Emergent particles of a dS universe:

Thermal interpretation of the stochastic formalism and beyond

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Introduction

- The stochastic formalism

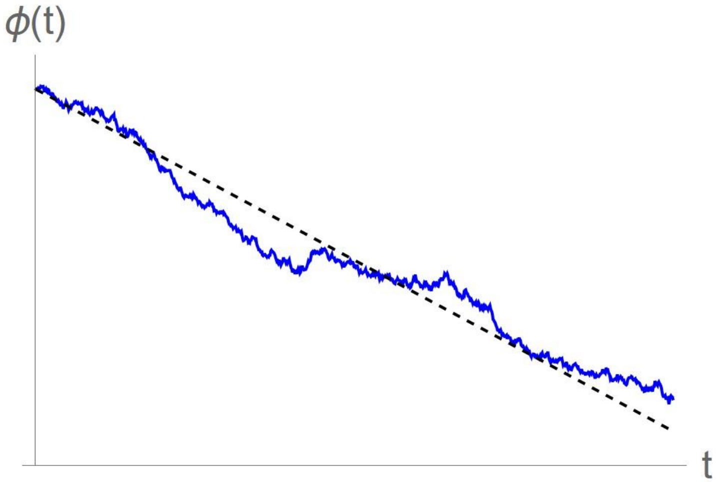
Starobinsky (1986)
Starobinsky, Yokoyama (1994)

  - Effective theory of a slow-rolling scalar field in inflation
    - Wave number below a cutoff scale at $k = \epsilon a H$ ($\epsilon \ll 1$); "coarse-grained field"
  - Classicality of superhorizon modes
    - Measurement outcome of quantum state = Ensemble of classical random fields

Polarski, Starobinsky (1996)
Lesgourgues, Polarski, Starobinsky (1997)
Kiefer, Polarski (1998)
Kiefer, Polarski (2009)

  - Langevin equation ("classical" random evolution)

$$d\phi = -\frac{V'(\phi)}{3H}dt + \sqrt{\frac{H^3}{4\pi}}\, dW$$



Introduction

Finelli, Marozzi, Starobinsky, Vacca, Venturi (2009)
Rigopoulos (2013) & (2016)

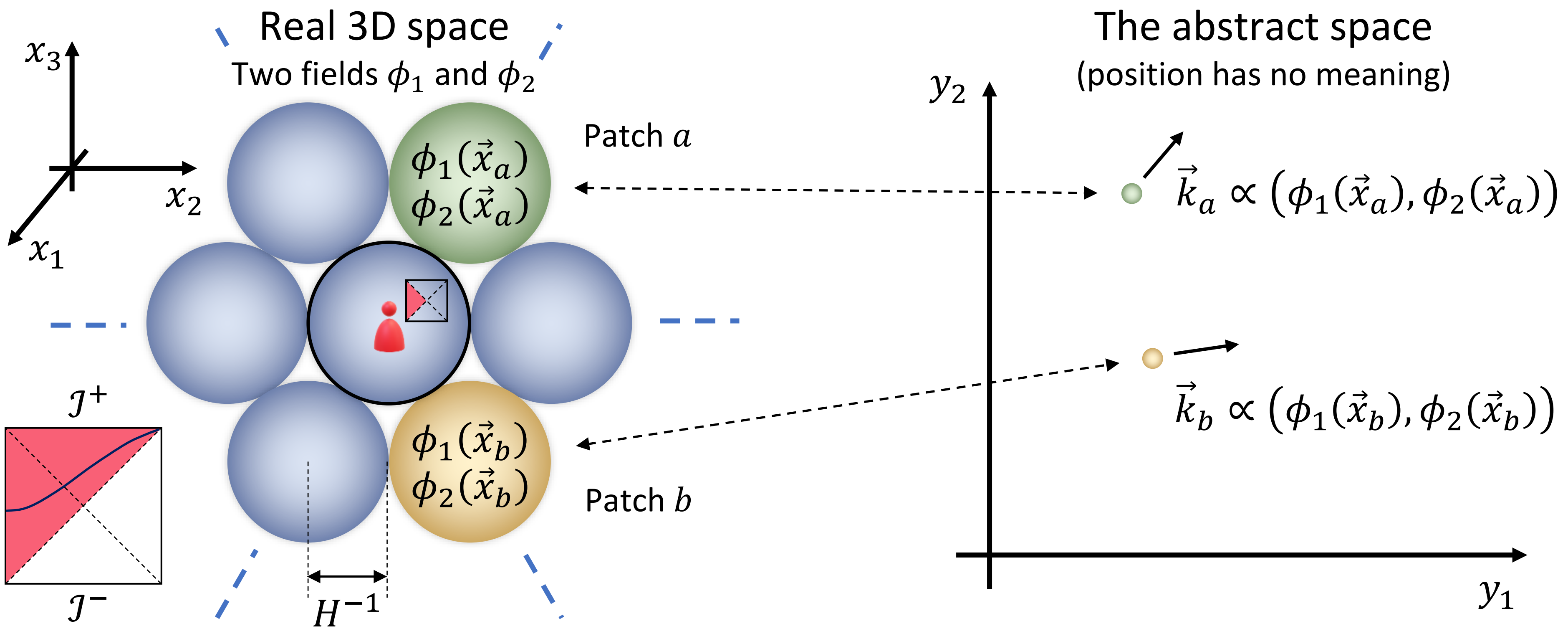
- Some similarities with thermal effects, but not a thermal effect
  - Despite the appearance of $T_{dS} = H/2\pi$ in many points…
    - Intuitive understanding of the formalism
  - It is for the field at the superhorizon limit, instead of at the horizon scale
    - T_{dS} originates from the inaccessibility of a local observer by the horizon
  - Has spin dependence; not a universal effect from spacetime structure
    - Happens only for spin 0 and 2 fields
  - The field's spectrum is scale invariant, not thermal ($\rho \sim e^{-\beta k}$)

