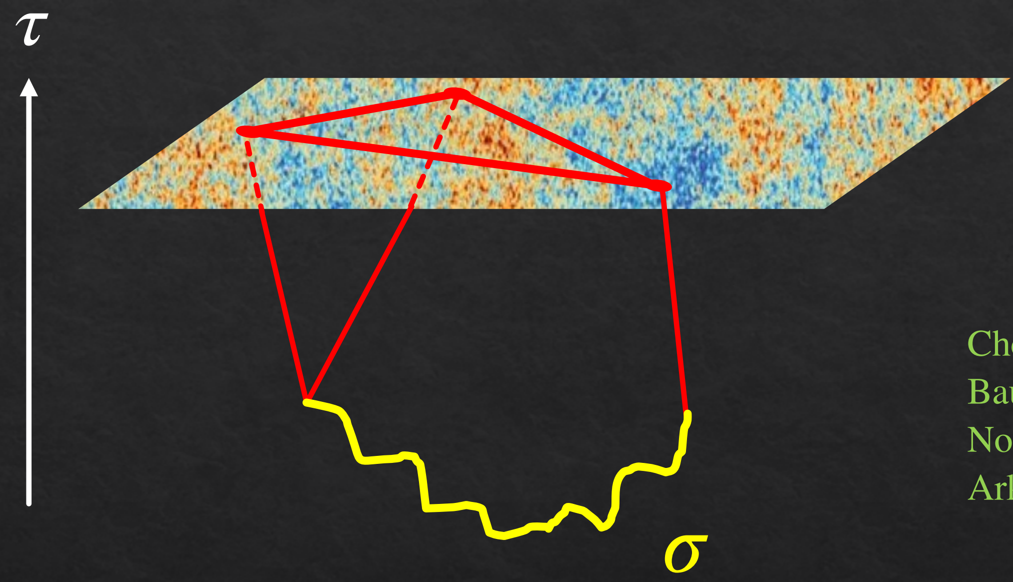

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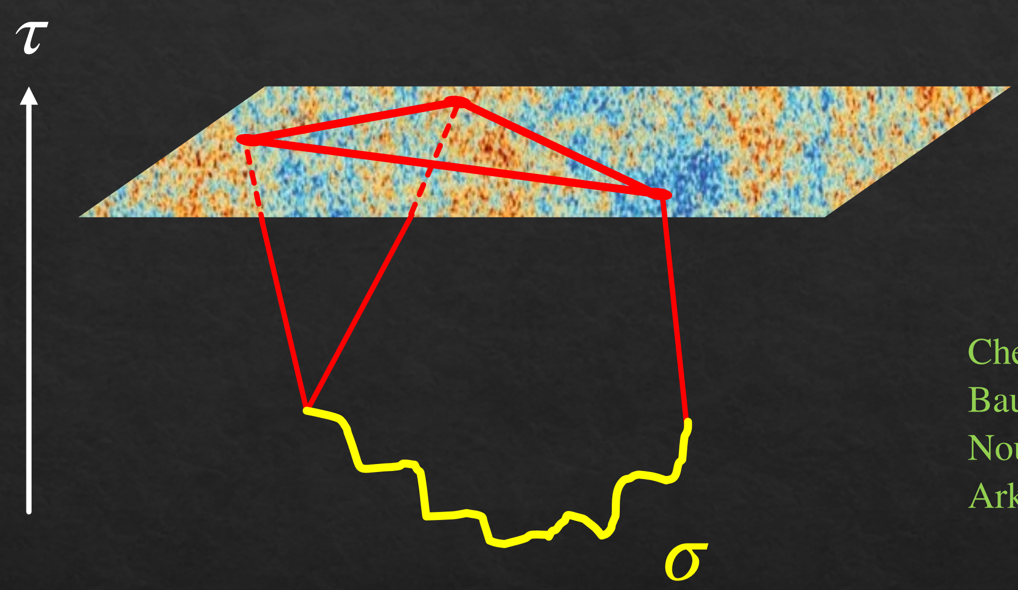

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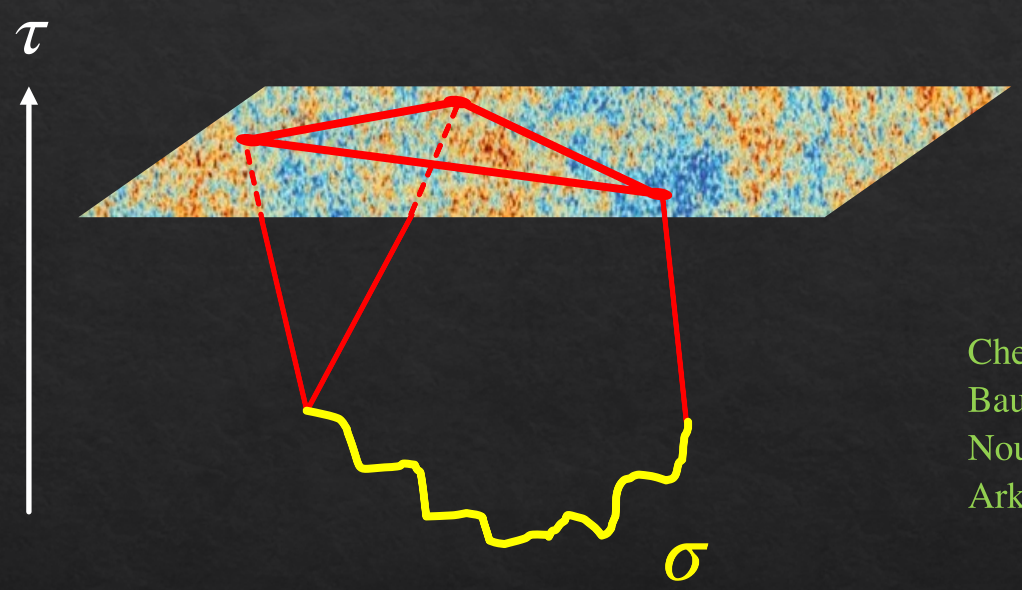
Cosmological collider (CC)



