Geometric view for the pNGB potential

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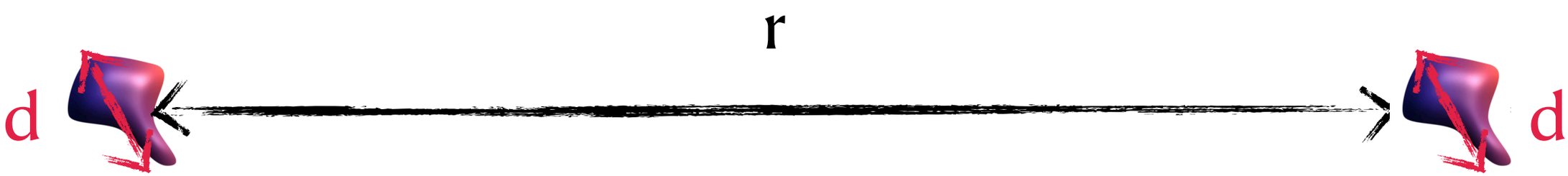
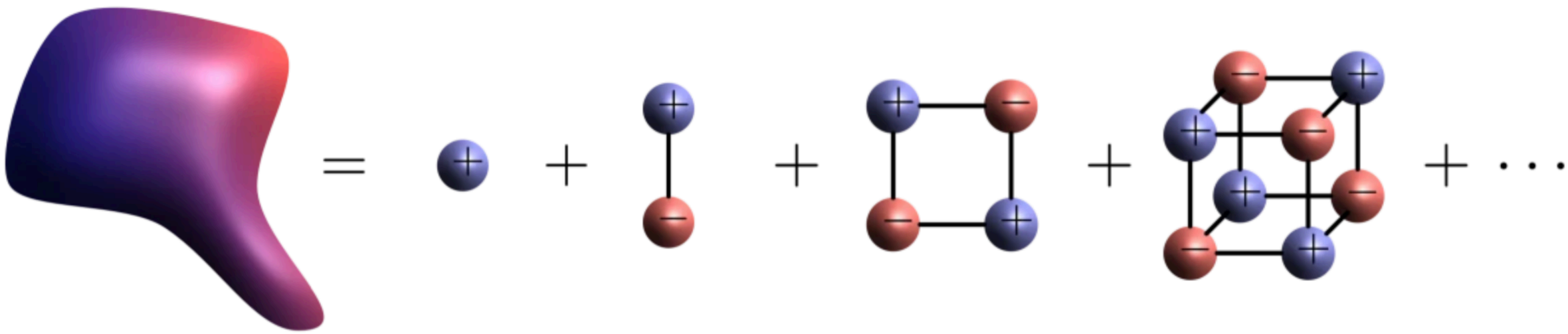
Based on work in progress with

Gautier Durieux, Matthew McCullough, and Jérémie Quevillon

Effective Field Theory



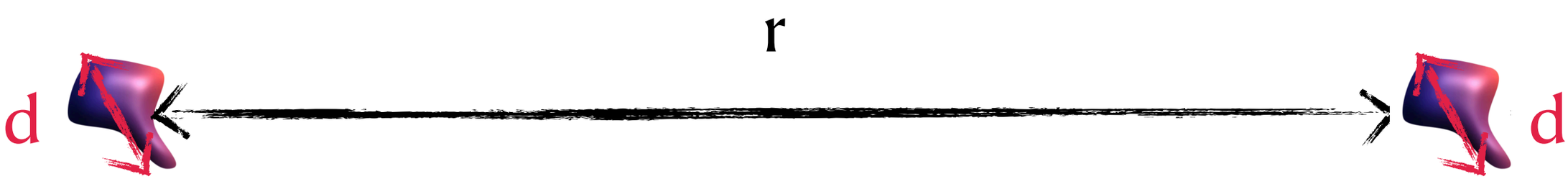
Effective Field Theory



$$V(r) = \frac{\text{monople}}{r} + \frac{\text{dipole}}{r^2} + \frac{\text{qudrupole}}{r^3} + \frac{\text{octupole}}{r^4} + \cdots$$

$$= \frac{a_0}{r} + \frac{a_1 d}{r^2} + \frac{a_2 d^2}{r^3} + \frac{a_3 d^3}{r^4} + \cdots$$

Effective Field Theory



$$V_{l_1 l_2}(r) = \frac{a_{l_1+l_2} d^{l_1+l_2}}{r^{l_1+l_2+1}}$$

Each multipole can be considered as a small “Spurion” parameter in an irrep which explicitly breaks spatial SO(3) symmetry.

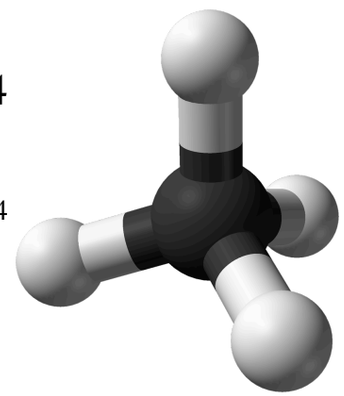
The charge configuration of molecule restrict the lowest multipole.

Multipoles and Symmetry

Lowest multipole allows is

CH4

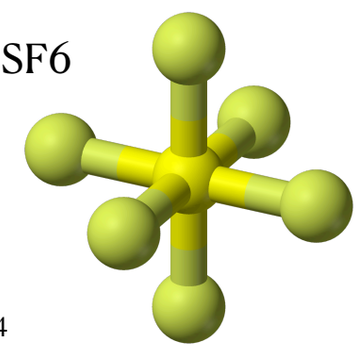
A_4



$$l = 3$$

SF6

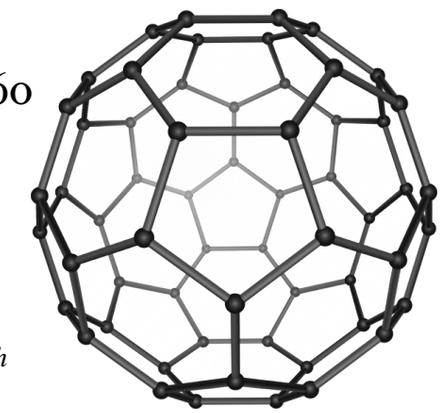
S_4



$$l = 4$$

C60

I_h



$$l = 6$$

[Gelessus, Thiel, Weber, 1995]

Goldstone Bosons

[Das, Hook 2020]

Consider a minimal example with U(1)

$$\mathscr{L} = \left|\partial_\mu \phi\right|^2 - \frac{\lambda}{4}\left(\left|\phi\right|^2 - \frac{f^2}{2}\right)^2$$

We get a single massless Goldstone boson.

To generate a potential we assume some small explicit breaking spurion with charge q

$$V_\epsilon = \epsilon \frac{\lambda}{f^{q-4}} \phi^q + h.c.$$

$$\propto \epsilon M^2 f^2 \cos\frac{q\Pi}{f}$$

$$\mathrm{U}(1) \to \mathbb{Z}_q$$

Goldstone Bosons

Consider an example with SO(N+1) →SO(N)

$$\mathscr{L} = \frac{1}{2}\partial_\mu \phi \cdot \partial^\mu \phi - \frac{\lambda}{4}\left(\phi \cdot \phi - \frac{f^2}{2}\right)^2$$

We get a N massless Goldstone boson

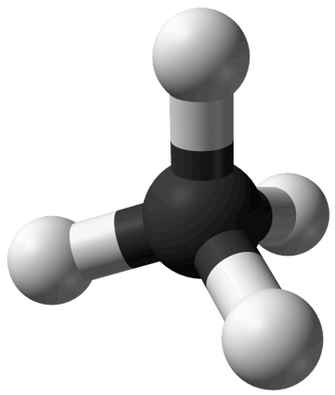
To generate a potential we assume some small explicit breaking spurion in a symmetric irrep with n-indices:

$$V_\epsilon = \epsilon \frac{M^2}{f^{n-2}} T^{a_1 \cdots a_n} \phi_{a_1} \cdots \phi_{a_n}$$

