Detecting Gravitational Wave Background by Electromagnetic Cavity

Chan Park (IBS-CTPU)

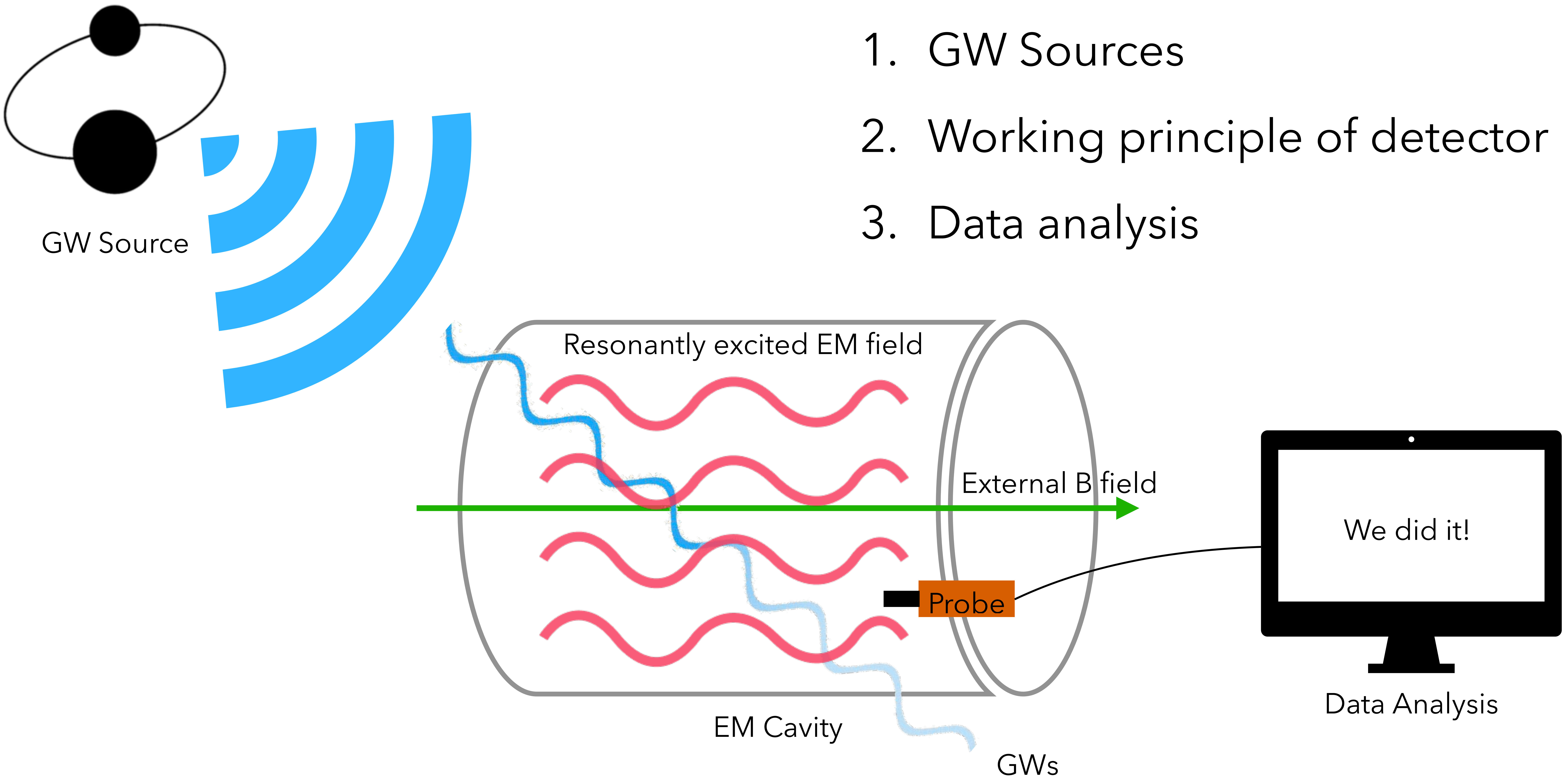
2023.08.28 @ KIAS

collaborated with Danho Ahn (IBS-CAPP) and Sang Hui Im (IBS-CTPU)

KIAS Seminar on Maxwell's Equations in Curved Spacetime

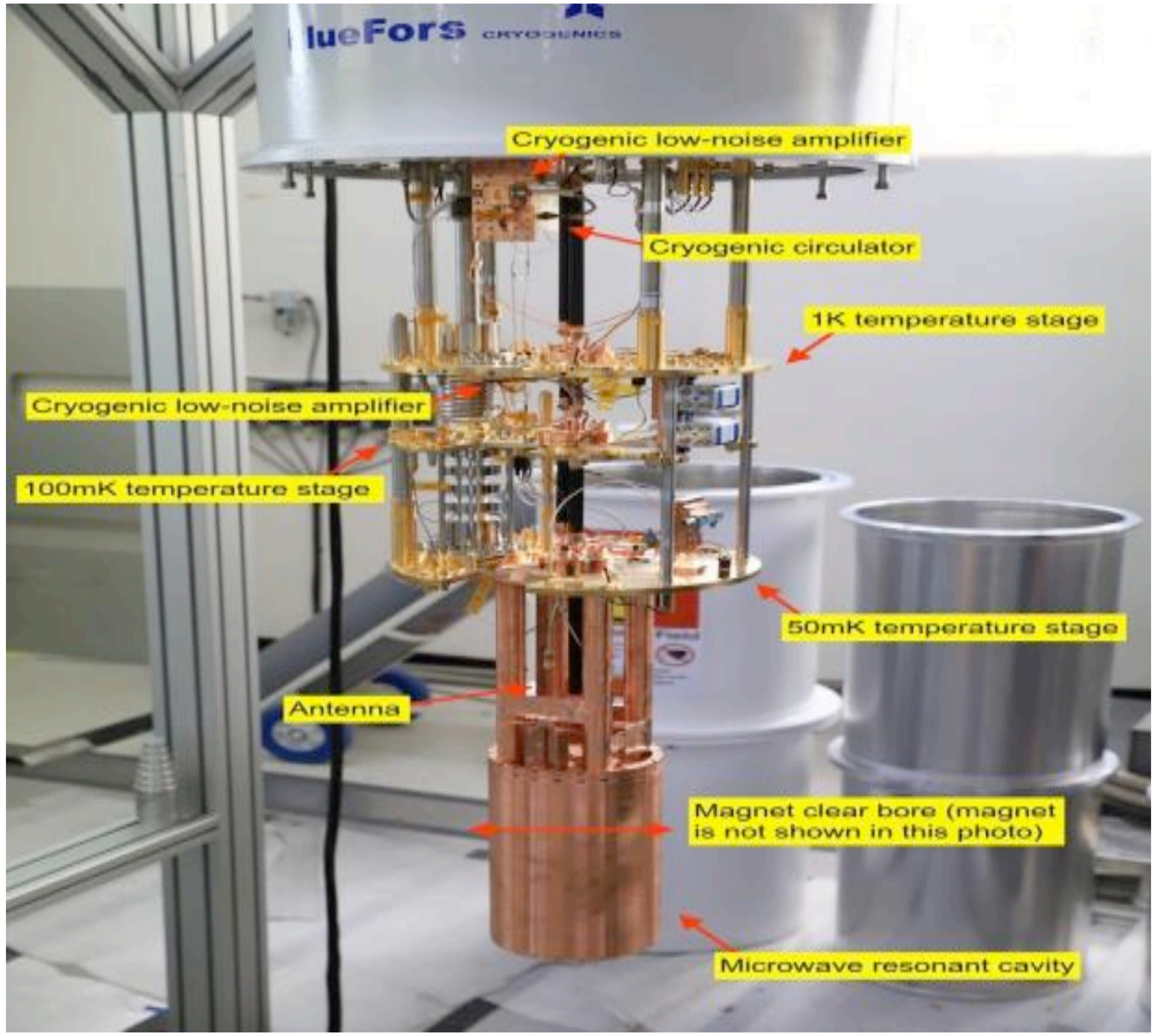
Overview

1. GW Sources
2. Working principle of detector
3. Data analysis



Why Electromagnetic Cavity?

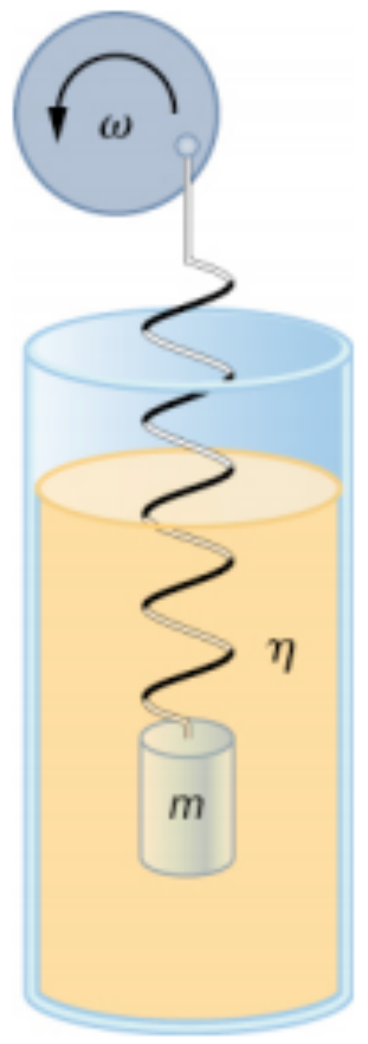
- There are many cavity experiments that have extreme sensitivity around the world to detect the axion that is a candidate for dark matter.
- There are recent studies that these cavity experiments have reached the level at which gravitational waves can be measured.
- Reference Papers
  - Detecting high-frequency gravitational waves with microwave cavities / A. Berline+ / 2022 PRD
  - Novel Search for High-Frequency Gravitational Waves with Low-Mass Axion Haloscopes / V. Domcke / 2022 PRL



Electromagnetic Cavity in IBS-CAPP

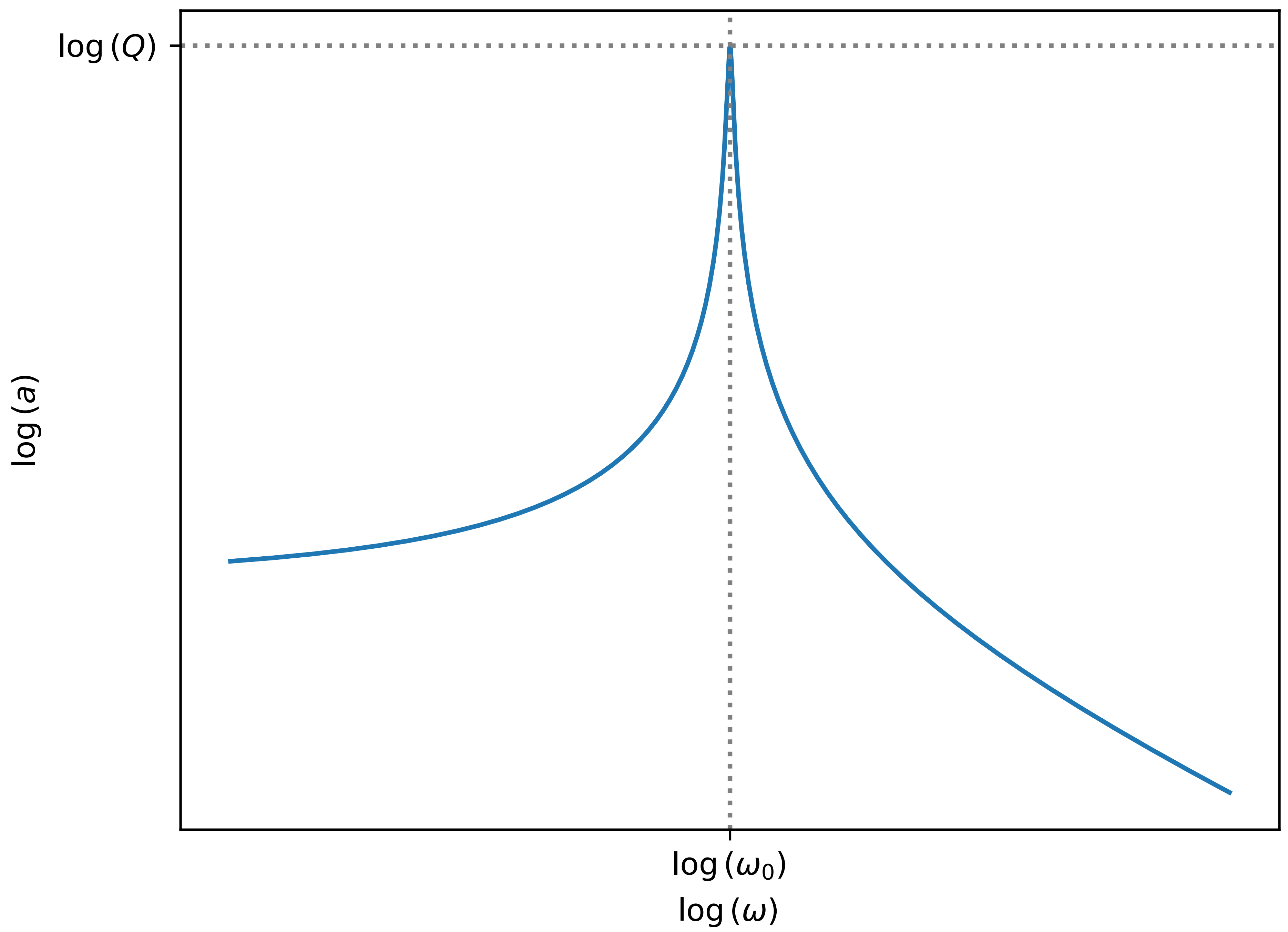
Example: Forced Oscillation

- Equation of Motion
  - $m\ddot{x} + b\dot{x} + kx = F(t)$
- Fourier Transformation
  - $x(t) = \int \frac{d\omega}{2\pi} \tilde{x}(\omega) e^{-i\omega t}$
  - $F(t) = \int \frac{d\omega}{2\pi} \tilde{F}(\omega) e^{-i\omega t}$
- Amplitude
  - $\tilde{x}(\omega) = a(\omega; \omega_0, Q) e^{i\alpha(\omega; \omega_0, Q)} \frac{1}{k} \tilde{F}(\omega)$



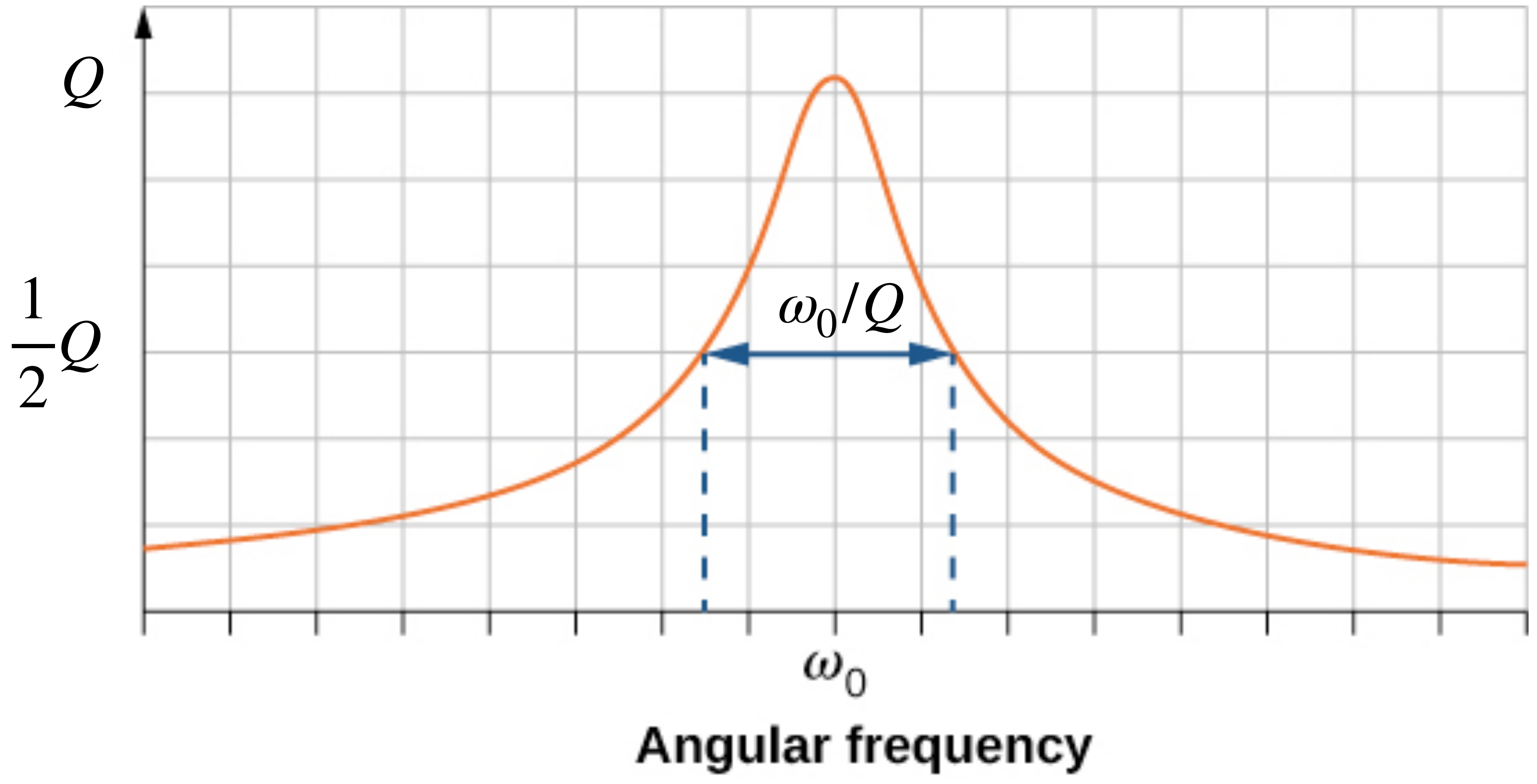
Forced Oscillation / Credit: LibreTexts

- Dimensionless resonance factor
  - $a(\omega; \omega_0, Q) = \left[\left((\omega/\omega_0)^2 - 1 \right)^2 + (\omega/\omega_0 Q)^2 \right]^{-1/2}$
  - where $\omega_0^2 = k/m$ and $Q = m\omega_0/b$

