Searching for High Frequency GW with Axion Detectors

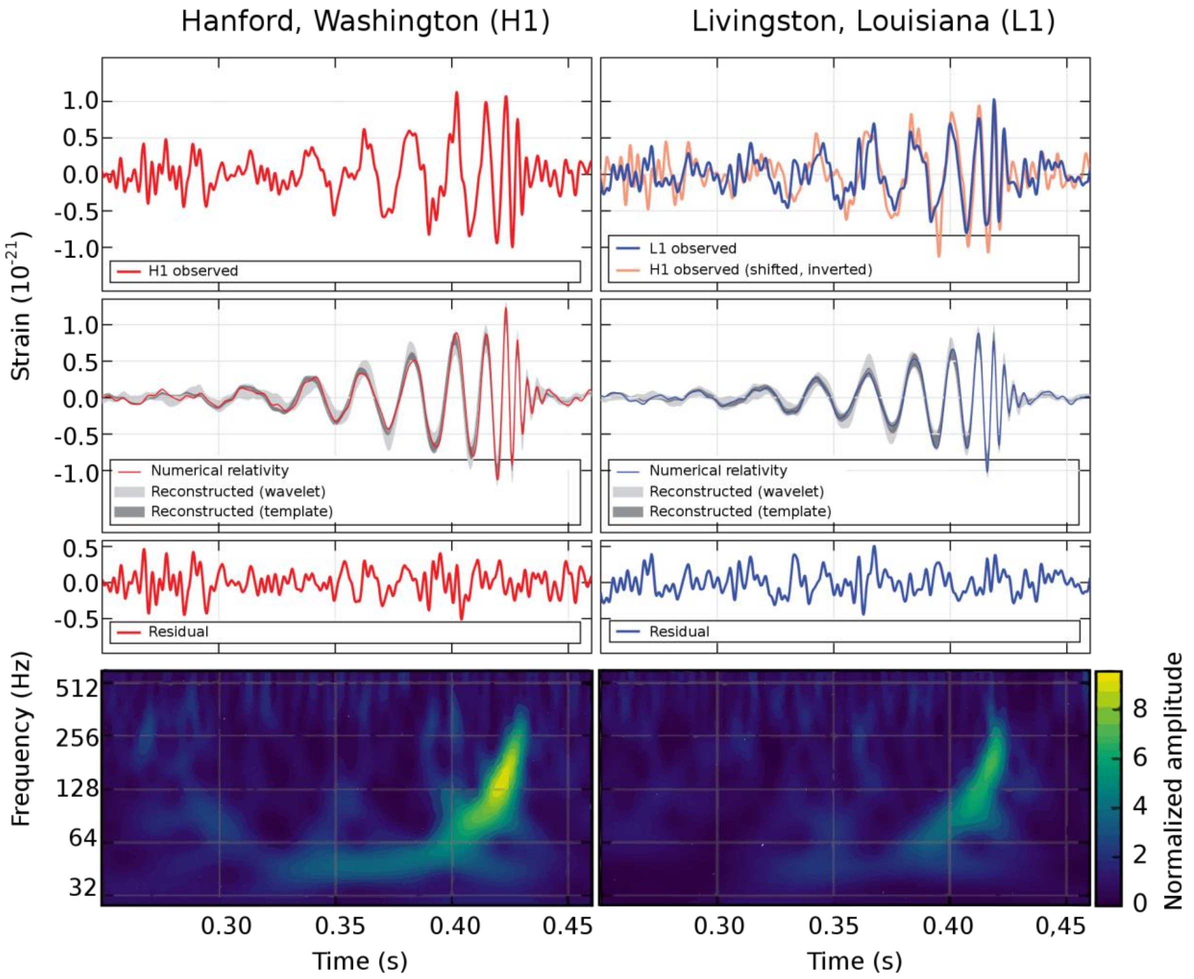
Sung Mook Lee

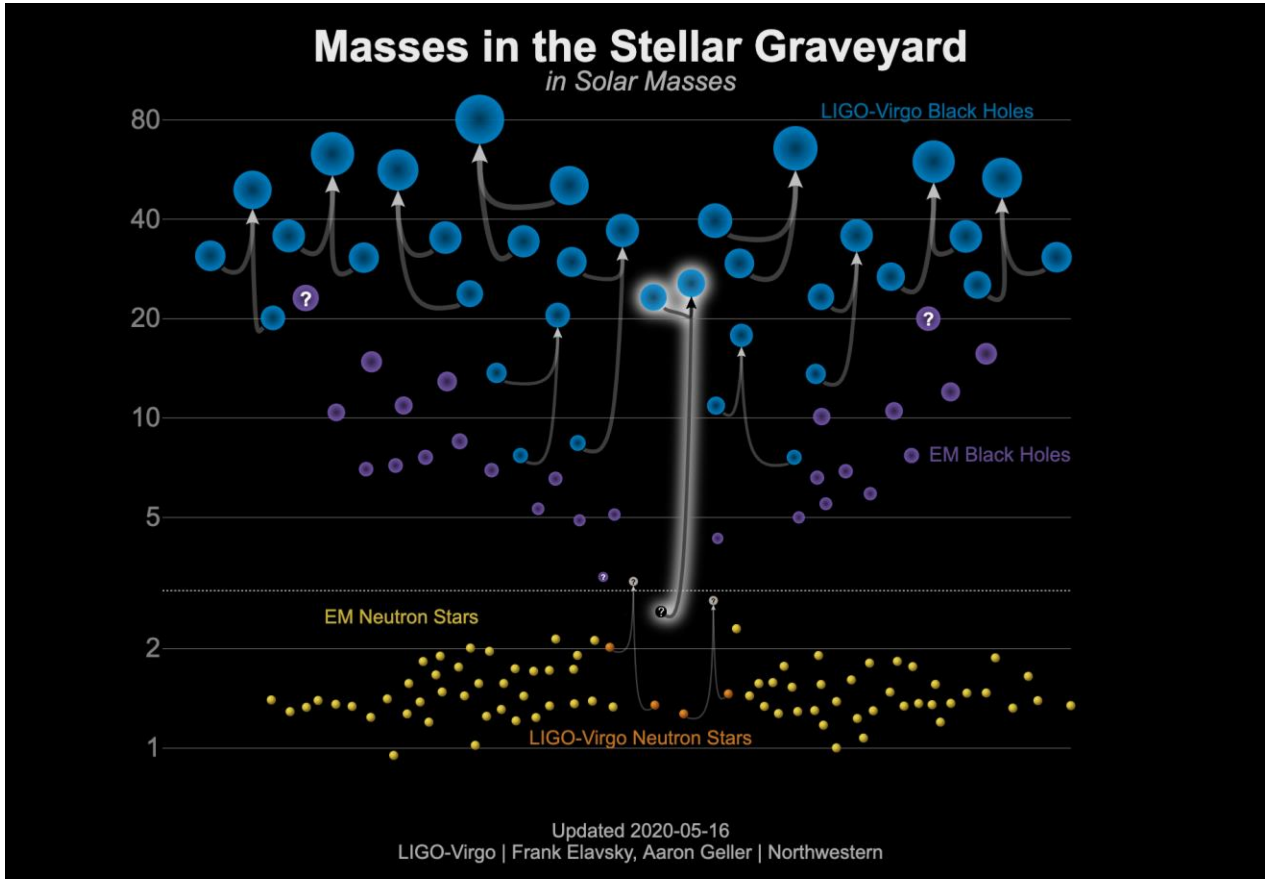
Yonsei U. → KAIST

2306.03125

with Valerie Domcke (CERN), Camilo Carcia-Cely (Valencia, IFIC), and Nicholas L. Rodd (CERN)

Detection of Gravitational Waves





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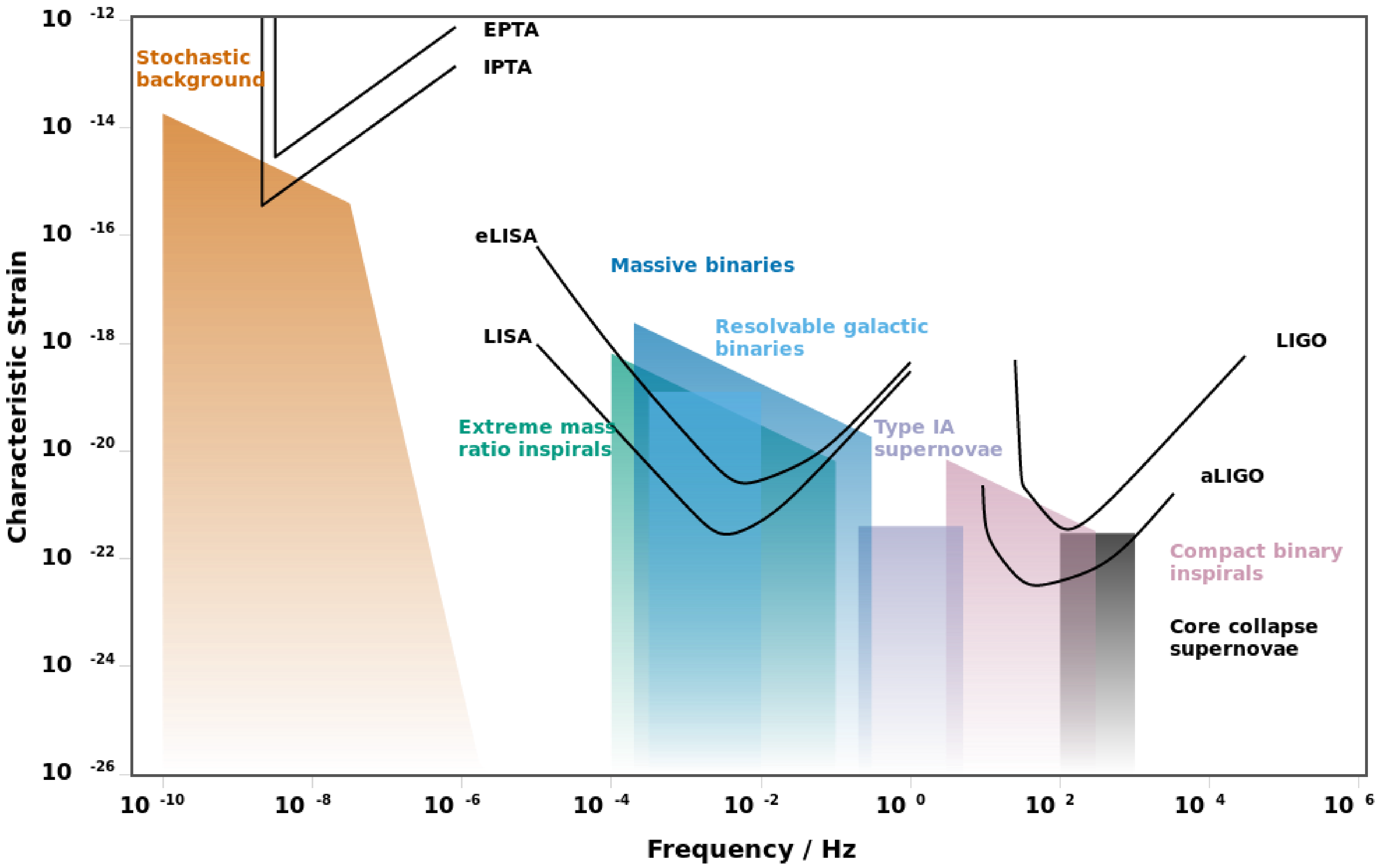
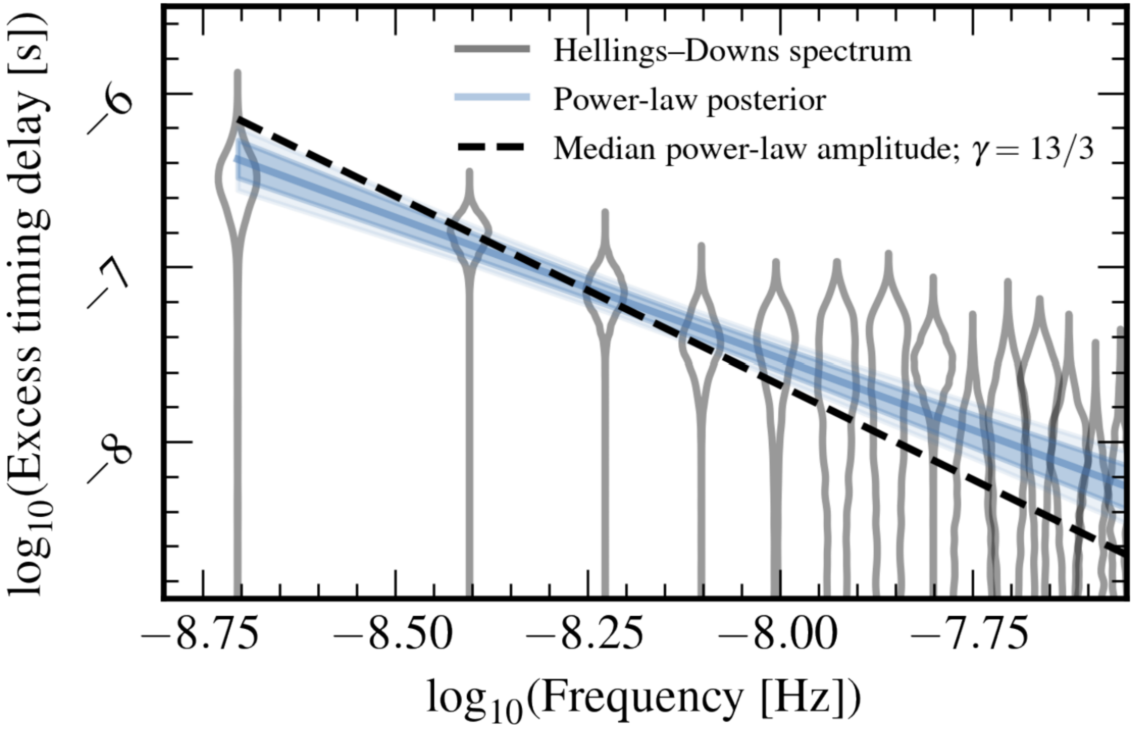
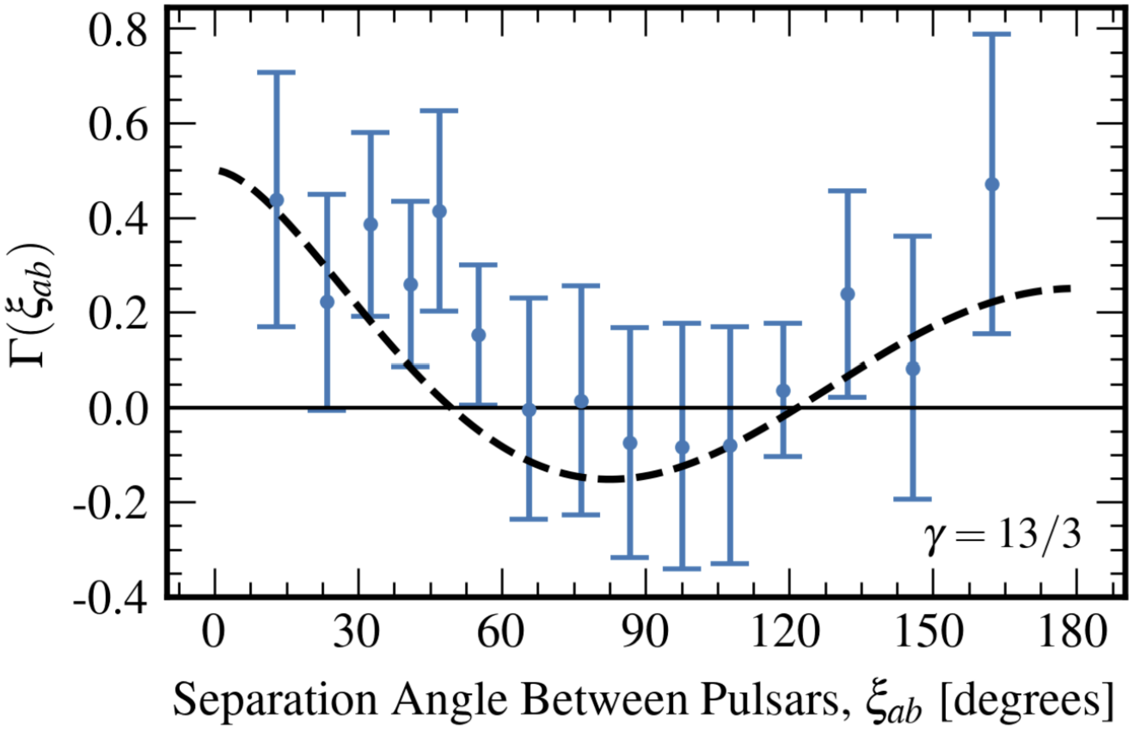


Detection of Gravitational Waves

- First Detector of Stochastic GW

[2306.16213]