Introduction

- Thermal interpretation by Rigopoulos (2013) & (2016)
  - Effective action of superhorizon modes has a term responsible for stochastic force
    - Schwinger-Keldysh formalism; effective action expressed in the Keldysh basis
  - Satisfies the fluctuation-dissipation relation with the Hubble friction at T_{dS}
  - Interprets stochastic evolution as Brownian motion in a medium at T_{dS}
- Can we reach the same conclusion from physical observations?
  - Proposing by a general formalism & a heat bath model

The general formalism

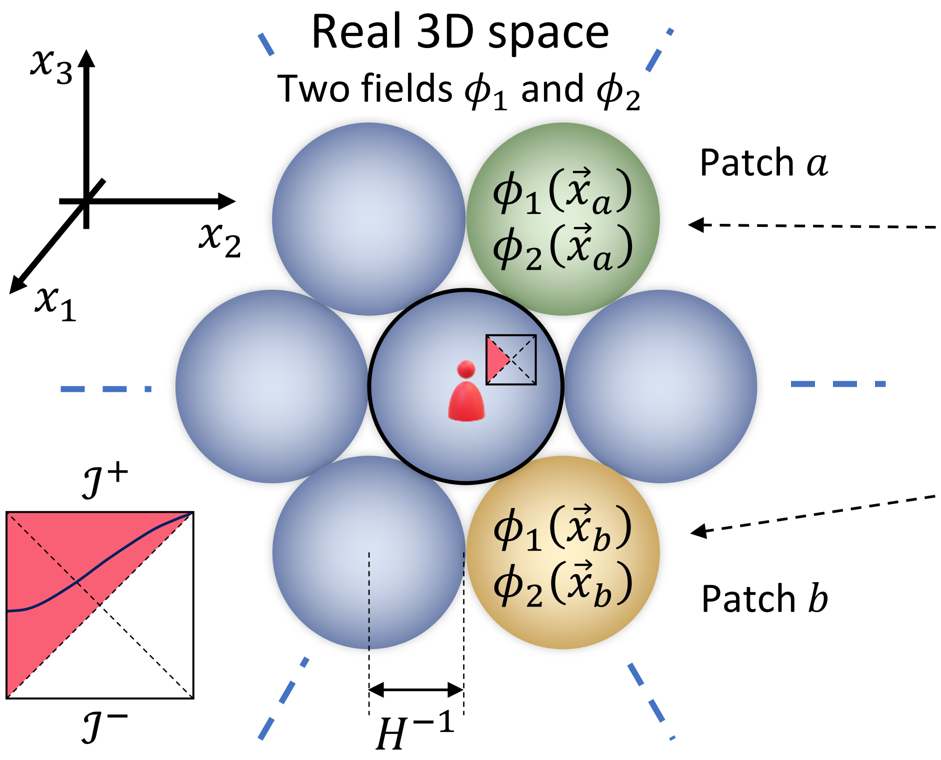
- We treat horizon-sized spatial regions as “particles” in a virtual space



