Phenomenology of discrete flavor symmetries beyond neutrino masses and mixing

Biswajit Karmakar
University of Silesia
Katowice, Poland

Based on 2203.08185, 2209.08610, , 230x.xxxxx

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Outline

- **Introduction:**
  - ∗ Neutrino physics and the known unknowns.
- **Flavor Symmetry and Lepton Masses and Mixing:**
  - ∗ Flavor symmetries, why?
  - ∗ General framework
  - ∗ Family symmetry, nonzero θ_{13} and nonzero δ_{CP}
  - ∗ Flavor symmetry and CP symmetries, higher order discrete Groups, GUT etc.
- **Implications of Flavor Symmetry in Various Frontiers**
  - ∗ Dark matter
  - ∗ Baryon asymmetry of the Universe
  - ∗ Collider physics
- **Recent Developements**
  - ∗ Modular symmetry
  - ∗ How to falsify flavor models?
- **Conclusion**

Neutrino parameters and the known unknowns

- Neutrinos are special!! It's flavor and mass eigenstates are related by : $|\nu_\alpha\rangle = \sum_i U_{\alpha i}|\nu_i\rangle$.
- Pontecorvo-Maki-Nakagawa-Sakata parametrization:

$$U_{PMNS} = \begin{bmatrix} C_{12}C_{13} & S_{12}C_{13} & S_{13}e^{-i\delta} \\ -S_{12}C_{23} - C_{12}S_{13}S_{23}e^{i\delta} & C_{12}C_{23} - S_{12}S_{13}S_{23}e^{i\delta} & C_{13}S_{23} \\ S_{12}S_{23} - C_{12}S_{13}C_{23}e^{i\delta} & -C_{12}S_{23} - S_{12}S_{13}C_{23}e^{i\delta} & C_{13}C_{23} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha_{21}/2} & 0 \\ 0 & 0 & e^{i\alpha_{31}/2} \end{bmatrix}$$

here $C_{ij} = \cos\theta_{ij}$ and $S_{ij} = \sin\theta_{ij}$.

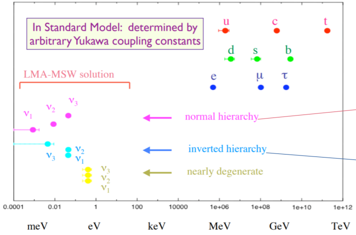
- Large Lepton Mixings

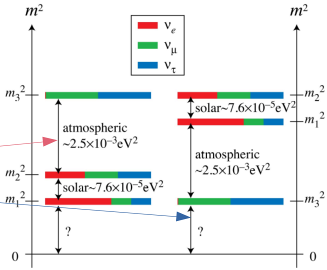
$$|U_{PMNS}| \sim \begin{pmatrix} 0.79-0.86 & 0.50-0.61 & 0.14-0.16 \\ 0.24-0.52 & 0.44-0.69 & 0.63-0.79 \\ 0.26-0.52 & 0.47-0.71 & 0.60-0.77 \end{pmatrix}$$

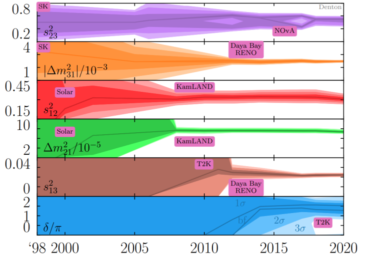
- Small Quark Mixings

$$|V_{CKM}| \sim \begin{pmatrix} 0.9745-0.9757 & 0.219-0.224 & 0.002-0.005 \\ 0.218-0.224 & 0.9736-0.9750 & 0.036-0.046 \\ 0.004-0.014 & 0.034-0.046 & 0.9989-0.9993 \end{pmatrix}$$

Neutrino parameters and the known unknowns







	Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 2.6$)	
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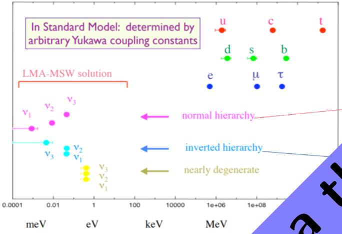
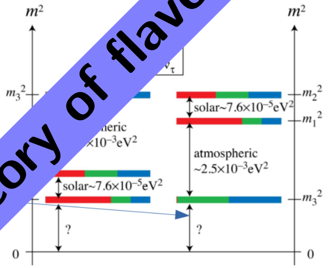
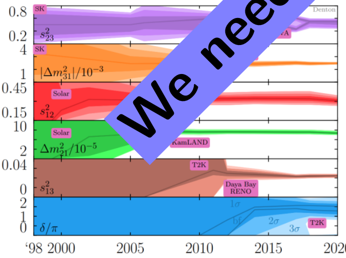
$\delta_{CP}/^\circ$	194^{+52}_{-25}	$105 \to 405$	287^{+27}_{-32}	$192 \to 361$
$\frac{\Delta m^2_{21}}{10^{-5}\ \text{eV}^2}$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$
$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \text{eV}^2}$	$+2.515^{+0.028}_{-0.028}$	$+2.431 \to +2.599$	$-2.498^{+0.028}_{-0.029}$	$-2.584 \to -2.413$

Neutrino parameters and the known unknowns

We need a theory of flavor!!







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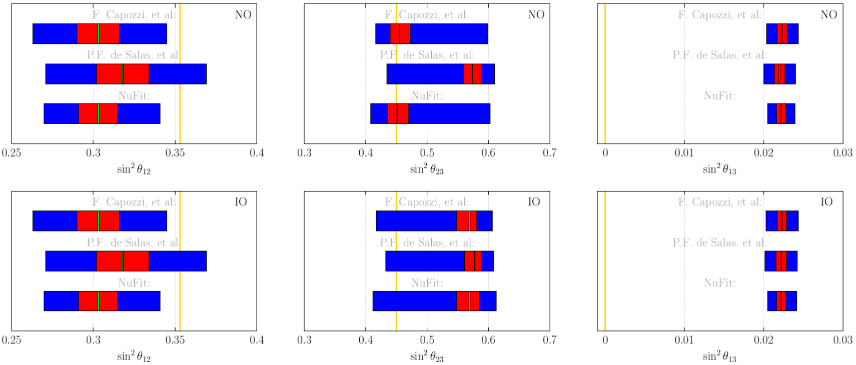
Flavor symmetries, why?



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$\Downarrow$

(Prior to 2012)

$$\boxed{s_{23} = 1/\sqrt{2}\ (\theta_{23} = 45^\circ) \text{ and } \theta_{13} = 0}$$

$\Downarrow$

$$U_0 = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -\frac{s_{12}}{\sqrt{2}} & \frac{c_{12}}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{s_{12}}{\sqrt{2}} & \frac{c_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}.$$

$\theta_{12} = 45^\circ (s_{12} = 1/\sqrt{2})$ Bimaximal Mixing; $\theta_{12} = 35.26^\circ (s_{12} = 1/\sqrt{3})$ Tribimaximal Mixing; $\theta_{12} = 31.7^\circ$ Golden Ratio Mixing; $\theta_{12} = 30^\circ (s_{12} = 1/2)$ Hexagonal Mixing

$$U_0 = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \frac{\varphi}{\sqrt{2+\varphi}} & \frac{1}{\sqrt{2+\varphi}} & 0 \\ \frac{-1}{\sqrt{4+2\varphi}} & \frac{\varphi}{\sqrt{4+2\varphi}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{4+2\varphi}} & \frac{-\varphi}{\sqrt{4+2\varphi}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \sqrt{\frac{3}{4}} & \frac{1}{2} & 0 \\ -\frac{1}{2\sqrt{2}} & \frac{\sqrt{3}}{2\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{2\sqrt{2}} & \frac{\sqrt{3}}{2\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Fukugita, Tanimoto, Yanagida PRD98; Harrison Perkins, Scott PLB02; Dutta,Ramond NPB03; Rodejohann et. al. EPJC10

(GR: $\tan\theta_{12} = 1/\phi$ where $\phi = (1+\sqrt{5})/2$)

Flavor symmetries, why?

- Using the diagonalization relation

$$m_\nu = U_0^\star \,\mathrm{diag}(m_1, m_2, m_3) U_0^\dagger ,$$

such a mixing matrices can easily diagonalize a $\mu - \tau$ symmetric (transformations $\nu_e \to \nu_e$, $\nu_\mu \to \nu_\tau$, $\nu_\tau \to \nu_\mu$ under which the neutrino mass term remains unchanged) neutrino mass matrix of the form

$$m_\nu = \begin{pmatrix} A & B & B \\ B & C & D \\ B & D & C \end{pmatrix} ,$$

With $A + B = C + D$ this matrix yields tribimaximal mixing pattern where $s_{12} = 1/\sqrt{3}$ *i.e.*, $\theta_{12} = 35.26^\circ$

- Observed mixing matrix :

$$U_{\mathrm{PMNS}} \simeq \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \epsilon \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}}(?) \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}}(?) \end{pmatrix}$$

New approximate symmetry?

General Framework

Anarchy

- Neutrino mixing anarchy is the hypothesis that the leptonic mixing matrix can be described as the result of a random draw from an unbiased distribution of unitary 3×3 matrices.
- Random analysis without imposing prior theories or symmetries on the mass and mixing matrices.
- This hypothesis does not make any correlation among the neutrino masses and mixing parameters

de Gouvea, Haba, Hall, Murayama : 9911341, 0009174, 1204.1249

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Texture

- More specific studies with imposed mass or mixing textures for which models with underlying symmetries can be sought.
- It's an intermediate approach
- Some texture zeros of neutrino mass matrices can be eliminated.

