Probing primordial black holes from a first order phase transition through pulsar timing and gravitational wave signals

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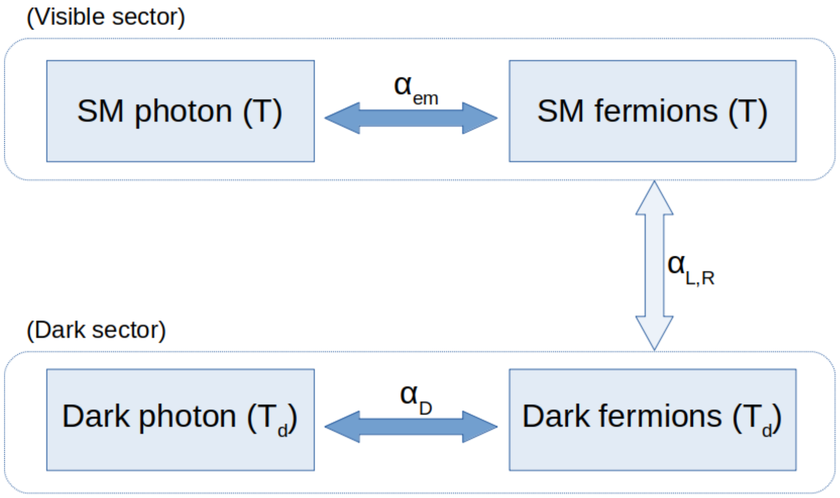
[JTA, Po-Yen Tseng (曾柏彥), JHEP 08 (2023) 117]

The 3rd International Joint Workshop on the Standard Model and Beyond

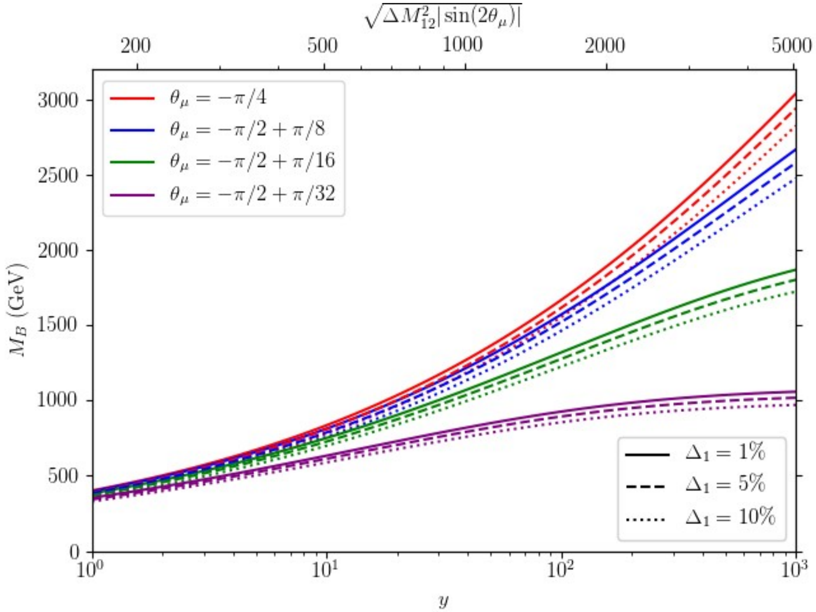
&

11th KIAS Workshop on Particle Physics and Cosmology

15 November 2023



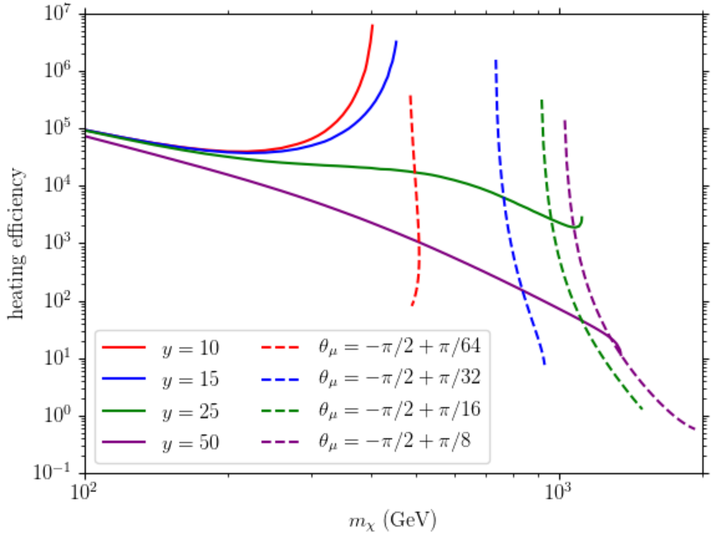
JTA, Marco Fabbrichesi, Piero Ullio (2020). Physical Review D, 102(8), 083009.



$$\mathcal{L} \supset m_Z^2 \sum_j \mathcal{S}_j^\dagger \left(\hat{T}_3 - \hat{Q}_{em} s_W^2\right) \mathcal{S}_j \left(1 + \frac{h}{v}\right)^2 - \frac{m_{LR}^2}{v} h(\tilde{\mu}_L \tilde{\mu}_R^\dagger + \text{h.c.})$$

JTA, Patrick Stengel, Piero Ullio (2022). Physical Review D, 105, 075007.

Deviation from MFV



$$T = (1600\,K)\, f^{1/4}$$

JTA, Patrick Stengel, Piero Ullio hep-ph/arxiv:2209.12552

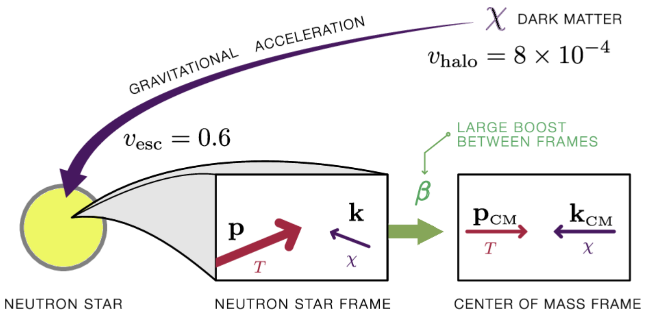


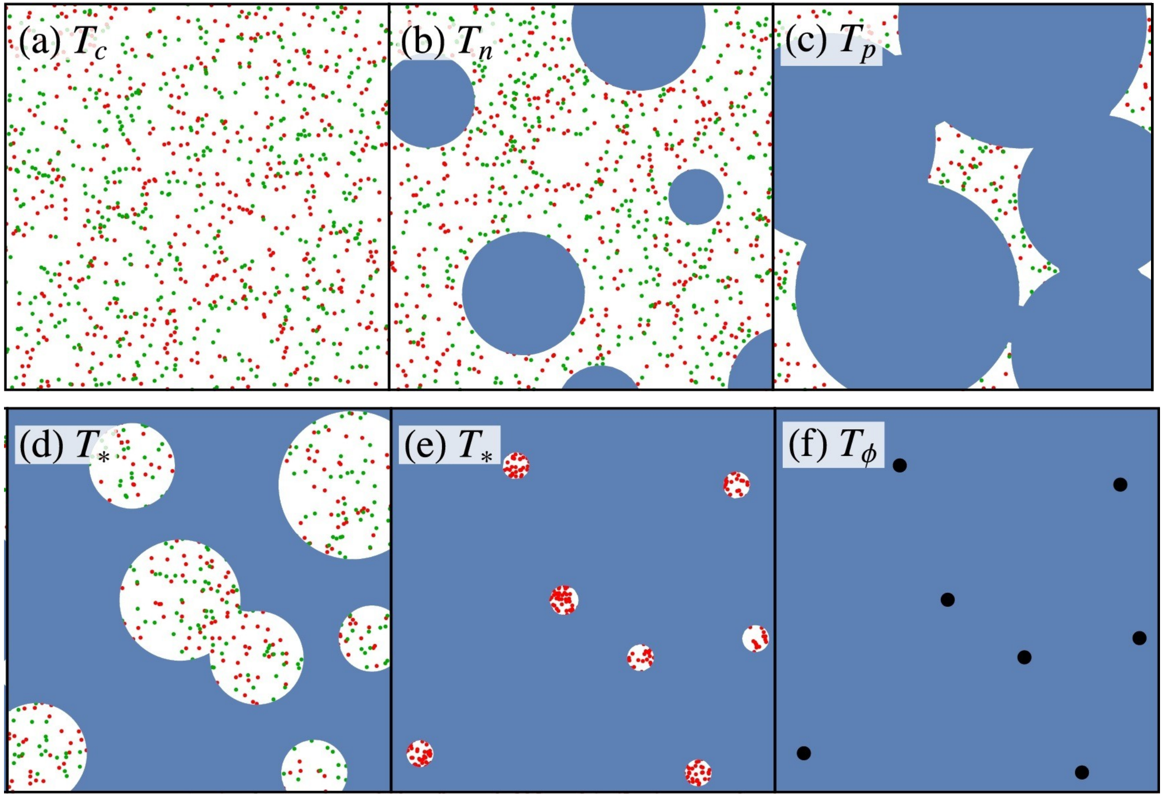
Figure taken from Phys. Rev. D **102**, 123002

Outline

- Primordial black holes (*see talk by Po-Yen Tseng, cf. Jongkuk Kim)*
- Novel PBH formation mechanism: Fermi ball collapse from dark FOPT

  (see talk by Po-Yen Tseng; cf. talks by Tomasz Dutka, Tzu-Chiang Yuan, Ryuusuke Jinno)
- Pulsar timing *(cf. talk by Xing-Yu Yang)*
  - Doppler and Shapiro
  - Constraints
- Complementary probe: stochastic gravitational waves (*cf. talks by Asuka Ito, Yue-Lin Sming Tsai, Kingman Cheung, Jinsu Kim, Liliana Velasco-Sevilla, Qiuyue Liang*)
- Generic quartic potential
- Summary

Novel PBH formation mechanism



FV bubbles split (f_{FV}=0.29)

Fermi ball formation

Collapse to PBH

$$L = \frac{1}{2}(\partial \phi)^2 - V_{eff}(\phi, T)$$
$$+ \bar{\chi}(i\gamma^\mu \partial_\mu - m_\chi)\chi - g_\chi \phi \bar{\chi}\chi$$

Percolation sets initial conditions:

- -low η_χ
- -low chemical potential

FB formation:

- -Q_{FB} is conserved
- -net χ-anti χ is larger
- -huge chemical potential

PBH formation:

- -Jeans like instability

Panels (a)-(c) taken from: Kawana, Kiyoharu, and Ke-Pan Xie. "Primordial black holes from a cosmic phase transition: The collapse of Fermi-balls." Physics Letters B 824 (2022): 136791.

Novel PBH formation mechanism

- Characterized by a continuous mass distribution
- PBH distribution determined by[2]
  - Generic FOPT parameters: α_{tr}, β/H_*, T_*, T_c
  - Derived FOPT parameters: bubble wall velocity (Chapman-Jouguet; detonations)
  - Other parameters: DM asymmetry parameter (η_χ), $\xi = T/T_{SM}$

$$P(M) = \frac{R_*}{3\left(12\pi^2 \Delta V_{eff}(T_*)\right)^{1/4}} \frac{dn}{dR_r}(t_*) \; \frac{1}{\eta_\chi s_v(t_*)}$$

$$\alpha_{tr} = \frac{\left(1 - T/4\, d/dT\right)\Delta V_{eff}}{\rho_R}$$

$$\Delta V_{eff} \approx \epsilon_c \left(1 - \frac{T}{T_c}\right)$$

$$\frac{\beta}{H_*} \simeq T_* \frac{d}{dT}\left[\frac{S_3}{T}\right]_{T=T_*}$$

$$v_w = \frac{1}{\sqrt{3}} \frac{1 + \sqrt{2\alpha_d + 3\alpha_d^2}}{1 + \alpha_d}$$

$$\alpha_d = \alpha_{tr} \rho_R / \rho_d$$

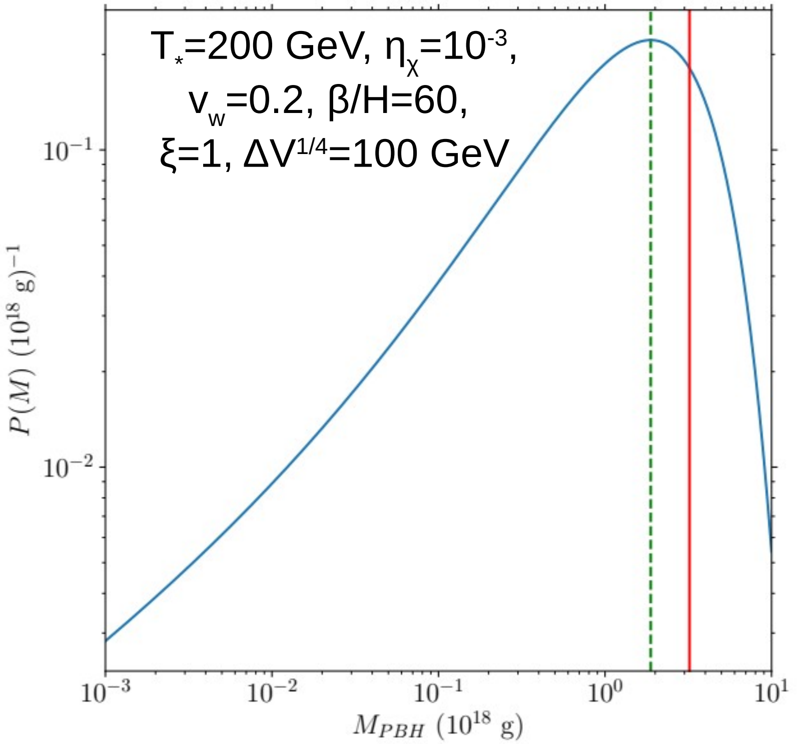
[2]Lu, Philip, Kiyoharu Kawana, and Ke-Pan Xie. "Old phase remnants in first-order phase transitions." Physical Review D 105.12 (2022): 123503.

