NANOGrav Signal & Primordial Black Hole from Modified Higgs Inflation

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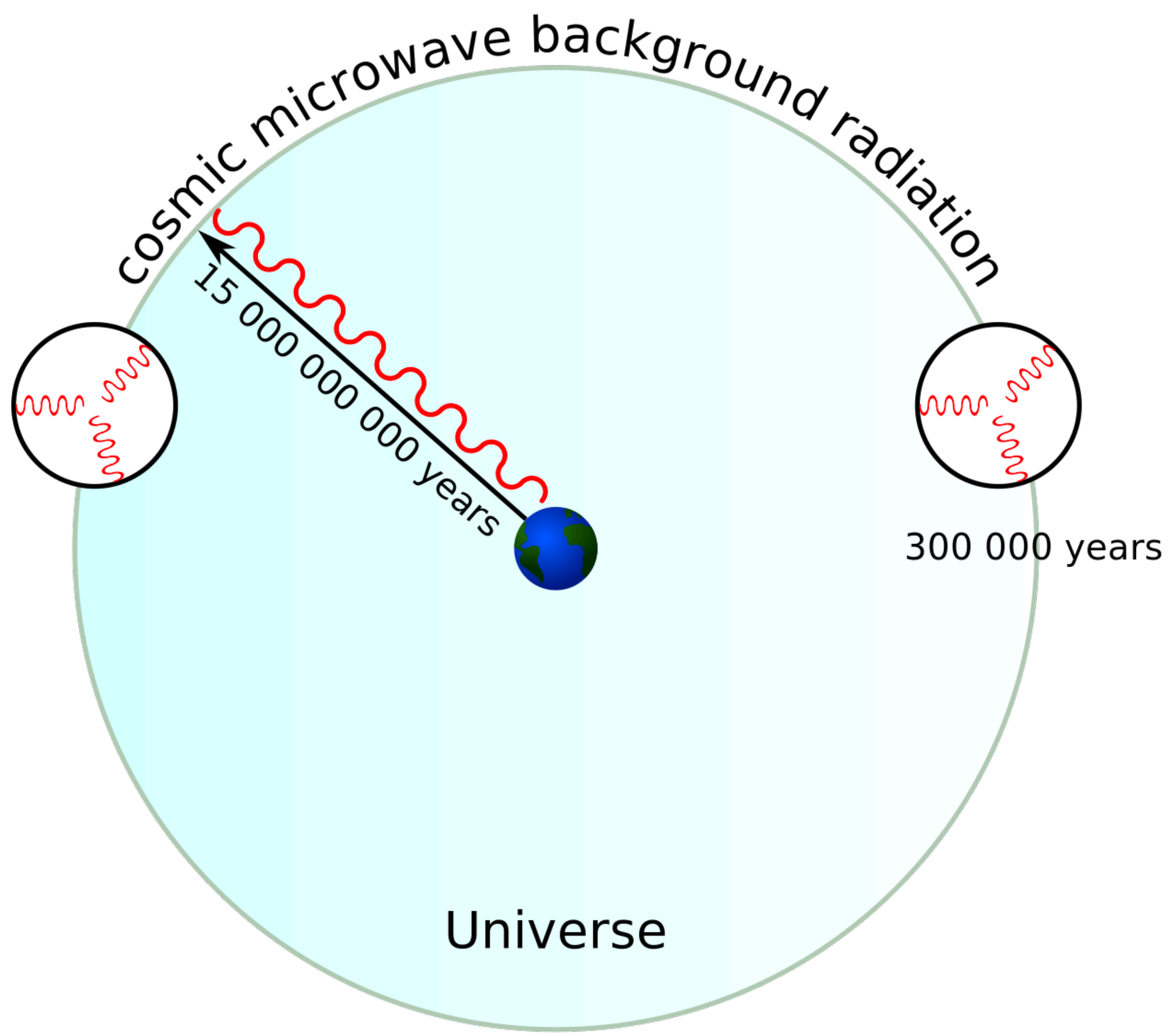
Outline

- Inflation
- Primordial Black Holes :: motivations
- Modified Higgs Inflation
- Stochastic Gravitational Wave

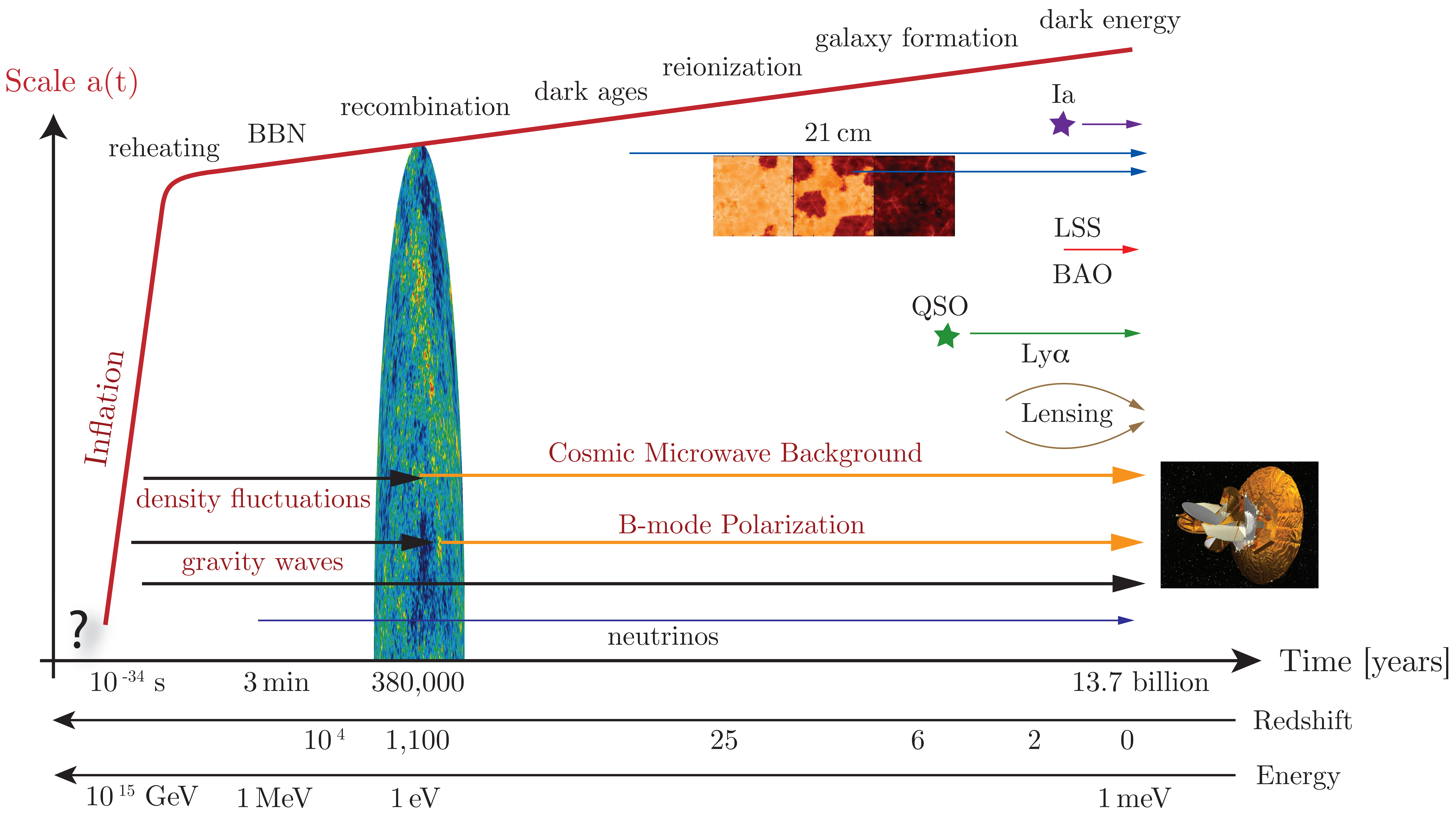
Idea of inflation solves a number of problems in Big Bang cosmology.