Alejandro Ibarra, Graham Ross: Phys.Lett.B 2003

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Symmetry

- Theoretical studies where some explicit symmetries at the Yukawa Lagrangian level are assumed and corresponding extended particle sector is defined.
- The symmetry-based approach to explain the non-trivial mixing in the lepton sector known as family symmetry or horizontal symmetry

Reviews: Tanimoto *et.al*. 1003.3552, Altarelli, Feruglio 1002.0211, King 1301.1340

General Framework: Symmetry based approach

Why we are interested in flavor symmetries?

SM flavor problem
- Why there are three families?
- Fermion mass hierarchy
- Different quark and lepton mixing

Flavor Symmetries :

→ Models based on **discrete flavor symmetric groups** : $A_4, S_3, S_4, \Delta(27)$ etc

↓ Models based on **continuous groups**: $G_F = U(1), SU(2)$ etc

General Framework: Symmetry based approach

Continuous flavor symmetry :

Froggatt-Nielsen models

Froggatt, Nielsen '79
Leurer, Seiberg, Nir '92

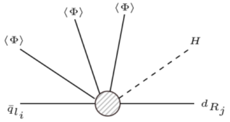
- SM fermions are charged under G_F
- Flavon fields (Φ) are introduced (charged under G_F)
- Forbids Yukawa couplings at the normalizable level
- Yukawa terms are allowed as higher dimensional operator:

$$\mathcal{L} \in \bar{q_{L_i}} d_{R_j} H \left(\frac{\Phi}{\Lambda}\right)^{n^d_{ij}} \Rightarrow m^d_{ij} = \left(\frac{\langle\Phi\rangle}{\Lambda}\right) \frac{v}{\sqrt{2}}$$

Λ : Cut-off scale introduced $\longrightarrow$ can be interpreted as the xxxxxx scale of new degree of freedom

$\downarrow$

might be related to new physics (neutrino mass, DM, baryogenesis etc.)

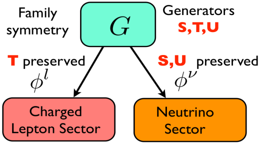


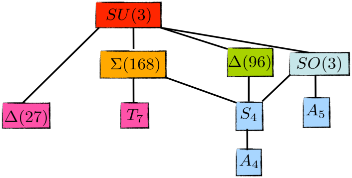
$$\langle\Phi\rangle < \Lambda \Rightarrow \epsilon = \frac{\langle\Phi\rangle}{\Lambda} \ : \ \text{small parameter}, \ n_{ij} \rightarrow \ \text{dictated by symmetry}$$

- quark mass hierarchy and mixing can be explained

General Framework: Symmetry based approach

- Fundamental symmetry in the lepton sector can easily explain the origin of neutrino mixing which is considerably different from quark mixing.
- Incidentally, both Abelian or non-Abelian family symmetries have potential to shade light on the Yukawa couplings.
- The Abelian symmetries (such as Froggatt-Nielsen symmetry) only points towards a hierarchical structure of the Yukawa couplings.
- Non-Abelian symmetries are more equipped to explain the non-hierarchical structures of the observed lepton mixing as observed by the oscillation experiments.





S. F. King 1301.1340

$\boxed{G_f \rightarrow G_e, G_\nu}$ typically, $G_e = Z_3$ and $G_\nu = Z_2 \times Z_2$.

An example:

- Let us consider $G_f = S_4$ as a guiding symmetry.
- Geometrically, it's a symmetry group of a rigid cube (group of permutation 4 objects).
- the order of the group is $4! = 24$ and the elements can be conveniently generated by the generators S, T and U satisfying the relation

$$S^2 = T^3 = U^2 = 1 \text{ and } ST^3 = (SU)^2 = (TU)^2 = 1.$$

- irreducible triplet representations:

$$S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix};\ T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix} \text{ and } U = \mp\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$T^\dagger M_\ell^\dagger M_\ell T = M_\ell^\dagger M_\ell,\ S^T M_\nu S = M_\nu \text{ and } U^T M_\nu U = M_\nu$$

$$[T, M_\ell^\dagger M_\ell] = [S, M_\nu] = [U, M_\nu] = 0$$

- The non-diagonal matrices S, U can be diagonalized by

$$U_{TBM} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix},$$

Tribimaximal Mixing: A_4- Ma, Rajasekaran 0106291; Altarelli, Feruglio 0504165; $\Delta(27)$-Varzielas, King, Ross-0607045; Bimaximal Mixing: Frampton, Petcov, Rodejohann 0401206; Golden Ratio Mixing: Feruglio, Paris 1101.0393; Hexagonal Mixing: Albright, Dueck, Rodejohann-1004.2798.

Non-zero θ_{13}

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Bimaximal Mixing | Tribimaximal Mixing | Golden Ratio Mixing | Hexagonal Mixing

$$U_0 = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \frac{\varphi}{\sqrt{2+\varphi}} & \frac{1}{\sqrt{2+\varphi}} & 0 \\ \frac{-1}{\sqrt{4+2\varphi}} & \frac{\varphi}{\sqrt{4+2\varphi}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{4+2\varphi}} & \frac{-\varphi}{\sqrt{4+2\varphi}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \sqrt{\frac{3}{4}} & \frac{1}{2} & 0 \\ -\frac{1}{2\sqrt{2}} & \frac{\sqrt{3}}{2\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{2\sqrt{2}} & \frac{\sqrt{3}}{2\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Decendents of fixed pattern mixing schemes

Non-zero θ_{13}: Decendents of tribimaximal mixing

$$U_{TBM} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}, \quad U_{\mathrm{PMNS}} \simeq \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \epsilon \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}}(?) \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}}(?) \end{pmatrix}$$

$$|U_{\mathrm{TM}_1}| = \begin{pmatrix} \frac{2}{\sqrt{6}} & * & * \\ \frac{1}{\sqrt{6}} & * & * \\ \frac{1}{\sqrt{6}} & * & * \end{pmatrix} \qquad |U_{\mathrm{TM}_2}| = \begin{pmatrix} * & \frac{1}{\sqrt{3}} & * \\ * & \frac{1}{\sqrt{3}} & * \\ * & \frac{1}{\sqrt{3}} & * \end{pmatrix},$$

- If S_4 is considered to be broken spontaneously into $Z_3 = \{1, T, T^2\}$ (for the charged lepton sector) $Z_2 = \{1, SU\}$ (for the neutrino sector) such that it satisfies : $[T, M_\ell^\dagger M_\ell] = [SU, M_\nu] = 0$

$$U_{\mathrm{TM}_1} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{c_\theta}{\sqrt{3}} & \frac{s_\theta}{\sqrt{3}} e^{-i\gamma} \\ -\frac{1}{\sqrt{6}} & \frac{c_\theta}{\sqrt{3}} - \frac{s_\theta}{\sqrt{2}} e^{i\gamma} & -\frac{s_\theta}{\sqrt{3}} e^{-i\gamma} - \frac{c_\theta}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{c_\theta}{\sqrt{3}} - \frac{s}{\sqrt{2}} e^{i\gamma} & -\frac{s_\theta}{\sqrt{3}} e^{-i\gamma} + \frac{c_\theta}{\sqrt{2}} \end{pmatrix}, U_{\mathrm{TM}_2} = \begin{pmatrix} \frac{2c_\theta}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{2s_\theta}{\sqrt{6}} e^{-i\gamma} \\ -\frac{c_\theta}{\sqrt{6}} + \frac{s}{\sqrt{2}} e^{i\gamma} & \frac{1}{\sqrt{3}} & -\frac{s_\theta}{\sqrt{3}} e^{-i\gamma} - \frac{c_\theta}{\sqrt{2}} \\ -\frac{c_\theta}{\sqrt{6}} + \frac{s}{\sqrt{2}} e^{i\gamma} & \frac{1}{\sqrt{3}} & -\frac{s_\theta}{\sqrt{3}} e^{-i\gamma} + \frac{c_\theta}{\sqrt{2}} \end{pmatrix}$$

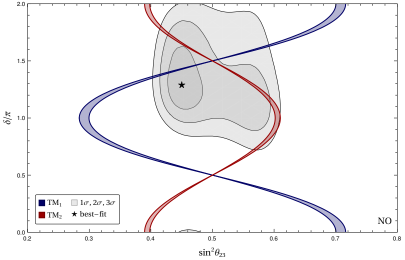
Non-zero θ_{13}: Decendents of tribimaximal mixing

- TM_1 vs TM_2

	TM_1	TM_2
$\lvert U_{e2}\rvert$	$\left\lvert\frac{\cos\theta}{\sqrt{3}}\right\rvert$	$\frac{1}{\sqrt{3}}$
$\lvert U_{e3}\rvert$	$\left\lvert\frac{\sin\theta}{\sqrt{3}}e^{-i\gamma}\right\rvert$	$\left\lvert\frac{2\sin\theta}{\sqrt{6}}e^{-i\gamma}\right\rvert$
$\lvert U_{\mu 3}\rvert$	$\left\lvert\frac{\cos\theta}{\sqrt{2}}+\frac{\sin\theta}{\sqrt{3}}e^{-i\gamma}\right\rvert$	$\left\lvert-\frac{\cos\theta}{\sqrt{2}}-\frac{\sin\theta}{\sqrt{6}}e^{-i\gamma}\right\rvert$
$\sin^2\theta_{12}$	$1-\frac{2}{3-\sin^2\theta}$	$\frac{1}{3-2\sin^2\theta}$
$\sin^2\theta_{13}$	$\frac{1}{3}\sin^2\theta$	$\frac{2}{3}\sin^2\theta$
$\sin^2\theta_{12}$	$\frac{1}{2}\left(1-\frac{\sqrt{6}\sin 2\theta\cos\gamma}{3-\sin^2\theta}\right)$	$\frac{1}{2}\left(1+\frac{\sqrt{3}\sin 2\theta\cos\gamma}{3-\sin^2\theta}\right)$
J_{CP}	$-\frac{1}{6\sqrt{6}}\sin 2\theta\sin\gamma$	$-\frac{1}{6\sqrt{3}}\sin 2\theta\sin\gamma$
$\sin\delta_{CP}$	$-\frac{(5+\cos 2\theta)\sin\gamma}{\sqrt{(5+\cos 2\theta)^2-24\sin^2 2\theta\cos^2\gamma}}$	$-\frac{(2+\cos 2\theta)\sin\gamma}{\sqrt{(2+\cos 2\theta)^2-3\sin^2 2\theta\cos^2\gamma}}$