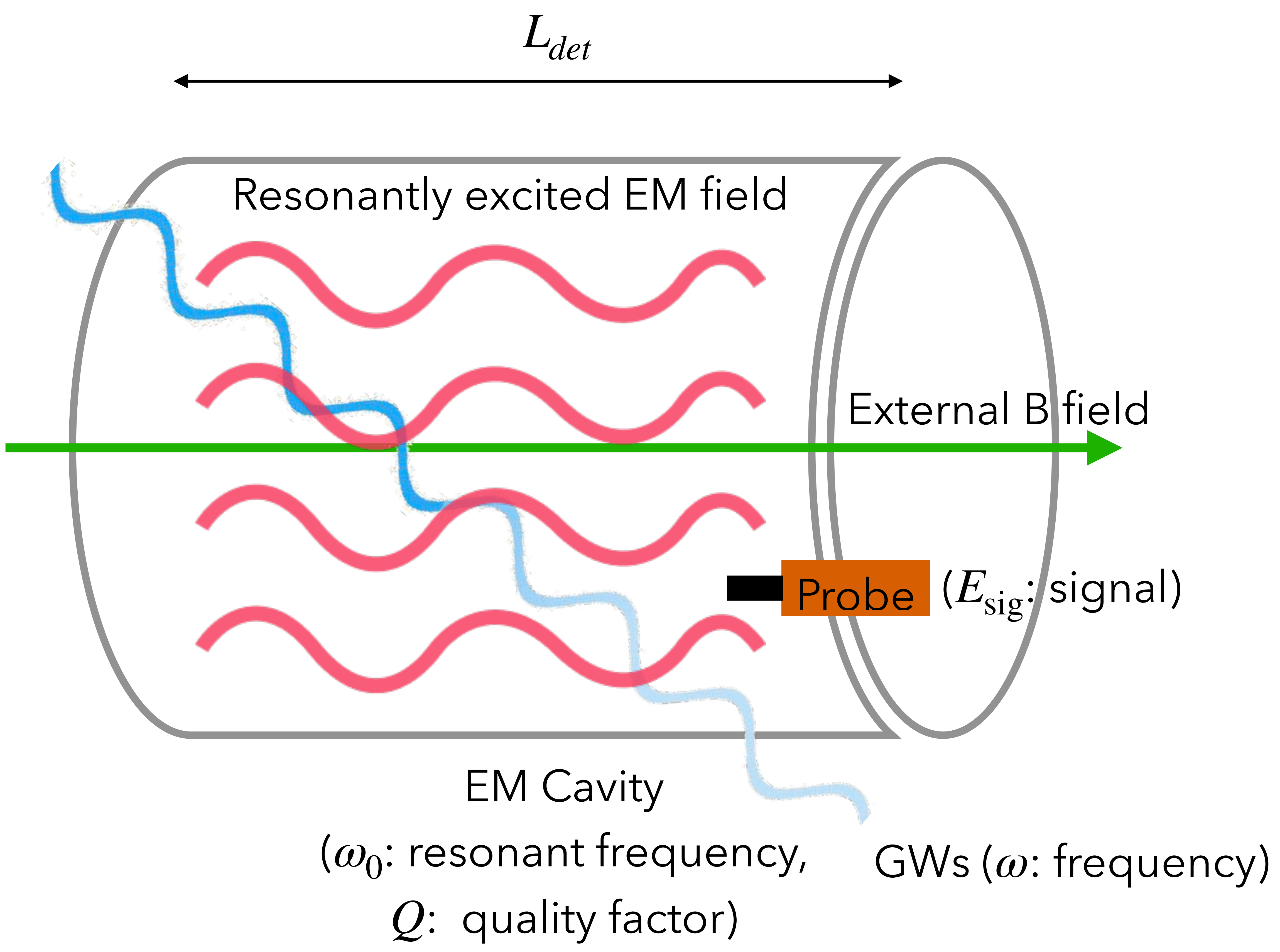


Brief Working Principle of the Detection

- GW induces a forced oscillation of EM field. When $\omega \to \omega_0$, the cavity is resonantly excited.
- Detectable GW frequency:
$$\omega \sim \omega_0 \sim \frac{1}{L_{det}} \sim 1\ \mathrm{GHz}$$
- Frequency Band: $\Delta\omega \sim \Delta\omega_0 \sim \dfrac{\omega_0}{Q} \sim 10\ \mathrm{kHz}$



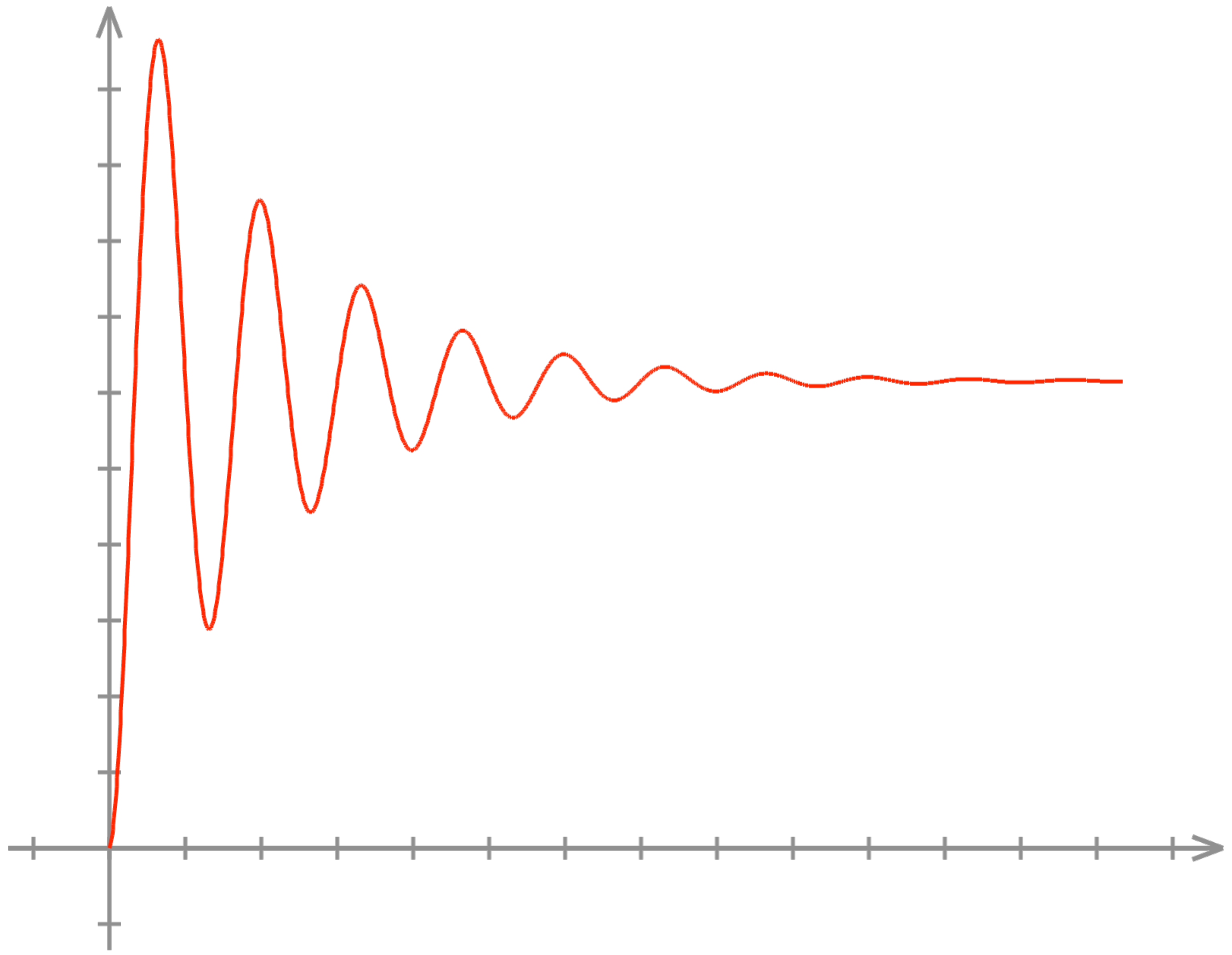
Curve for resonance factor $a\left(\omega; \omega_0, Q\right)$



Sources of GWs

Source of Gravitational Waves (GWs)

- It should be extreme event to emit GWs as strong as enough to detect.
- Source Type
  - Transient Source
  - Stochastic Source

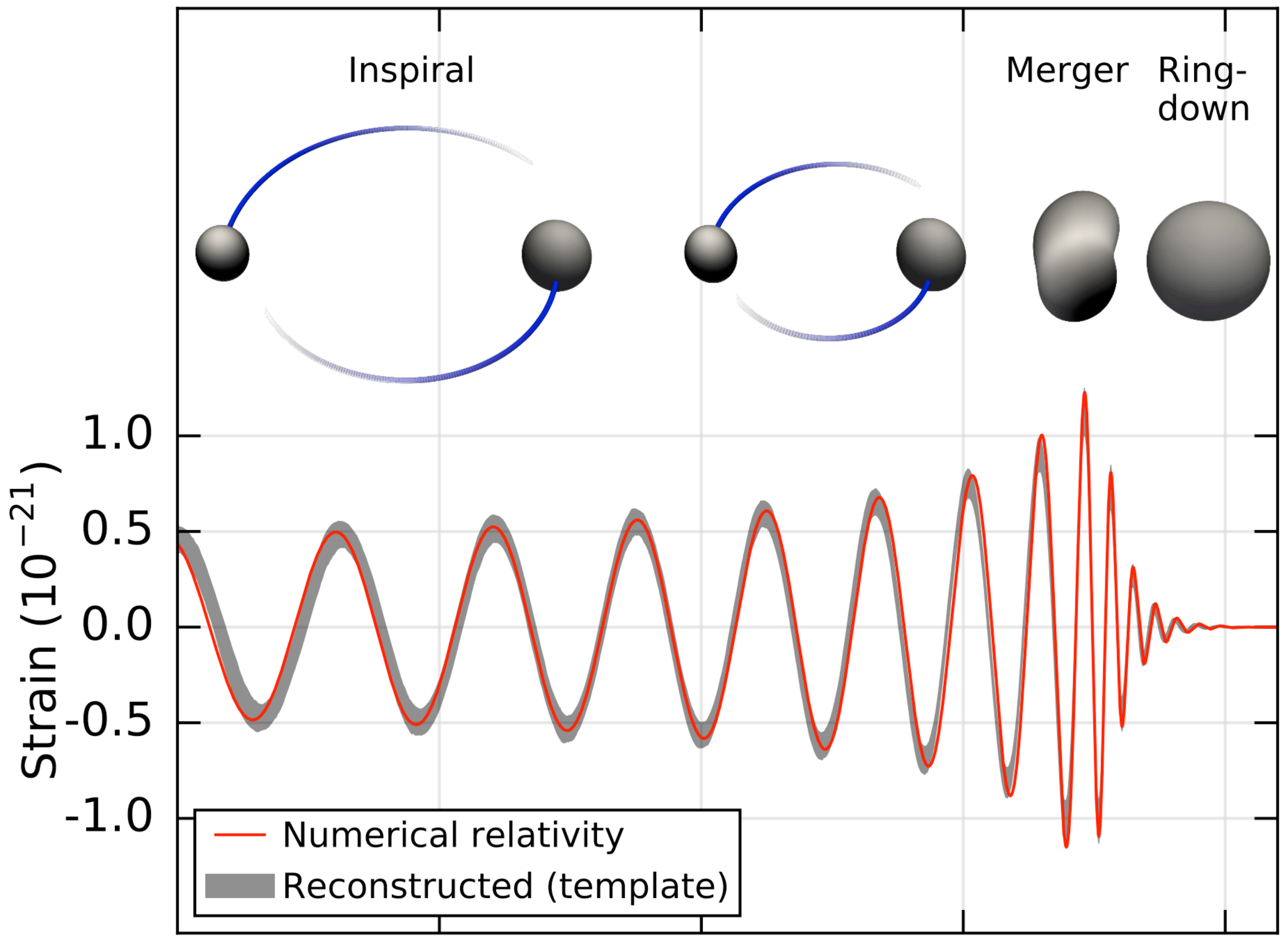


Transient Signal

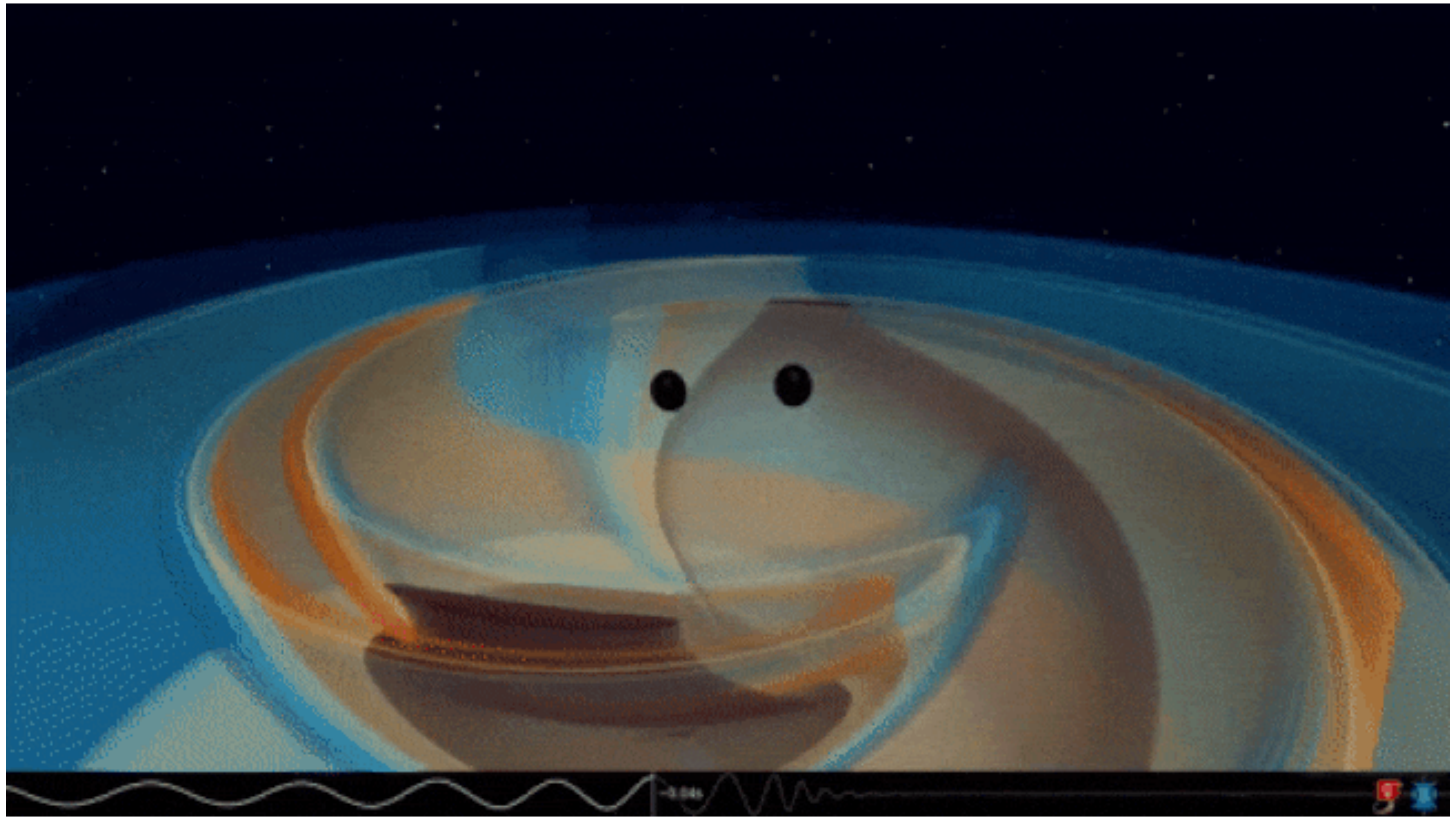
Stochastic Water Waves from Distribution of Rain Drops

Sources of GWs: Transient GWs

- Merger of Binary Compact Stars
  - Binary Black Holes (BH) Merger
  - Binary Neutron Star (NS) Merger
  - BH-NS Merger



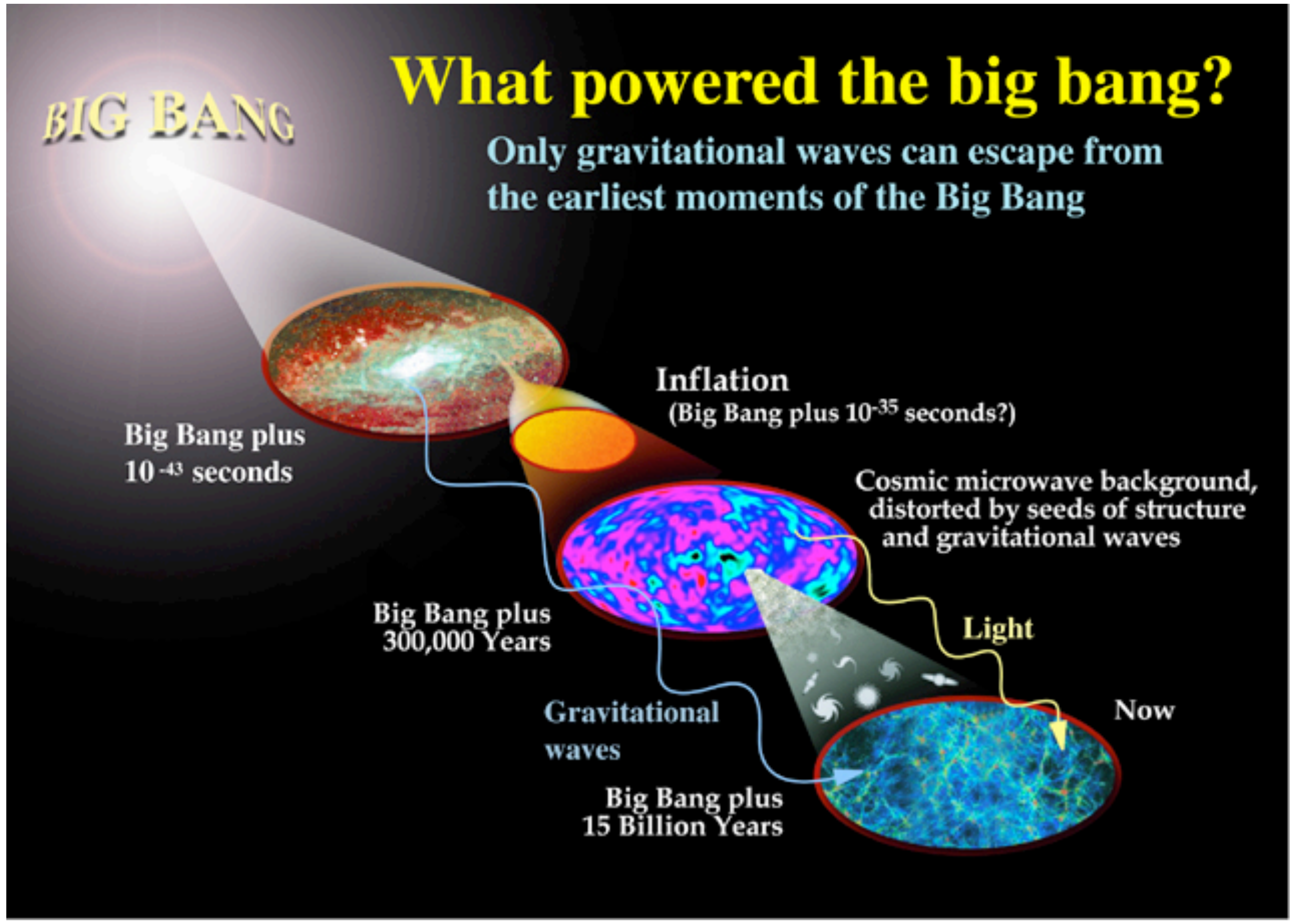
GW Waveform of a Transient Source / PRL 116, 061102 (2016)