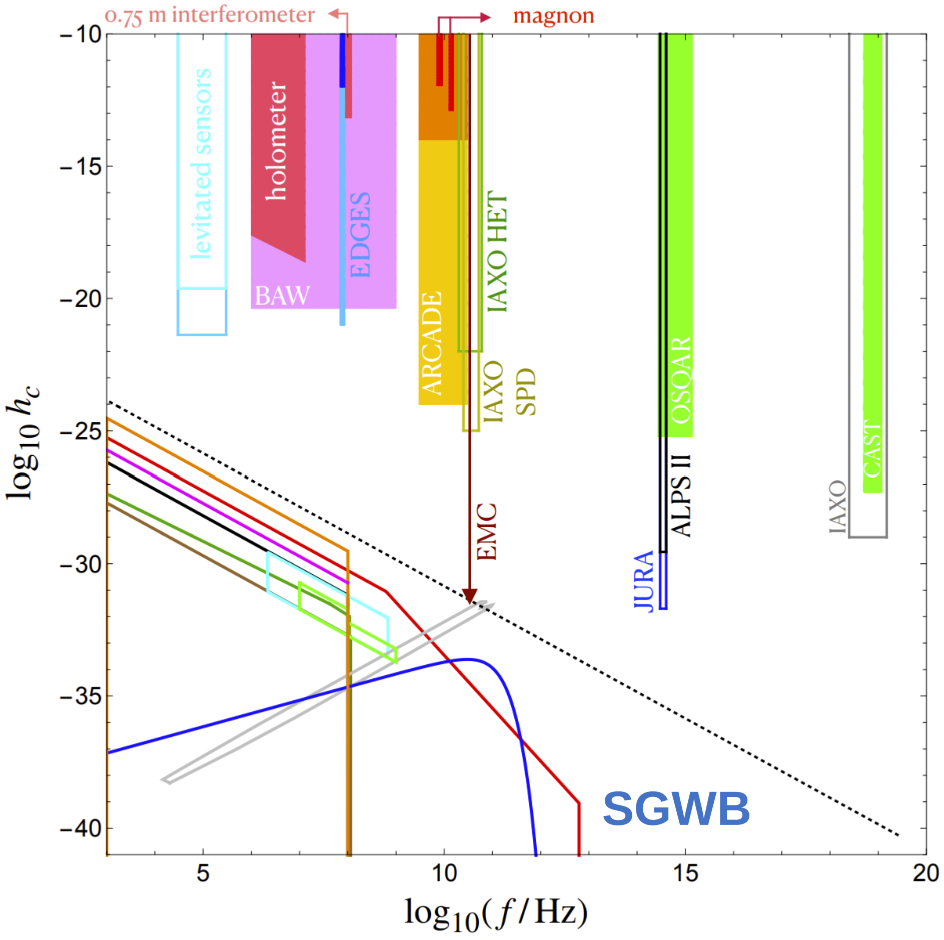
High Frequency Gravitational Wave

High Frequency Gravitational Waves

- **Stochastic Gravitational Wave Background**
  - High frequency = Early universe

$$f_{\rm GW} \gtrsim O(1)\ \mathrm{MHz} \left(\frac{T_*}{10^8\ \mathrm{GeV}}\right)$$

[2011.12414 for review]



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$$\left(\frac{\rho_{\mathrm{GW}}}{\rho_\gamma} \right) \leq \frac{7}{8} \left(\frac{4}{11} \right)^{4/3} \Delta N_{\mathrm{eff}} \lesssim 0.05$$

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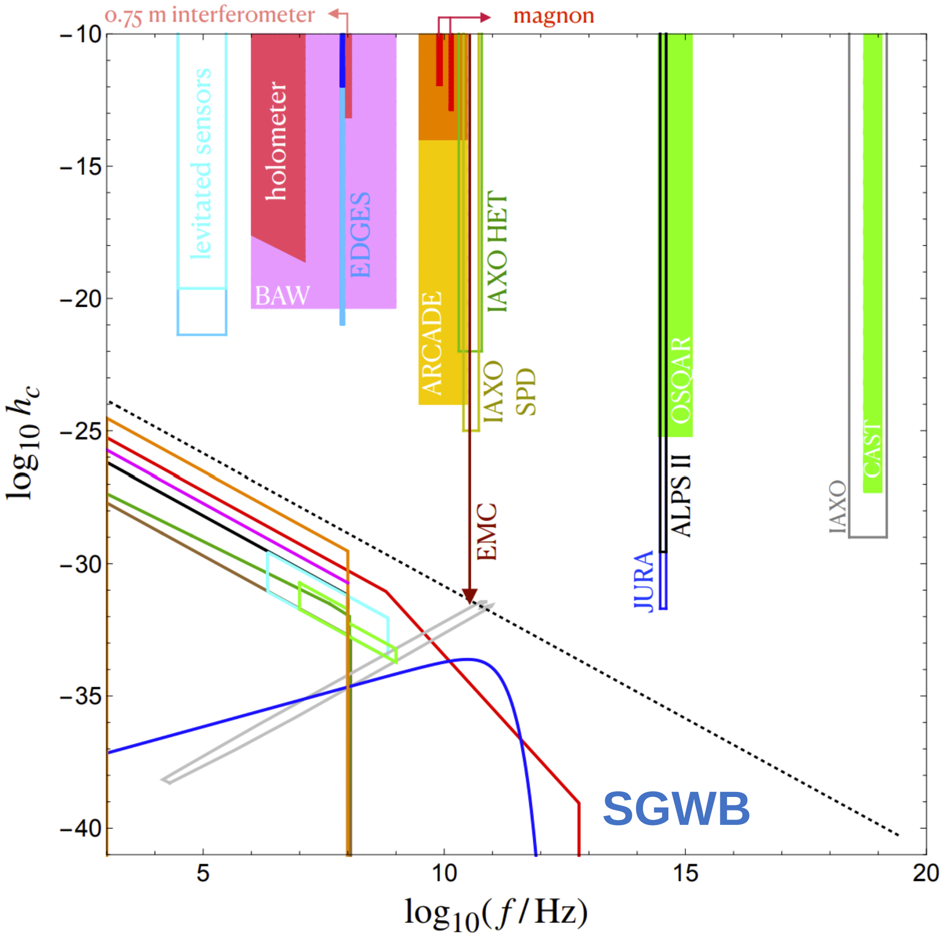

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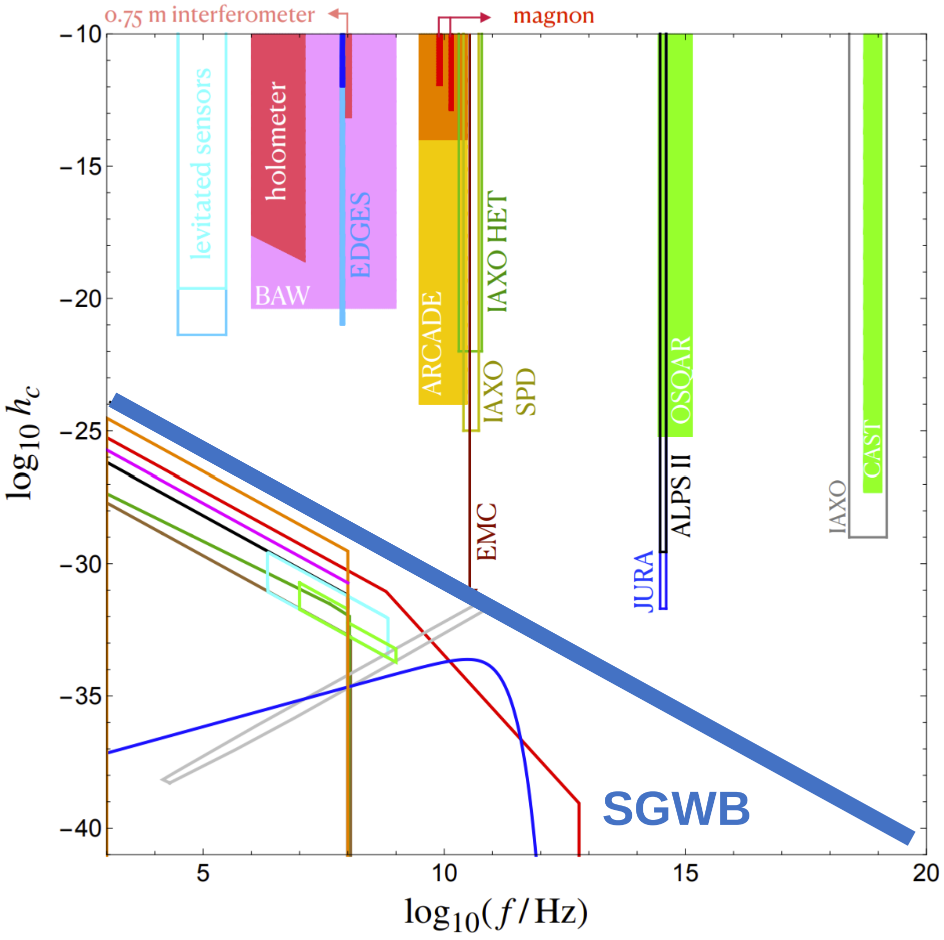


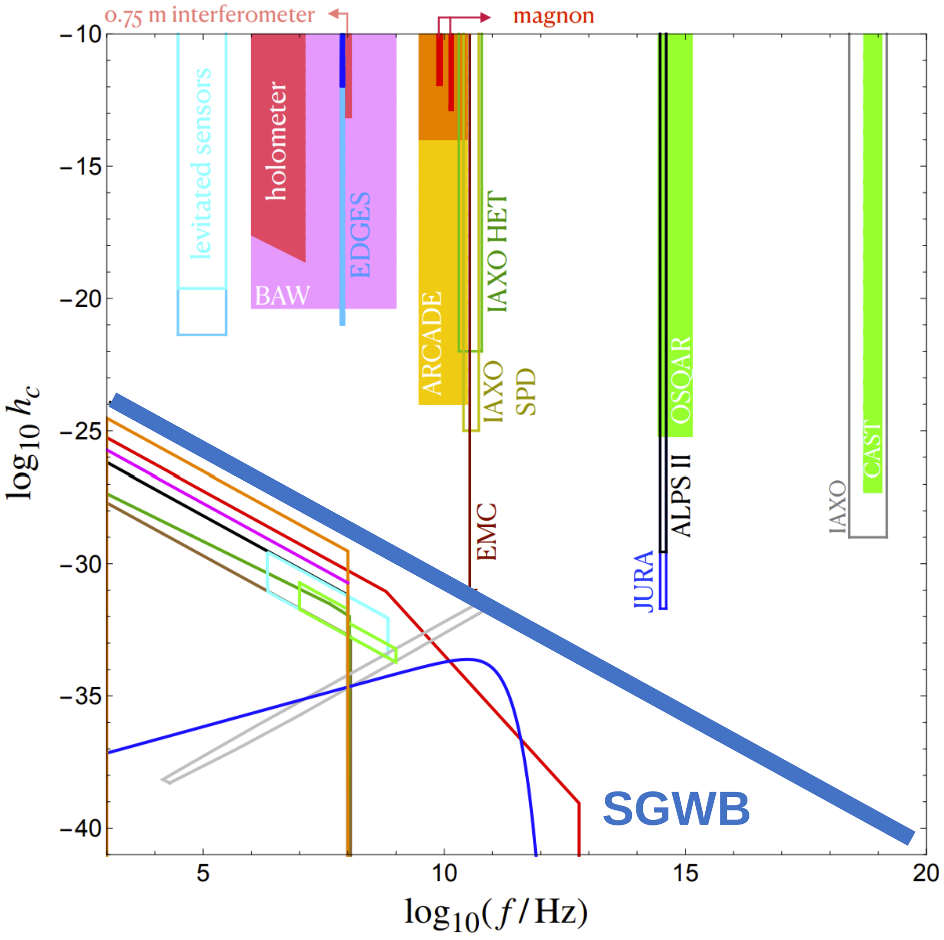
High Frequency Gravitational Waves

- **Localized Sources**
  - PBH binaries / exotic compact objects

$$f \simeq 220\ \mathrm{MHz}\left(\frac{10^{-5} M_\odot}{m_{\mathrm{PBH}}}\right)$$

  - Larger signals expected

[2011.12414 for review]



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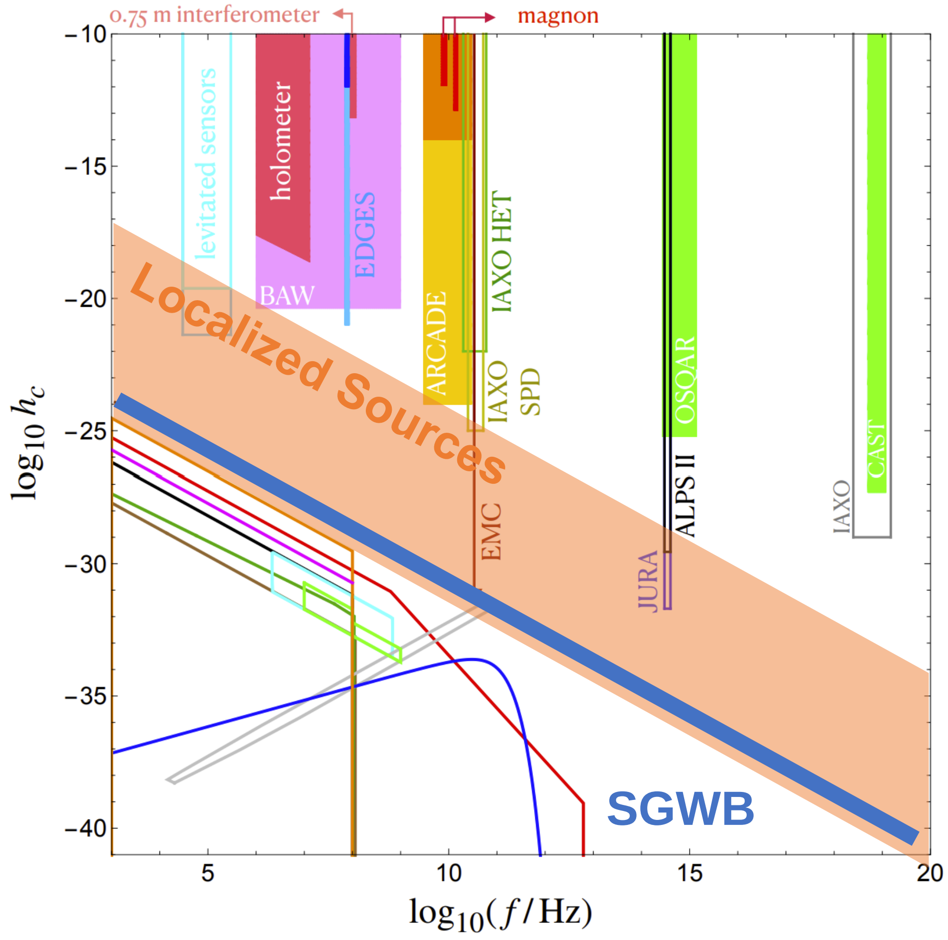