Non-zero θ_{13}: Decendents of tribimaximal mixing

- TM_1, TM_2 Vs Current data: B.K. et al. 230x.xxxxx

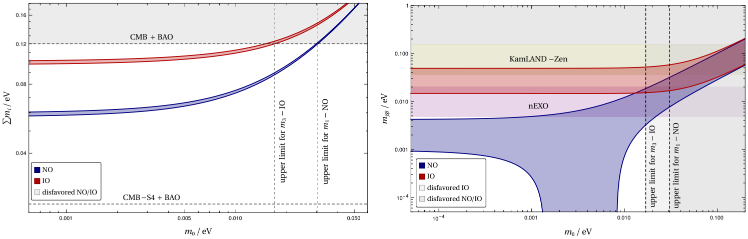


NuFit 5.2

Origin of neutrino mass?

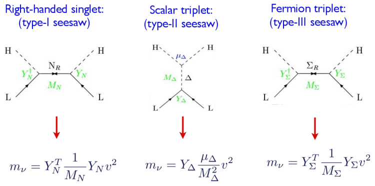
Dirac or Majorana Particle??

Neutrino Mass : Cosmology to $0\nu\beta\beta$



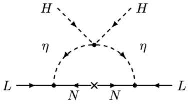
Neutrino Mass Generation

Seesaw frameworks



• **Type-I Seesaw, Type-II Seesaw, Type-III Seesaw, etc.:** Minkowski 77; Gellman, Ramond, Slansky 80; Glashow, Yanagida 79; Mohapatra, Senjanovic 80; Lazarides, Shafi, Schechter, Valle 80, 82; Mohapatra, Senjanovic 81; Lazarides, Shafi, Wetterich 81; Foot, Lew, He, Joshi 89; Ma 98; Bajc, Senjanovic 07....

Radiative neutrino mass



• **Radiative models, started in 80s:** Zee 80, Cheng, Li 80; Zee 86; Babu 88; Ma 06;

• **For a review of radiative models:** Cai, Herrero-Garcia, Schmidt, Vicente, Volkas 17;

Hybrid Scenarios??

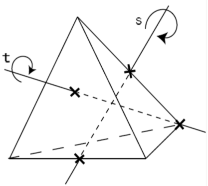
Are they connected?

Caldwell, Mohapatra 1993; Asaka, Blanchet, Shaposhnikov 2005; Boehm 2008; Kubo, Ma, Suematsu 2006; Hambye, Kannike, Ma, Raidal 2007; Lindner, Schmidt, Schwetz 2011; Borah, Adhikari 2012; Restrepo, Zapata, Yaguna 2013; Huang, Deppisch 2014; Escudero, Rius, Sanz 2016; Borah, Karmakar, Nanda 2018;..many more..

Standard Model with A_4 discrete flavor symmetry

Standard Model with A_4 discrete flavor symmetry

- A_4 is considered to be a favored symmetry in the neutrino sector
- Even permutation of 4 objects/invariant group of a tetrahedron
- Minimal group which contains 3 dim. representation (can accommodate three flavors of leptons)
- Product rule: $3 \otimes 3 = 1 \oplus 1' \oplus 1'' \oplus 3_A \oplus 3_S$
- $1 \otimes 1 = 1,\ 1' \otimes 1' = 1'', 1' \otimes 1'' = 1$
  $1'' \otimes 1'' = 1'$ etc



Flavor symmetric scoto-seesaw mechanism

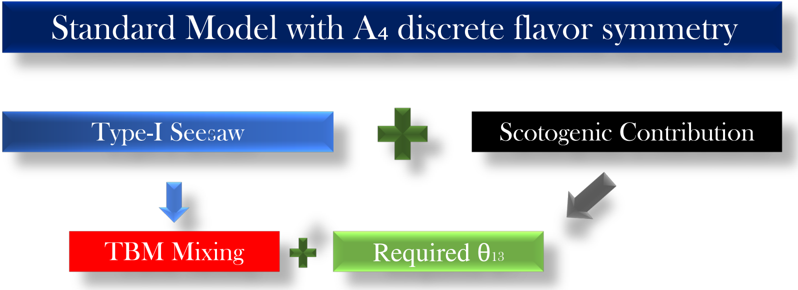
Standard Model with A_4 discrete flavor symmetry

Type-I Seesaw

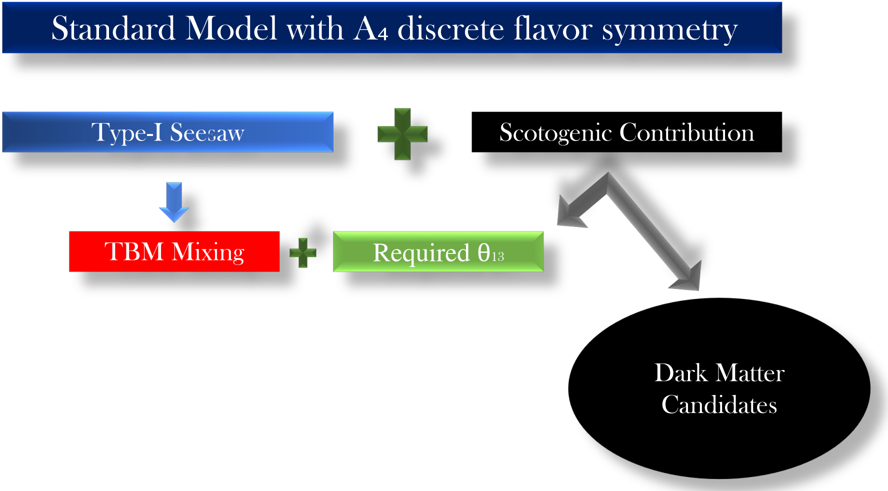
↓

TBM Mixing

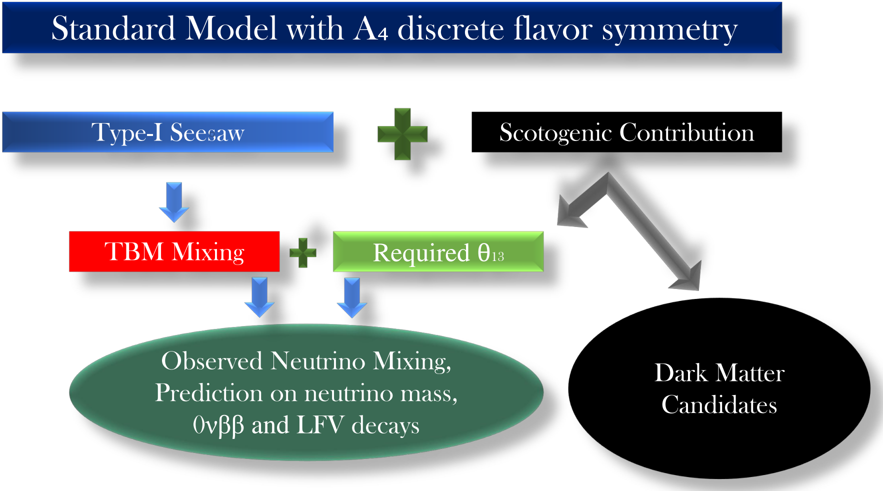
Flavor symmetric scoto-seesaw mechanism



Flavor symmetric scoto-seesaw mechanism



Flavor symmetric scoto-seesaw mechanism



J. Ganguly, J. Gluza and B. Karmakar, JHEP 11 (2022) 074, arXiv: 2209.08610

Flavor symmetric scoto-seesaw mechanism:

Type-I Seesaw contribution:

$$\mathcal{L}_{\mathrm{TREE}} = \frac{y_{N_1}}{\Lambda}(\bar{L}\phi_s)\tilde{H}N_{R_1} + \frac{y_{N_2}}{\Lambda}(\bar{L}\phi_a)\tilde{H}N_{R_2} + \frac{1}{2}M_{N_1}\bar{N}^c_{R_1}N_{R_1} + \frac{1}{2}M_{N_2}\bar{N}^c_{R_2}N_{R_2} + h.c.,$$

- L, ϕ_a and ϕ_s $\rightarrow$ A_4 triplets; H, N_{R_1}, N_{R_2} $\rightarrow$ A_4 singlets

- A_4 multiplication rules: If we have two triplets (a_1, a_2, a_3) and (b_1, b_2, b_3), their products are given by $\Rightarrow 3 \otimes 3 = 1 + 1' + 1'' + 3_A + 3_S$

$$\begin{aligned} 1 \quad &\sim \quad a_1b_1 + a_2b_3 + a_3b_2, 1' \sim a_3b_3 + a_1b_2 + a_2b_1, 1'' \sim a_2b_2 + a_3b_1 + a_1b_3, \\ 3_S \quad &\sim \quad \begin{bmatrix} 2a_1b_1 - a_2b_3 - a_3b_2 \\ 2a_3b_3 - a_1b_2 - a_2b_1 \\ 2a_2b_2 - a_1b_3 - a_3b_1 \end{bmatrix}, 3_A \sim \begin{bmatrix} a_2b_3 - a_3b_2 \\ a_1b_2 - a_2b_1 \\ a_3b_1 - a_1b_3 \end{bmatrix}. \end{aligned}$$