- Horizon problem.
- Flatness problem.
- Monopole problem — magnetic monopoles are predicted to be produced in GUT theories.
- Inflation suggested that the Universe that we see today came from a very small patch of space-time.

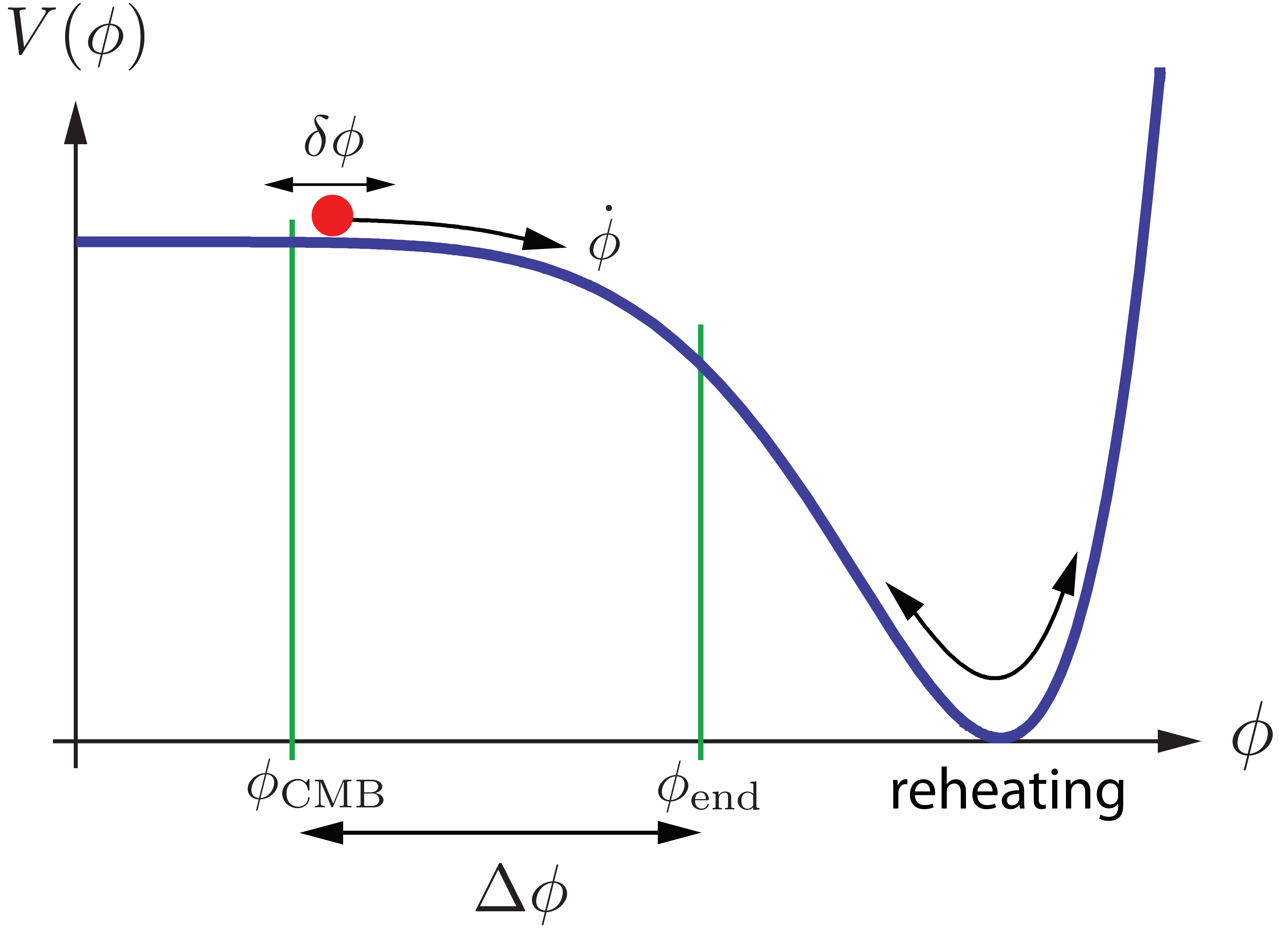


These 2 patches are not casually connected. The light emitted when the Universe is 300,000 yr old could only reach as far as the smaller circles. There is no reason that they are very similar to each other.

Daniel Baumann TASI lecture



Scalar Field Dynamics



Slow roll parameters: $\epsilon(\phi) = \frac{M_{PL}^2}{2}\left(\frac{dU/d\phi}{U}\right)^2$, $\eta(\phi) = M_{PL}^2\left(\frac{d^2U/d\phi^2}{U}\right)$

Slow roll condition: $\epsilon,\ \eta \ll 1$, as long as $\epsilon < 1$ accelerated expansion occurs.

Higgs Inflation Models

- Introducing a non-minimal coupling between the SM Higgs and the Ricci scalar R with a non-minimal coupling strength ξ. The effective action:

.

$$\mathcal{S}_{\mathcal{J}} = \int d^4x \sqrt{-g}\Big[-\frac{M_{\rm PL}^2+\xi h^2}{2}\mathcal{R}+\frac{\partial_\mu h\partial^\mu h}{2}-\frac{\lambda}{4}(h^2-v^2)^2\Big] .$$

- A conformal transformation: $\hat{g}^{\mu\nu} = \Omega g^{\mu\nu}$ with $\Omega = 1 + \xi h^2/M_{PL}^2$ followed by the transformation $h \rightarrow \phi$:

$$\frac{d\phi}{dh} = \sqrt{\frac{\Omega}{\Omega^2}+\frac{3M_{\rm PL}^2}{2}\frac{(\frac{d\Omega}{dh})^2}{\Omega^2}}$$

- Then obtain the Action in Einstein frame:

$$\mathcal{S}_{\mathcal{E}} = \int d^4x\sqrt{-\hat{g}}\Big[-\frac{M_{\rm PL}^2}{2}\hat{\mathcal{R}} + \frac{\partial_\mu\phi\partial^\mu\phi}{2} - U(\phi)\Big]. \qquad U(\phi) = \frac{\lambda M_{PL}^2}{4\xi^2}\left(1+\exp\Big(-\frac{2\phi}{\sqrt{6}M_{PL}}\Big)\right)^{-2}$$