Back to the multipoles

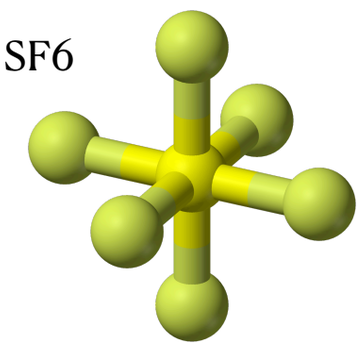
Lowest multipole allows is

CH4



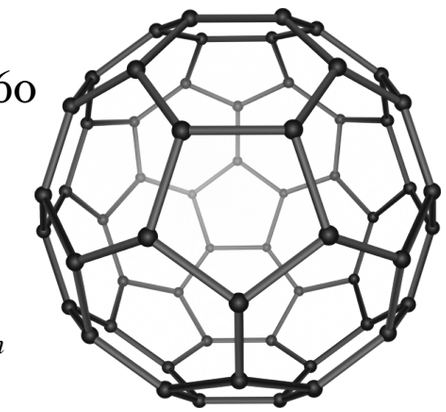
A_4

SF6



S_4

C60



I_h

$$l = 3 \qquad l = 4 \qquad l = 6$$

Discrete symmetry only allows dim-l explicit symmetry breaking operator.

$$\frac{M^2}{f}\phi_1\phi_2\phi_3 \qquad \frac{M^2}{f^2}(\phi_1^4 + \phi_2^4 + \phi_3^4) \qquad \frac{M^2}{f^3}(\phi_1^2 - \phi_2^2)(\phi_1^2 - \phi_3^2)(\phi_2^2 - \phi_3^2)$$

[Enguita-Vileta, Gavela, Houthz, Quinze 2022]

Internal Symmetries

We want to be as general as possible

- Looking to Different breaking patterns
- Preserving some continuous symmetries
- Trying to capture as many potential physical applications as possible
- Focusing on the IR and pNGBs as much as possible

One could consider a standard CCWZ construction in terms of G and H

$$V_{pNGB} = \cdots + \epsilon \frac{M^2}{f^{n-2}} T^{a_1 \cdots a_n} \phi_{a_1} \cdots \phi_{a_n}$$

But pNGB don't care about G and H, only G/H

Question

- If we live in the IR and have some set of light pNGBs with a scalar potential
- How do we essentially “organize” or structurally understand this potential without recourse to the UV symmetries and irreps?

Sphere, sphere, sphere, ...

- Goldstone bosons parameterise coordinates on manifold G/H.

- Notably, lots of pNGB examples live on the sphere

$$G/H \simeq S^N$$

$$\begin{aligned}
\mathrm{SO}(n)\,/\,\mathrm{SO}(n-1) &\simeq S^{n-1} \\
U(n)\,/\,U(n-1) &\simeq S^{2n-1} \\
\mathrm{SU}(n)\,/\,\mathrm{SU}(n-1) &\simeq S^{2n-1} \\
\mathrm{Sp}(n)\,/\,\mathrm{Sp}(n-1) &\simeq S^{4n-1} \\
\mathrm{Sp}(n)\cdot\mathrm{SO}(2)\,/\,\mathrm{Sp}(n-1)\cdot\mathrm{SO}(2) &\simeq S^{4n-1} \\
\mathrm{Sp}(n)\cdot\mathrm{Sp}(1)\,/\,\mathrm{Sp}(n-1)\cdot\mathrm{Sp}(1) &\simeq S^{4n-1} \\
G_2\,/\,\mathrm{SU}(3) &\simeq S^6 \\
\mathrm{Spin}(7)\,/\,G_2 &\simeq S^7 \\
\mathrm{Spin}(9)\,/\,\mathrm{Spin}(7) &\simeq S^{15}
\end{aligned}$$

- We can understand functions on a manifold with harmonic functions!

Sphere and Harmonics

Harmonic functions are eigenfunctions of the Laplace-Beltrami operator

$$\Delta_{S^N} V_\epsilon(\Pi)$$

We have

$$\frac{\partial^2 V_\epsilon(\Pi)}{\partial \Pi^2} + \frac{N-1}{f} \cot \frac{\Pi}{f} \frac{\partial V_\epsilon(\Pi)}{\partial \Pi}$$

And the eigenfunctions are

$$\Delta_{S^N} G_n^{(N-1)/2}(\cos \Pi/f) = -\frac{n(n+N-1)}{f^2} G_n^{(N-1)/2}(\cos \Pi/f)$$

Gegenbauer polynomials

Decomposing pNGB Potentials

For any spontaneous symmetry breaking with

$$G/H \simeq S^N$$

for which there is also explicit symmetry breaking that preserves an SO(N) subgroup in H, we can decompose any pNGB potential as a sum of Gegenbauer polynomials

$$V(\Pi) = \epsilon M^2 f^2 \sum_n a_n G_n^{\frac{N-1}{2}}(\cos \Pi/f)$$

Gegenbauer polynomials form a complete basis of the manifold.

Again back to the multipoles

For the case of SO(N+1) →SO(N) can shown that each polynomial uniquely corresponds to n-index symmetric irrep spurion, i.e. multipole!

$$\epsilon \frac{M^2}{f^{n-2}} T^{a_1 \cdots a_n} \phi_{a_1} \cdots \phi_{a_n} \to G_n^{\frac{N-1}{2}}(\cos \Pi / f)$$

Gegenbauer polynomials are n-dimensional generalisation of Legendre polynomials which is familiar 3D angular momentum eigenfunctions.

$$G_n^{\frac{N-1}{2}}(\cos \Pi / f) \sim \quad n, \mathbf{0}\rangle$$

We can interpret as “angular momentum” in the internal manifold.

Organising a pNGB Potential

As an explicit example, suppose SO(N+1) →SO(N) spontaneous breaking with a small source of explicit SO(N+1) →SO(N) breaking,

$$\epsilon T^{a_1 \cdots a_n}$$

We can organize the potential in epsilon order

$$V_\epsilon = \epsilon M^2 f^2 G_n^{\frac{N-1}{2}}(\cos \Pi/f)$$

$$V_{\epsilon^2} = \epsilon^2 M^2 f^2 \sum_{j=0}^{2n} a_j G_j^{\frac{N-1}{2}}(\cos \Pi/f)$$

$$V_{\epsilon^3} = \epsilon^3 M^2 f^2 \sum_{j=0}^{3n} b_j G_j^{\frac{N-1}{2}}(\cos \Pi/f)$$

Tensor Products

Internal "angular momentum" adds in the same way as in 3D.

$$n, \mathbf{0}\rangle \otimes \ \ n, \mathbf{0}\rangle = \sum_{j=0}^{2n} a_j \ \ j, \mathbf{0}\rangle$$

or, more generally

$$(\ \ n, \mathbf{0}\rangle)^P = \sum_{j=0}^{Pn} a_j \ \ j, \mathbf{0}\rangle$$

So with no work we know which IR operators arise and, for SO(N+1) case, we know which Wilson coefficients in UV are allowed by symmetry

Quantum Corrections

We learn how to write general pNGB potential

$$V(\Pi) = \epsilon M^2 f^2 \left(G_n^{\frac{N-1}{2}}(\cos \Pi/f) + \epsilon \sum_{j=0}^{2n} a_j G_j^{\frac{N-1}{2}}(\cos \Pi/f) + \mathcal{O}(\epsilon^2) + \cdots \right)$$

What are the quantum corrections to the pNGB potential?