- flavon fields get VEVs along $\langle\phi_s\rangle = (0, v_s, -v_s)$, $\langle\phi_a\rangle = (v_a, v_a, v_a)$

$$\frac{y_{N_1}}{\Lambda}(\bar{L}\phi_s)_1\tilde{H}N_{R_1} = \frac{y_{N_1}}{\Lambda}(\bar{L_1}\phi_{s1} + \bar{L_2}\phi_{s3} + \bar{L_3}\phi_{s2})_1\tilde{H}N_{R_1} = \frac{y_{N_1}}{\Lambda}(0 - \bar{L_2}v_s + \bar{L_3}v_s)_1\tilde{H}N_{R_1}$$

$$\frac{y_{N_2}}{\Lambda}(\bar{L}\phi_a)_1\tilde{H}N_{R_2} = \frac{y_{N_2}}{\Lambda}(\bar{L}_1\phi_{a1} + \bar{L}_2\phi_{a3} + \bar{L}_3\phi_{a2})_1\tilde{H}N_{R_2} = \frac{y_{N_2}}{\Lambda}(\bar{L}_1v_a + \bar{L}_2v_a + \bar{L}_3v_a)_1\tilde{H}N_{R_2}$$

- Dirac neutrino mass matrix :

$$M_D = \frac{v}{\Lambda}\begin{pmatrix} 0 & y_{N_2}v_a \\ -y_{N_1}v_s & y_{N_2}v_a \\ y_{N_1}v_s & y_{N_2}v_a \end{pmatrix} = vY_N, \quad M_R = \begin{pmatrix} M_{N_1} & 0 \\ 0 & M_{N_2} \end{pmatrix}.$$

Flavor symmetric scoto-seesaw mechanism:

Scotogenic contribution:

$$\begin{aligned}
\mathcal{L}_{\rm LOOP} &= \frac{y_s}{\Lambda^2}(\bar{L}\phi_s)\xi i\sigma_2\eta^* f + \frac{1}{2}M_f\bar{f}^c f + h.c.,\\
(M_\nu)_{\rm LOOP} &= \mathcal{F}(m_{\eta_R}, m_{\eta_I}, M_f)M_f Y_f^i Y_f^j.\\
Y_F &= (Y_F^e, Y_F^\mu, Y_F^\tau)^T = (y_s\frac{v_s}{\Lambda}\frac{v_\xi}{\Lambda}, 0, -y_s\frac{v_s}{\Lambda}\frac{v_\xi}{\Lambda})^T.
\end{aligned}$$

Therefore, the corresponding mass matrix takes the form

$$(M_\nu)_{\rm LOOP} = C\begin{pmatrix} 1 & 0 & -1\\ 0 & 0 & 0\\ -1 & 0 & 1\end{pmatrix}, \quad C = \mathcal{F}(m_{\eta_R}, m_{\eta_I}, M_f)y_s^2\frac{v_s^2 v_\xi^2}{\Lambda^4}.$$

Here $\mathcal{F}(m_{\eta_R}, m_{\eta_I}, M_f)$ is the loop function

- **Effective neutrino mass matrix:**

$$\begin{aligned}
M_\nu &= -M_D M_R^{-1} M_D^T + (M_\nu)_{\rm LOOP}\\
&= (M_\nu)_{\rm TREE} + (M_\nu)_{\rm LOOP}\\
&= \begin{pmatrix} -B+C & -B & -B-C\\ -B & -A-B & A-B\\ -B-C & A-B & -A-B+C\end{pmatrix}.
\end{aligned}$$

- **After rotation by TBM matrix:**

$$\begin{aligned}
M'_\nu &= U_{TB}^T M_\nu U_{TB}\\
&= \frac{1}{2}\begin{pmatrix} 3C & 0 & -\sqrt{3}C\\ 0 & -6B & 0\\ -\sqrt{3}C & 0 & -4A+C\end{pmatrix},
\end{aligned}$$

Flavor symmetric scoto-seesaw mechanism:

- Effective neutrino mixing matrix (TM_2 mixing):

$$U_\nu = \begin{pmatrix} \sqrt{\frac{2}{3}}\cos\theta & \frac{1}{\sqrt{3}} & \sqrt{\frac{2}{3}}e^{i\phi}\sin\theta \\ -\frac{\cos\theta}{\sqrt{6}} + \frac{e^{i\phi}\sin\theta}{\sqrt{2}} & \frac{1}{\sqrt{3}} & -\frac{\cos\theta}{\sqrt{2}} - \frac{e^{i\phi}\sin\theta}{\sqrt{6}} \\ -\frac{\cos\theta}{\sqrt{6}} - \frac{e^{i\phi}\sin\theta}{\sqrt{2}} & \frac{1}{\sqrt{3}} & \frac{\cos\theta}{\sqrt{2}} - \frac{e^{i\phi}\sin\theta}{\sqrt{6}} \end{pmatrix} U_m.$$

- Corelations:

$$\tan\phi = \frac{\alpha\sin\phi_{AC}}{1-\alpha\cos\phi_{AC}}, \quad \tan 2\theta = \frac{\sqrt{3}}{\cos\phi + 2\alpha\cos(\phi_{AC}+\phi)}.$$

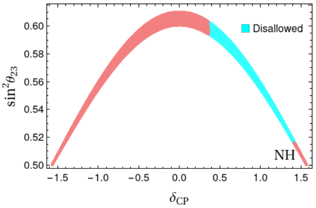
- Comparing with U_{PMNS}:

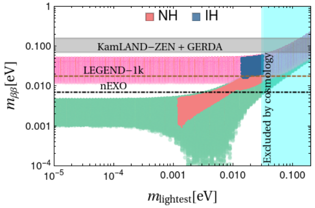
$$\sin\theta_{13}e^{-i\delta_{\rm CP}} = \sqrt{\frac{2}{3}}e^{-i\phi}\sin\theta, \quad \tan^2\theta_{12} = \frac{1}{2-3\sin^2\theta_{13}},$$

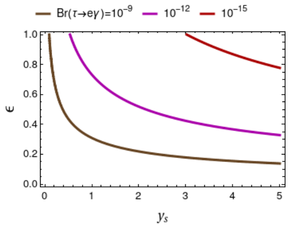
$$\tan^2\theta_{23} = \frac{\left(1+\frac{\sin\theta_{13}\cos\phi}{\sqrt{2-3\sin^2\theta_{13}}}\right)^2 + \frac{\sin^2\theta_{13}\sin^2\phi}{(2-3\sin^2\theta_{13})}}{\left(1-\frac{\sin\theta_{13}\cos\phi}{\sqrt{2-3\sin^2\theta_{13}}}\right)^2 + \frac{\sin^2\theta_{13}\sin^2\phi}{(2-3\sin^2\theta_{13})}}.$$

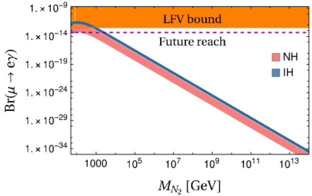
Ganguly, Gluza, BK, 2209.08610

- Predictions:



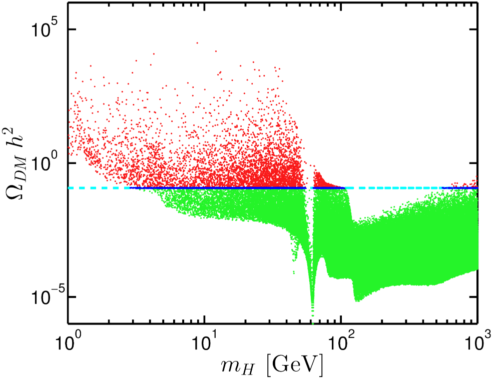






Dark Matter Phenomenology: Preliminary

- 2 viable DM candidates ⇒ the **lightest neutral scalar** and the **singlet fermion**.
- **Scalar dark matter:**

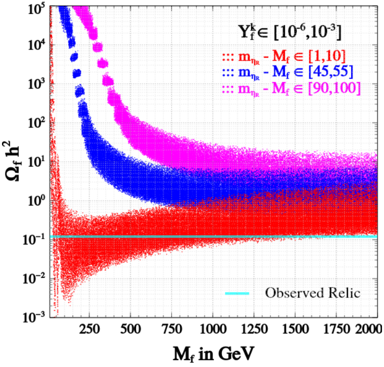


Dolle, Su 2009; Diaz, Koch, Urrutia-Quiroga 2015;

Dark Matter Phenomenology: Preliminary

- 2 viable DM candidates $\Rightarrow$ the **lightest neutral scalar** and the **singlet fermion**.
- **Fermionic dark matter: WIMP**

$$\frac{dn_f}{dt} + 3Hn_f = -\langle\sigma v\rangle_{\text{eff}}(n_f^2 - (n_f^{eq})^2)$$



Ganguly, Gluza, Karmakar, Mahapatra 230x:xxxxx

Non-zero θ_{13}:Cobimaximal Mixing

- $\mu - \tau$ permutation symmetry : $\nu_e \to \nu_e,\ \nu_\mu \to \nu_\tau,\ \nu_\tau \to \nu_\mu$
- $\mu - \tau$ symmetry + CP : $\nu_e \to \nu_e^c,\ \nu_\mu \to \nu_\tau^c,\ \nu_\tau \to \nu_\mu^c$
- The mixing matrix satisfy the condition :

$$|U_{\mu i}| = |U_{\tau i}| \ \text{ with } i = 1, 2, 3.$$

- Predicts specific values for the atmospheric mixing angle $\theta_{23} = 45^\circ$ and Dirac CP phase $\delta = -90^\circ$.
- The neutrino mixing matrix can be parametrized as

$$U_0 = \begin{pmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ v_1^* & v_2^* & v_3^* \end{pmatrix},$$

where the entries in the first row, u_i's are real (and non-negative) with trivial values of the Majorana phases.