- The slow-roll parameters can be obtained:

$$\epsilon = \frac{M_{\rm PL}^2}{2}\left(\frac{\frac{\partial U}{\partial\phi}}{U}\right)^2 = \frac{4M_{\rm PL}^4}{3\xi^2 h^4} \qquad \eta = M_{\rm PL}^2\left(\frac{\frac{\partial^2 U}{\partial\phi^2}}{U}\right) = -\frac{4M_{\rm Pl}^2}{3\xi h^2}.$$

- The slow roll ends at $\epsilon = 1$, the no. of e-folds, and the initial field value

$$h_{end} = \Big(\frac{4}{3}\Big)^{\frac{1}{4}}\Big(\frac{M_{\rm PL}}{\sqrt{\xi}}\Big). \qquad N_e = \int_{h_{end}}^{h_{int}} \frac{1}{M_{\rm PL}^2}\left(\frac{U}{\frac{\partial U}{\partial h}}\Big(\frac{\partial\phi}{\partial h}\Big)^2\right)dh \qquad h_{int} = \frac{2M_{\rm PL}}{\sqrt{3\xi}}\Big[\frac{\sqrt{3}}{2} + N_e\Big]^{1/2}.$$

Fine-tuning in Higgs inflation

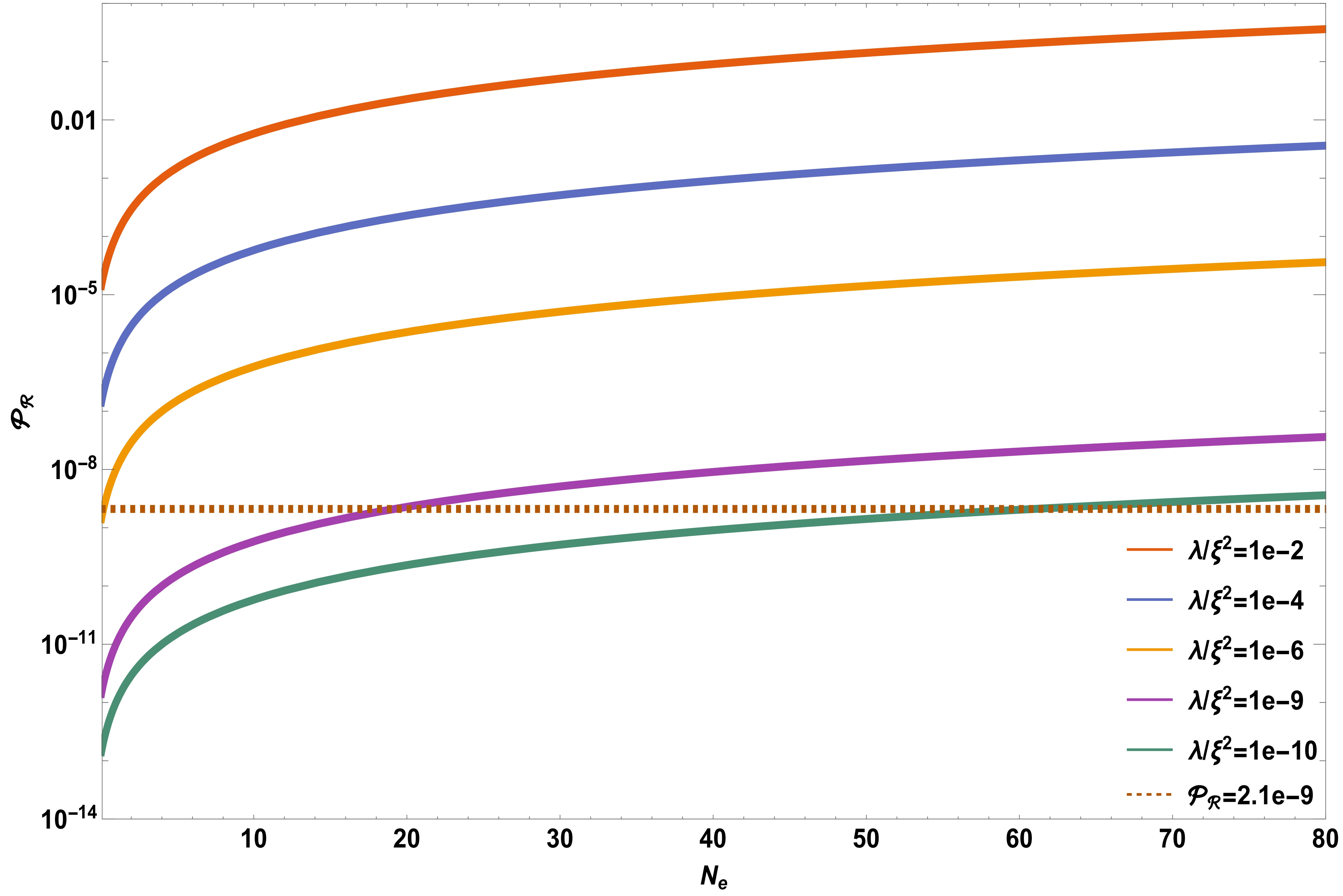
- λ: Higgs self coupling $\quad \xi$: non-minimal coupling
- For enough e-fold $N_e \approx 60 \Rightarrow \lambda/\xi^2 \sim 4.4 \times 10^{-10}$
- The scalar spectral index n_s and tensor-to-scalar ratio r

$$n_s = 1 - 6\epsilon + 2\eta \approx 0.9633, \qquad r = 16\epsilon = 0.0032$$

Well within the Planck bound.

- With $\lambda = O(1)$ the non-minimal coupling $\xi \simeq 10^5$ is very large

The Scalar Power Spectrum



$$\mathcal{P}_{\mathcal{R}} = \frac{1}{12\pi^2} \frac{U^3(\phi)}{M_{PL}^6 U'^2(\phi)}$$