Consider Coleman-Weinberg with coset metric:

$$V^{CW} = \frac{1}{2} Tr \int \frac{d^4 p}{(2\pi)^4} \log \left[p^2 + g^{-1} \left(\frac{\delta^2 V}{\delta \varphi^2} - \frac{\delta V}{\delta \varphi} \Gamma \right) \right]$$

[t'Hooft 1973]
[Alonso, Jenkins, Manohar 2016]

However the last term is Laplace-Beltrami for the coset manifold

$$g^{-1} \left(\frac{\delta^2 V}{\delta \varphi^2} - \frac{\delta V}{\delta \varphi} \Gamma \right) = \Delta_{\mathcal{M}} V$$

Quantum Corrections

For the SO(N+1) scenario,

$$
\begin{aligned}
V(\Pi) &= V^{Tr} + V^{CW} + V^{CT} \\
&\approx \epsilon \left[V_\epsilon(\Pi) + \frac{M^2}{32\pi^2}\Delta_{S^N} V_\epsilon(\Pi) + V_\epsilon^{CT}(\Pi) \right] + \\
&\quad \epsilon^2 \left[V_{\epsilon^2}(\Pi) + \frac{1}{64\pi^2}(\Delta_{S^N} V_\epsilon(\Pi))^2 \log\left(\frac{\Delta_{S^N} V_\epsilon(\Pi)}{M^2} \right) \right. \\
&\quad \left. + \frac{M^2}{32\pi^2}\Delta_{S^N} V_{\epsilon^2}(\Pi) + V_{\epsilon^2}^{CT}(\Pi) \right] + \mathcal{O}(\epsilon^3) + \cdots
\end{aligned}
$$

The required countqerterms and allowed RG of coefficients is fixed by symmetry.

Thermal Corrections

[Koutroulis, McCullough, Merchand, Pokorski, Sakurai, 2023]

For the SO(N+1) scenario,

$$
\begin{aligned}
V(\Pi) &= V^{Tr} + V^{CW} + V^{CT} + V^{T} \\
&\approx \epsilon \left[V_\epsilon(\Pi) + \frac{M^2}{32\pi^2}\Delta_{S^N} V_\epsilon(\Pi) + V_\epsilon^{CT}(\Pi) \right] + \\
&\quad \epsilon^2 \left[V_{\epsilon^2}(\Pi) + \frac{1}{64\pi^2}(\Delta_{S^N} V_\epsilon(\Pi))^2 \log\left(\frac{\Delta_{S^N} V_\epsilon(\Pi)}{M^2} \right) \right. \\
&\quad \left. + \frac{M^2}{32\pi^2}\Delta_{S^N} V_{\epsilon^2}(\Pi) + V_{\epsilon^2}^{CT}(\Pi) \right] + \mathcal{O}(\epsilon^3) + \cdots
\end{aligned}
$$

$$
V^{\mathrm{T}} \approx -N\frac{\pi^2}{90}T^4 + \varepsilon\frac{T^2}{24}\Delta_{\mathcal{S}^N} V_\varepsilon - \frac{T}{12\pi}\left(\varepsilon V_\varepsilon''\right)^{3/2} - (N-1)\frac{T}{12\pi}\left(\varepsilon \cot\frac{\Pi}{f}\frac{V_\varepsilon'}{f}\right)^{3/2} + \ldots
$$

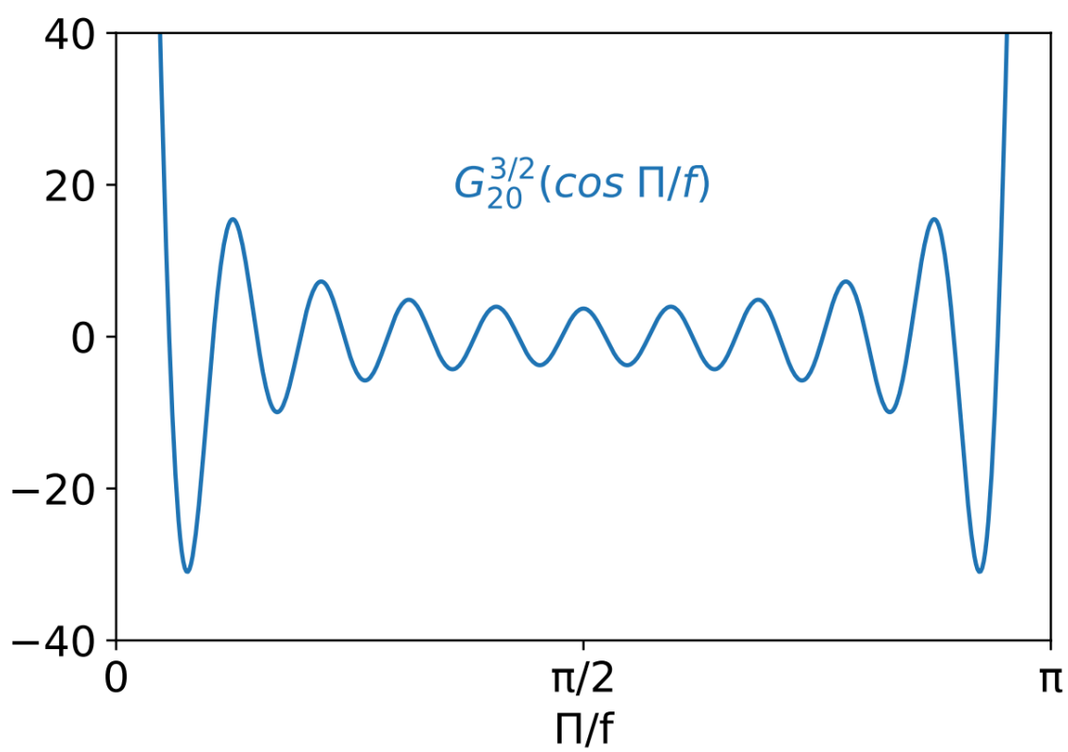
Gegenbauer Higgs

[Duriex, McCullough, Salvioni 2021]