- The mass matrix leading to the above mixing matrix can be written as

$$m_\nu = \begin{pmatrix} a & b & b^\star \\ b & c & d \\ b^\star & d & c^\star \end{pmatrix},$$

where b and c are in general complex while c and d remain real.

Fukuura, Miura, Takasugi, Yoshimura PRD 99; Miura, Takasugi, Yoshimura PRD01; Harrison, Scott PLB02; Grimus, Lavoura PLB04; Babu, Ma, Valle, PLB03

Cobimaximal Mixing: A flavor model

	$\ell_{e,\mu,\tau}$	e_R, μ_R, τ_R	H	N_R	$\phi_{1,2,3}$	ξ	ϕ_S
A_4	$1, 1', 1''$	$1, 1'', 1'$	1	3	3	1	3
Z_3	1	1	1	ω^2	ω	ω^2	ω^2
Z_4	$-i,-1,i$	$i,-1,-i$	1	1	$i,-1,-i$	1	1

BK, arXiv:230x.xxxxx

- **Neutrinos:**

$$-\mathcal{L}_\nu = \frac{y_1}{\Lambda}(\bar{\ell_e})_{\mathbf{1}}\tilde{H}(N_R\phi_1)_{\mathbf{1}} + \frac{y_2}{\Lambda}(\bar{\ell_\mu})_{\mathbf{1}'}\tilde{H}(N_R\phi_2)_{\mathbf{1}''} + \frac{y_3}{\Lambda}(\bar{\ell_\tau})_{\mathbf{1}''}\tilde{H}(N_R\phi_3)_{\mathbf{1}'}$$
$$+(y_x\xi + y_\phi\phi_S)\overline{N_R^c}N_R + h.c.$$

- **Light neutrino mass via type-I seesaw:**

$$m_\nu \sim -m_D M^{-1} m_D^T$$
$$\sim\lambda\begin{pmatrix} 1-\kappa_1^2 & (\kappa_1\kappa_2-\kappa_2)\omega & (\kappa_1\kappa_2-\kappa_2)\omega^2 \\ (\kappa_1\kappa_2-\kappa_2)\omega & (1-\kappa_2^2)\omega^2 & \kappa_2^2-\kappa_1 \\ (\kappa_1\kappa_2-\kappa_2)\omega^2 & \kappa_2^2-\kappa_1 & (1-\kappa_2^2)\omega \end{pmatrix};$$

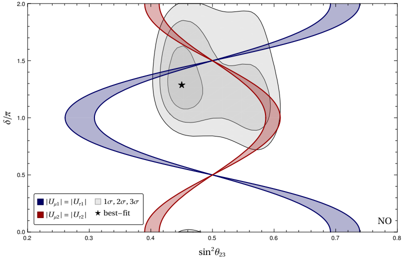
$$m_\nu = U^\star \mathrm{diag}(m_1, m_2, m_3)U^\dagger$$

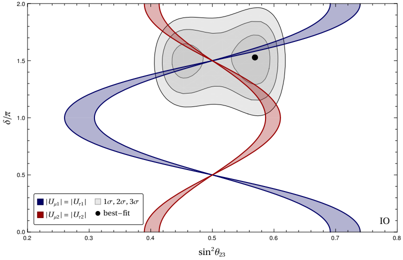
$$U == \begin{pmatrix} \cos\vartheta_{12}\cos\vartheta_{13} & -\sin\vartheta_{12}\cos\vartheta_{13} & -\sin\vartheta_{13} \\ \frac{\sin\vartheta_{12}-i\cos\vartheta_{12}\sin\vartheta_{13}}{\sqrt{2}} & \frac{\cos\vartheta_{12}+i\sin\vartheta_{12}\sin\vartheta_{13}}{\sqrt{2}} & -\frac{i\cos\vartheta_{13}}{\sqrt{2}} \\ \frac{\sin\vartheta_{12}+i\cos\vartheta_{12}\sin\vartheta_{13}}{\sqrt{2}} & \frac{\cos\vartheta_{12}-i\sin\vartheta_{12}\sin\vartheta_{13}}{\sqrt{2}} & \frac{i\cos\vartheta_{13}}{\sqrt{2}} \end{pmatrix}.$$

$$\delta = \arcsin\left[\frac{\mathrm{Im}[U_{23}U_{13}^\star U_{12}U_{22}^\star]}{\sin 2\theta_{12}\sin 2\theta_{23}\sin 2\theta_{13}\cos\theta_{13}}\right] = -\pi/2;\quad \sin^2\theta_{23} = \frac{|U_{23}|^2}{1-|U_{13}|^2} = \frac{1}{2}$$

Beyond Cobimaximal Mixing ($\mu - \tau$ Reflection Symmetry)

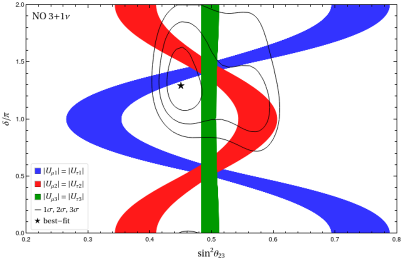
- **Partial $\mu - \tau$ Reflection Symmetry** B.K. et al. 230x.xxxxx





Partial $\mu - \tau$ Reflection Symmetry

- **3+1 neutrino scenario** B.K. et al. 230x.xxxxx



Flavor symmetry with CP invariance:

- $\mu - \tau$ symmetry + CP : $\nu_e \rightarrow \nu_e^c,\ \nu_\mu \rightarrow \nu_\tau^c,\ \nu_\tau \rightarrow \nu_\mu^c$.
- Residual symmetries with CP transformations may lead to new invariance conditions on the mass matrices.
- The cobimaximal matrix

$$M_0 = \begin{pmatrix} a & b & b^\star \\ b & c & d \\ b^\star & d & c^\star \end{pmatrix}$$

is invariant under

$$\mathcal{S}^T M_0 \mathcal{S} = M_0^*,$$

where the transformation matrix is given by

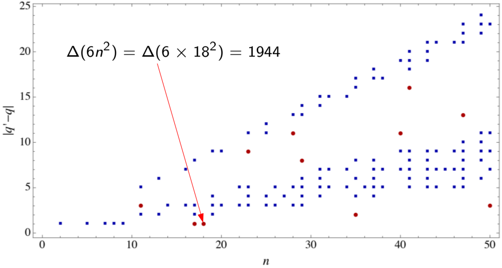
$$\mathcal{S} \quad = \quad \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

and such transformations are usually referred to as **generalized CP symmetry transformation**.

- The existence of both discrete flavor and generalized CP symmetries determines the possible structure of the generalized CP symmetry matrices and predictions involving Dirac and Majorana CP phases are made.
- For further readings: Feruglio, Hagedorn 1211.5560; Nishi 1306.0877; Li, Ding 1408.0785; Ding, King 1510.03188; Penedo Petcov, Titov 1803.11009; Iura, López-Ibáñez Meloni 1811.09662
- **Flavor symmetry and GUT** S. F. King, Unified Models of Neutrinos, Flavour and CP Violation, 1701.04413

Flavor symmetry and Higher Order Discrete Groups:

- Fixed mixing schemes such as BM, TBM, GR, HG are dead after measurement of non-zero θ_{13}
- Mixing schemes such as TM_1 ,TM_2, CBM are still consistent with observations.
- Smaller discrete groups such as S_3, A_4, S_4, A_5, $\Delta(27)$ etc. can be used to reproduce TM_1 ,TM_2, CBM or to generate appropriate "**clever/ugly**" modifications to BM, TBM, GR, HG mixings.
- Lepton mixing with larger groups : $G_f \to G_e, G_\nu$, G_f any higher order group.
- Example : $G_e = Z_3 \;\; G_\nu = Z_2$



Holthausen, Lim, Lindner 1212.2411; Joshipura, Patel 1610.07903

- The values of $n \leq 50$ and $|q' - q|$ ($q, q' = 0, 1, \ldots, n-1$) leading to the viable columns of leptonic mixing matrix. The blue squares (red dots) indicate that the corresponding prediction is consistent with the first (third) column of U_{PMNS} matrix within 3σ. Each point represents a unique solution obtained by the smallest possible values of n and $|q' - q|$.