Novel PBH formation mechanism

$$\langle M \rangle \simeq \boxed{(4.07\times 10^{-8} M_\odot)} \left(\frac{10.63}{g_*}\right)^{1/4} \left(\frac{0.1\,\mathrm{MeV}}{T_*}\right)^2 \left(\frac{\xi}{0.1}\right)^2$$
$$\times \left(\frac{\eta_\chi}{10^{-7}}\right) \left(\frac{v_w}{1}\right)^3 \left(\frac{2.5\times 10^2}{\beta/H_*}\right)^3 \left(\frac{\alpha_{tr}}{0.1}\right)^{1/4} \left[\frac{\mathcal{F}(T_*/T_c)}{0.308}\right]^{1/4}$$

$$\omega_{PBH,*} \simeq \boxed{0.434} \left(\frac{\alpha_{tr}}{0.1}\right)^{1/4} \left(\frac{g_*}{10.63}\right)^{1/4} \left(\frac{T_*}{0.1\,\mathrm{MeV}}\right)$$
$$\left(\frac{0.1}{\xi}\right) \left(\frac{\eta_\chi}{10^{-7}}\right) \left[\frac{\mathcal{F}(T_*/T_c)}{0.308}\right]^{1/4}$$

$$F(x) \equiv \frac{1-x}{1-3x/4}$$



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- ~~Primordial black holes~~
- ~~Novel PBH formation mechanism: Fermi ball collapse from dark FOPT~~
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  - Doppler and Shapiro
  - Constraints
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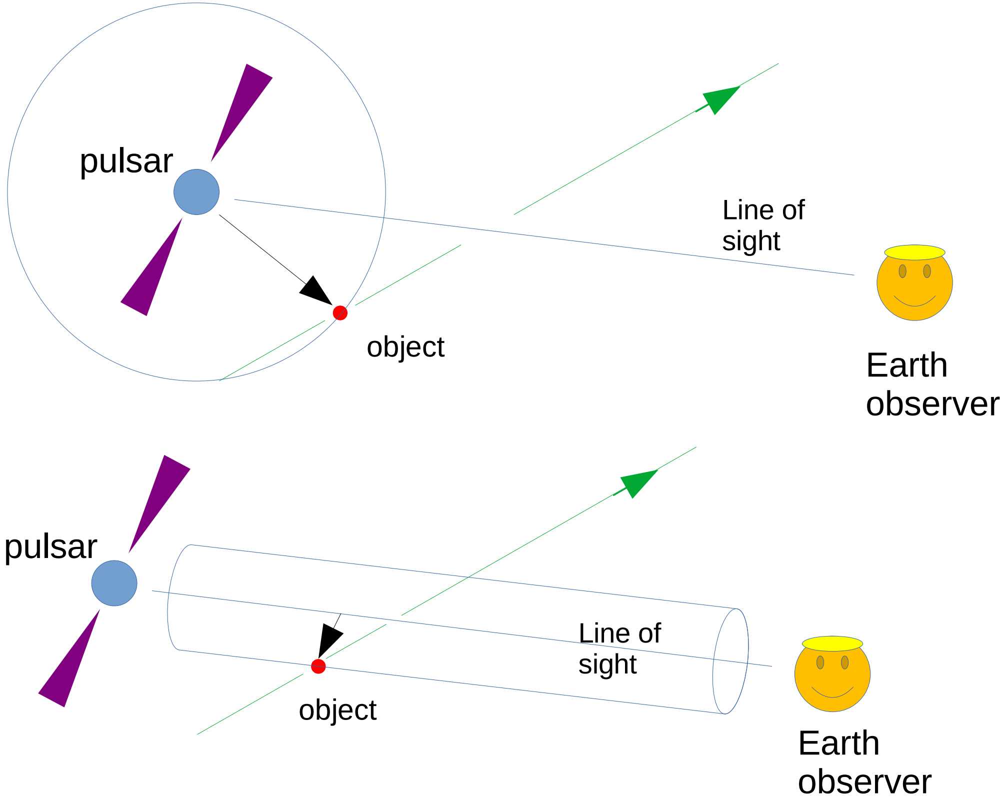
Pulsar timing: Doppler & Shapiro

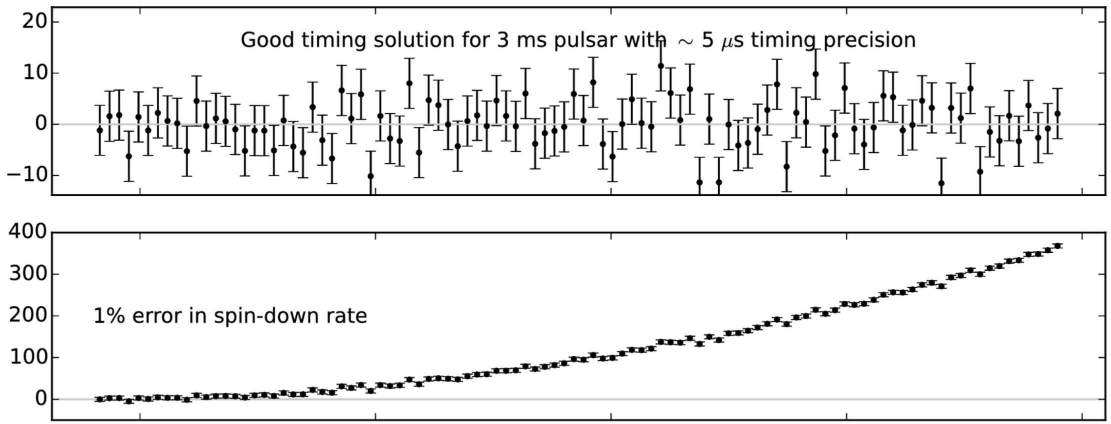


$$\left(\frac{\delta\nu}{\nu}\right)_D = \frac{1}{c}\hat{d}\cdot\int \vec{\nabla}\Phi \; dt$$

$$\left(\frac{\delta\nu}{\nu}\right)_S = -\frac{2}{c^3}\vec{v}\cdot\int \vec{\nabla}\Phi \; dz$$

$$\delta\phi = \int^t dt'\,\delta\nu(t')$$

Pulsar timing: Detector properties



Retrieved from: https://www.cv.nrao.edu/~sransom/web/Ch6.html

PTA property	Value
No. of pulsars	200
RMS timing residual	50 ns
Cadence	2 weeks
Total observation time	20 years

$$\phi(t) = \phi_0 + \nu t + \frac{\dot{\nu}}{2} t^2 + \frac{\ddot{\nu}}{6} t^3 + \dots$$

$$SNR = \frac{|\ddot{\nu}/\nu|}{\sigma_{\ddot{\nu}/\nu}}$$ (?)

$$\sigma_{\ddot{\nu}/\nu} = 6\sqrt{\frac{2800\,\Delta t}{T}\frac{t_{rms}}{T^3}}$$

(~2.8 x 10^{-33} Hz^2)

[3]Uncertainty in $\ddot{\nu}/\nu$

Dynamical (τ<<T) vs Static (τ>>T)[4]

[4]Dror, Jeff A., et al. "Pulsar timing probes of primordial black holes and subhalos." Physical Review D 100.2 (2019): 023003.

[3]Liu, X. J., C. G. Bassa, and B. W. Stappers. "High-precision pulsar timing and spin frequency second derivatives." Monthly Notices of the Royal Astronomical Society 478.2 (2018): 2359-2367.

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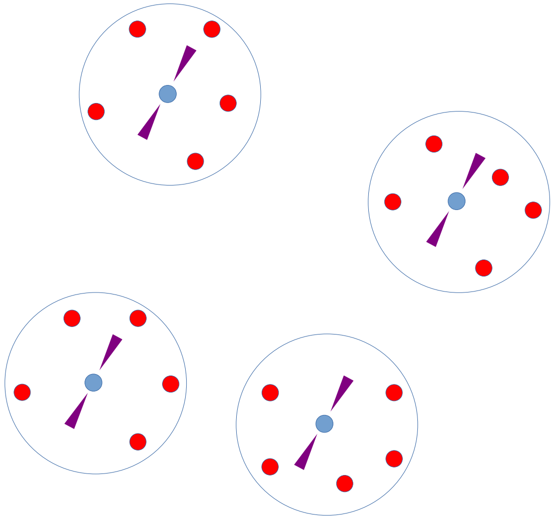
Pulsar timing: sensitivity limits[5,6]

$$\delta\phi = \int^{t} dt'\,\delta\nu(t')$$

$$\mathrm{SNR}_I^2 = \frac{1}{\nu_I^2 t_{rms}^2 \Delta t}\int_0^{T_{obs}} dt\; h_I^2(t).$$

$$h_I(t) = \sum_{i=1}^{N} \delta\phi_{I,i}(t) - \delta\phi_{0,I}(t)$$

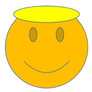
(f,M) -> size of simulation volume



$f\rho_{DM}/M$ -> # density

Maxwell-Boltzmann -> velocity assignment

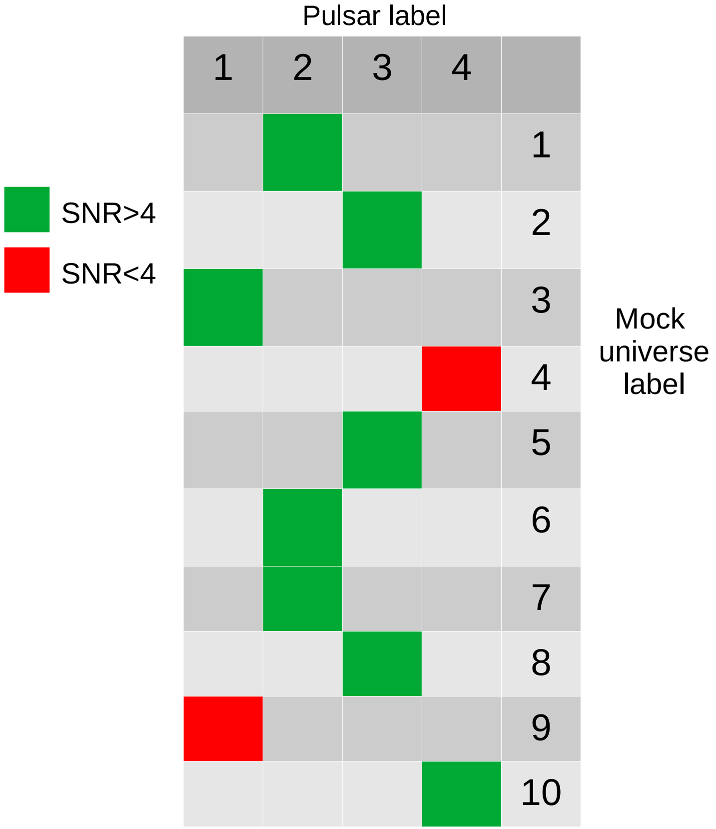
NB: We developed a parallelizable FORTRAN code to perform the simulation on a 72-core cluster



[5]Lee, Vincent SH, et al. "Probing small-scale power spectra with pulsar timing arrays." Journal of High Energy Physics 2021.6 (2021): 1-30

[6]Ramani, Harikrishnan, Tanner Trickle, and Kathryn M. Zurek. "Observability of dark matter substructure with pulsar timing correlations." Journal of Cosmology and Astroparticle Physics 2020.12 (2020): 033.