- CMB suggested at pivot scale

$$\mathscr{P}_R = 2.1 \times 10^{-9}$$

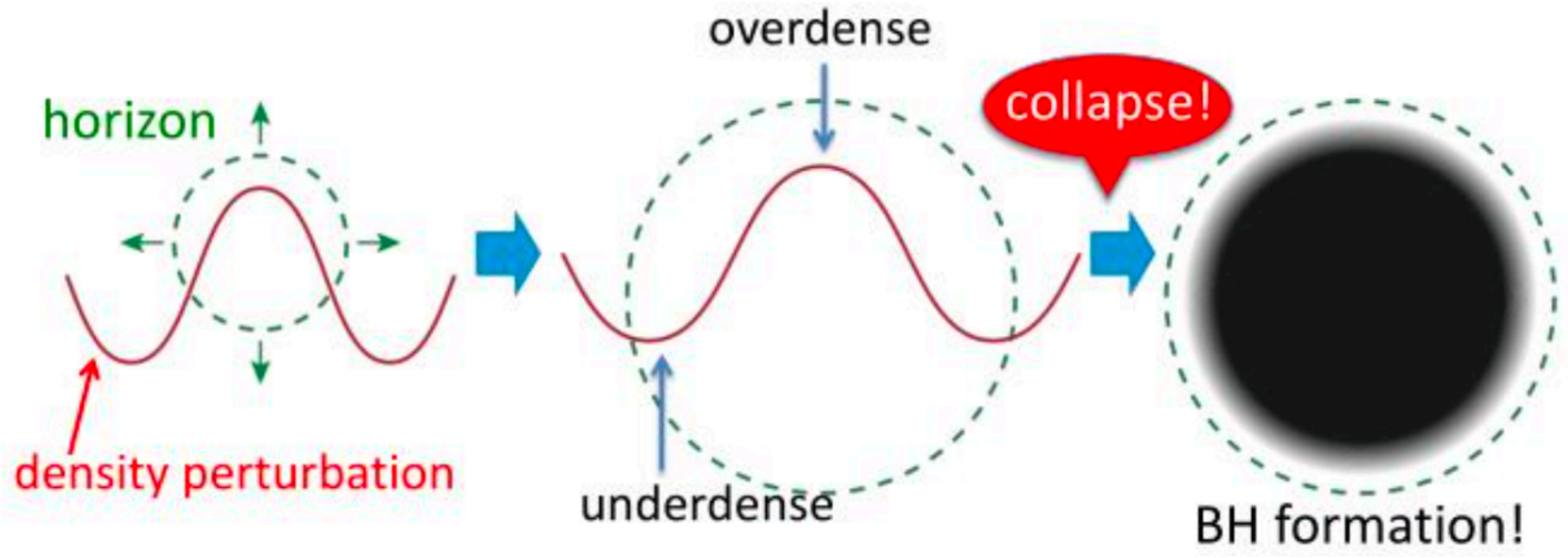
- Achievable for $\lambda/\xi^2 \sim 10^{-10}$

At $N_e = 60$

Story for Primordial Black Holes

- Stellar BH originates from the death of a massive star toward the end of its life — supernova explosion. The remnant is so heavy that it collapses into BH.

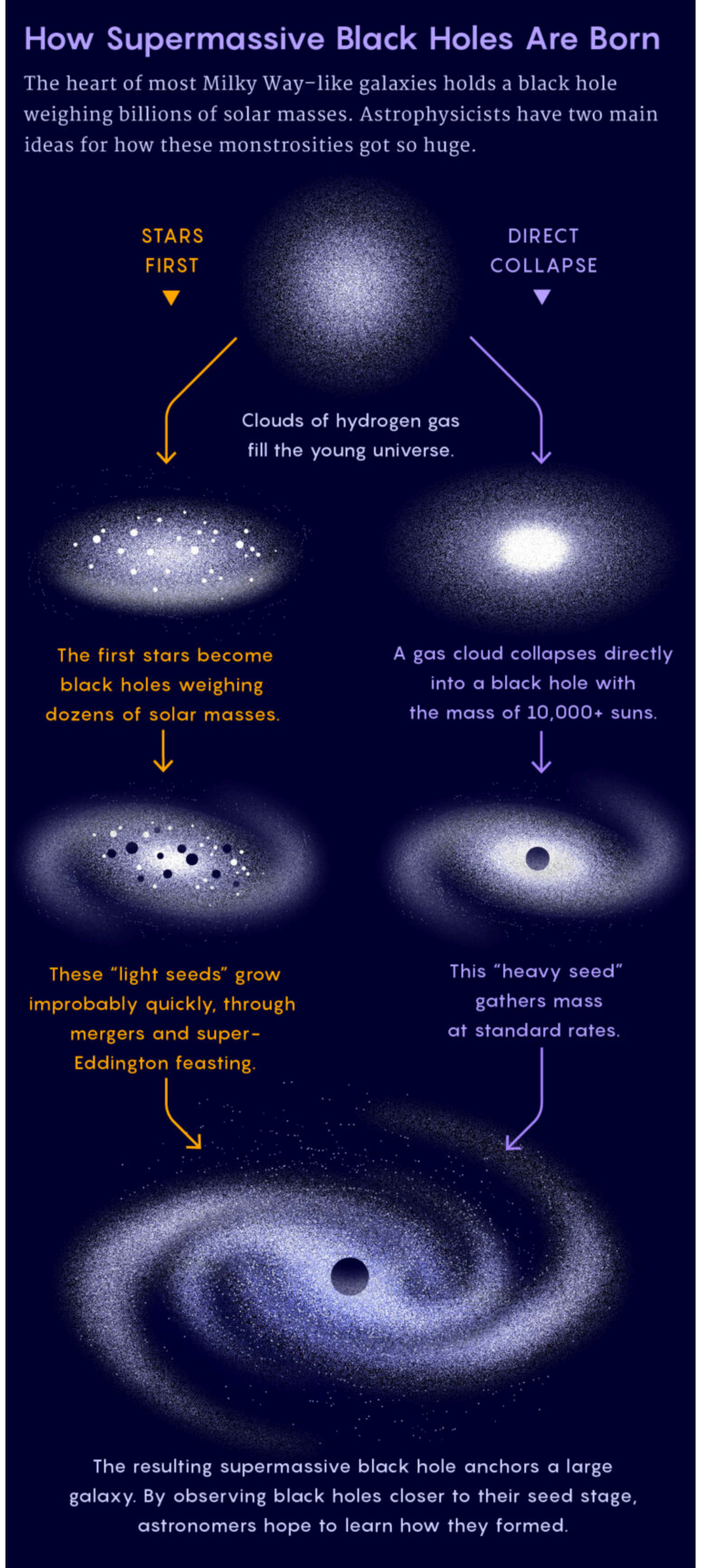
- Shortly after Big Bang when the Universe is filled up with radiation and fundamental particles such as quarks and gluons. Natural density perturbation will cause some regions with more and some with less. Those regions with enough radiation and particles can collapse itself into BH with various mass and size — Primordial Black Hole (PBH). Typically, the density contrast of $\frac{\delta\rho}{\rho} \sim 0.1$ is required.



ASTROPHYSICS

JWST Spots Giant Black Holes All Over the Early Universe

Giant black holes were supposed to be bit players in the early cosmic story. But recent James Webb Space Telescope observations are finding an unexpected abundance of the beasts.