The general formalism

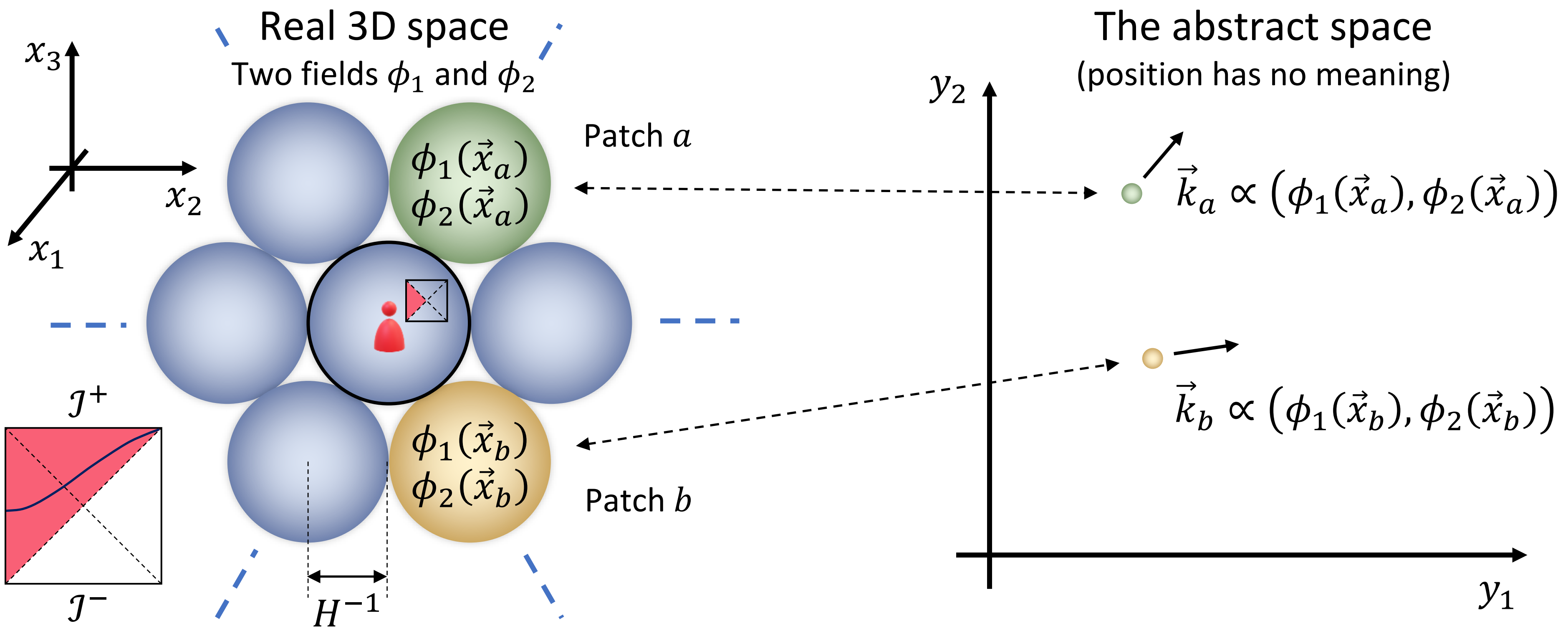
Time slice of dS universe
= flat slicing of dS space (in this talk!)

Causal patch
$\simeq$ Hubble volume



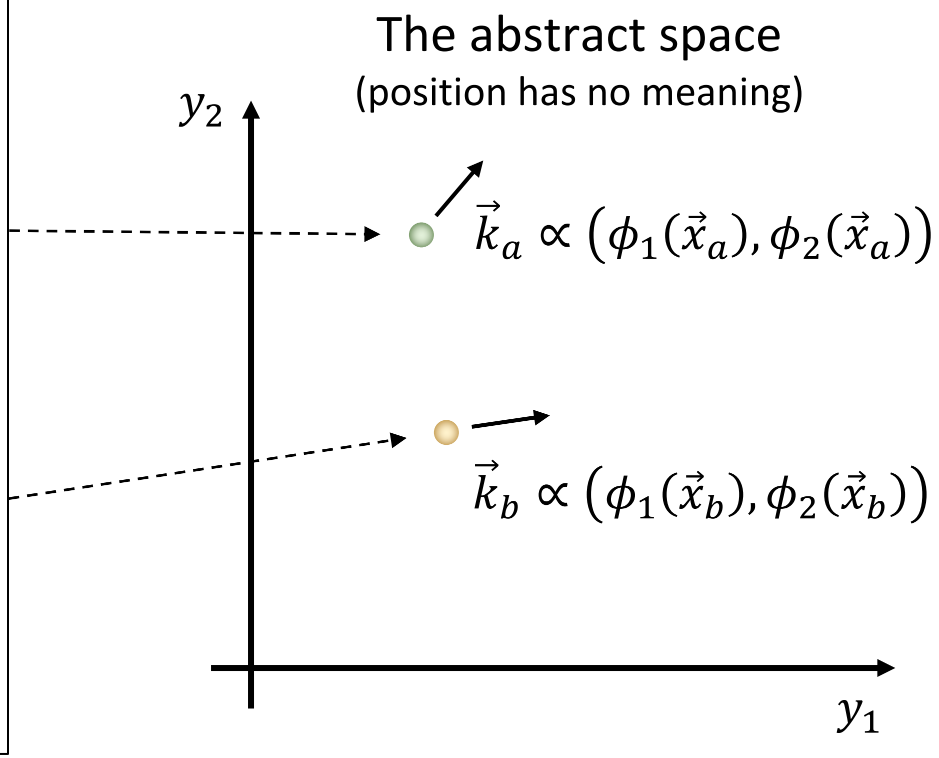
- The formalism depicts the space composed of many causal patches
  - "Horizon in an objective manner"
  - Note: We are developing a formalism, NOT a new theory!
- Each patch has its field value for the coarse-grained fields