Pulsar timing: sensitivity limits[5,6]

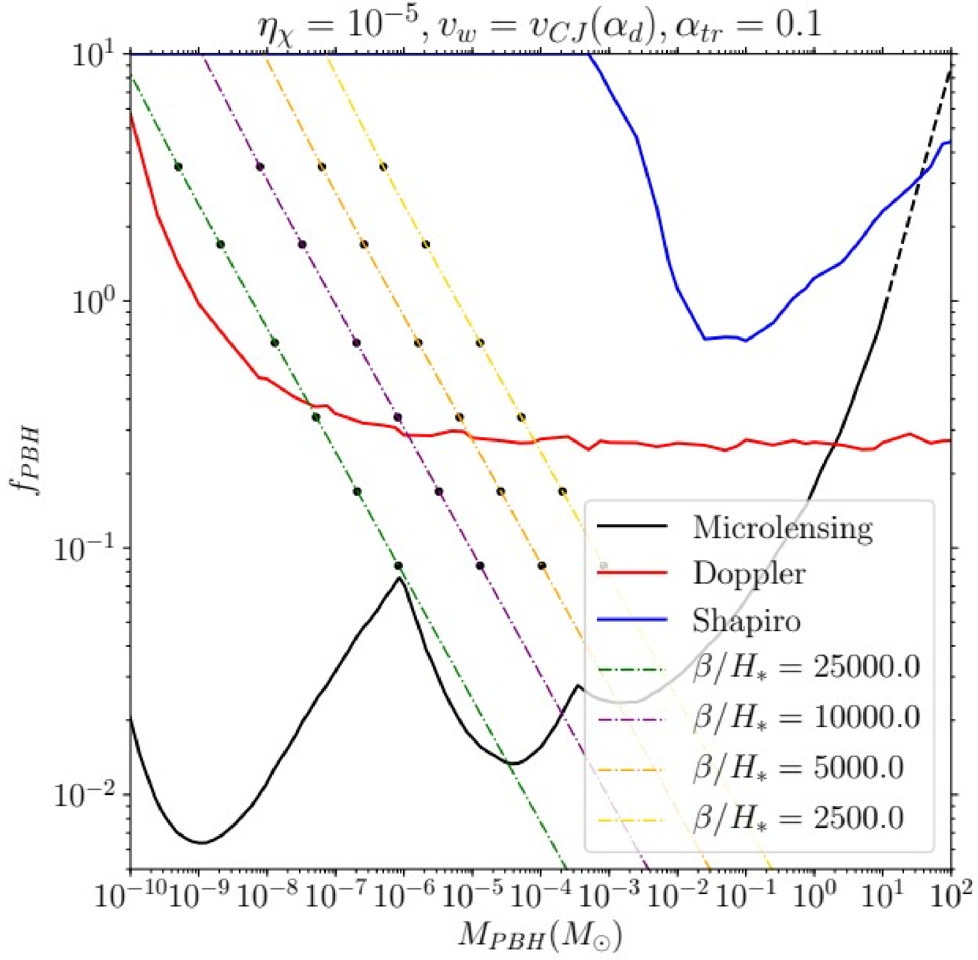


Sensitivity criterion: 90% of mock universes have max SNR > 4

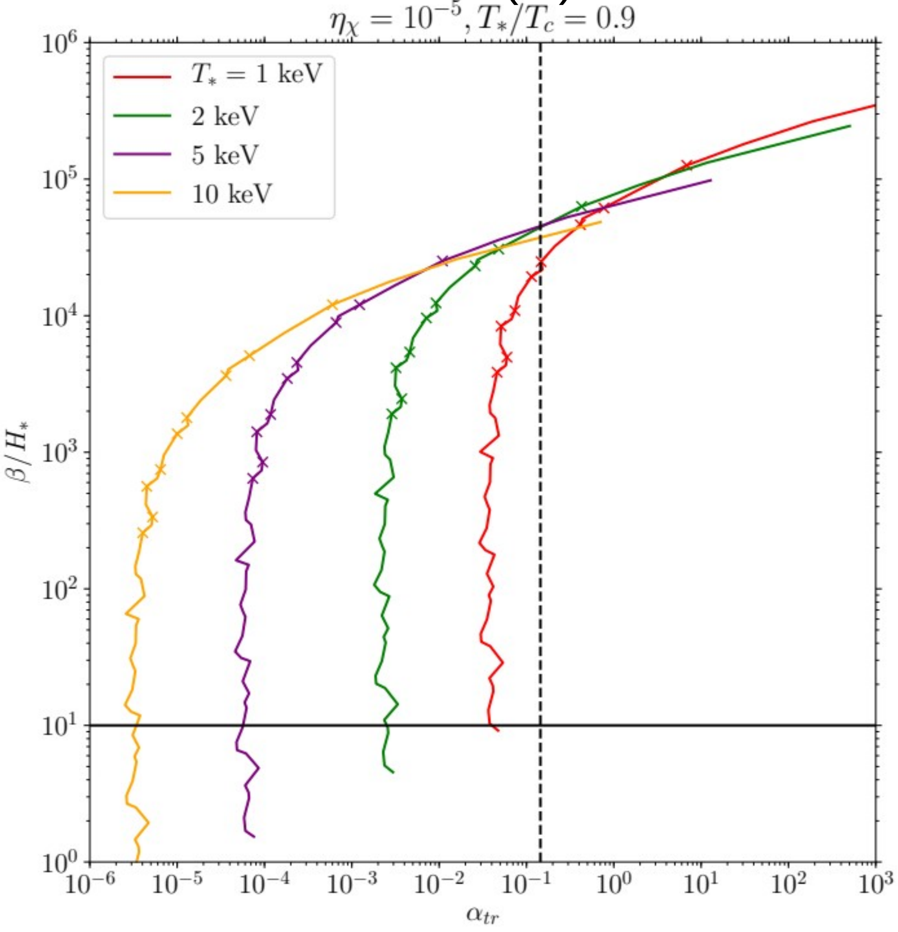
[5]Lee, Vincent SH, et al. "Probing small-scale power spectra with pulsar timing arrays." Journal of High Energy Physics 2021.6 (2021): 1-30

[6]Ramani, Harikrishnan, Tanner Trickle, and Kathryn M. Zurek. "Observability of dark matter substructure with pulsar timing correlations." Journal of Cosmology and Astroparticle Physics 2020.12 (2020): 033.

Monochromatic PBH mass (benchmark)



Novel PBH formation scenario w/ P(M)



[4]Dror, Jeff A., et al. "Pulsar timing probes of primordial black holes and subhalos." Physical Review D 100.2 (2019): 023003.
[7]D. Croon, D. McKeen, N. Raj and Z. Wang, Subaru-HSC through a different lens: Microlensing by extended dark matter structures, Phys. Rev. D 102 (2020) 083021 [2007.12697]

Plots from: JTA, Po-yen Tseng JHEP 08 (2023) 117

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Complementary signal: Stochastic GWs

- GW through sound waves, nonrunaway regime
- Assess sensitivity reach using some SNR
- Peak-integrated sensitivity curves (PISC)[8] as a means to calculate SNR

$$\Omega_s(f)h^2 = \Omega_s^{peak}h^2 \; \mathcal{S}_s(f, f_s)$$

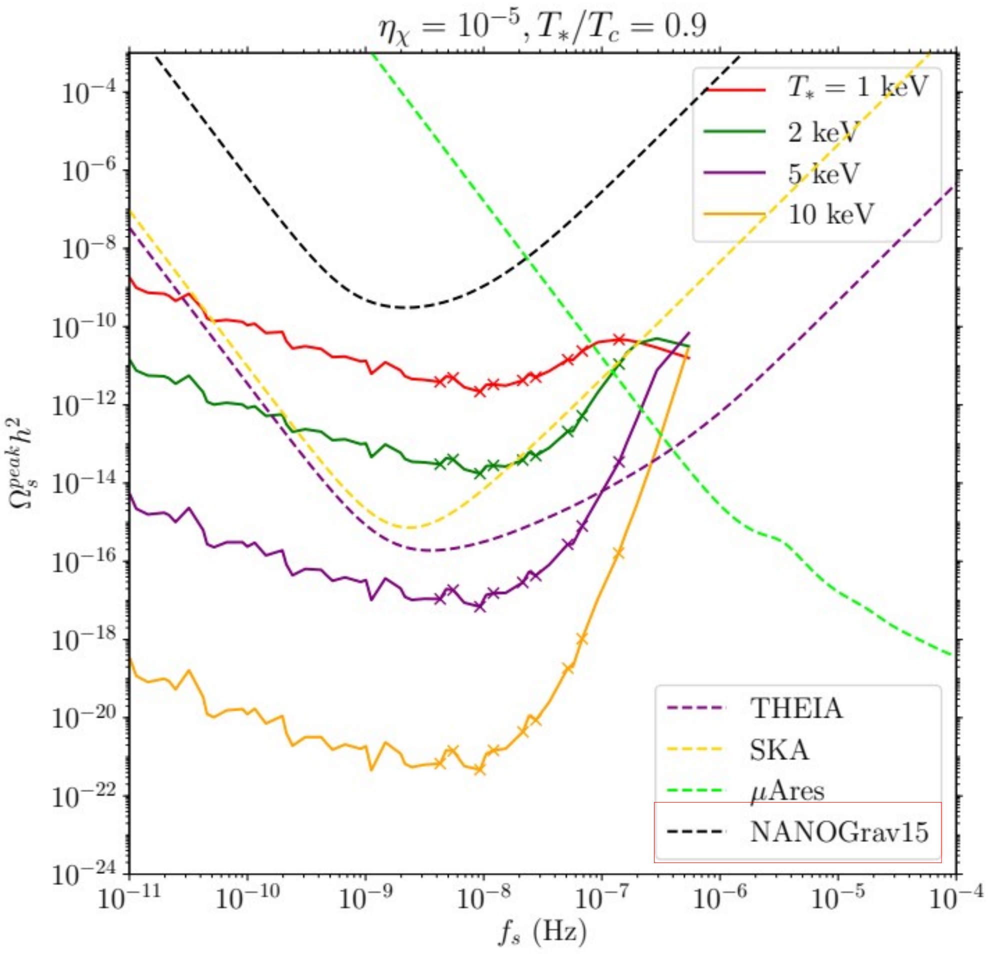
$$\mathcal{S}_s(f, f_s) = \left(\frac{f}{f_s}\right)^3 \left[\frac{7}{4+3(f/f_s)^2}\right]^{7/2}$$

$$\rho^2 \equiv n_{det}\tau_{obs,GW} \int_{f_{min}}^{f_{max}} df \; \left[\frac{\Omega_{sig}(f)h^2}{\Omega_{noise}(f)h^2}\right]^2$$

$$\rho(f_s) = \frac{\Omega_{peak}h^2}{\Omega_{PIS}(f_s)h^2}$$

$$\left(\Omega_{PIS}h^2\right)^{-2}(f_s) \equiv n_{det}\tau_{obs,GW} \int_{f_{min}}^{f_{max}} df \; \left[\frac{\mathcal{S}(f, f_s)}{\Omega_{noise}(f)h^2}\right]^2$$

[8] Schmitz, Kai. "New sensitivity curves for gravitational-wave signals from cosmological phase transitions." Journal of High Energy Physics 2021.1 (2021): 1-62.



Sensitivity criterion: sGW SNR > 1

$$\Omega_s^{peak} h^2 \simeq \boxed{2.65 \times 10^{-6}} \left(\frac{v_w}{\beta/H_*}\right) \left[\frac{100}{g_{*\rho,v}(T_*)}\right]^{1/3} \left(\frac{\kappa_s \alpha_{tr}}{1+\alpha_{tr}}\right)^2 \left(1 + \frac{g_{*\rho,d}}{g_{*\rho,v}}\xi^4\right)$$

$$f_s \simeq \boxed{1.9 \times 10^{-2}\,\mathrm{mHz}} \left[\frac{g_{*v}(T_*/\xi)}{100}\right]^{1/6} \left(\frac{T_*}{100\,\mathrm{GeV}}\right) \left(\frac{\beta/H_*}{v_w}\right) \left(1 + \frac{g_{*\rho,d}}{g_{*\rho,v}}\xi^4\right)^{1/2} \frac{1}{\xi}$$

SKA is sensitive to ~1 keV (~0.1 keV) for $\eta_\chi = 10^{-5}$ (10^{-4})

Plot from: JTA, Po-yen Tseng
JHEP 08 (2023) 117

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Generic quartic potential

$$V_{eff}(\phi, T) = D\left(T^2 - T_0^2\right)\phi^2 - (AT + C)\phi^3 + \frac{\lambda}{4}\phi^4$$

cf. Ref. [9]

$$-B = -DT_0^2\phi_0^2 - C\phi_0^3 + \frac{\lambda}{4}\phi_0^4$$
$$0 = -2DT_0^2 - 3C\phi_0 + \lambda\phi_0^2,$$