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- NG can contain information of heavy (new) particle σ with specific oscillation signal (frequency = m_σ)
- $m_\sigma \sim H \sim 10^{13}$(GeV) $\gg$ energy scale of terrestrial experiment
- Signal can be large!!
- CC = new tool for searching high energy physics

Many works

✓SUSY (Baumann, Green, ’12)
✓EFT approach (Noumi, Yamaguchi, Yokoyama, ’13)
✓Spinning particle (H. Lee, D. Baumann, and G. L. Pimentel, ’16, S. Kim, T. Noumi, K. Takeuchi, Siyi Zhou, ’19, …)
✓Neutrino (Chen, Wang, Xianyu, ’16)
✓Leptogenesis (Cui, Xianyu ’21)
✓GUT (Maru, Okawa, ’21)
…

Question

Can we really distinguish BSM from CC signal??

Personal view: still far from practical application

Many technical problems:
time integral of special functions, evaluation of loop diagram, etc.
($\Leftarrow$ "Cosmological Bootstrap" may help but still in development)

Question

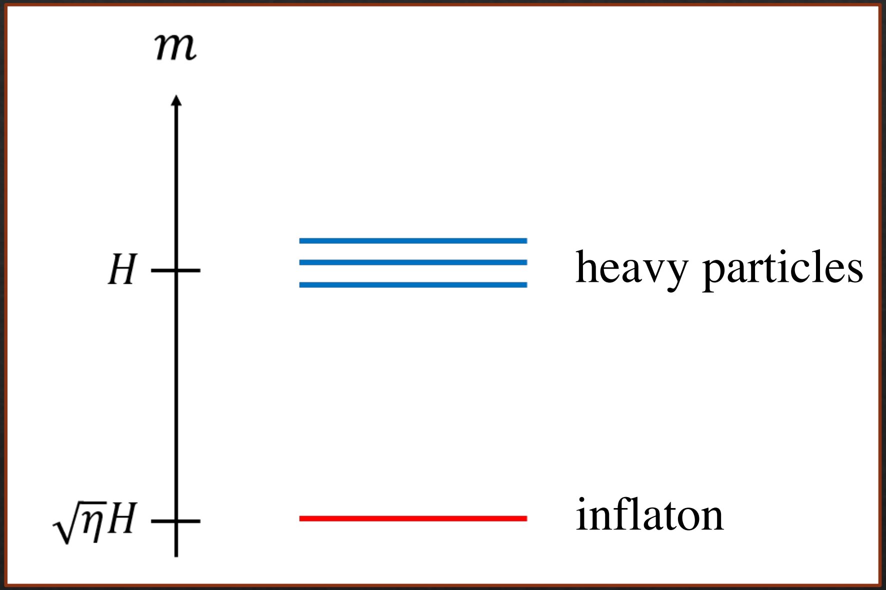
Can we really distinguish BSM from CC signal??

However, even at this stage, there are some models
that predict distinctly different signals that are distinguishable from others.

$\Rightarrow$ Today's topic: Continuous spectrum on CC & time-dependent mass

Multifield on CC

In realistic model building,
natural setup = existence of multiple heavy states



Example:
SUSY → Sparticles with soft masses ~H
Extra dimensions → infinite tower of KK states