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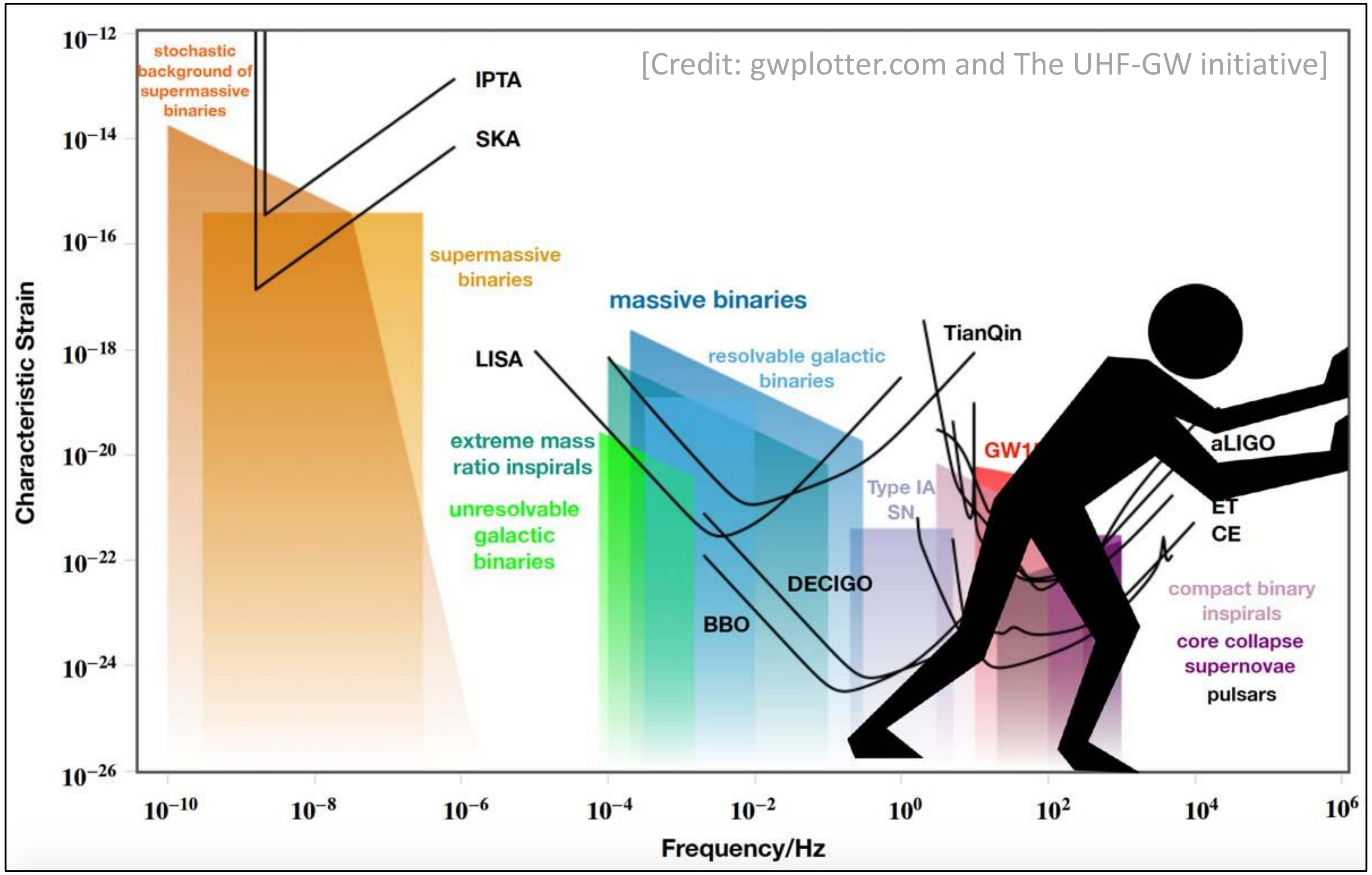
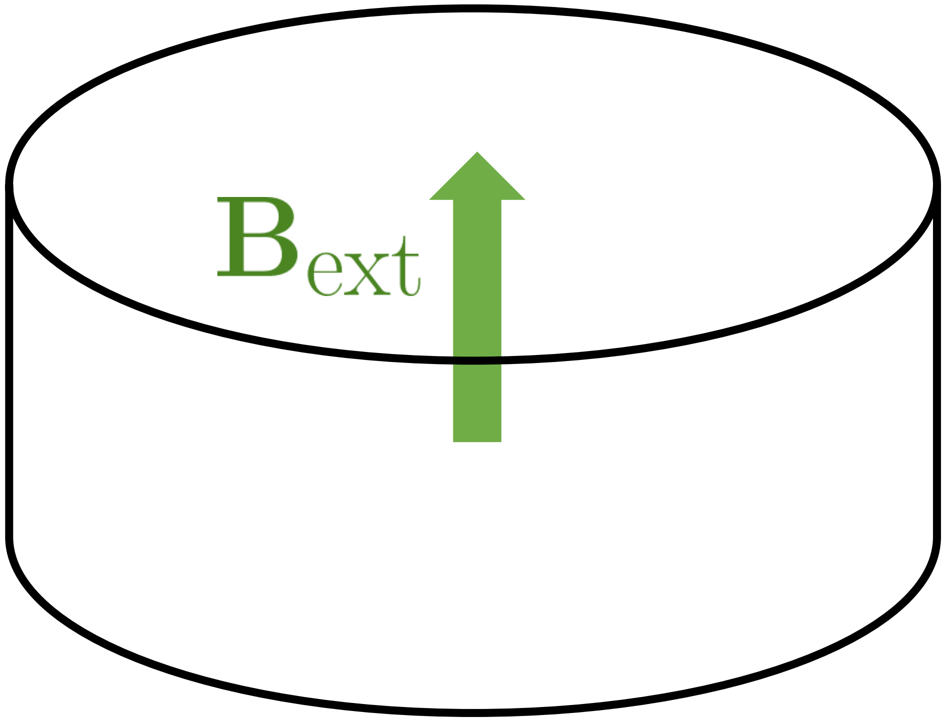


How can we detect GW with f > O(1MHz)?

Key Idea



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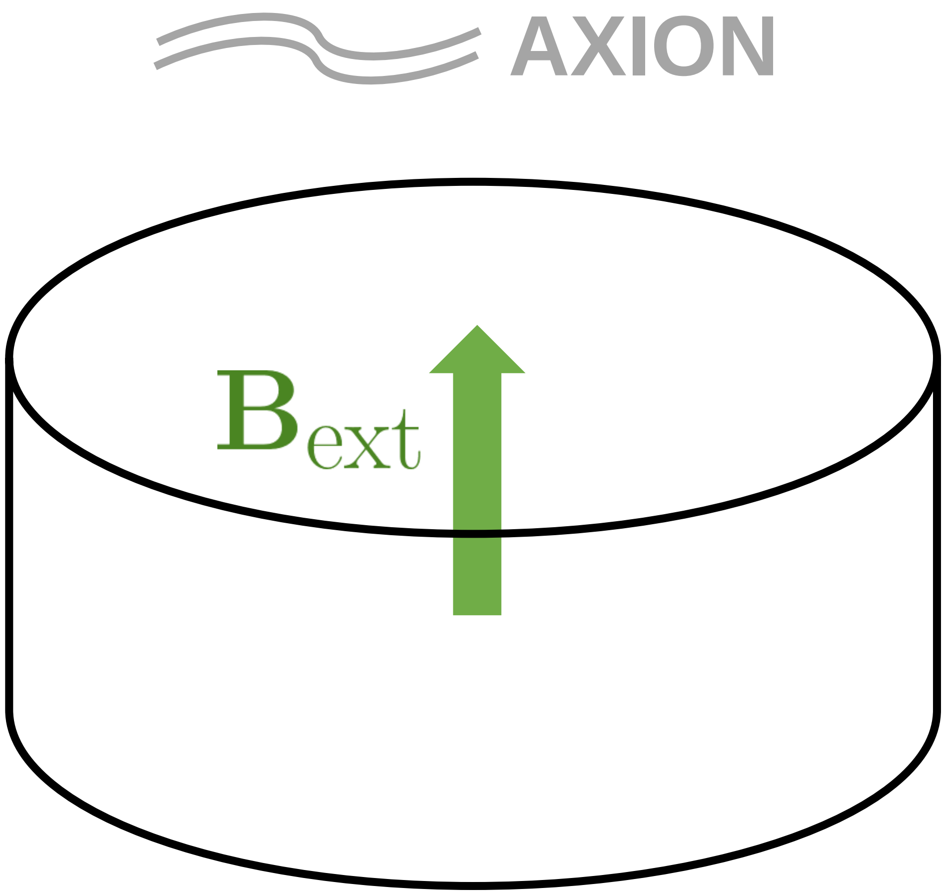

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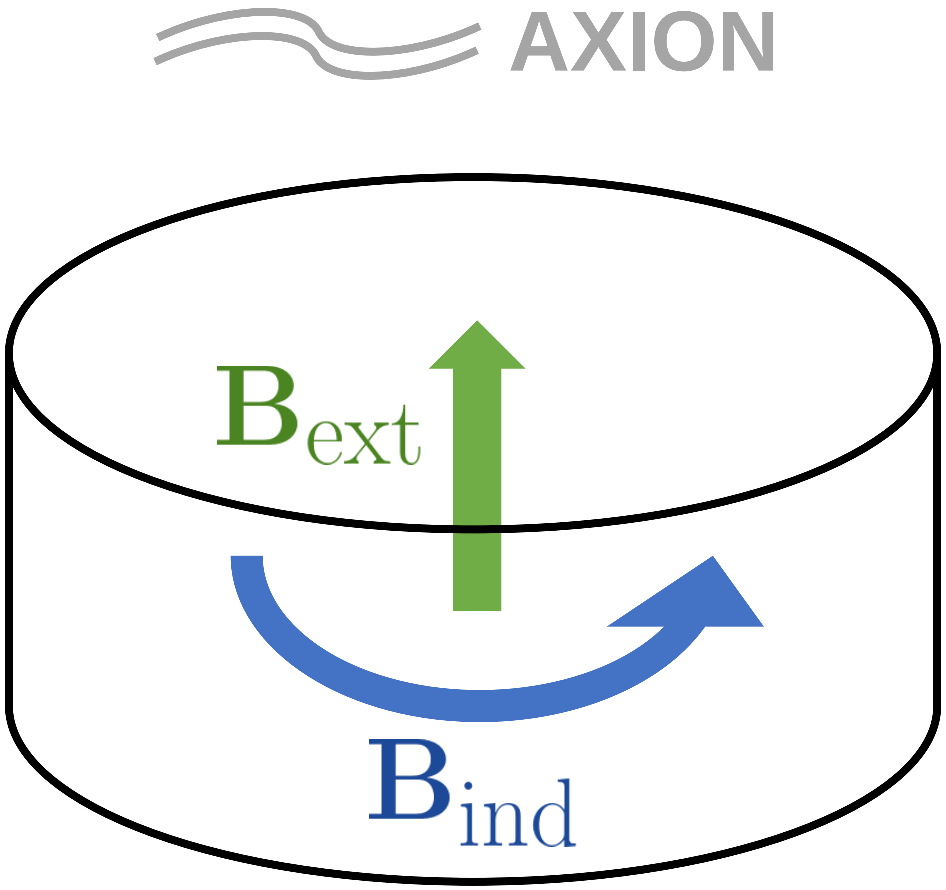

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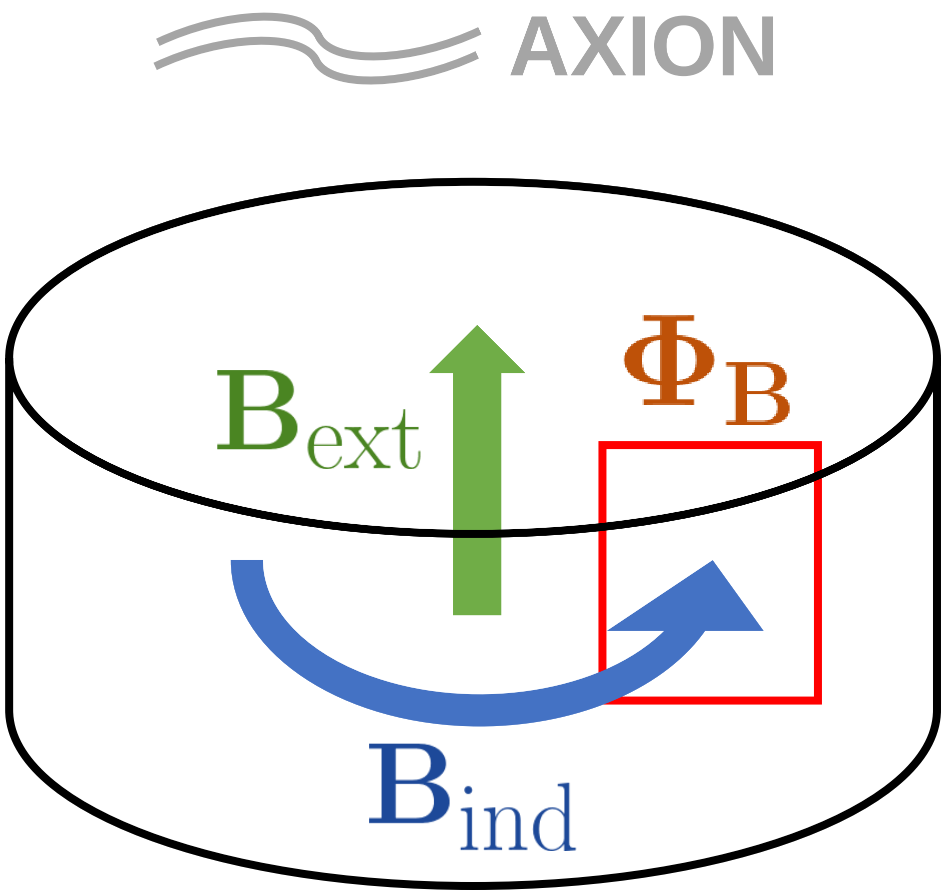
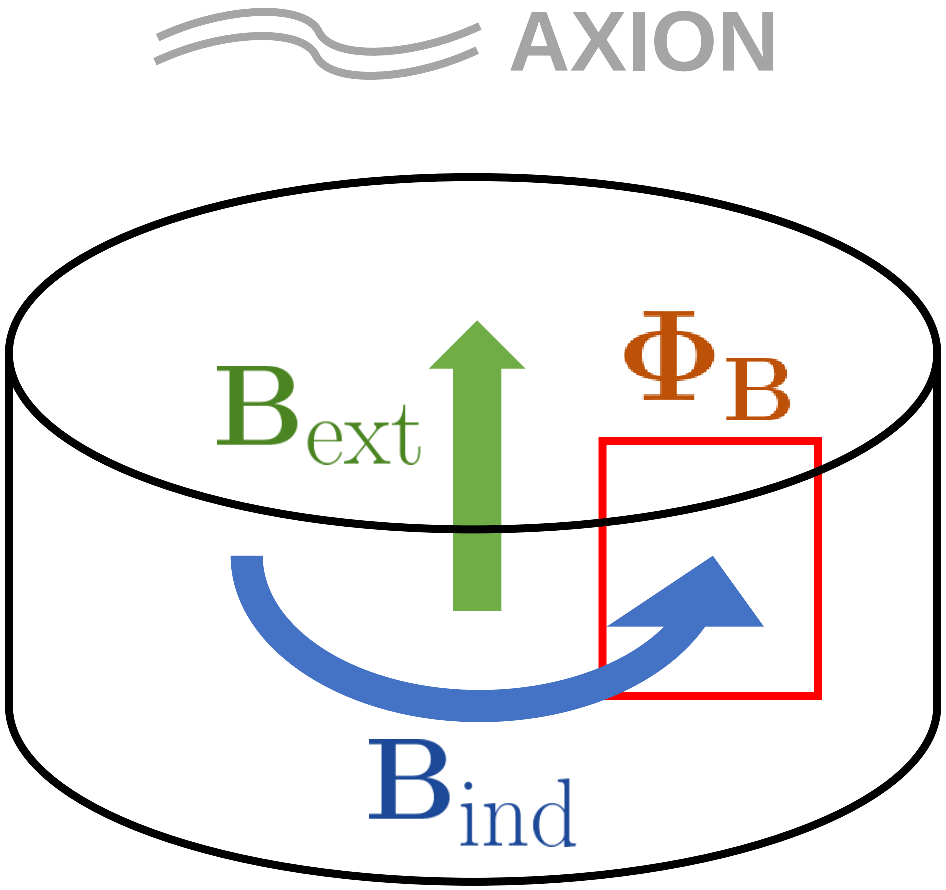


Key Idea

$$\Phi_{\text{Axion,Solenoid}} = \frac{1}{4} g_{a\gamma\gamma} \dot{a} B_0 l R^2$$



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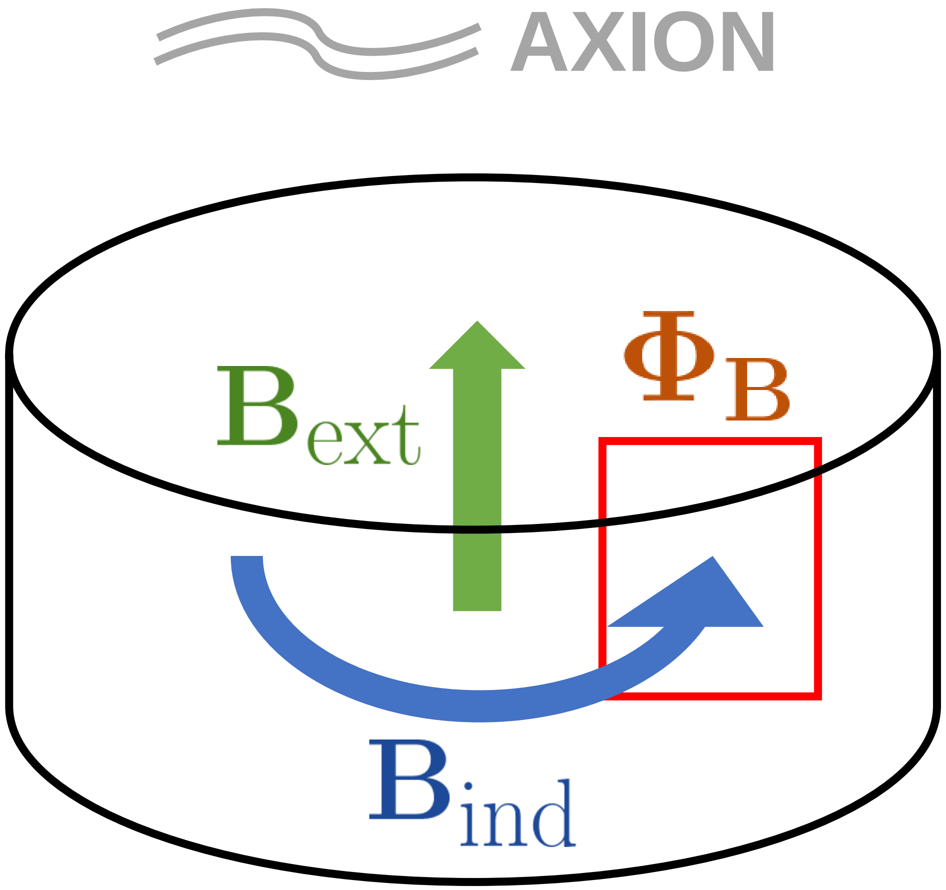