The general formalism



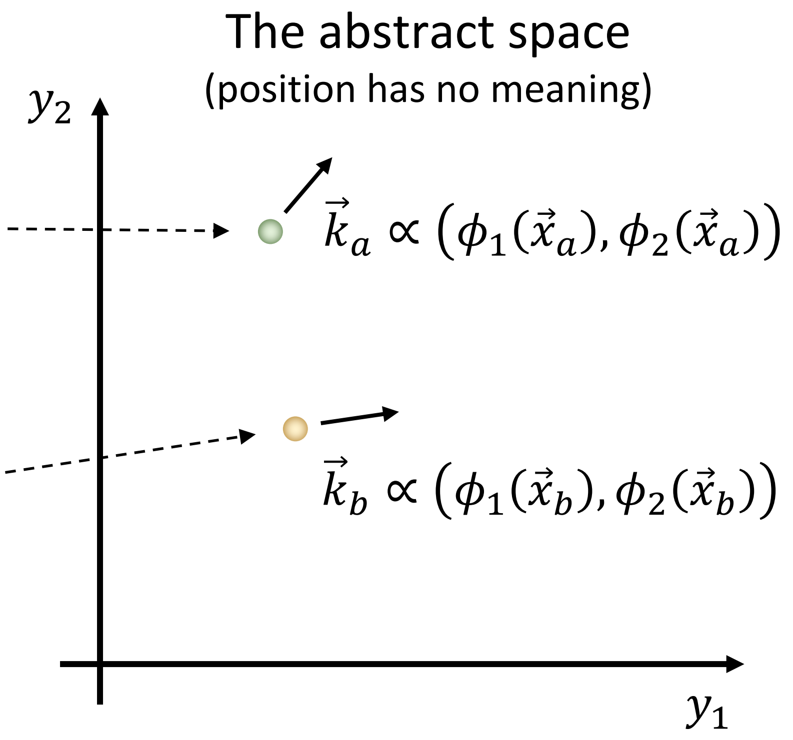
The general formalism

- Each patch is regarded as a particle in another space
  - "Emergent particle"
  - "The abstract space"



The general formalism

- Two principles assumed:
  1. Field value ∝ momentum
     - "Dual description"
  2. The usual classical mechanics in the abstract space
     - Set of relations between physical quantities in there



The general formalism

- We translate several aspects of dS universe into the dynamics in the abstract space
  - Stochastic field evolution
  - 1st slow-roll condition and Hubble expansion
- We stick to the “minimal non-minimal” setup
  - $V_0 = 3M_P^2 H^2$: Unspecified background energy density
  - ϕ: One real minimally coupled slow-rolling spectator scalar field (coarse-grained)

The general formalism

- Identifying physical quantities in the abstract space
  - Uniquely identified through the assumed classical mechanics

Causal patch in dS	Emergent particle in the Abstract space	Equation
Field value ϕ	Momentum k	$k = \frac{4\pi M_P m_\phi}{\sqrt{3}H^2}\phi$
Potential V_ϕ	Kinetic energy E_k	$E_k = \frac{4\pi}{3H^3}V_\phi$
Background energy V_0	Mass M	$M = \frac{4\pi M_P^2}{H}$
Potential slope V'_ϕ	Velocity v	$v = \frac{1}{\sqrt{3}M_P H m_\phi}V'_\phi$

