Sources for pulsar timing array observations

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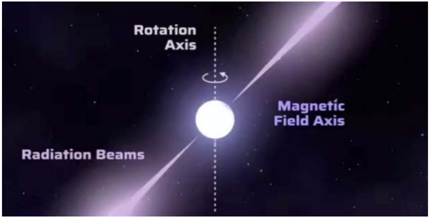
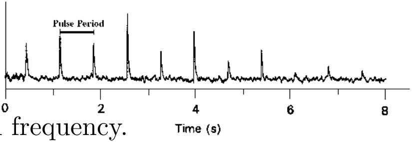
L. Bian, R.-G. Cai, J. Liu, XYY, R. Zhou [2009.13893] Phys. Rev. D 103, L081301 (2021)
S. Sun, XYY, Y.-L. Zhang [2112.15593] Phys. Rev. D 106, 066006 (2022)
L. Bian, S. Ge, J. Shu, B. Wang, XYY, J. Zong [2307.02376]
C. Fu, J. Liu, XYY, W.-W. Yu, Y. Zhang [2308.15329]

Pulsar

A highly magnetized rotating neutron star that emits beams of electromagnetic radiation out of its magnetic poles.

$\sim$ Lighthouse



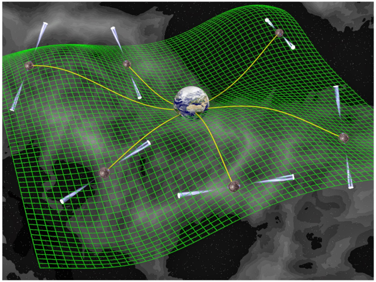


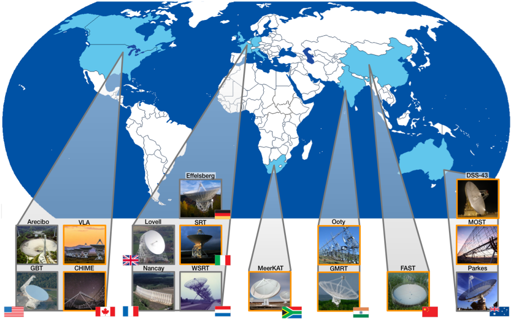
- Millisecond pulsar: extremely stable ratation frequency.

$$\nu(t) = \nu_0 + \dot{\nu}_0 t, \qquad \dot{\nu}_0/\nu_0 \sim 10^{-23} - 10^{-20}\,\mathrm{Hz}$$

- Pulse timing modulation
  - at source: binary rotation, spin-down, ...
  - at path: interstellar medium, gravitational waves, ...
  - at observer: solar-system motion, instrumental noise, ...
  - ...

A set of galactic pulsars that is monitored and analysed to search for correlated signatures in the pulse arrival times on Earth.





- North American Nanohertz Observatory for GWs (NANOGrav)
- Parkes PTA (PPTA)
- European PTA (EPTA)
- Indian PTA (InPTA)
- Chinese PTA (CPTA)
- MeerKAT
- Square Kilometre Array (SKA, in future)

Gravitational waves

- Timng residual

$$\delta t = t_{\rm TOA} - t_{{\rm TOA},p}$$

The pulsar arrival time $t_{{\rm TOA},p}$ can be accurately determined by sophisticated timing model.

Gravitational waves $\Rightarrow$ Pulsar timing residual

$$\sim 1 - 100\,{\rm nHz}$$

Stochastic background of GWs $\Rightarrow$ Timing-residual cross-power spectral density

Isotropic
Stationary
Gaussian
Unpolarized

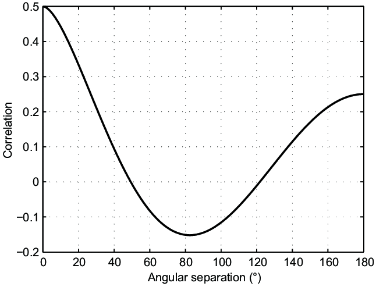
$$P_{ab}^{\rm GWB}(f) = P^{\rm GWB}(f)\Gamma(\zeta_{ab})$$