Multifield on CC

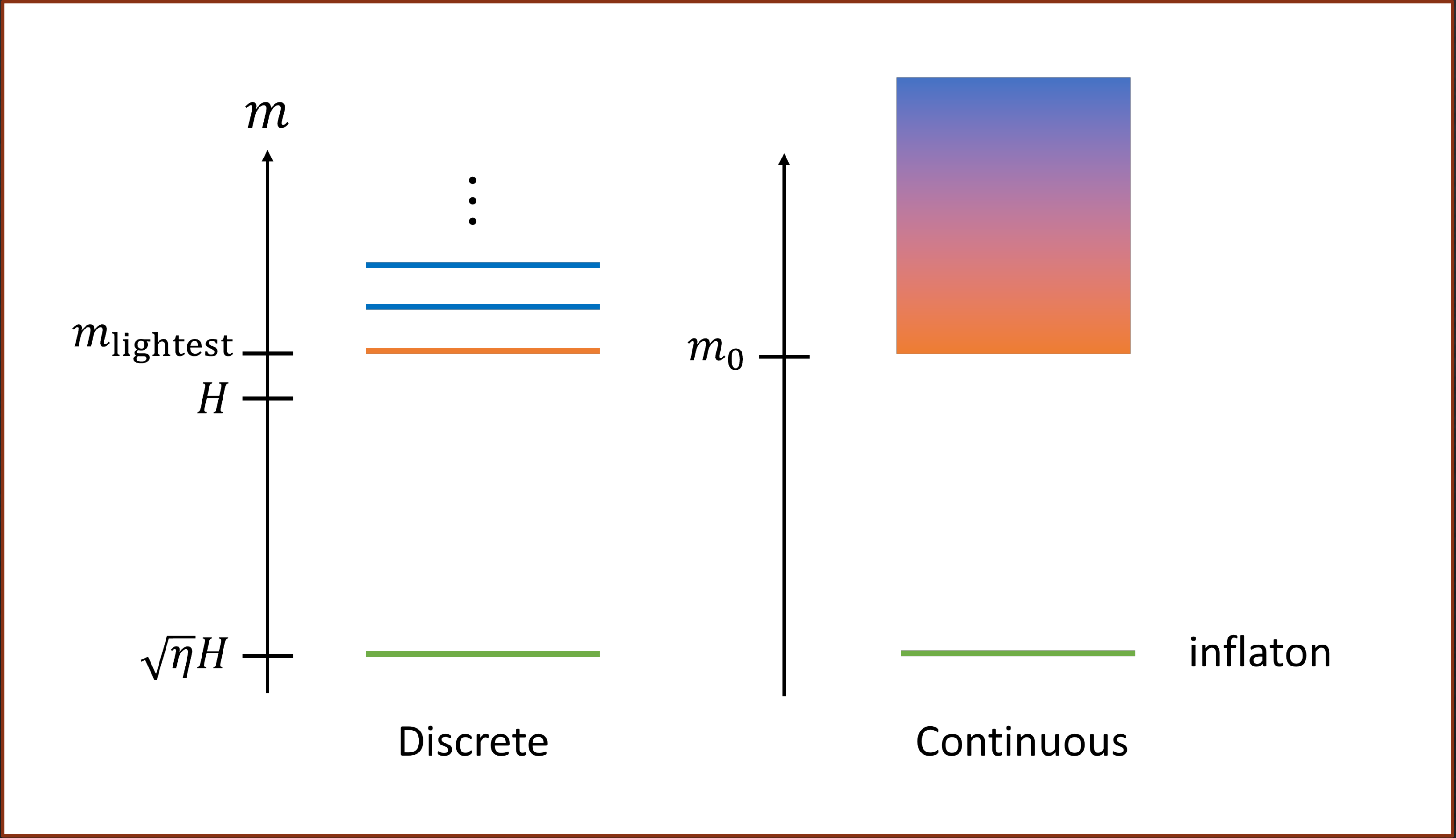
Whole signal is governed by lightest particle even if there are several heavy particles due to Boltzmann suppression $\sim e^{-\pi m/H}$

$\Rightarrow$ almost the same signal by a single heavy state

Exception: degenerate case

SA, M. Yamaguchi, JHEP 04 (2021) 127
L. Pinol, SA, S. Renaux-Petel, M. Yamaguchi, 2112.05710

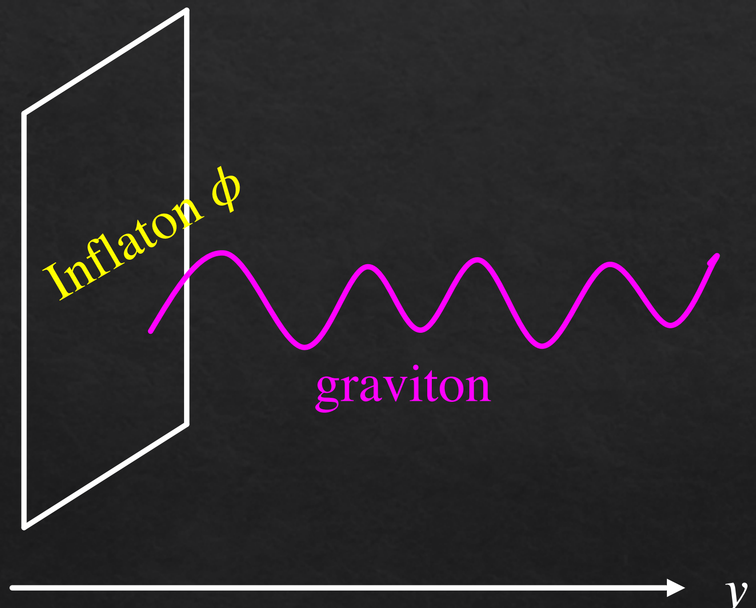
Continuous spectrum??



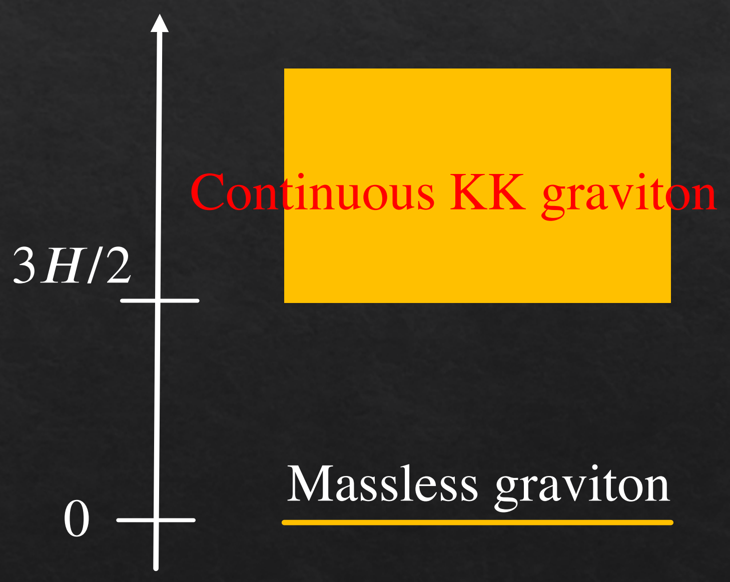
Example from Extra dimension

S. Kumar and R. Sundrum' 18

RS2-like setup (5 dim.)



