Dark Matter Capture, Multiple Scattering and Thermalsation in Compact Stars

Based on 1807.02840, 1904.09803, 2004.14888, 2010.1325, 2012.08918, 2104.1436, 2108.0252, 2312:xxxxx, 2402:xxxxx

Giorgio Busoni

The Australian National University

Australian National University

NATURAM PRIMUM COGNOSCERE RERUM

Australian Government
Australian Research Council

ARC CENTRE OF EXCELLENCE FOR DARK MATTER PARTICLE PHYSICS

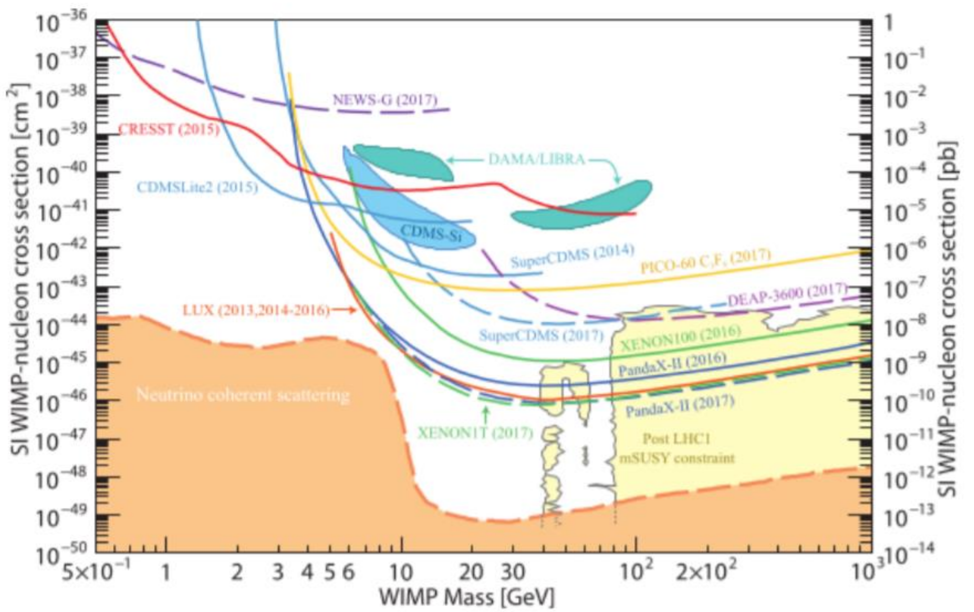
Outline

1. Motivations
2. DM Capture: from the Sun to NS and WD
3. Multiple Scattering
4. Thermalisation
5. Conclusions

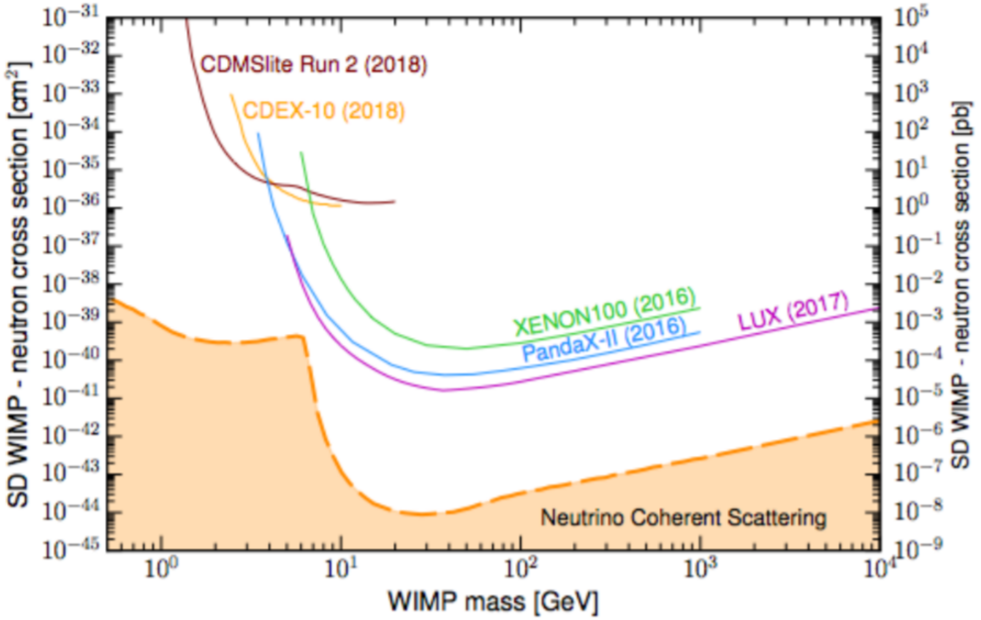
1.Motivations

Direct Detection

- Constraints depend strongly on interaction type
- Strong target dependence
- Some operators are suppressed by kinematics (momentum/velocity suppressed)
- Recoil energy is small, nonrelativistic kinematics
- Experimental detectors have recoil energy thresholds
- Probes the high-energy part of the DM speed distribution