Overlap reduction function

- Hellings-Downs correlation

$$\Gamma(\zeta_{ab}) = \frac{1}{2} - \frac{1}{4}x_{ab} + \frac{3}{2}x_{ab}\ln(x_{ab}) + \frac{1}{2}\delta_{ab}$$

$$x_{ab} = \frac{1-\cos\zeta_{ab}}{2}$$



SMBHBs

Supermassive black hole binaries $\Rightarrow$ Stochastic background of GWs

- Characteristic GW strain

$$h_c(f) = A\left(\frac{f}{f_{\rm yr}}\right)^{\alpha}$$

with $\alpha = -2/3$ for a population of inspiraling SMBHBs in circular orbits whose evolution is dominated by GW emission.

- Timing-residual cross-power spectral density

$$P^{\rm GWB}(f) = \frac{A^2}{12\pi^2}\left(\frac{f}{f_{\rm yr}}\right)^{-\gamma} f_{\rm yr}^{-3}$$

$\gamma = 3 - 2\alpha$, for SMBHBs $\gamma = 13/3$.

Bayesian analysis

- Bayesian model fitting: finding the posterior of parameters when data and model are known.

$$\underbrace{P(\theta|D,M)}_{posterior} = \frac{\overbrace{P(D|\theta,M)}^{likelihood}\overbrace{P(\theta|M)}^{prior}}{\underbrace{P(D|M)}_{evidence}}$$

- Bayesian model comparison: finding the model evidence when data are known.

$$P(M|D) = P(D|M)\frac{P(M)}{P(D)}$$

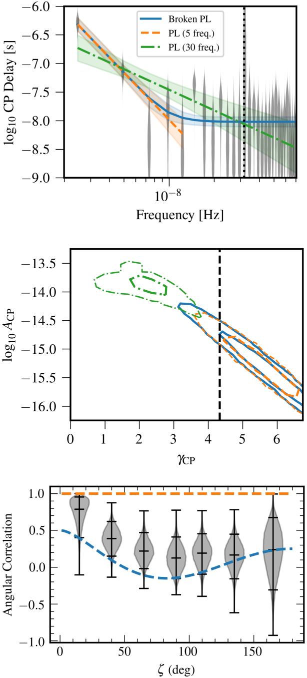
Odds ratio between two alternative models

$$O_{ij} \equiv \frac{P(M_i|D)}{P(M_j|D)} = \frac{P(D|M_i)}{P(D|M_j)}\frac{P(M_i)}{P(M_j)} = B_{ij}\frac{P(M_i)}{P(M_j)}$$

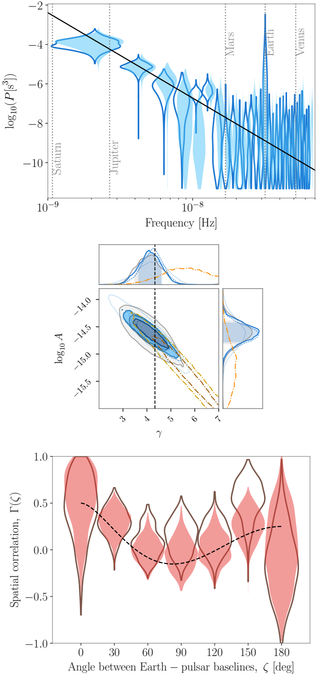
- $B_{ij} \equiv P(D|M_i)/P(D|M_j)$: Bayes factor for M_i versus M_j.
- $P(M_i)/P(M_j)$: ratio of prior odds, usually assumed to be near unity.

