17th August, KIAS

Walls, Bubbles & Doom

the Cosmology of HEFT.

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in collaboration with Rodrigo Alonso, Juan Carlos Criado & Rachel Houtz

Aims

① What is HEFT & why should I care about it?

② What does the LHC not tell us?

③ What can cosmology tell us that the LHC can't?

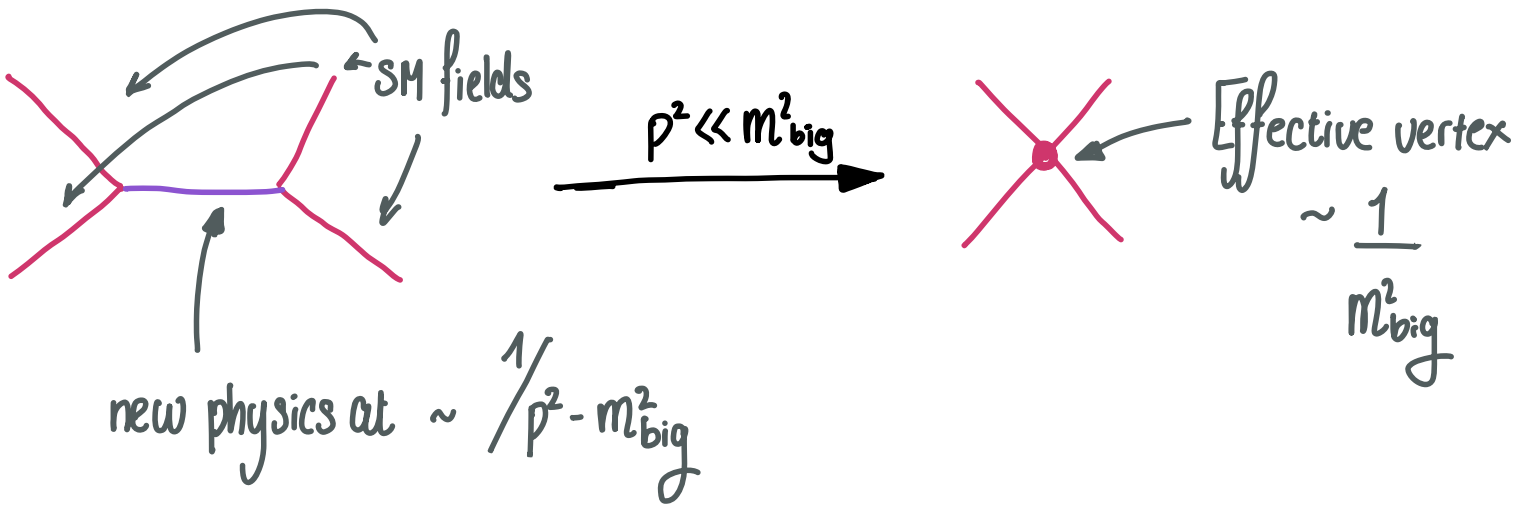
④ Some pheno! (ie. what could HEFT look like in gravitational wave format?)

1 What is HEFT?

Effective Field Theory.

Take the SM to be a *low-energy, effective* theory: all new physics is at

$$E \gg \Lambda \sim \text{TeV}$$



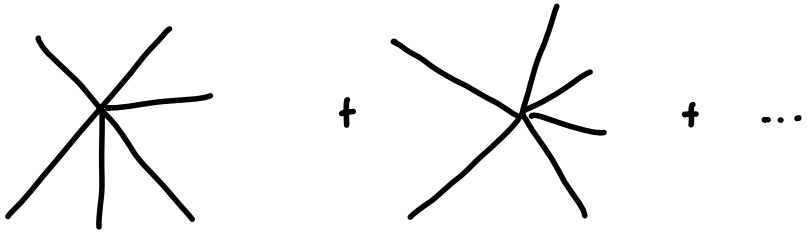
Think 4-Fermi theory!!

Since we know 'integrating out' UV physics makes these 'effective', higher-point vertices, we can write down all the ways this can happen...

$$\mathcal{L}_{EFT} = \mathcal{L}_{low\text{-}energy} + \sum_{d=4} \frac{C_i}{\Lambda^{d-4}} Q_i^{(d)}$$

Wilson coefficients

Operators made of SM fields.
(obeying SM symmetry group)



SMEFT

Linearly realised ew symmetry

$$SU(3)_c \times SU(2)_L \times U(1)_Y$$

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_4 + i\phi_3 \end{pmatrix} \sim \frac{(h+v)}{\sqrt{2}} U(\varphi)$$

$\frac{(h+v)}{\sqrt{2}}$: radial; $U(\varphi)$: 'angular'

V_{SMEFT} is analytic function of $H^\dagger H$

HEFT

Non-linearly realised ew symmetry

$$SU(3)_c \times U(1)_{em}$$

most general EFT parameterisation of low energy physics involving SM DoF.

h and $\vec{\varphi} = \begin{pmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \\ \sqrt{v^2 - \vec{\varphi}\cdot\vec{\varphi}} \end{pmatrix}$

$h \to h$ and $\vec{\varphi} \to O\vec{\varphi}$

$$V_{HEFT} = V(h)$$

$$\mathcal{L}_{HEFT} = \frac{1}{2}(\partial h)^2 + \frac{v^2 F(h)^2}{2}(\partial\vec{\varphi})^2$$

flare function $F(h) = 1 + a\frac{h}{v} + b\frac{h^2}{v^2} + \dots$

$$+ \frac{v^2}{8}F(h)^2\left[2g^2 W^+_\mu W^-_\mu + (g^2+g'^2) Z_\mu Z^\mu\right]$$

$$- \frac{m_h^2}{2}h^2 - \frac{m_h\sqrt{\lambda}}{2}\gamma_4(1-\epsilon)h^3 - \frac{\lambda}{8}\gamma_4^2 h^4 + \dots$$

$$\underbrace{\qquad\qquad\qquad V(h) \qquad\qquad\qquad}$$

+ higher derivatives which we will neglect

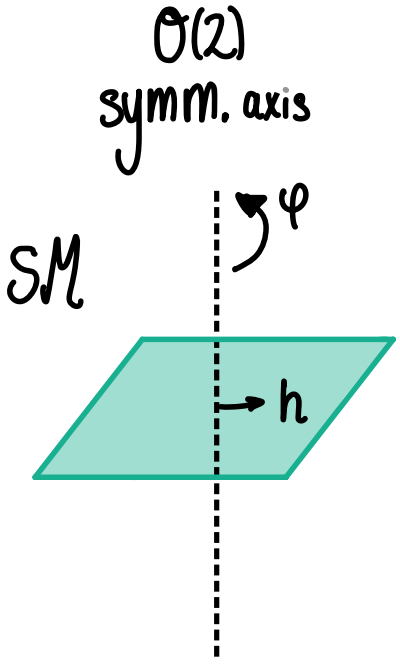
NB for the SM, $F(h) = 1 + \frac{h}{v}$ and $h \in [0, \infty]$.

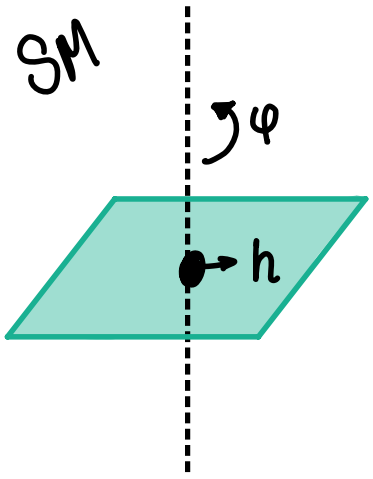
EFT theories as Manifolds: [arXiv: 1605.03602]

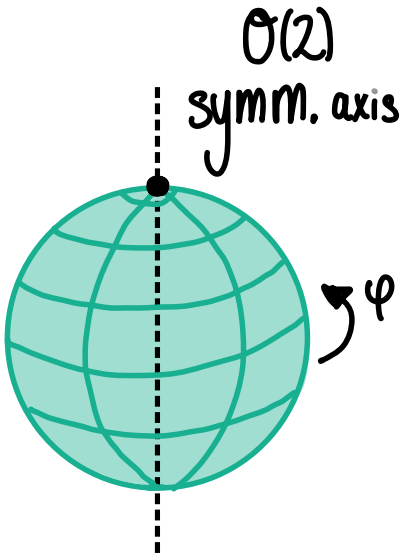
1: Symmetric axis around which $\vec{\varphi} \to O\vec{\varphi}$