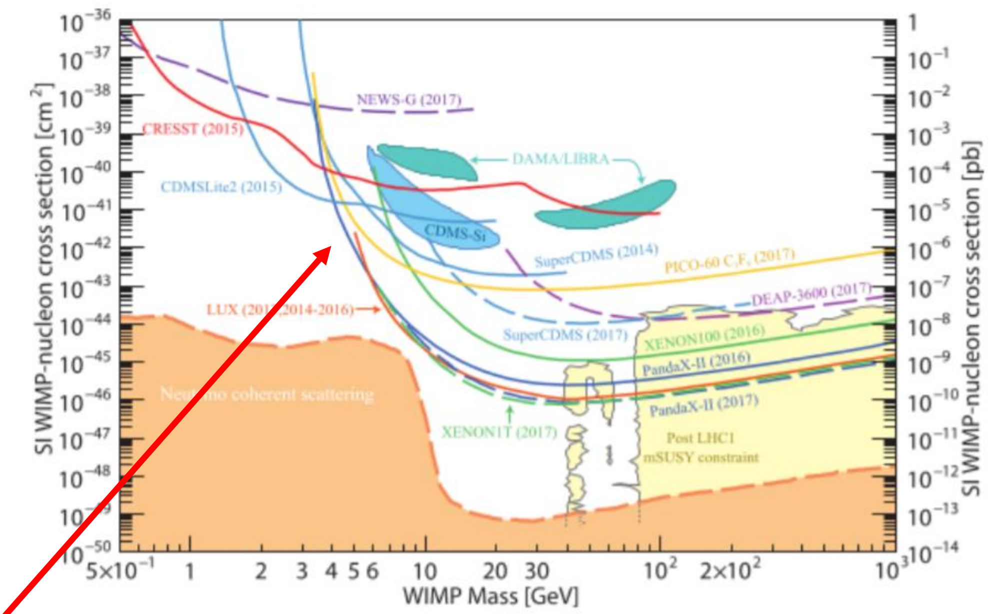


M. Tanabashi et al. (PDG) 2018

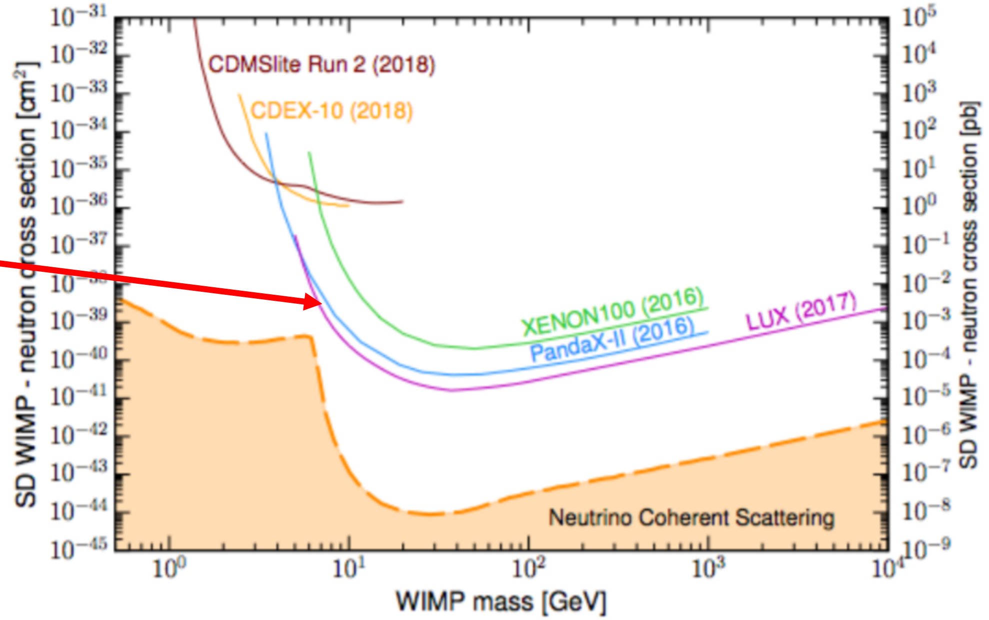


Direct Detection

- Constraints depend strongly on interaction type
- Strong target dependence
- Some operators are suppressed by kinematics (momentum/velocity suppressed)
- Recoil energy is small, nonrelativistic kinematics
- Experimental detectors have recoil energy thresholds
- Probes the high-energy part of the DM speed distribution

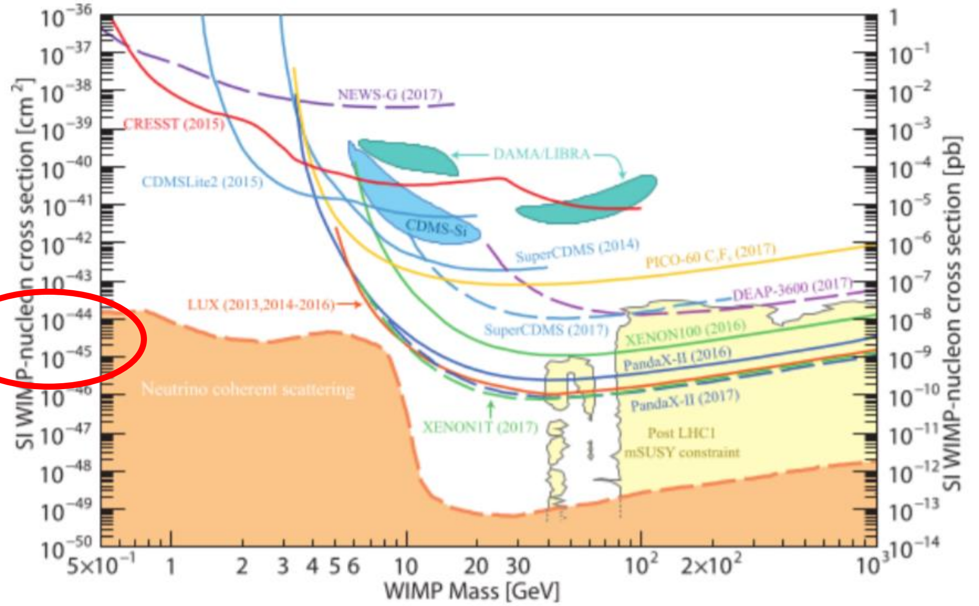


M. Tanabashi et al. (PDG) 2018

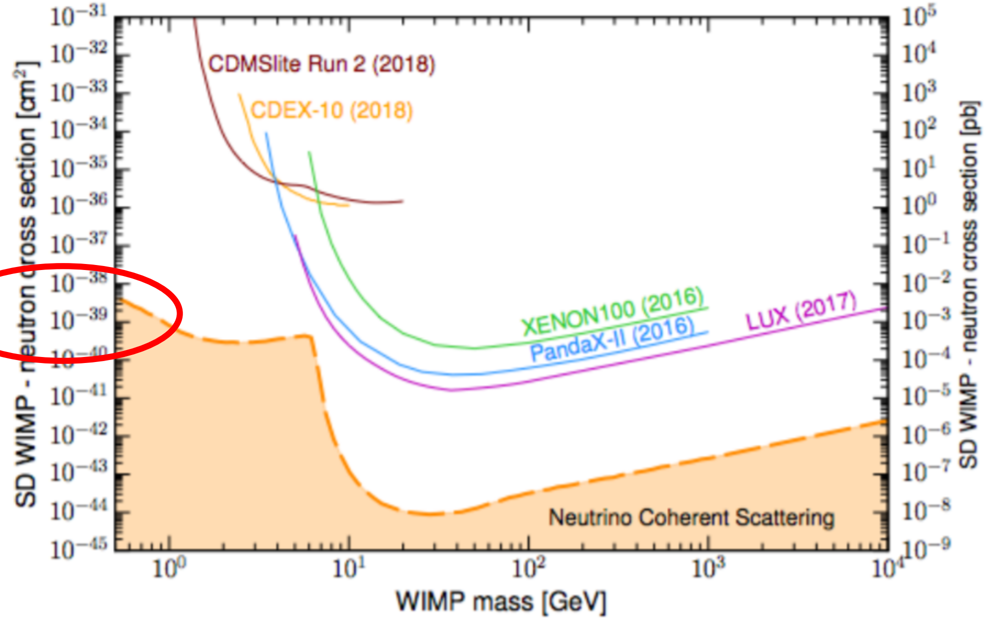


Direct Detection

- Constraints depend strongly on interaction type
- Strong target dependence
- Some operators are suppressed by kinematics (momentum/velocity suppressed)
- Recoil energy is small, nonrelativistic kinematics
- Experimental detectors have recoil energy thresholds
- Probes the high-energy part of the DM speed distribution