Heat bath model

- Langevin equation of ϕ → Brownian motion of emergent particle

$$d\phi = -\frac{V'(\phi)}{3H} dt + \sqrt{\frac{H^3}{4\pi}}\, dW \quad \Longrightarrow \quad dk = -\frac{4\pi M_P^2 m_\phi^2}{3H^2}\, v\, dt + \sqrt{\frac{4M_P^2 m_\phi^2}{3H}}\, dW$$

  - Drag force $\propto -v$
  - Continuous random impulses (momentum kicks)

- Classical Brownian motion in a medium at a finite temperature
  - We build a concrete particle model of the heat bath

Heat bath model

- Abstract space filled with a heat bath of another type of particle
- Successful model: massless bath particles
  - c: "Speed of light (massless particles)"
  - T: Bath temperature
  - λ: Number density of bath particle
  - Bath particles are absorbed by emergent particles upon collision
    - Just like photons in our world

Bath particle

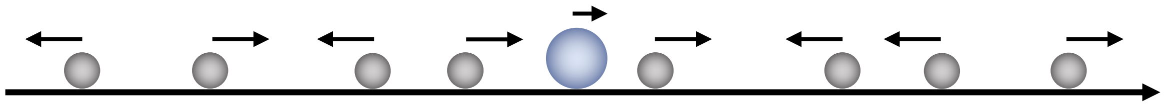
$$f(p) \propto e^{-\beta c|p|}$$

$$v = \pm c$$

Emergent particle

$$k = \frac{4\pi M_P m_\phi}{\sqrt{3}H^2}\phi$$

$$v = \frac{1}{\sqrt{3}M_P H m_\phi}V'_\phi$$



The abstract space (1D)

Heat bath model

- Momentum conservation at each collision
  - Kinetic theory of bath particles + central limit theorem gives
    - $\Delta t \gtrsim \left(H^2/M_P^2 \ll 1\right) \times \frac{1}{H}$ is sufficient to have Gaussianity

$$\Delta k = -\left[2\lambda \int_0^\infty p f(p) dp\right] v\, \Delta t + \left[2\lambda c \int_0^\infty p^2 f(p) dp\right]^{1/2} \Delta W$$

    - Deterministic force $\propto -v$ and Gaussian-distributed random kicks
      - Desired thermal motion, hence thermal interpretation is now available
  - Three model parameters, two equations from coefficients?

Heat bath model

- Energy conservation?
  - Bath particles are absorbed. Kinetic energy is lost.
  - On the other hand, the Hubble expansion is massive particle production
  - By virtue of energy conservation, we equate the amount of kinetic energy loss from collisions to the one required for Hubble expansion

$$-\frac{\langle \Delta E \rangle}{\Delta t} \simeq 2\lambda c^2 \int_0^\infty p f(p) dp = 12\pi M_P^2$$

Heat bath model

- Parameters of the successful model
  - $c = \frac{3H}{m_\phi}$
  - $T = \frac{H}{2\pi} = T_{dS}$
  - $\lambda = \frac{8\pi^2 M_P^2 m_\phi}{H^2}$

$$dk = -\frac{4\pi M_P^2 m_\phi^2}{3H^2}\, v\, dt + \sqrt{\frac{4M_P^2 m_\phi^2}{3H}}\, dW$$

$$d\phi = -\frac{V'(\phi)}{3H} dt + \sqrt{\frac{H^3}{4\pi}}\, dW$$

Giving thermal interpretation

Stochastic field evolution ⟷ Brownian motion in the abstract space

- Superhorizon fluctuation modes
- Quantum field evolution

- Heat bath of massless particles
- Thermal motion

↑

Emergent particle formalism

1. Dual description of scalar field
2. Usual classical mechanics in the abstract space

$$k = \frac{4\pi M_P m_\phi}{\sqrt{3}H^2}\phi$$

$$E_k = \frac{4\pi}{3H^3}V_\phi$$

$$M = \frac{4\pi M_P^2}{H}$$

$$v = \frac{1}{\sqrt{3}M_P H m_\phi}V'_\phi$$

Giving thermal interpretation

- Arrived at the same conclusion of Rigopoulos (2013) & (2016)
  - Brownian motion in a medium at T_{dS}
- But our approach is simply started from sudden assumptions
  - Although we set the principles based on physical observations of dS universe…
  - We can always give a thermal interpretation for any Langevin equation if we "declare" the LHS to be a momentum
    - E.g. Thermal interpretation of stock market price? Nonsense.

And beyond (1st slow-roll condition)

- The physical significance is given by the reappearance of other seemingly unrelated quantities and phenomena in consistent ways
- $c = \frac{3H}{m_\phi}$: the "speed of light" in the abstract space
  - Would be the speed limit for massive particles
    - Already assumed in deriving the model parameters
  - What would be the value of c when reverted to the usual field variables?