Consider the SO(5) → SO(4) pNGB EFT

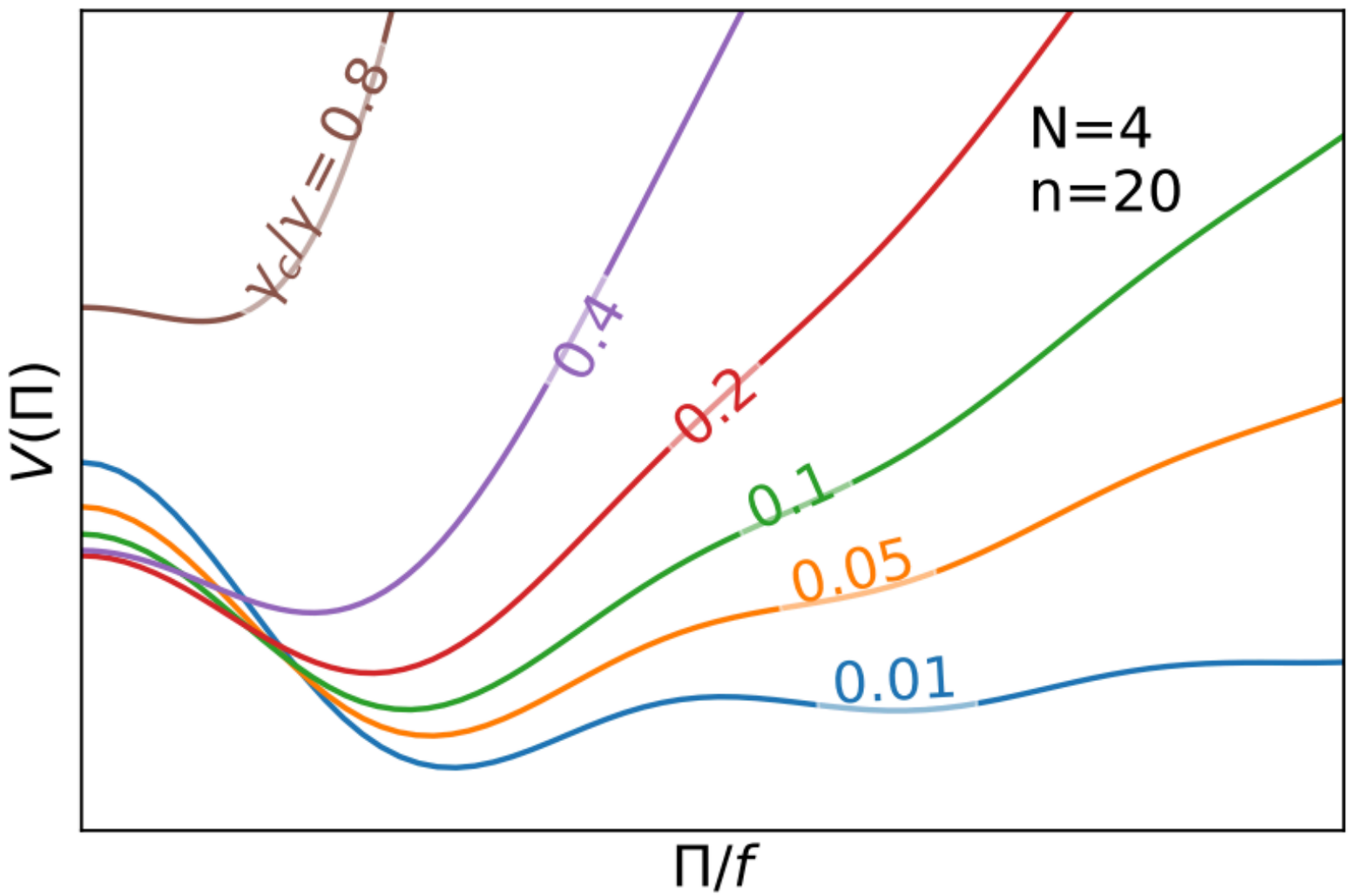
Explicit SO(5) → SO(4) breaking by an irrep spurion

$$T^{a_1 \cdots a_n} \phi_{a_1} \cdots \phi_{a_n} = G_n^{3/2}(\cos \Pi / f)$$



Gegenbauer Higgs

$$V(\Pi) = \kappa \frac{N_c}{16\pi^2} y_t^2 f^2 M_T^2 \left[\sin^2 \Pi/f + \gamma G_n^{3/2}(\cos \Pi/f)\right]$$



Beyond SO(N+1)→ SO(N)

Consider the spontaneous breaking SO(N) → SO(N-1) and explicit breaking of SO(N) → SO(N-k),

For k =1, we already know the corresponding harmonic function (eigenfunction) is

$$G_n^{\overline{N-2}}(\cos\Pi/f) \sim \quad n, \mathbf{0}\rangle$$

What is the corresponding eigenfunction for arbitrary k smaller than N ?

$$l_{N-1}, l_{N-2}, \cdots, l_{N-k}, \mathbf{0}\rangle$$

Beyond SO(N+1)→ SO(N)

- Hyperspherical harmonics

$$l_N, l_{N-1}, \cdots, l_2, l_1\rangle$$

The indices are organized according to the chain of subgroups SO(N) ⊃ SO(N – 1) ⊃ · · · ⊃ SO(2).

Beyond SO(N+1)→ SO(N)

- Hyperspherical harmonics

$$l_N, l_{N-1}, \cdots, l_2, l_1\rangle$$

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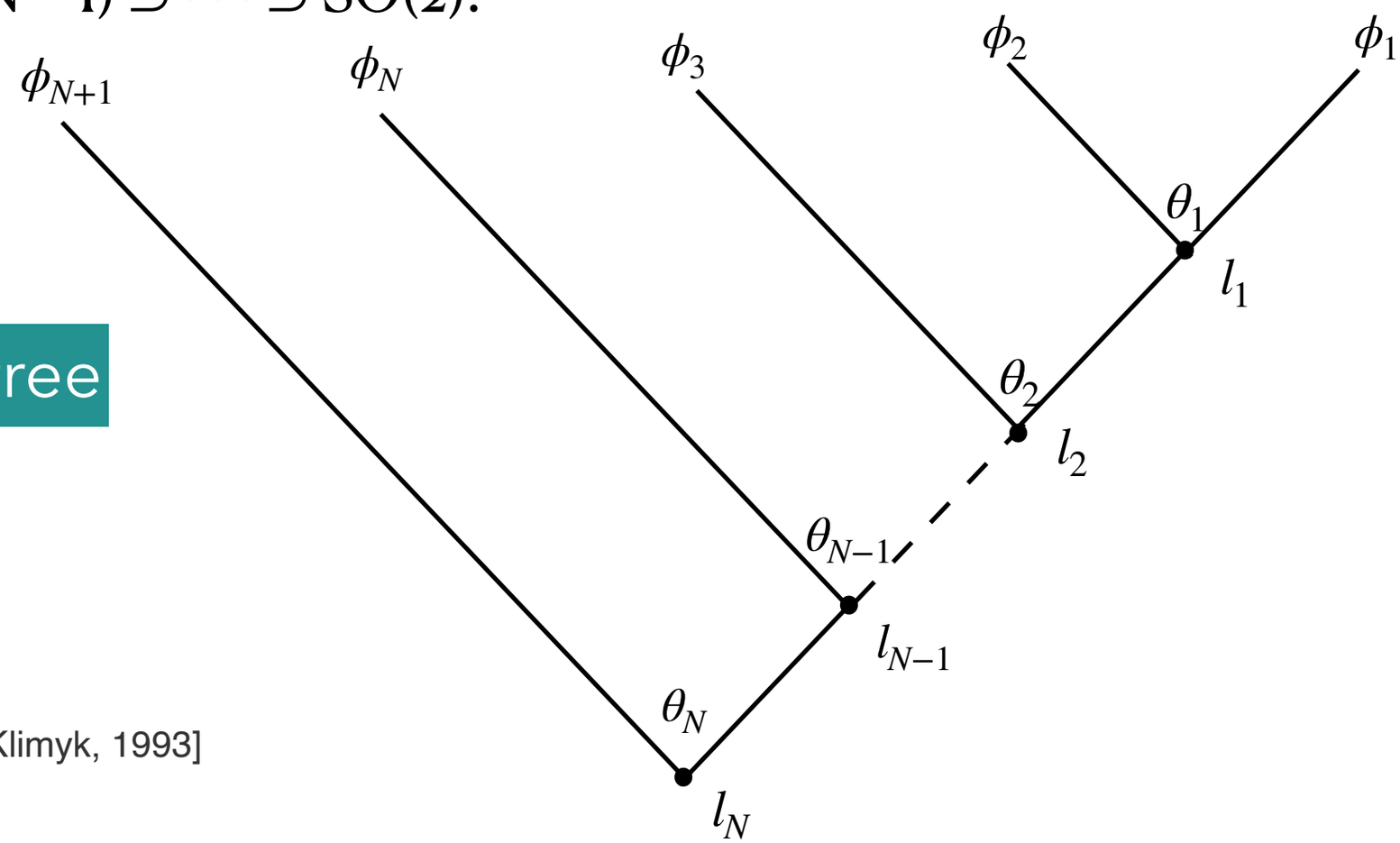

Standard tree

[N. Ja. Vilenkin, A. U. Klimyk, 1993]

Beyond SO(N+1)→ SO(N)

- Hyperspherical harmonics

$$l_N, l_{N-1}, \cdots, l_2, l_1\rangle \quad = \prod_{k=2}^{N} \left((\sin\theta_k)^{l_{k-1}} G^{\frac{k-1}{2}+l_{k-1}}_{l_k - l_{k-1}} (\cos\theta_k) \right) e^{i l_1 \theta_1}$$