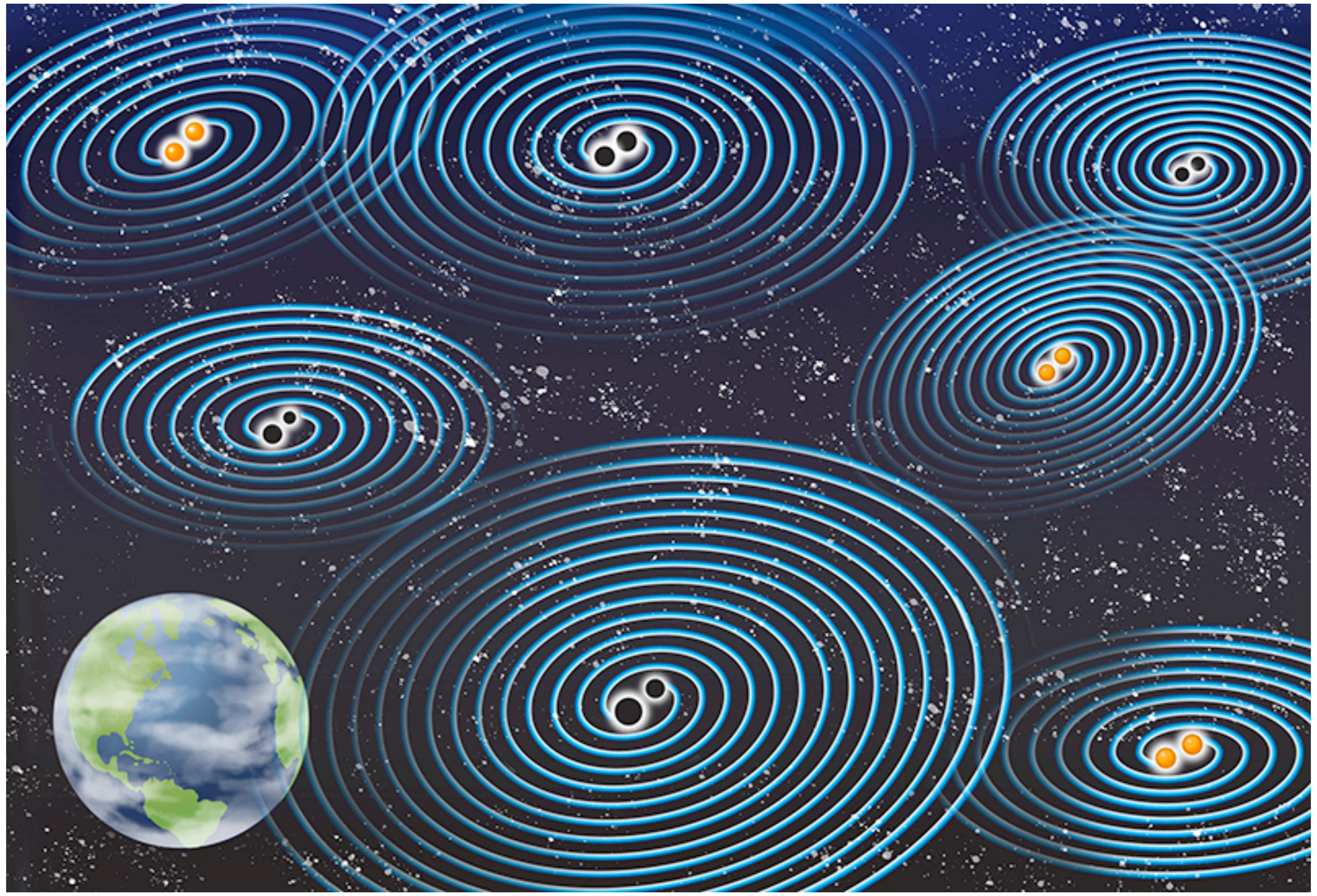
Merger of Binary Black Holes and Its Gravitational Waves / Credit: Simulating Extreme Spacetimes

Sources of GWs: Stochastic GWs

- Cosmological origin: Quantum state in early universe
- Astrophysical origin: Distribution of compact binaries



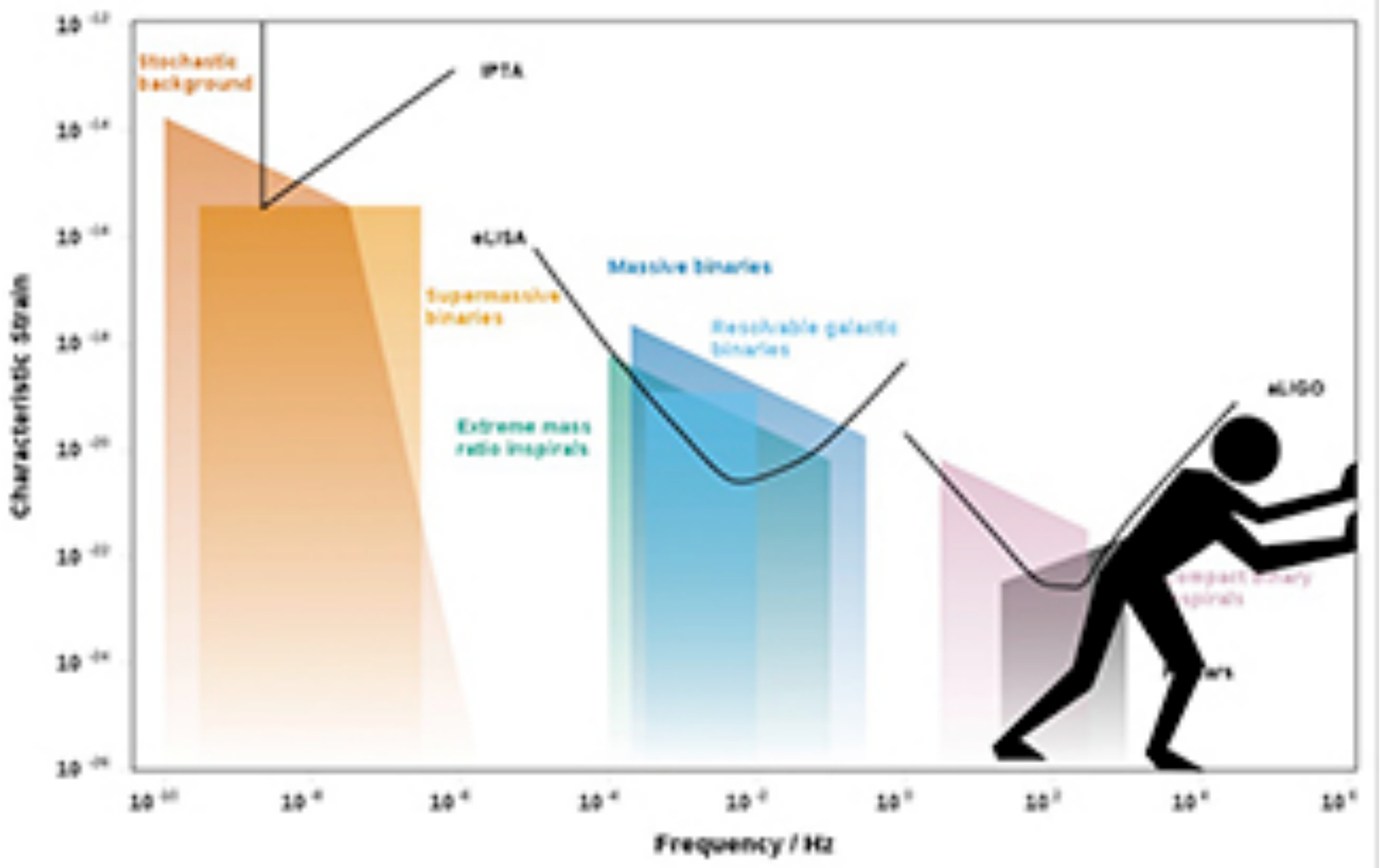
Stochastic GW from Big Bang / Credit: NASA



Distribution of Compact Binaries / Credit: APS

Ultra-High-Frequency (~GHz) GWs

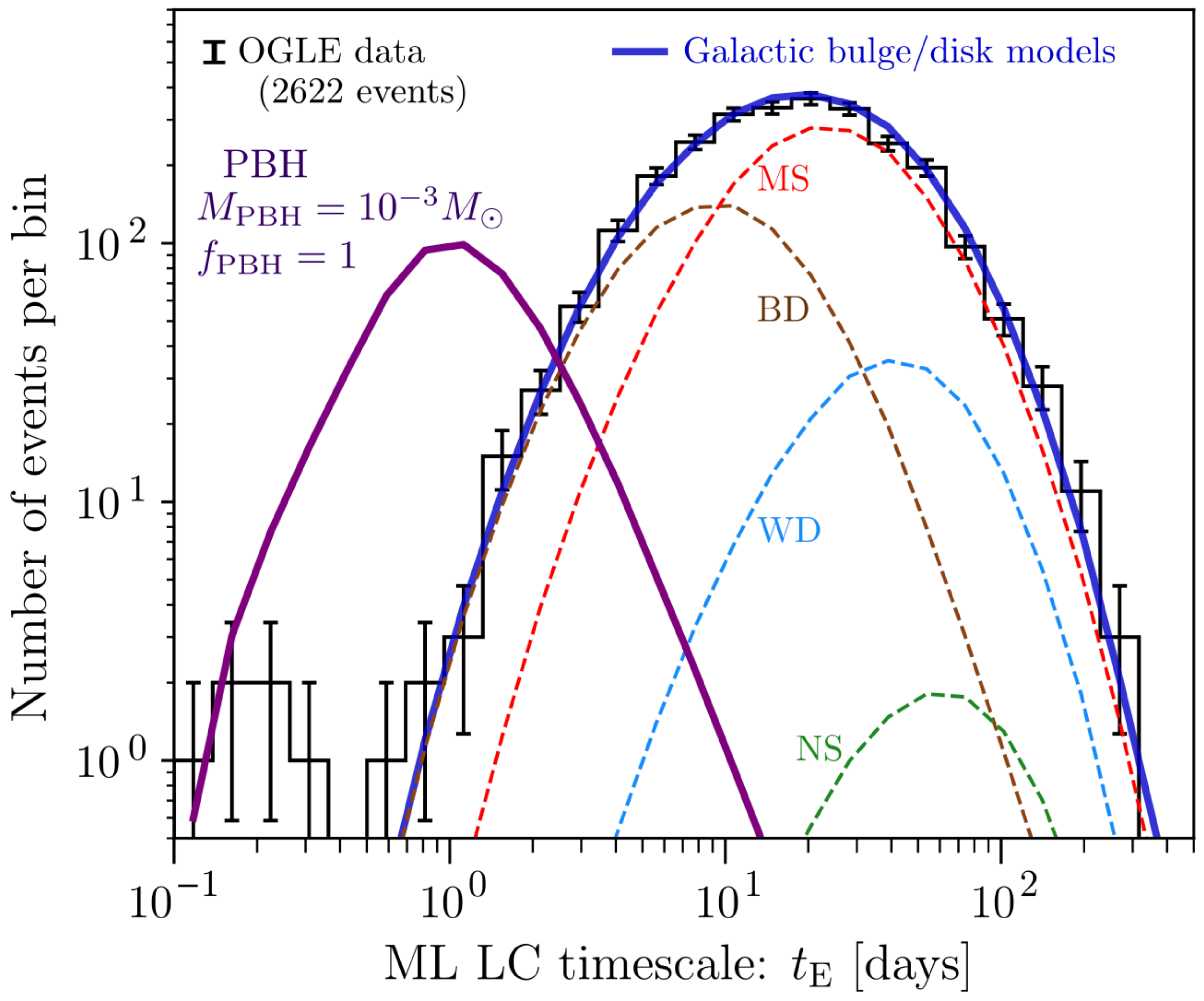
- Mergers of compact objects with subsolar masses
  - Primordial black holes of $\sim 10^{-5} M_{\odot}$ ~ earth mass
  - Exotic compact objects: boson stars, fermion stars, gravitino stars, gravistars, dark matter blobs
- Cosmological stochastic GWs
  - first-order phase transition, cosmic string, inflation, preheating
- We are welcome to your novel scenario!