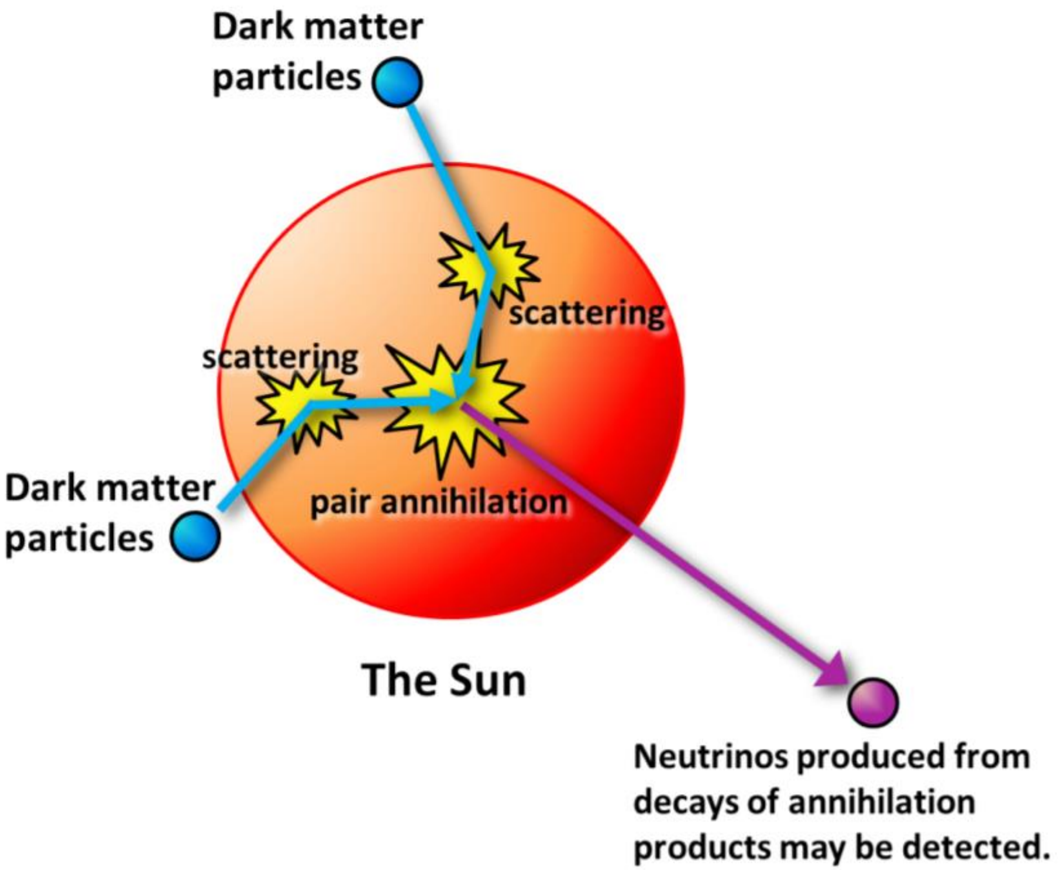


M. Tanabashi et al. (PDG) 2018



DM Capture in the Sun

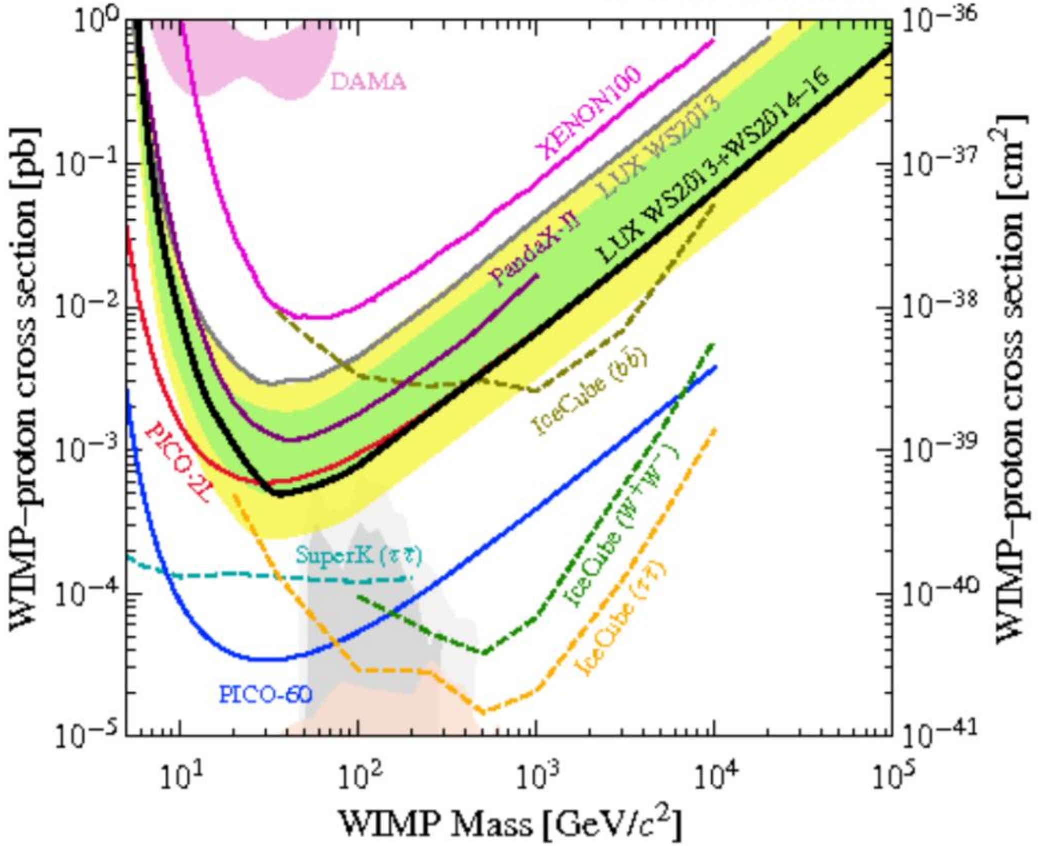
- DM can be captured and accumulate in Stars:
- Dark matter infalls, scatters, loses energy, becomes gravitationally bound to star
- Continues to scatter and loose energy, falling more and more towards the core, and accumulates in centre of Sun
- Can potentially annihilate at the center
- At equilibrium Capture=Annihilation
- Probes same observables as DD



DM Capture in the Sun

- Huge DM Detector/huge exposure
- Energy threshold for capture
- Probes different part of the DM speed distribution
- Limited sensitivity at low DM mass
- Evaporation mass ($m \approx 3GeV$) sets hard limit on minimum testable DM
- Comparing sensitivities with DD:
- SI: DD wins
- SD: DM in Sun wins
- DM in Sun requires some few more assumptions, like that it annihilates, and the annihilation channel

arXiv:1705.03380



Some other ways to infer indirectly DM presence in the Sun: modified energy transport (see 1411.6626 , 1703.07784)