Flavor Symmetries in Various Frontiers: Leptogenesis

- The origin of tiny neutrino mass is often best explained by various seesaw mechanisms.
- New heavy fermions and scalar are introduced to justify lightness of the active neutrinos.
- Out-of-equilibrium decay of these heavy particles can generate observed matter anti-matter asymmetry
- Type-I seesaw, heavy right-handed neutrinos are introduced.
- The CP-violating out-of-equilibrium decay of RH neutrinos into lepton and Higgs doublets in the early universe produces a net lepton asymmetry Fukugita, Yanagida, 1986; Covi, Roulet, Vissani 9605319
- The CP asymmetry parameter :

$$\epsilon_i^\alpha = \frac{\Gamma(N_i \to \ell_\alpha H) - \Gamma(N_i \to \bar{\ell}_\alpha \bar{H})}{\Gamma(N_i \to \ell_\alpha H) + \Gamma(N_i \to \bar{\ell}_\alpha \bar{H})} = \frac{1}{8\pi} \sum_{j \neq i} \frac{\mathrm{Im}\left[\left((\hat{Y}_\nu^\dagger \hat{Y}_\nu)_{ij}\right)^2\right]}{(\hat{Y}_\nu^\dagger \hat{Y}_\nu)_{ii}} f\left(\frac{m_i^2}{m_j^2}\right),$$

$$f(x) = \sqrt{x}\left[\frac{2-x}{1-x} - (1-x)\ln\left(1+\frac{1}{x}\right)\right] \text{ with } x = m_i^2/m_j^2$$

- Flavor symmetry dictates the structure of Y_ν and M_R, hence leaves its imprint on leptogenesis
- **(Altarelli-Feruglio) models with tribimaximal mixing:**

$$\begin{aligned} \hat{Y}_{\nu 0}^\dagger \hat{Y}_{\nu 0} &\propto |y^2|\mathbf{1} \\ \epsilon_i &= 0 \end{aligned}$$

Jenkins, Manohar 0807.4176

Flavor Symmetries in Various Frontiers: Leptogenesis

- Possible remedy: NLO correction in Yukawa sector
- Relevant contribution Yukawa sector:

$$y(LN^c)H_u + x_C N^c(L\phi_T)_{3_S} H_u/\Lambda + x_D N^c(L\phi_T)_{3_A} H_u/\Lambda$$

BK, Sil PRD 2015

- Yukawa matrix and $\hat{Y}_\nu \hat{Y}_\nu^\dagger$:

$$\begin{aligned} Y_\nu &= Y_{\nu 0} + \delta Y_\nu \\ &= y \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} + \frac{x_C v_T}{\Lambda} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix} + \frac{x_D v_T}{\Lambda} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}, \end{aligned}$$

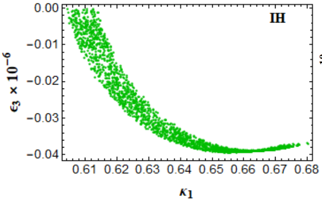
- Charged lepton mass-matrix remains diagonal

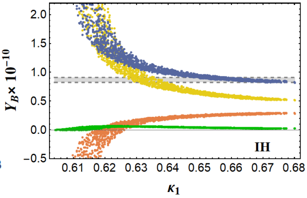
$$\begin{aligned} \epsilon_1 = \frac{-1}{2\pi}\left(\frac{v_T}{\Lambda}\right)^2 \Bigg[& \sin\alpha_{21}\left(2\mathrm{Re}(x_C)^2\cos^2\theta + \frac{2\mathrm{Re}(x_D)^2}{3}\sin^2\theta + \frac{2\mathrm{Re}(x_C)\mathrm{Re}(x_D)}{\sqrt{3}}\sin 2\theta\right) f\left(\frac{m_1}{m_2}\right) \\ & + \sin\alpha_{31}\left(\mathrm{Re}(x_C)^2\sin^2 2\theta + \frac{\mathrm{Re}(x_D)^2}{3}\cos^2 2\theta + \frac{\mathrm{Re}(x_C)\mathrm{Re}(x_D)}{\sqrt{3}}\sin 4\theta\right) f\left(\frac{m_1}{m_3}\right)\Bigg] \end{aligned}$$

and similar expressions for ϵ_2 and ϵ_3.

Flavor Symmetries in Various Frontiers: Leptogenesis

- Leptogenesis with cobimaximal mixing (**BK, arXiv:230x.xxxxx**):

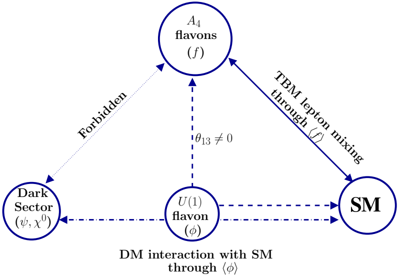


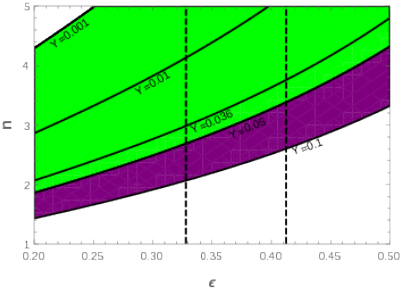


Flavor Symmetries in Various Frontiers: Dark Matter

- Can we extend flavor symmetry to the dark sector as well?
- Can discrete symmetry play any role to ensure the stability of dark matter?
- Example :

$$\mathcal{L}_{int} = \left(\frac{\phi}{\Lambda}\right)^n \bar{\psi}\tilde{H}\chi^0 + \frac{(HL^T LH)\phi\eta}{\Lambda^3} \text{ with } Y = \left(\frac{\phi}{\Lambda}\right)^n = \epsilon^n$$



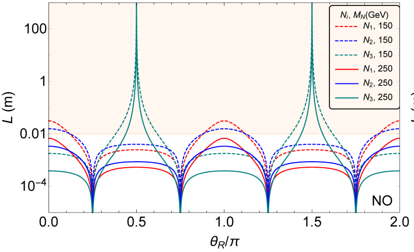


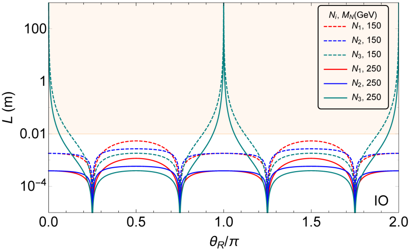
- A schematic representation of dark matter (ψ, χ^0) interaction with SM to generate non-zero θ_{13} in the presence of the $U(1)$ flavor symmetry. The A_4 flavons help in generating base TBM mixing.

S. Bhattacharya , B.K., N. Sahu, A. Sil 1603.04776

Flavor Symmetries in Various Frontiers: Collider Physics

- The high-energy CP phases present in Y_D that are responsible for leptogenesis are in general unrelated to the low-energy CP phases in U_{PMNS}.
- Since the experiments are only sensitive to the low-energy CP phases
- As discussed earlier, incorporating residual flavor and CP symmetries the high- and low-energy CP phases can be related.
- Since in this case the PMNS mixing matrix depends on a single free parameter, this turns out to be highly constraining and predictive for both low- and high-energy CP phases as well as the lepton mixing angles
- Example : $\Delta(6n^2) \times CP$





G. Chauhan, P. S. Bhupal Dev 2112.09710

Flavor Symmetry : Drawbacks

Flavor Symmetry : Drawbacks

Criticism for conventional model building with flavor symmetry :

- Traditionally discrete flavor symmetry groups are very useful to explain correct neutrino masses and mixing due to its high predictability.
- The spectrum of the models here is so large that it is difficult to obtain clear clue of the underlying flavour symmetry.
- Often introduces many parameters and auxiliary symmetries $\rightarrow$ non-minimal.

Reason for non-minimality:

Introduce flavons (gauge singlet scalars) to discuss dynamics of flavours. Write down an effective Lagrangian including flavons. Flavour symmetry is broken spontaneously by VEV of flavons.

- Possible Origin $\longrightarrow$ Unknown

New Ideas: Modular Symmetry

Possible Origin:

Superstring theory on certain compactifications may lead to Modular groups. In fact, torus compactification leads to Modular symmetery, which includes S_3, A_4, S_4, A_5 as its congruence subgroup.

Use of Modular Symmetry:

- Very recently, it has been showed that neutrino mass might be of modular form (F. Feruglio, [arXiv:1706.08749 [hep-ph]]), introducing modular invariance approach to the lepton sector.
- The primary advantage is that the flavon fields might not be needed and the Yukawa couplings are written as modular forms, functions of only one complex parameter.
- T. Kobayashi, K. Tanaka, T. H. Tatsuishi 1803.10391, J. T. Penedo, S. T. Petcov 1806.11040,F. J. de Anda, S. F. King, E. Perdomo 1812.05620, Wang, Zhou 2102.04358

Rich phenomenology : Yet to be explored

How to falsify flavor models

- Neutrino Oscillation Experiments
- Neutrinoless Double Beta Decay Experiments

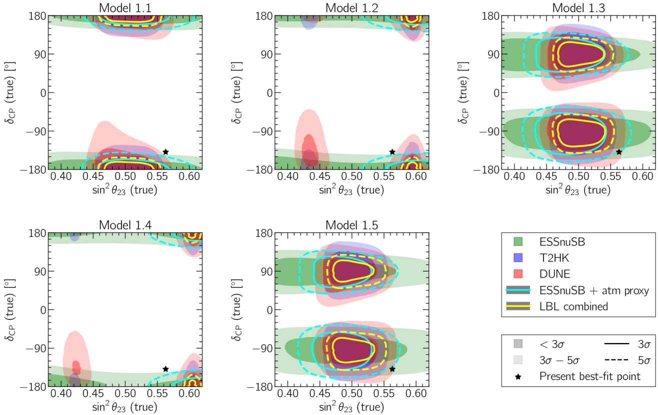
Flavor Symmetry and Oscillation Experiments:

- We need to test the existence underlying flavor symmetry G_f, if any.
- We look for the possibilities of testing its predictions at the current and future neutrino experiments.
- Such studies crucially depend on the breaking pattern of G_f into its residual subgroups for charged lepton sector G_e and neutrino sector G_ν.
- Example : $G_e = Z_k, k > 2$ or $Z_m \times Z_n$, $m, n \geq 2$ and $G_\nu = Z_2 \times CP$
- Correlations among $\theta_{23}, \theta_{12}, \theta_{12}$ and δ_{CP} are obtained and studied in the context of various experiments.