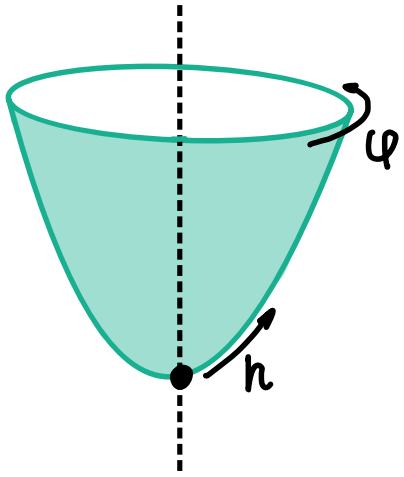
↳ Goldstone bosons act as angular modes

2: Higgs boson in the 'radial' direction





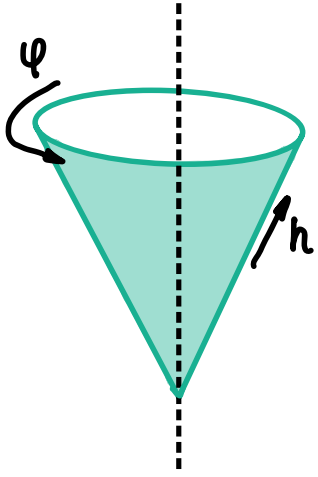


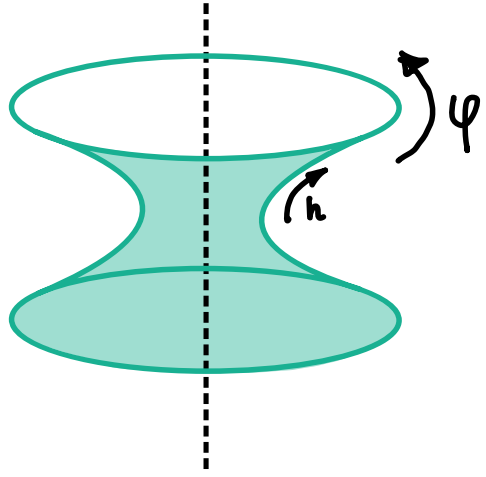


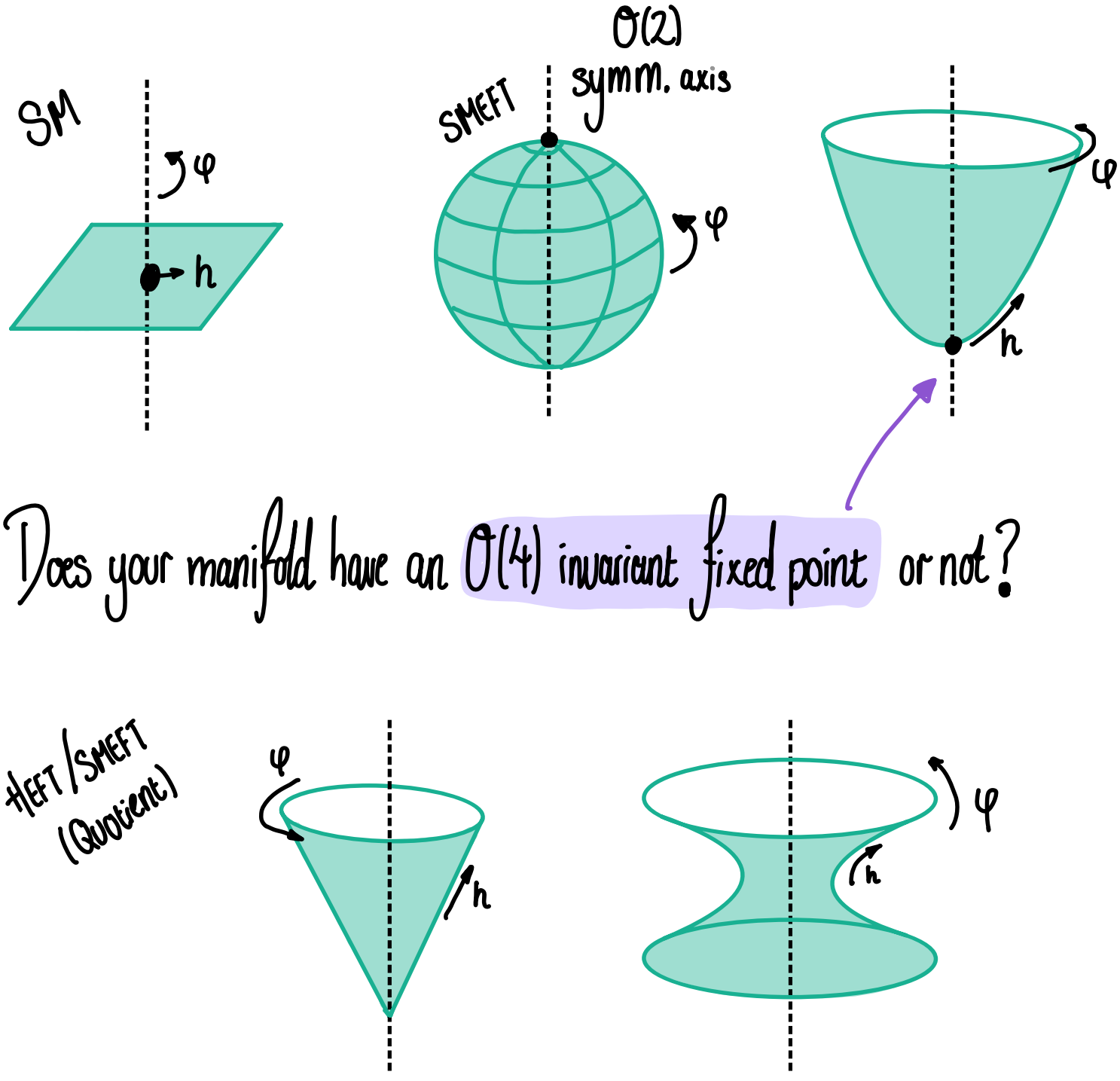
Examples of manifolds you can draw!

See eg:
[arXiv: 2109.1329
R. Alonso & MW]

[arXiv: 2008.08597; T. Cohen,
N. Craig, X. Lu, D. Sutherland]

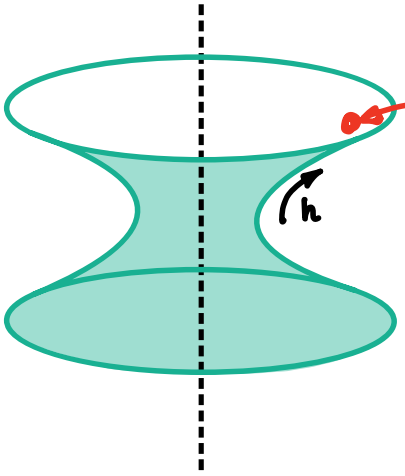




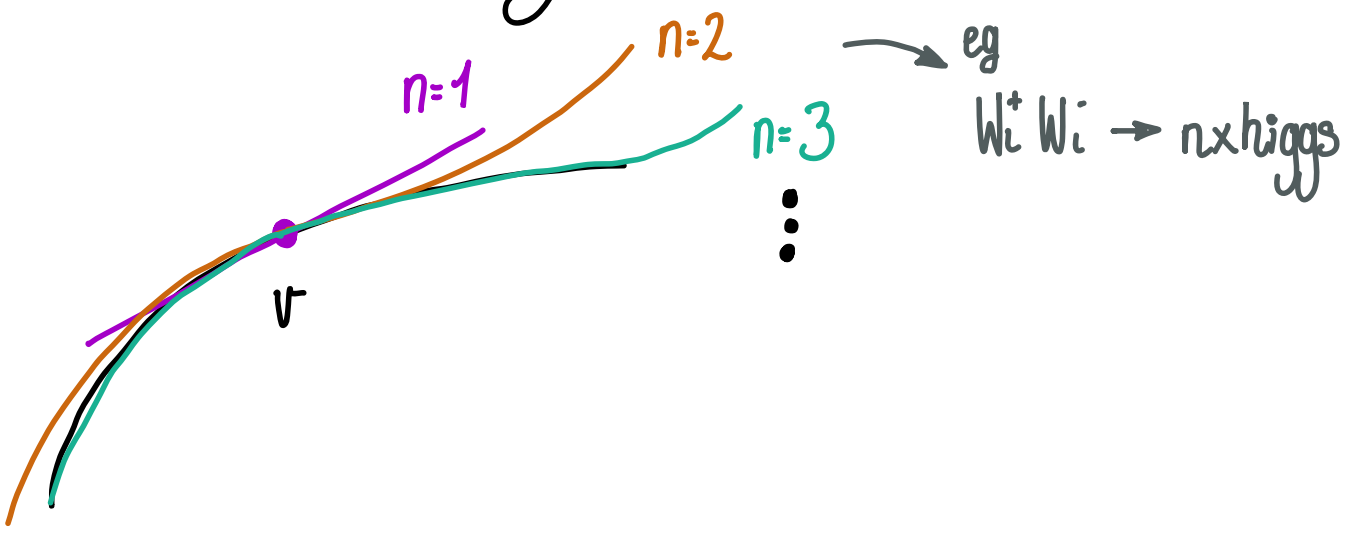

SM
φ
h
O(2)
symm. axis
SMEFT
φ
φ
h
HEFT
Does your manifold have an O(4) invariant fixed point or not?
HEFT/SMEFT
(Quotient)
φ
h
φ
h

But how can we look for a hole or singularity? → [S. Kanemura, R. Nagai, M. Tanaka, arXiv:2202.12774]

THIS TALK



Scattering experiments look locally around the vacuum.



What do we do?

a) Panic and/or give up?

Too stubborn

b) Hope that an answer miraculously descends to us from the sky.

Sounds about right!

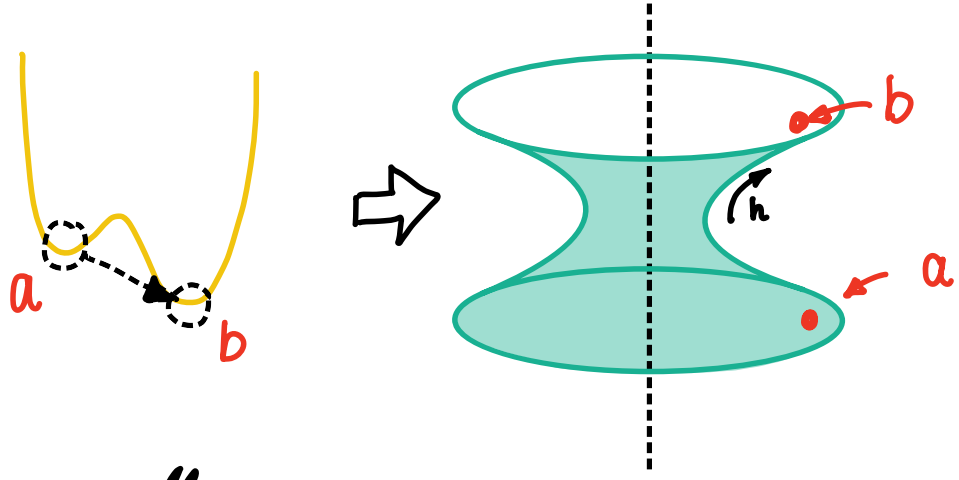
Look at Cosmology for a global approach

1: Sphaleron Tunnelling

2: Vacuum Evolution

↳ Walls, Bubbles & Doom.

We're interested in whether the universe may have tunnelled between vacua as the universe cooled...



We need to build ourselves an *effective potential*

↳ Tree level (quartic) potential.

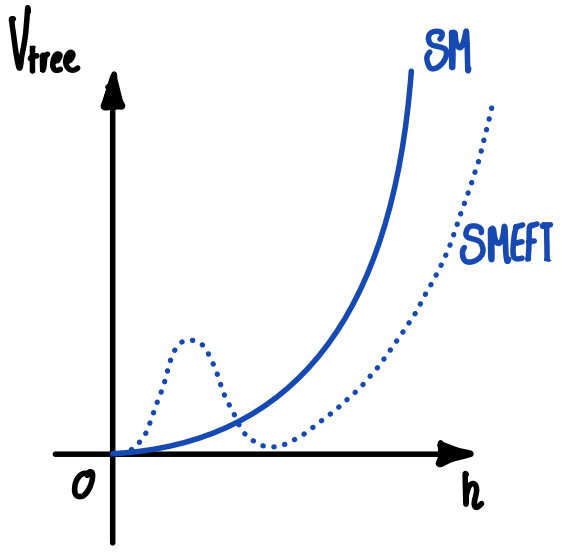
↳ Temperature in the early universe modifies couplings.

↳ Loop corrections modifies couplings

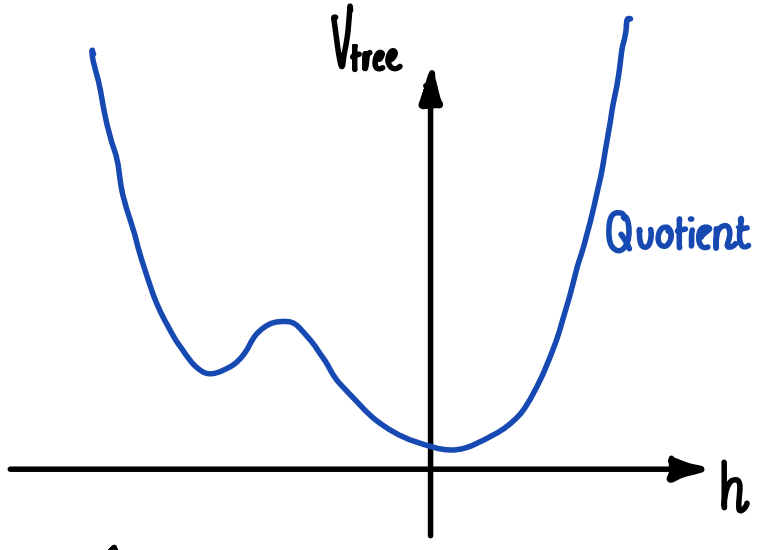
$$V_{eff} = V_{tree} + V_{thermal} + V_{loops}$$

HEFT Potential at Tree Level

SM and SMEFT



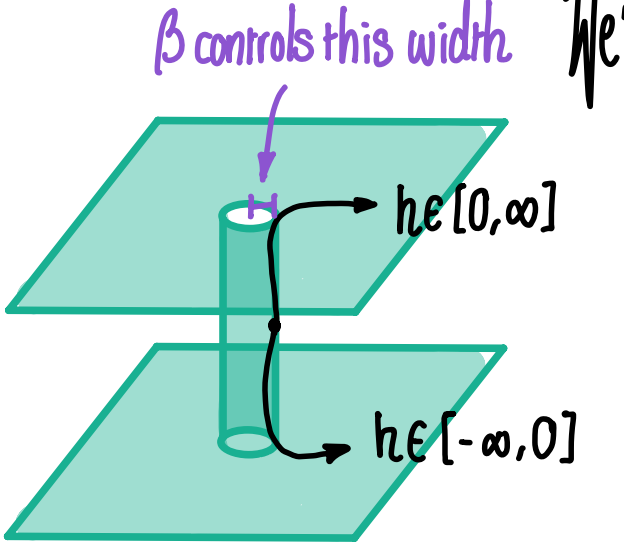
Quotient Theory



Quotient theories can allow for $h \in [-\infty, \infty]$, revealing possible *new minima*

Quotient Effective Potential

We would like to open up the *domain of the Higgs h* to reveal new minima...



We've done this by gluing together two SM manifolds..

$h \in [-\infty, \infty]$

$$F(h) = \sqrt{\sin^2\beta + \cos^2\beta\,(1+\gamma_a h/v)^2}$$

NB// Keep in mind that we've restricted quotient space!