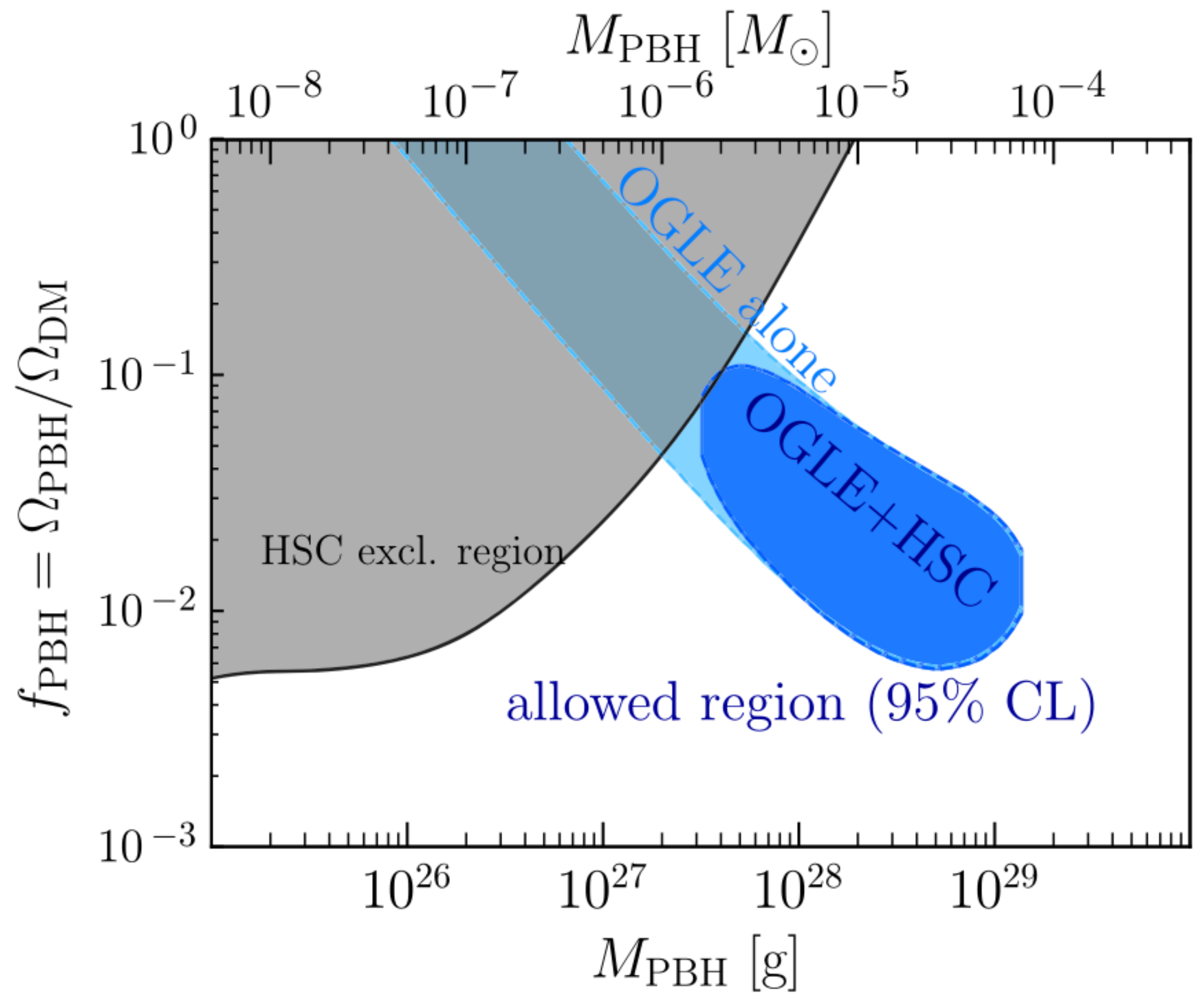


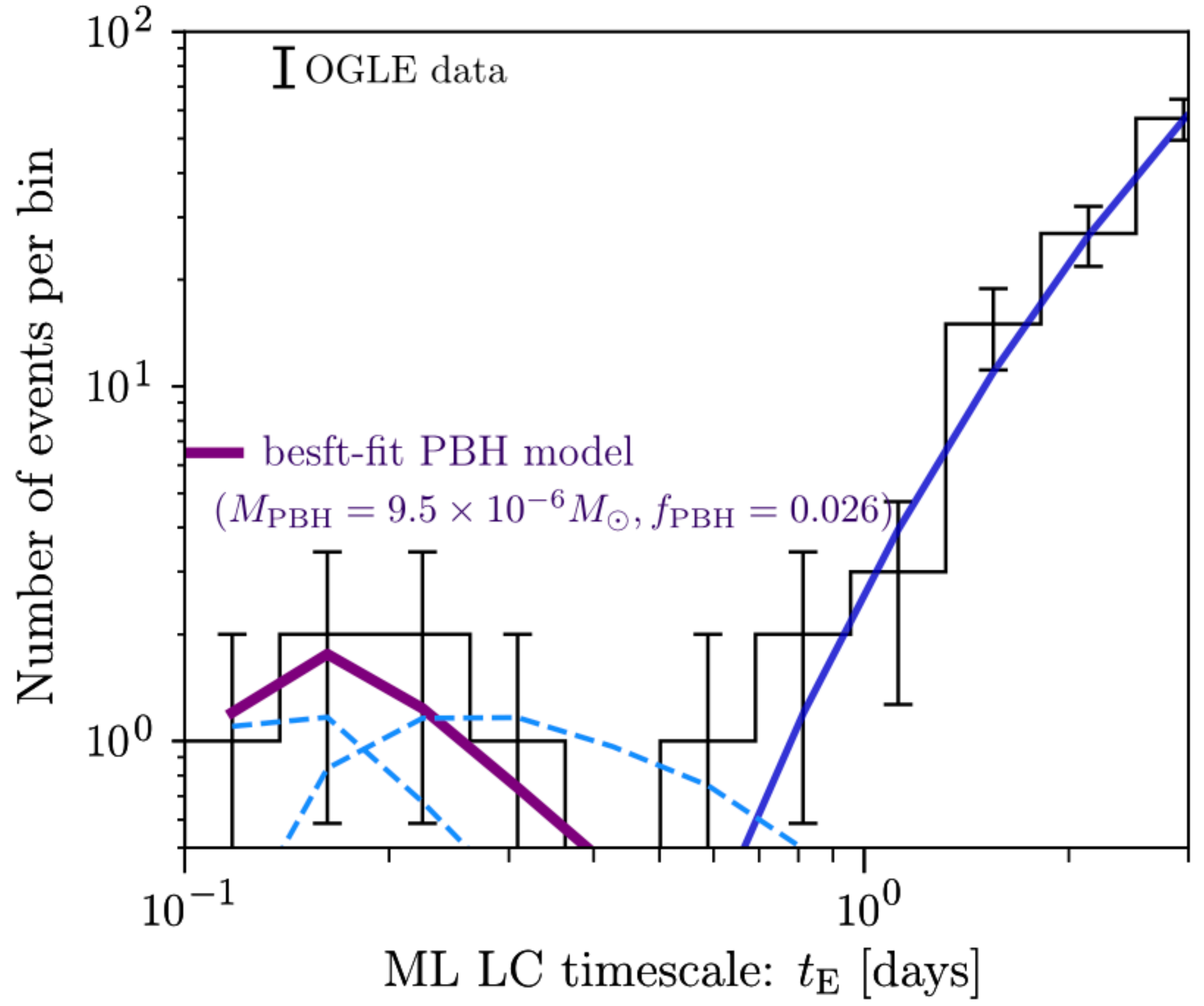
Credit: UHF-GW Initiative

OGLE Ultrashort-Timescale Events

- Niikura+ PRD 99, 083503 (2019)
- t_E = Einstein Radius / Speed on Lens Plane





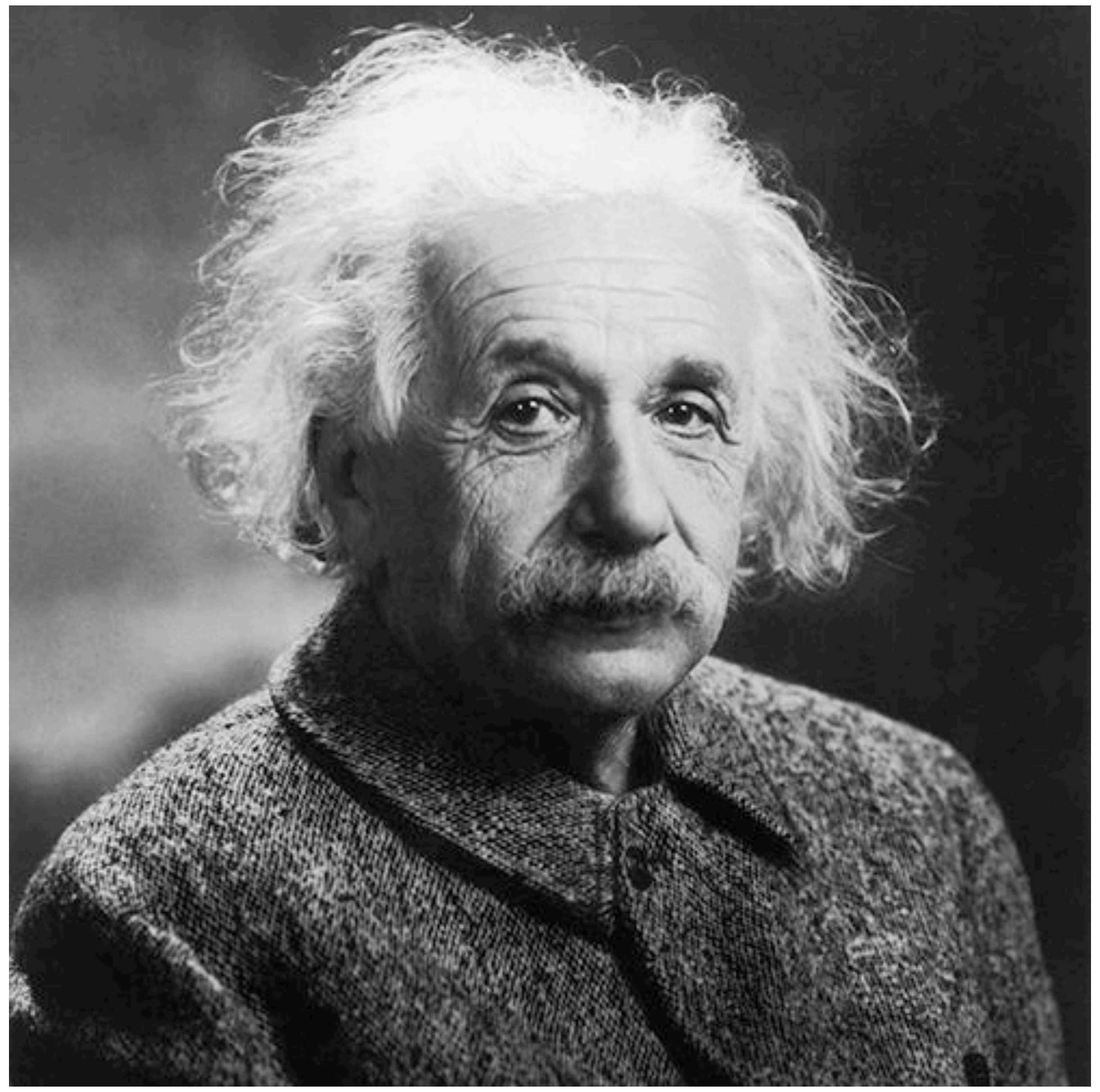


Working Principle:
Physical Laws in Curved Spacetime

Einstein Equation

- Einstein Equation
  - $G_{ab} = 8\pi G T_{ab}$

    We set $c = 1$: Natural Unit
- where
  - $G_{ab} = R_{ab} - \frac{1}{2} g_{ab} R^c{}_c$
  - $R_{ab} = R^c{}_{acb}$
  - $R^a{}_{bcd} v^b = 2 \nabla_{[c} \nabla_{d]} v^a$ for a vector v
  - ∇: Levi-Civita connection



Albert Einstein

Maxwell's Equations

- Fundamental Quantity for Electromagnetism
  - A: Electromagnetic Potential (not field strength F)
  - ex: Aharonov-Bohm effect
- Lagrangian
  - $\mathscr{L}_{\mathrm{EM}} = -\frac{1}{4} g^{ac} g^{bd} F_{ab} F_{cd} + A_a J^a$
  - where $F = dA$

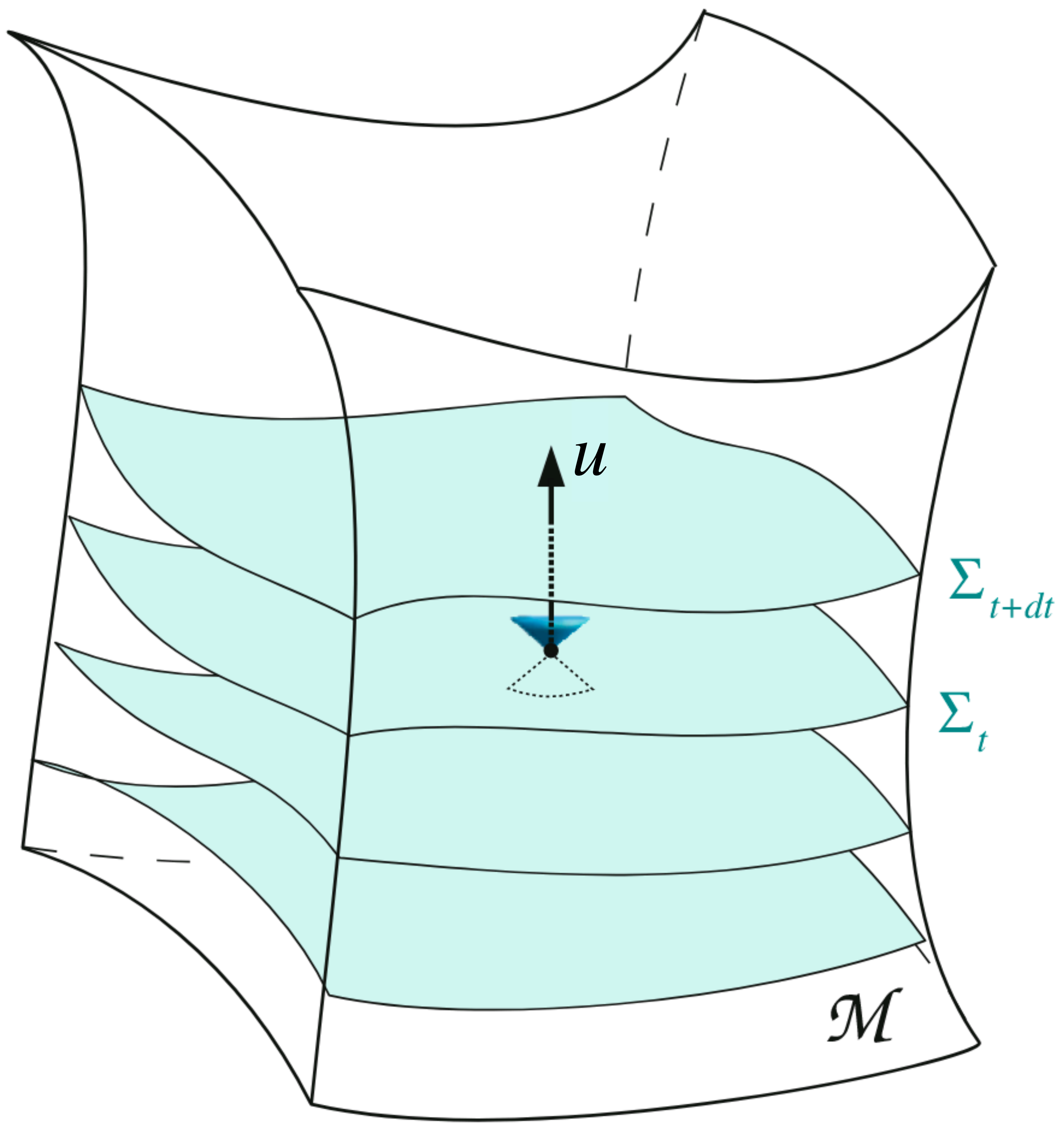
We set $\epsilon_0 = 1$:Heaviside-Lorentz Unit

James C. Maxwell

- Maxwell Equation
  - $g^{bc} \nabla_c F_{ab} = J_a$
  - $dF = 0$ (trivial because $F = dA$)
- For observer with 4-velocity u
  - $E_a = F_{ab} u^b$
  - $B_a = \frac{1}{2} \epsilon_{dbca} u^d F_{bc}$
- Wave equation form
  - $\nabla^c \nabla_c F_{ab} = -2 \nabla_{[a} J_{b]} - F_{cd} R^{cd}{}_{ab} - 2 F_{d[a} R^d{}_{b]}$

Globally Hyperbolic Spacetime

- Let us consider a globally hyperbolic spacetime $(\mathscr{M}, g)$ foliating by spacelike Cauchy hypersurfaces Σ_t.
- u: the normal vector field of Σ_t such that $g_{ab}u^a u^b = -1$.

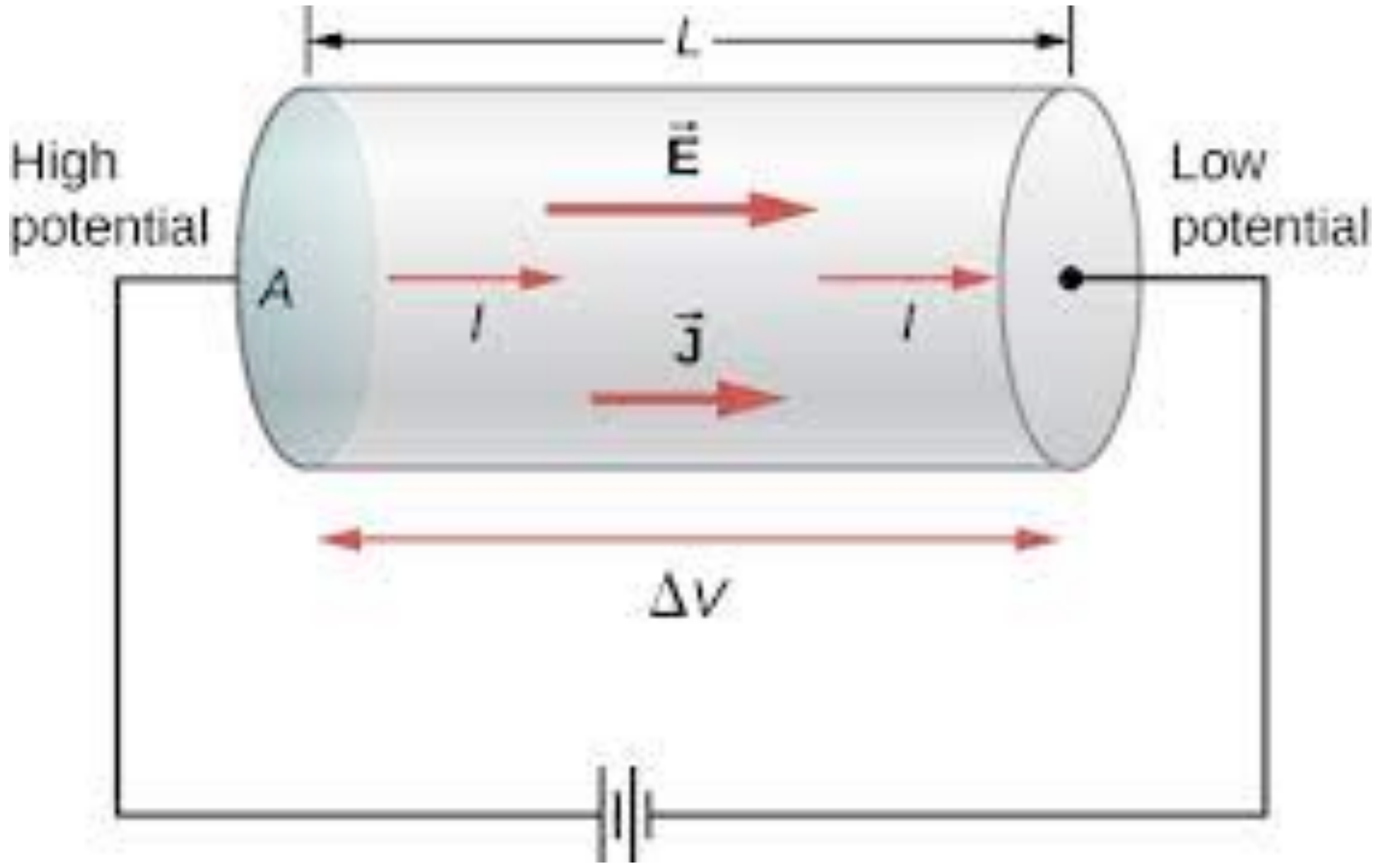


Credit: E. Gourgoulhon

Ohm's Law in Conductor

- $\gamma^a{}_b J^b = \sigma g^{ab} E_b$
- where
- σ: conductivity
- u: 4-velocity of the conductor.
- $E_a = F_{ab} u^b$: E field with respect to the conductor
- $\gamma^a{}_b = \delta^a{}_b + u^a u_b$: projection operator to Σ_t
- This law would be broken when the charge current is relativistic. We consider only situations in which the law is sufficiently satisfied.