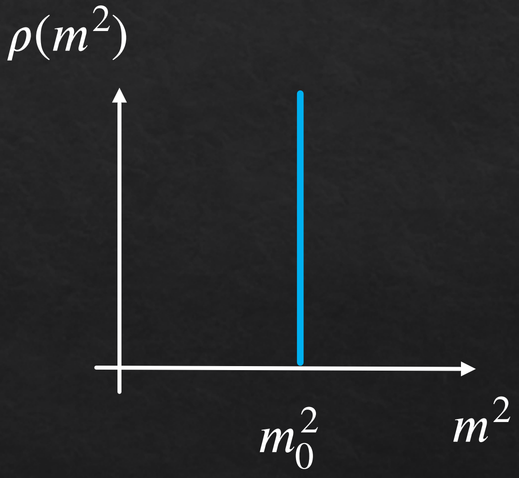
KK graviton spectrum



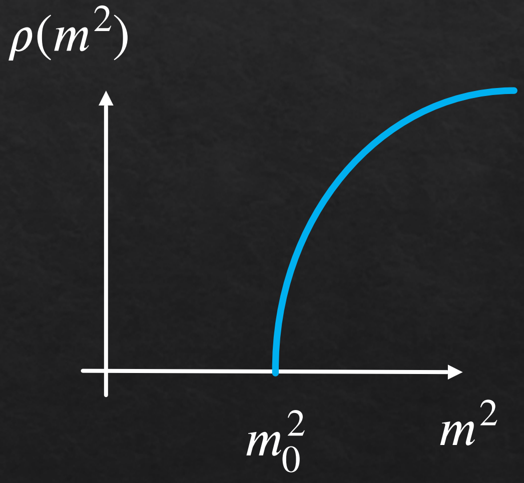
Setup

ϕ: inflaton

σ: field with continuous spectrum
characterized by spectral density $\rho(m^2)$



Discrete (or single)



Continuous

Setup

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{\rm Pl}^2}{2} R - \frac{1}{2} f(\sigma) \left(\partial_\mu \phi\right)^2 - \frac{1}{2} \left(\partial_\mu \sigma\right)^2 - V(\phi) - U(\sigma) \right]$$

ϕ: inflaton
σ: field with continuous spectrum

Setup

$$S = \int d^4x \sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}R - \frac{1}{2}f(\sigma)\left(\partial_\mu\phi\right)^2 - \frac{1}{2}\left(\partial_\mu\sigma\right)^2 - V(\phi) - U(\sigma)\right]$$

ϕ: inflaton

σ: field with continuous spectrum

Quantization of fluctuations

$$\delta\phi = \int \frac{d^3\boldsymbol{k}}{(2\pi)^3}\left(u_k a_{\boldsymbol{k}} + u_k^* a_{-\boldsymbol{k}}^\dagger\right) e^{i\boldsymbol{k}\cdot\boldsymbol{x}},$$

$$\delta\sigma = \boxed{\int \frac{dm^2}{2\pi}\sqrt{\rho\left(m^2\right)}}\int \frac{d^3\boldsymbol{k}}{(2\pi)^3}\left(v_{k,m} b_{\boldsymbol{k},m} + v_{k,m}^* b_{-\boldsymbol{k},m}^\dagger\right) e^{i\boldsymbol{k}\cdot\boldsymbol{x}},$$