And beyond (1st slow-roll condition)

- $v = \frac{1}{\sqrt{3}M_P H m_\phi} V'_\phi$. What is the potential slope when $v = c$ is reached?

$$V'_\phi\Big|_{v=c} = 3\sqrt{3}M_P H^2$$

- 1st (potential) slow-roll parameter $\epsilon_V = \left(V'_\phi / 3\sqrt{2}M_P H^2\right)^2$

- Surprisingly, $V'_\phi\Big|_{v=c}$ is where $\epsilon_V \simeq 1$ (only $\sqrt{2/3} \approx 0.82$ difference)

And beyond (1st slow-roll condition)

- Unexpected agreement but consistent with the picture
  - Abs. space: c is the speed of light (speed limit for massive particles)
    - This would be the point at which the physics in $v \ll c$ regime breaks down
  - Our space: $\epsilon_V \simeq 1$ is the potential slope that (quasi-) dS expansion is terminated
  - The two are connected by $v = \frac{1}{\sqrt{3}M_P H m_\phi} V'_\phi$ from the general formalism

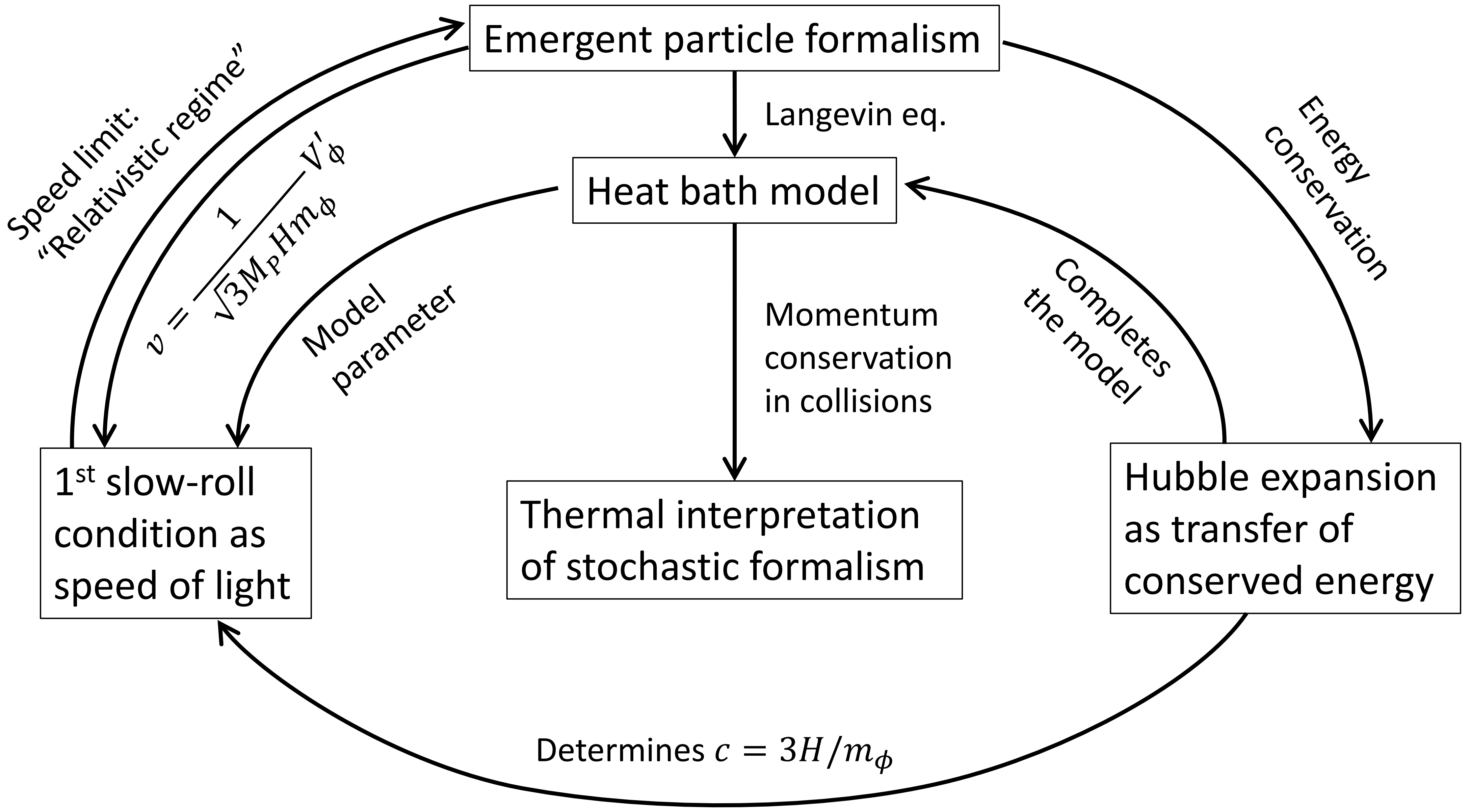
∴ 1st slow-roll condition is reinterpreted as the speed of light (speed limit) in the abstract space