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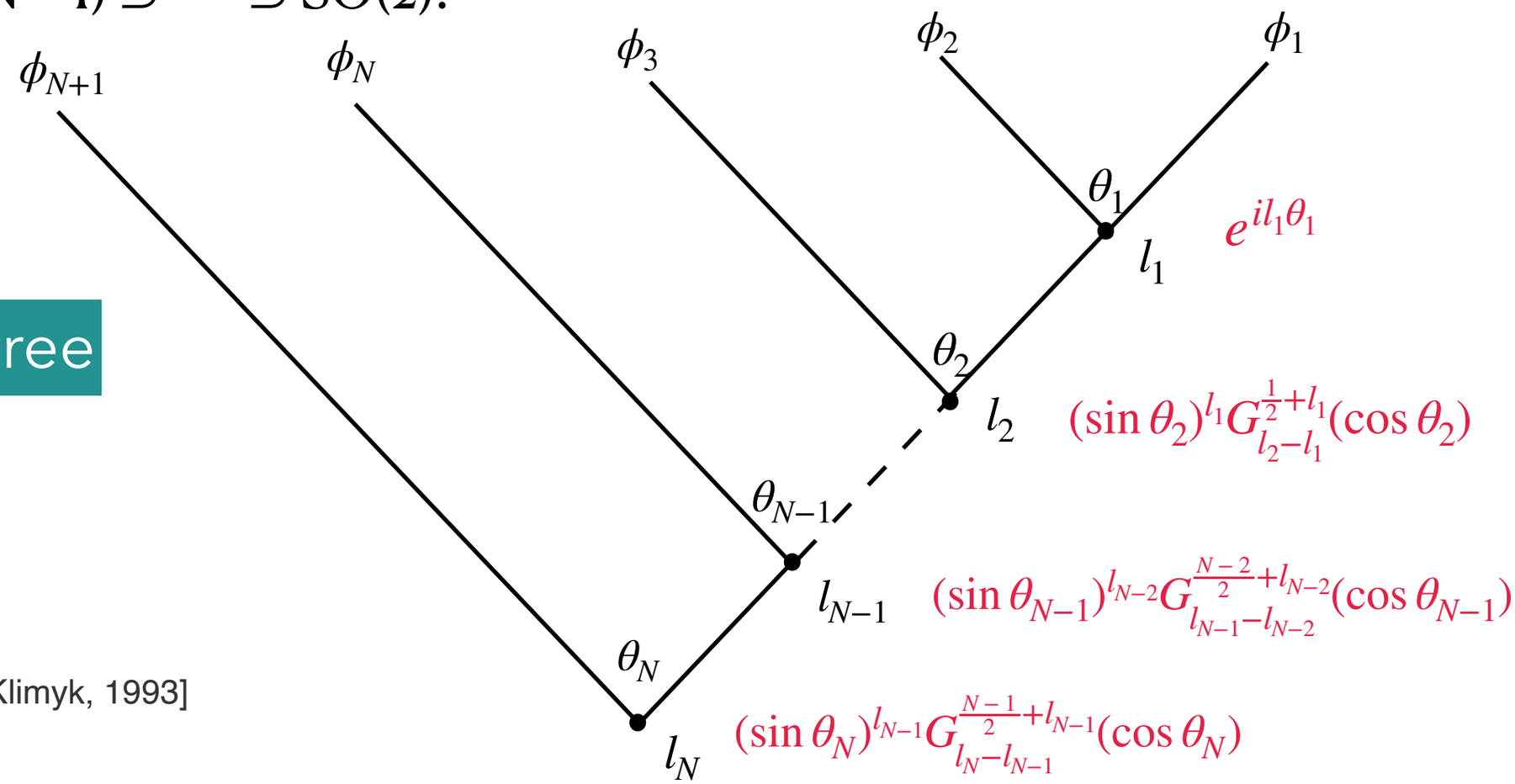

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Beyond SO(N+1)→ SO(N)

Consider the spontaneous breaking SO(N) → SO(N-1) and explicit breaking of SO(N) → SO(k)×SO(N-k),

What is the corresponding eigenfunction?

Beyond SO(N+1)→ SO(N)

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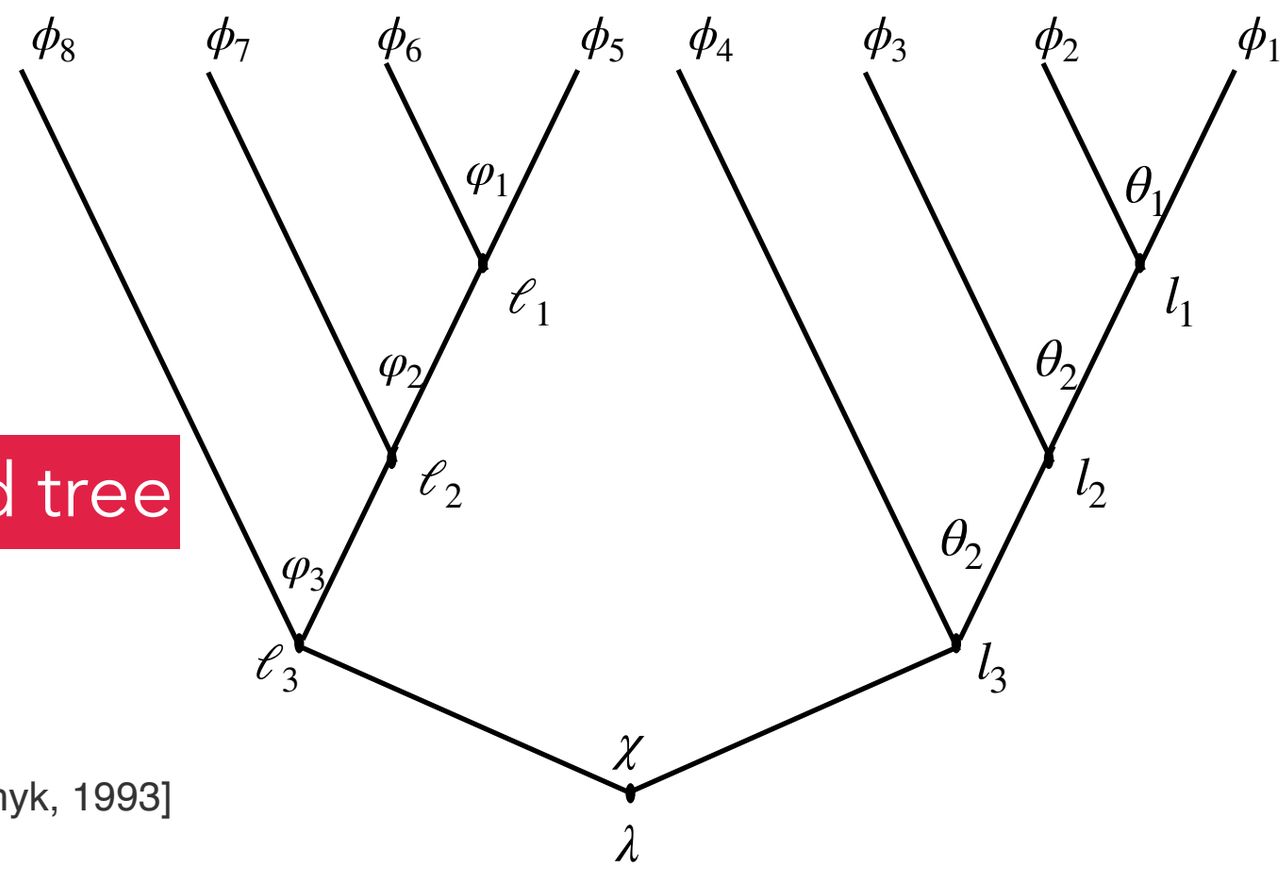
Polyspherical harmonics