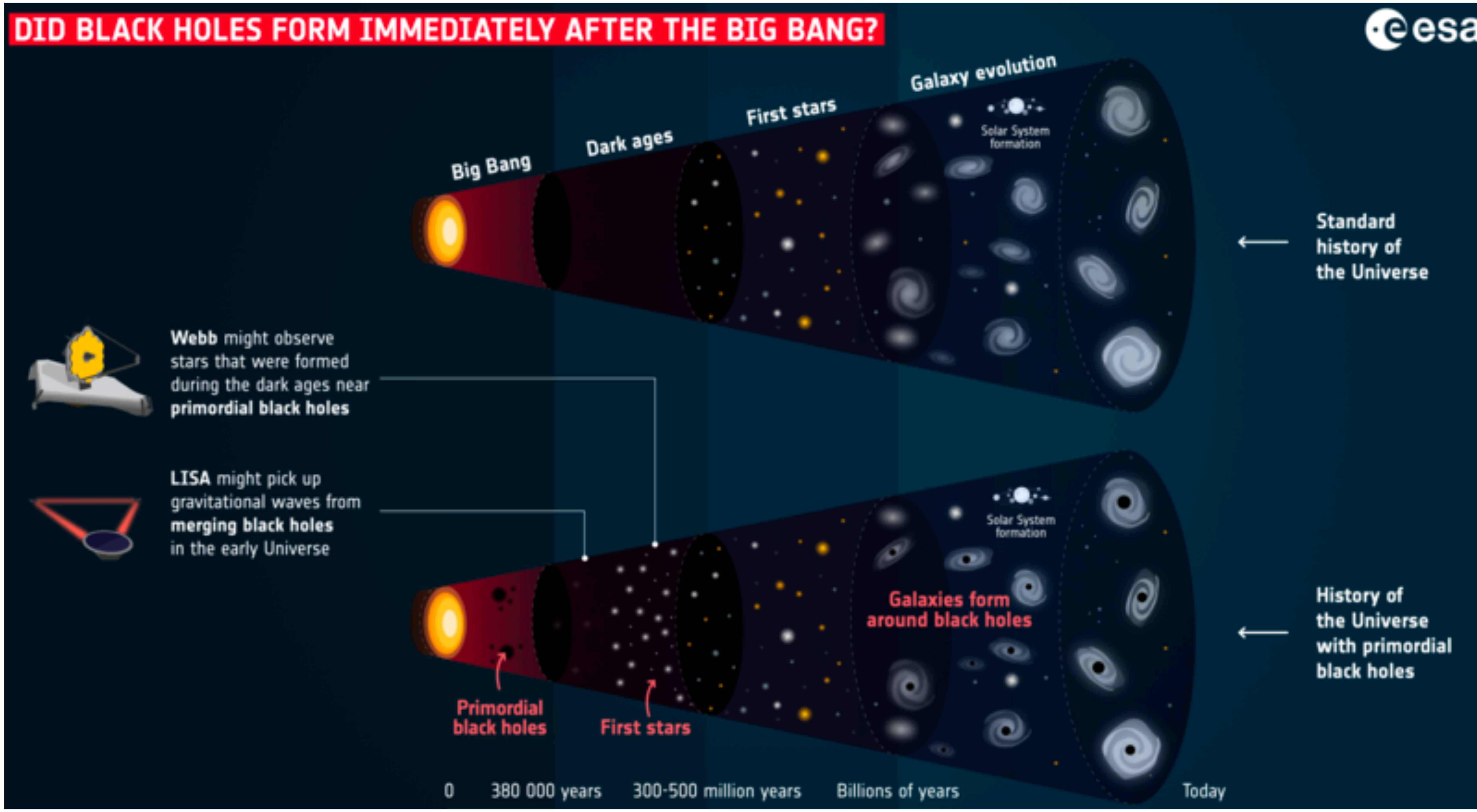


Story of PBH ….

- PBH could account for all or part of the dark matter (DM) of the Universe, explaining the gravitational lensing effects of galaxies and galactic clusters.

- Some PBH could be very massive, forming SMBH fueling formation of galaxy earlier than expected.

- Observed LIGO BH merger can be accounted for by the PBH binaries.



Conditions for PBH Formation

- Need substantial scalar fluctuation during inflationary period, which can lead to significant density fluctuation ⇒ PBH formation

- To produce PBH in early Universe the curvature power spectrum

$$\mathscr{P}_R \sim 10^{-3} - 10^{-2}$$

- In order to have a successful inflation, $\mathscr{P}_R \approx 2.1 \times 10^{-9}$ at the CMB scale.

- The Higgs inflation model with a non-minimal coupling cannot address both inflation and PBH simultaneously.

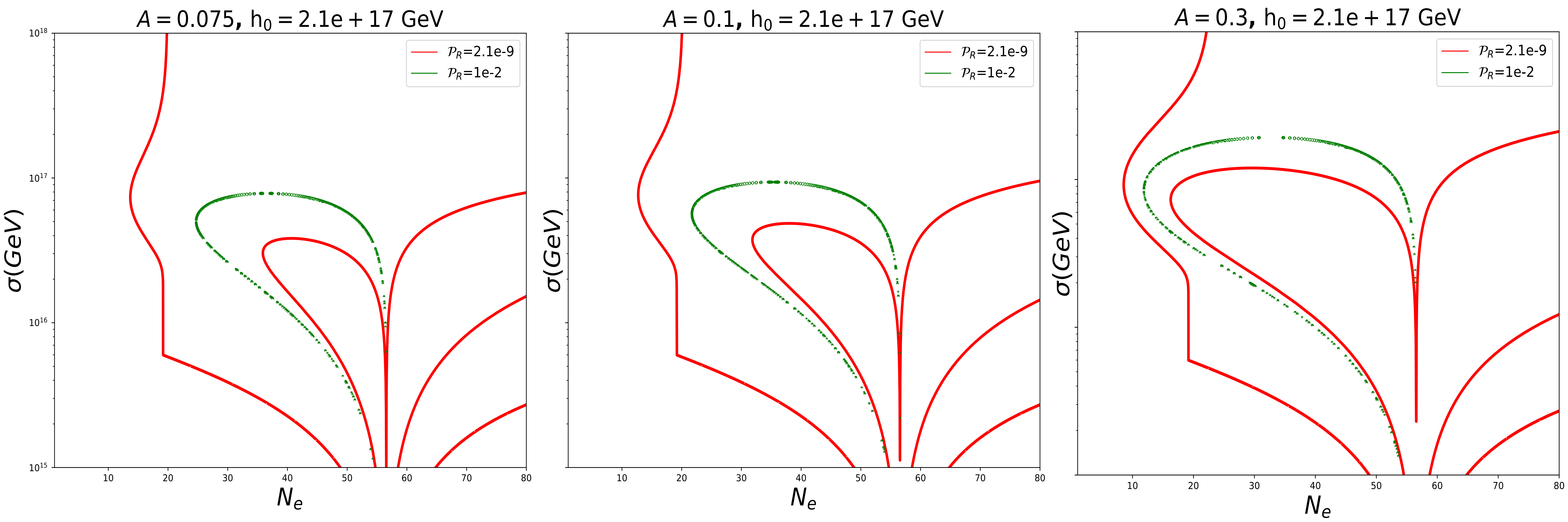
- We want the PBH formation during the radiation epoch before the CMB scale.