DD vs DM Capture in the Sun

DD

- Constraints depend strongly on interaction type
- Strong target dependence
- Some operators are suppressed by kinematics (momentum/velocity suppressed)
- Recoil energy is small, nonrelativistic kinematics
- Experimental detectors have recoil energy thresholds
- Probes the high-energy part of the DM speed distribution

DM Capture in the Sun

- Constraints depend strongly on interaction type
- Strong target dependence
- Some operators are suppressed by kinematics (momentum/velocity suppressed)
- Recoil energy is small, nonrelativistic kinematics
- Recoil energy threshold for capture
- Probes the low-energy part of the DM speed distribution

DM Capture in NS

- Very large density means very efficient capture
- Whole DM flux can be captured already for $\sigma \ \sim \ 10^{-45} cm^2$

Possible signatures

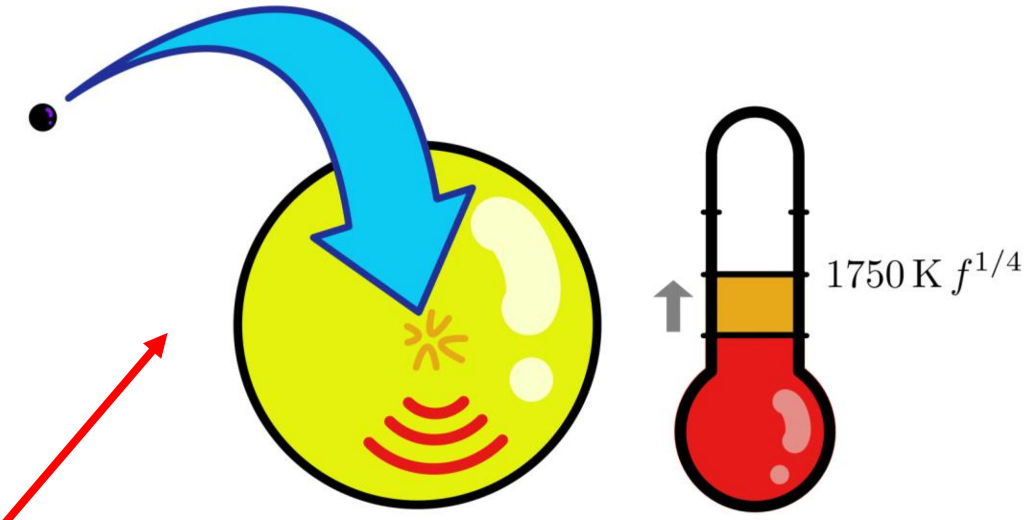
- NS to BH collapse (more likely for bosonic DM)

Possible signatures

- NS to BH collapse (more likely for bosonic DM)
- Gravitational waves: DM increases tidal deformability (1803.03266)

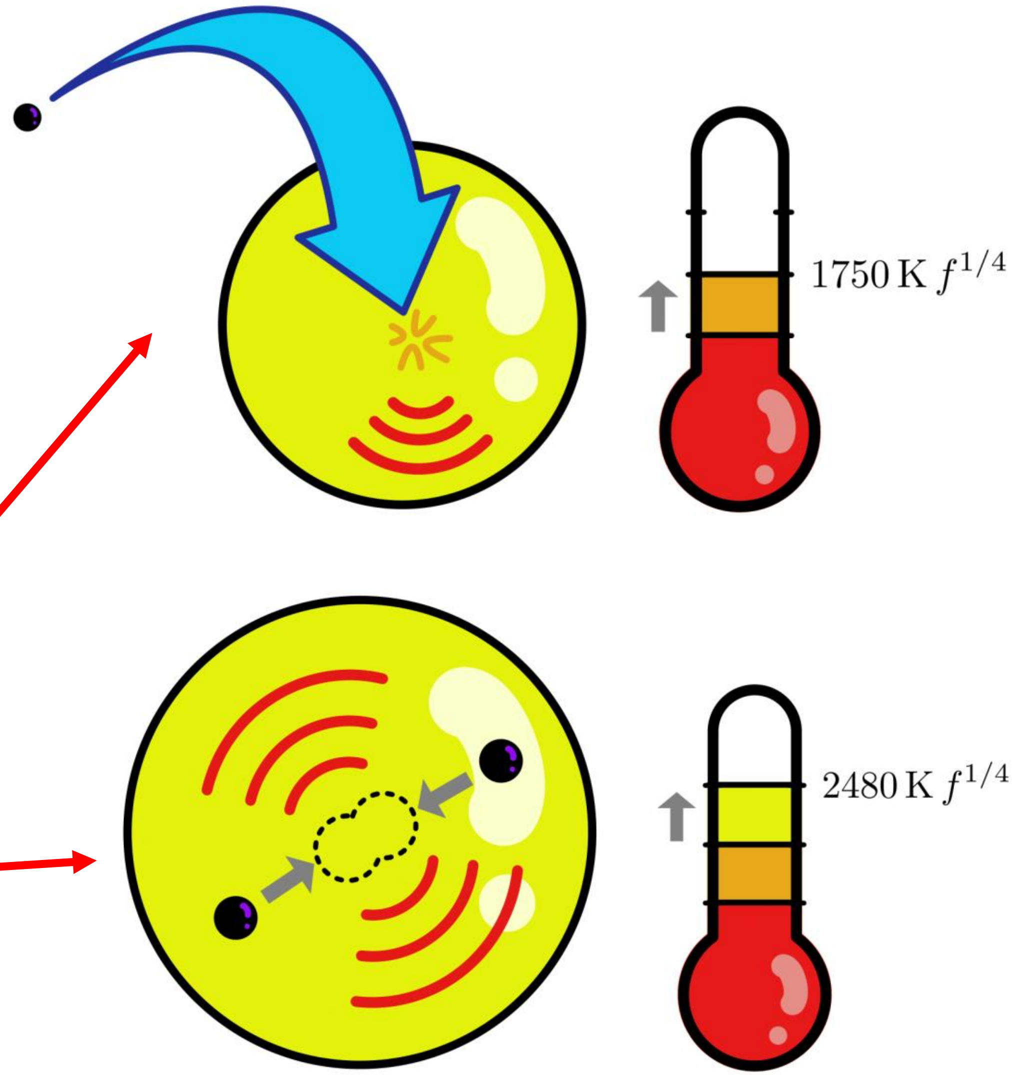
Possible signatures

- NS to BH collapse (more likely for bosonic DM)
- Gravitational waves: DM increases tidal deformability (1803.03266)
- Kinetic Heating (M. Baryakhtar et al. PRL 119, 131801 (2017) arXiv:1704.01577)