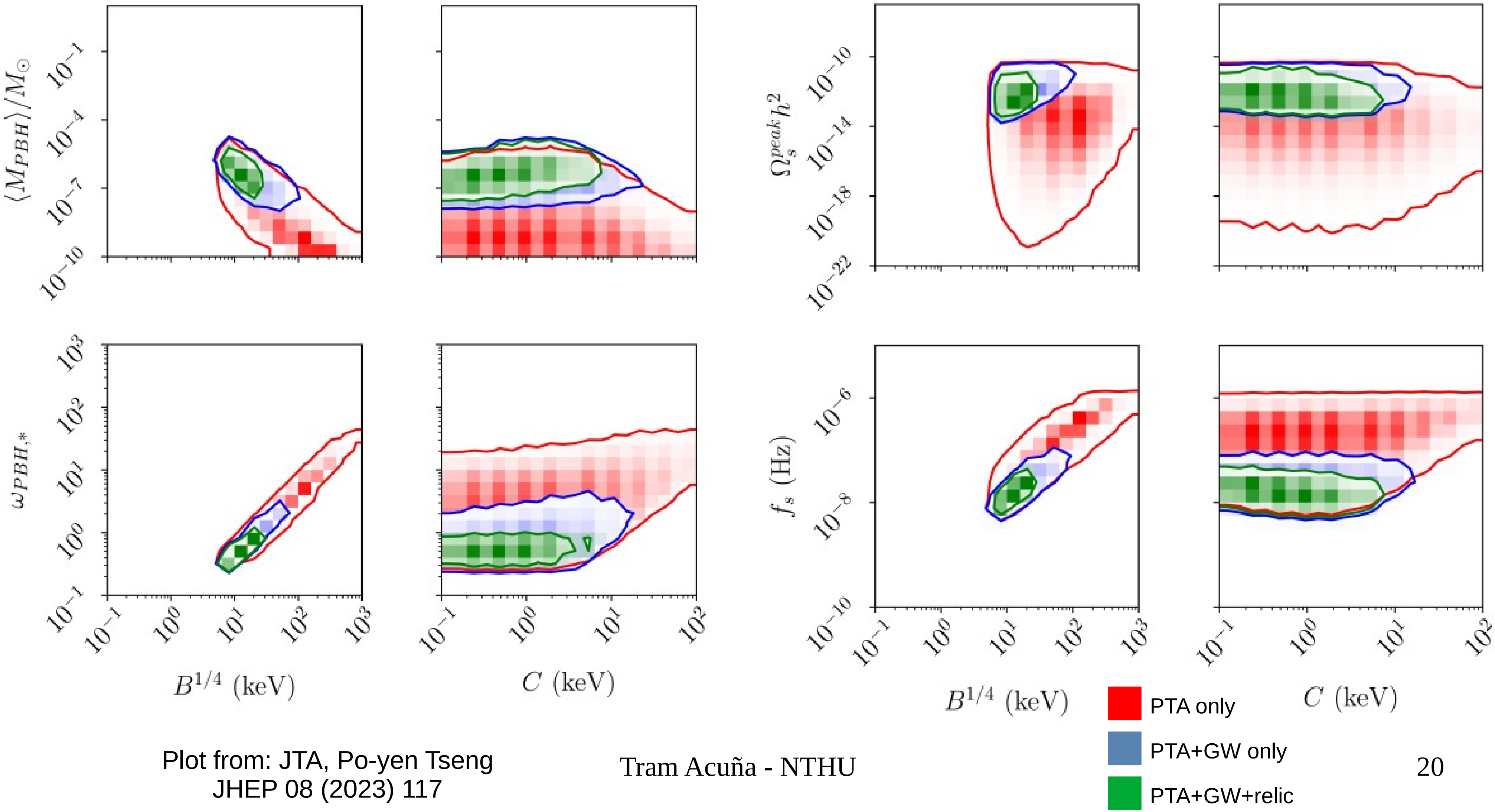
$\{B^{1/4}, C, D, \lambda\} \longrightarrow \{\eta_\chi, T_*, \alpha_{tr}, T_c, \xi, \beta/H_*, v_w\} \longrightarrow$ Observables:

Effective potential parameters

FOPT parameters

Observables:
-Ave. PBH mass
-PBH fraction
-Peak GW abundance
-Peak GW frequency

[9]Marfatia, Danny, and Po-Yen Tseng. "Correlated signals of first-order phase transitions and primordial black hole evaporation." Journal of High Energy Physics 2022.8 (2022): 1-14.



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Summary

- Presented dark FOPT scenario to produce PBHs and sGWs
- PTA facility can be used to also search for PBHs
- Parameter region: PBH mass of 10^{-8}~10^{-4}, GW frequency of nHz~μHz
- Parameter region: keV-scale FOPT, FOPT rate of 10^{3}~10^{4}, FOPT strength from 10^{-6}~0.1
- Obtained a viable class of generic quartic potentials

Acknowledgments

- NSTC grant # NSTC 111-2811-M-007-018-MY2
- NTHU IoA CICA cluster

Thank you for your attention!

고맙습니다 !

感謝各位的聆聽！

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Extra slides

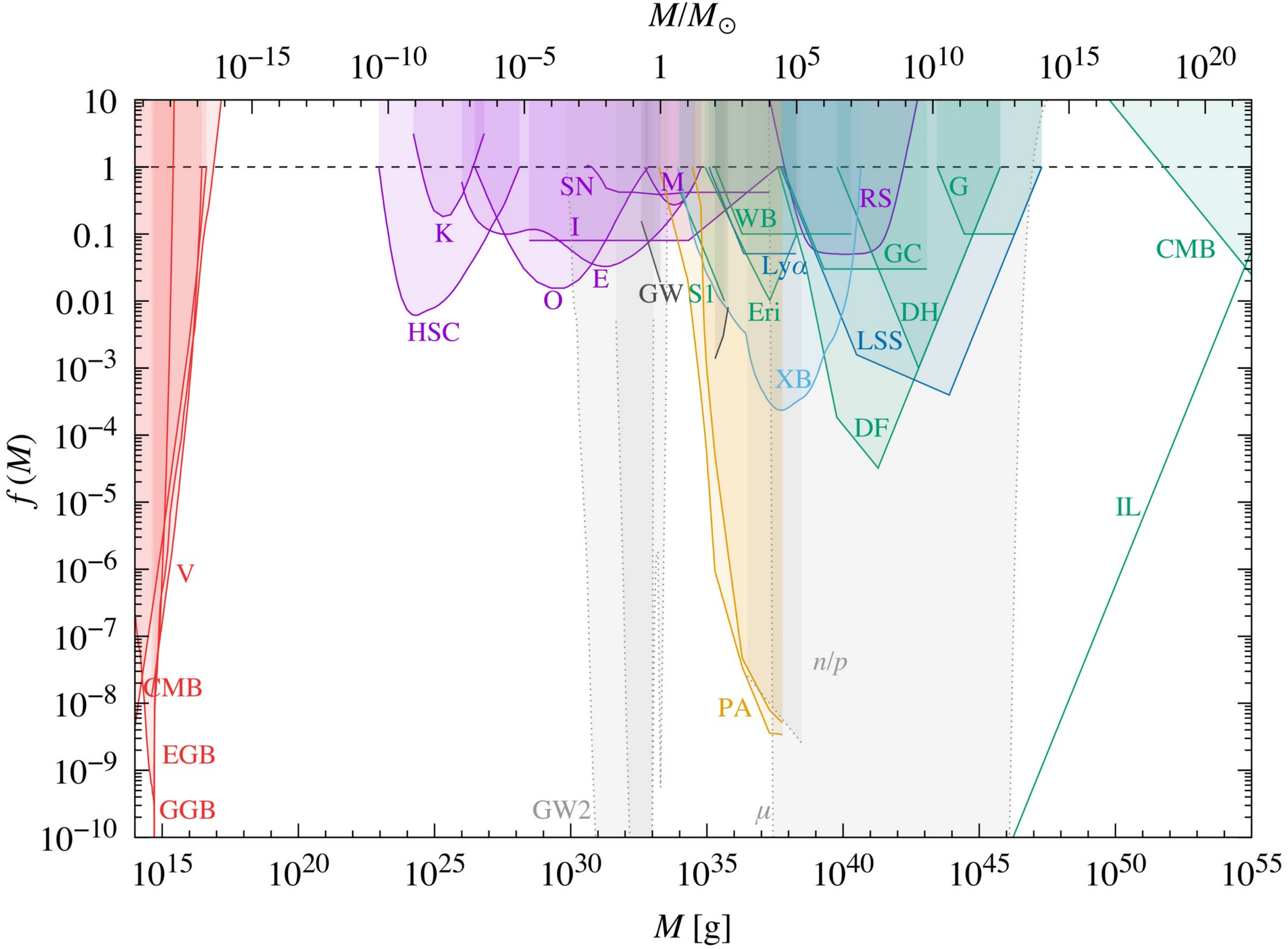


Figure from: B. Carr, K. Kohri, Y. Sendouda and J. Yokoyama, Constraints on primordial black holes, Rept. Prog. Phys. 84(11), 116902 (2021)

Primordial black holes

- Recall: Po-yen's PBH talk
- Overdensities in the early Universe may trigger collapse to PBH
- M>10^{-18} M$_{sol}$ survive until today
- Potential DM candidate
- Formation mechanism:

Collapse of Fermi balls from filtered out DM during <u>dark</u> FOPT[1]

$$\rho_{BH}=\frac{3}{8\pi G_N}\frac{1}{R_s^2}$$

$$\bar{\rho}=\frac{3}{8\pi G_N}\frac{1}{(1/H)^2}$$

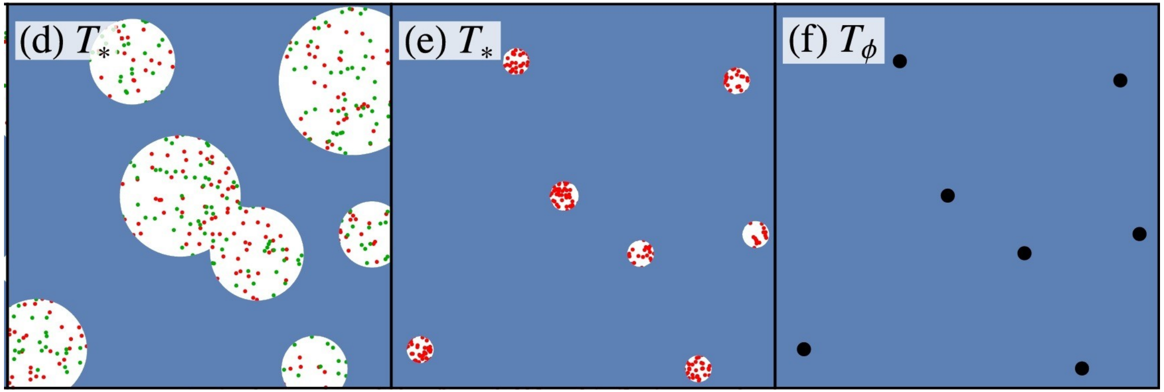
$$\tau=10^{64}\,y\,(M/M_{sol})^3$$

$$\xi(t)=\frac{T(t)}{T_{SM}(t)}$$

$$\frac{g_{*s,d}(T(t))T^3(t)}{g_{*s,v}(T_{SM}(t))T_{SM}^3(t)}=const.$$

[1]Kawana, Kiyoharu, and Ke-Pan Xie. "Primordial black holes from a cosmic phase transition: The collapse of Fermi-balls." Physics Letters B 824 (2022): 136791.

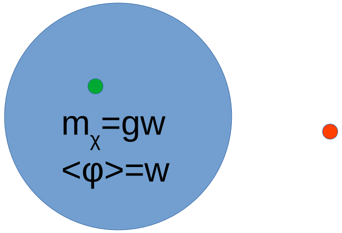
Extra: Novel PBH formation mechanism



FV bubbles split (f_{FV}=0.29)[4]

Fermi ball formation

Collapse to PBH



m_χ=0
<φ>=0

gw >> T_*

$$U_{tot}=\frac{3\pi}{4}\left(\frac{3}{2\pi}\right)^{2/3}\frac{Q_{FB}^{4/3}}{R}+\frac{4\pi}{3}\Delta V R^3$$

$$Q_{FB}=\frac{\eta_\chi s_v(t_*)}{f_{FV}(t_*)}A\frac{4\pi R_*^3}{3}$$

U(1) charge in FV bubble

$$M_{FB}=Q_{FB}\left(12\pi^2\Delta V_{eff}(T_*)\right)^{1/4}$$

$$f_{\bar{\chi},\chi}=\frac{1}{\exp[(p\pm\mu)/T]+1}$$

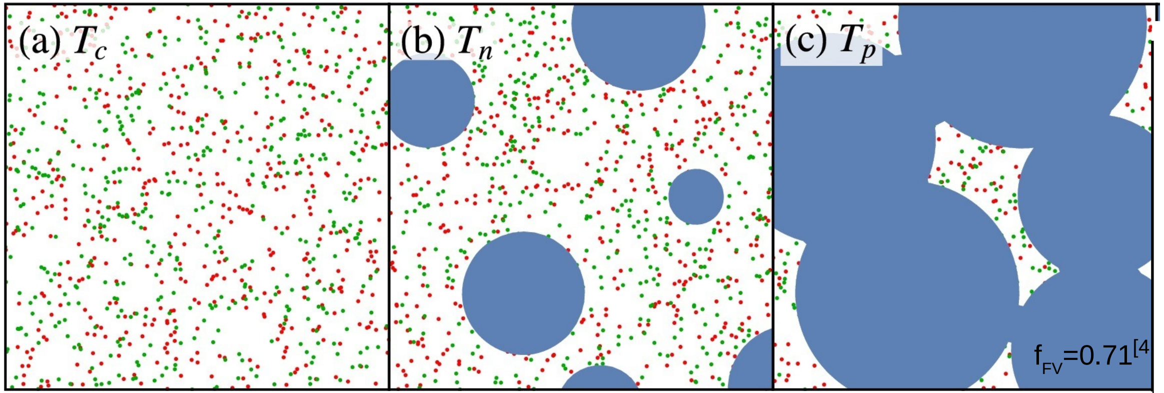
$$\rho=\frac{3}{4}\left(\frac{3}{\pi^2}\right)^{1/4}(n_\chi-n_{\bar{\chi}})^{3/4}$$

-FB cools down via χ → χ f f*
-Collapse via Jeans instability from additional Yukawa attractive force (see also 2110.00005 where they included gravity effects)
-What if you gauge dark U(1)? Additional repulsive force, cooling can be achieved by emitting the dark photon

Panels (d)-(f) taken from: Kawana, Kiyoharu, and Ke-Pan Xie. "Primordial black holes from a cosmic phase transition: The collapse of Fermi-balls." Physics Letters B 824 (2022): 136791.

[4]Rintoul, Mark D., and Salvatore Torquato. "Precise determination of the critical threshold and exponents in a three-dimensional continuum percolation model." Journal of physics a: mathematical and general 30.16 (1997): L585.