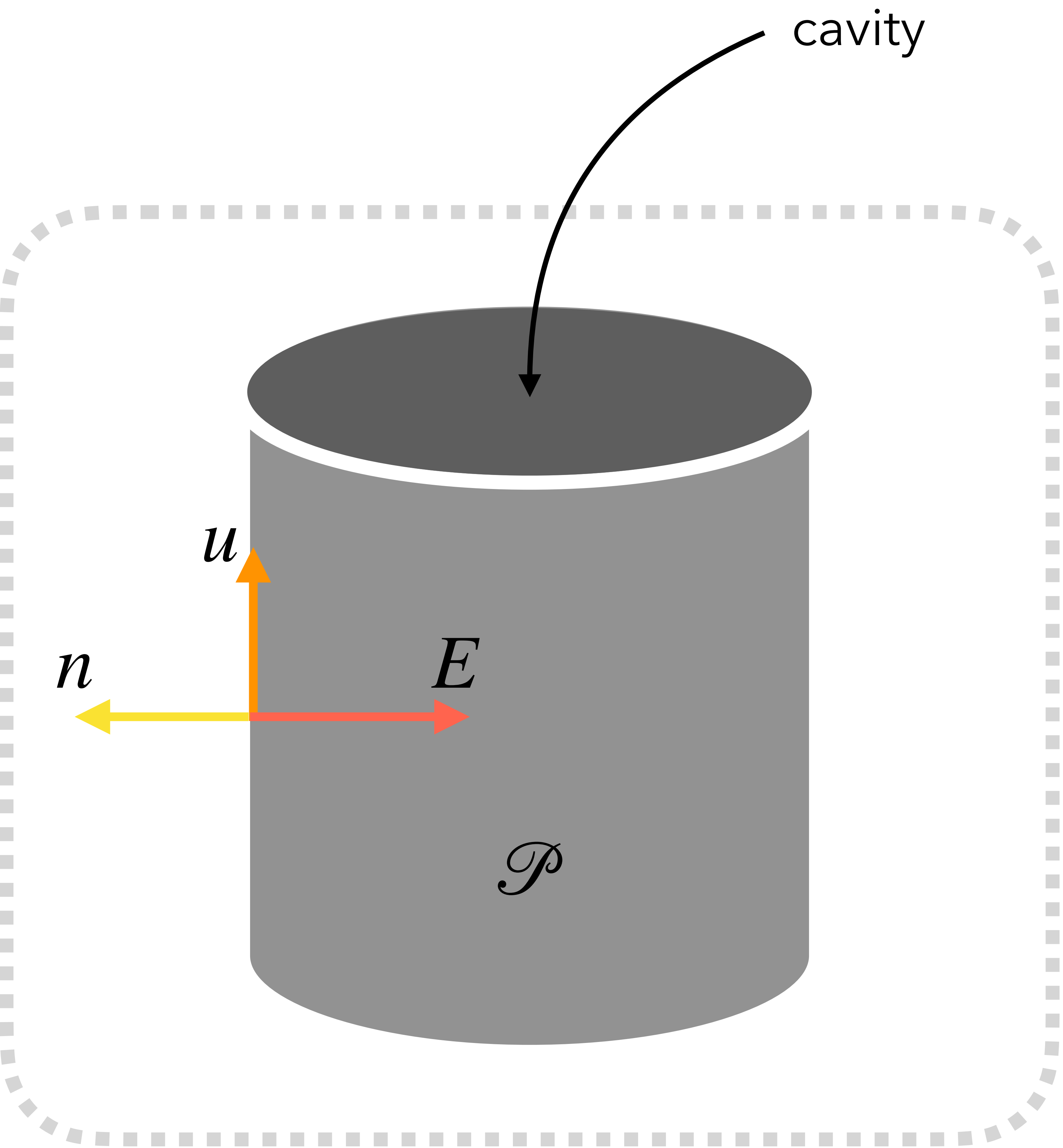
Georg Ohm



Resistance of Conductor

Boundary Condition for Surface of Conductor

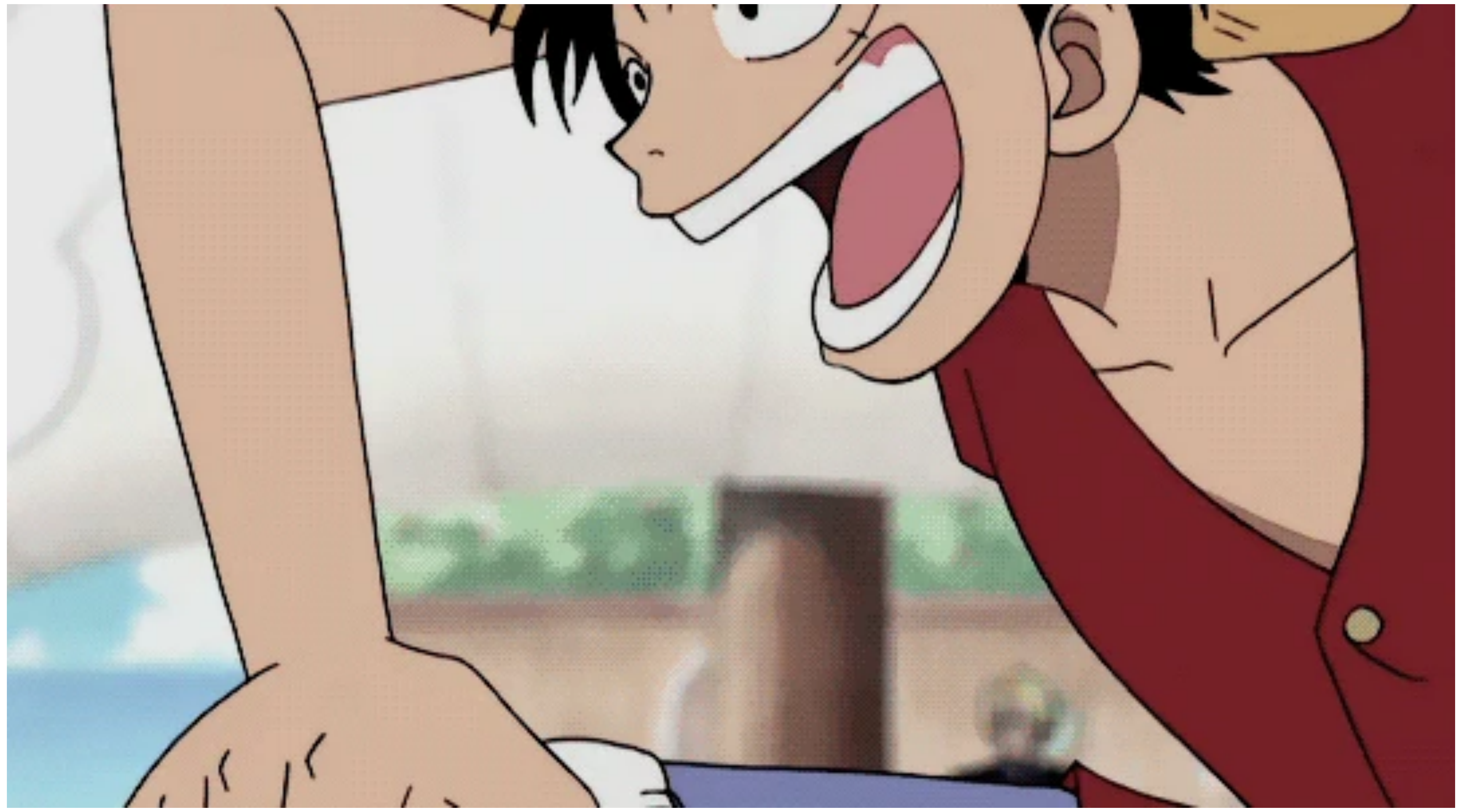
- Let us consider a conductor and its motion in a spacetime.
- $\mathscr{P}$: 3-dimensional timelike hypersurface for the surface of conductor
- If the conductor is perfect
- $\left(E_{\parallel}\right)_a = P^b{}_a E_b = 0$ on $\mathscr{P}$
- where
- n: normal vector to $\mathscr{P}$ (it is also orthogonal to u)
- $P^a{}_b \equiv \gamma^a{}_b - n^a n_b$: Projection operator orthogonal to n^a.



Spacetime diagram for cavity conductor

Elasticity

- Equation of Motion
  - $u^b \nabla_b u^a = f^a$
- The behavior u is determined by the elasticity theory (Newton's law) with a boundary condition. (in progress ...)
- The elasticity theory would be break when the motion of conductor with respect to the equilibrium point is relativistic. We consider only situations in which the law is sufficiently satisfied.

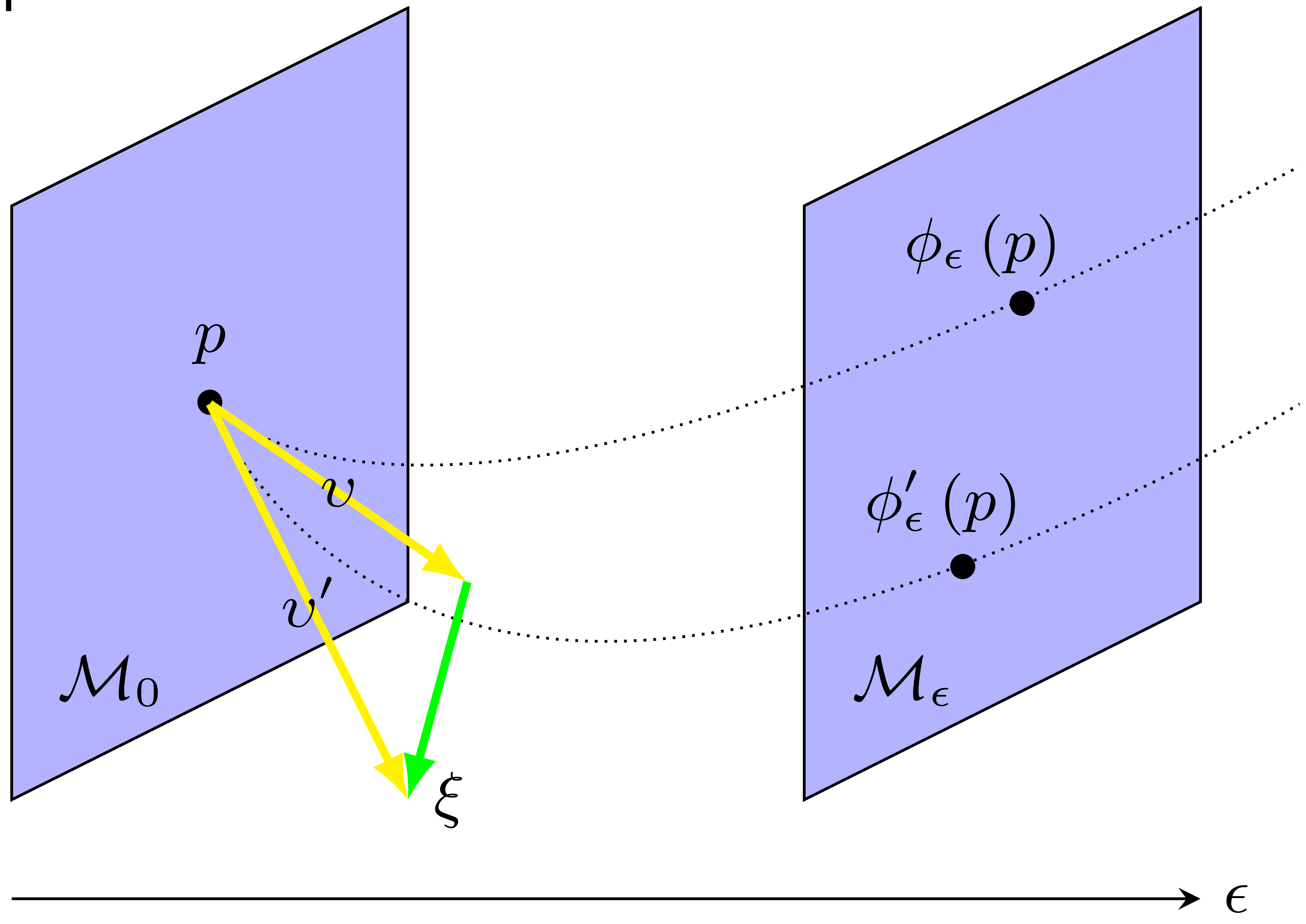


Example of elasticity (?)
/ Credit: TOEI ANIMATION

Working Principle: Perturbations

Perturbation

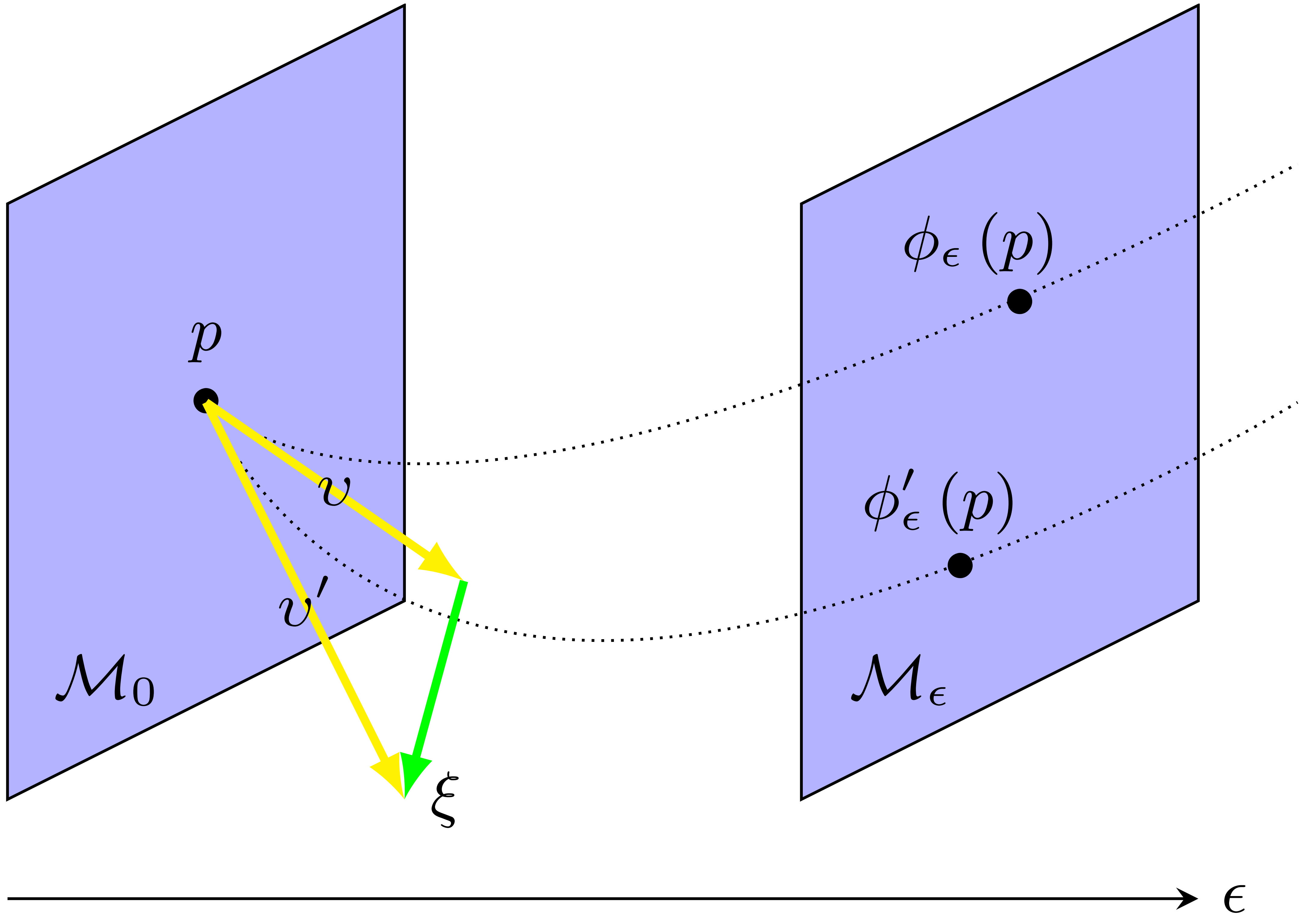
- For a one-parameter foliation of spacetimes $\mathscr{M}_\epsilon$
- ϕ_ϵ: one-parameter group of diffeomorphism
- $\tilde{Q}(\epsilon)$: perturbed Q in $\mathscr{M}_0$
  - $\tilde{Q}(\epsilon) = \phi^*_{-\epsilon} Q(\epsilon) = \epsilon^n \left[Z + \epsilon \delta Z + O\left(\epsilon^2\right) \right]$
  - n: an integer power for the strength
- δZ: perturbation of Z
  - $\delta Z = \lim_{\epsilon \to 0} \frac{1}{\epsilon} \left(\tilde{Z}(\epsilon) - Z \right) = \mathscr{L}_v Z$



Credit: C. Park / ApJ 940 58

Gauge for Perturbation

- We have gauges to choose ϕ_ϵ.
- Gauge Transformation
  - $\mathscr{L}_{v'}Z - \mathscr{L}_{v}Z = \mathscr{L}_{\xi}Z$ for $\xi \in T\left(\mathscr{M}_0\right)$
- Stewart-Walker theorem
  - δZ is gauge-invariant if and only if $\mathscr{L}_{\xi}Z = 0$ for all ξ.
  - if and only if Z is zero, constant scalar, or constructed by identity endomorphism with constant coefficients.



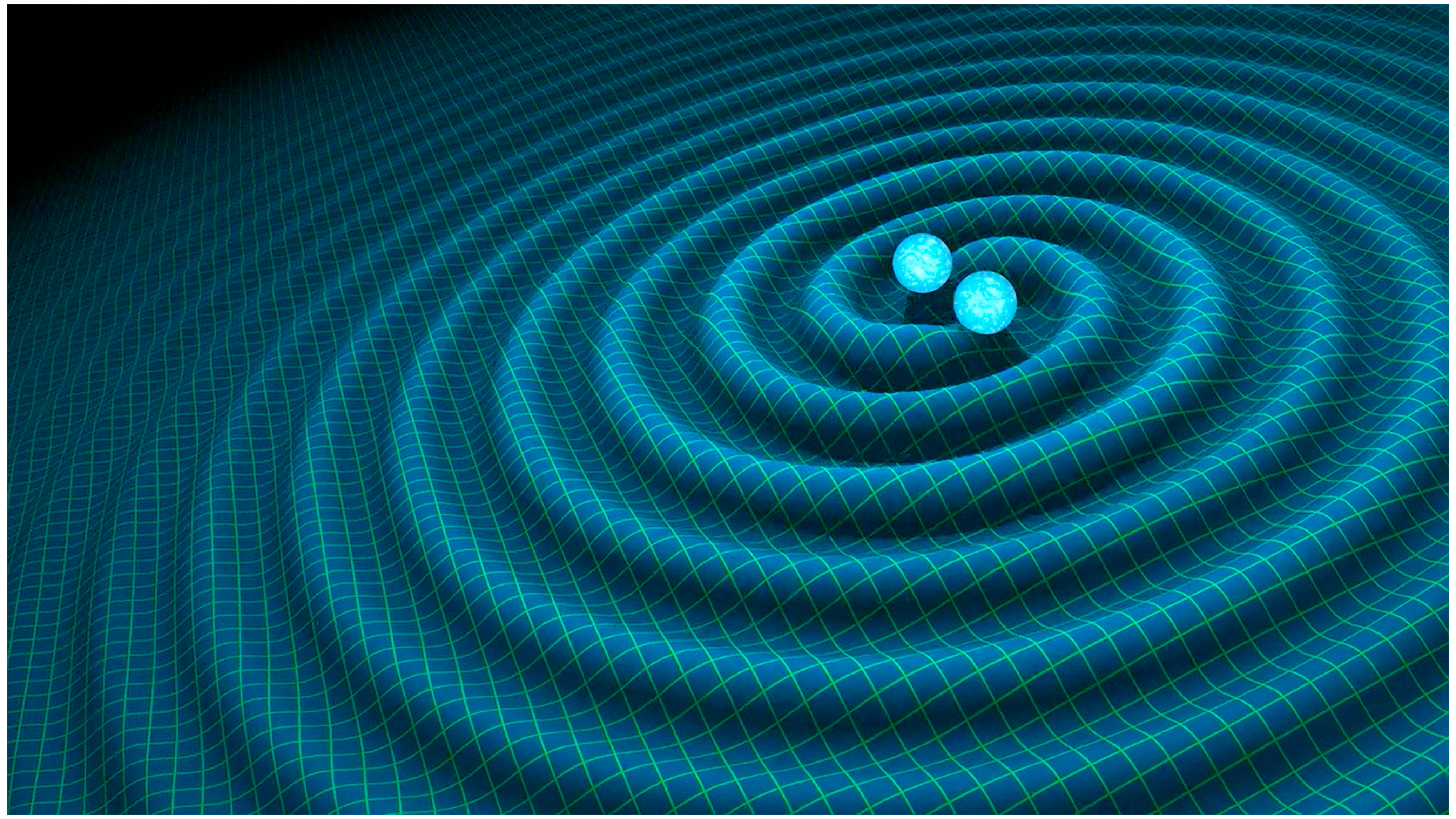
Credit: C. Park / ApJ 940 58

Metric

- We introduce the Minkowski spacetime in which
- ${}^{\epsilon}g_{ab} = g_{ab} + \epsilon h_{ab} + O\left(\epsilon^2\right)$
- where
- g: the flat metric
- $R^a{}_{bcd} = 0$: Riemann tensor associated with g

- Gauge Conditions
  - $\nabla^b h_{ab} = 0 \qquad h^a{}_a = 0$
  - where ∇ is the Levi-Civita connection associated with g
- Vacuum up to the First-order
  - ${}^{\epsilon}T_{ab} = O\left(\epsilon^2\right)$
- Perturbed Einstein Equation
  - $\nabla^c \nabla_c h_{ab} = 0$
  - Note that it is the wave equation