Modified Higgs Inflation Model

- Adding a Gaussian dip or bump to the Higgs potential: $\pm\left[Ae^{-\frac{(h(\phi)-h_0(\phi))^2}{2\sigma^2}}\right]$
- After the conformal transformation, the inflationary potential is

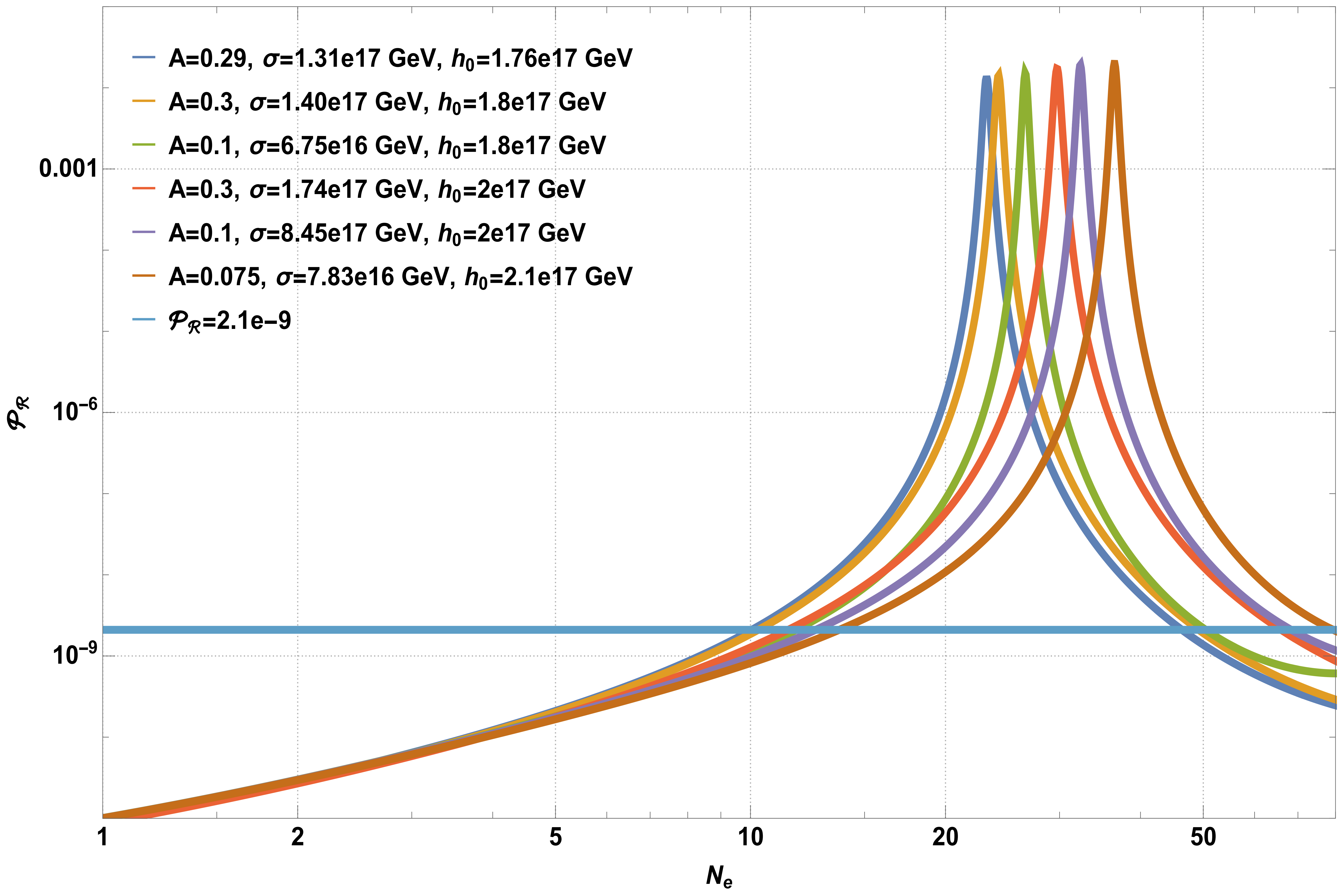
$$U_{eff}(\phi) = \frac{\lambda h^4(\phi)}{4}\frac{\left(1 \pm Ae^{-\frac{(h(\phi)-h_0(\phi))^2}{2\sigma^2}}\right)}{\left(\frac{h^2(\phi)\xi}{M_{PL}^2}+1\right)^2}.$$

- The bump (dip) is featured by A, position h_0, and width σ. The power spectrum $\mathscr{P}_R[A, \sigma, h_0, \lambda, \xi]$ that needs $\mathscr{P}_R = 10^{-3} - 10^{-2}$ for PBH formation and $\mathscr{P}_R = 2.1 \times 10^{-9}$ at the CMB scale for successful inflation.



Effects of adding a DIP.

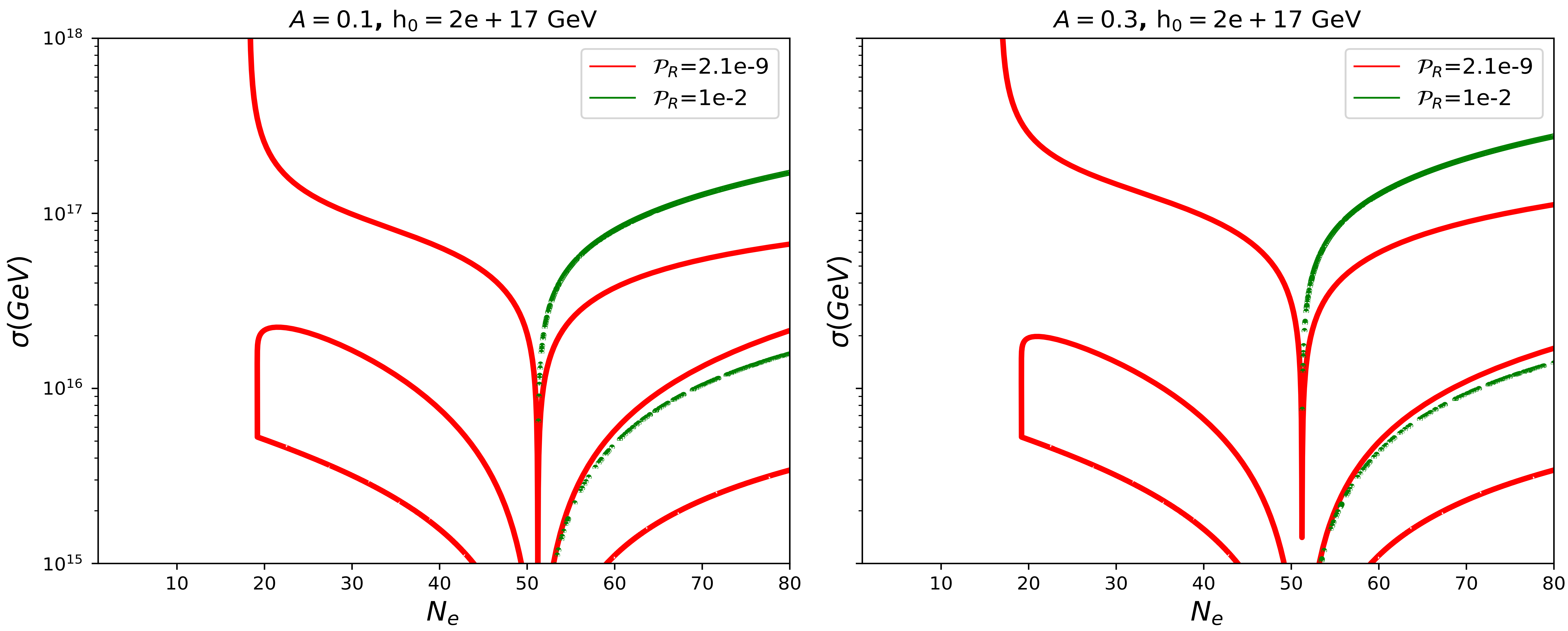
Contour: red $\mathscr{P}_R = 2.1 \times 10^{-9}$, green: $\mathscr{P}_R = 10^{-2}$. It is possible to find σ such that PBH requirement is fulfilled before the end of inflation at $N_e \approx 60$



Power Spectrum features

A peak $\mathscr{P}_R \sim 10^{-2}$ before

e-fold reaches 60.