Non-standard tree



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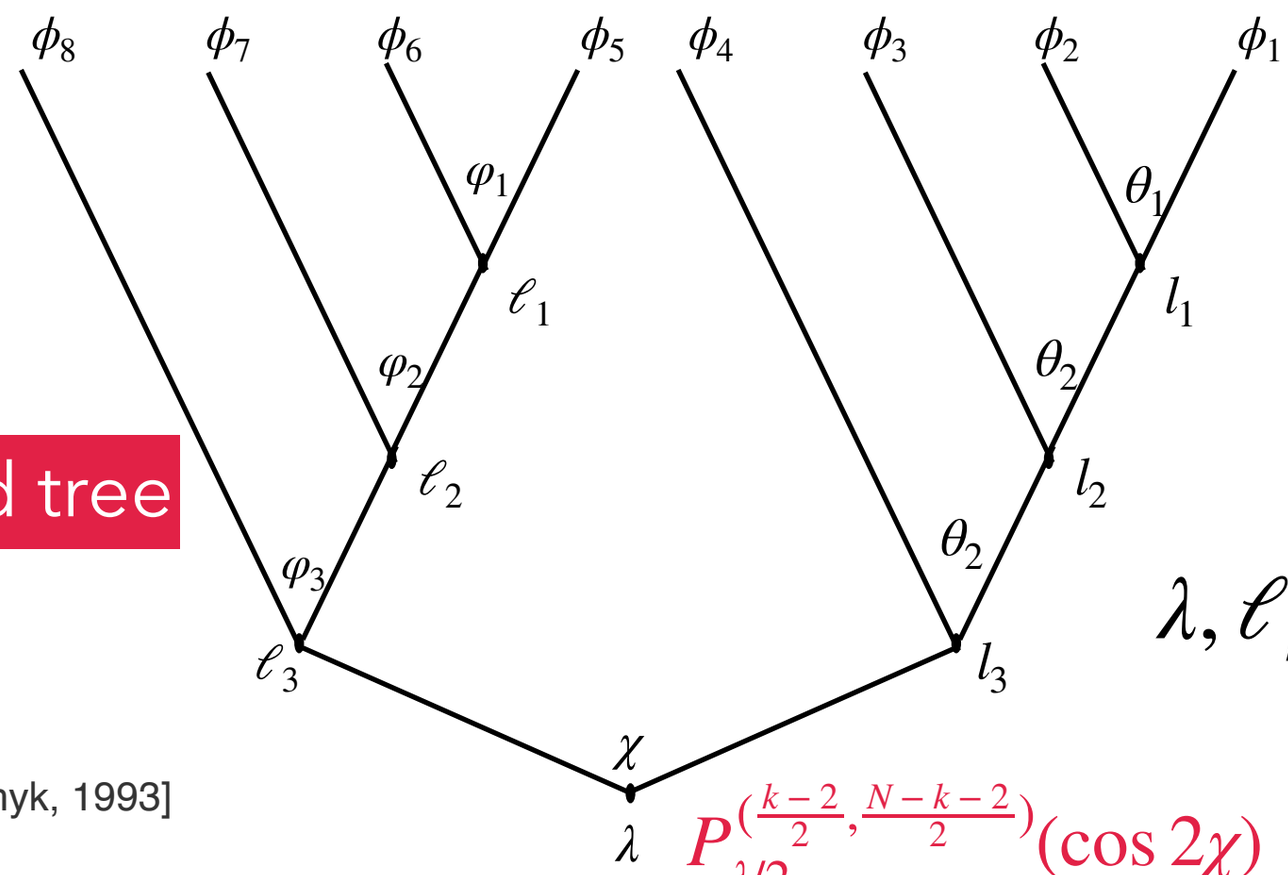
Non-standard tree



$$\lambda, \ell_{k-1}, \cdots, l_{N-k-1}, \cdots\rangle$$

$$P_{\lambda/2}^{(\frac{k-2}{2},\frac{N-k-2}{2})}(\cos 2\chi)$$

[N. Ja. Vilenkin, A. U. Klimyk, 1993]

Beyond SO(N+1)→ SO(N)

Consider the spontaneous breaking SO(N) → SO(N-1) and explicit breaking of SO(N) → SO(k)×SO(N-k),

What is the corresponding eigenfunction? $n, \mathbf{0}, \mathbf{0}\rangle$

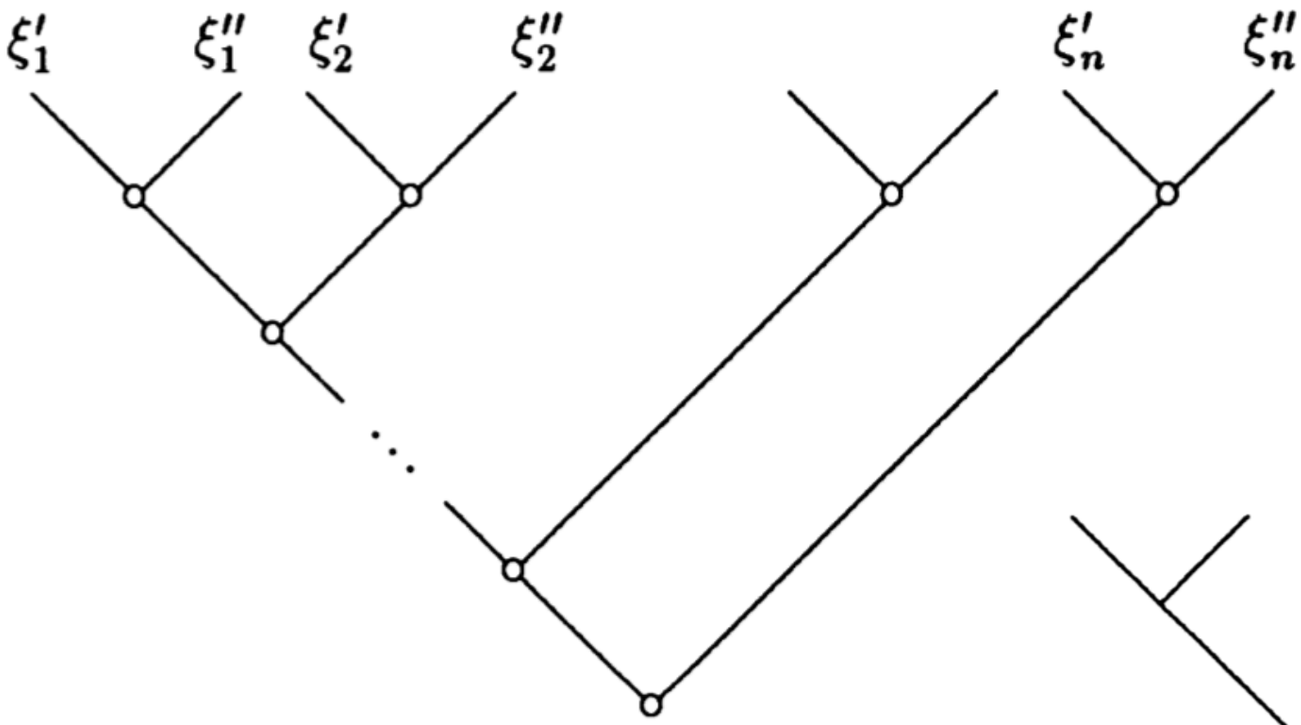
$$V(\Pi) \sim P_{n/2}^{(\frac{k-2}{2},\frac{N-k-2}{2})}(\cos 2\Pi/f)$$

which is the Jacobi polynomial, generalized version of Gegenbauer polynomial.