Gravitational Waves



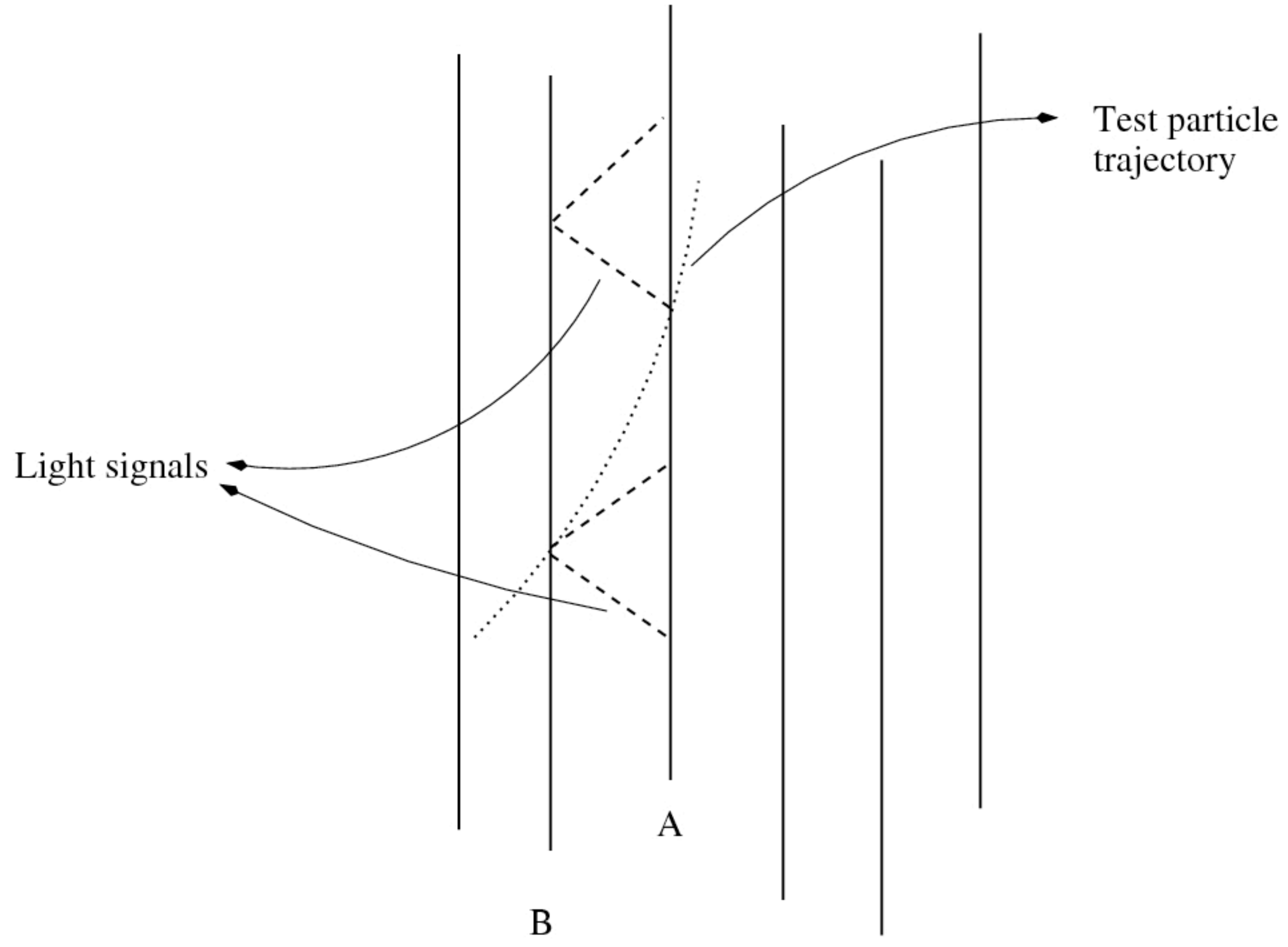
Credits: R. Hurt (Caltech-JPL)

- $h_{ab}\left(t,\vec{x}\right) = \int d^2\kappa \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \tilde{h}_{ab}(\omega,\kappa)\, e^{i\omega(-t+\kappa\cdot\vec{x})}$
  - where $\int d^2\kappa$: integration over all directions
- For $\kappa\left(\theta,\phi\right) = \sin\theta\cos\phi\hat{x} + \sin\theta\sin\phi\hat{y} + \cos\theta\hat{z}$
  - $\int d^2\kappa = \int_0^{2\pi} d\phi \int_0^{\pi} d\theta\sin\theta$
- From the gauge we chose
  - $0 = \tilde{h}_{ab}(\omega,\kappa)\, u^b$ $\quad$ $0 = \tilde{h}_{ab}(\omega,\kappa)\,\kappa^b$ $\quad$ $0 = \tilde{h}^a{}_a(\omega,\kappa)$
- The perturbation of Riemann tensor becomes
  - $(\delta R)^{ab}{}_{cd}\left(t,\vec{x}\right) = 2\int d^2\kappa \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} k^{[a}k_{[c}\tilde{h}^{b]}{}_{d]}(\omega,\kappa)\, e^{i\omega(-t+\kappa\cdot\vec{x})}$

Vector Field of 4-velocities

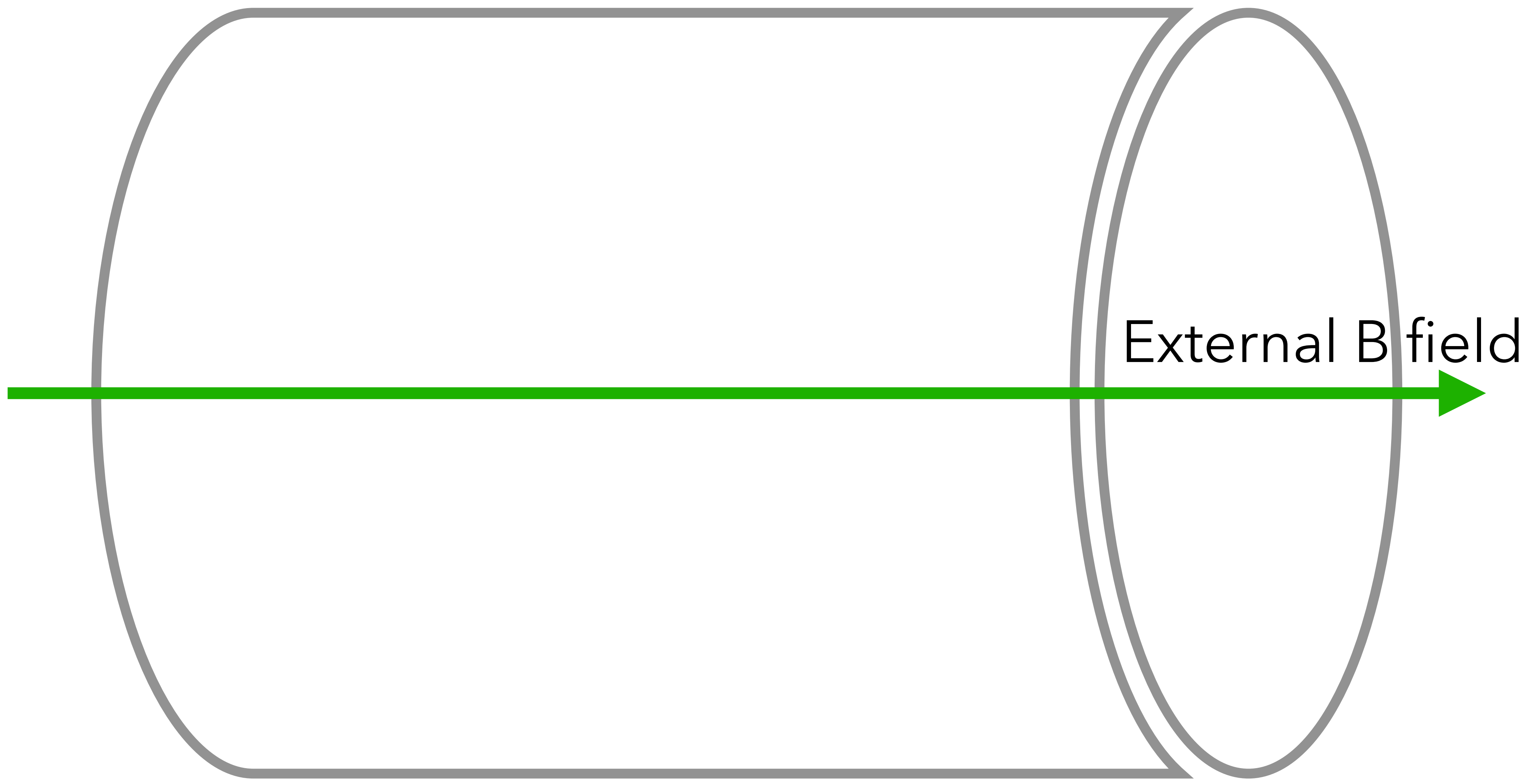
- ${}^{\epsilon}u^a = u^a + \epsilon\,(\delta u)^a + O\left(\epsilon^2\right)$
- where
- $u^a = \left(\partial/\partial t\right)^a$ such that $\nabla_b u_a = 0$
- $\{t, \vec{x}\}$: globally inertial coordinate system
- We additionally introduce the radiation gauge condition as
- $h_{ab}u^b = 0.$

- Then, we have Transverse-Traceless (TT) gauge.



A congruence of geodesics for inertial observers. They are fixed at the spatial coordinate even if GWs pass. / Credit: M. Nouri-Zonoz and A. Nourizonoz

Electromagnetic Field and Electric Current



- ${}^{\epsilon}F_{ab} = \epsilon\left\{F_{ab} + \epsilon\,(\delta F)_{ab} + O\left(\epsilon^2\right)\right\}$
- ${}^{\epsilon}J_a = \epsilon\left\{\cancel{J_a} + \epsilon\,(\delta J)_a + O\left(\epsilon^2\right)\right\}$
- where
- $F_{ab} = \epsilon^{c}{}_{ab}B_c = \text{const}$ and $\nabla_c F_{ab} = 0$
- $J^a = 0$
- δJ and δE is gauge-invariant because their background vanishes.
- Note that we set ${}^{\epsilon}F = O(\epsilon)$ and ${}^{\epsilon}J = O\left(\epsilon^2\right)$ then ${}^{\epsilon}T = O\left(\epsilon^2\right)$ that does not spoil the vacuum of Minkowski spacetime in the background and does not give the source term to the wave equation of GWs.

Maxwell Equation

- $\nabla^b (\delta F)_{ab} = (\delta J)_a - F_{bc} \nabla^b h^c{}_a$
- $\nabla_{[a} (\delta F)_{bc]} = 0$
- $\Box (\delta F)_{ab} = -2 \nabla_{[a} (\delta J)_{b]} - F_{cd} (\delta R)^{cd}{}_{ab}$
- We have cancelled all terms with $\nabla_c F_{ab}$

We are in tough section. 😵‍💫

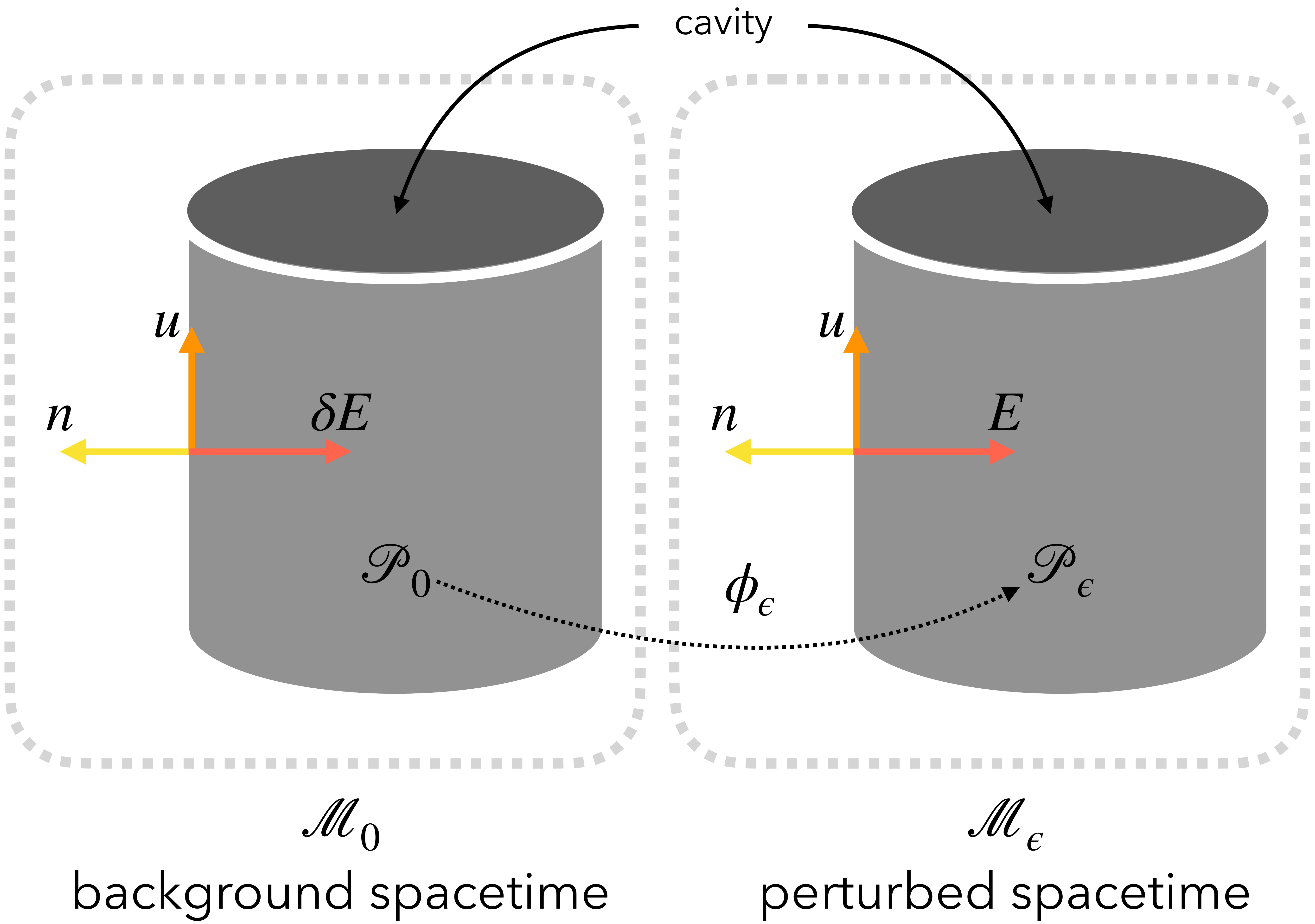
Electric Field

- The electric field with respect to u
  - $(\delta E)_a = (\delta F)_{ab}\, u^b + F_{ab}\, (\delta u)^b$
  - δE is spatial.
  - Note that δE is gauge-invariant because $E_a = F_{ab} u^b = 0$.
- Maxwell Equation for δE
  - $$\begin{aligned}\Box\, (\delta E)_a = D_c D^b\, (\delta E)_b + \partial_t \left(\gamma^b{}_a\, (\delta J)_b\right) - F_{cd}(\delta R)^{cd}{}_{ab} u^b \\ +(\cdots)\,(\delta u)\end{aligned}$$
- Ohm's Law
  - $\gamma^a{}_b\, (\delta J)^b + \cancel{J}^b \left(\delta \gamma\right)^a{}_b = \sigma\, (\delta E)_a + \left((\delta\sigma)\, g^{ab} - \sigma h^{ab}\right) \cancel{E}_b$

We are in tough section. 😵‍💫

Boundary Condition for $\mathscr{P}_0$

- Let us consider a gauge with $\phi_\epsilon : \mathscr{M}_0 \to \mathscr{M}_\epsilon$ such that $\phi_\epsilon \left[\mathscr{P}_0\right] = \mathscr{P}_\epsilon$.
- $\left(\delta' E_\parallel\right)_a = P^b{}_a (\delta' E)_b + \cancel{E_b} (\delta' P)^b{}_a = 0$ on $\mathscr{P}_0$ by the perturbation with ϕ_ϵ.
- Because δE is gauge-invariant, in our TT gauge, we also have
- $P^b{}_a (\delta' E)_b = P^b{}_a (\delta E)_b = 0$ on $\mathscr{P}_0$
- Due to the boundary condition, the eigenmode expansion is possible.

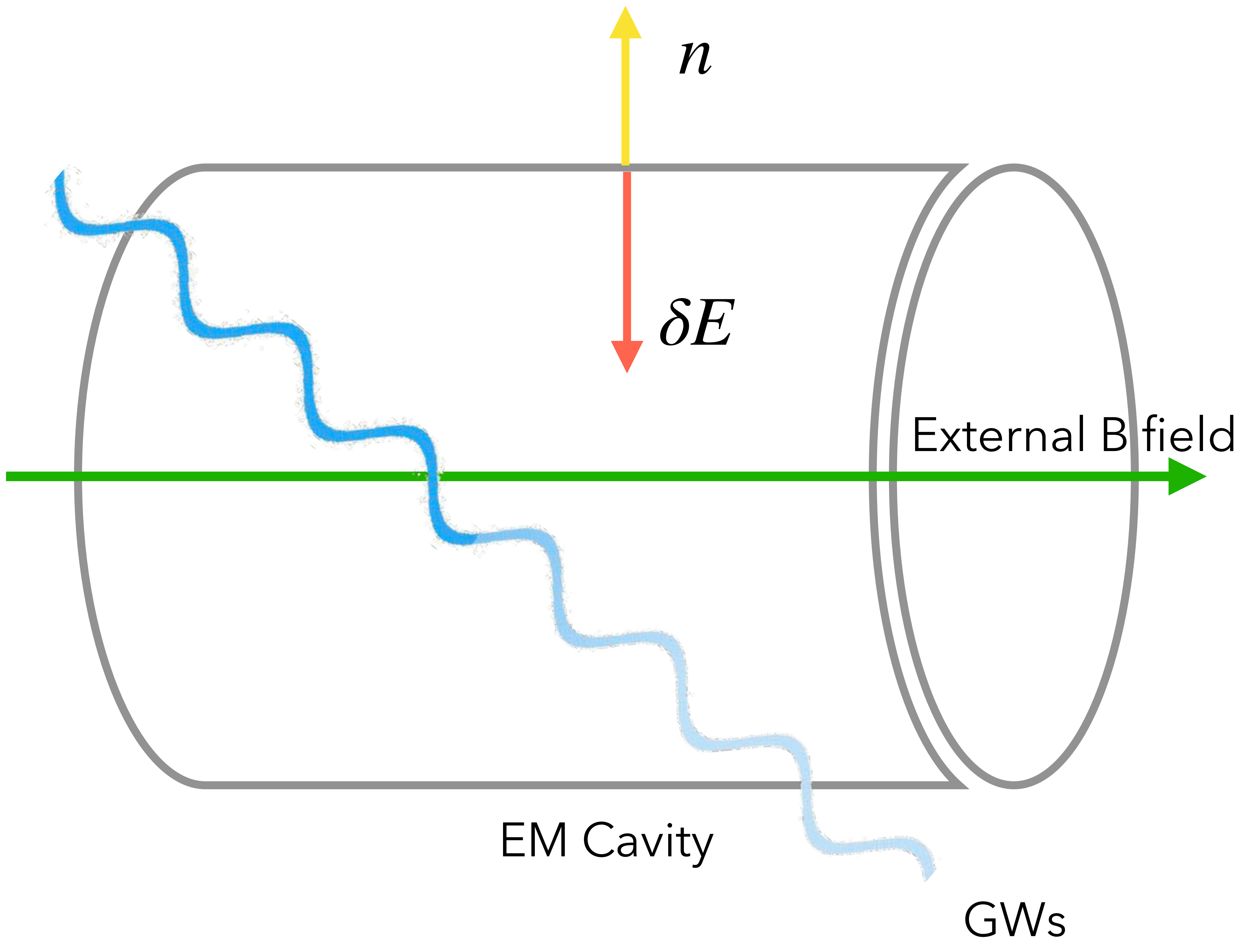


Elasticity

- Equation of Motion
  - $u^b \nabla_b (\delta u)^a + \cancel{C^a{}_{bc} u^b u^c} = \left(\delta f\right)^a$
- The behavior δu is determined by the elasticity theory. (in progress ...)
- The governed equation will be the wave equation of δu with dissipation term.
  - $\left(-\partial_t^2 - \frac{\omega_0}{Q}\partial_t + v^2 \partial^i \partial_i\right) \delta u^a = \cdots$
  - where v is the speed of the acoustic wave.
- Because v is much smaller than the speed of light ($v \ll 1$), its resonance frequency will be much smaller than the one of EM field.
  - $\omega_0^{\text{acoustic}} \sim \frac{v}{L_{\text{det}}} \ll \frac{1}{L_{\text{det}}} \sim \omega_0^{\text{em}}$
- We expect that the effect of the acoustic oscillation is negligible in the frequency band of our interest ($\sim \omega_0^{\text{em}}$).