And beyond (Hubble expansion)

- $c = \frac{3H}{m_\phi}$ relies on the energy conservation with Hubble expansion
  - Result of the energy conservation in the abstract space

∴ Hubble expansion is reinterpreted as transfer of conserved energy in the abstract space

- Thermal interpretation is extended also to the Hubble expansion
- Particle creation should be realized in the quantum version



Emergent particle formalism
Langevin eq.
Heat bath model
Momentum conservation in collisions
Thermal interpretation of stochastic formalism
Speed limit: “Relativistic regime”
$v = \frac{1}{\sqrt{3}M_P H m_\phi} V'_\phi$
Model parameter
1st slow-roll condition as speed of light
Energy conservation
Completes the model
Hubble expansion as transfer of conserved energy
Determines $c = 3H/m_\phi$

Summary

- Stochastic formalism for slow-rolling fields in inflation has similarities with thermodynamics but not a thermal effect
- We arrived at the thermal interpretation through the emergent particle formalism and the Heat bath model
- Consistent reinterpretation of the 1st slow-roll condition and the Hubble expansion are also achieved

Thank you for the attention!

THK; 2310.15216 [gr-qc]

Backup slides

Appearance of T_{dS} in the stochastic formalism

- $\langle \Delta\phi^2 \rangle \sim T_{dS}$ per Hubble time
- $d\rho/d\ln k \sim T_{dS}^4$ at horizon crossing
- $\langle V(\phi) \rangle \sim T_{dS}^4$ after reaching the equilibrium
- Background spacetime having only one energy scale H

Deducing the formalism

- After depicting the time slice as composed of causal patches
- Similarities with quantum particles
  - All equivalent
  - Can be created (Hubble expansion)
- Proposal for properties of the abstract space
  - Paralleling our situation to basic QFT

Deducing the formalism

- Case 1: minimal dS universe
  - A positive cosmological constant, no other fields
  - Space is homogeneous (nothing to fluctuate), all the causal patches are identical
  - The corresponding emergent particles are identical too
- Case 2: with n independent scalar fields
  - n fields vary their values on positions
  - The causal patches equivalent but no longer identical
  - Each emergent particle has n continuous degrees of freedom

Deducing the formalism

- The abstract space
  - Number of scalar fields → dimension
  - Field value of a patch → momentum of the emergent particle
    - "Dual description" of the usual field space approach

Deducing the Abstract space variables

- Starting point is the dual description: $k \propto \phi$
- If $k \propto \phi$, then $E_k \propto V_\phi$.
  - Energy density depending on ϕ → energy depending on k = kinetic energy
  - A conversion factor with volume dimension is needed.
  - We "choose" to use the Hubble volume (we also checked how our results varies according to this choice)
  - $E_k = \frac{4\pi}{3H^3} V_\phi$

Deducing the Abstract space variables

- If $E_k \propto V_\phi$, then $M \propto V_0$
  - Energy density depending on ϕ vs energy density independent on ϕ

    → Energy depending on k vs energy independent of k
  - We “choose” to use the same volume factor
  - $M = \frac{4\pi M_P^2}{H}$

Deducing the Abstract space variables

- Work – energy theorem
  - We assumed the same classical mechanics in the abstract space
  - $E_k = W = \int F\, dx = \int v\, dk$ while $E_k = \frac{4\pi}{3H^3} V_\phi = \frac{4\pi}{3H^3} \int V'_\phi\, d\phi$
    - Following the dual description, if we have $k \propto \phi$ then $v \propto V'_\phi$
  - Coefficients are fixed by the "Newtonian" form near $\phi = 0$:

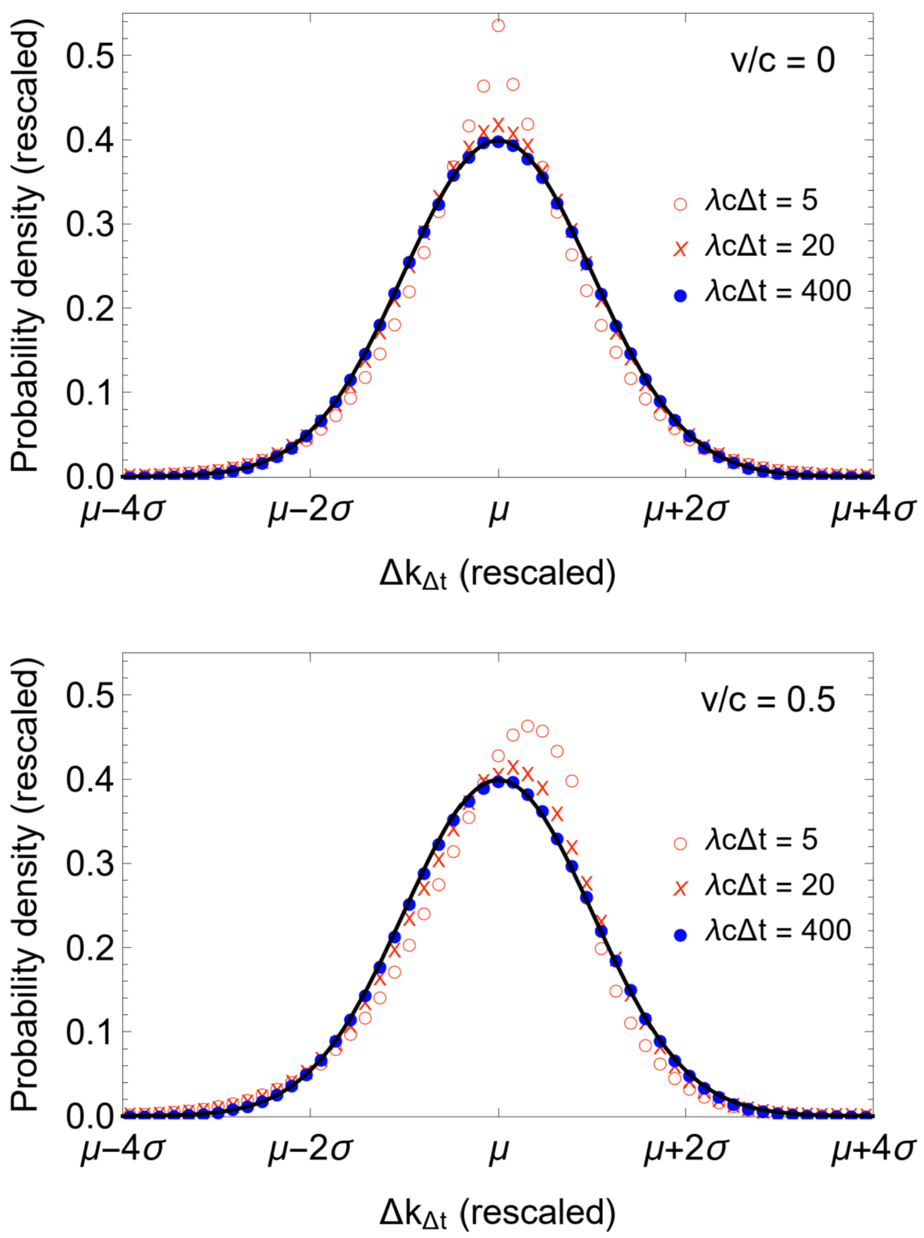
$$E_k \simeq \frac{1}{2M}\left(\frac{4\pi M_P m_\phi}{\sqrt{3}H^2}\phi\right)^2 = \frac{1}{2}M\left(\frac{m_\phi}{\sqrt{3}M_P H}\right)^2$$

  - $k = \frac{4\pi M_P m_\phi}{\sqrt{3}H^2}\phi$ and $v = \frac{1}{\sqrt{3}M_P H m_\phi}V'_\phi$

Achieving the Gaussianity

- Numerical simulation result
- $\lambda c\, \Delta t \simeq$ total number of collisions in Δt
- Gaussianity achieved for $\lambda c\, \Delta t \gtrsim \mathcal{O}(100)$
  - Holds even for $v \simeq c$
  - Expressing in terms of field variables gives

    $\Delta t \gtrsim \left(H^2/M_P^2\right) \times \frac{1}{H}$



Conjecture for quantum emergent particles

- Quantum field theoretical description of emergent particles
- Two interacting quantum fields in the abstract space
  - Emergent particle field absorbs energy from bath particle field
  - Particle creation should happen
- Classical (MB) statistics with quantum particles:
  - Reproduces the equilibrium distribution of the Fokker-Planck equation
- 1st law of thermodynamics: $S_{dS} = \frac{8\pi^2 M_P^2}{H^2}$ is entropy per emergent particle