Higgs-Dependent Masses

$$\tilde{D} \sim p^2 - m^2(h)$$

- effective potential: $V_{eff}(\phi_c) = V_0(\phi_c) + i \int \frac{d^4k}{(2\pi)^4} \, \mathrm{Tr} \log i \tilde{D}^{-1}(\phi_c, k) + \dots$

acts over all Lorentz & gauge indices.

$$\mathcal{L}_{HEFT} = \frac{v^2}{8} F(h)^2 \left[2g^2 W^+_\mu W^-_\mu + (g^2 + g'^2) Z_\mu Z^\mu \right] + V(h) + \dots$$

$m_h^2 = V''(h)$

$m^2_{W/Z}(h) = m^2_{W/Z} F^2(h)$

& for simplicity also $m_t^2(h) = m_t^2 F^2(h)$

If only it were that simple!

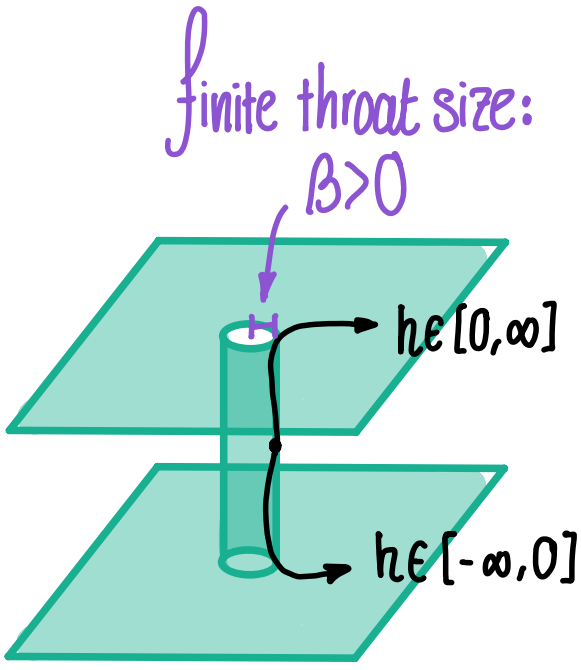
1710.06848 : R. Alonso, K. Kanshin, S. Saa
1511.00724 : R. Alonso, E. Jenkins, A. Manohar

- Want to compute in *on-shell gauge* → m_h behaves nicely!
- $W_L^\pm$ and Z_L get a contribution from the *Goldstones*.

Higgs-dependent masses:

$$m_h^2(h) = V''_{tree}(h) \qquad m_t^2(h) = m_t^2(0)\,F^2(h)$$

$$m^2_{\varepsilon W,Z3_L}(h) = (\log F)' V' + m_{\varepsilon W,Z3}(h) \qquad m^2_{\varepsilon W,Z3}(h) = m^2_{\varepsilon W,Z3}(0)\,F^2(h)$$



Bear in mind for $\beta > 0$,

$$F(h) = \sqrt{\sin^2\beta + \cos^2\beta \left(1 + \gamma_a h/v\right)^2}$$

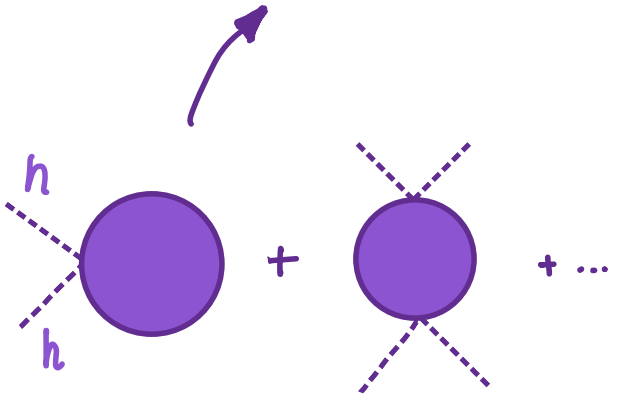
$$\min\left(F(h)\right) = \sin\beta$$

$$\min\left(M_{w/z}(h)\right) = M_{w/z}\sin^2\beta$$

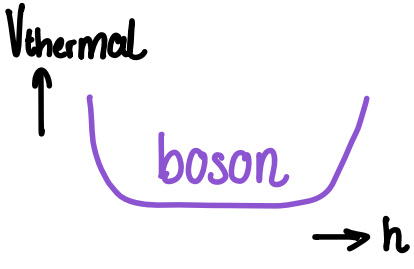
Loop Corrections in on-shell scheme...

$$V_{1\text{-loop}} = \frac{1}{64\pi^2} \sum_i \left\{ m_i^4(h) \left(\log \frac{m_i^2(h)}{m_i^2(0)} - \frac{3}{2} \right) + 2 m_i^2(h) m_i^2(0) \right\}$$

$i \to t, W^\pm, Z, h$



$V_{1\text{-loop}} \sim$



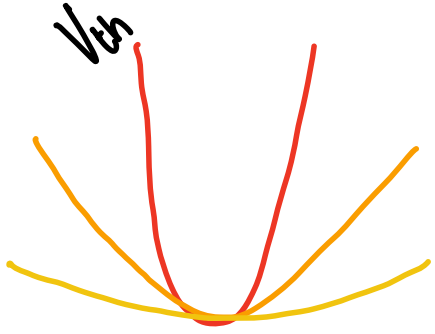
Thermal Corrections

[arXiv: 9901312 : M. Quiros]

$$V_{thermal} = \frac{T^4}{2\pi^2} \sum_i n_i J_{B/F}\left(\frac{m_i^2(h)}{T^2}\right) \sim$$

$W^\pm, Z, h, t$

$$J_{B/F} = \pm \int_0^\infty dx\, x^2 \log\left[1 \mp e^{-\sqrt{x^2 + m^2/T^2}}\right]$$

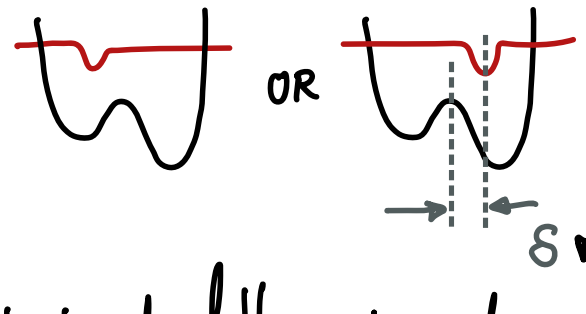


high T

low T

Changing γ_a changes where the thermal corrections are wrt V_{tree} :

$$F(h) = \sqrt{\sin^2\beta + \cos^2\beta \left(1 + \gamma_a \frac{h}{v}\right)^2}$$



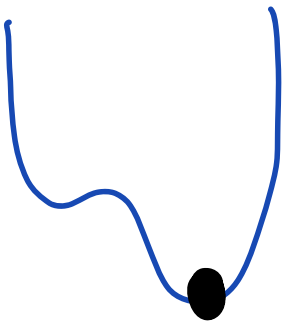
the sign - delta tells us which side of V_{tree} thermal corrections go at tree level.

$$\delta = \begin{cases} \gamma_4^{-1}\gamma_\epsilon - \gamma_a^{-1} & \epsilon \leq \sqrt{9/8 - 1} \\ \gamma_4^{-1}\sqrt{2} - \gamma_a^{-1} & \epsilon > \sqrt{9/8 - 1} \end{cases}$$

V_{eff}

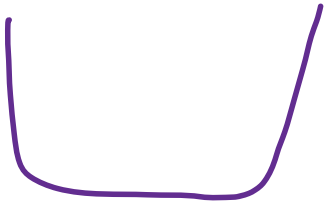
=

V_{tree}



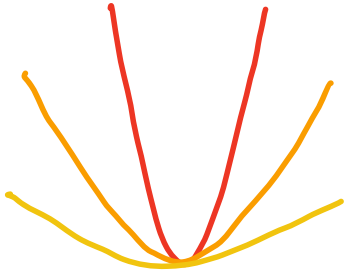
+

V_{1loop}



+

$V_{thermal}$



Validity of Vthermal

- Linde's Infrared Problem

A. Linde, 1980

2009.10080: D. Croon, O. Gould, et al

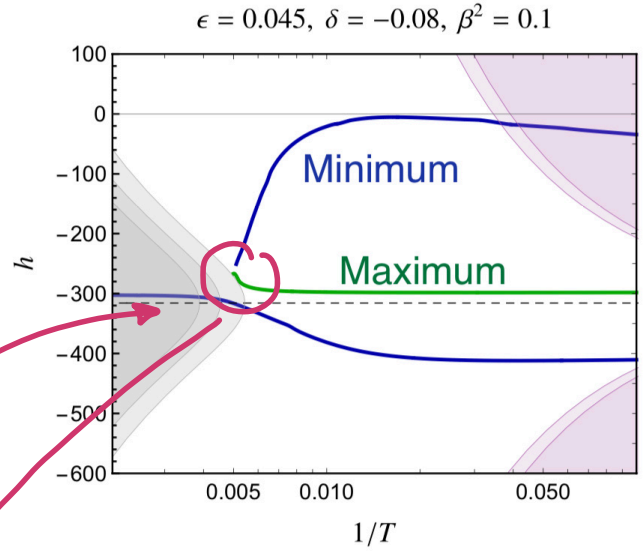
At high temperatures, IR bosonic modes are highly occupied.

$$\epsilon_{IR} = \frac{g^2 T}{\pi m_{ew}(h)}$$

effective expansion parameter

Define IR-unsafe band

$$\left(\frac{h_t v/\gamma_a}{T}\right)^2 > \frac{1}{\gamma_a}\left[\left(\frac{2g}{C_\Pi \cos\beta}\right)^2 - \tan^2\beta \frac{v_h^2}{T}\right]$$

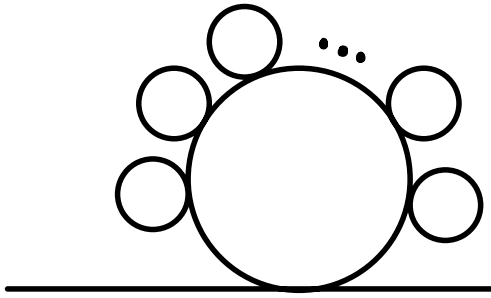


If these appear OUTSIDE band, we're OK.

If not, we say nothing about this point

What advantages does this have over daisy corrections?

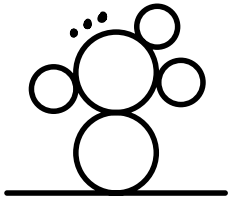
ie we *resum* the most IR divergent diagrams



$\sim (T/m_{ew}(h))^{2n}$

1// Only alleviates the problem

Super daisies diverge next



$\sim (T/m_{ew}(h))^{2n-2}$

2// Maintain a consistent loop expansion

[D. Curtin, P. Meade, H. Ramani arXiv: 1612.00466]

3// If turns out that in our quotient theories (since $m_W(h)/m_Z(h)$ cannot get too small), most interesting phenomenology is outside the band.

And dimensional reduction? 2009.10080: D. Croon, O. Gould, et al.

↳ Takes advantage of a *hierarchy of scales* to dimensionally reduce to a 3D effective theory.

It's much simpler.

- UV cutoff.

Because we're working with an EFT:

- $|h| < 4\pi v_{ew} \sim 4\pi v$
- $T < 4\pi v_{ew} \sim 4\pi v$

→ T. Cohen, N. Craig, X. Lu, D. Sutherland [arXiv: 2108.03240]

- Boundedness from below before $h = 4\pi v \sim 3\,\text{TeV}$

$V_{1\text{loop, top}} \sim$

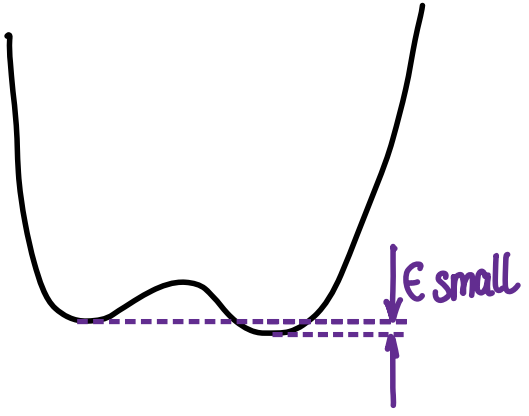
Minima Evolution.