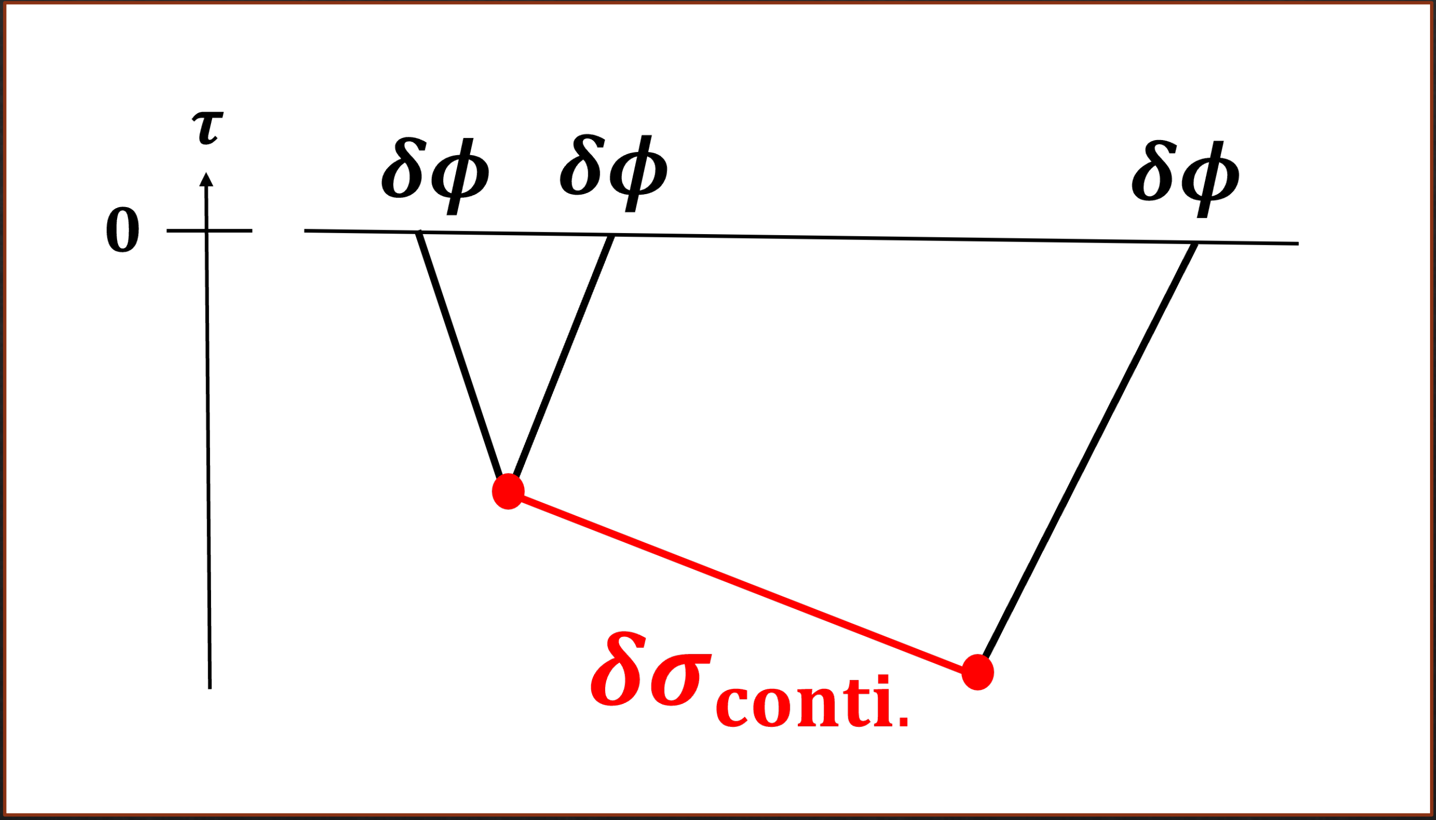
E. Megıas, M. Quirsos,'19

C. Csaki, S. Hong, G. Kurup, S. J. Lee, M. Perelstein, W. Xue, '22

$\rho(m^2)$: spectral density

Interactions

$$\mathcal{L}_{2,\mathrm{int}} = a^3 f_\sigma \dot{\phi}_0 \delta\sigma \delta\dot{\phi},$$
$$\mathcal{L}_{3,\mathrm{int}} = \frac{a^3}{2} f_\sigma \delta\sigma \left((\delta\dot{\phi})^2 - \frac{1}{a^2} (\partial\delta\phi)^2 \right),$$



Observable:

$$\left\langle \delta\phi^3 \right\rangle |_{\kappa \equiv k_{1,2}/k_3 \gg 1} \propto F$$

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Usual case
(single particle with
definite mass):

$$F_{\text{particle}} = e^{-\pi\mu_\sigma} \mathcal{A}(\mu_\sigma) \sin\left(\mu_\sigma \log\kappa + \varphi(\mu_\sigma)\right)$$

$$\mu_\sigma = \sqrt{\left(\frac{m_\sigma}{H}\right)^2 - \frac{9}{4}}$$

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Continuous case:

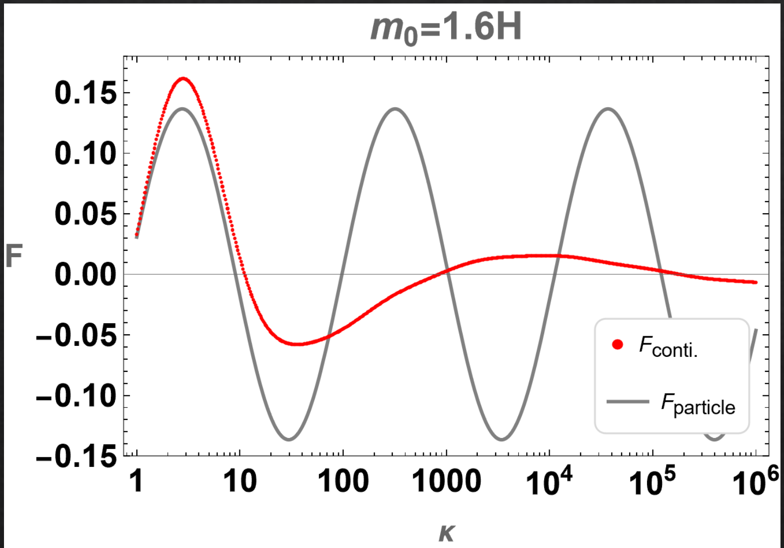
$$F_{\text{conti.}} = \boxed{\int_{m_0^2}^{\infty} \frac{dm^2}{2\pi}\rho\left(m^2\right)} e^{-\pi\mu}\mathcal{A}(\mu)\sin\left(\mu\log\kappa + \varphi(\mu)\right)$$