Effects of adding a BUMP.

Contour: red $\mathscr{P}_R = 2.1 \times 10^{-9}$, green: $\mathscr{P}_R = 10^{-2}$. The PBH requirement is fulfilled only AFTER the end of inflation.

PBH Formation

- PBH formation requires $\mathscr{P}_R \sim O(0.01)$, the curvature perturbation $\mathscr{R}_k$ is related to density contrast $\delta(t,k)$ by

$$\delta(t,k) = \frac{2(1+\omega)}{5+3\omega}\frac{k}{aH}\mathcal{R}_k$$

- The over-dense region in Universe with $\delta > \delta_c \approx 0.4$ would collapse into PBH

- The mass fraction $\beta(M_{PBH})$ is given in terms of $\delta_c, \mathscr{P}_R$, and related to the abundance f_{PBH}.

$$\beta(M_{PBH}) = \gamma\sqrt{\frac{2}{\pi}}\,\frac{4\mathcal{P}_R(k)}{9\delta_c}exp\Big(-\frac{81\delta_c^2}{32\mathcal{P}_R(k)}\Big)$$

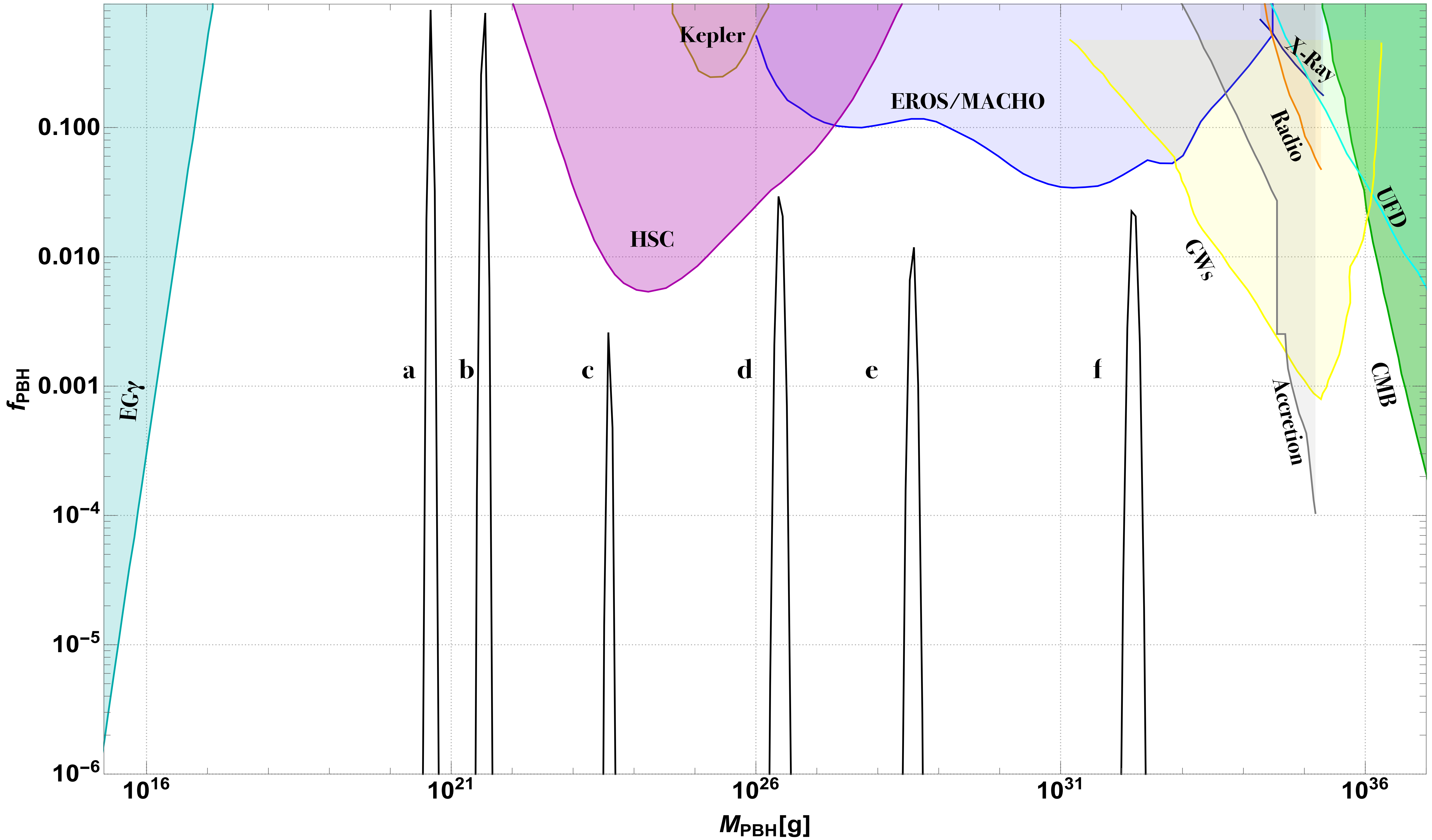
- The mass of PBH at formation is a fraction of the horizon mass:

$$M_{\rm PBH} = \gamma\, M_{\rm H}\Big|_{\text{at formation}} = \gamma \frac{4\pi M_P^2}{H_N} e^{2N}$$

- The mass fraction $\beta(M_{PBH})$ of PBHs can be related to the abundance f_{PBH} as follows when considering PBH as a fraction of DM:

$$\beta(M_{PBH}) = 3.7 \times 10^{-9} \left(\frac{\gamma}{0.2}\right)^{-1/2} \times \left(\frac{g_{*form}}{10.75}\right)^{1/4} \left(\frac{M_{PBH}}{M_\odot}\right)^{1/2} f_{PBH}$$

Here g_{*form} is the relativistic dot at time of formation, γ is chosen to be 0.4. N is the e-fold during horizon exit. H_N is the Hubble expansion rate at the inflection point.



Constraints

- EGγ
- Subaru HSC microlensing
  Kepler microlensing
  EROS/MACHO microlensing
- Dynamical heating of ultra Faint dwarf galaxy (UFD)
- X-ray / radio
- Accretion constraint by CMB

Region	A	h_0(GeV)	σ(GeV)	N	n_s	r
a	0.29	1.76×10^{17}	1.31×10^{17}	N=47	0.950907	0.0242479
b	0.3	1.8×10^{17}	1.40×10^{17}	N=50	0.980819	0.0244824
c	0.1	1.8×10^{17}	6.75×10^{16}	N=51	0.988732	0.0300489
d	0.3	2×10^{17}	1.74×10^{17}	N=65	0.98924	0.0206994
e	0.1	2×10^{17}	8.45×10^{16}	N=68	0.98953	0.0279209
f	0.075	2.1×10^{17}	7.83×10^{16}	N=78	0.989654	0.0275894

Parameters of Regions

a, b the PBH makes up

All DM of Universe

Stochastic Gravitational Wave Background

- Large density perturbation not only produce PBH, but also generate the second-order gravitational wave.

- GW then propagates freely through subsequent epochs of the Universe.