Domain Walls

[G. Gelmini, M. Gleiser, E. Kolb 1989]

- If $V(h)$ is nearly symmetric around $(h + v/\sqrt{6}) \to -(h + v/\sqrt{6})$, domain walls form.

- A small bias ($|\epsilon| > 0$) in the potential will let the walls evaporate, leaving behind a GW signal.



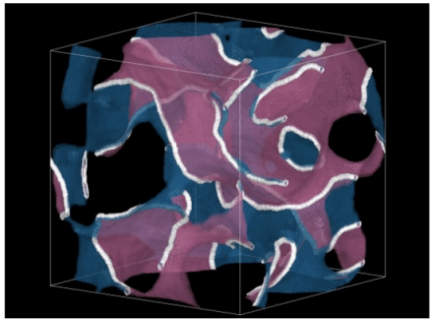
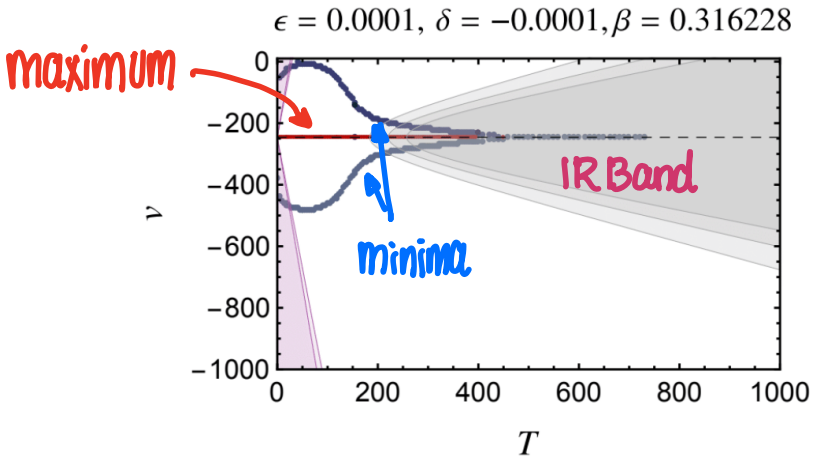


Image lifted from Hiramatsu, Kawasaki, Saikawa, Sekiguchi, arXiv:1207.3166

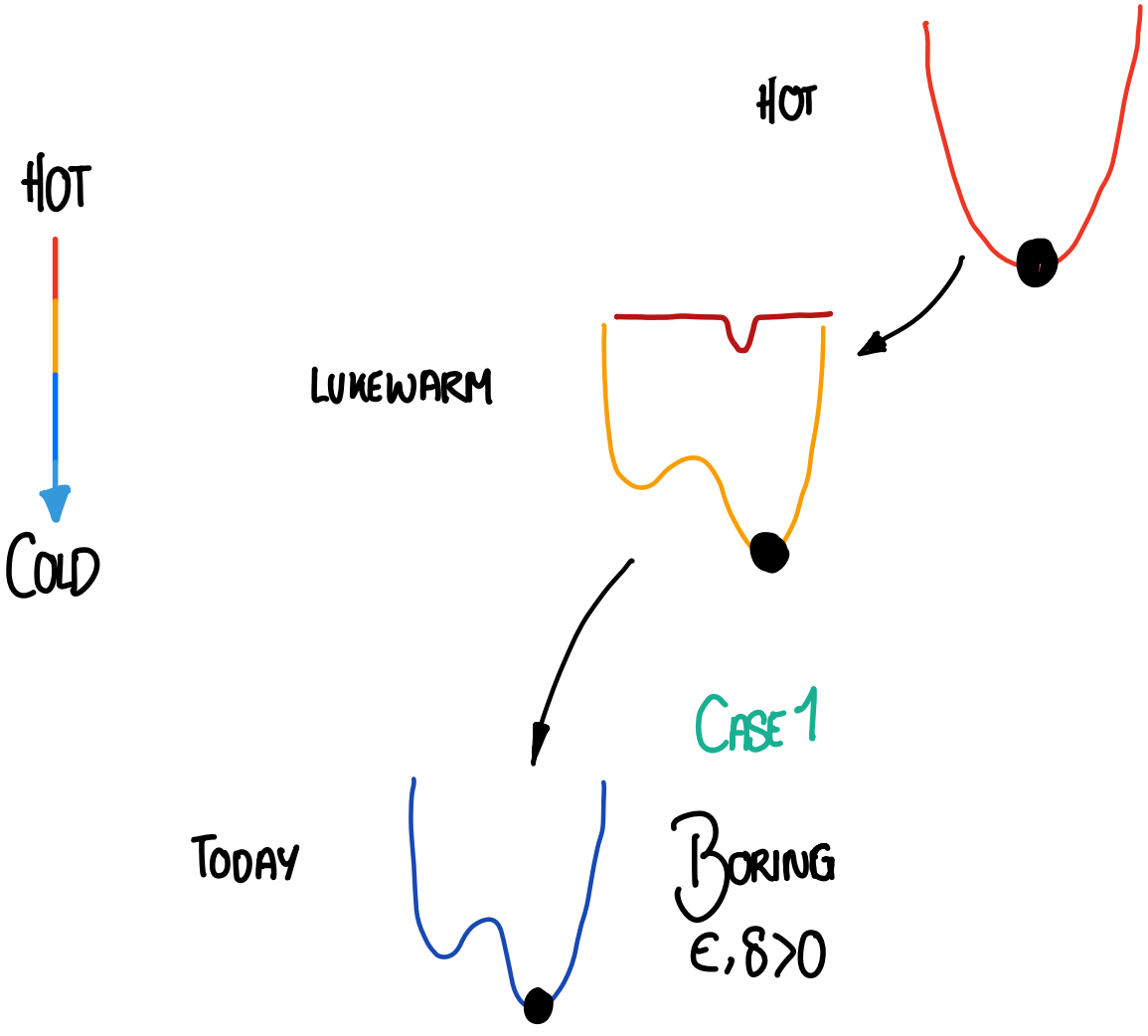
2009.01903 : G. Gelmini, S. Pascoli, E. Vitagliano, Y-L. Zhou

- Note that walls do not have the same IR criterion.

- Due to approximate symmetry of V(h).

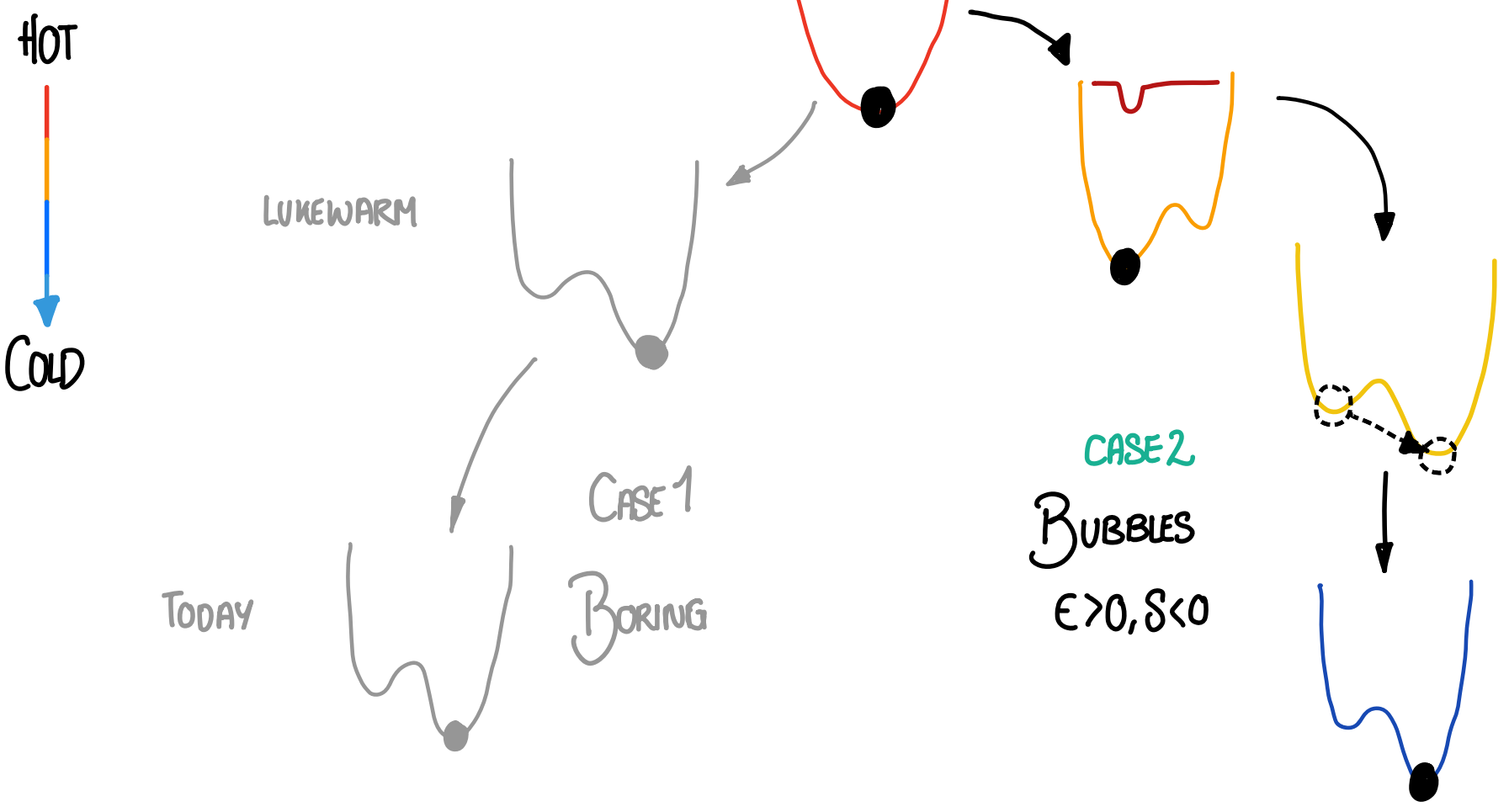


Bubbles & Doom



Define ourselves to be in RHS minimum today.

Minima Evolution



Define ourselves to be in RHS minimum today.