Forced Oscillation Equation

- As a result, we get the forced oscillation equation
- For solenoidal mode: $D^a (\delta E)_a = 0$
  - $\left(-\partial_t^2 - \sigma\partial_t + \partial^i\partial_i\right)(\delta E)_a = -F_{cd}(\delta R)^{cd}{}_{ab}u^b$
- For irrotational mode: $\epsilon^{abc}D_b (\delta E)_c = 0$
  - $\left(-\partial_t^2 - \sigma\partial_t\right)(\delta E)_a = -F_{cd}(\delta R)^{cd}{}_{ab}u^b$
- Boundary Condition
  - $P^b{}_a (\delta E)_b = 0$ on $\mathscr{P}_0$

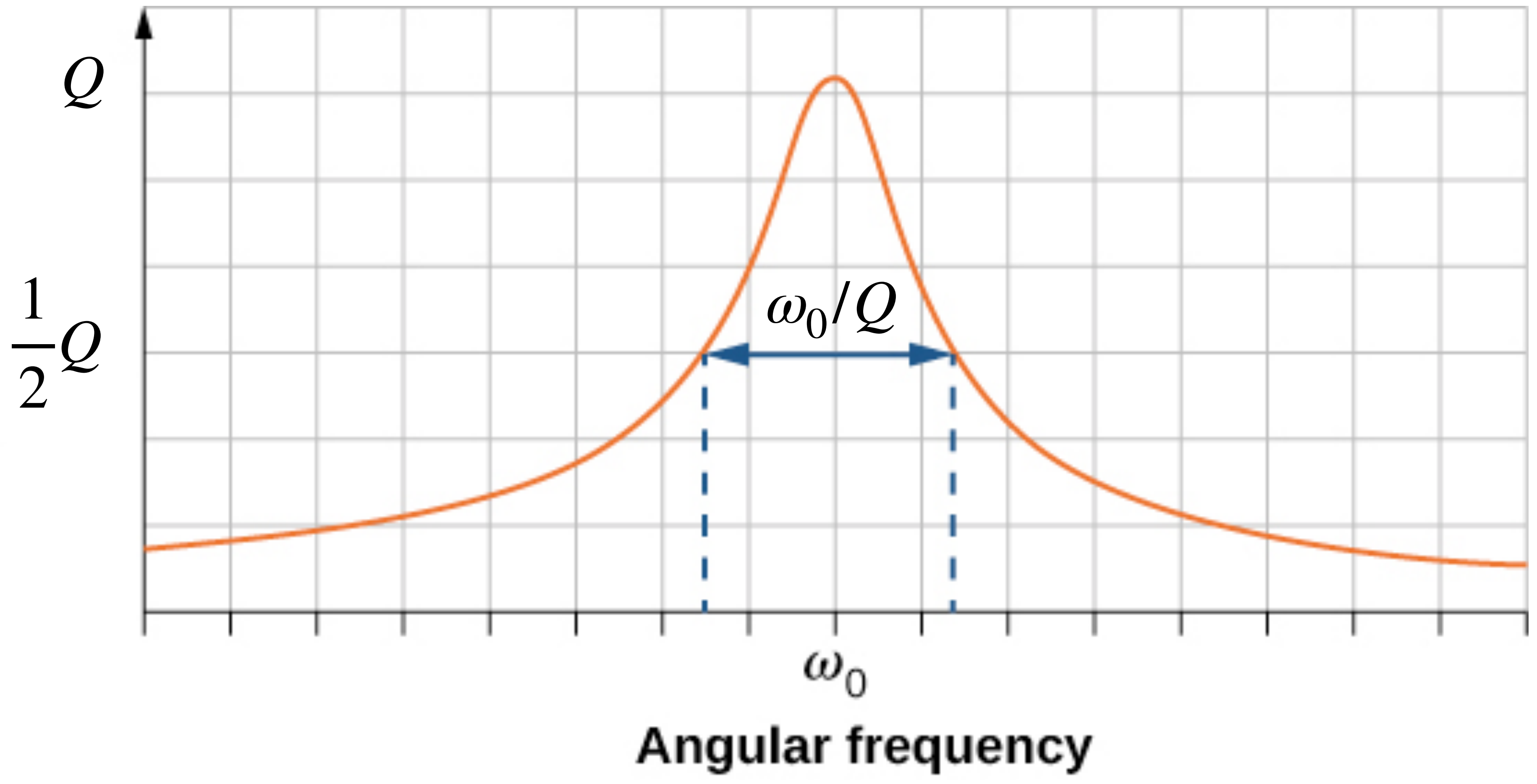


Eigenmode Expansion

- Eingenmode Expansion (depends on the shape of cavity)
  - $(\delta E)_a\left(t,\vec{x}\right) = \sum_n \mathscr{E}_n(\omega)\, e_a^n$
- where real basis $e^n\left(\vec{x}\right)$ satisfies
  - $D^a e_a^n = 0$ (solenoidal mode)
  - $\Delta e_a^n = -\,\omega_n^2 e_a^n$ (dispersion)
  - $\int_{\mathscr{V}} d\mathscr{V}\; e^n \cdot e^m = \delta_{nm} V$ (orthogonality)

Eigenmode Expansion

- Equation for n-th mode
  - $\ddot{\mathscr{E}}_n + \frac{\omega_n}{Q_n}\dot{\mathscr{E}}_n + \omega_n^2 \mathscr{E}_n = \left|B\right| \int_{-\infty}^{\infty} \frac{d\omega}{2\pi}\, \omega^2 \tilde{h}_n(\omega)\, e^{-i\omega t}$
  - ω_n: resonance frequency
  - Q_n: quality factor
  - $\tilde{h}_n(\omega) = \int d^2\kappa\, \tilde{h}_{ab}(\omega, \kappa) \left(\hat{B} \times \kappa\right)^a \frac{1}{V} \int_{\mathscr{V}} d\mathscr{V}\, e_n^a(\vec{x})\, e^{i\omega\kappa\cdot\vec{x}}$

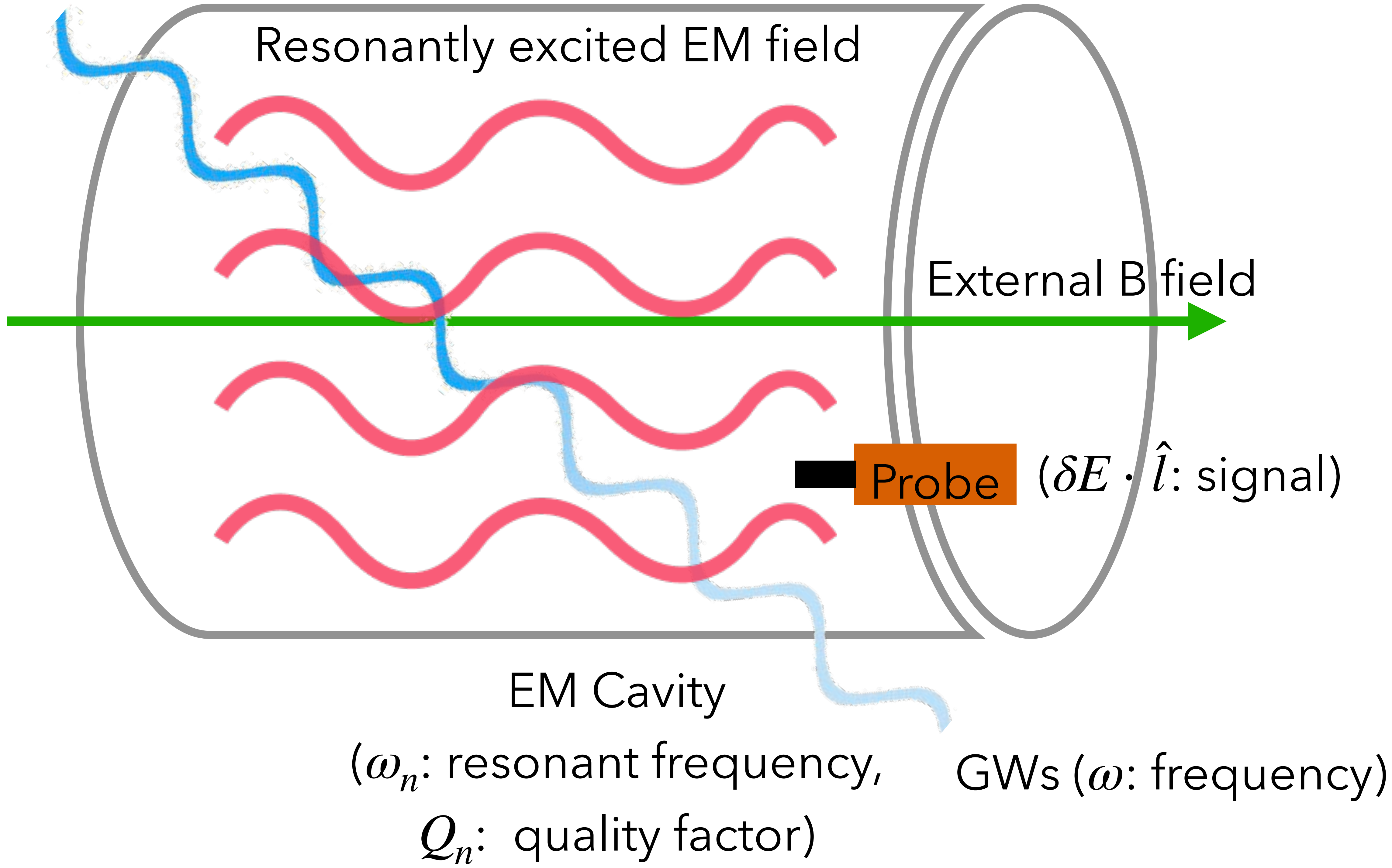


Curve for resonance factor $a\left(\omega; \omega_0, Q\right)$

- Amplitude of forced oscillation
  - $\tilde{\mathscr{E}}_n(\omega) = T(\omega)\, \tilde{h}_n(\omega)$
  - $T(\omega) = a\left(\omega; \omega_n, Q_n\right) e^{i\alpha\left(\omega; \omega_0, Q\right)} \left(\omega/\omega_0\right)^2 \left|B\right|$: transfer function

GW Signal

- Let us consider the measurement of δE using the probe at $\vec{x}_{\rm p}$ directed to $\hat{l}$
  - $\delta\left(E\cdot\hat{l}\right) = (\delta E)\cdot\hat{l} + \cancel{E}\cdot\delta\hat{l}$
  - $\simeq \int_{|\omega|=\omega_1}^{\omega_2} \frac{d\omega}{2\pi}\,\tilde{\mathscr{E}}_n(\omega)\left(e_n\left(\vec{x}_{\rm p}\right)\cdot\hat{l}\right)e^{-i\omega t}$
  - where we choose a specific mode with n and frequency band $\omega_1 < \omega_n < \omega_2$.
- From the measurement, we can determine $\tilde{\mathscr{E}}_n(\omega)$ in $\left(\omega_1, \omega_2\right)$

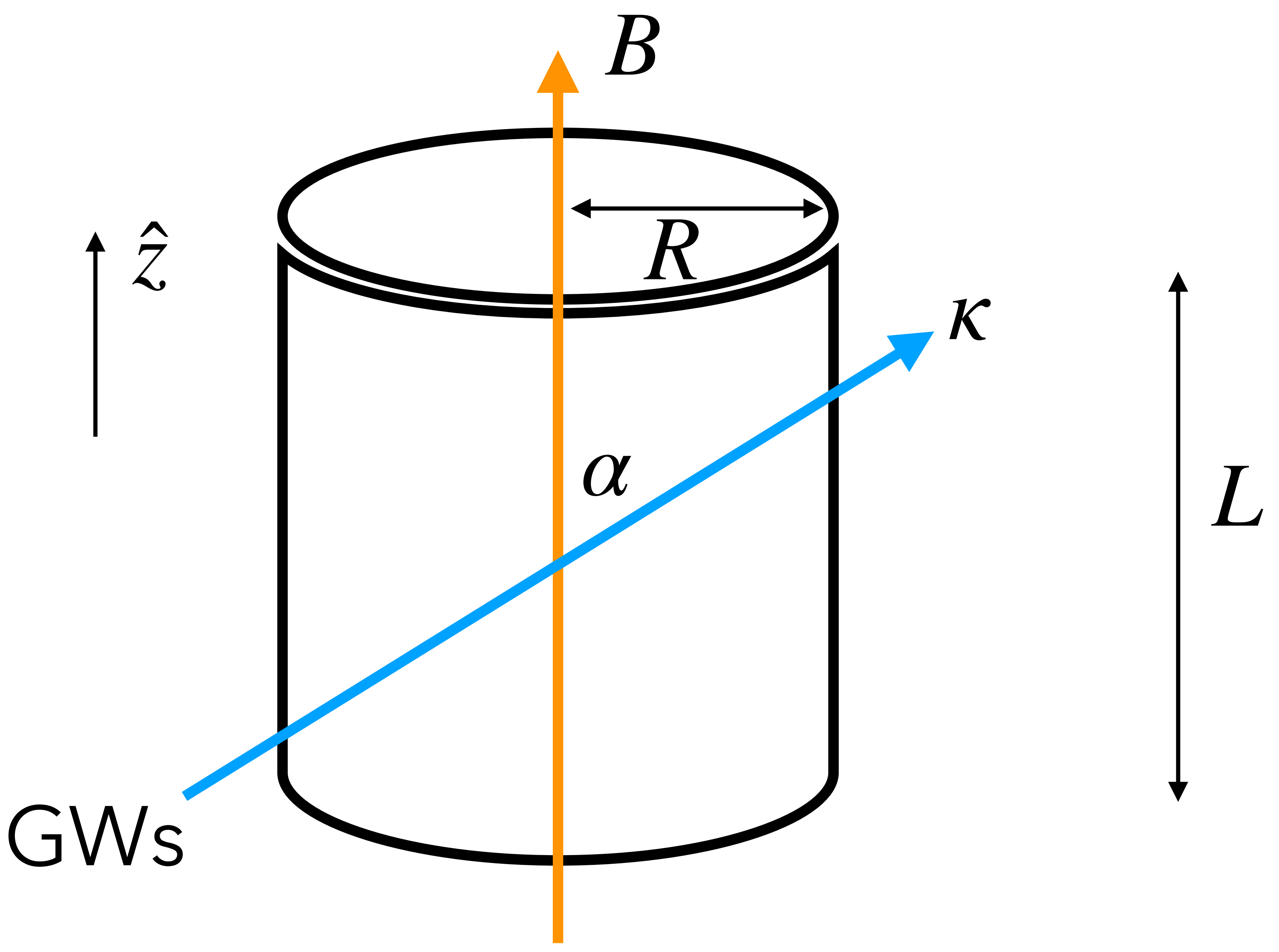


Antenna Pattern

- Orthogonalization for real and imaginary parts of $\tilde{h}$
  - $\tilde{h}_{ab} = H\left(\cos\eta\, e^{+}_{ab} + i\sin\eta\, e^{\times}_{ab}\right) e^{i\delta}$
  - H: GW strength
  - η: ellipticity
  - δ: phase adjustment
- Pattern Function
  - $\tilde{\mathscr{E}}_n(\omega) = \int d^2\kappa\, H(\omega,\kappa)\, \tilde{F}_n(\omega,\kappa)$
  - $\tilde{F}_n(\omega,\kappa) = \left(\cos\eta(\omega,\kappa)\, e^{+}_{ab}(\omega,\kappa) + i\sin\eta(\omega,\kappa)\, e^{\times}_{ab}(\omega,\kappa)\right)\left(\hat{B}\times\kappa\right)^a \bar{e}^b_n(\omega,\kappa)\, e^{i\delta(\omega,\kappa)}$

Example: TM010 mode of Cylindrical Cavity

- $e_{010}(\rho, \phi, z) = |B| \, J_0(\omega_{010}\rho)\,\hat{z}$
  - B: strength of external magnetic field
  - $\omega_{010} = j_{0,1}/R$: resonance frequency
  - $j_{0,1}$: first zero of Bessel function J_0
  - $\mathscr{A} = L/R$: aspect ratio
  - R: radius of cylinder