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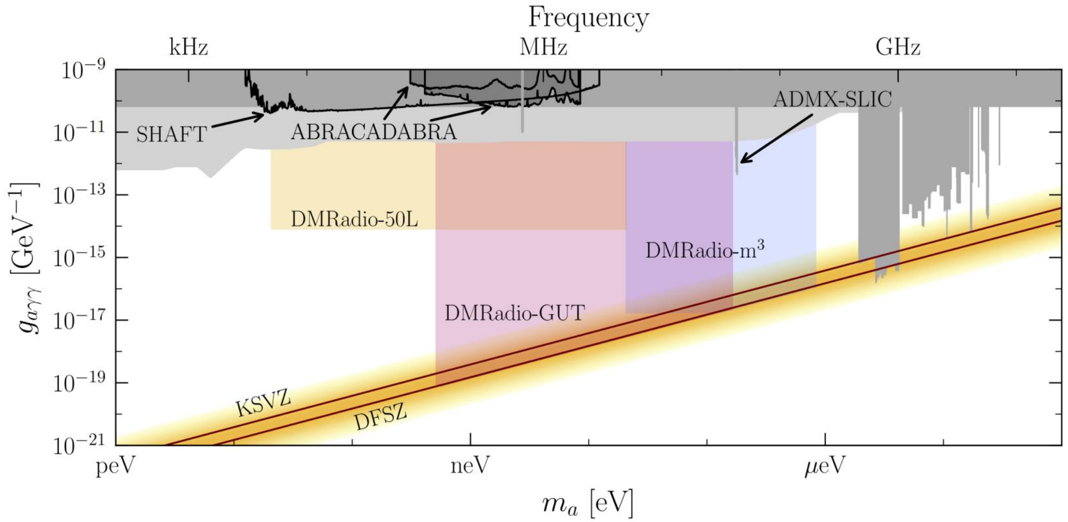

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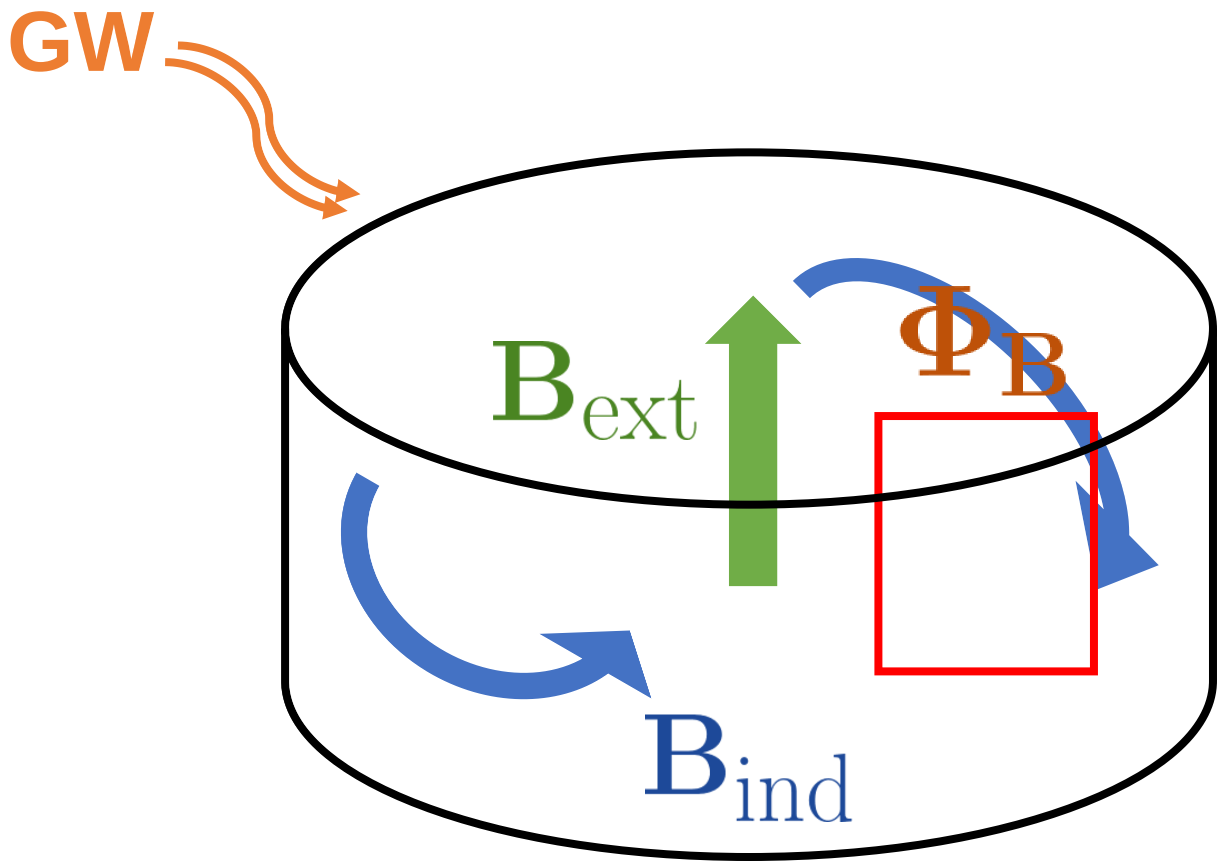

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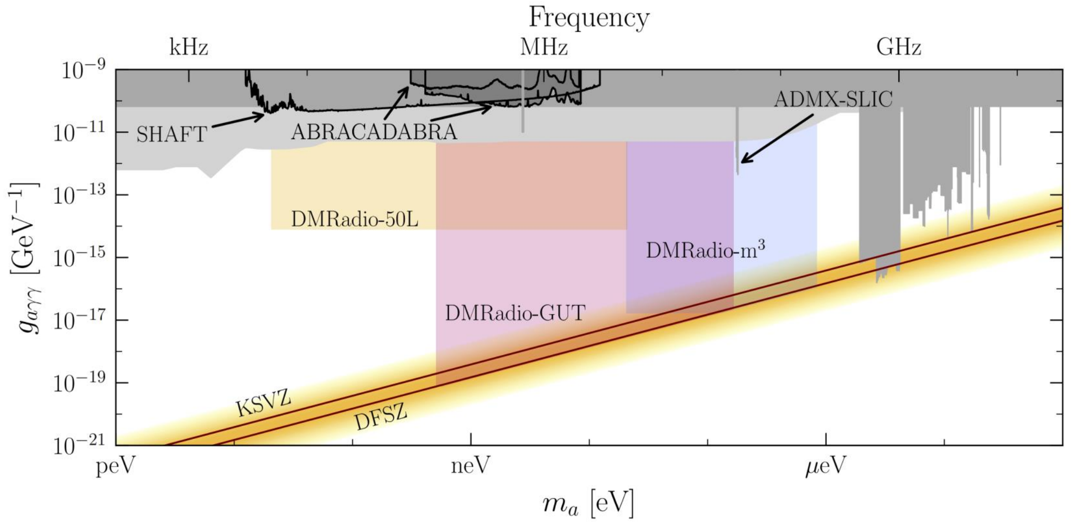

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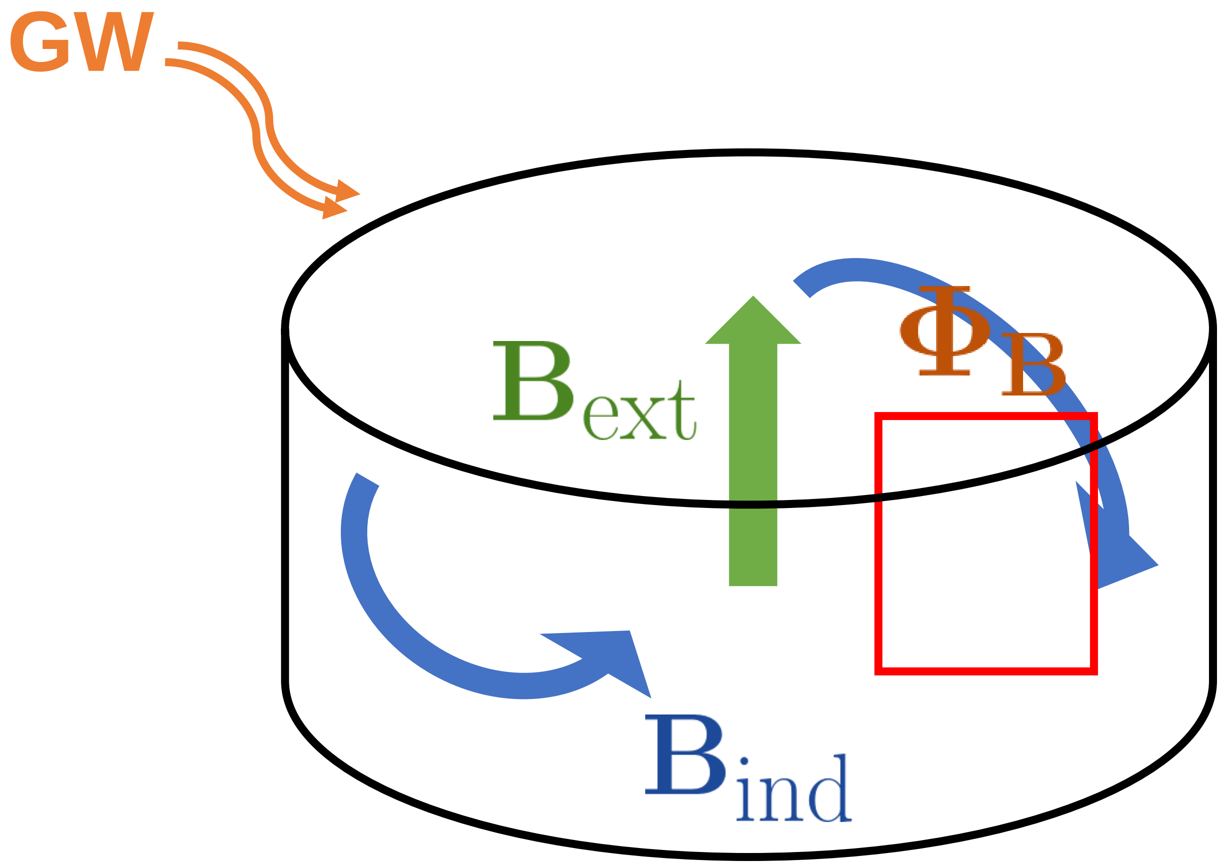
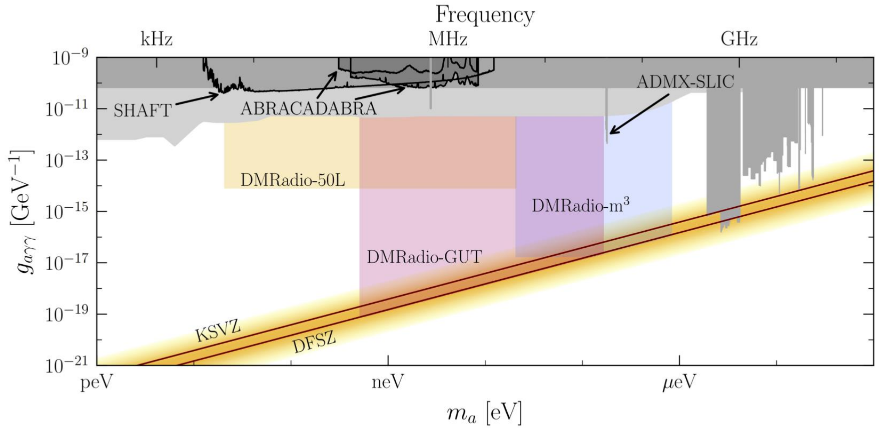
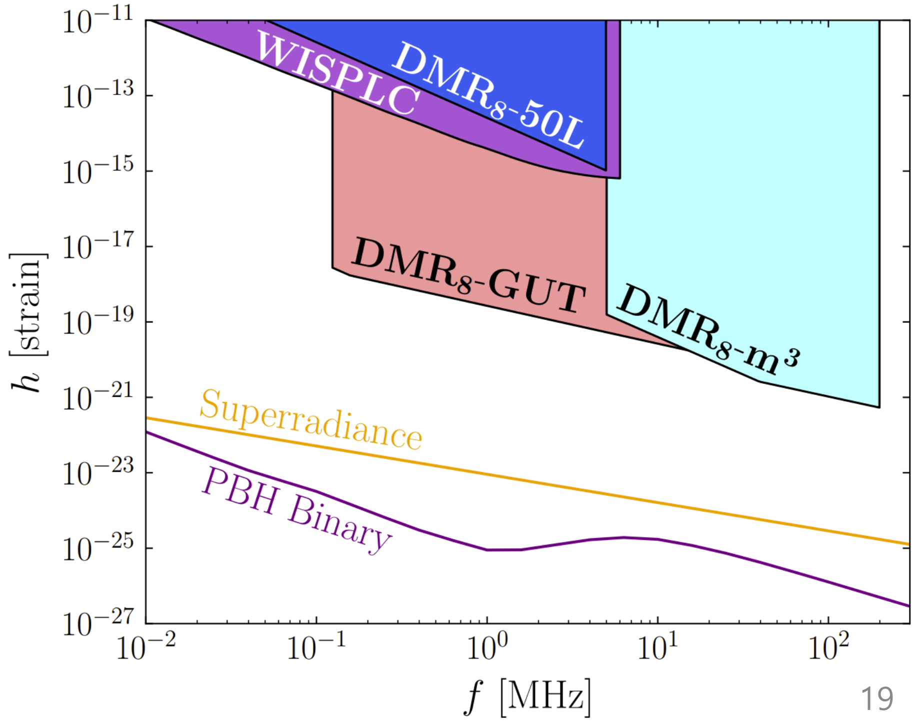
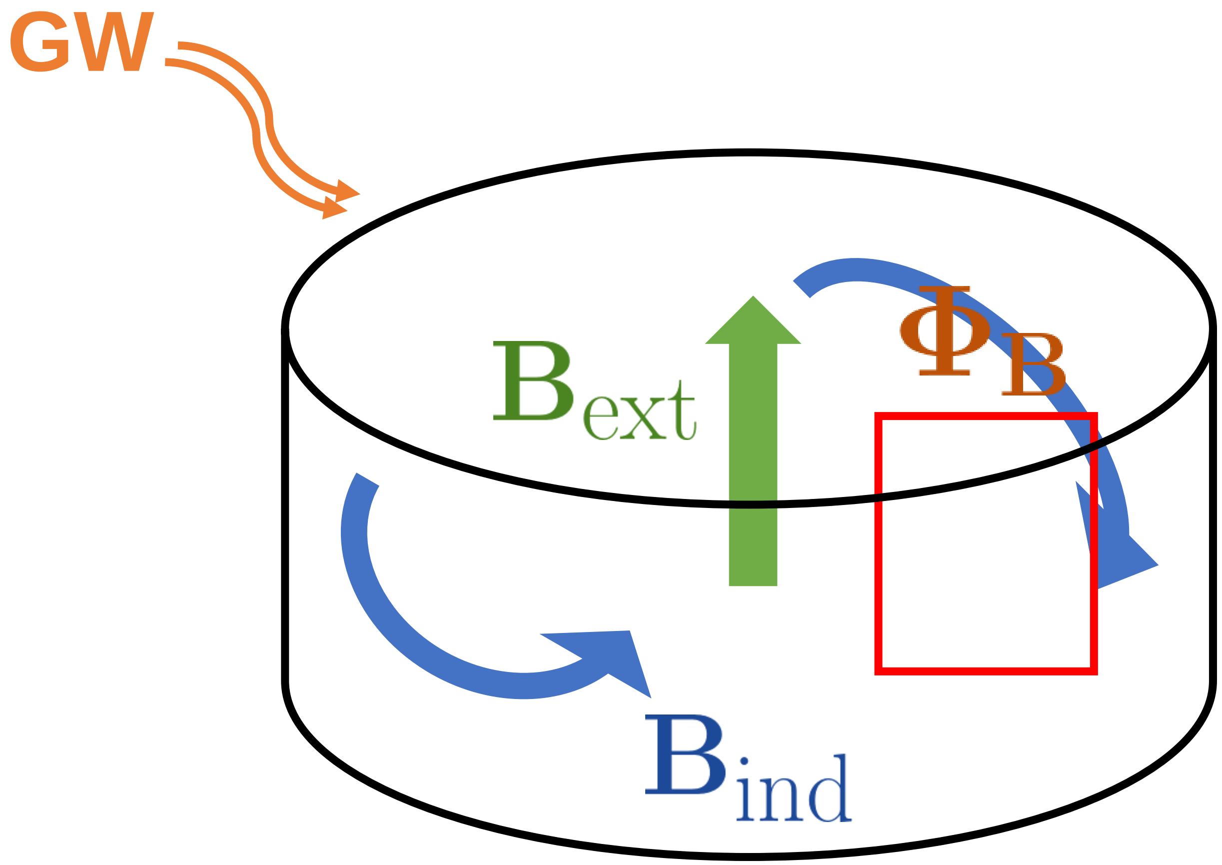

GW
Bext
ΦB
Bind

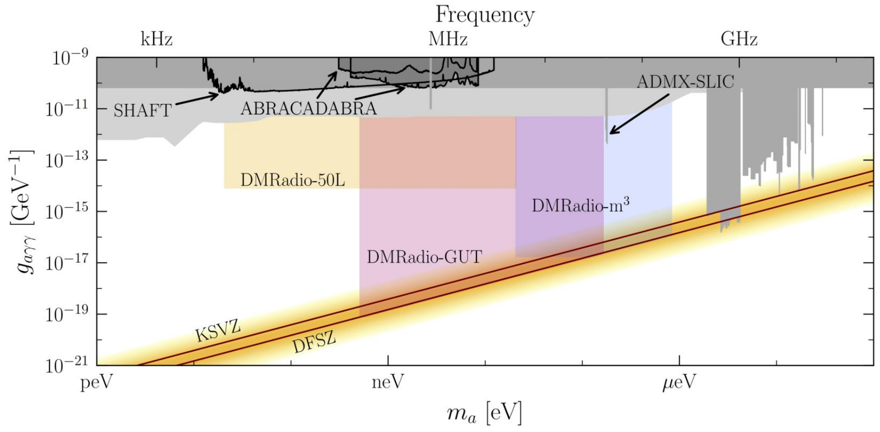

Frequency
kHz
MHz
GHz
SHAFT
ABRACADABRA
ADMX-SLIC
DMRadio-50L
DMRadio-m³
DMRadio-GUT
KSVZ
DFSZ
$g_{a\gamma\gamma}$ [GeV^{-1}]
peV
neV
μeV
m_a [eV]