Minima Evolution

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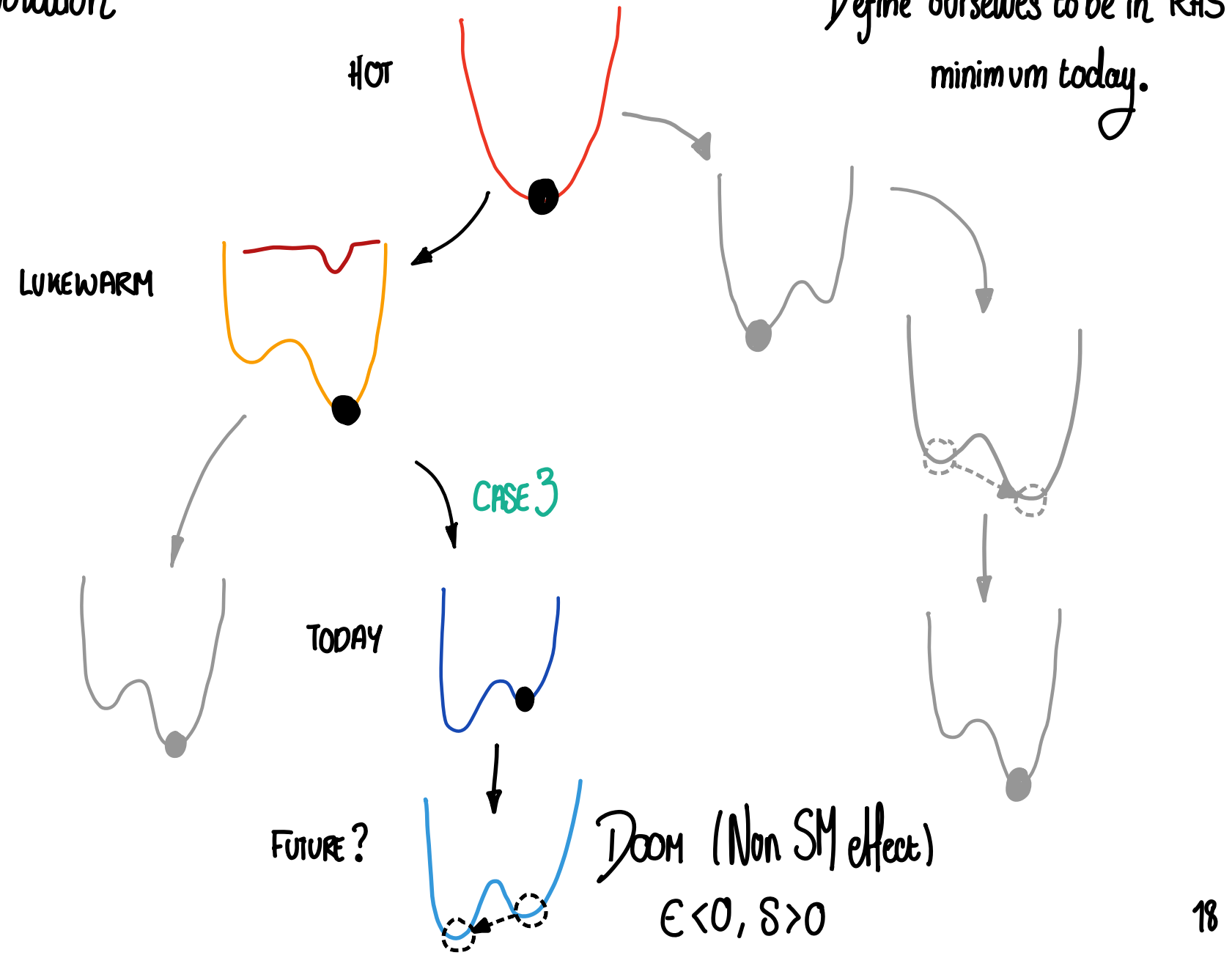
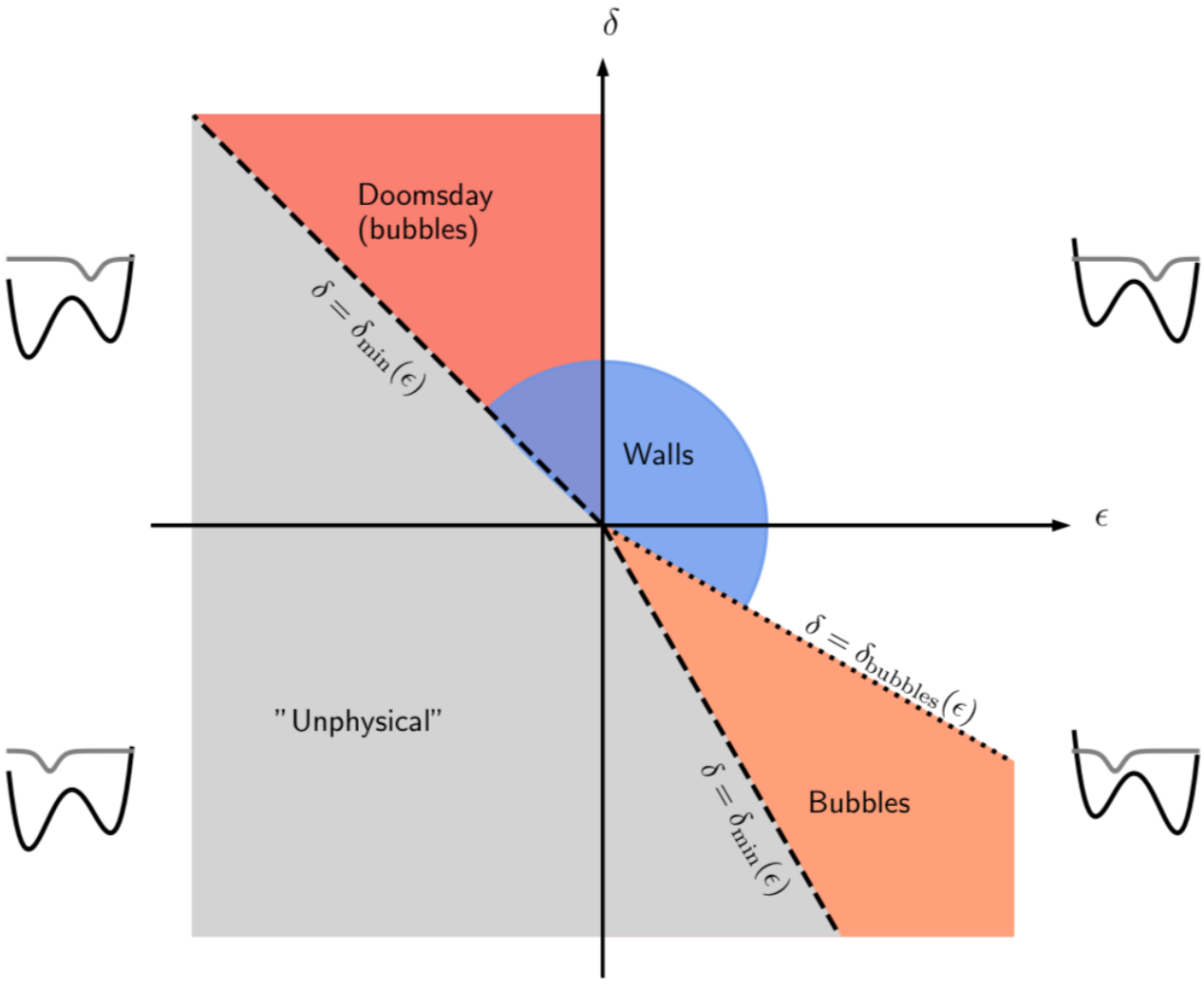
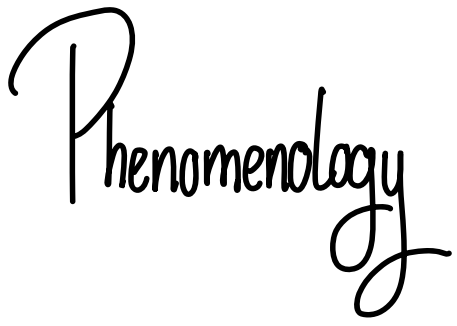


Doom (Non SM effect)

$\epsilon < 0, S > 0$

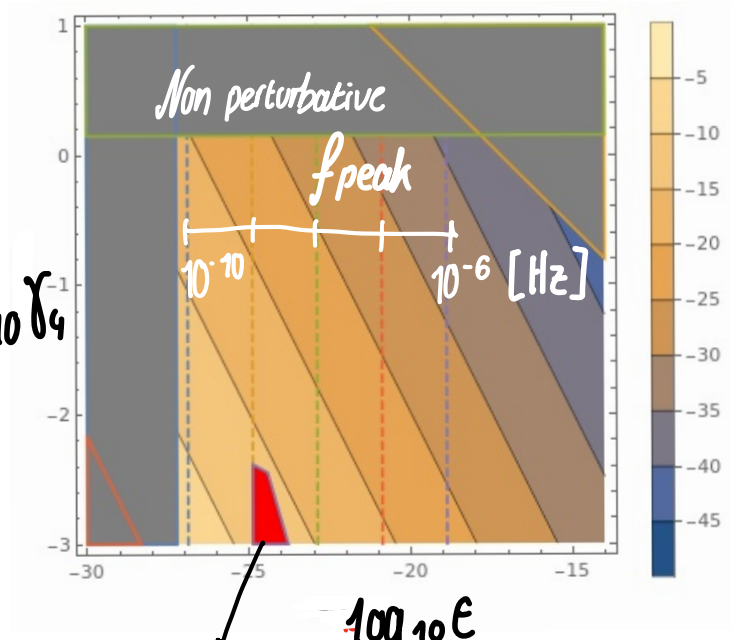
Schematic Summary...




Phenomenology

Walls

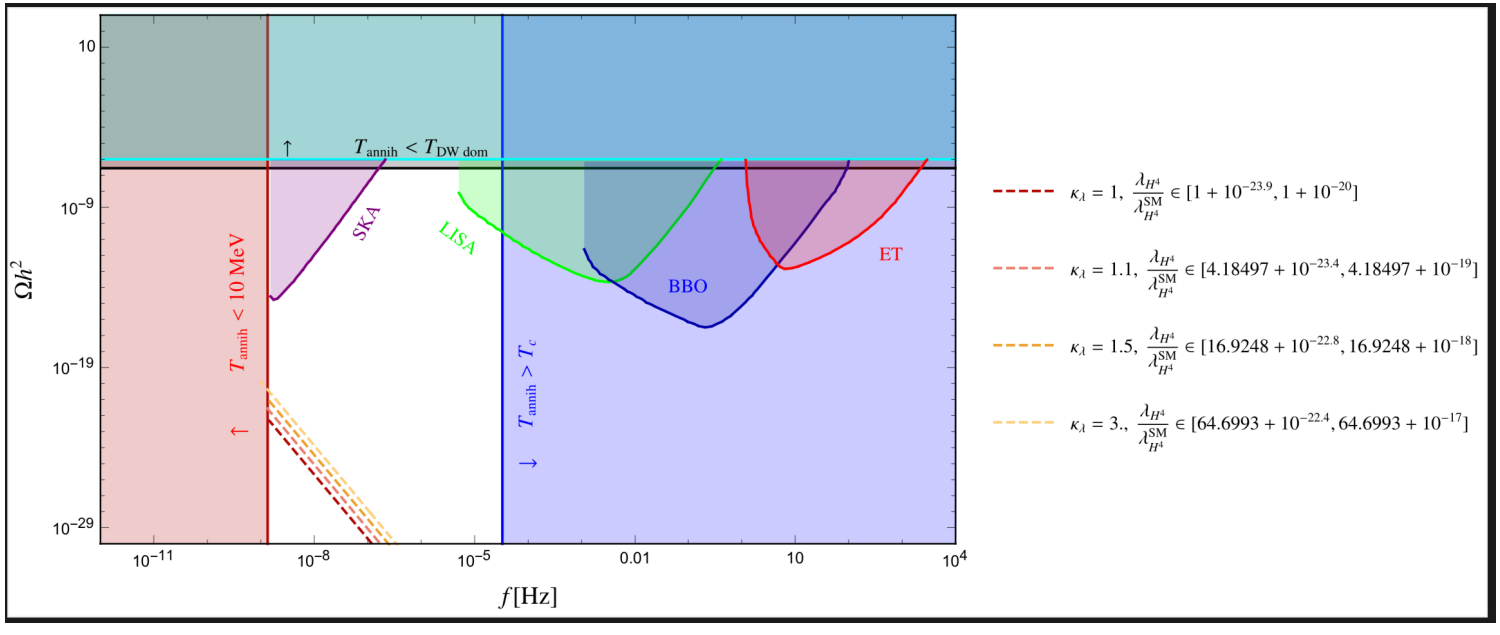
[Preliminary]