Possible signatures

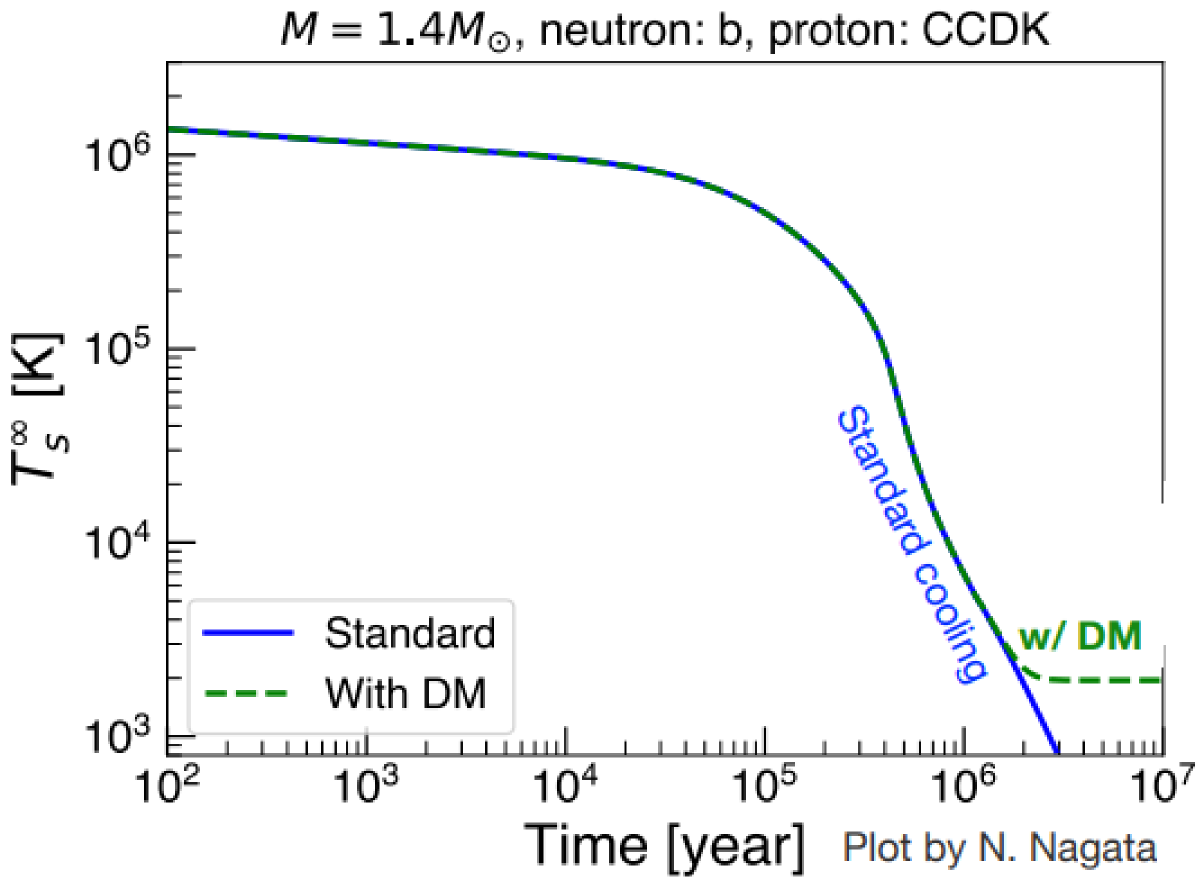
- NS to BH collapse (more likely for bosonic DM)
- Gravitational waves: DM increases tidal deformability (1803.03266)
- Kinetic Heating (M. Baryakhtar et al. PRL 119, 131801 (2017) arXiv:1704.01577)
- Kinetic + Annihilation heating (Bramante, Delgardo and Martin 1704.01577)



1707.09442

NS temperature evolution

- NS have no know large heating sources
- Lose energy by neutrino and photon emission
- Neutrino dominates early stages of NS life, photon the late stages
- In absence of other heating sources, one expects $T \sim 1000K$ after $10M\ yr$ and $T \sim 100K$ after $1Gyr$
- Kinetic heating: sets a maximum equilibrium temperature of $T_{eq,max} \approx 1700K$ if whole DM flux is captured
- Kin+Ann heating: maximum equilibrium temperature is raised to $T_{eq,max} \approx 2400K$



Equilibrium Temperature

- Maximum capture rate:

$$C_{geom} = \frac{\pi R^2(1 - B(R))}{v_s B(R)} \frac{\rho_\chi}{m_\chi} Erf\left[\sqrt{\frac{3}{2}}\frac{v_s}{v_d}\right]$$

- Equilibrium condition

$$C_{geom}\langle\delta E\rangle = \sigma_{SB} T_{eq,max}^4 \pi R^2$$

$$\langle\delta E\rangle_{kin} = \left(\frac{1}{\sqrt{B(R)}} - 1\right) m_\chi, \langle\delta E\rangle_{ann} = \frac{m_\chi}{\sqrt{B(R)}}$$

- Dependence on R, m_χ simplifies out!

Equilibrium Temperature

Metric element $ds^2 = B\,dt^2 - \cdots$

- Maximum capture rate:

$$C_{geom} = \frac{\pi R^2 (1 - B(R))}{v_s B(R)} \frac{\rho_\chi}{m_\chi} Erf\left[\sqrt{\frac{3}{2}} \frac{v_s}{v_d}\right]$$

- Equilibrium condition

$$C_{geom}\langle \delta E \rangle = \sigma_{SB} T_{eq,max}^4 \pi R^2$$

$$\langle \delta E \rangle_{kin} = \left(\frac{1}{\sqrt{B(R)}} - 1\right) m_\chi, \langle \delta E \rangle_{ann} = \frac{m_\chi}{\sqrt{B(R)}}$$

- Dependence on R, m_χ simplifies out!

Equilibrium Temperature

- What if capture rate is less than maximal?
- At small cross section, C is linear in σ
- Define

$$C = C_{geom} \frac{\sigma}{\sigma_{th}}$$

One gets

$$T \geq T_{eq} = T_{eq,max} \left(\frac{\sigma}{\sigma_{th}} \right)^{1/4}$$

- Upper bound on T_{eq} becomes upper bound on cross section!