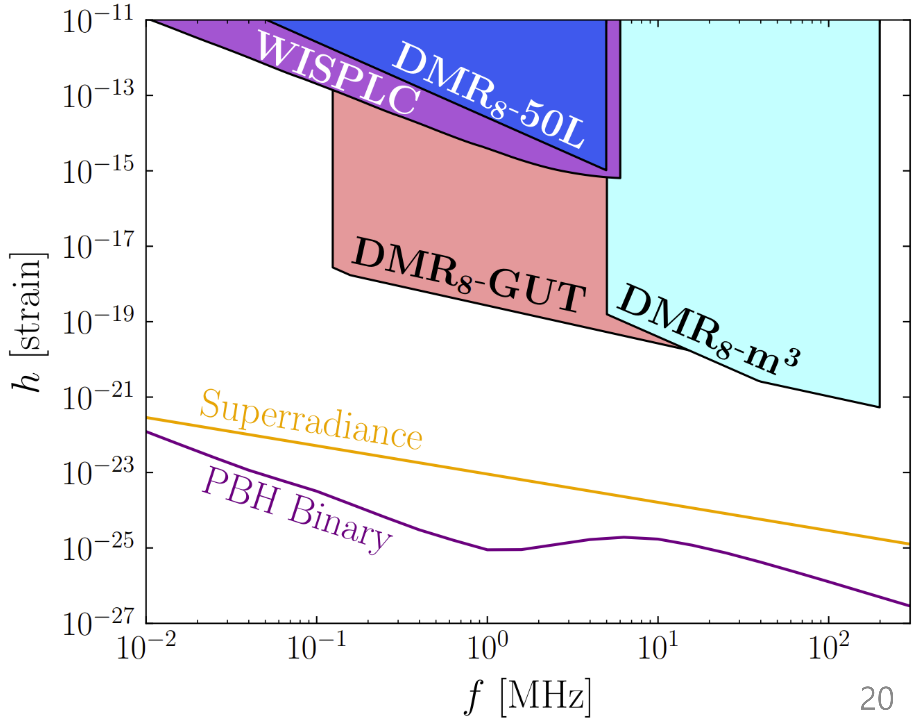

WISPLC
DMR8-50L
DMR8-GUT
DMR8-m³
Superradiance
PBH Binary
h [strain]
f [MHz]

Key Idea

You do not need a new detector!







Axion Detectors

Electromagnetism with Axion

- **Axion-Photon Coupling**

$$\mathcal{L}_{\text{int}} = -\frac{1}{4} g_{a\gamma\gamma} a F_{\mu\nu} \tilde{F}^{\mu\nu}$$

- **Effective Current**

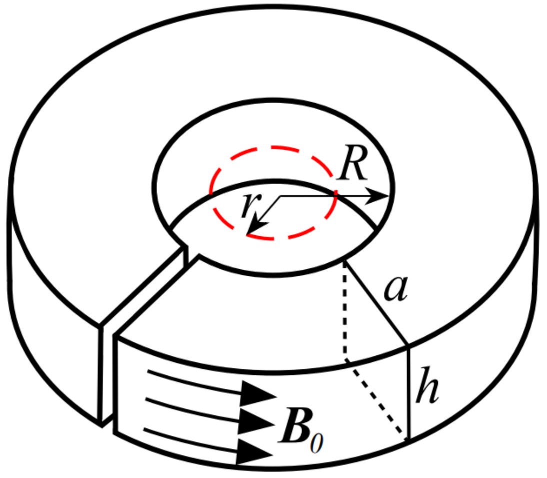
$$\partial_\nu F^{\mu\nu} = \partial_\nu \left(g_{a\gamma\gamma} a \tilde{F}^{\nu\mu} \right) = g_{a\gamma\gamma} (\partial_\nu a) \tilde{F}^{\nu\mu} \equiv j^\mu_{\text{eff}}$$

  - Axion is mainly sensitive to the magnetic field

$$\mathbf{j}_{\text{eff}} = g_{a\gamma\gamma} \sqrt{2\rho_{\text{DM}}} \cos(m_a t) \mathbf{B} \qquad \rho_{\text{DM}} \simeq 0.3 \text{ GeV}/\text{cm}^3$$

Axion Detectors

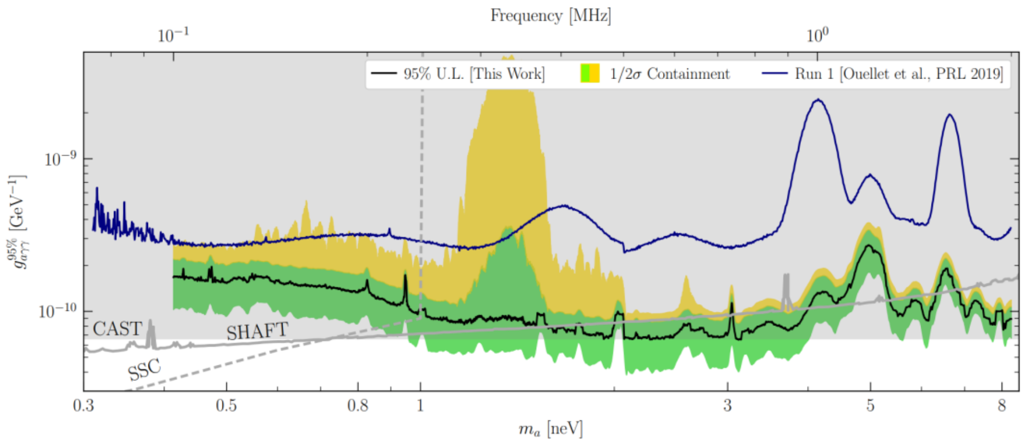
- **Axion haloscope experiments** [2203.14923 for review]



$$\Phi = e^{-i\omega t} \boxed{g_{a\gamma\gamma}} \sqrt{2\rho_{\rm DM}} B_{\rm max} \pi r^2 R \ln(1 + a/R)$$

measurement

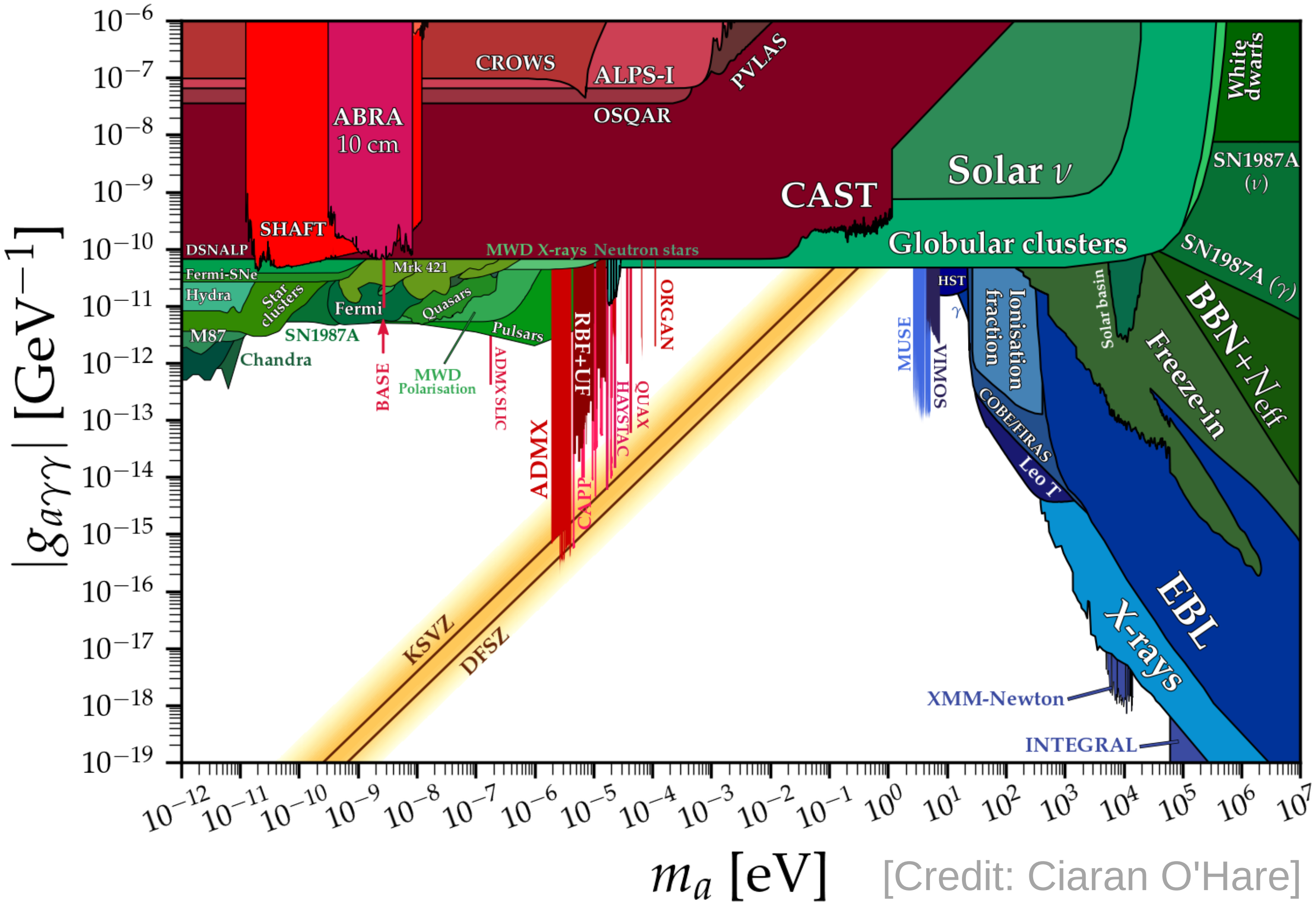
constraints



[1810.12257]

- Solenoidal: ADMX-SLIC, BASE, DMRadio-m^3
- Toroidal: ABRACADABRA, SHAFT

Axion Experiment Zoo



[Credit: Ciaran O'Hare]

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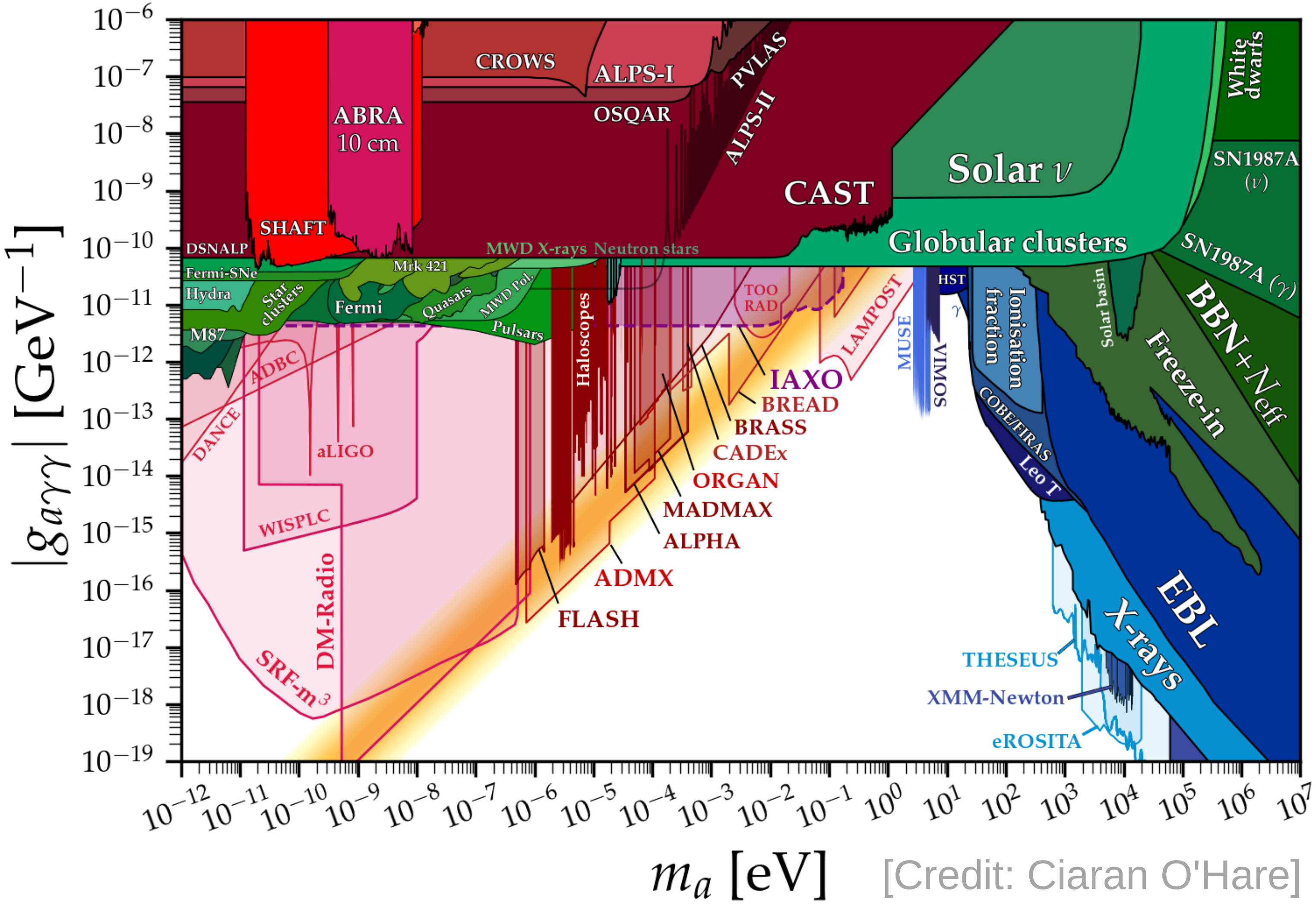
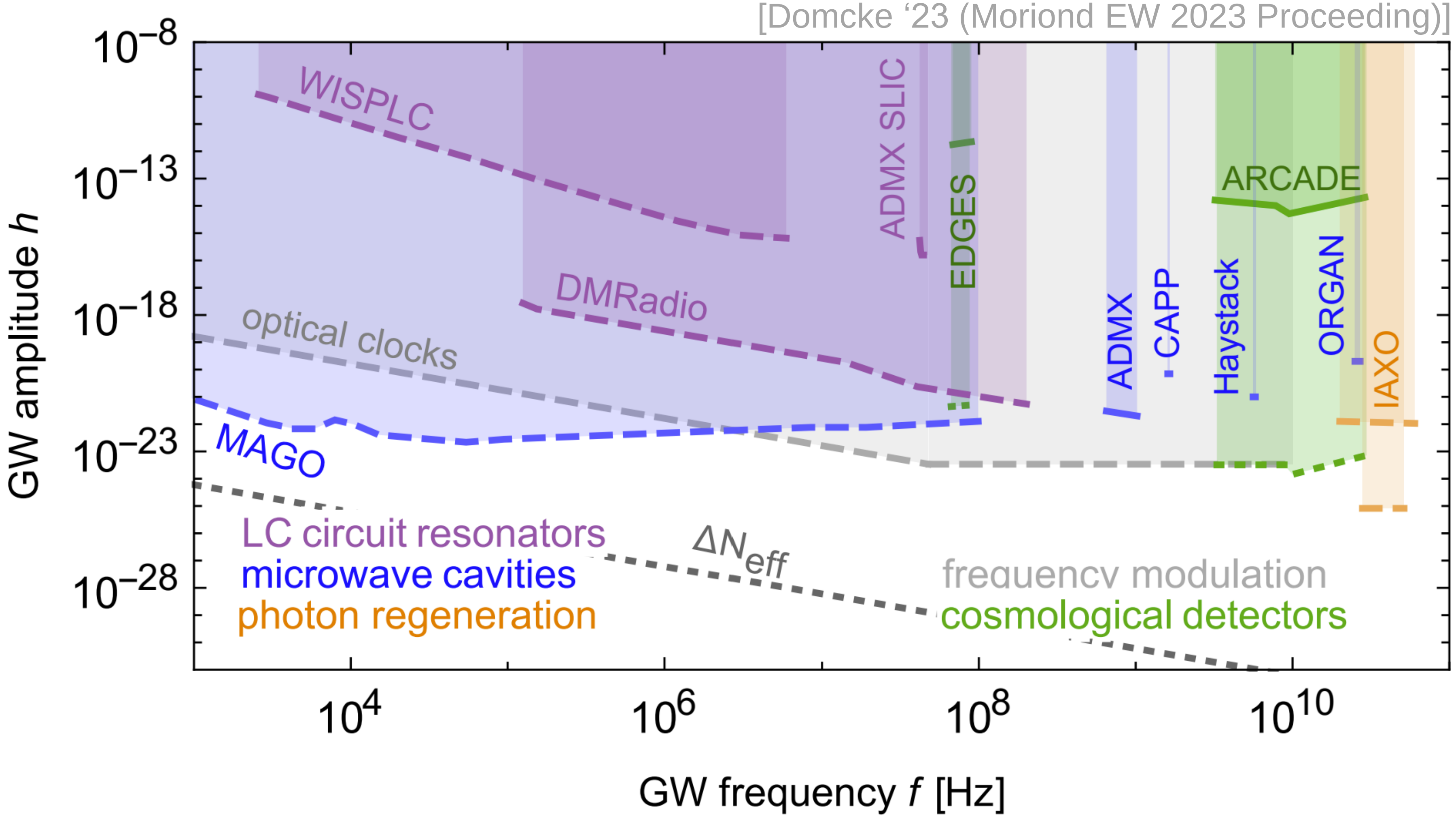