- The frequency of the GW corresponds to the Hubble mass at that time. Since the mass of PBH $\propto$ Hubble mass, the present-day frequency of the GW is related to M_{PBH}

$$f_{GW} \simeq 10^{-9} \left(\frac{M_{PBH}}{30 M_{\odot}} \right)^{-\frac{1}{2}} \text{ Hz}$$

- The generation of gravitational waves due to the initial scalar perturbations at the second order: $g_{\alpha\beta} = \bar{g}_{\alpha\beta} + \delta g_{\alpha\beta} + \delta^2 g_{\alpha\beta}$

- We focus on second-order tensor perturbation h_{ij}, the perturbed metric:

$$ds^2 = a^2(\eta)\left[-(1+\Phi)d\eta^2 + \left[(1-2\Psi)\delta_{ij} + \frac{1}{2}h_{ij}\right]dx^i dx^j \right]$$

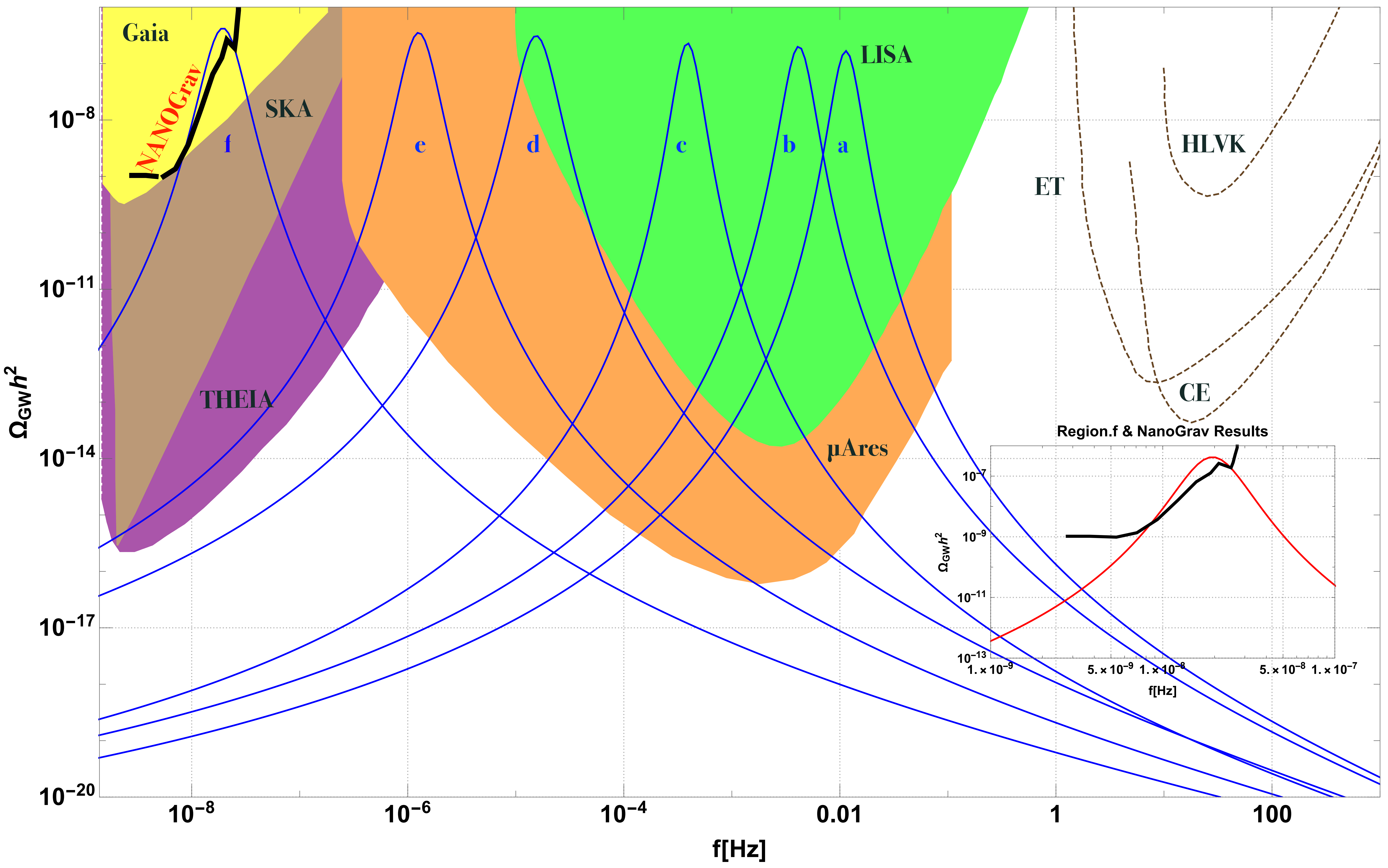
- After manipulation the h_{ij} satisfies, with the source term:

$$\frac{d^2 h(\vec{k},\tau)}{d\tau^2} + \frac{2}{\tau}\frac{dh(\vec{k},\tau)}{d\tau} + k^2 h((\vec{k}),\tau) = \mathcal{S}((\vec{k}),\tau),$$

$$\mathcal{S} = \int \frac{d^3\widetilde{k}}{(2\pi)^{3/2}}\widetilde{k}^2\left[1 - \left(\frac{\vec{k}\cdot\vec{\widetilde{k}}}{k\widetilde{k}}\right)^2\right] \times \Bigg(12\Phi(\vec{k}-\vec{\widetilde{k}},\tau)\Phi(\vec{\widetilde{k}},\tau) + 8\left[\tau\Phi(\vec{k}-\vec{\widetilde{k}},\tau) + \frac{\tau^2}{2}\frac{d\Phi(\vec{k}-\vec{\widetilde{k}},\tau)}{d\tau}\right]\frac{d\Phi(\vec{k}-\vec{\widetilde{k}},\tau)}{d\tau}\Bigg)$$

- After solving, the energy density of GW is

$$\Omega_{GW} = 10\mathcal{P}_{\mathcal{R}}^2 a_{eq} \qquad a_{eq} = \frac{a_0}{3.1\times 10^4\ \Omega_M h^2},$$



Region f:

$$A = 0.075$$

$$\sigma = 7.83 \times 10^{16}\ \mathrm{GeV}$$

$$h_0 = 2.1 \times 10^{17}\ \mathrm{GeV}$$

$$M_{PBH} \approx 0.1 M_\odot$$

$$f_{GW} \simeq 10^{-9}\left(\frac{M_{PBH}}{30 M_\odot}\right)^{-\frac{1}{2}}\ \mathrm{Hz}$$

$$\Omega_{GW}h^2 = \frac{10\,\mathscr{P}_R^2\,a_0}{3.1 \times 10^4\,\Omega_M} \text{ with } \Omega_M = 0.3,\ H_0 = 67.27\,km/s^2$$

Conclusions

- A modified Higgs inflation model featured a dip structure could account for PBH formation during the radiation epoch before the end of inflation.
- PBH's can constitute a fraction of the DM and could potentially explain the BH mergers detected by LIGO.
- Gravitational wave can be formed simultaneously with the PBH formation. For certain parameter space, the GW can account for the data observed by NANOgrav.