Advantages of NS vs DD/DM Capture in the Sun

- Constraints depend strongly on interaction type
- Strong target dependence
- Some operators are suppressed by kinematics (momentum/velocity suppressed)
- Recoil energy is small, nonrelativistic kinematics
- Experimental detectors have recoil energy thresholds
- Probes the high/low-energy part of the DM speed distribution

- Constraints are similar for all interaction types
- Neutrons and protons gives similar responses
- DM accelerated to $O(0.5c)$: any non-relativistic suppression is washed out
- Large recoil energy, relativistic kinematics
- No Recoil energy threshold
- Probes the whole DM distribution

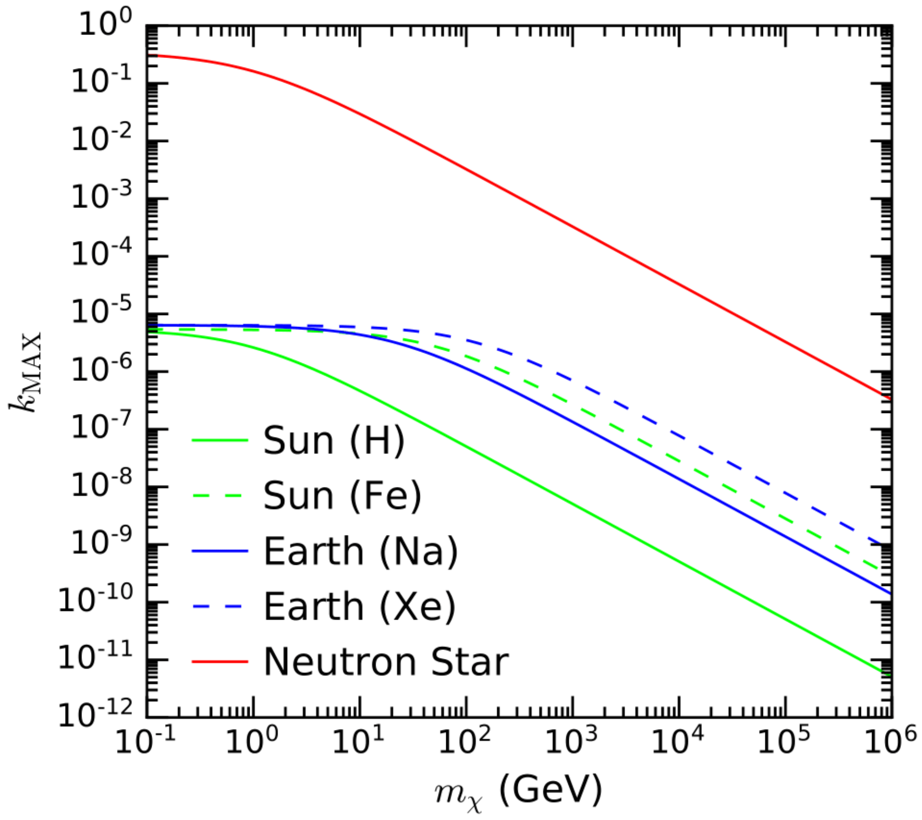
Example Scenario 1: Inelastic DM (Pseudo-Dirac)

- Consider 2 dark states $\chi_{1,2}$, separated by small mass splitting δm:

$$m_2 = m_1 + \delta m$$

- Only interaction allowed at tree level $\chi_1 N \longrightarrow \chi_2 N$
- NS allows scatterings with mass splitting 3 to 5 more orders of magnitudes in larger

$$\delta m = k m_1$$

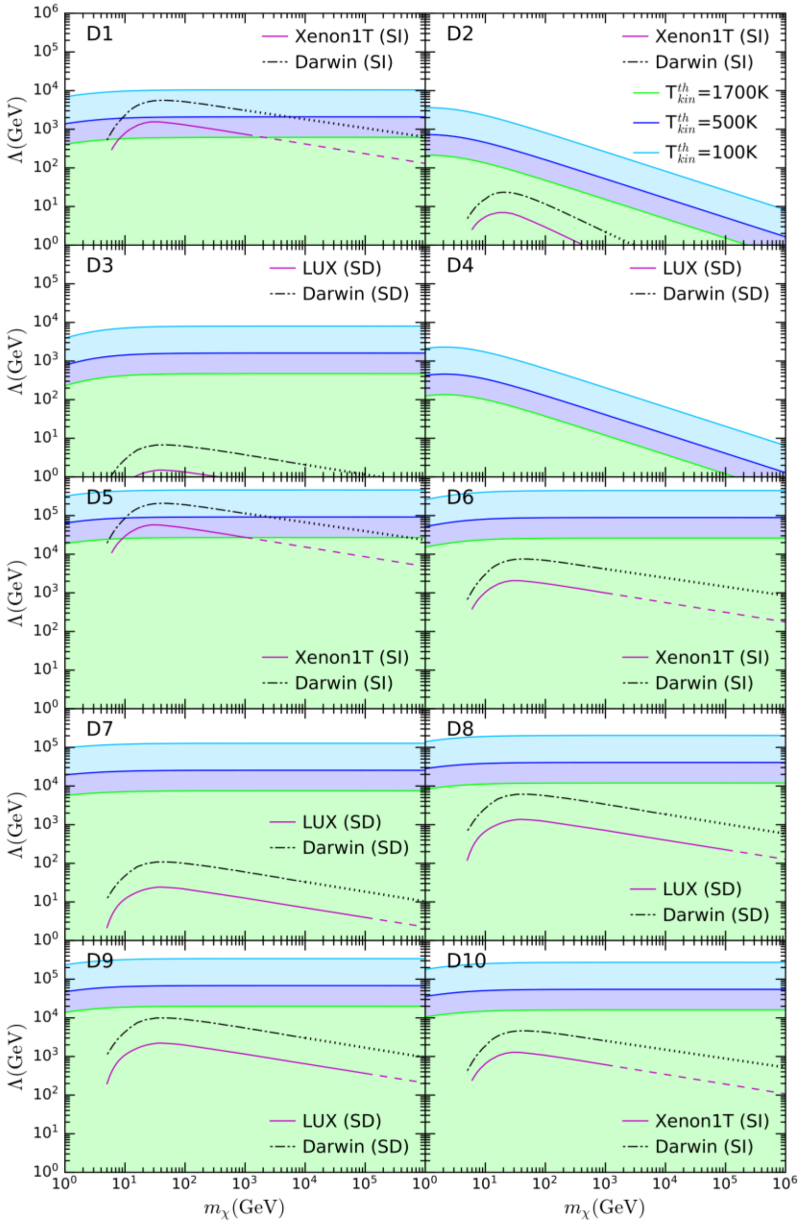


Example Scenario 2: Momentum suppressed operators

- Consider full set of dim 6 EFT operators

$$D_i = \frac{c_i}{\Lambda^2} O_i$$

- Direct Detection (dashed): limits on operator scales very different depending on the operator
- NS (solid): all operators get very similar constraints