[Credit: Ciaran O'Hare]

EM-HFGW Program

[Domcke '23 (Moriond EW 2023 Proceeding)]



Detection of GW

Electromagnetism in Curved Spacetime

[Gertsenshtein ‘62]
[Boccaletti, Sabbata, Fortini, Gualdi ’70]

$$\partial_\nu F^{\mu\nu} = j^\mu$$

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$$\partial_\nu F^{\mu\nu} = j^\mu$$

$$\Big\downarrow \; \partial \to \nabla$$

$$\nabla_\nu F^{\mu\nu} = \frac{1}{\sqrt{-g}} \partial_\nu \left(\sqrt{-g} F^{\mu\nu} \right) = j^\mu$$

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$$\Big\downarrow \; \text{perturbation}$$

$$\partial_\nu F^{\mu\nu} = \left(1 + \frac{1}{2}h^\mu{}_\mu\right) j^\mu + \underbrace{\partial_\nu\left(-\frac{1}{2}h^\alpha{}_\alpha F^{\mu\nu} + F_\alpha{}^\nu h^{\alpha\mu} + F^\mu{}_\alpha h^{\alpha\nu}\right)}_{\text{‘effective current’}} + O(h^2)$$

Electromagnetism in Curved Spacetime

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[Boccaletti, Sabbata, Fortini, Gualdi '70]

$$\partial_\rho F_{\mu\nu} + \partial_\nu F_{\rho\mu} + \partial_\mu F_{\nu\rho} = 0$$

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$$= \partial_\rho F_{\mu\nu} + \partial_\nu F_{\rho\mu} + \partial_\mu F_{\nu\rho}$$

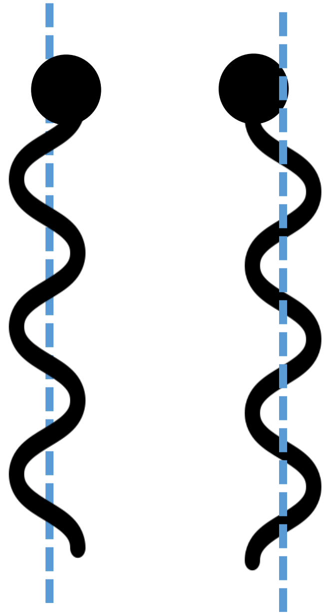
Note on Frame

- **TT frame** $h_{ij}^{TT} = (h^+ e_{ij}^+(\phi_h, \theta_h) + h^\times e_{ij}^\times(\phi_h, \theta_h))e^{i(\mathbf{k}\cdot\mathbf{r}-\omega\mathbf{t})}$

  - Coordinates fixed by geodesic of freely falling test masses

  - GW takes the simple form $h_{0\mu} = 0, h_i^i = 0, \partial_j h^{ij} = 0$

  - Detector (rigid body) looks oscillating in presence of GWs
    → makes the experimental setup & observables obscure



Note on Frame

[Berlin, Blas, Tito D'Agnolo, Ellis, Harnik, Kahn, Schutte-Engel '21]
[Domcke, Carcia-Cely, Rodd '22]

- **Proper detector frame**
  - Coordinates fixed by laboratory frame
  - More involved form

$$h_{00} = \omega^2 e^{-\mathrm{i}\omega t} F(\mathbf{k}\cdot\mathbf{r})\, r_m r_n \sum_{A=+,\times} h^A e^A_{mn}(\hat{\mathbf{k}}), \qquad F(\xi) = (e^{i\xi} - 1 - i\xi)/\xi^2$$

$$h_{0i} = \frac{1}{2}\omega^2 e^{-\mathrm{i}\omega t}[F(\mathbf{k}\cdot\mathbf{r}) - \mathrm{i}F'(\mathbf{k}\cdot\mathbf{r})][\hat{\mathbf{k}}\cdot\mathbf{r}\; r_m\delta_{ni} - r_m r_n \hat{k}_i] \sum_{A=+,\times} h^A e^A_{mn}(\hat{\mathbf{k}}),$$

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  - Description of the experimental setup and observables is straightforward

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Leading order : $O(\omega^2 L^2)$

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Axion Detectors as GW Detectors

- **Strategy**

[Domcke, Carcia-Cely, Rodd '22]

$$\Phi_{\mathrm{Axion}}(g_{a\gamma\gamma}) \quad = \quad \Phi_{\mathrm{GW}}(h_{+,\times};\theta_h,\phi_h)$$

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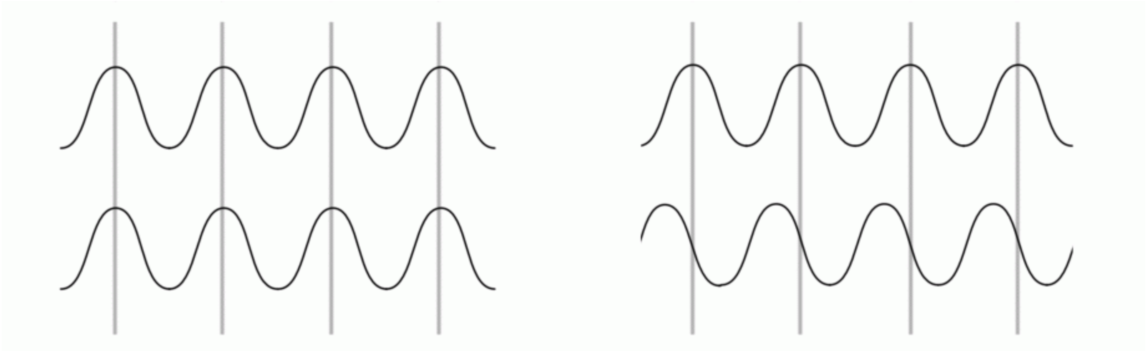

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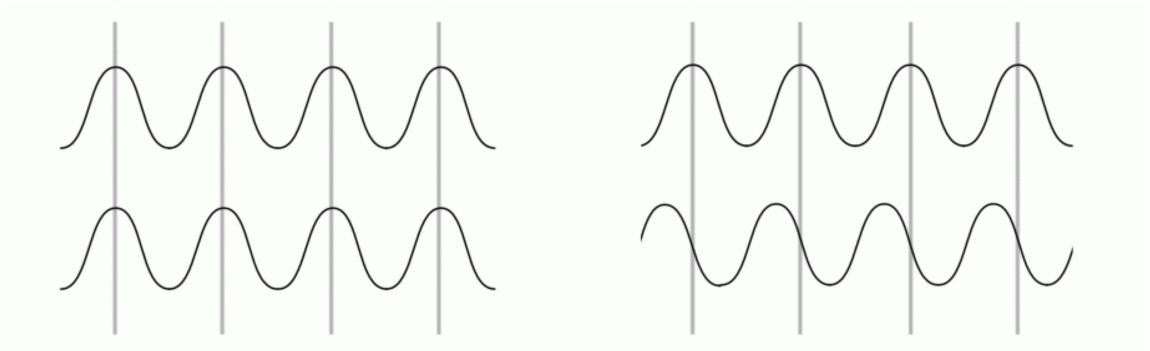
  - *Coherence Ratio Factor*

$$R_c = \left(\frac{Q_a}{Q_h}\right)^{1/4}$$

(persistent signal)

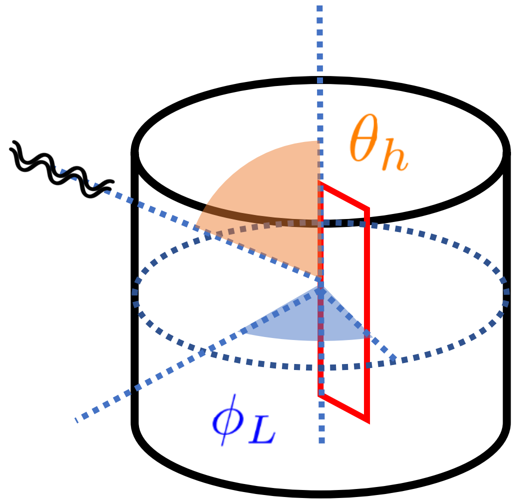


For more general cases, [Domcke, Carcia-Cely, **SML**, Rodd 23']

Example: Solenoidal Geometry

- For single pickup loop [Domcke, Carcia-Cely, **SML**, Rodd ‘23]

$$\Phi_{\rm GW} = \frac{e^{-i\omega t}}{144}\omega^2 B_z l r(30R^2 - 13r^2)\sin\theta_h(h^+\cos\theta_h\sin\phi_L + h^\times\cos\phi_L) + \mathcal{O}[(\omega L)^3]$$



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leading order

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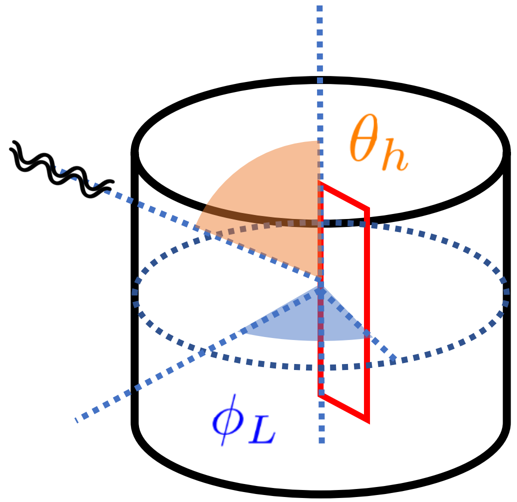

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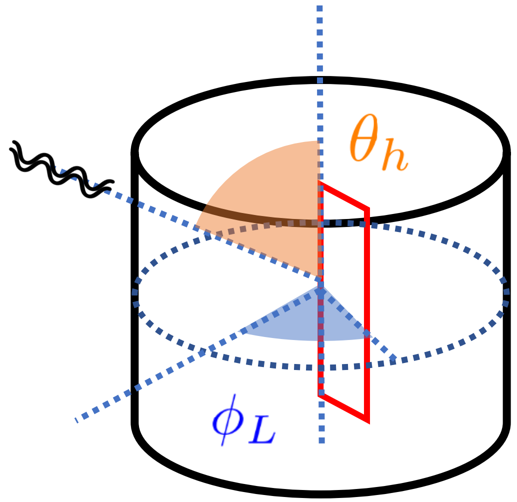

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leading order volume effect angular dependence

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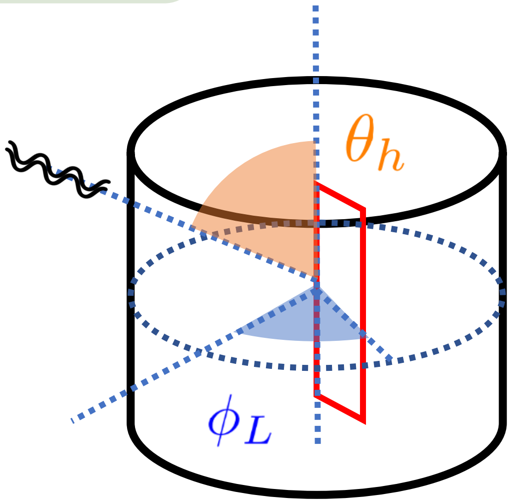

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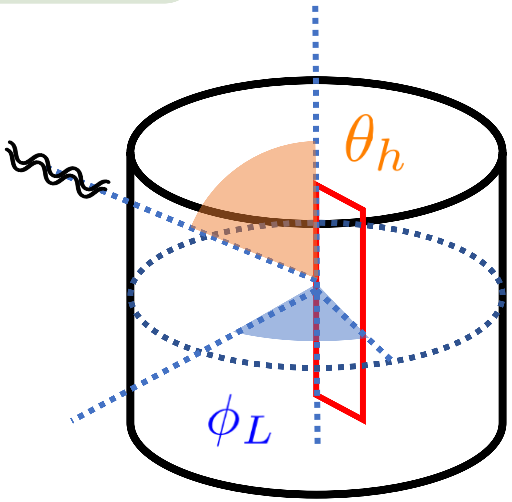

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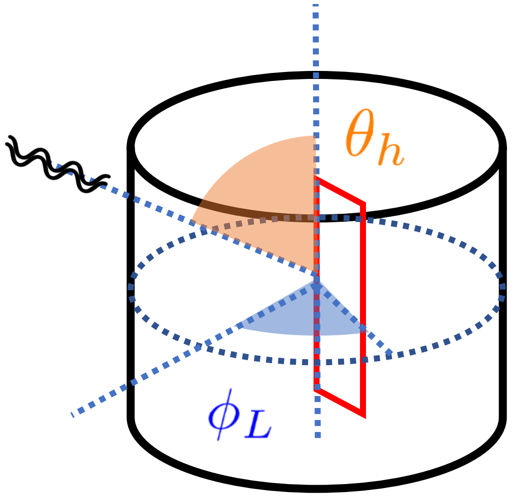


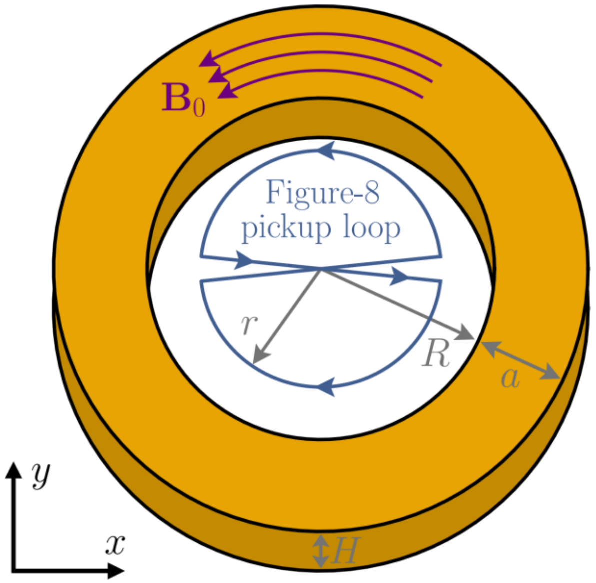
- Toroidal loop (ϕ_L integration) : $\Phi_{\rm GW,Sol} = O(\omega^3)$

This cancellation *always* happens for *cylindrically symmetric* axion detectors

Example: Toroidal Geometry

- Normal Loop

$$\Phi_{\rm GW} = \frac{ie^{-i\omega t}}{48}\omega^3 B_{\rm max}\pi r^2 Ra(a+2R)h^\times \sin^2\theta_h$$



[2202.00695]

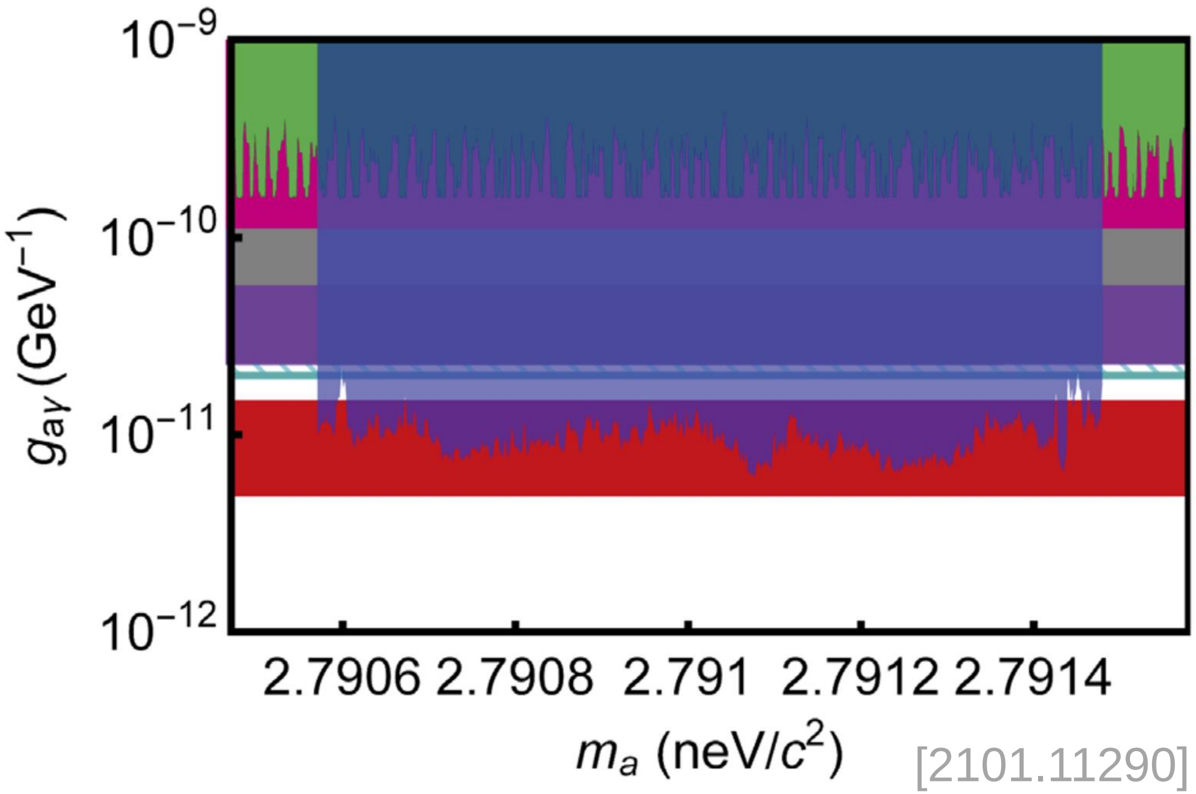
- Figure-8 Loop

$$\Phi_{\rm GW,Fig-8} = \frac{e^{-i\omega t}}{3}\omega^2 B_{\rm max} r^3 R \ln\left(1+\frac{a}{R}\right) s_{\theta_h}(h^\times s_{\phi_h} - h^+ c_{\theta_h} c_{\phi_h})$$