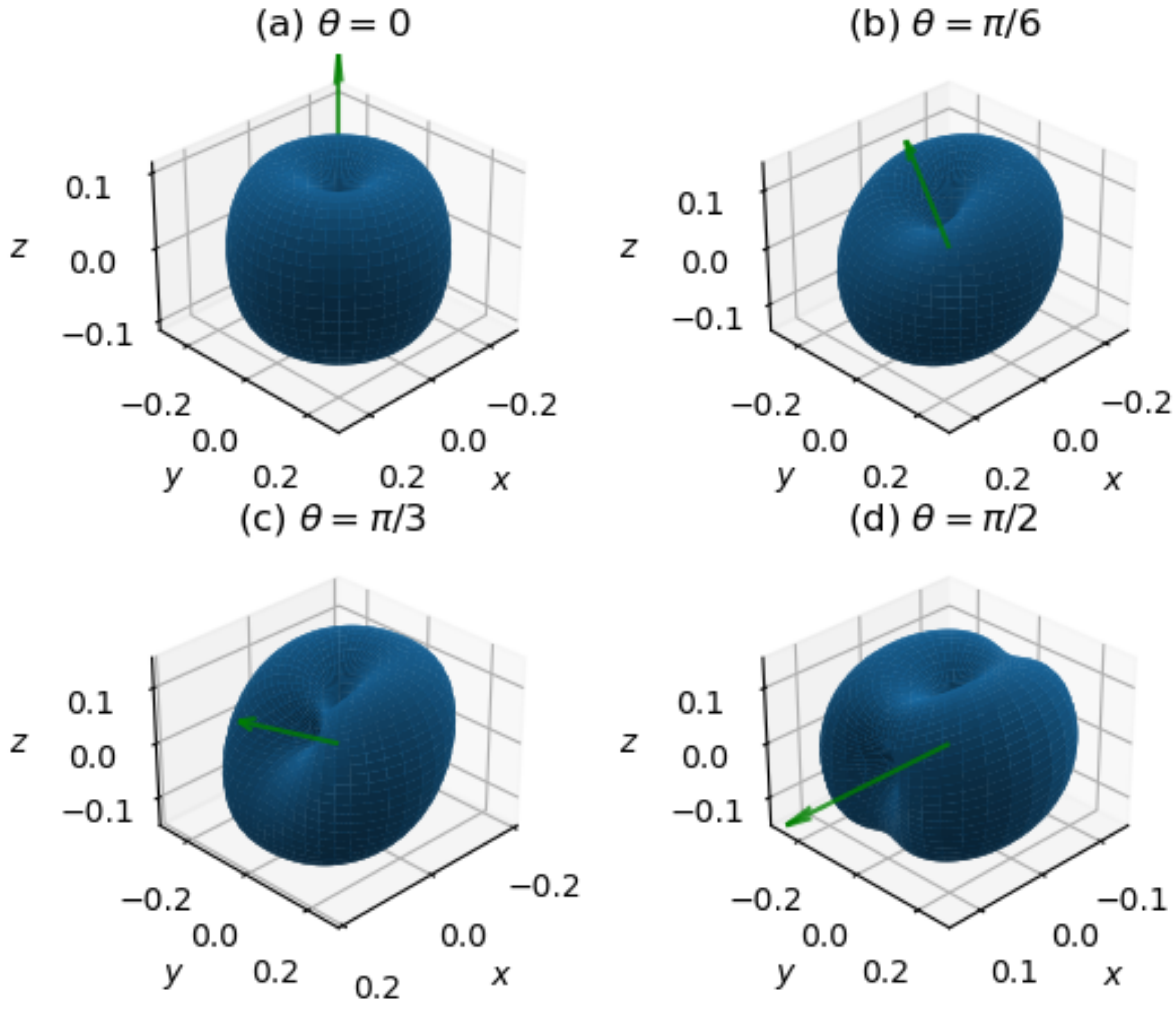


$$\tilde{F}(\omega_{010}, \kappa) = 2\frac{\sin\left(\frac{1}{2}\mathscr{A} j_{0,1}\cos\alpha\right)}{\frac{1}{2}\mathscr{A} j_{0,1}\cos\alpha}\frac{J_0\left(j_{0,1}\sin\alpha\right)}{j_{0,1}\cos^2\alpha}$$

- $$\times\left(\cos\eta e^{+}_{ab}(\omega_{010}, \kappa) + i\sin\eta e^{\times}_{ab}(\omega_{010}, \kappa)\right)\left(\hat{B}\times\kappa\right)^a \hat{z}^b e^{i\delta}$$

Example: TM010 mode of Cylindrical Cavity

- $\hat{B} = B/\left|B\right| = \sin\theta\hat{x} + \cos\theta\hat{z}$
- $\kappa = \sin\alpha\cos\beta\hat{x} + \sin\alpha\sin\beta\hat{y} + \cos\alpha\hat{z}$
- $u = \cos\left(2\psi\right)u_0 + \sin\left(2\psi\right)v_0$
- $v = -\sin\left(2\psi\right)u_0 + \cos\left(2\psi\right)u_0$
- $F\left(\alpha,\beta;\theta\right) = \sqrt{\left\langle \tilde{F}\tilde{F}^* \right\rangle_\psi}$



$F\left(\alpha,\beta;\theta\right)$ for $\theta \in \left\{0,\pi/6,\pi/3,\pi/2\right\}$

Data Analysis for GWs

Thermal Noise

- Equation for n-th mode with thermal noise

  $$\frac{V}{\omega_n^2}\left(\ddot{\mathscr{E}}_n + \frac{\omega_n}{Q_n}\dot{\mathscr{E}}_n + \omega_n^2\mathscr{E}_n\right) = \frac{V\left|B\right|}{\omega_n^2}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\,\omega^2\tilde{h}(\omega)\,e^{-i\omega t} + f(t)$$

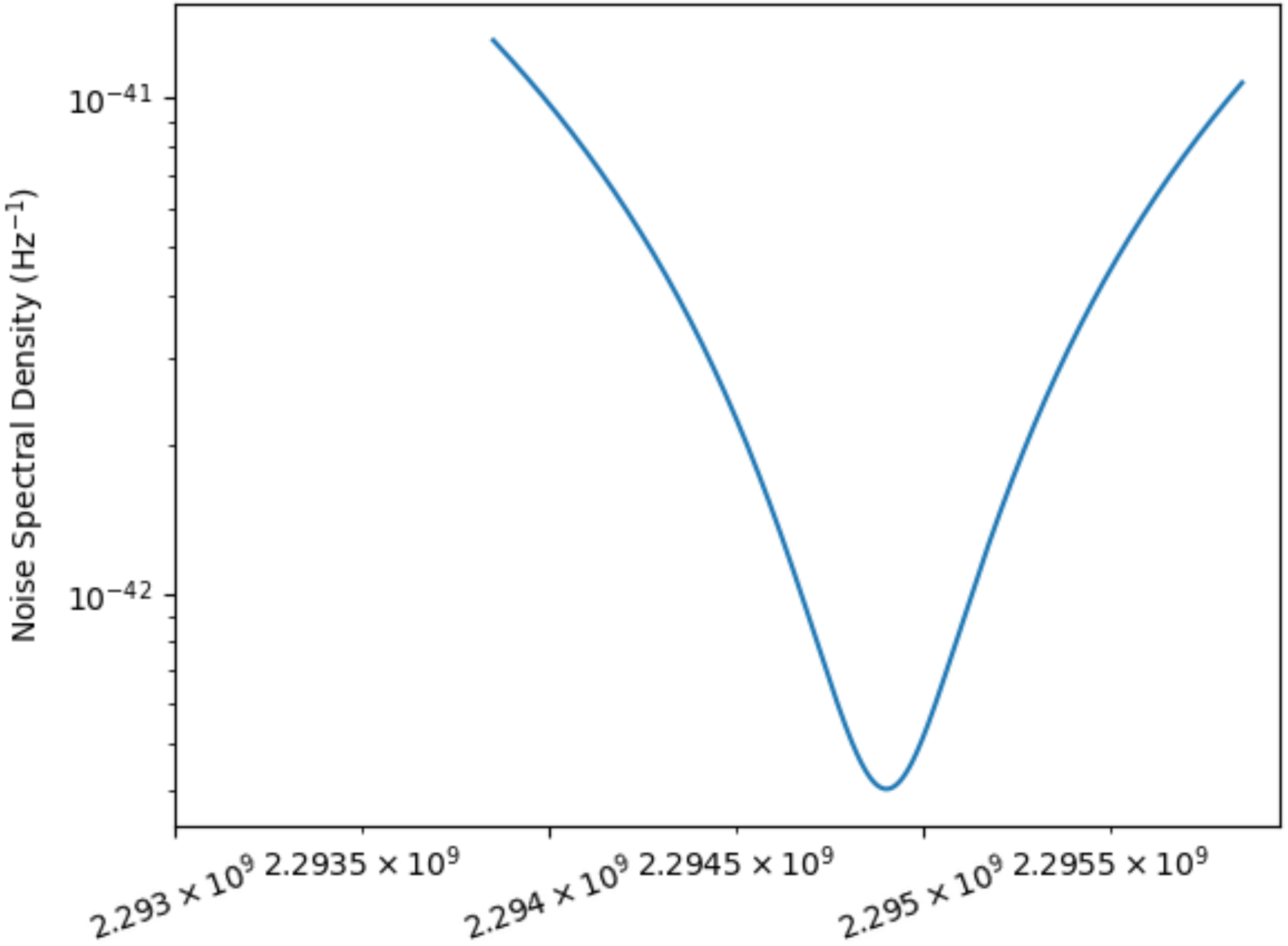
- $$= \frac{V\left|B\right|}{\omega_n^2}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\,\omega^2\left(\tilde{h}(\omega) + \tilde{n}(\omega)\right)e^{-i\omega t}$$

- Fluctuation-Dissipation Theorem
  - $S_f(\omega) = 4\dfrac{V}{\omega_n Q_n}\hbar\omega\left(\dfrac{1}{2} - \dfrac{1}{e^{\hbar\omega/kT} - 1}\right)$
  - $S_n(\omega) = 4\dfrac{\omega_n^3}{VB^2Q_n\omega^4}\hbar\omega\left(\dfrac{1}{2} - \dfrac{1}{e^{\hbar\omega/kT} - 1}\right)$

- Total Noise
  - $S_n(\omega) = 4\dfrac{\omega_n^3}{VB^2Q_n\omega^4}\left[\hbar\omega\left(\dfrac{1}{2} - \dfrac{1}{e^{\hbar\omega/kT} - 1}\right) + kT_{\mathrm{add}}\right]$

Example: TM010 mode of Cylindrical Cavity

- $R \sim 5\,\mathrm{cm}$
- $R/L \sim 1$
- $V = \pi R^2 L \sim 3.1\,\mathrm{L}$
- $B \sim 8\,\mathrm{T}$
- $Q_{010} \sim 10^5$
- $f_{010} = \frac{1}{2\pi} j_{0,1} c/R \sim 2.29\,\mathrm{GHz}$
- $T_{\mathrm{cav}} \sim 60\,\mathrm{mK}$
- $T_{\mathrm{add}} \sim 150\,\mathrm{mK}$

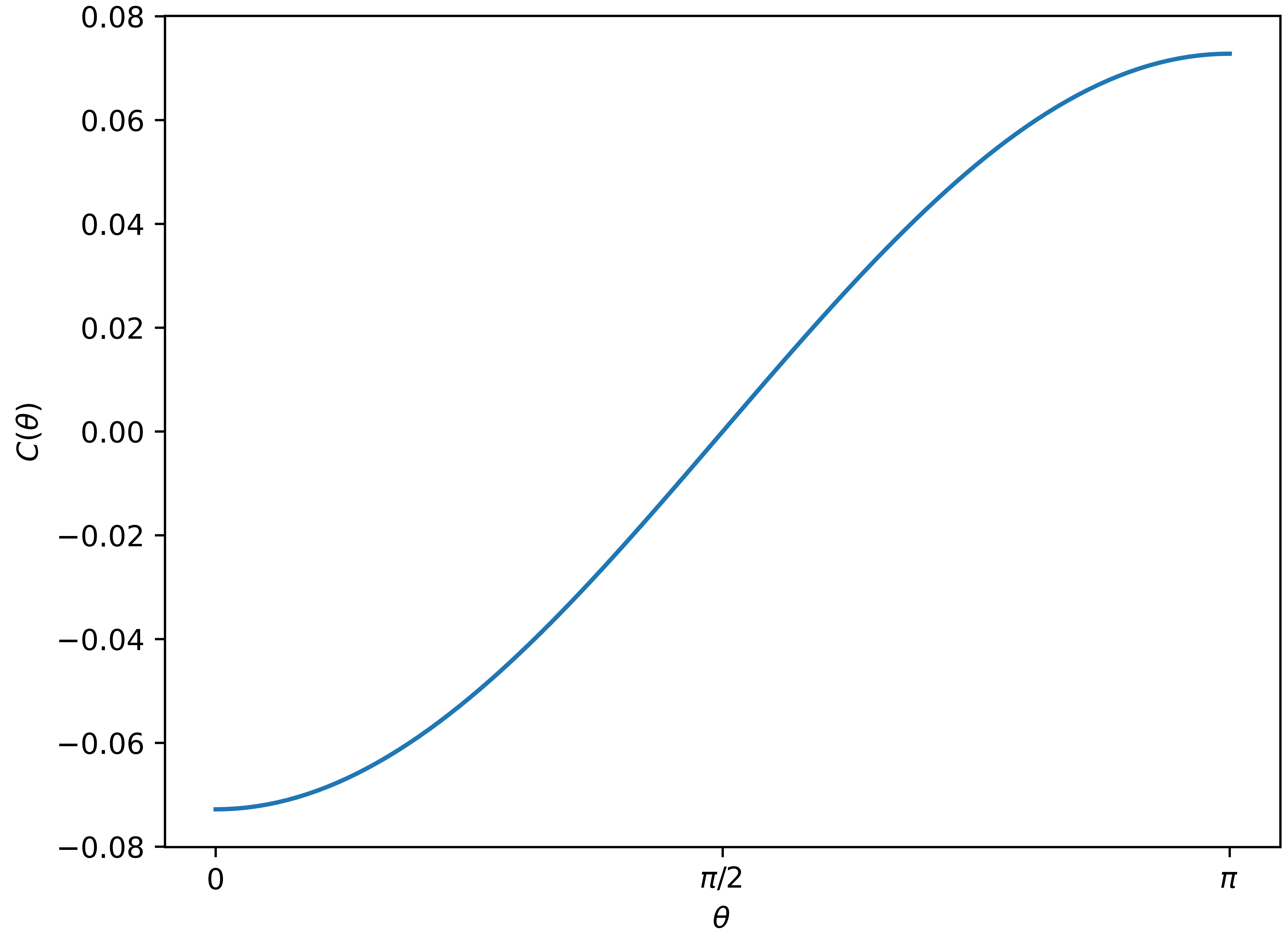


2-Detector Correlation

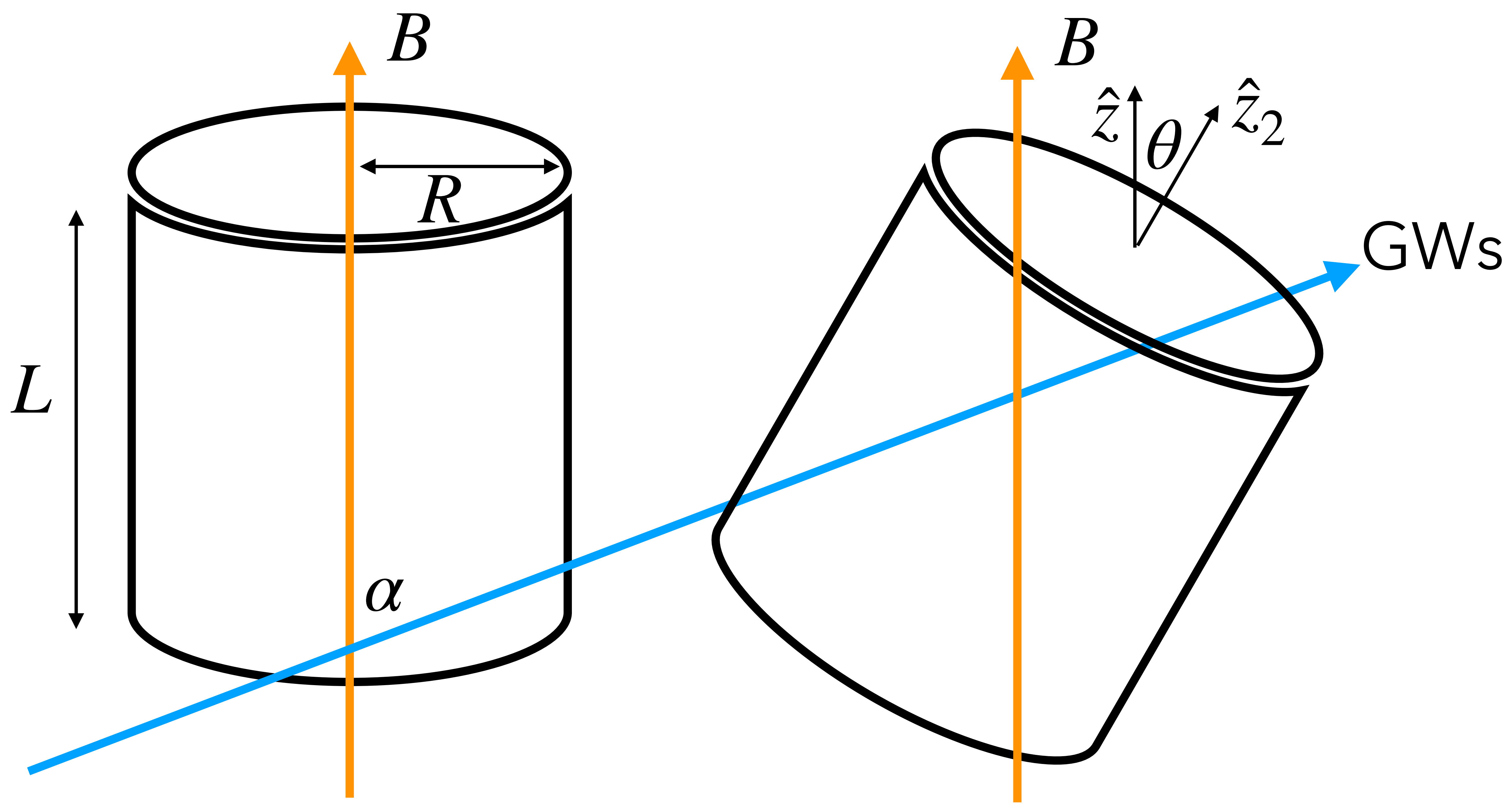
- Maximal Signal to Noise Ratio for 2-Detector Correlation
- $\frac{S}{N} = \sqrt{T}\left[\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\left|\tilde{\Gamma}(\omega)\right|^2\frac{S_h^2(\omega)}{S_{n,1}(\omega)\,S_{n,2}(\omega)}\right]^{1/2}$
- T: observation time
- $S_{n,i}(\omega)$: noise spectral density for i-th detector
- $\tilde{\Gamma}(\omega)$: overlap reduction that depends on a configuration of cavities.

Example: Two Identical Cavities with TM010

- $\tilde{\Gamma}(\omega) = C(\theta)$



Ahn-Im-Park Curve



Discussions

- Can EM cavities be utilized to detect GWs?
  - Yes. I have reviewed intensively.
- Is there any source of GWs that can be observed by the EM cavity detector?
  - Maybe there is. Earth mass PBH?
- Is there any possible cavity to apply our concepts?
  - We can propose our idea to Center for Axion and Precision Physics Research (CAPP) in IBS. One of our colleagues is associated with CAPP-IBS and is experts on EM cavity experiment.
- Is the sensitivity of detector enough to detect GWs?
  - We need more analysis.
- Is there data analysis method for our detector concept?
  - We have developed a correlation method for the detector.
- Thank you for listening 😄