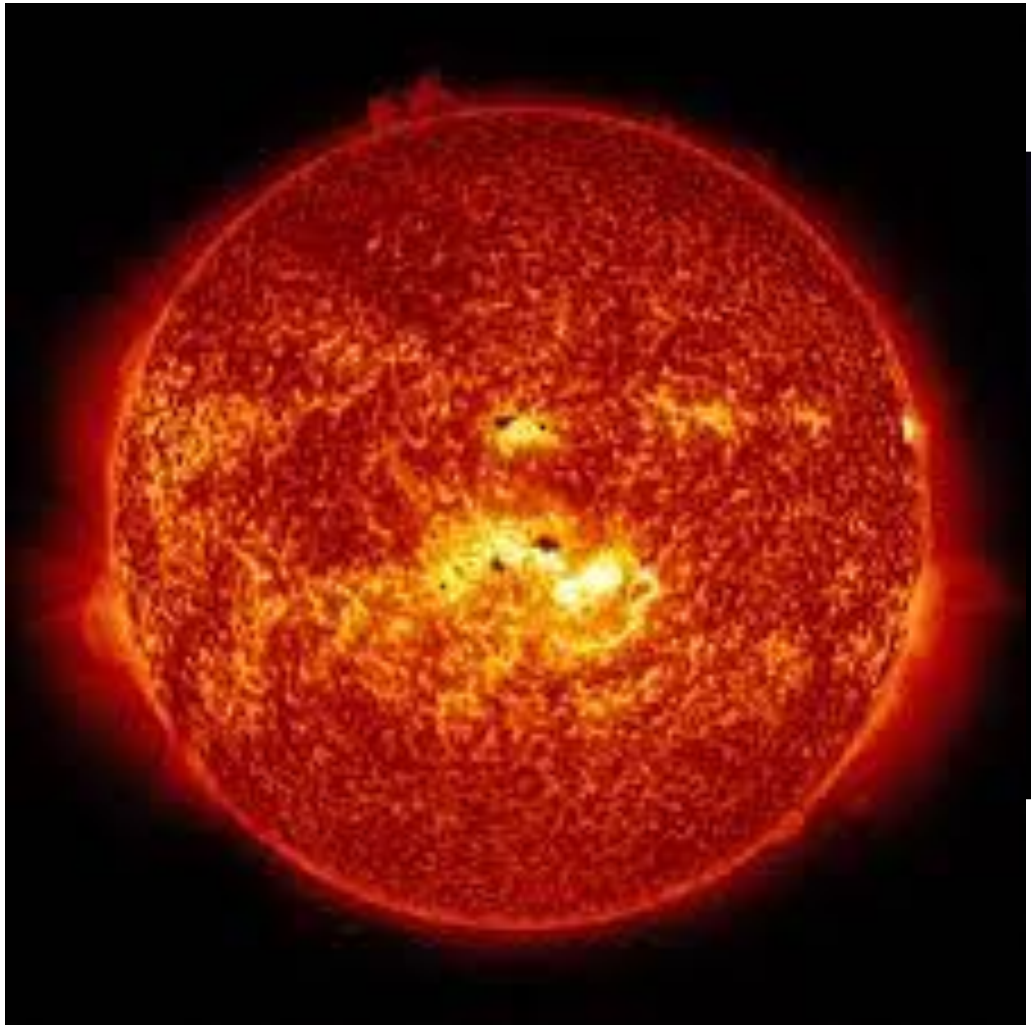


2. DM Capture: from the Sun to NS and WD

Calculating C in NS (degenerate targets)

- DM capture in the Sun is well established (Press and Spergel ’85, Griest and Seckel ’86, Gould ’87, Goldman et.al. ’89, Gould ‘89)
- Increased popularity of DM effects in NS, but no formalism developed until 10 years ago
- First attempts to use apply Gould Formalism to NS

Calculating C in NS (degenerate targets)



$\neq$

Calculating C in NS (degenerate targets)

- DM capture in the Sun is well established (Press and Spergel ’85, Griest and Seckel ’86, Gould ’87, Goldman et.al. ’89, Gould ‘89)
- Ordinary stars and NS **very different** objects, many quantities living at very different scales
- Need to update the formalism!

Calculating C in NS (degenerate targets)

$$C = \int_0^{R_\odot} dr 4\pi r^2 \eta(r) \int_0^\infty du \frac{w}{u} f_\odot^{cap}(u) \sum_i \Omega_i^-(w), \tag{3.1}$$

where $f_\odot^{cap}(r)$ is the DM speed distribution in the reference frame of the Sun, normalised to the local DM number density

$$f_\odot^{cap}(u) = \frac{\rho_\chi}{m_\chi} \lim_{T\to 0} f_{cap}(u) = \frac{\rho_\chi}{m_\chi} \sqrt{\frac{3}{2\pi}} \frac{u}{v_\odot v_d} \left[e^{-\frac{3(u-v_\odot)^2}{2v_d^2}} - e^{-\frac{3(u+v_\odot)^2}{2v_d^2}} \right]. \tag{3.2}$$

$$\Omega^-(w) = \int_0^{v_e} R^-(w \to v) \mathrm{d}v \tag{C.13}$$

$$R^-(w \to v) = \int_0^\infty \mathrm{d}s \int_0^\infty \mathrm{d}t \frac{32\mu_+^4}{\sqrt{\pi}} k^3 n_i \frac{d\sigma_i}{d\cos\theta}(s,t,v,w) \frac{vt}{w} e^{-k^2 v_N^2} \times\Theta(t+s-w)\Theta(v-|t-s|), \tag{C.14}$$

where we have defined

$$k^2 = \frac{m_i}{2T}, \tag{C.15}$$

$$\lim_{T\to 0} \frac{8\mu_+^2}{\sqrt{\pi}} k^3 t\mu e^{-k^2 v_N^2} \Theta(t+s-w) \to \delta\left(s - \frac{w\mu}{2\mu_+}\right) \delta\left(t - \frac{w}{2\mu_+}\right), \tag{C.16}$$

Calculating C in NS (degenerate targets)

Sun	**NS**
Newtonian gravity	GR
Sun structure from Standard Solar Model	NS structure from EOS
Non-relativistic kinematics	Relativistic kinematics
Atomic Nuclei Targets	Baryon and Lepton targets
Non-relativistic matrix element	Relativistic matrix element
MB distribution for targets	FD distribution for targets
Capture probability $\neq 1$	Capture probability = $1*$
Star opacity	Star opacity
MS requires MC approach	MS can be treated (semi)analytically
Targets have FF	Targets have FF
Fixed Target mass	Density-dependent Target mass

Calculating C in NS (degenerate targets)

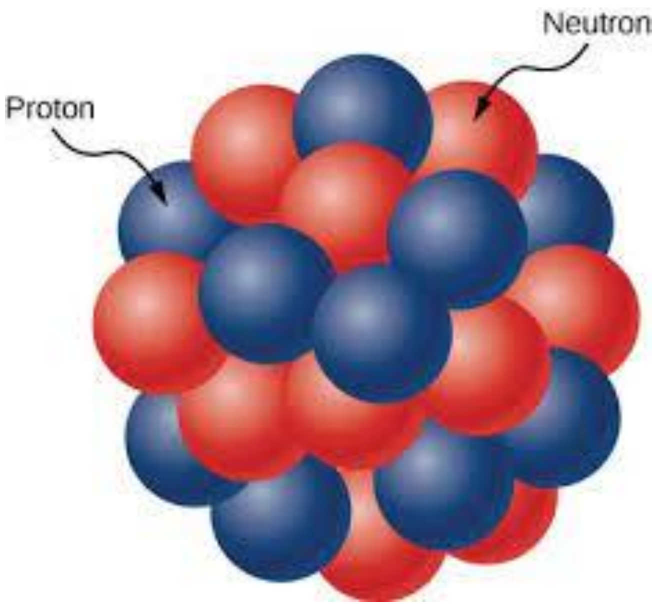
Sun	**NS**
Newtonian gravity	GR
Sun structure from Standard Solar Model	NS structure from EOS
Non-relativistic kinematics	Relativistic kinematics
Atomic Nuclei Targets	Baryon and Lepton targets
Non-relativistic matrix element	Relativistic matrix element
MB distribution for targets	FD distribution for targets
Capture probability $\neq 1$	Capture probability = $1*$
Star opacity	Star opacity
MS requires MC approach	MS can be treated (semi)analytically
Targets have FF	Targets have FF
Fixed Target mass	Density-dependent Target mass