Extra: Novel PBH formation mechanism

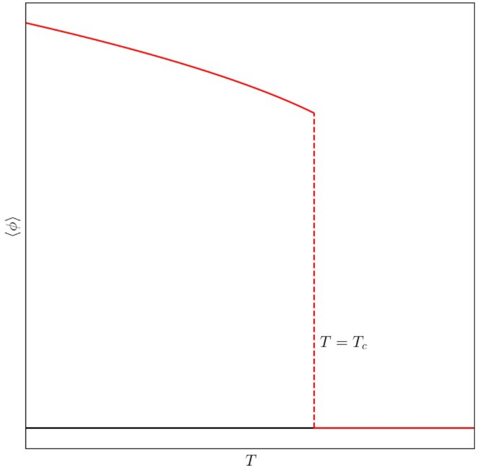


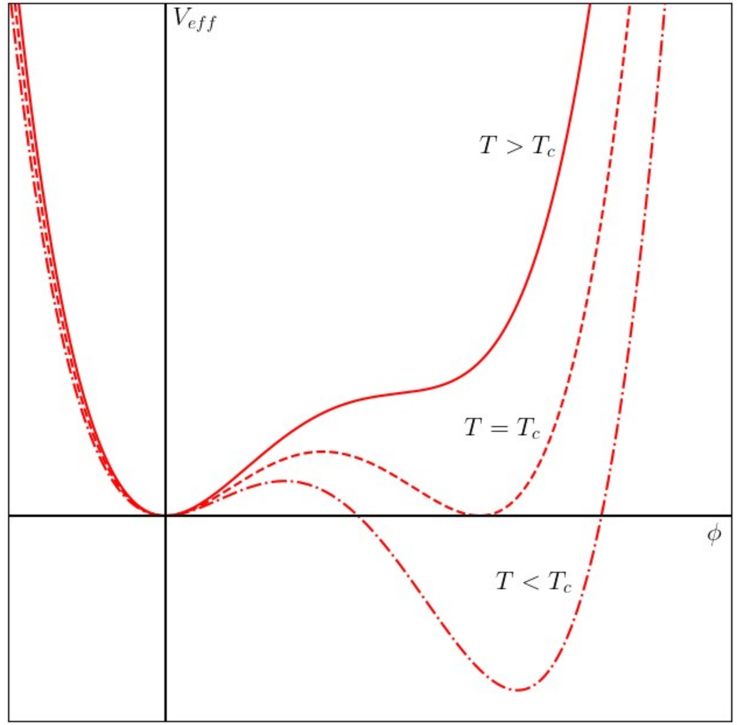
$$1 = \int_{t_c}^{t_n} dt' \frac{\Gamma(t')}{H^3(t')}$$

$$\Gamma = T^4 \left[\frac{S_3(T)}{2\pi T}\right]^{3/2} \exp\left[\frac{-S_3(T)}{T}\right]$$

$$f_{FV}(t) \equiv \exp[-I(t)]$$

$$I(t) \simeq \frac{4\pi}{3} v_w^3 \int_{t_c}^{t} dt''(t-t'')^3 \Gamma(T(t''))$$





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Extra: Novel PBH formation mechanism

- Mass distribution comes from distribution of radii of FV remnants

$$\frac{dn}{dR_r} = \frac{\beta^4}{192 v_w^4} I_*^4 \exp\left(4\beta R_r / v_w\right) \exp\left[-I(t)\right]\left\{1 - \exp\left[-I(t)\right]\right\}$$

[5]Lu, Philip, Kiyoharu Kawana, and Ke-Pan Xie. "Old phase remnants in first-order phase transitions." Physical Review D 105.12 (2022): 123503.

Extra: Novel PBH formation mechanism

- Characterized by a continuous mass distribution
- PBH distribution determined by[5]
  - Generic FOPT parameters: α_{tr}, β/H_*, T_*, T_c
  - Derived FOPT parameters: bubble wall velocity (Chapman-Jouguet; detonations)
  - Other parameters: DM asymmetry parameter (η_χ), $\xi = T/T_{SM}$

$$\alpha_{tr} = \frac{\left(1 - T/4\, d/dT\right)\Delta V_{eff}}{\rho_R}$$

$$\Delta V_{eff} \approx \epsilon_c \left(1 - \frac{T}{T_c}\right)$$

$$\frac{\beta}{H_*} \simeq T_* \frac{d}{dT}\left[\frac{S_3}{T}\right]_{T=T_*}$$

$$v_w = \frac{1}{\sqrt{3}} \frac{1 + \sqrt{2\alpha_d + 3\alpha_d^2}}{1 + \alpha_d}$$

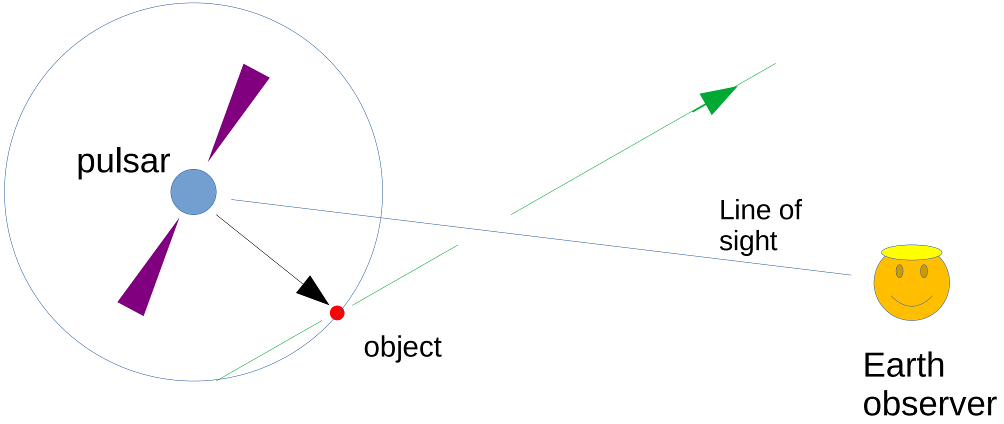
$$\alpha_d = \alpha_{tr} \rho_R / \rho_d$$

Deflagration: wall drags behind shock
Detonation: shock drags behind wall
Jouguet: $v_- = c_s$

-Numerical simulation of bubble wall dynamics requires us to solve hydrodynamical equations to track the fluid evolution, and KG equation + friction term due to heavy species colliding with the bubble wall
-Classification of steady state solutions:

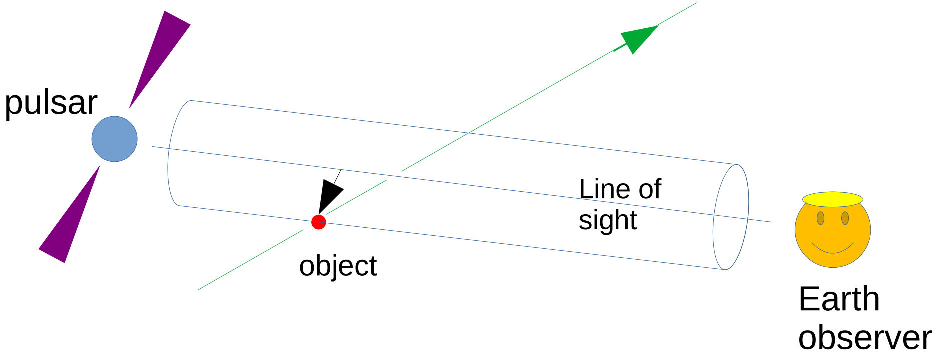
	$v_-^2 < 1/3$	$v_-^2 > 1/3$
$v_+^2 > 1/3$	Weak deflagration	Strong deflagration
$v_+^2 < 1/3$	Strong detonation	Weak detonation

Pulsar timing: Doppler & Shapiro



$$\left(\frac{\delta\nu}{\nu}\right)_D = \frac{1}{c}\hat{d}\cdot\int \vec{\nabla}\Phi\ dt$$

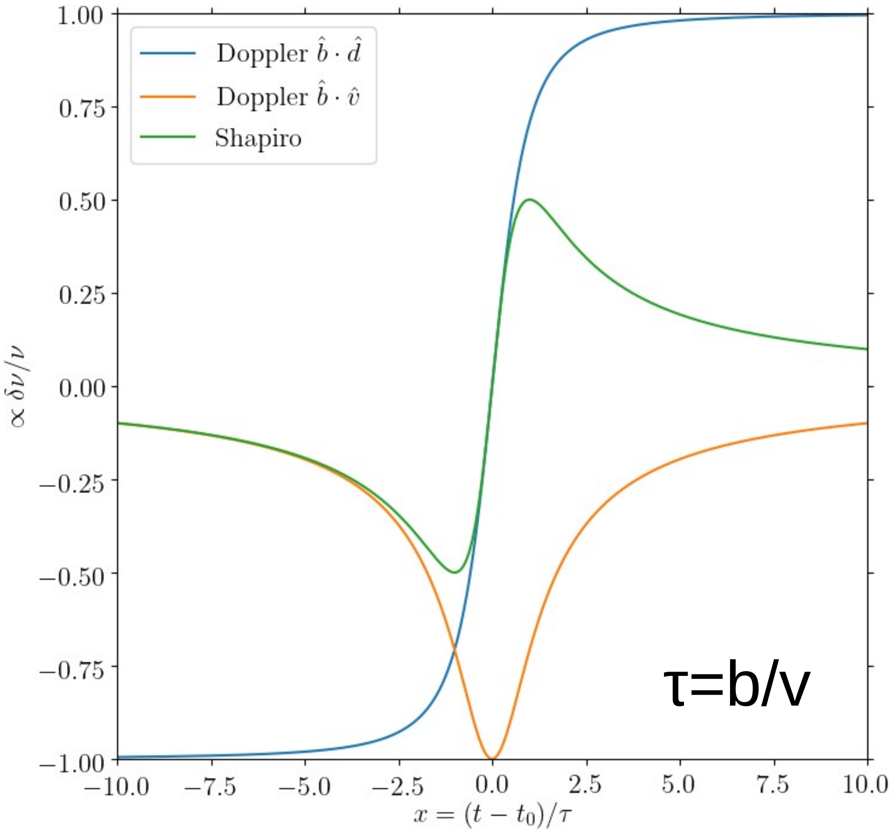
$$\Gamma_D \sim \pi \nu n^{1/3} \sim 0.1 f^{1/3} \left(\frac{M}{10^{-8} M_{sun}}\right)^{-1/3} yr^{-1}$$



$$\left(\frac{\delta\nu}{\nu}\right)_S = -\frac{2}{c^3}\vec{v}\cdot\int \vec{\nabla}\Phi\ dz$$

$$\Gamma_S \sim \pi^{1/2} \nu (nL)^{1/2} \sim 0.17 f^{1/2} \left(\frac{M}{10^{-4} M_{sun}}\right)^{-1/2} \left(\frac{L}{10\,kpc}\right)^{1/2} yr^{-1}$$

Example: monochromatic, pointlike PBHs



$$\left(\frac{\delta\nu}{\nu}\right)_D = \frac{G_N M_{PBH}}{v^2 c \tau_D} \frac{1}{\sqrt{1+x_D^2}} (x_D \hat{b} - \hat{v}) \cdot \hat{d}$$

$$\left(\frac{\delta\nu}{\nu}\right)_S = \frac{4 G_N M_{PBH}}{c^3 \tau_S} \frac{x_S}{1+x_S^2}.$$

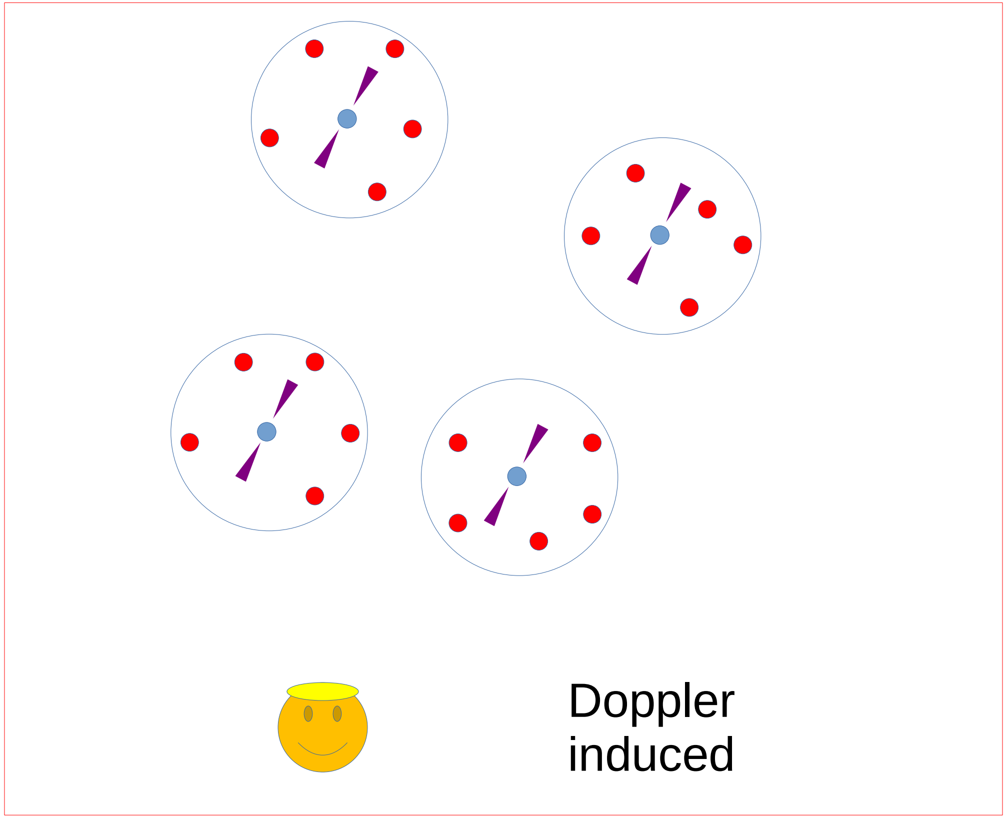
$$SNR = \frac{|\ddot{\nu}/\nu|}{\sigma_{\ddot{\nu}/\nu}}$$ (~10^{-34} N_pf Hz^2)