Common-spectrum process

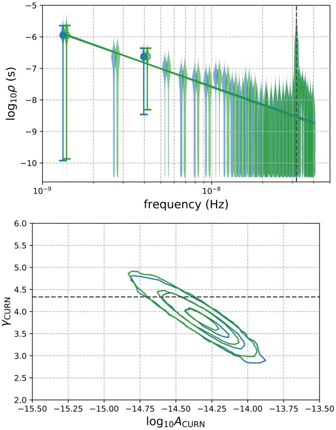
NANOGrav [2009.04496]



PPTA [2107.12112]



EPTA [2110.13184]



$$P_{ab}^{\mathrm{GWB}}(f) = P^{\mathrm{GWB}}(f)\Gamma(\zeta_{ab})$$

Common-spectrum process

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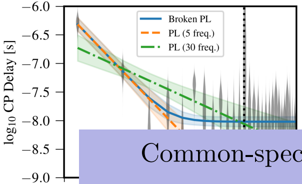
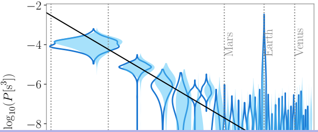
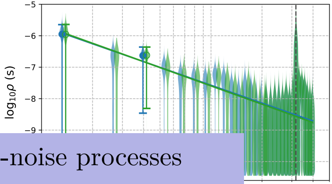
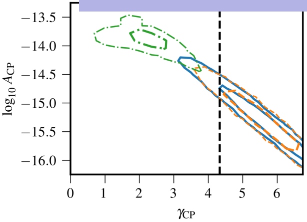


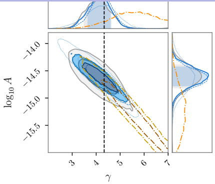


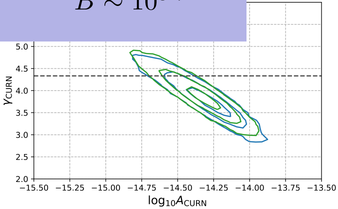


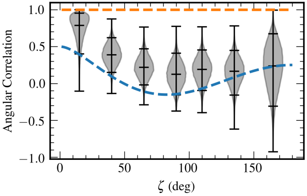
Common-spectrum process vs Independent red-noise processes

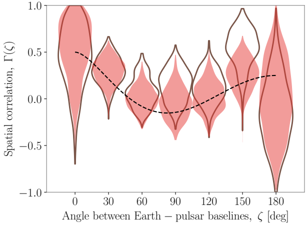
$B > 10000$ $B > 3 \times 10^6$ $B \sim 10^{3.7}$











$$P_{ab}^{\rm GWB}(f) = P^{\rm GWB}(f)\Gamma(\zeta_{ab})$$

Common-spectrum process

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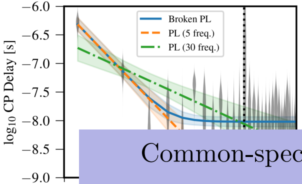

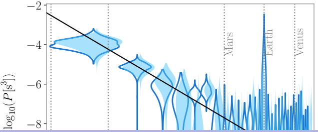

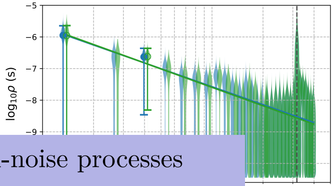

Common-spectrum process vs Independent red-noise processes

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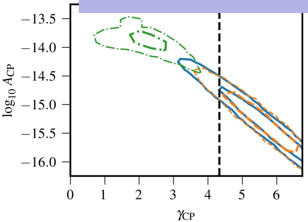
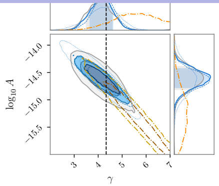
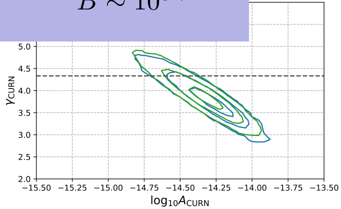
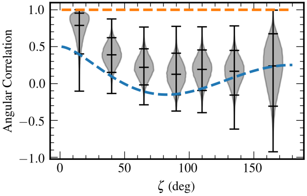
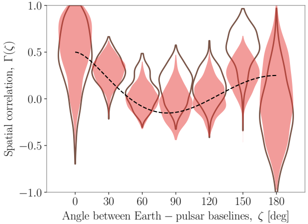