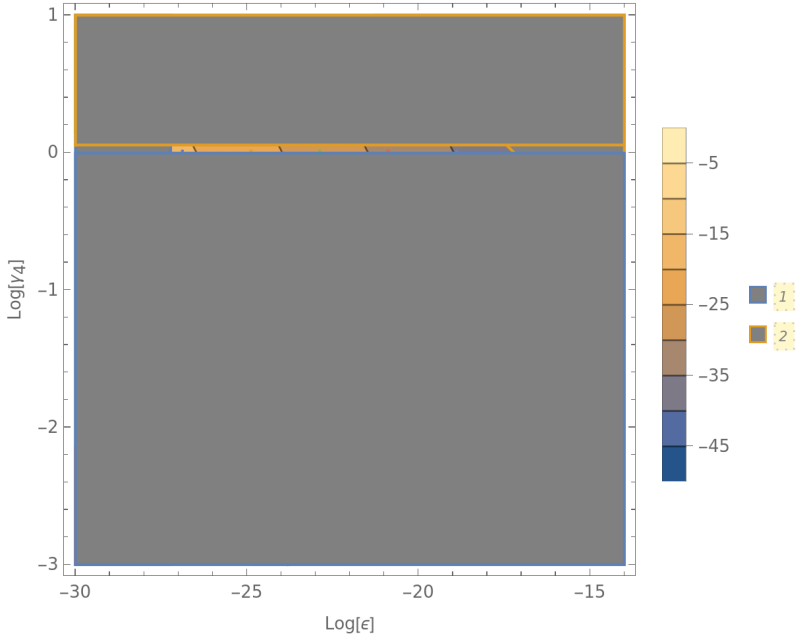
Visible at SKA

Reduced hubble NOT Higgs

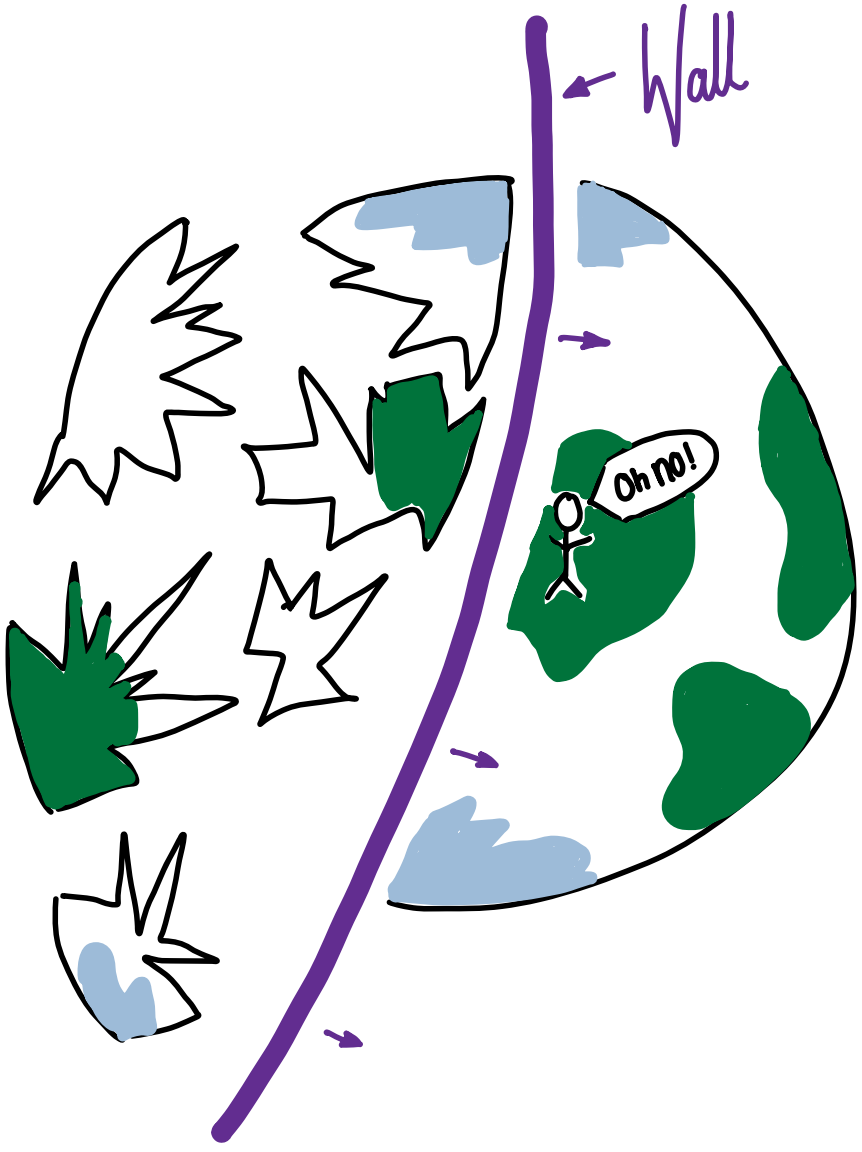
$\log_{10} h^2\Omega_{GW}$



Adding in LHC bounds...

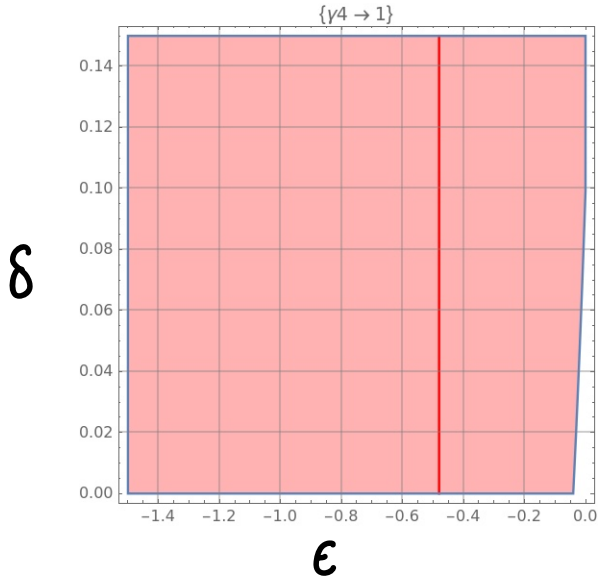


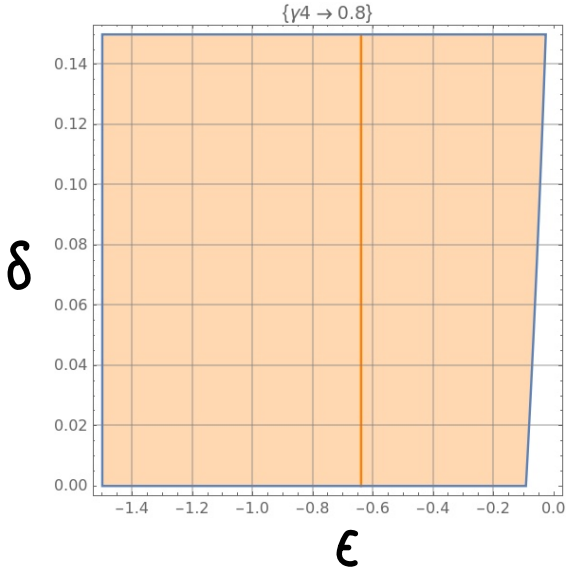
Doom

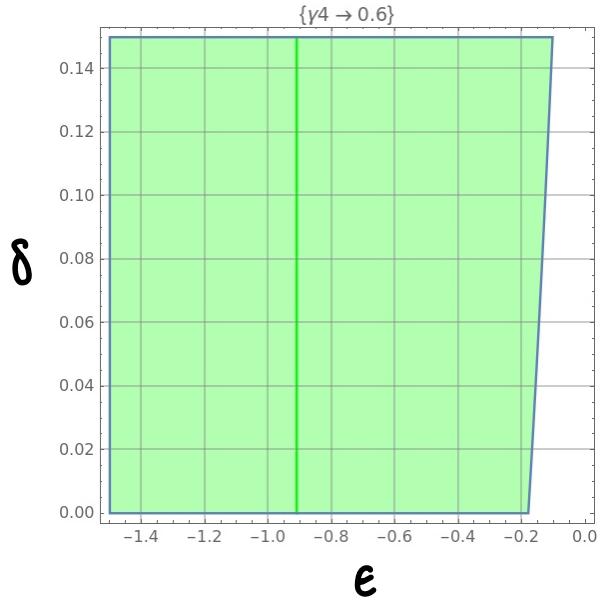


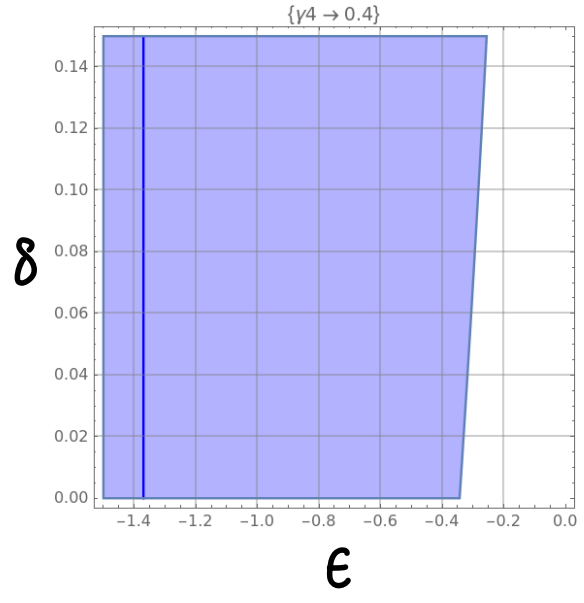
Luckily This Isn't Likely!!

- Shaded is excluded by perturbativity
- Solid line means $P(\text{decay today}) = 1$.

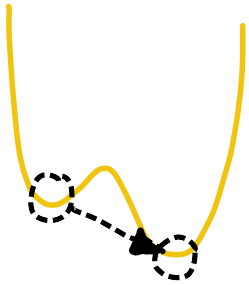


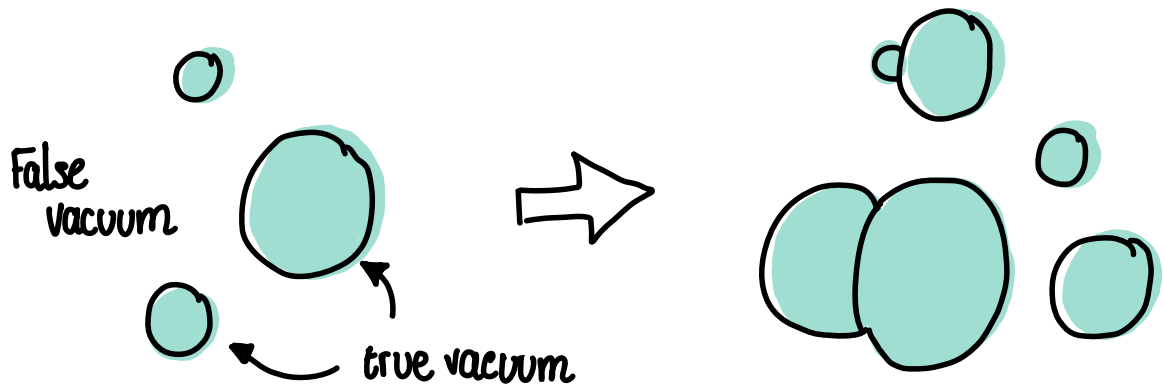






Bubble nucleation in the Early Universe.