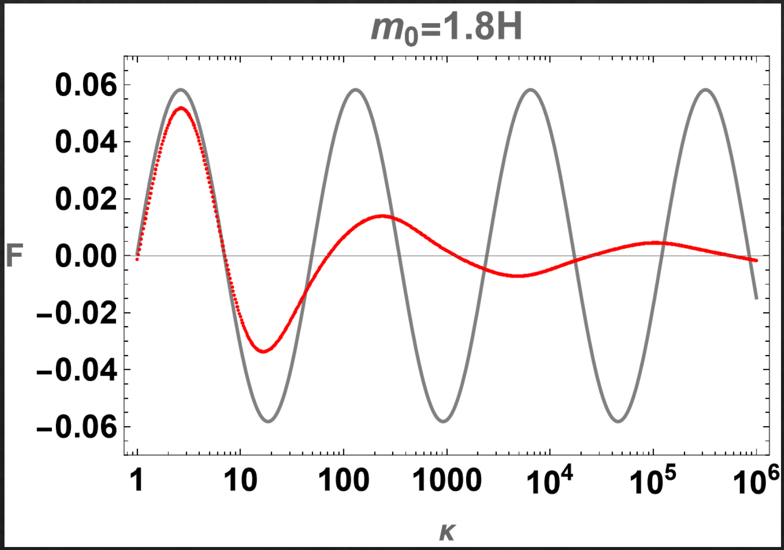
$$\mu = \sqrt{\left(\frac{m}{H}\right)^2 - \frac{9}{4}}.$$

Result

choose a specific spectral density $\rho\left(m^2\right) = \frac{2\pi}{m_0^2}\sqrt{\frac{m^2}{m_0^2} - 1},$

E. Megıas, M. Quirsos,'19
C. Csaki, S. Hong, G. Kurup, S. J. Lee, M. Perelstein, W. Xue, '22





✓Damping amplitude = signal of continuous spectrum

✓ $\int \mathrm{d}m \sin(m\kappa) \rightarrow 1/\kappa$ factor (consequence of integration)

Time dependent mass

In general, inflationary background gives a time-dependence to σ-mass

$$\mathcal{L}_{\rm int} \supset -\frac{1}{2} g(\phi)\sigma^2.$$

$$m_{\rm eff}^2 = m_0^2 + g(\phi)$$

Time-dependent mass
(scale dependence)

Effects on CC signal??

So far, studied only by numerical simulation

M. Reece, L.T. Wang, Z.Z. Xianyu' 2022

Other possibilities $-\xi R\sigma^2$ & $(\partial\phi)^2\sigma^2/\Lambda^2$

Analytic approach

SA, T. Noumi, F. Sano, M.Yamaguch, 2309.XXXX

Linear app.
$$\phi_0(\tau) = \phi_{0*} - \sqrt{2\epsilon} M_{\rm Pl} \left(1 - \frac{\tau}{\tau_*}\right) + \cdots ,$$

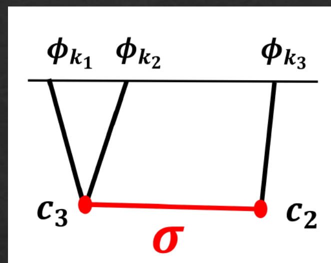
$$m_{\rm eff}^2(\tau) = g_* - g_{\phi,*} \sqrt{2\epsilon} M_{\rm Pl} \left(1 - \frac{\tau}{\tau_*}\right) + \cdots ,$$

Mode function of σ :
$$v_k = \frac{e^{\pi\kappa/2}}{\sqrt{2k}} H(-\tau) W_{-{\rm i}\kappa, {\rm i}\mu}(2{\rm i}k\tau),$$

$$\kappa(v) = -\frac{\sqrt{2\epsilon}\, g_{\phi,*}\big|(v) M_{\rm Pl}}{2H^2}$$

$\kappa = 0 \rightarrow$ constant mass

Analytic approach



$$= -2c_2c_3H \cdot \frac{1}{8k_1k_2k_3^4}\lim_{k_4\to0}\sum_{\mathrm{a,b}=\pm}\mathcal{I}^{0,-2}_{\mathrm{ab}} + 2\text{per}..$$

$$\begin{aligned}
&\mathcal{I}^{p_1p_2}_{\pm\mp}\left(u_1,1,v(k_3)\right)/H^2 \\
&= \left\{ \frac{-e^{\mp \mathrm{i}\frac{\pi}{2}(p_1-p_2)} e^{\pi\kappa}\cosh\left[\pi(\mu-\kappa)\right]}{2^{\frac52+p_2}\operatorname{sech}(2\pi\mu)} \Gamma\left[\begin{matrix} -2-p_2\mp \mathrm{i}\kappa,\ \frac52+p_2\pm \mathrm{i}\mu \\ \frac12+i\mu\mp \mathrm{i}\kappa,\ \frac12 - \mathrm{i}\mu\mp \mathrm{i}\kappa,\ -\frac32-p_2\pm \mathrm{i}\mu\end{matrix}\right] + (\mu\to-\mu)\right\} \\
&\times\left\{2^{\pm \mathrm{i}\mu}\left(\frac{u_1}{2}\right)^{\frac52+p_1\pm \mathrm{i}\mu}\pi\operatorname{csch}(2\pi\mu)\,{}_2\mathcal{F}_1\left[\left.\begin{matrix}p_1+\frac52\pm \mathrm{i}\mu,\ \frac12\pm \mathrm{i}\mu\mp \mathrm{i}\kappa\\ 1\pm 2\mathrm{i}\mu\end{matrix}\right| u_1\right] + (\mu\to-\mu)\right\},
\end{aligned} \tag{3.51}$$