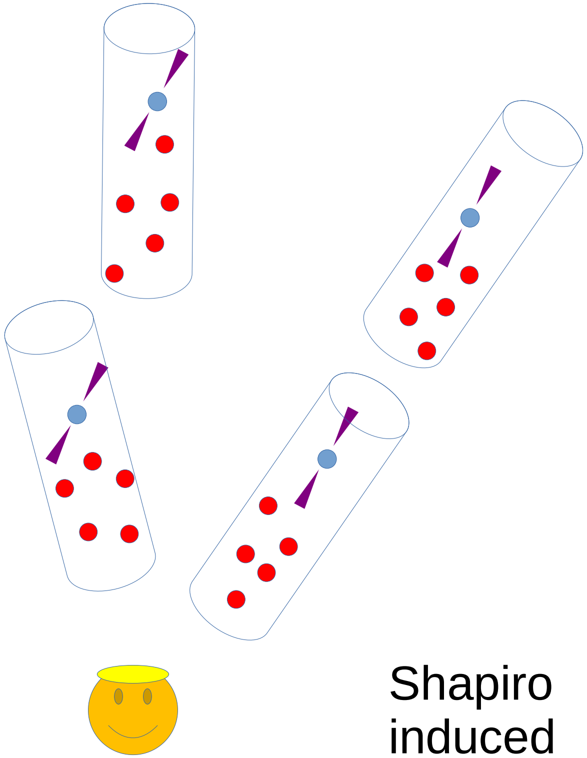
$$\sigma_{\ddot{\nu}/\nu} = 6\sqrt{\frac{2800\,\Delta t}{T}}\frac{t_{rms}}{T^3}$$ (~2.8 x 10^{-33} Hz^2)

Dynamical (τ<<T) vs Static (τ>>T)

[4]Dror, Jeff A., et al. "Pulsar timing probes of primordial black holes and subhalos." Physical Review D 100.2 (2019): 023003.

Pulsar timing array

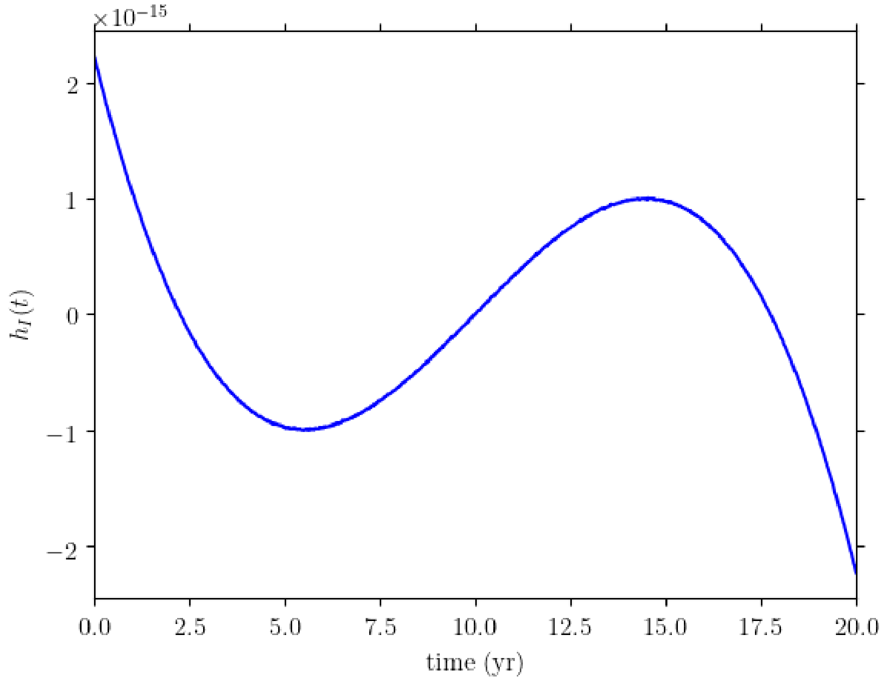




Pulsar timing: sensitivity limits[4,5]

$$h_I(t) = \sum_{i=1}^{N} \delta\phi_{I,i}(t) - \delta\phi_{0,I}(t)$$

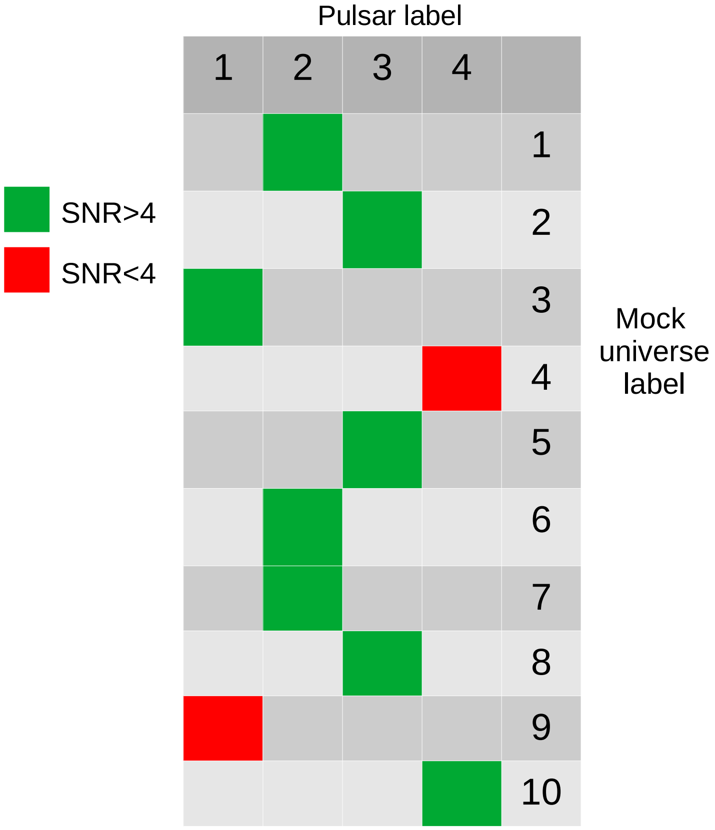
Typical signal waveform



[4]Lee, Vincent SH, et al. "Probing small-scale power spectra with pulsar timing arrays." Journal of High Energy Physics 2021.6 (2021): 1-30

[5]Ramani, Harikrishnan, Tanner Trickle, and Kathryn M. Zurek. "Observability of dark matter substructure with pulsar timing correlations." Journal of Cosmology and Astroparticle Physics 2020.12 (2020): 033.

Why this choice of SNR?



$$\mathrm{SNR}_I^2 = \frac{1}{\nu_I^2 t_{rms}^2 \Delta t} \int_0^{T_{obs}} dt\ h_I^2(t).$$

Similar to matched filter analysis in GW physics: desired signal is known, so perform an appropriate convolution using some kernel to extract it

$$T = \langle \int dt\,(h(t)+n(t))Q(t)\rangle = \int df\, h(f)\,Q(f)$$

$$N^2 = \int\int dt\, dt'\, Q(t)Q(t')\langle n(t)n(t')\rangle = \nu^2 t_{rms}^2 \Delta t \int df\, Q^2(f)$$

$$SNR^2 = \frac{T^2}{N^2}$$

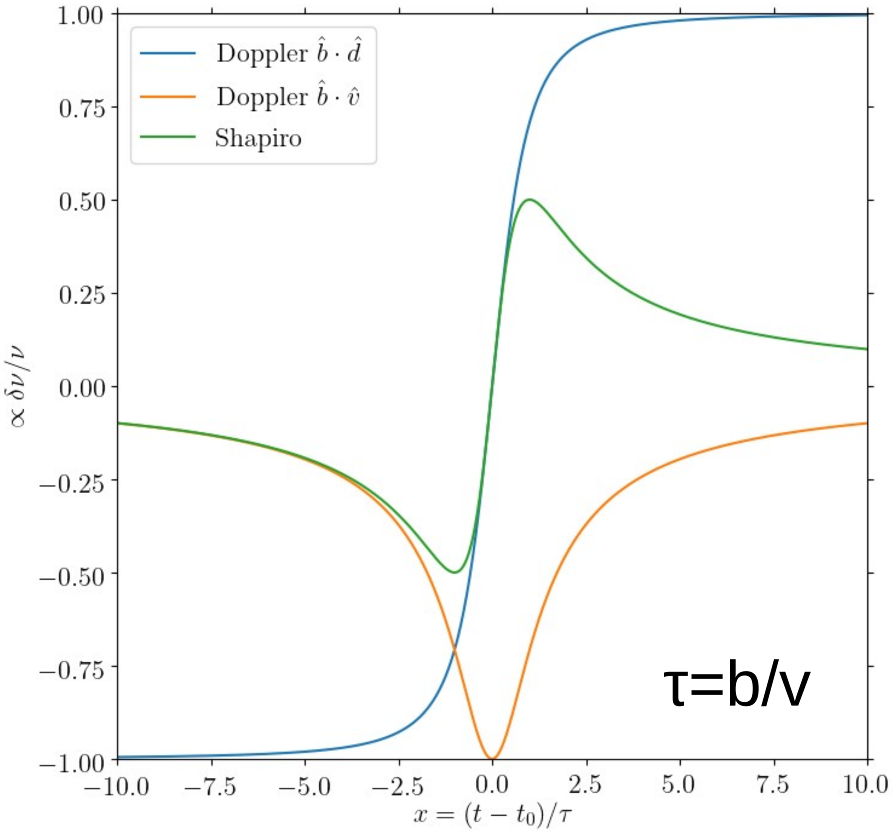
$$\langle n(f)n(f')\rangle = \nu^2 t_{rms}^2 \Delta t\, \delta(f-f')$$

Why threshold SNR=4?

-Null hypothesis: SNR follows a one-sided Gaussian, p-value 0.05 and 200 pulsars gives SNR=4

[7]Lee, Vincent SH, et al. "Probing small-scale power spectra with pulsar timing arrays." Journal of High Energy Physics 2021.6 (2021): 1-30
[8]Ramani, Harikrishnan, Tanner Trickle, and Kathryn M. Zurek. "Observability of dark matter substructure with pulsar timing correlations." Journal of Cosmology and Astroparticle Physics 2020.12 (2020): 033.

Example: monochromatic, pointlike PBHs



$$SNR=\frac{|\ddot{\nu}/\nu|}{\sigma_{\ddot{\nu}/\nu}}$$

(~10^{-34} $N_p f$ Hz^2)

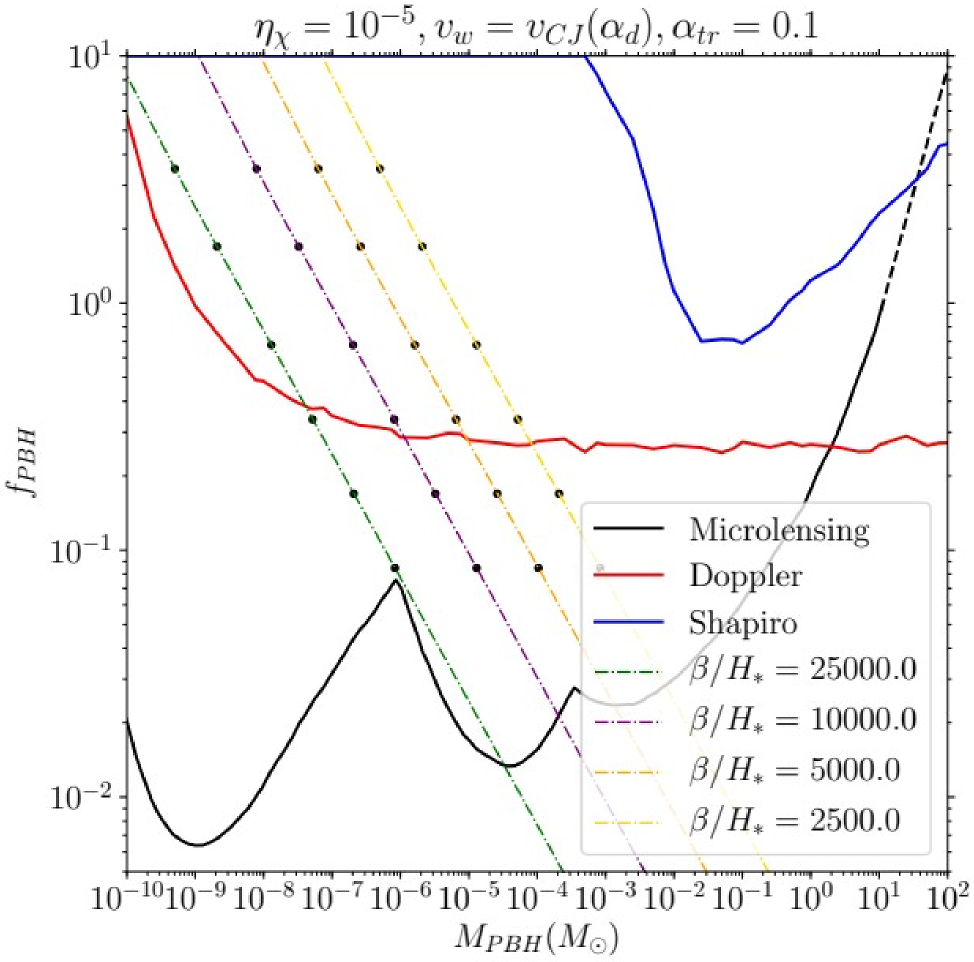
$$\sigma_{\ddot{\nu}/\nu}=6\sqrt{\frac{2800\,\Delta t}{T}}\,\frac{t_{rms}}{T^3}$$