3 gravitational wave production mechanisms:

1/ Bubble collisions
2/ Sound waves from motion of plasma w/ wall.
3/ Turbulance from shocks in the plasma

Transition Parameters

- $S_3 = \int d^3x \left[\frac{1}{2} (\partial_\mu h)^2 + V(h) \right]$

Calculated with:

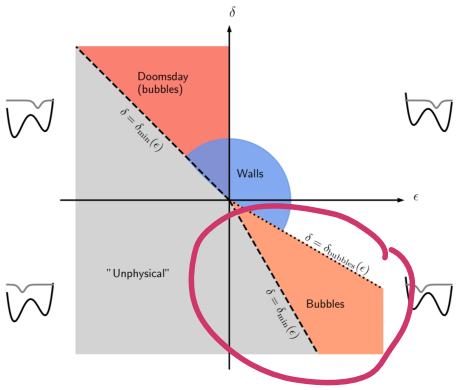
- CosmoTransitions [C. Wainwright arXiv: 1109.4189]
- AnyBubble [A. Masoumi et. al. arXiv: 1610.06594]

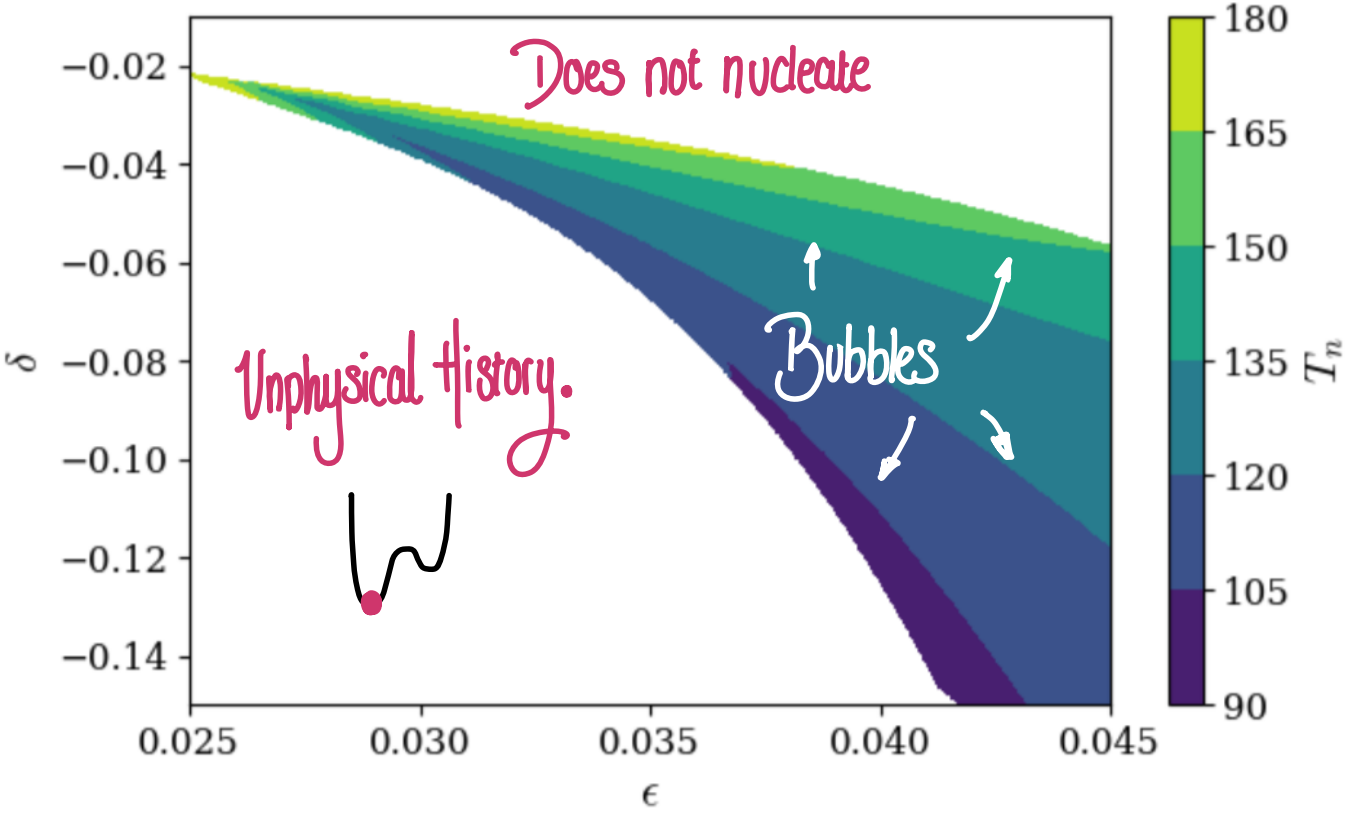
- Assuming *spherically* symmetric bubbles, eom is $\frac{d^2h}{d\rho^2} + \frac{2}{\rho}\frac{dh}{d\rho} = \nabla V(h)$
- Boundary conditions are *false* & *true* minima & non-singularity at $\rho = 0$.

Nucleation occurs when $\frac{S_3}{T_N} \sim 140$.

1 bubble per Hubble Volume

Bottom right quadrant





GW Spectra for Bubbles.

The gravitational wave signal can be estimated using thermal parameters calculated from the tunelling action.

Latent Heat

$$\alpha = \frac{1}{\rho_{rad}} \left(\Delta V - T \frac{d(\Delta V)}{dT} \right) \Bigg|_{T=T_N}$$

Duration

$$\frac{\beta}{H_*} = T \frac{d(S_3/T)}{dT} \Bigg|_{T=T_N}$$

[M. Lewicki, M. Merchand, M. Zych arXiv: 2111.02393]

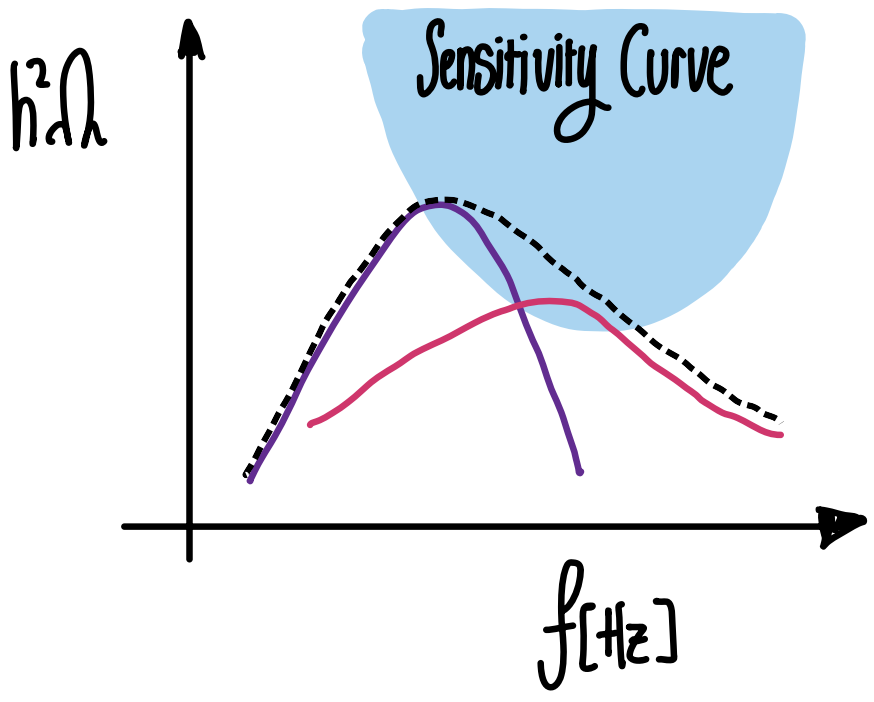
Wall Velocity

$$v_w = \begin{cases} \sqrt{\Delta V/\alpha \rho_r} & \sqrt{\Delta V/\alpha \rho_r} < v_J(\alpha) \\ 1 & \sqrt{\Delta V/\alpha \rho_r} \geq v_J(\alpha) \end{cases}$$

What Do the Spectra Look Like?

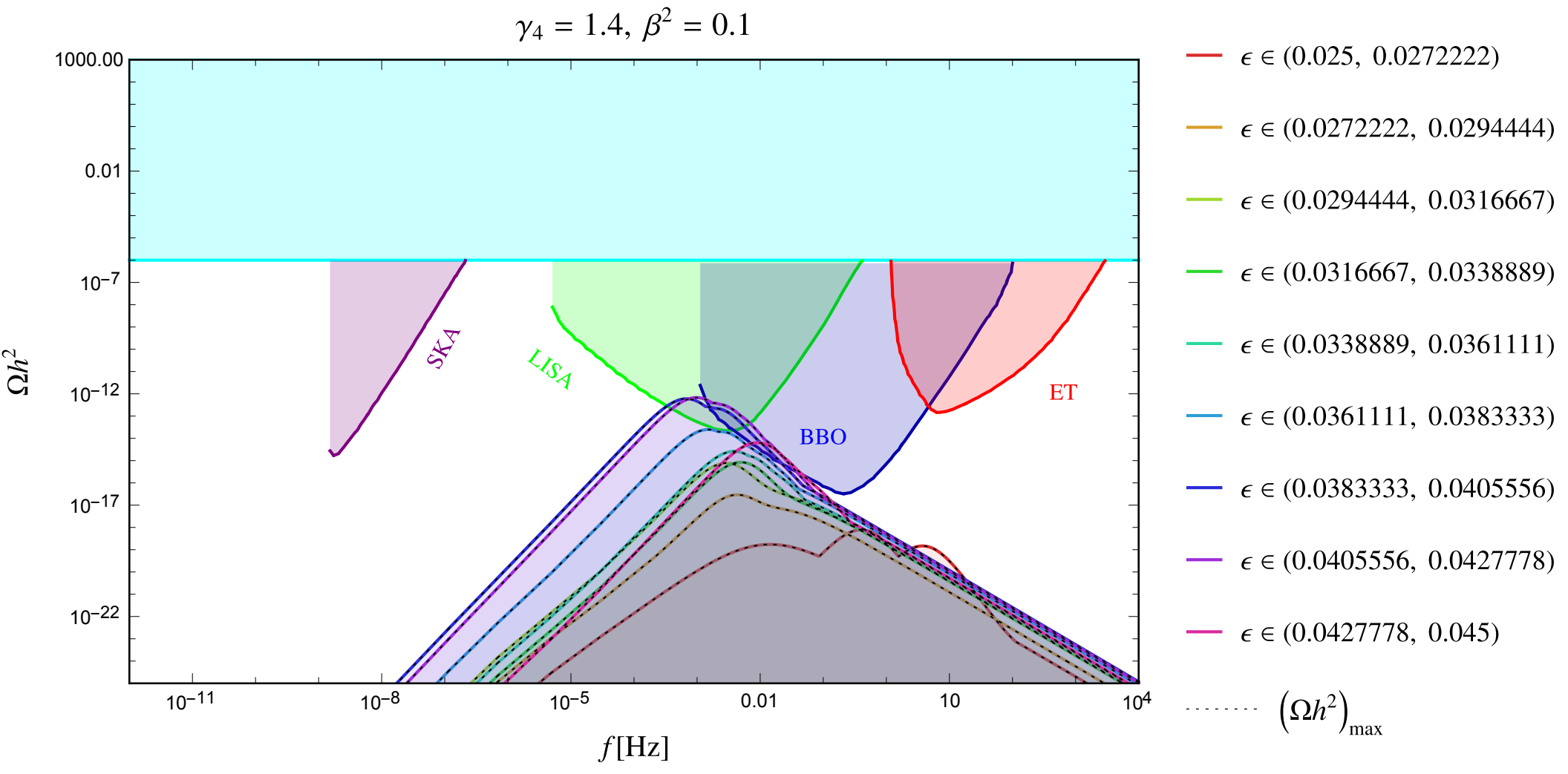
$$\Omega_{GW} = \Omega_{\text{Sound Waves}} + \Omega_{\text{Turbulance}} + \Omega_{\text{collisions}}$$

neglect



$$h^2\Omega_{\rm sw}(f) = 2.59\times10^{-6}h^2\left(\frac{100}{g_*}\right)^{1/3}\Gamma^2\bar{U}_{fl}^4\left(\frac{H_*}{\beta}\right)v_w\mathrm{Max}[H_*\tau_{\rm sh},1]S_{\rm sw}(f)\,,$$

$$h^2\Omega_{\rm tu}(f) = 3.35\times10^{-4}\left(\frac{H_*}{\beta}\right)\left(\frac{\kappa_{\rm tu}\alpha_{T_*}}{1+\alpha_{T_*}}\right)^{3/2}\left(\frac{100}{g_*}\right)^{1/3}v_{\rm w}S_{\rm tu}(f)\,,$$


$\gamma_4 = 1.4,\ \beta^2 = 0.1$
1000.00
0.01
10^{-7}
10^{-12}
10^{-17}
10^{-22}
Ωh^2
SKA
LISA
BBO
ET
10^{-11}
10^{-8}
10^{-5}
0.01
10
10^4
f[Hz]
$\epsilon \in (0.025,\ 0.0272222)$
$\epsilon \in (0.0272222,\ 0.0294444)$
$\epsilon \in (0.0294444,\ 0.0316667)$
$\epsilon \in (0.0316667,\ 0.0338889)$
$\epsilon \in (0.0338889,\ 0.0361111)$
$\epsilon \in (0.0361111,\ 0.0383333)$
$\epsilon \in (0.0383333,\ 0.0405556)$
$\epsilon \in (0.0405556,\ 0.0427778)$
$\epsilon \in (0.0427778,\ 0.045)$
$\left(\Omega h^2\right)_{\max}$