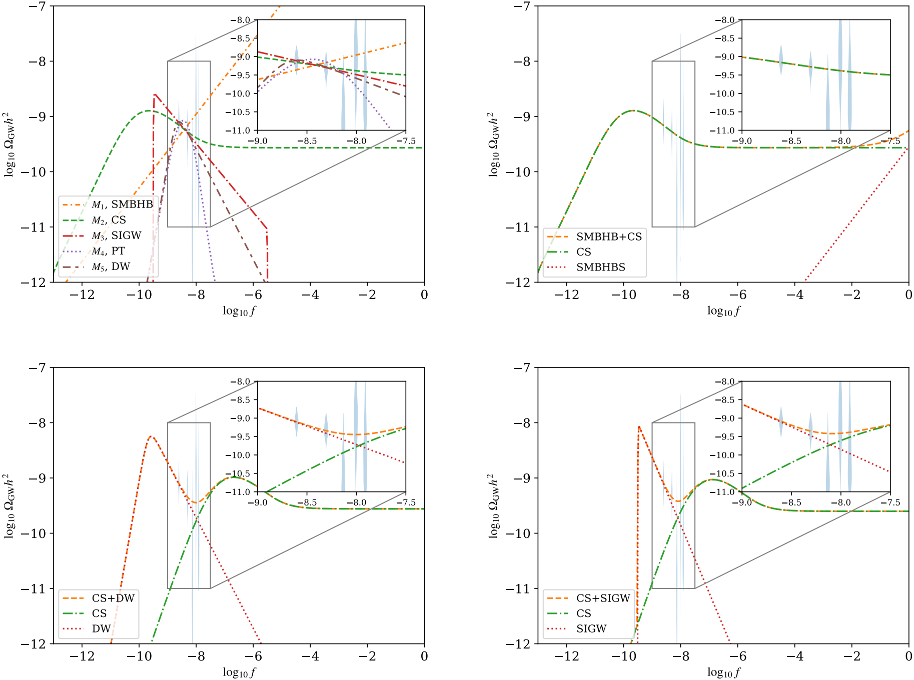


$$P_{ab}^{\rm GWB}(f) = P^{\rm GWB}(f)\Gamma(\zeta_{ab})$$

GWs or not ?

2009.13893

Evidence for different GW sources



Evidence for different GW sources

SMBHB, CS, SIGW, PT, DW, SMBHB + CS, CS+SIGW, CS+DW

$$B_{ij} = \begin{pmatrix} 1 & 0.09 & 0.37 & 0.28 & 0.83 & 0.16 & 0.12 & 0.17 \\ 10.8 & 1 & 3.96 & 3.01 & 8.93 & 1.75 & 1.32 & 1.84 \\ 2.73 & 0.25 & 1 & 0.76 & 2.26 & 0.44 & 0.33 & 0.47 \\ 3.6 & 0.33 & 1.32 & 1 & 2.97 & 0.58 & 0.44 & 0.61 \\ 1.21 & 0.11 & 0.44 & 0.34 & 1 & 0.2 & 0.15 & 0.21 \\ 6.18 & 0.57 & 2.26 & 1.72 & 5.11 & 1 & 0.76 & 1.05 \\ 8.17 & 0.76 & 2.99 & 2.27 & 6.75 & 1.32 & 1 & 1.39 \\ 5.86 & 0.54 & 2.15 & 1.63 & 4.85 & 0.95 & 0.72 & 1 \end{pmatrix}$$

B_{ij}	Evidence in favor of M_i against M_j
$1-3$	Weak
$3-20$	Positive
$20-150$	Strong
≥ 150	Very strong