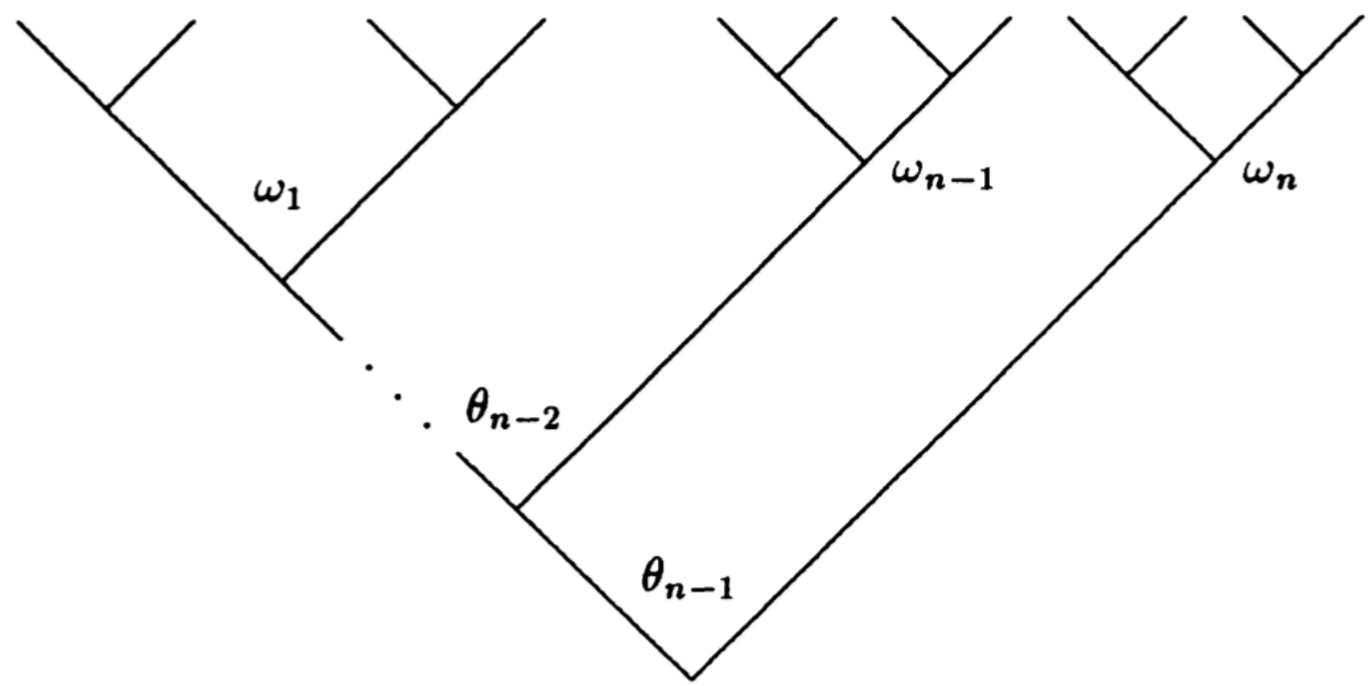
$$G_n^{\alpha}(x) = \frac{(2\alpha)_n}{(\alpha+1/2)_n} P_n^{(\alpha-1/2,\alpha-1/2)}(x)$$

Beyond SO(N+1)→ SO(N)

SU(N) ~ S^{2N-1}



Sp(N) ~ S^{4N-1}

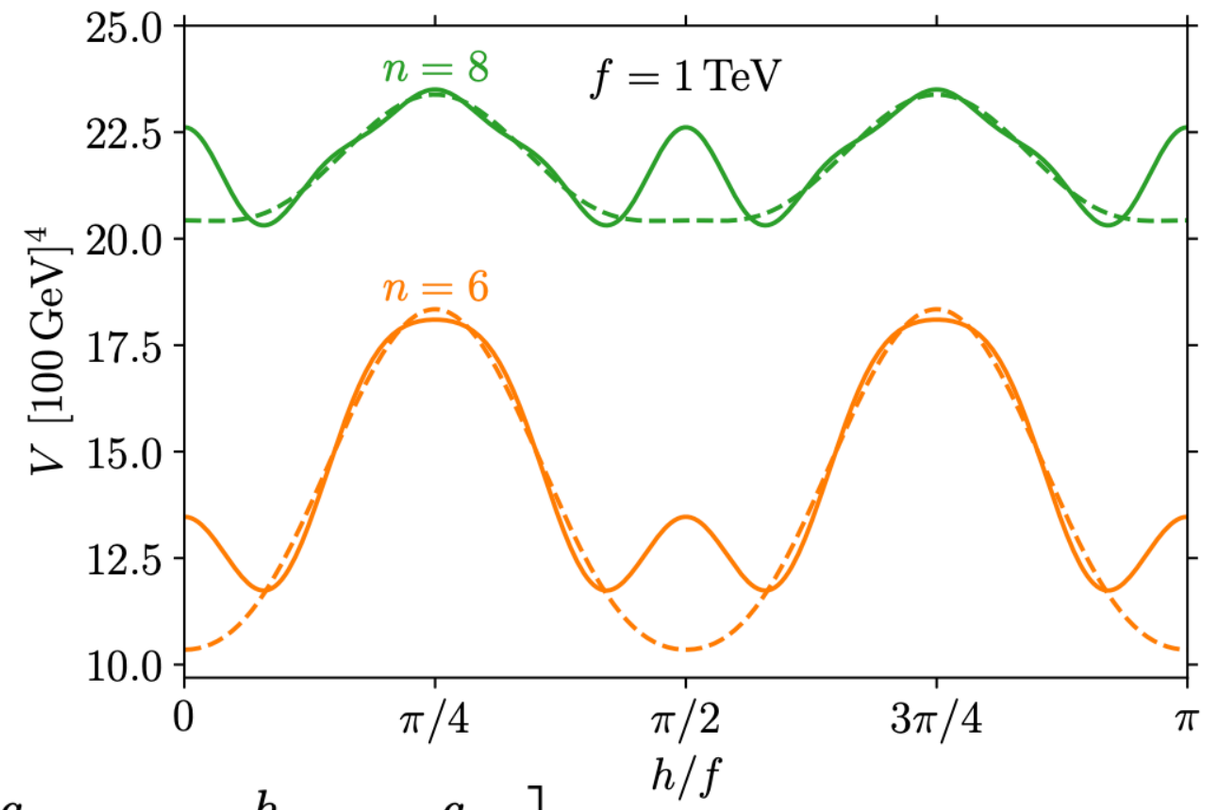


Gegenbauer's Twin

[Duriex, McCullough, Salvioni 2022]

From a symmetry perspective the Twin Higgs is essentially SO(8) → SO(7) spontaneous breaking with SO(8) → SO(4) x SO(4) explicit breaking.

$$V(\Pi) \sim \epsilon M^2 f^2 P^{(1,1)}_{n/2}(\cos 2\Pi/f)$$
$$\sim \epsilon M^2 f^2 G^{3/2}_{n/2}(\cos 2\Pi/f)$$



$$V_t \approx \beta f^4 \left[\sin^4 \frac{h}{f} \log \frac{a}{\sin^2 \frac{h}{f}} + \cos^4 \frac{h}{f} \log \frac{a}{\cos^2 \frac{h}{f}} \right]$$

Beyond the Sphere?

But, the coset is just S^N. Could there be alternative models, with different G and H, but same G/H, like SU(N), or generalisations that are interesting phenomenologically?

$$
\begin{aligned}
\mathrm{SO}(n)\,/\,\mathrm{SO}(n-1) &\simeq S^{n-1} \\
U(n)\,/\,U(n-1) &\simeq S^{2n-1} \\
\mathrm{SU}(n)\,/\,\mathrm{SU}(n-1) &\simeq S^{2n-1} \\
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\mathrm{Spin}(9)\,/\,\mathrm{Spin}(7) &\simeq S^{15}
\end{aligned}
$$

GXG/G
SO(n)/SO(n-k)XSO(k)
U(n)/U(n-k)XU(k)
SU(n)/SO(n)
.....