  - Kills axion sensitivity

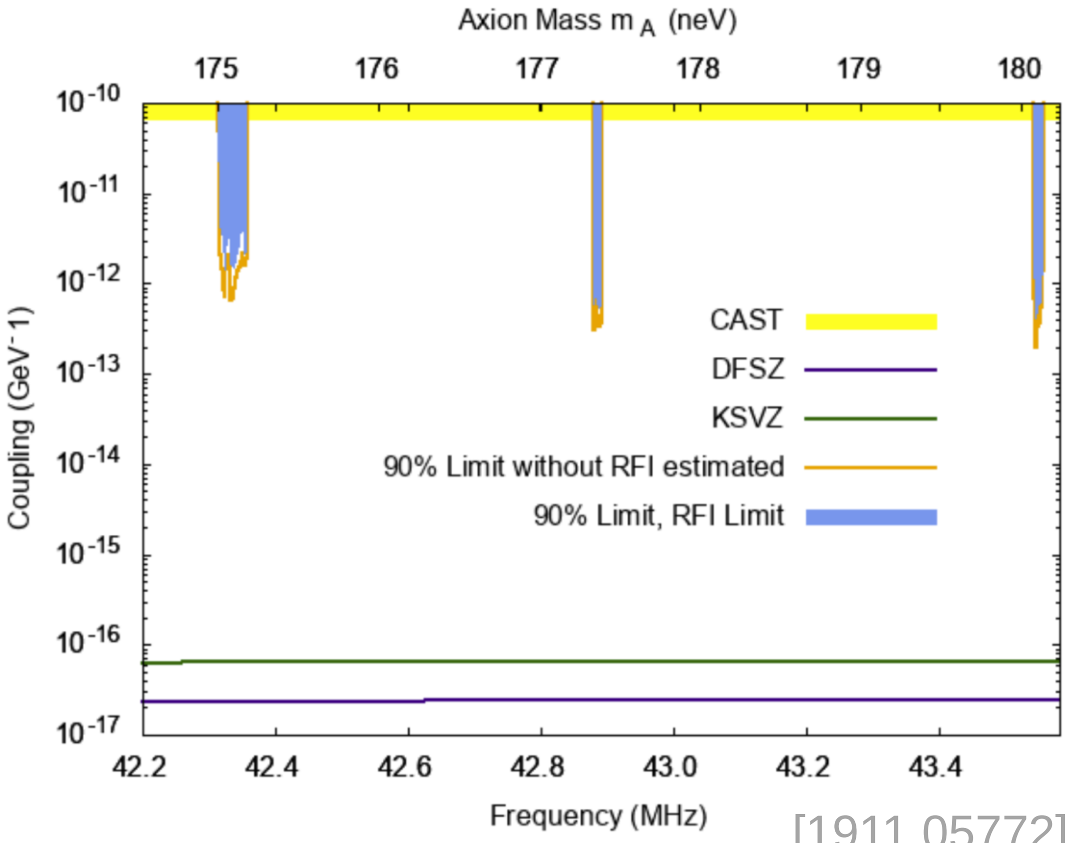
Result: Reinterpreting Axion Detectors

- **BASE**



[2101.11290]

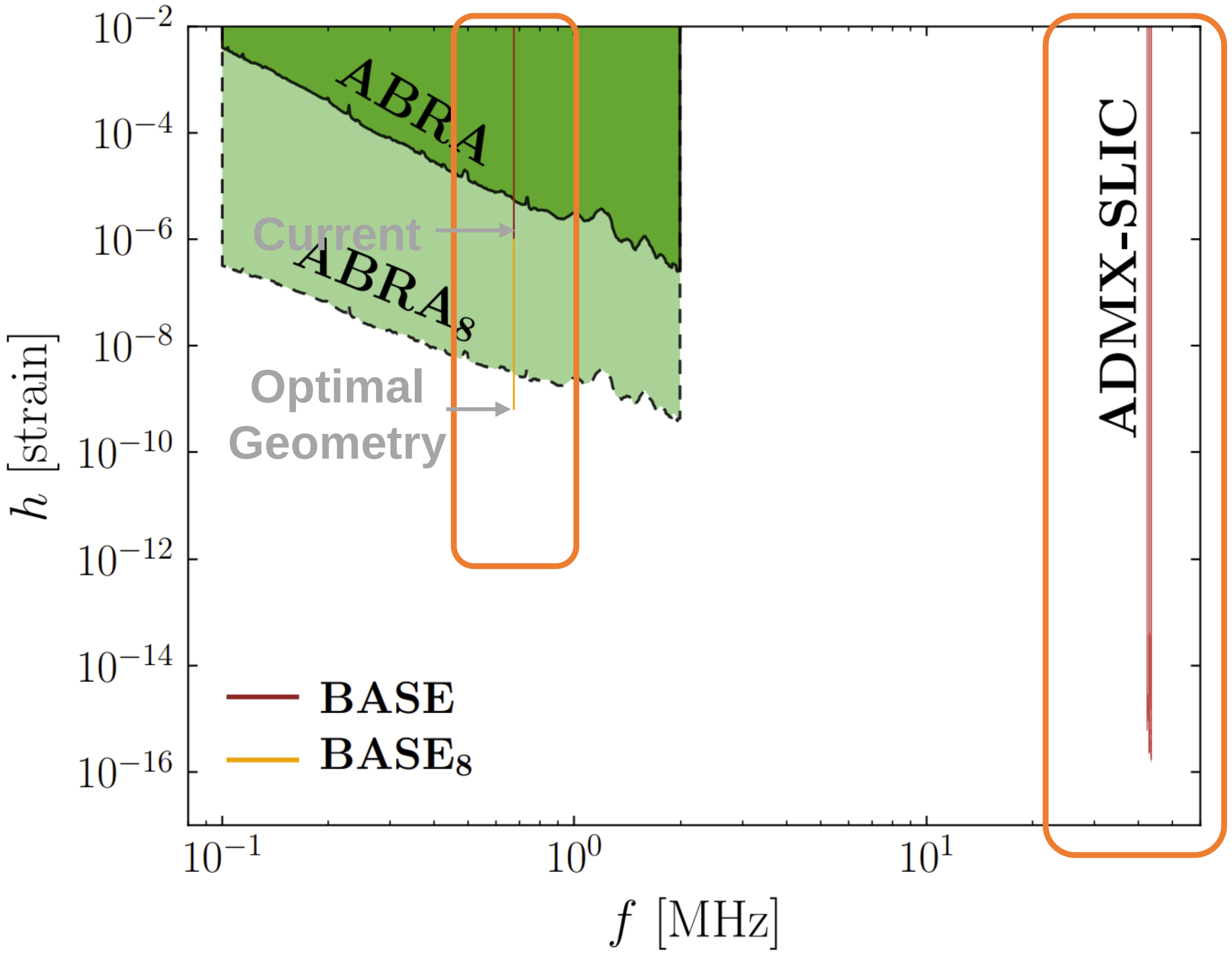
- **ADMX-SLIC**



[1911.05772]

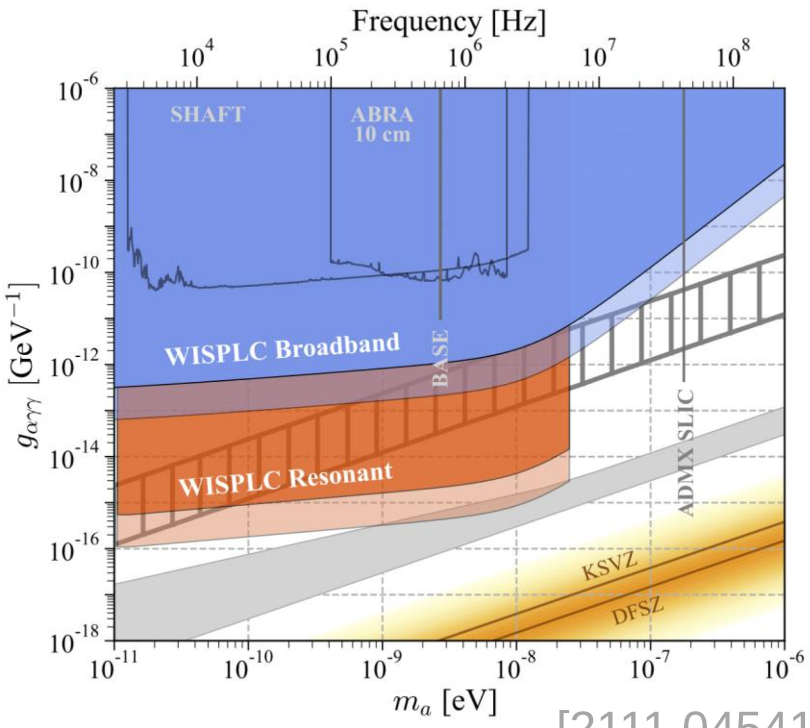
Result: Reinterpreting Axion Detectors

[Domcke, Carcia-Cely, **SML**, Rodd '23]



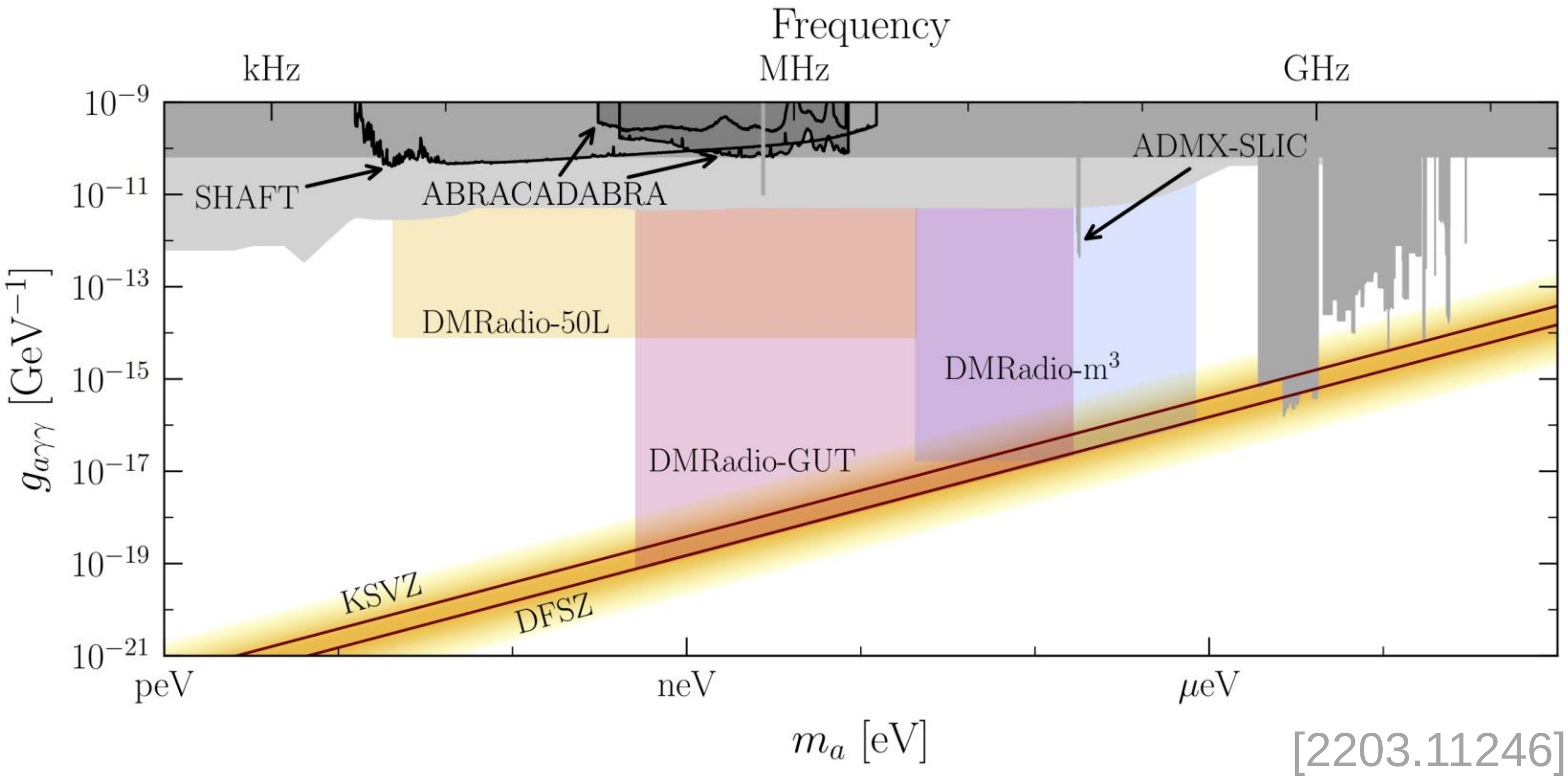
Future Prospects

- **WISPLC**



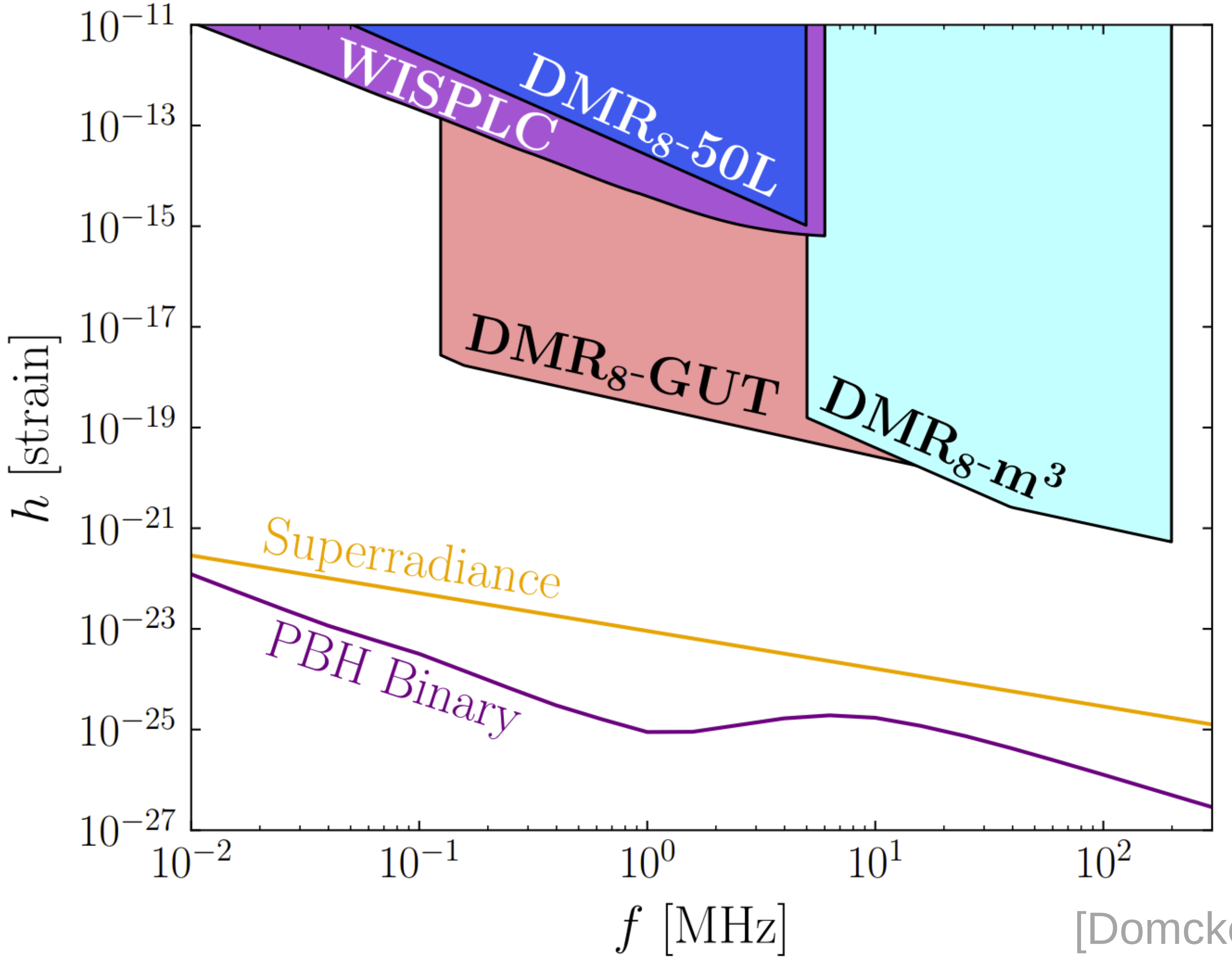
[2111.04541]