⋆ Positive evidence in favor of the cosmic strings against the others

Ultralight dark matter

Oscillation of ultralight DM	$\sim \phi(\boldsymbol{x})\cos[mt+\theta(\boldsymbol{x})]$
$\Downarrow$	$\Downarrow$
Oscillation of gravitational potential	$\sim \bar{\Psi}(\boldsymbol{x}) + \Psi_\phi \cos[2mt + 2\theta(\boldsymbol{x})]$
$\Downarrow$	$\Downarrow$
Pulsar timing residual	$\sim R_\phi \cos(2mt + \theta_\phi + \theta_0 - mD)$

Monochromatic ultralight DM $\Rightarrow$ Monochromatic signal

$$f \sim 4.8\,\mathrm{nHz}\left(\frac{m}{10^{-23}\,\mathrm{eV}}\right)$$

Multifield $\Rightarrow$ Wideband mass spectrum $\Rightarrow$ Wideband frequency spectrum

- Marcenko-Pastur distribution

$$P(m) = \frac{m}{\pi\beta\delta^2}\sqrt{\left(1-\frac{m_-^2}{m^2}\right)\left(\frac{m_+^2}{m^2}-1\right)}$$

- Rayleigh distribution

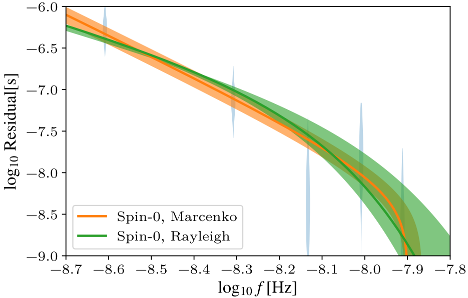
$$P(m) = \frac{m}{\sigma^2}e^{-\frac{m^2}{2\sigma^2}}$$

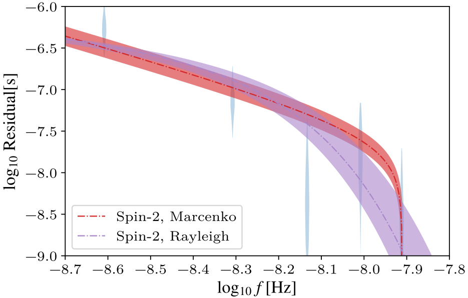
Ultralight dark matter

Spin-0: $h_c(f) = \frac{\alpha_0}{M_{\rm P}^2}\frac{\sqrt{3}\rho_{\rm DM}}{4\pi f}P(\pi f)$

Spin-1: $h_c(f) = \frac{\alpha_1}{M_{\rm P}^2}\frac{\sqrt{3}\rho_{\rm DM}}{12\pi f}P(\pi f)$

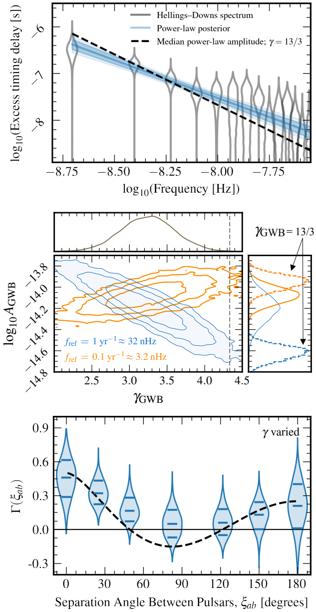
Spin-2: $h_c(f) = \frac{\alpha_2}{M_{\rm P}}\frac{2\sqrt{\rho_{\rm DM}}}{\sqrt{5}}P(2\pi f)$



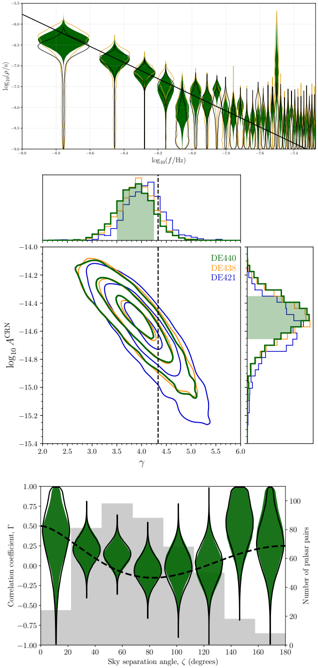


Gravitational-wave background ?