(~2.8 x 10^{-33} Hz^2)

Dynamical (τ<<T) vs Static (τ>>T)

$$vT\sim 6\times 10^{-3}\,pc\left(\frac{v}{300\,km/s}\right)\left(\frac{T}{20\,yr}\right)$$

$$b\sim 10^{-3}\,pc\,(N_p f)^{-1/3}\left(\frac{M}{10^{-9}\,M_{sun}}\right)^{1/3}$$



Microlensing

- multiple images are formed, but not resolved as separate
- change the magnification
- microlensing event is registered if magnification is >1.34

Number of expected microlensing events:

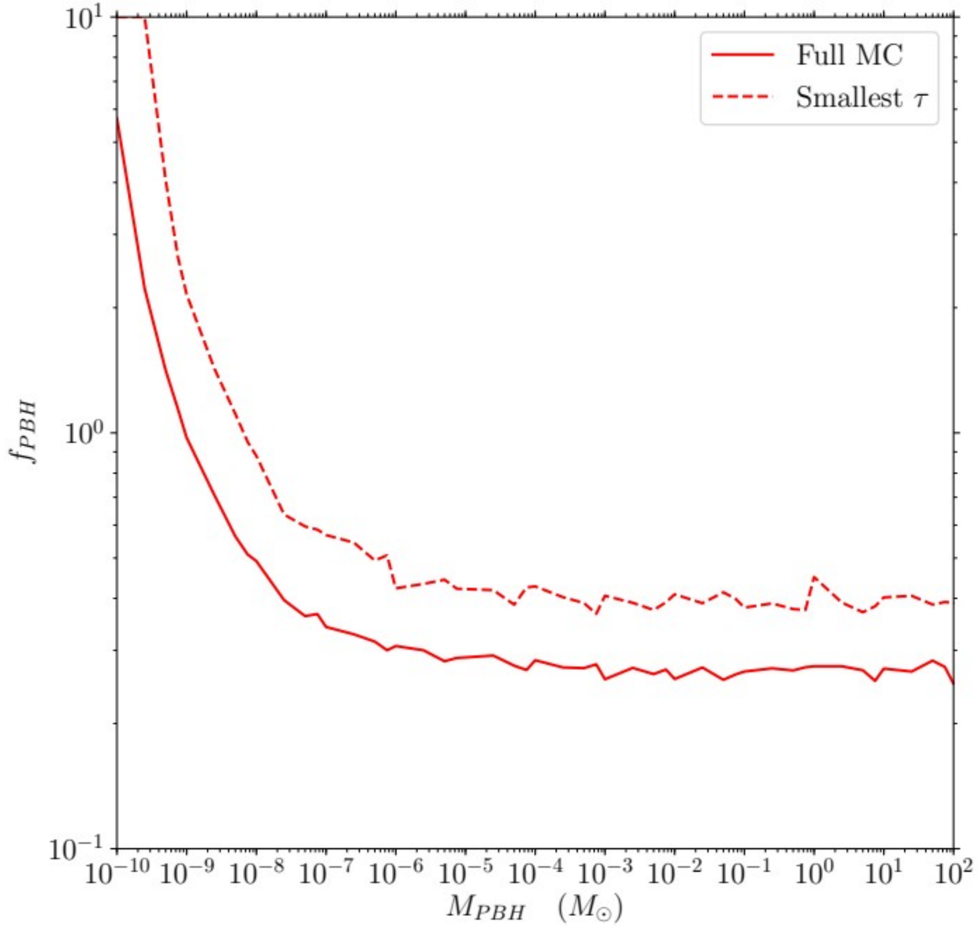
- $N_* T_{obs}$ (rate per source star) (transit time)
- rate per source star ~ 1/M

[7]D. Croon, D. McKeen, N. Raj and Z. Wang, Subaru-HSC through a different lens: Microlensing by extended dark matter structures, Phys. Rev. D 102 (2020) 083021 [2007.12697]

Plots from: JTA, Po-yan Tseng
JHEP 08 (2023) 117

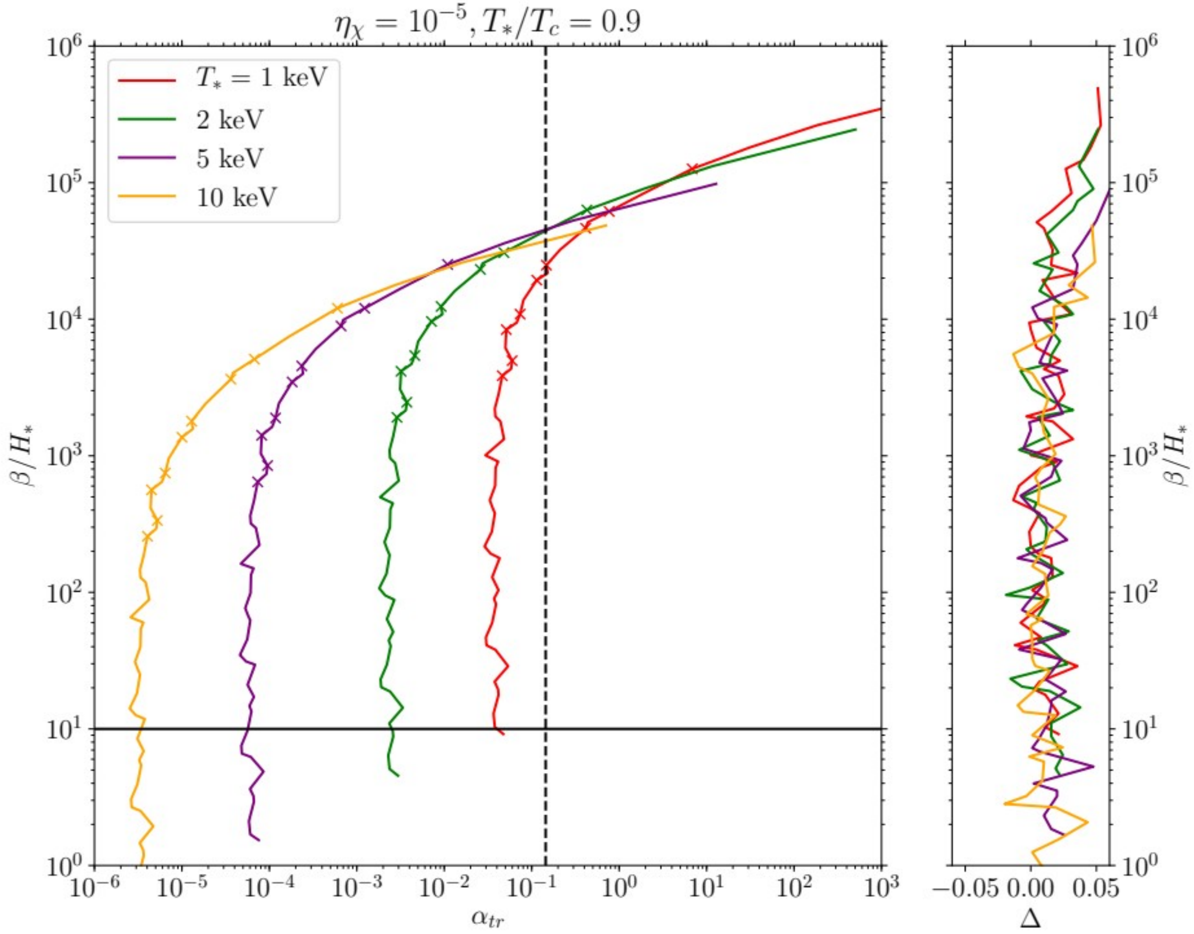
[9]Dynamical regime (blips): τ (=b/v) << T
Static regime (contribution to 2nd derivative of ν): τ >> T

Trend:
-low M_{PBH}: dynamical regime, and tugging acceleration too small
-intermediate M_{PBH}: static regime, and 3rd order phase shift scales as M/b^3~Mn~f ρ



Plots from: JTA, Po-yan Tseng JHEP 08 (2023) 117

[4]Dror, Jeff A., et al. "Pulsar timing probes of primordial black holes and subhalos." Physical Review D 100.2 (2019): 023003.



Pulsar timing limits: Novel PBH scenario

Recall:

-f depends on: $\alpha^{1/4}$ T_*

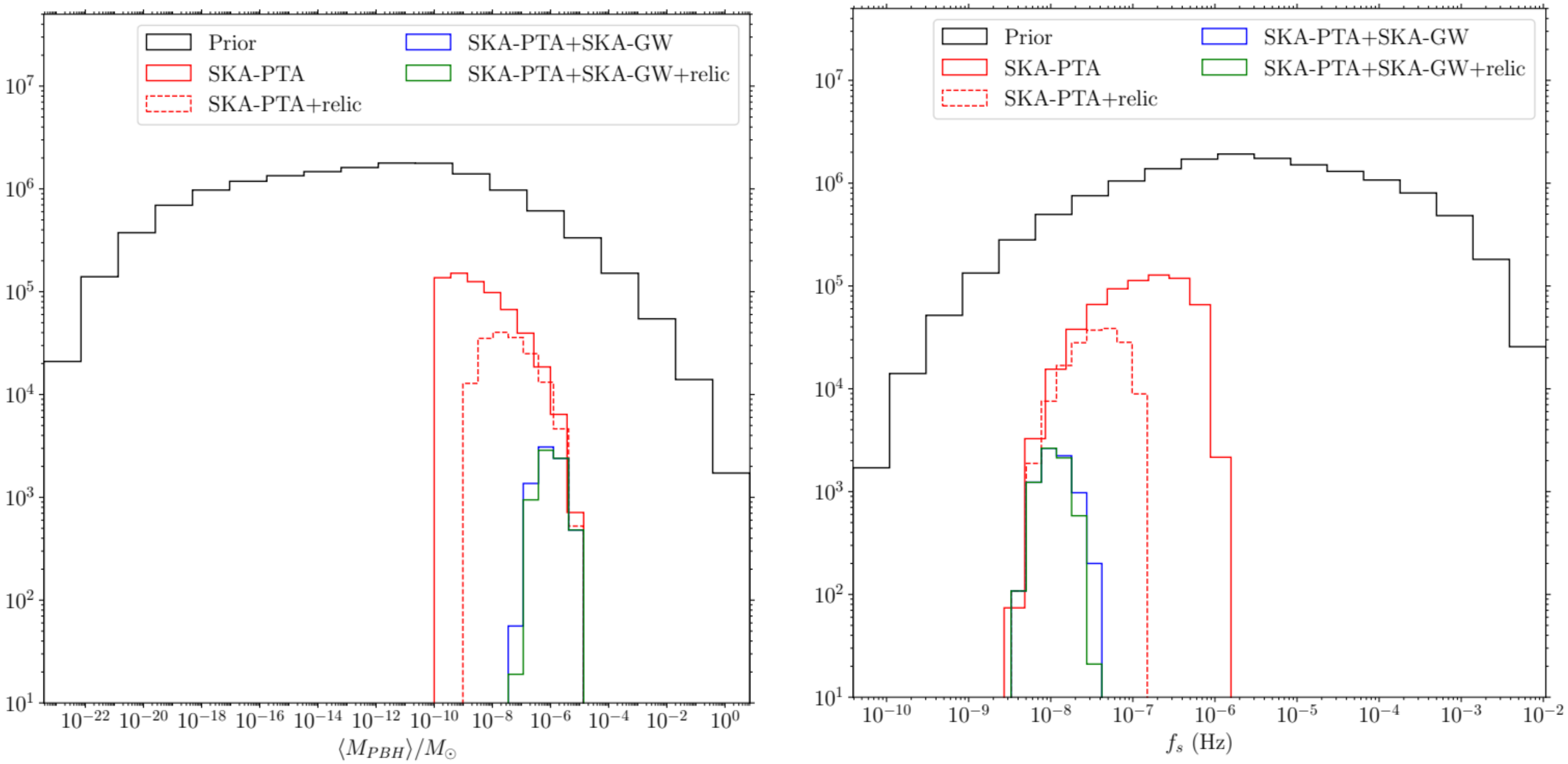
-<M> depends on $\alpha^{1/4}$, s/β^3

Plot from: JTA, Po-yan Tseng
JHEP 08 (2023) 117

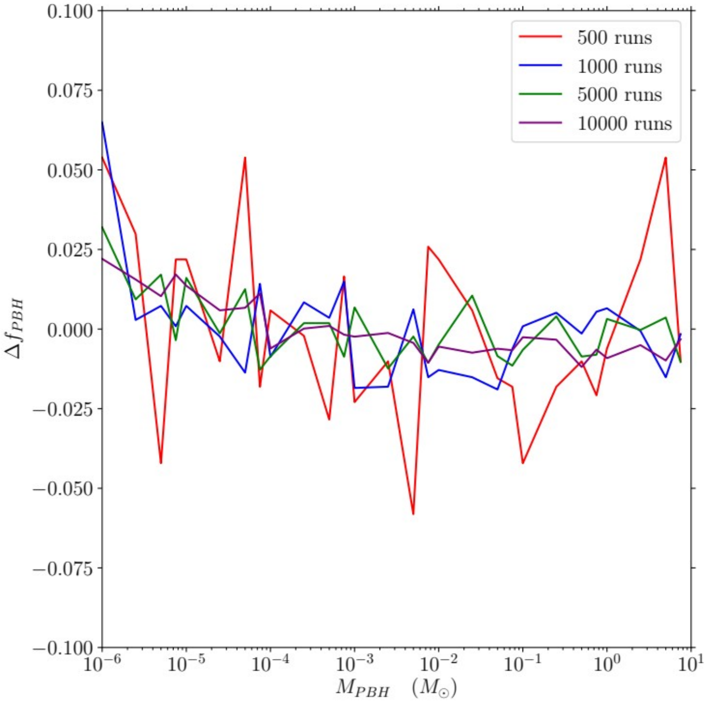
Complementary signal: Stochastic GWs

- GW through
  - Bubble collisions
  - Turbulence
  - Compression waves

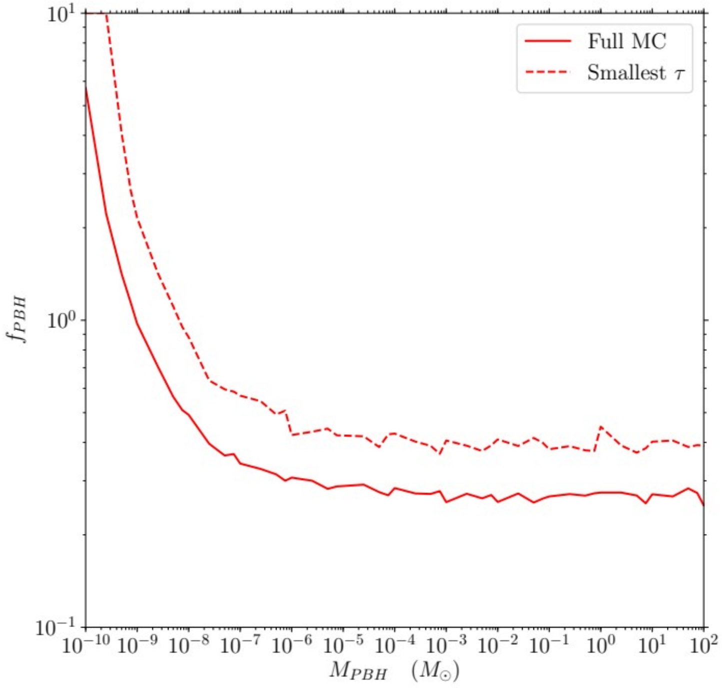
[8] Schmitz, Kai. "New sensitivity curves for gravitational-wave signals from cosmological phase transitions." Journal of High Energy Physics 2021.1 (2021): 1-62.


Prior
SKA-PTA
SKA-PTA+relic
SKA-PTA+SKA-GW
SKA-PTA+SKA-GW+relic
$\langle M_{PBH} \rangle / M_\odot$
f_s (Hz)

Why jagged?



500 runs
1000 runs
5000 runs
10000 runs
Δf_{PBH}
M_{PBH} $(M_\odot)$



Full MC
Smallest τ
f_{PBH}
M_{PBH} $(M_\odot)$

Generic quartic potential

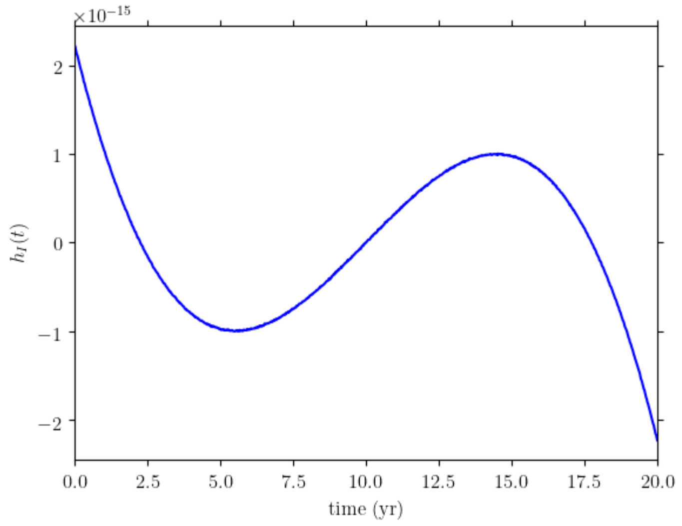
- (NB: The GW spectrum we are using is based on a fit on the numerical results of 1504.03291, using the same generic quartic potential in our paper, setting C = 0)

Generic quartic potential

- Potential connection with nonperturbative dynamics: proper calculation of GW spectrum from a generic quartic potential [JHEP 04 (2021) 055,JHEP04(2021)057]

Pulsar timing

- How do you get sensitivity limits?[6,7]
  - Monte Carlo simulation
    - Pick PBH fraction and PBH mass (f,M)
    - 1 simulation: assign random positions and velocities to a fixed number of PBHs
    - Calculate total phase shift, subtract away intrinsic phase shift
    - Calculate SNR per pulsar, take the max SNR in the simulation
    - Repeat for N simulations, pulsar timing is sensitive to (f,M) if >90% of the simulations have SNR above threshold



Take away point: signal manifests at 3rd order

$$\delta\phi_0 = \sum_{n=0}^{2} f_n(t)\left[\frac{1}{T_{obs}}\int_0^{T_{obs}} dt'\ \delta\phi(t')f_n(t')\right], \quad f_n(t) \equiv \sqrt{2n+1}P_n(2t/T_{obs}-1)$$

$$h_I(t) = \sum_{i=1}^{N} \delta\phi_{I,i}(t) - \delta\phi_{0,I}(t), \quad \delta\phi_{0,I} \equiv \sum_{n=0}^{2} f_n(t)\left[\frac{1}{T_{obs}}\int_0^{T_{obs}} dt'\ \sum_{i=1}^{N}\delta\phi_{I,i}(t')f_n(t')\right]$$

$$\mathrm{SNR}_I^2 = \frac{1}{\nu_I^2 t_{rms}^2 \Delta t}\int_0^{T_{obs}} dt\ h_I^2(t).$$

[6]Lee, Vincent SH, et al. "Probing small-scale power spectra with pulsar timing arrays." Journal of High Energy Physics 2021.6 (2021): 1-30
[7]Ramani, Harikrishnan, Tanner Trickle, and Kathryn M. Zurek. "Observability of dark matter substructure with pulsar timing correlations." Journal of Cosmology and Astroparticle Physics 2020.12 (2020): 033.

Stochastic GWs

- FOPT produces GW
  - Bubble wall collisions
  - Sound waves
  - Turbulence
- Complementary signal for probing FOPT
- Relevant experiments
  - SKA, THEIA, muAres
  - Look for correlated angular displacements of stars (THEIA), or even changes in timing signal (SKA)
- Assess sensitivity reach using some SNR

$$\Omega_s(f)h^2 = \Omega_s^{peak}h^2\ \mathcal{S}_s(f,f_s),$$

$$\Omega_s^{peak}h^2 \simeq \boxed{2.65\times 10^{-6}}\left(\frac{v_w}{\beta/H_*}\right)\left[\frac{100}{g_{*\rho,v}(T_*)}\right]^{1/3}\left(\frac{\kappa_s\alpha_{tr}}{1+\alpha_{tr}}\right)^2\left(1+\frac{g_{*\rho,d}}{g_{*\rho,v}}\xi^4\right)$$

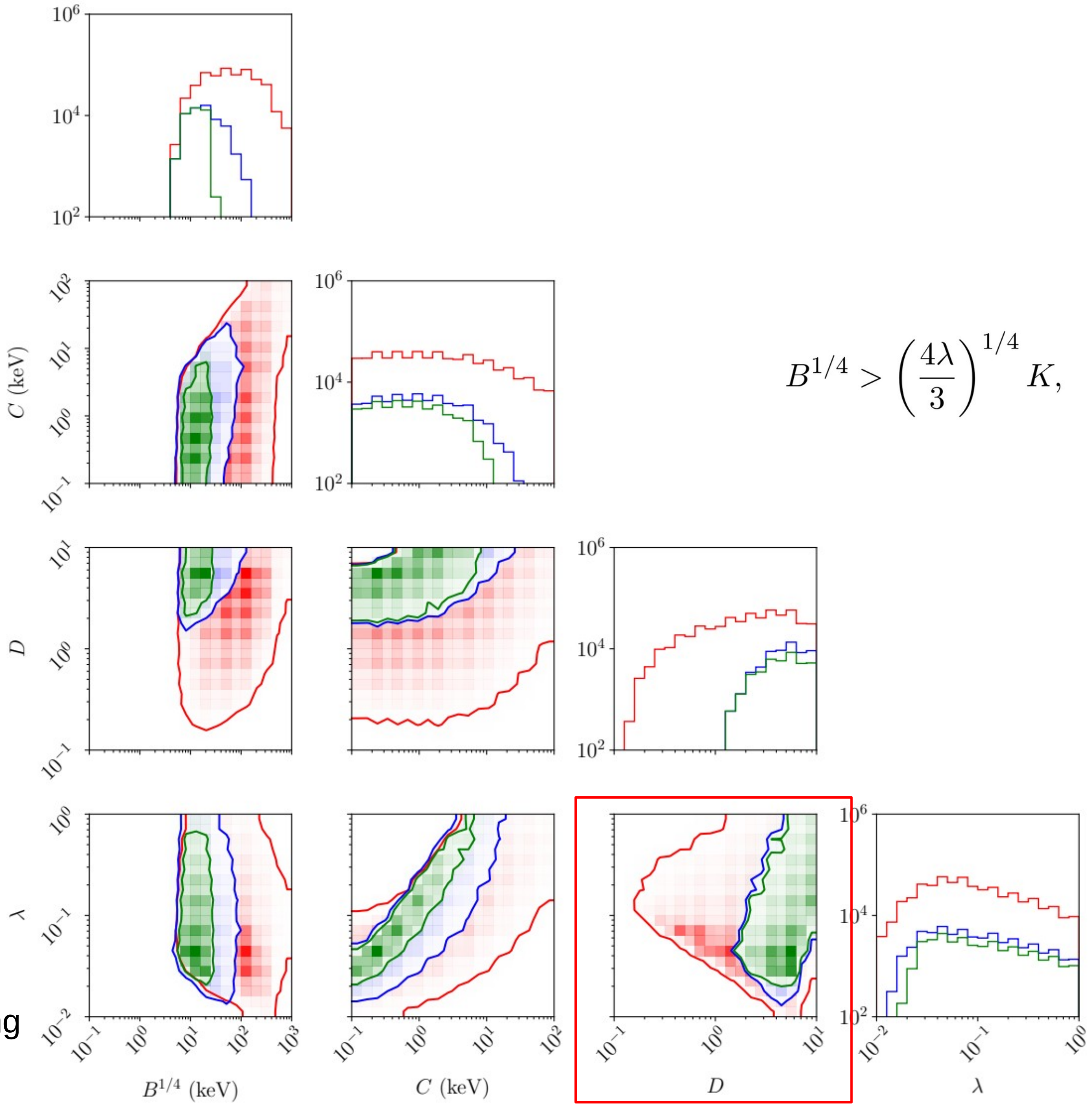
$$\mathcal{S}_s(f,f_s) = \left(\frac{f}{f_s}\right)^3\left[\frac{7}{4+3(f/f_s)^2}\right]^{7/2}$$

$$f_s \simeq \boxed{1.9\times 10^{-2}\,\mathrm{mHz}}\left[\frac{g_{*v}(T_*/\xi)}{100}\right]^{1/6}\left(\frac{T_*}{100\,\mathrm{GeV}}\right)\left(\frac{\beta/H_*}{v_w}\right)\left(1+\frac{g_{*\rho,d}}{g_{*\rho,v}}\xi^4\right)^{1/2}\frac{1}{\xi}$$

$$\kappa_s(\alpha_{tr}) \simeq \begin{cases}\frac{\alpha_{tr}}{0.73+0.083\sqrt{\alpha_{tr}}+\alpha_{tr}}, & v_w \geq v_{w,\alpha}\\ \frac{6.9\alpha_{tr}v_w^{6/5}}{1.36-0.037\sqrt{\alpha_{tr}}+\alpha_{tr}}, & v_w \leq v_{w,\alpha}\end{cases}$$

$$v_{w,\alpha} \equiv \left[\frac{1.36-0.037\sqrt{\alpha_{tr}}+\alpha_{tr}}{6.9\left(0.73+0.083\sqrt{\alpha_{tr}}+\alpha_{tr}\right)}\right]^{5/6}$$

[9] Schmitz, Kai. "New sensitivity curves for gravitational-wave signals from cosmological phase transitions." Journal of High Energy Physics 2021.1 (2021): 1-62.



$$B^{1/4} > \left(\frac{4\lambda}{3}\right)^{1/4} K, \quad A^2 - \lambda D < \left(\frac{C}{T_0}\right)^2, \quad K \equiv \frac{3C}{2\lambda}$$

PTA only

PTA+GW only

PTA+GW+relic

Plot from: JTA, Po-yan Tseng
JHEP 08 (2023) 117