Also, could the top sector be embedded into a larger multipole too?
Had just assumed the old n=1 spurion for fermion spurion Yukawas.

Homogeneous Space

N-sphere, S^N is a typical example of the homogeneous spaces.
All homogeneous space are quotient space.

If the coset space G/H is homogeneous space, the pNGB potential is functions on G/H , i.e., H-invariant functions on G.

examples of homogeneous spaces

space X	group G	stabilizer H
spherical space $\mathbb{S}^{n-1}$	$\mathrm{O}(n)$	$\mathrm{O}(n-1)$
oriented $\mathbb{S}^{n-1}$	$\mathrm{SO}(n)$	$\mathrm{SO}(n-1)$
projective space $\mathbb{PR}^{n-1}$	$\mathrm{PO}(n)$	$\mathrm{PO}(n-1)$
Euclidean space $\mathbb{E}^n$	$\mathrm{E}(n)$	$\mathrm{O}(n)$
oriented $\mathbb{E}^n$	$\mathrm{E}^+(n)$	$\mathrm{SO}(n)$
hyperbolic space $\mathbb{H}^n$	$\mathrm{O}^+(1,n)$	$\mathrm{O}(n)$
oriented $\mathbb{H}^n$	$\mathrm{SO}^+(1,n)$	$\mathrm{SO}(n)$
anti-de Sitter space AdS_{n+1}	$\mathrm{O}(2,n)$	$\mathrm{O}(1,n)$
Grassmannian $\mathrm{Gr}(r,n)$	$\mathrm{O}(n)$	$\mathrm{O}(r)\times\mathrm{O}(n-r)$
affine space $\mathbb{A}(n,K)$	$\mathrm{Aff}(n,K)$	$\mathrm{GL}(n,K)$

Ex) G = SO(N+1), H = SO(N)

The Gegenbauer polynomial $G_n^{\frac{N-1}{2}}(\cos\Pi/f)$ is SO(N) invariant functions on SO(N+1).

Future directions

- Main goal throughout has ultimately been concerned with trying to understand Higgs naturalness.
- Could there be anything to learn in condensed matter systems with pNGBs, or for spacetime pNGBs?
- Finite temperature correction of the scalar potential
- What is the origin of the large multipoles?
- Large-J expansion?

Summary

- Supposedly no exact GBs in nature, only pNGBs
- For the physics of pNGBs, their scalar potential is more phenomenological relevant than are super high dimension derivative operators
- Historically it has been assumed that the explicit symmetry breaking that generates pNGB potential is minimal, with minimal multipoles
- That may not be the case and there could be great richness structure in the pNGB potentials with significant phenomenological implications for their applications
- This structure can be organized and understood, even radiatively
- Studying operators in large irreps (multipoles) in theories with spontaneous symmetry breaking may have interesting theory lessons, perhaps like the abelian version with the large charge expansion.

Thank you for your attention

[B. Bellazzini, C. Csáki, J. Serra 2014]

$\mathcal{G}$	$\mathcal{H}$	C	N_G	$\mathbf{r}_{\mathcal{H}} = \mathbf{r}_{\mathrm{SU}(2)\times\mathrm{SU}(2)}\,(\mathbf{r}_{\mathrm{SU}(2)\times\mathrm{U}(1)})$	Ref.
SO(5)	SO(4)	✓	4	$\mathbf{4} = (\mathbf{2},\mathbf{2})$	[11]
SU(3) × U(1)	SU(2) × U(1)		5	$\mathbf{2}_{\pm 1/2} + \mathbf{1}_0$	[10, 35]
SU(4)	Sp(4)	✓	5	$\mathbf{5} = (\mathbf{1},\mathbf{1}) + (\mathbf{2},\mathbf{2})$	[29, 47, 64]
SU(4)	$[\mathrm{SU}(2)]^2 \times \mathrm{U}(1)$	✓*	8	$(\mathbf{2},\mathbf{2})_{\pm 2} = 2\cdot(\mathbf{2},\mathbf{2})$	[65]
SO(7)	SO(6)	✓	6	$\mathbf{6} = 2\cdot(\mathbf{1},\mathbf{1}) + (\mathbf{2},\mathbf{2})$	–
SO(7)	G_2	✓*	7	$\mathbf{7} = (\mathbf{1},\mathbf{3}) + (\mathbf{2},\mathbf{2})$	[66]
SO(7)	SO(5) × U(1)	✓*	10	$\mathbf{10}_0 = (\mathbf{3},\mathbf{1}) + (\mathbf{1},\mathbf{3}) + (\mathbf{2},\mathbf{2})$	–
SO(7)	$[\mathrm{SU}(2)]^3$	✓*	12	$(\mathbf{2},\mathbf{2},\mathbf{3}) = 3\cdot(\mathbf{2},\mathbf{2})$	–
Sp(6)	Sp(4) × SU(2)	✓	8	$(\mathbf{4},\mathbf{2}) = 2\cdot(\mathbf{2},\mathbf{2})$	[65]
SU(5)	SU(4) × U(1)	✓*	8	$\mathbf{4}_{-5} + \bar{\mathbf{4}}_{+5} = 2\cdot(\mathbf{2},\mathbf{2})$	[67]
SU(5)	SO(5)	✓*	14	$\mathbf{14} = (\mathbf{3},\mathbf{3}) + (\mathbf{2},\mathbf{2}) + (\mathbf{1},\mathbf{1})$	[9, 47, 49]
SO(8)	SO(7)	✓	7	$\mathbf{7} = 3\cdot(\mathbf{1},\mathbf{1}) + (\mathbf{2},\mathbf{2})$	–
SO(9)	SO(8)	✓	8	$\mathbf{8} = 2\cdot(\mathbf{2},\mathbf{2})$	[67]
SO(9)	SO(5) × SO(4)	✓*	20	$(\mathbf{5},\mathbf{4}) = (\mathbf{2},\mathbf{2}) + (\mathbf{1}+\mathbf{3},\mathbf{1}+\mathbf{3})$	[34]
$[\mathrm{SU}(3)]^2$	SU(3)		8	$\mathbf{8} = \mathbf{1}_0 + \mathbf{2}_{\pm 1/2} + \mathbf{3}_0$	[8]
$[\mathrm{SO}(5)]^2$	SO(5)	✓*	10	$\mathbf{10} = (\mathbf{1},\mathbf{3}) + (\mathbf{3},\mathbf{1}) + (\mathbf{2},\mathbf{2})$	[32]
SU(4) × U(1)	SU(3) × U(1)		7	$\mathbf{3}_{-1/3} + \bar{\mathbf{3}}_{+1/3} + \mathbf{1}_0 = 3\cdot\mathbf{1}_0 + \mathbf{2}_{\pm 1/2}$	[35, 41]
SU(6)	Sp(6)	✓*	14	$\mathbf{14} = 2\cdot(\mathbf{2},\mathbf{2}) + (\mathbf{1},\mathbf{3}) + 3\cdot(\mathbf{1},\mathbf{1})$	[30, 47]
$[\mathrm{SO}(6)]^2$	SO(6)	✓*	15	$\mathbf{15} = (\mathbf{1},\mathbf{1}) + 2\cdot(\mathbf{2},\mathbf{2}) + (\mathbf{3},\mathbf{1}) + (\mathbf{1},\mathbf{3})$	[36]

Table 1: Symmetry breaking patterns $\mathcal{G} \to \mathcal{H}$ for Lie groups. The third column denotes whether the breaking pattern incorporates custodial symmetry. The fourth column gives the dimension N_G of the coset, while the fifth contains the representations of the GB's under $\mathcal{H}$ and $\mathrm{SO}(4) \cong \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ (or simply $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ if there is no custodial symmetry). In case of more than two SU(2)'s in $\mathcal{H}$ and several different possible decompositions we quote the one with largest number of bi-doublets.