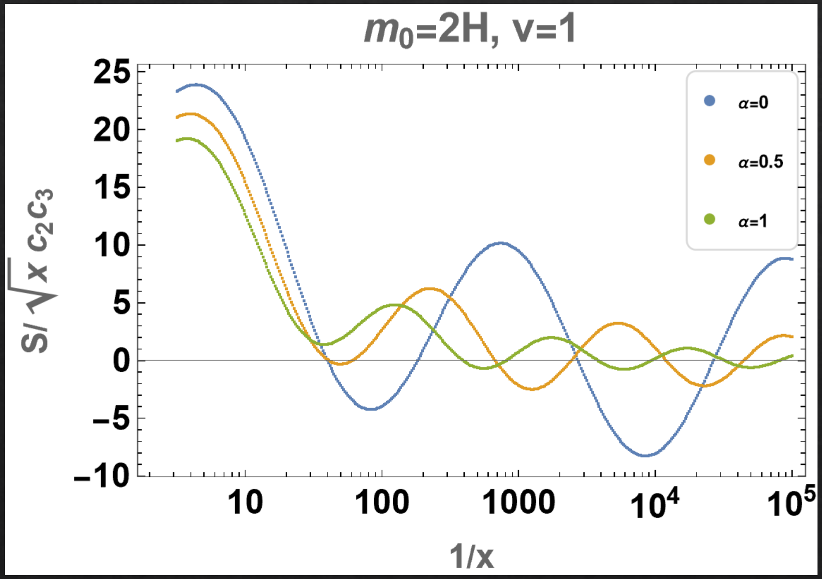
and

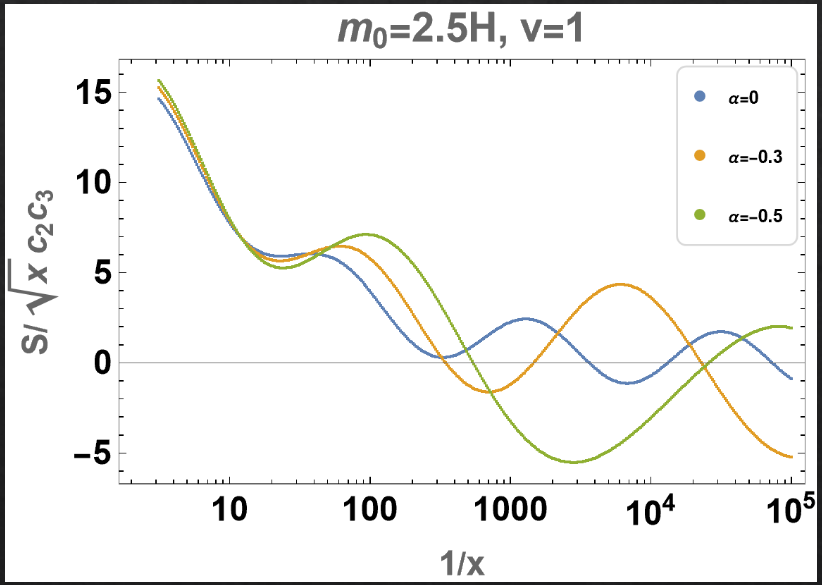
Effects of time-dependence

$$\begin{aligned}
&\mathcal{I}^{p_1p_2}_{\pm\pm}\left(u_1,1,v(k_3)\right)/H^2 \\
&= \frac{e^{\mp \mathrm{i}\frac{\pi}{2}p_{12}}\Gamma\left(5+p_{12}\right)}{2^{5+p_{12}}\left[\mu^2+\left(p_2+\frac52\right)^2\right]} u_1^{5+p_{12}}\,{}_3\mathrm{F}_2\left[\left.\begin{matrix}1,3+p_2\mp \mathrm{i}\kappa\ 5+p_{12}\\ \frac72+p_2-\mathrm{i}\mu,\ \frac72+p_2+\mathrm{i}\mu\end{matrix}\right| u_1\right] \\
&\mp \mathrm{i}\left\{\frac{e^{\mp \mathrm{i}\frac{\pi}{2}p_{12}}e^{\pi\kappa}\cosh\left[\pi(\mu+\kappa)\right]}{2^{\frac52+p_2}\operatorname{sech}(2\pi\mu)}\Gamma\left[\begin{matrix}-2-p_2\pm \mathrm{i}\kappa,\ \frac52+p_2\pm \mathrm{i}\mu\\ -\frac32-p_2\pm \mathrm{i}\mu\end{matrix}\right] + (\mu\to-\mu)\right\} \\
&\times\left\{2^{\pm \mathrm{i}\mu}\left(\frac{u_1}{2}\right)^{\frac52+p_1\pm \mathrm{i}\mu}\frac{e^{\pi\mu}\cosh\left[\pi(\mu-\kappa)\right]}{\sinh(2\pi\mu)}\,{}_2\mathcal{F}_1\left[\left.\begin{matrix}p_1+\frac52\pm \mathrm{i}\mu,\ \frac12\pm \mathrm{i}\mu\mp \mathrm{i}\kappa\\ 1\pm 2\mathrm{i}\mu\end{matrix}\right| u_1\right] + (\mu\to-\mu)\right\},
\end{aligned} \tag{3.52}$$