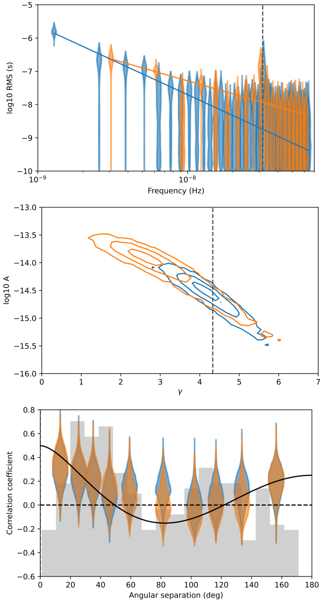
NANOGrav [2306.16213]



PPTA [2306.16215]



EPTA [2306.16214]



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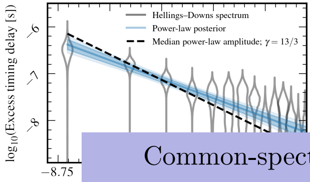

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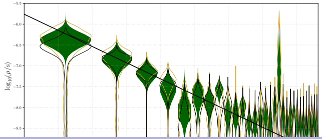

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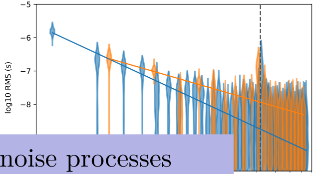
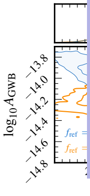
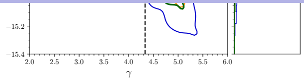
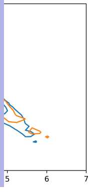






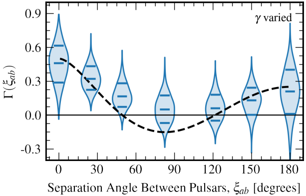


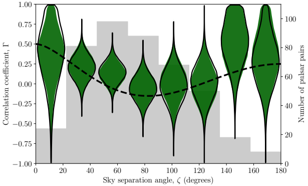
Common-spectrum process vs Independent red-noise processes

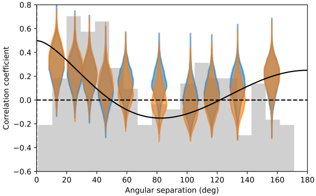
$B > 10^{14}$ $B > 10^{4.8}$

Gravitational-wave background vs Uncorrelated common-spectrum

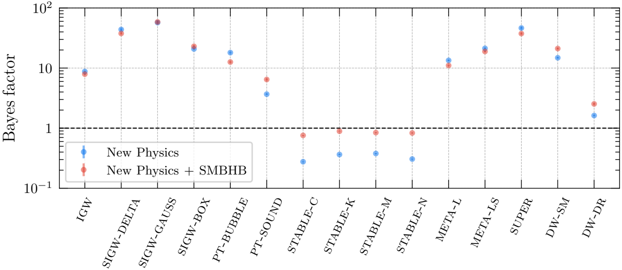
$B \sim 200 - 1000$ $B \sim 1.5$ $B \sim 4$

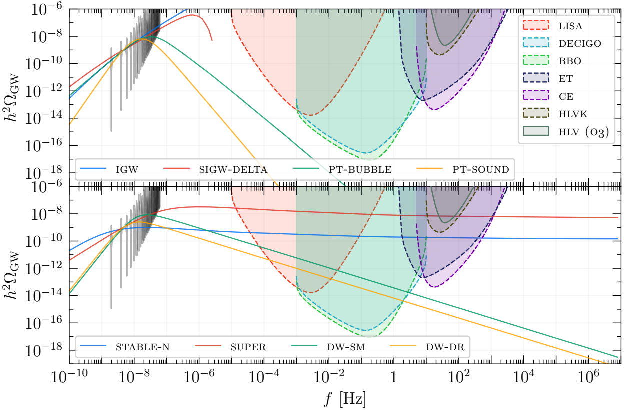






NANOGrav [2306.16219]





Inflationary GW
Scalar-induced GW
Phase transiton
Cosmic string
Domain wall
vs
SMBHBs

Many cosmological models seem to provide a better fit resulting in Bayes factors in the range from 10 to 100.