Flavor Symmetry and Oscillation Experiments:

Model	Case [Ref.]	Group	$\sin^2\theta_{12}$	$\sin^2\theta_{23}$	δ_{CP}	χ^2_{min}
1.1	VII-b [25]	$A_5 \rtimes$ CP	0.331	0.523	180°	5.37
1.2	III [25]	$A_5 \rtimes$ CP	0.283	0.593	180°	5.97
1.3	IV [24]	$S_4 \rtimes$ CP	0.318	1/2	±90°	7.28
1.4	II [24]	$S_4 \rtimes$ CP	0.341	0.606	180°	8.91
1.5	IV [25]	$A_5 \rtimes$ CP	0.283	1/2	±90°	11.3

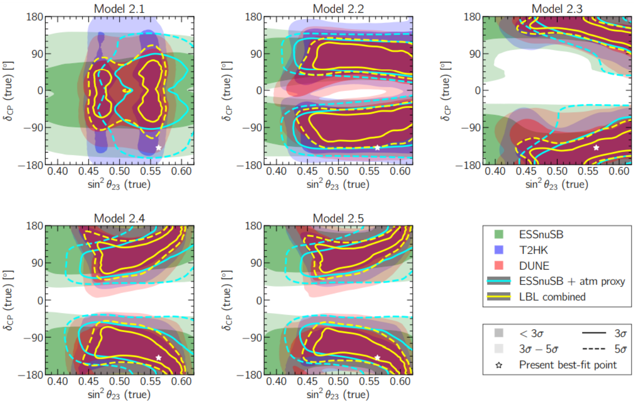
M. Blennow, M. Ghosh, T. Ohlsson, A. Titov 2005.12277



Flavor Symmetry and Oscillation Experiments:

2.1	A1 [28]	A_5	—	0.554	$f_1(\theta_{12})$	0.151
2.2	B2 [28]	S_4	0.318	—	$f_2(\theta_{23})$	0.386
2.3	B2 [28]	A_5	0.330	—	$f_3(\theta_{23})$	2.49
2.4	B1 [28]	A_5	0.283	—	$f_4(\theta_{23})$	4.40
2.5	B1 [28]	$A_4/S_4/A_5$	0.341	—	$f_5(\theta_{23})$	5.67

M. Blennow, M. Ghosh, T. Ohlsson, A. Titov 2005.12277



Flavor Symmetry and Oscillation Experiments:

2.1	A1 [28]	A_5	—	0.554	$f_1(\theta_{12})$	0.151
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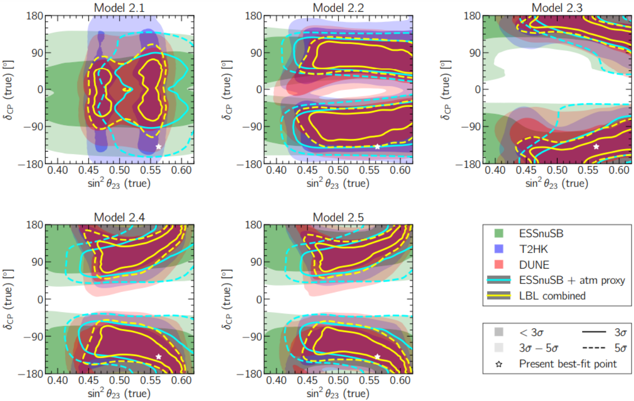
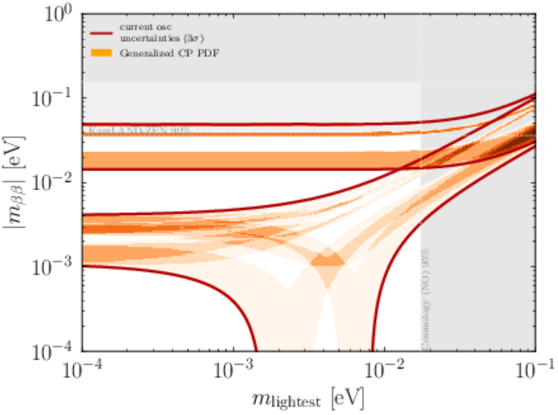
M. Blennow, M. Ghosh, T. Ohlsson, A. Titov 2005.12277



Flavor Symmetry and $0\nu\beta\beta$ Experiments:



Models with generalized CP Denton, Gehrlein 2308.09737

Flavor Symmetry and $0\nu\beta\beta$ Experiments:

- Generalised mass sum rules:

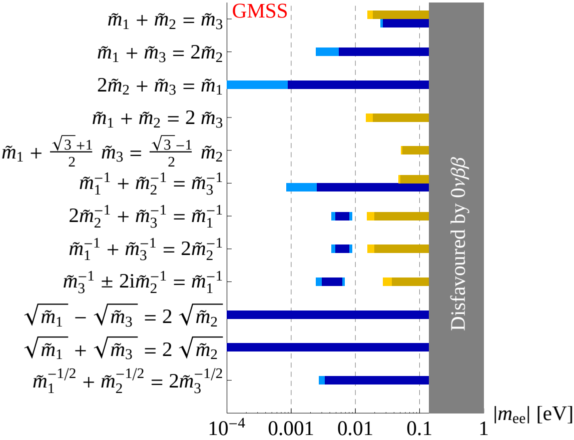
$$A_1 \tilde{m}_1^p e^{i\chi_1} + A_2 \tilde{m}_2^p e^{i\chi_2} + A_3 \tilde{m}_3^p e^{i\chi_3} = 0$$

where $p \neq 0, \chi_1 \in [0, 2\pi], A_i > 0$

- Simplified Sum Rules obtained from various flavor models:

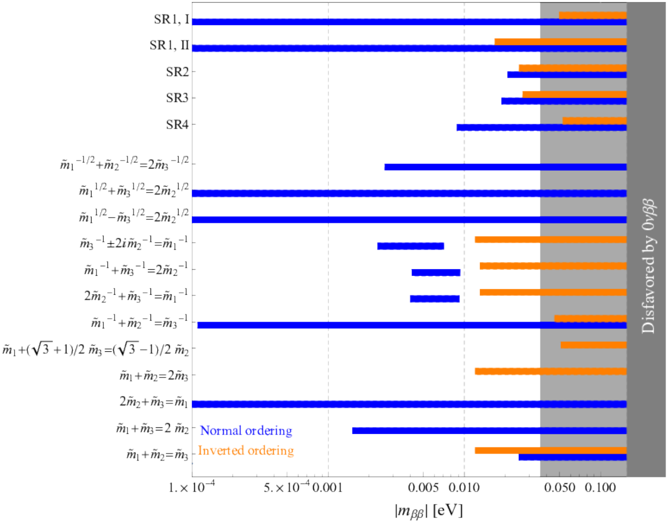
Sum Rule	Group	Seesaw Type
$\tilde{m}_1 + \tilde{m}_2 = \tilde{m}_3$ $\tilde{m}_1 + \tilde{m}_2 = \tilde{m}_3$	A_4; S_4; A_5 $\Delta(54)$; S_4	Weinberg Type II
$\tilde{m}_1 + 2\tilde{m}_2 = \tilde{m}_3$	S_4	Type II
$2\tilde{m}_2 + \tilde{m}_3 = \tilde{m}_1$ $2\tilde{m}_2 + \tilde{m}_3 = \tilde{m}_1$	A_4 S_4; T'; T_7 A_4	Weinberg Type II
$\tilde{m}_1 + \tilde{m}_2 = 2\tilde{m}_3$ $\tilde{m}_1 + \tilde{m}_2 = 2\tilde{m}_3$	S_4 $L_e - L_\mu - L_\tau$	Dirac Type II
$\tilde{m}_1 + \frac{\sqrt{3}+1}{2}\tilde{m}_3 = \frac{\sqrt{3}-1}{2}\tilde{m}_2$	A_5'	Weinberg
$\tilde{m}_1^{-1} + \tilde{m}_2^{-1} = \tilde{m}_3^{-1}$ $\tilde{m}_1^{-1} + \tilde{m}_2^{-1} = \tilde{m}_3^{-1}$	A_4; S_4; A_5 S_4	Type I Type III
$2\tilde{m}_2^{-1} + \tilde{m}_3^{-1} = \tilde{m}_1^{-1}$ $\tilde{m}_1^{-1} + \tilde{m}_3^{-1} = 2\tilde{m}_2^{-1}$	A_4; T' A_4; T'	Type I Type I
$\tilde{m}_3^{-1} \pm 2i\tilde{m}_2^{-1} = \tilde{m}_1^{-1}$	$\Delta(96)$	Type I
$\tilde{m}_1^{1/2} - \tilde{m}_3^{1/2} = 2\tilde{m}_2^{1/2}$ $\tilde{m}_1^{1/2} + \tilde{m}_3^{1/2} = 2\tilde{m}_2^{1/2}$	A_4 A_4	Type I Scotogenic
$\tilde{m}_1^{-1/2} + \tilde{m}_2^{-1/2} = 2\tilde{m}_3^{-1/2}$	S_4	Inverse

Flavor Symmetry and $0\nu\beta\beta$ Experiments:

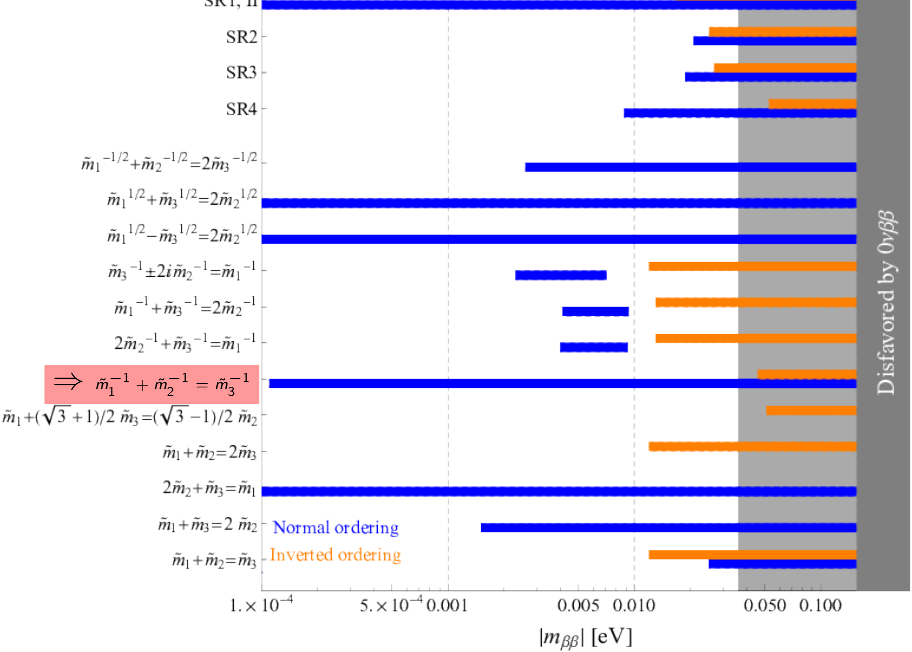


Models with Sum Rules; King, Marle, Stuart 1307.2901

Flavor Symmetry and $0\nu\beta\beta$ Experiments:



Models with Sum Rules ; Snowmass White paper Cirigliano et al. 2203.12169


SR1, II
SR2
SR3
SR4
$\tilde{m}_1^{-1/2}+\tilde{m}_2^{-1/2}=2\tilde{m}_3^{-1/2}$
$\tilde{m}_1^{1/2}+\tilde{m}_3^{1/2}=2\tilde{m}_2^{1/2}$
$\tilde{m}_1^{1/2}-\tilde{m}_3^{1/2}=2\tilde{m}_2^{1/2}$
$\tilde{m}_3^{-1}\pm 2i\tilde{m}_2^{-1}=\tilde{m}_1^{-1}$
$\tilde{m}_1^{-1}+\tilde{m}_3^{-1}=2\tilde{m}_2^{-1}$
$2\tilde{m}_2^{-1}+\tilde{m}_3^{-1}=\tilde{m}_1^{-1}$
$\Longrightarrow \tilde{m}_1^{-1}+\tilde{m}_2^{-1}=\tilde{m}_3^{-1}$
$\tilde{m}_1+(\sqrt{3}+1)/2\,\tilde{m}_3=(\sqrt{3}-1)/2\,\tilde{m}_2$
$\tilde{m}_1+\tilde{m}_2=2\tilde{m}_3$
$2\tilde{m}_2+\tilde{m}_3=\tilde{m}_1$
$\tilde{m}_1+\tilde{m}_3=2\,\tilde{m}_2$
$\tilde{m}_1+\tilde{m}_2=\tilde{m}_3$
Normal ordering
Inverted ordering
Disfavored by $0\nu\beta\beta$
$1.\times 10^{-4}$
$5.\times 10^{-4}$
0.001
0.005
0.010
0.050
0.100
$|m_{\beta\beta}|$ [eV]

Flavor Symmetry and $0\nu\beta\beta$ Experiments:

Sum Rule	Group	Seesaw Type
$\tilde{m}_1 + \tilde{m}_2 = \tilde{m}_3$	A_4; S_4; A_5	Weinberg
$\tilde{m}_1 + \tilde{m}_2 = \tilde{m}_3$	$\Delta(54)$; S_4	Type II
$\tilde{m}_1 + 2\tilde{m}_2 = \tilde{m}_3$	S_4	Type II
$2\tilde{m}_2 + \tilde{m}_3 = \tilde{m}_1$	A_4	Weinberg
	S_4; T'; T_7	
$2\tilde{m}_2 + \tilde{m}_3 = \tilde{m}_1$	A_4	Type II
$\tilde{m}_1 + \tilde{m}_2 = 2\tilde{m}_3$	S_4	Dirac
$\tilde{m}_1 + \tilde{m}_2 = 2\tilde{m}_3$	$L_e - L_\mu - L_\tau$	Type II
$\tilde{m}_1 + \frac{\sqrt{3}+1}{2}\tilde{m}_3 = \frac{\sqrt{3}-1}{2}\tilde{m}_2$	A'_5	Weinberg
$\tilde{m}_1^{-1} + \tilde{m}_2^{-1} = \tilde{m}_3^{-1}$	A_4; S_4; A_5	Type I
$\tilde{m}_1^{-1} + \tilde{m}_2^{-1} = \tilde{m}_3^{-1}$	S_4	Type III
$2\tilde{m}_2^{-1} + \tilde{m}_3^{-1} = \tilde{m}_1^{-1}$	A_4; T'	Type I
$\tilde{m}_1^{-1} + \tilde{m}_3^{-1} = 2\tilde{m}_2^{-1}$	A_4; T'	Type I
$\tilde{m}_3^{-1} \pm 2i\tilde{m}_2^{-1} = \tilde{m}_1^{-1}$	$\Delta(96)$	Type I
$\tilde{m}_1^{1/2} - \tilde{m}_3^{1/2} = 2\tilde{m}_2^{1/2}$	A_4	Type I
$\tilde{m}_1^{1/2} + \tilde{m}_3^{1/2} = 2\tilde{m}_2^{1/2}$	A_4	Scotogenic
$\tilde{m}_1^{-1/2} + \tilde{m}_2^{-1/2} = 2\tilde{m}_3^{-1/2}$	S_4	Inverse

Conclusion

- Is there any guiding principle behind the observed pattern of lepton mixing?
- (Discrete) flavor symmetry is one such potential candidate.
- What is the origin of such symmetries?
- What additional role they can play?
- How to falsify this plethora of models?
- If flavor symmetry is not the guiding principle, what else?

Thank you for your attention!!

Gravitational wave signatures from discrete flavor symmetries

Graciela B. Gelmini,[a] **Silvia Pascoli,**[b,c] **Edoardo Vitagliano**[a] **and Ye-Ling Zhou**[d]

[a]Department of Physics and Astronomy, University of California, Los Angeles
475 Portola Plaza, Los Angeles, CA 90095-1547, USA

[b]Institute for Particle Physics Phenomenology, Department of Physics, Durham University, Lower Mountjoy, South Rd, Durham DH1 3LE, United Kingdom

[c]Dipartimento di Fisica e Astronomia, Università di Bologna,
Via Irnerio 46, I-40126 Bologna, Italy

[d]Department of Physics and Astronomy, University of Southampton
University Rd, Southampton SO17 1BJ, United Kingdom

E-mail: gelmini@physics.ucla.edu, silvia.pascoli@durham.ac.uk, edoardo@physics.ucla.edu, ye-ling.zhou@soton.ac.uk

Abstract. Non-Abelian discrete symmetries have been widely used to explain the patterns of lepton masses and flavor mixing. In these models, a given symmetry is assumed at a high scale and then is spontaneously broken by scalars (the flavons), which acquire vacuum expectation values. Typically, the resulting leading order predictions for the oscillation parameters require corrections in order to comply with neutrino oscillation data. We introduce such corrections through an explicit small breaking of the symmetry.

- Multiplication Rules:

It has four irreducible representations: three one-dimensional and one three dimensional which are denoted by $\mathbf{1}, \mathbf{1}', \mathbf{1}''$ and $\mathbf{3}$ respectively. The multiplication rules of the irreducible representations are given by

$$\mathbf{1} \otimes \mathbf{1} = \mathbf{1}, \mathbf{1}' \otimes \mathbf{1}' = \mathbf{1}'', \mathbf{1}' \otimes \mathbf{1}'' = \mathbf{1}, \mathbf{1}'' \otimes \mathbf{1}'' = \mathbf{1}', \mathbf{3} \otimes \mathbf{3} = \mathbf{1} + \mathbf{1}' + \mathbf{1}'' + \mathbf{3_a} + \mathbf{3_s} \tag{2}$$

where **a** and **s** in the subscript corresponds to anti-symmetric and symmetric parts respectively. Now, if we have two triplets as $A = (a_1, a_2, a_3)^T$ and $B = (b_1, b_2, b_3)^T$ respectively, their direct product can be decomposed into the direct sum mentioned above. The product rule for this two triplets in the S diagonal basis[1] can be written as

$$(A \times B)_{\mathbf{1}} \backsim a_1 b_1 + a_2 b_2 + a_3 b_3, \tag{3}$$

$$(A \times B)_{\mathbf{1}'} \backsim a_1 b_1 + \omega^2 a_2 b_2 + \omega a_3 b_3, \tag{4}$$

$$(A \times B)_{\mathbf{1}''} \backsim a_1 b_1 + \omega a_2 b_2 + \omega^2 a_3 b_3, \tag{5}$$

$$(A \times B)_{\mathbf{3_s}} \backsim (a_2 b_3 + a_3 b_2, a_3 b_1 + a_1 b_3, a_1 b_2 + a_2 b_1), \tag{6}$$

$$(A \times B)_{\mathbf{3_a}} \backsim (a_2 b_3 - a_3 b_2, a_3 b_1 - a_1 b_3, a_1 b_2 - a_2 b_1), \tag{7}$$

here ω $(= e^{2i\pi/3})$ is the cube root of unity

[1] Here S is a 3×3 diagonal generator of A_4.

$$Y_B \approx \sum Y_{Bi} \tag{8}$$

where

$$Y_{Bi} \simeq -1.48 \times 10^{-3} \epsilon_i \eta_{ii}. \tag{9}$$

Y_{Bi}'s are coming from decay of each RH neutrinos and η_{ii} stands for efficiency factor [hep-ph/0310123] when $M_i < 10^{14}$ GeV,

$$\frac{1}{\eta_{ii}} \approx \frac{3.3 \times 10^{-3}\ \mathrm{eV}}{\tilde{m}_i} + \left(\frac{\tilde{m}_i}{0.55 \times 10^{-3}\ \mathrm{eV}}\right)^{1.16}, \tag{10}$$

with washout mass parameter, $\tilde{m}_i = \frac{(\hat{Y}_\nu^\dagger \hat{Y}_\nu)_{ii} v_u^2}{M_i}$.