Results (preliminary)

$$g(\phi) = m_0^2 \left(1 + \alpha \frac{\phi}{M_{\rm Pl}}\right) \quad \Rightarrow \kappa \sim \alpha$$





$x \equiv k_3/k_{1,2}$

Large deviation from standard (constant) mass
→ existence of time-dependent mass

Why??

Squeezed limit:
correlation between a long mode (k_3) and short ones ($k_{1,2}$)

Exit horizon at $\tau_{\rm early}$

at $\tau_{\rm late}$

Super-horizon evolution of long mode:

$$v_{k3}(\tau_{\rm late}) \propto v_{k3}(\tau_{\rm early}) \times Exp \underbrace{\left\{ -\frac{\pi}{2} \frac{m\left(\tau_{\rm late}\right) - m\left(\tau_{\rm early}\right)}{H} \right\}}_{\text{enhancement/suppression depending on sign of } \alpha}$$

enhancement/suppression
depending on sign of α

Summary

Cosmological collider (CC)
is a new attractive tool for exploring high energy

1. Continuous spectrum on CC
(motivated by some extra dimensional models)

✓damping effects in deep squeezed limit
✓universal feature independently of $\rho(m^2)$
✓the signal cannot be mimicked by single particle with a definite mass → Strong evidence of continuous spectrum
✓interesting to think about embedding into concrete UV setup

Summary

Cosmological collider (CC)
is a new attractive tool for exploring high energy

2. Time-dependent mass on CC
(coming from inflaton couplings)

✓non-derivative coupling → sizable effects
✓analytic estimation
✓amplitude enhancement/suppression & change of wavelength in squeezed limit
✓large deviation from the standard case (constant mass)

Thank you for your attention !!

Details

Observable:

Curvature perturbation $\zeta \sim -\frac{H}{\dot{\phi}_0}\delta\phi$

def. of shape function ($\sim f_{\mathrm{NL}}$)

$$\langle \zeta_{\mathbf{k_1}} \zeta_{\mathbf{k_2}} \zeta_{\mathbf{k_3}} \rangle = (2\pi)^7 P_\zeta^2 \frac{1}{(k_1 k_2 k_3)^2} \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) \boxed{S(k_1, k_2, k_3)},$$

Result

Result in squeezed limit ($\kappa \equiv k_1/k_3 \gg 1$)

$$S = -\frac{\pi}{4} M_{\rm Pl}^2 \epsilon \left(\frac{f_\sigma}{f}\right)^2 \kappa^{-1/2} \boxed{\int_{m_0^2}^{\infty} \frac{dm^2}{2\pi} \rho\left(m^2\right)} e^{-\pi\mu} \mathrm{Im}\left\{\left(\frac{1}{\kappa}\right)^{-i\mu} J_-(i\mu) + \left(\frac{1}{\kappa}\right)^{i\mu} J_+(i\mu)\right\}$$

New !!

$$\mu = \sqrt{\left(\frac{m}{H}\right)^2 - \frac{9}{4}}.$$

coefficient (analytic expression)

$$J_+(i\mu) = -\frac{\sqrt{\pi} 2^{-1-2i\mu} \left(e^{\pi\mu} - i\right) \Gamma\left(i\mu + \frac{7}{2}\right) \left(\mathrm{csch}(\pi\mu) + \mathrm{sech}(\pi\mu)\right)}{(2\mu - i)\Gamma(i\mu + 1)},$$

$$J_-(i\mu) = J_+(-i\mu) e^{2\pi\mu}.$$

Result

$$S = -\frac{\pi}{4} M_{\rm Pl}^2 \epsilon \left(\frac{f_\sigma}{f}\right)^2 \kappa^{-1/2} F_{\rm conti.}$$

Continuous case:

$$F_{\rm conti.} = \boxed{\int_{m_0^2}^{\infty} \frac{dm^2}{2\pi} \rho\left(m^2\right)} e^{-\pi\mu} \mathcal{A}(\mu) \sin\left(\mu \log \kappa + \varphi(\mu)\right)$$

$$\mathcal{A}(\mu) = \left[\left\{{\rm Im}J_-(i\mu) + {\rm Im}J_+(i\mu)\right\}^2 + \left\{{\rm Re}J_-(i\mu) - {\rm Re}J_+(i\mu)\right\}^2\right]^{1/2},$$
$$\varphi(\mu) = \arctan\left(\frac{{\rm Im}J_-(i\mu) + {\rm Im}J_+(i\mu)}{{\rm Re}J_-(i\mu) - {\rm Re}J_+(i\mu)}\right).$$

Usual case
(single particle with
definite mass):

$$F_{\rm particle} = e^{-\pi\mu_\sigma} \mathcal{A}(\mu_\sigma) \sin\left(\mu_\sigma \log \kappa + \varphi(\mu_\sigma)\right)$$