However, these results strongly depend on modeling assumptions about the cosmic SMBHB population and, at this stage, should not be regarded as evidence for new physics.

Bayes factor

SMBHB, PT, CS, DW, SIGW

$$B_{ij}^{\rm NG15} = \begin{pmatrix} 1 & 0.49 & 0.55 & 5.19 & 1.34 \\ 2.03 & 1 & 1.12 & 10.55 & 2.72 \\ 1.82 & 0.90 & 1 & 9.46 & 2.44 \\ 0.19 & 0.09 & 0.11 & 1 & 0.26 \\ 0.75 & 0.37 & 0.41 & 3.88 & 1 \end{pmatrix}$$

$$B_{ij}^{\rm PPTA} = \begin{pmatrix} 1 & 0.27 & 0.32 & 2.53 & 0.58 \\ 3.64 & 1 & 1.16 & 9.2 & 2.10 \\ 3.13 & 0.86 & 1 & 7.92 & 1.81 \\ 0.40 & 0.11 & 0.13 & 1 & 0.23 \\ 1.73 & 0.48 & 0.55 & 4.37 & 1 \end{pmatrix}$$

$$B_{ij}^{\rm EPTA} = \begin{pmatrix} 1 & 0.67 & 0.47 & 6.87 & 1.65 \\ 1.50 & 1 & 0.70 & 10.30 & 2.47 \\ 2.15 & 1.43 & 1 & 14.75 & 3.53 \\ 0.15 & 0.10 & 0.07 & 1 & 0.24 \\ 0.61 & 0.41 & 0.28 & 4.18 & 1 \end{pmatrix}$$

Current datasets do not display a strong preference for any specific SGWB source based on Bayesian analysis.

GWs in parity-violating gravity

- Nieh-Yan modified Teleparallel Gravity

$$S = \int d^4x\sqrt{-g}\left[-\frac{R}{2} + \frac{\alpha\phi}{4}\mathcal{T}_{\sigma\mu\nu}\tilde{\mathcal{T}}^{\sigma\mu\nu} + \frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi - V(\phi)\right] + S_{\text{other}}$$

$\mathcal{T}^{\sigma}_{\mu\nu}$: torsion two form, $\tilde{\mathcal{T}}^{\sigma\mu\nu}$: dual of torsion two form, ϕ : dynamical scalar field.

- Background evolution is identical to that in GR, with the spatially flat FLRW metric.

- Gauge invariant scalar perturbations corresponding to ϕ vanishes completely due to the coupling of the ϕ field with the Nieh-Yan term.

- Velocity birefringence for tensor perturbations

$$\ddot{h}^A_k + 3H\dot{h}^A_k + \frac{k}{a}\left(\frac{k}{a} + \lambda_A\alpha\dot{\phi}\right)h^A_k = 0$$

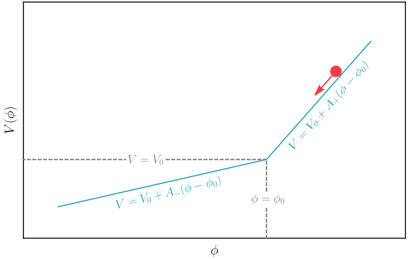
$A = R(L)$ denotes the right(left)-handed polarization with $\lambda_R = 1$ and $\lambda_L = -1$.
⋆ Tachyonic instability for one of two polarizations: $(k/a)(k/a + \lambda_A\alpha\dot{\phi}) < 0$