Complimentarity With LHC

[arXiv: 2301.03212, ATLAS]

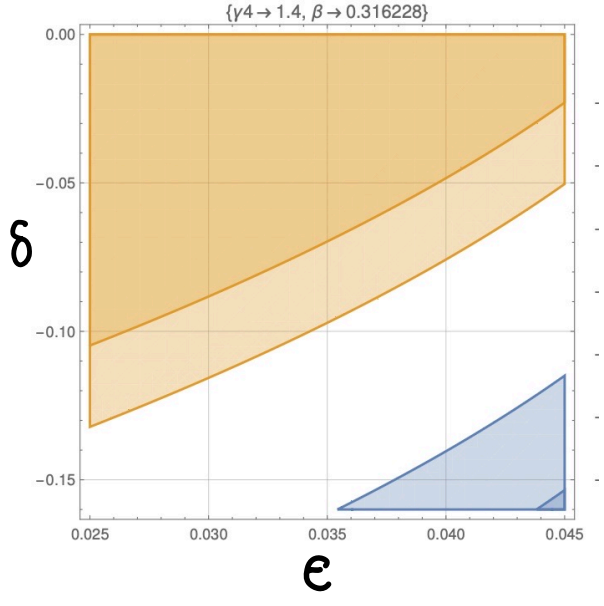
[ATLAS CONF-NOTE 2021-053]

$$\mathcal{A}(W_L^+W_L^- \to W_L^+W_L^-) = 1 - c_\beta^4 \gamma_a^2 = -.080^{+1.0}_{-1.1}$$

$$\mathcal{A}(W_L^+W_L^- \to hh) = -\gamma_a^2 s_\beta^2 c_\beta^2 = .080^{+0.12}_{-0.13}$$

ie we get a restriction on the ranges of δ, ϵ for a given γ_4

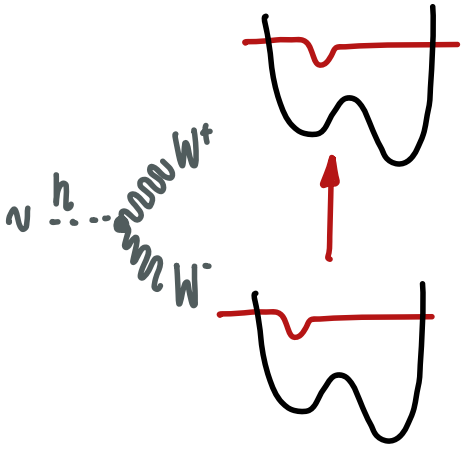
$$\delta = \begin{cases} \gamma_4^{-1}\gamma_\epsilon - \gamma_a^{-1} & \epsilon \leq \sqrt{9/8 - 1} \\ \gamma_4^{-1}\sqrt{2} - \gamma_a^{-1} & \epsilon > \sqrt{9/8 - 1} \end{cases}$$

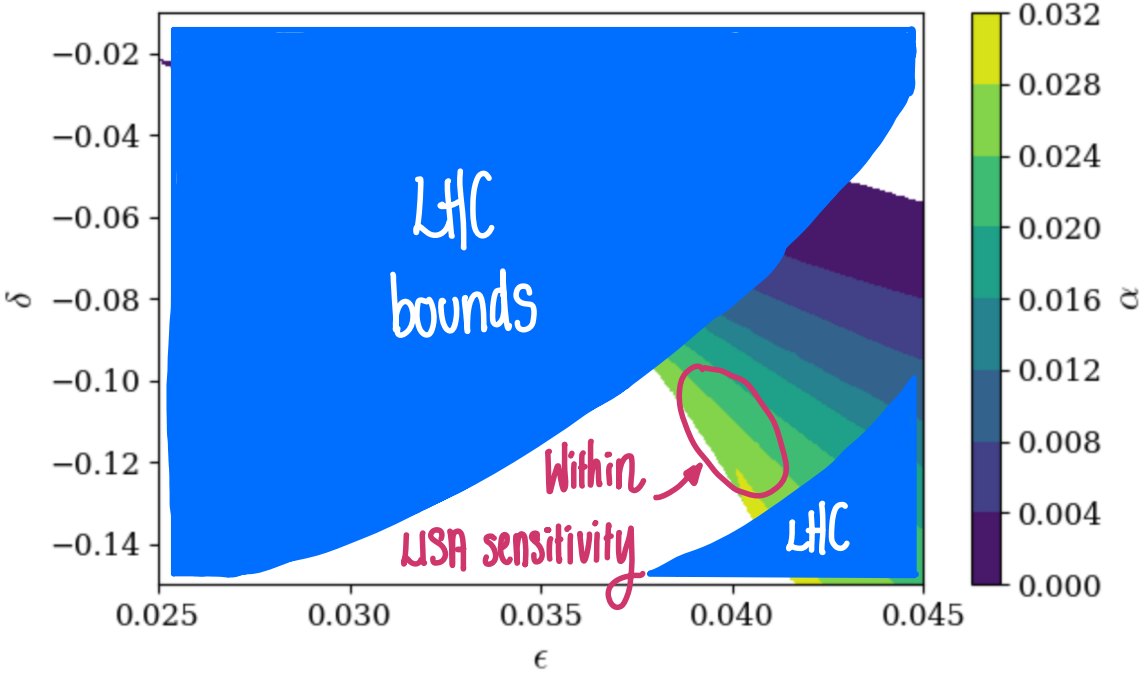


Summary Plot !

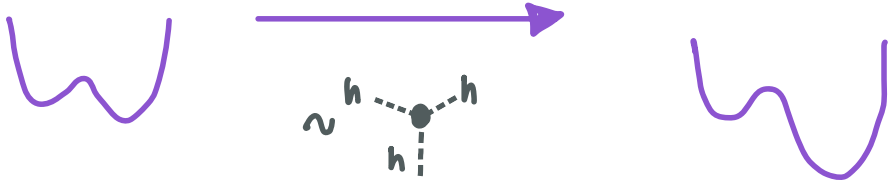
$\gamma_4 = 1.4$; $\beta = \sqrt{0.1}$

(Preliminary & Schematic)





Latent Heat of phase transition



Conclusions

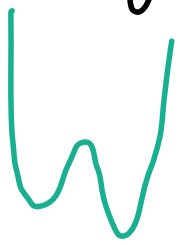
- Increasingly pertinant question: Which EFT should we use to describe nature?

- It is possible that cosmology could tell us about the global properties of quotient theories.

- Bubbles seem to be a promising phenomenological avenue.

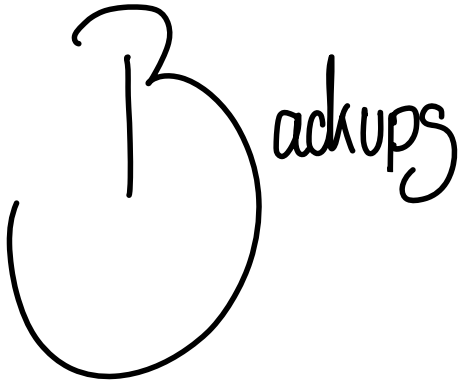
Outlook / Interesting Questions...

- What happens when you change β?

$\beta > \sqrt{.1}$ 

$\beta < \sqrt{.1}$ 

- What sort of UV physics could give a wormhole manifold?
- How about more complicated V_{tree}'s? w/ Multi-step transitions? Different $f(h)$'s?
- What happens when $F_{boson}(h) \neq F_{fermion}(h)$?


Backups

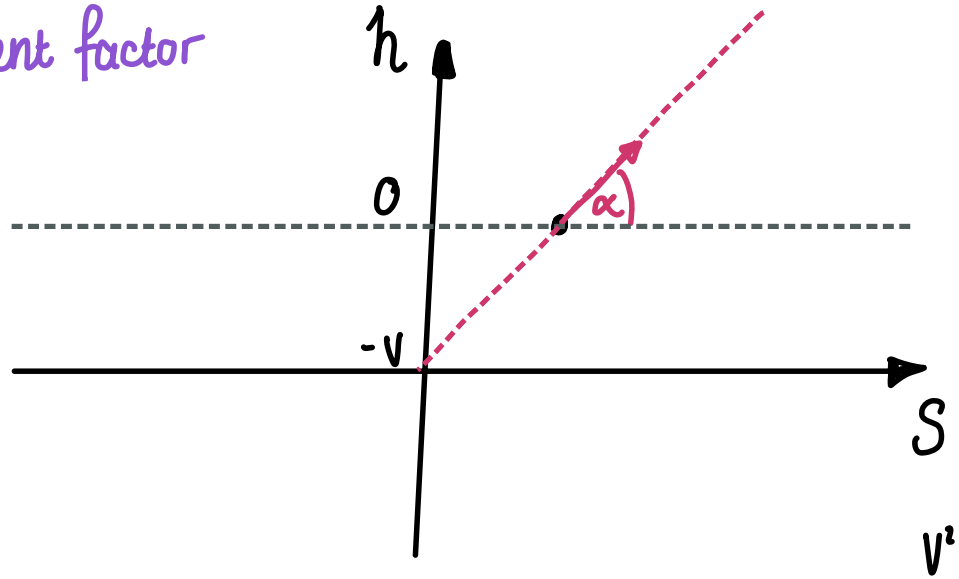
UV Physics : SM + singlet s which mix.

$$\mathcal{L} = \frac{1}{2}(\partial S)^2 + \frac{1}{2} D_\mu \underline{h}^T D^\mu \underline{h} - V(\underline{h}, S)$$

$\underline{h} = h\, U(\varphi)$

$v^2 = v_h^2 + v_S^2 C_S$

representation dependent factor



$h \& S$ mix with angle α.

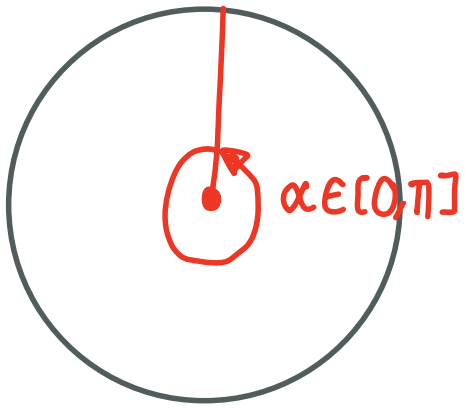
$$\begin{pmatrix} h \\ S \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \tilde{h} \\ h \end{pmatrix}$$

$$v^2 F(h) \sim v^2 + 2 c_\beta^2 \gamma_a v h + c_\beta^2 \gamma_a^2 h^2 + \dots$$

Sphalerons

- Static tunnelling solution to the equations of motion, between topologically distinct vacua.

- Maximum energy point of a minimum-energy non-contractible loop through scalar & gauge boson field space.



Why am I drawing pictures?

- Recall the LSZ formula where amplitudes are invariant under field-space redefinitions.
  - ↳ think coordinate redefinition invariance
  - ↳ treating t/EFT as one would GR saves a lot of trouble!

- Eg "best basis" (Riemann normal coordinates) where (ignoring $V(h)$), no 3pt $W_i W_j h$ vertex.

- Amplitudes relate to *curvature invariants*.

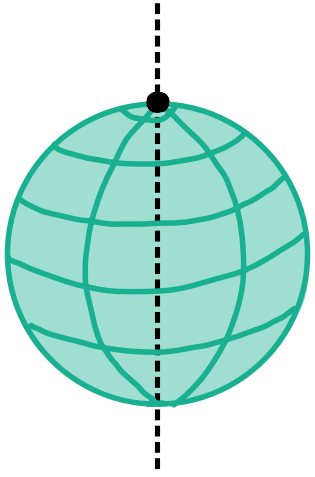
'Linearly realised' and 'non-linearly realised' ew symmetry is a *field-space redefinition-dependent* statement.

→ Think Cartesian & Polar coordinates

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \to R \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$r = \sqrt{x^2+y^2+z^2} \to r$$

$$\begin{pmatrix} \phi \\ \theta \end{pmatrix} \to \begin{pmatrix} a\phi \\ b\theta \end{pmatrix}.$$



SMEFT theories *admit* a linear representation.