- **DMRadio Proposal**



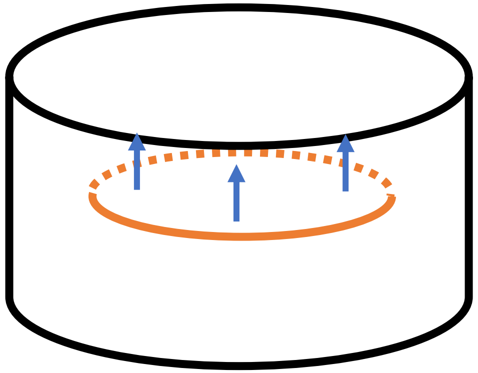
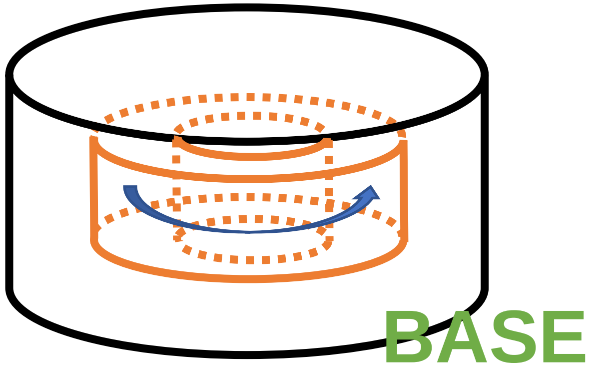
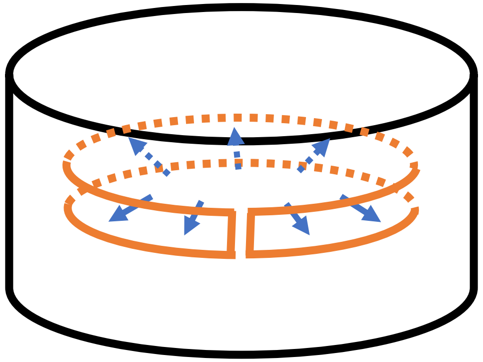
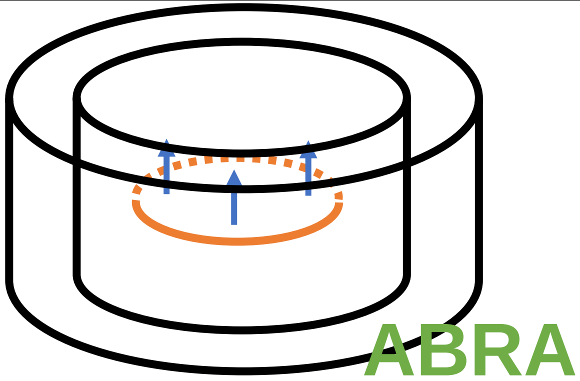
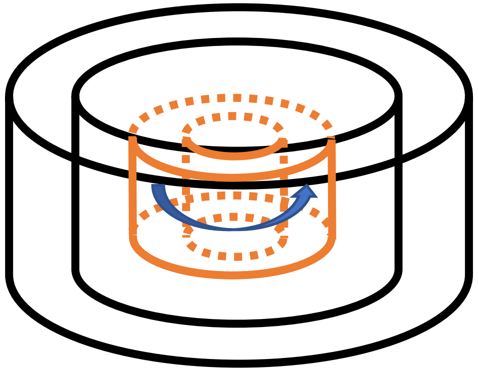
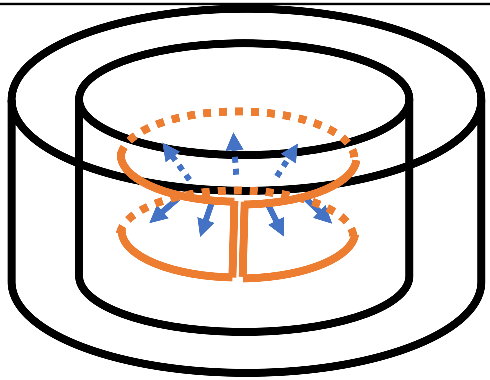
[2203.11246]

Future Prospects



[Domcke, Carcia-Cely, **SML**, Rodd '23]

Different Geometries

		$\hat{n}'$		
		$\hat{e}_z$	$\hat{e}_\phi$	$\hat{e}_\rho$
B	$\hat{e}_z$ (Sol)			
	$\hat{e}_\phi$ (Toro)			

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$$\Phi_{\mathrm{GW}}(\hat{\mathbf{k}}) = \Phi_{\mathrm{GW}}(R_z(\varphi)\hat{\mathbf{k}})$$

$$\Phi_{\mathrm{GW}}(\hat{\mathbf{k}}) = \frac{1}{2\pi}\int_0^{2\pi} d\varphi\ \Phi_{\mathrm{GW}}(R_z(\varphi)\hat{\mathbf{k}}) \propto D^{mn}\int_0^{2\pi} d\varphi\ \sum_A h_A e^A_{mn}(R_z\hat{\mathbf{k}})$$

$$\propto e^{-i\omega t} h^+ \sin^2\theta_h D^{mn} \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{pmatrix}_{mn}$$

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Selection Rule 3: For an instrument with full cylindrical symmetry, Φ_h will contain only even or odd powers of ω.

Different Geometries

- **Leading Orders**

[Domcke, Carcia-Cely, **SML**, Rodd ’23]

		$\hat{\boldsymbol{n}}'$		
		$\hat{\boldsymbol{e}}_{\boldsymbol{z}}$	$\hat{\boldsymbol{e}}_{\boldsymbol{\phi}}$	$\hat{\boldsymbol{e}}_{\boldsymbol{\rho}}$
$\boldsymbol{B}$	$\hat{\boldsymbol{e}}_{\boldsymbol{z}}$ **(Sol)**	h_+, even : $O[(\omega L)^2]$	$h_\times$, odd : $O[(\omega L)^3]$ **BASE**	h_+, odd : $O[(\omega L)^3]$ off-center: $O[(\omega L)^2]$
	$\hat{\boldsymbol{e}}_{\boldsymbol{\phi}}$ **(Toro)**	$h_\times$, odd : $O[(\omega L)^3]$ **ABRA**	h_+, even : $O[(\omega L)^{\cancel{2}\;4}]$	$h_\times$, even : $O[(\omega L)^4]$ off-center: $O[(\omega L)^3]$

- ***Optimal axion detection forbids optimal GW detection***

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- Symmetry is always good for theory, but sometimes bad for experiments.
  - Need to break cylindrical symmetry

Back Ups

Axion/Scalar Electrodynamics

$$\partial_\nu F_\varphi^{\mu\nu} = \partial_\nu (g_{\varphi\gamma\gamma} \varphi F_0^{\nu\mu}) = g_{\varphi\gamma\gamma} (\partial_\nu \varphi) F_0^{\nu\mu} - g_{\varphi\gamma\gamma} \varphi j^\mu,$$

$$\partial_\nu F_a^{\mu\nu} = \partial_\nu (g_{a\gamma\gamma} a \tilde{F}_0^{\nu\mu}) = g_{a\gamma\gamma} (\partial_\nu a) \tilde{F}_0^{\nu\mu},$$

$$\nabla \cdot \mathbf{E}_\varphi = -g_{\varphi\gamma\gamma} \mathbf{E} \cdot \nabla\varphi - g_{\varphi\gamma\gamma} \varphi \rho,$$

$$\nabla \cdot \mathbf{E}_a = -g_{a\gamma\gamma} \mathbf{B} \cdot \nabla a,$$

$$\nabla \times \mathbf{B}_\varphi = \partial_t \mathbf{E}_\varphi - g_{\varphi\gamma\gamma} (\nabla\varphi) \times \mathbf{B} + g_{\varphi\gamma\gamma} (\partial_t \varphi) \mathbf{E} - g_{\varphi\gamma\gamma} \varphi \mathbf{j},$$

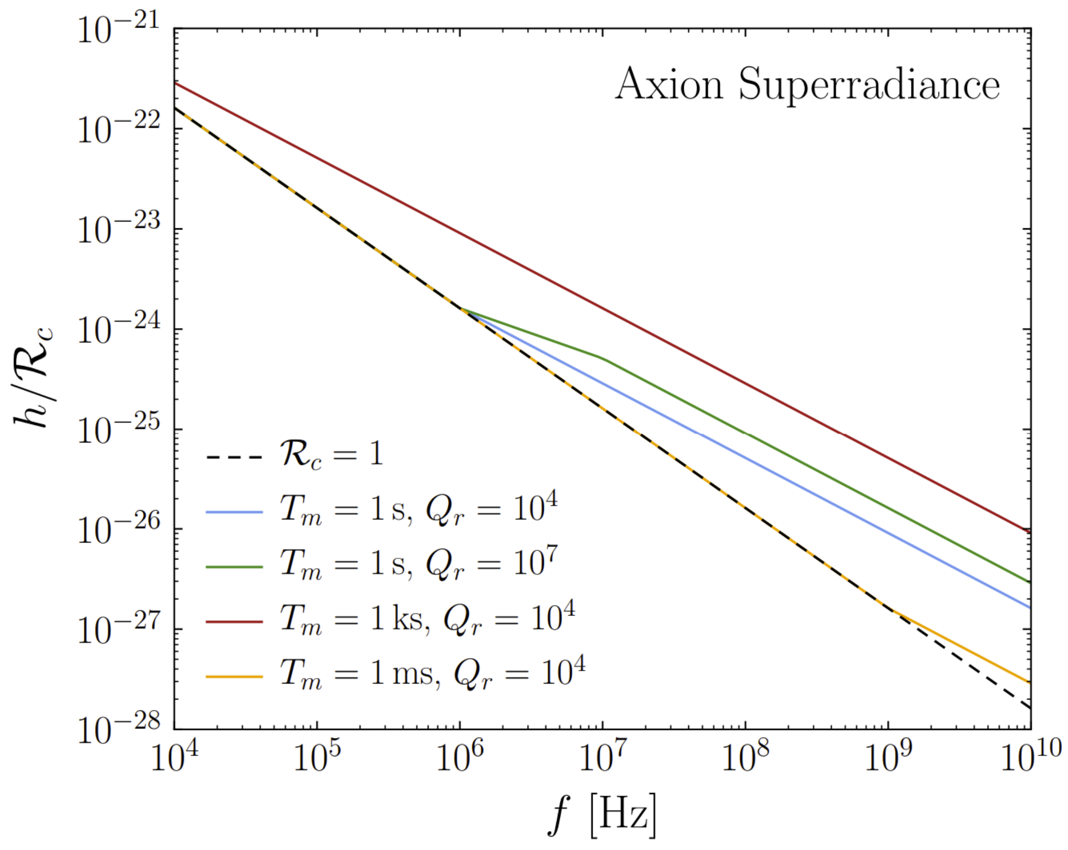
$$\nabla \times \mathbf{B}_a = \partial_t \mathbf{E}_a + g_{a\gamma\gamma} (\nabla a) \times \mathbf{E} + g_{a\gamma\gamma} (\partial_t a) \mathbf{B}.$$

Axion/Scalar Electrodynamics

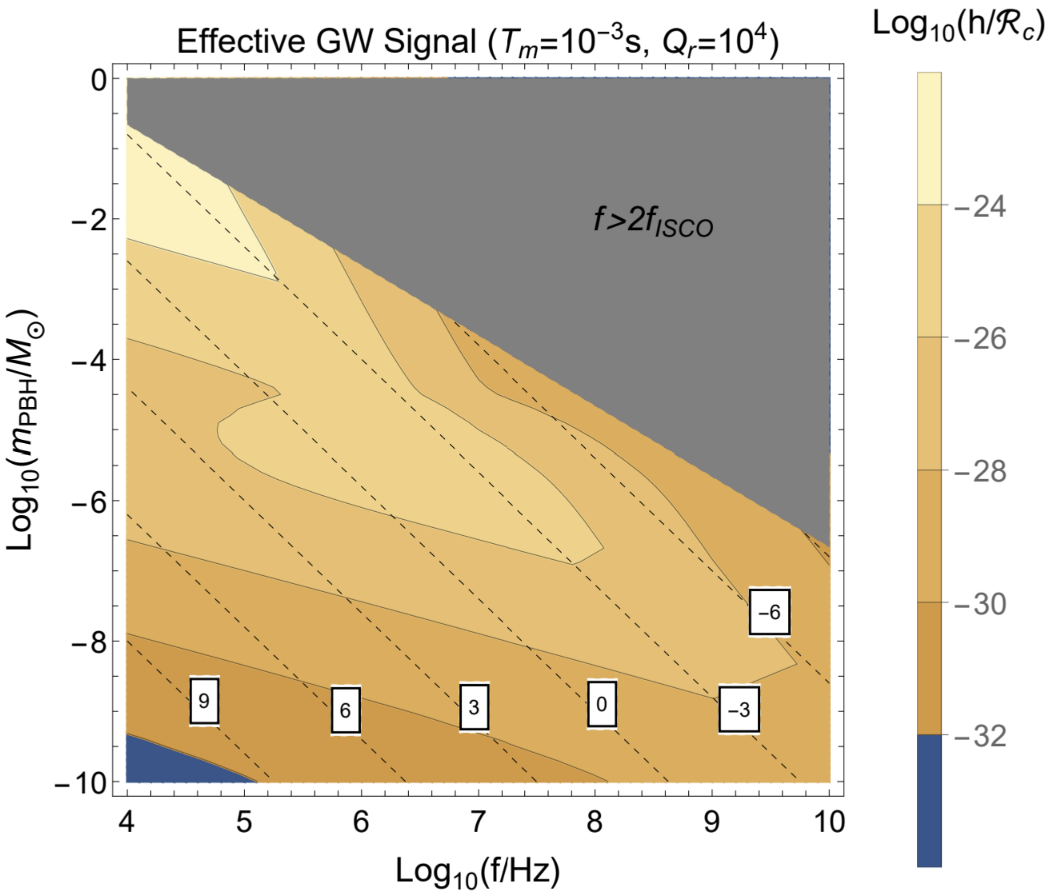
	Solenoid: $\mathbf{B}_0 \propto \hat{\mathbf{e}}_z$	Toroid: $\mathbf{B}_0 \propto \hat{\mathbf{e}}_\phi$
$\hat{\mathbf{n}}' \propto \hat{\mathbf{e}}_z$	scalar $\Phi_a \equiv 0,\ \Phi_\varphi \neq 0$	axion (ABRA) $\Phi_a \neq 0,\ \Phi_\varphi \equiv 0$
$\hat{\mathbf{n}}' \propto \hat{\mathbf{e}}_\phi$	axion (BASE) $\Phi_a \neq 0,\ \Phi_\varphi = 0$	scalar $\Phi_a \equiv 0,\ \Phi_\varphi \neq 0$
$\hat{\mathbf{n}}' \propto \hat{\mathbf{e}}_\rho$	scalar $\Phi_a \equiv 0,\ \Phi_\varphi = 0$	axion $\Phi_a = 0,\ \Phi_\varphi = 0$

Benchmark Signals

- Superradiance



- PBH



Benchmark Signals

$$\text{(B1)} \quad T_h = \tau_h = 1\ \text{s},\ Q_h = f_* \tau_h$$

$$\text{(B2)} \quad T_h \gg T_m,\ Q_h = 10^{10}$$

	Q_r	T_m	f_*	$\mathcal{R}_c^{(\mathrm{B1})}$	$\mathcal{R}_c^{(\mathrm{B2})}$
ADMX SLIC [16]	3×10^3	320 s[15]	50 MHz	1.6	0.1
BASE [17]	4×10^4	1 min	0.7 MHz	3.0	0.39
WISPLC [19]	10^4	1 min	(30 kHz, 5 MHz)	(6.7, 1.9)	(0.86, 0.24)
DMRadio [21]	2×10^7	(8 mins, 60 ns)	(100 kHz, 30 MHz)	(787, 1)	(0.18, 1)

Coherence Ratio Factor

- Persistent signal and a long interrogation time

$$\mathcal{R}_c = \sqrt{\frac{Q_a}{Q_h}} \left(\frac{\max[Q_h,\, Q_r]}{\max[Q_a,\, Q_r]}\right)^{1/4} \left(\frac{1}{\max[1,\, \min[Q_a,\, Q_r]/Q_h]}\right)^{1/4}$$
$$= \begin{cases} (Q_a/Q_h)^{1/2} & Q_a < Q_h < Q_r, \\ (Q_a^2/Q_r Q_h)^{1/4} & Q_a < Q_r < Q_h, \\ (Q_a/Q_h)^{1/4} & \text{otherwise.} \end{cases}$$

- Transient signal of equal duration and coherence time

$$\mathcal{R}_c = \sqrt{\frac{\tau_a}{T_{m,h}} \frac{\tau_r}{\min[T_{m,h},\, \tau_r]}} \left(\frac{T_{m,a}}{\max[\tau_a,\, \tau_r]}\right)^{1/4}$$