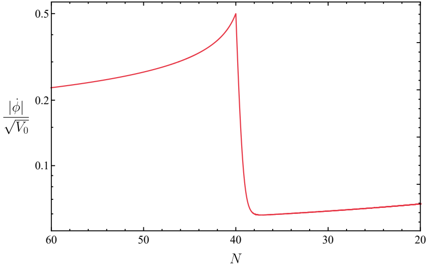
GWs in parity-violating gravity

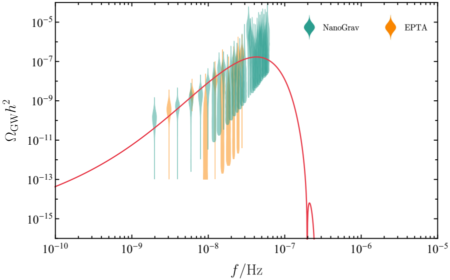
Scenario 1) ϕ : inflaton, $S_{\rm other}$: curvaton

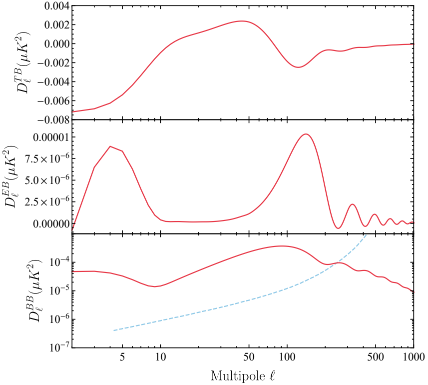
Starobinsky’ s linear potential

$$V(\phi) = \begin{cases} V_0 + A_+(\phi - \phi_0), & \text{for } \phi > \phi_0, \\ V_0 + A_-(\phi - \phi_0), & \text{for } \phi \leq \phi_0, \end{cases}$$









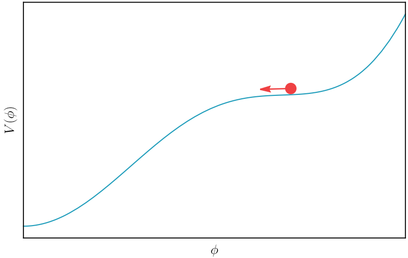
Circular polarization on large scale

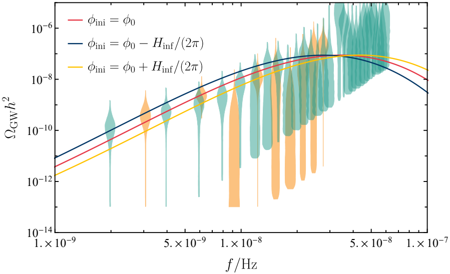
GWs in parity-violating gravity

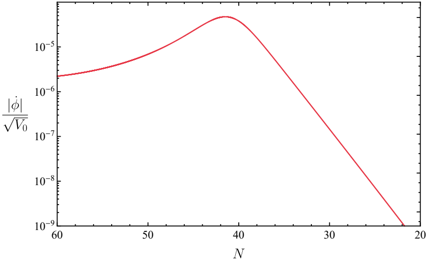
Scenario 2) ϕ : spectator , $S_{\rm other}$:inflaton

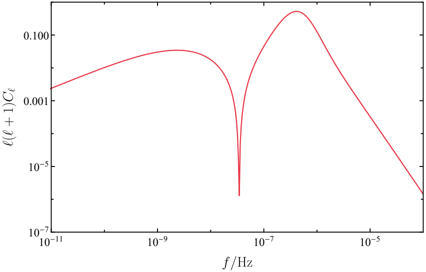
Axion potential

$$V(\phi) = \frac{1}{2}m^2\phi^2 + \Lambda^4 \frac{\phi}{\sigma}\sin\left(\frac{\phi}{\sigma}\right)$$









Anisotropies

Summary

- Signals in PTA observations = GWs ?
  - More evidence but not conclusive
  - = GWs
    * Supermassive black hole binary
    * Phase transiton
    * Cosmic string
    * Domain wall
    * Scalar-induced GWs
    * Parity-violating gravity
    * ...
  - $\neq$ GWs
    * Ultralight dark matter
    * ...

⋆ More data and more accurate theoretical predictions are crucial to conclusively settle down the question.