Targets Form Factors

Sun

Nuclei Composite Objects$\longrightarrow$ have a form factor

$$F(E_R) \propto e^{-E_R/E_0}$$

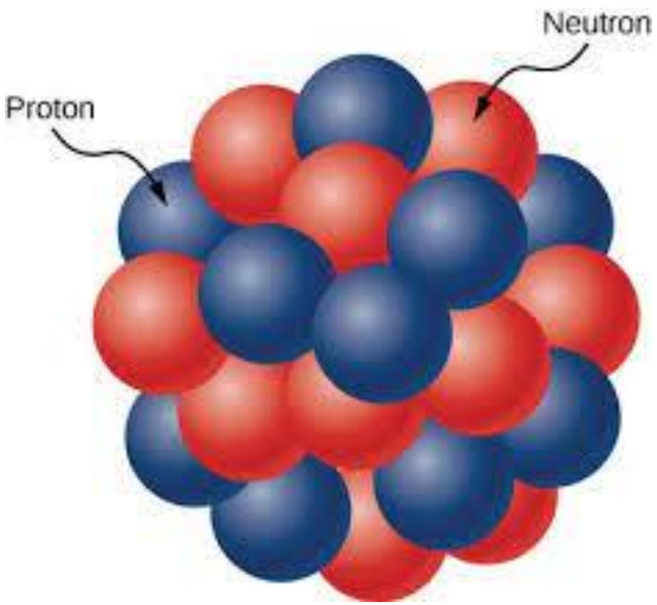


Targets Form Factors

Sun

Nuclei Composite Objects⟶ have a form factor

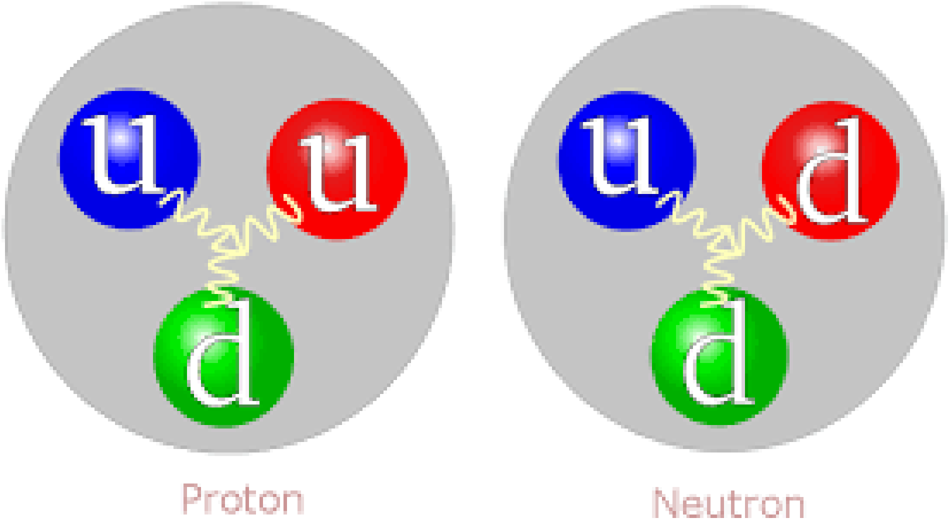
$$F(E_R) \propto e^{-E_R/E_0}$$



NS

Hadrons are also composite objects!

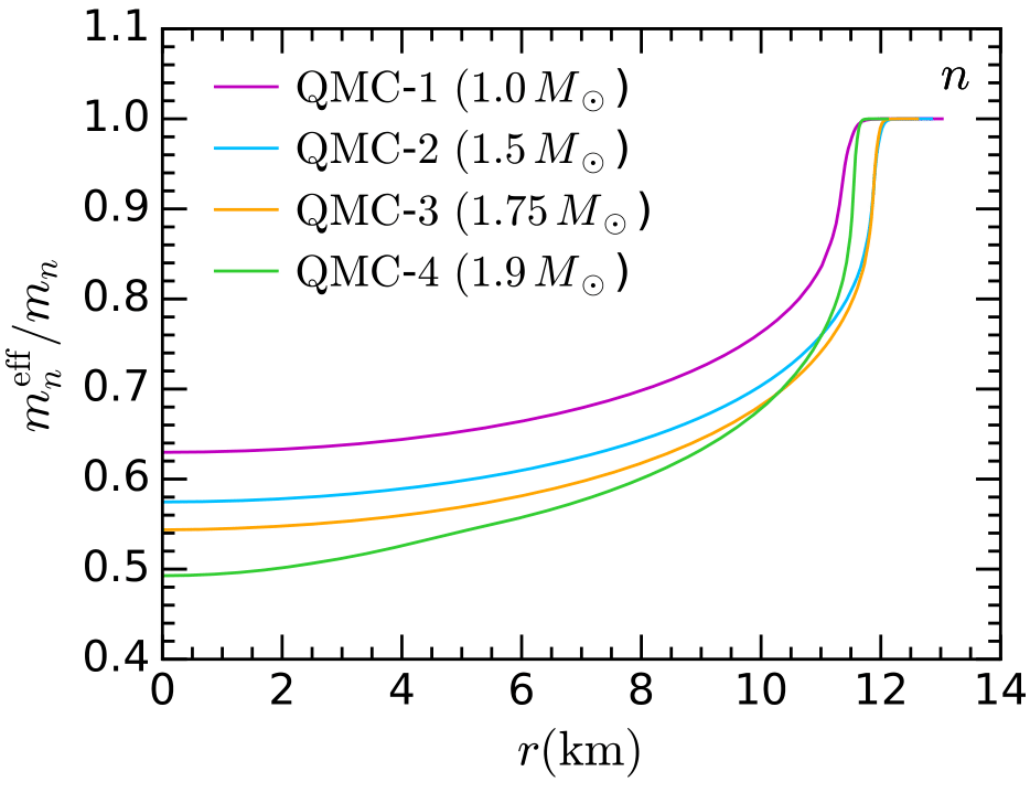
$$F(t) \propto \frac{1}{\left(1 - \frac{t}{Q_0}\right)^4}$$



Phys.Rev.Lett. 127 (2021) 11, 111803

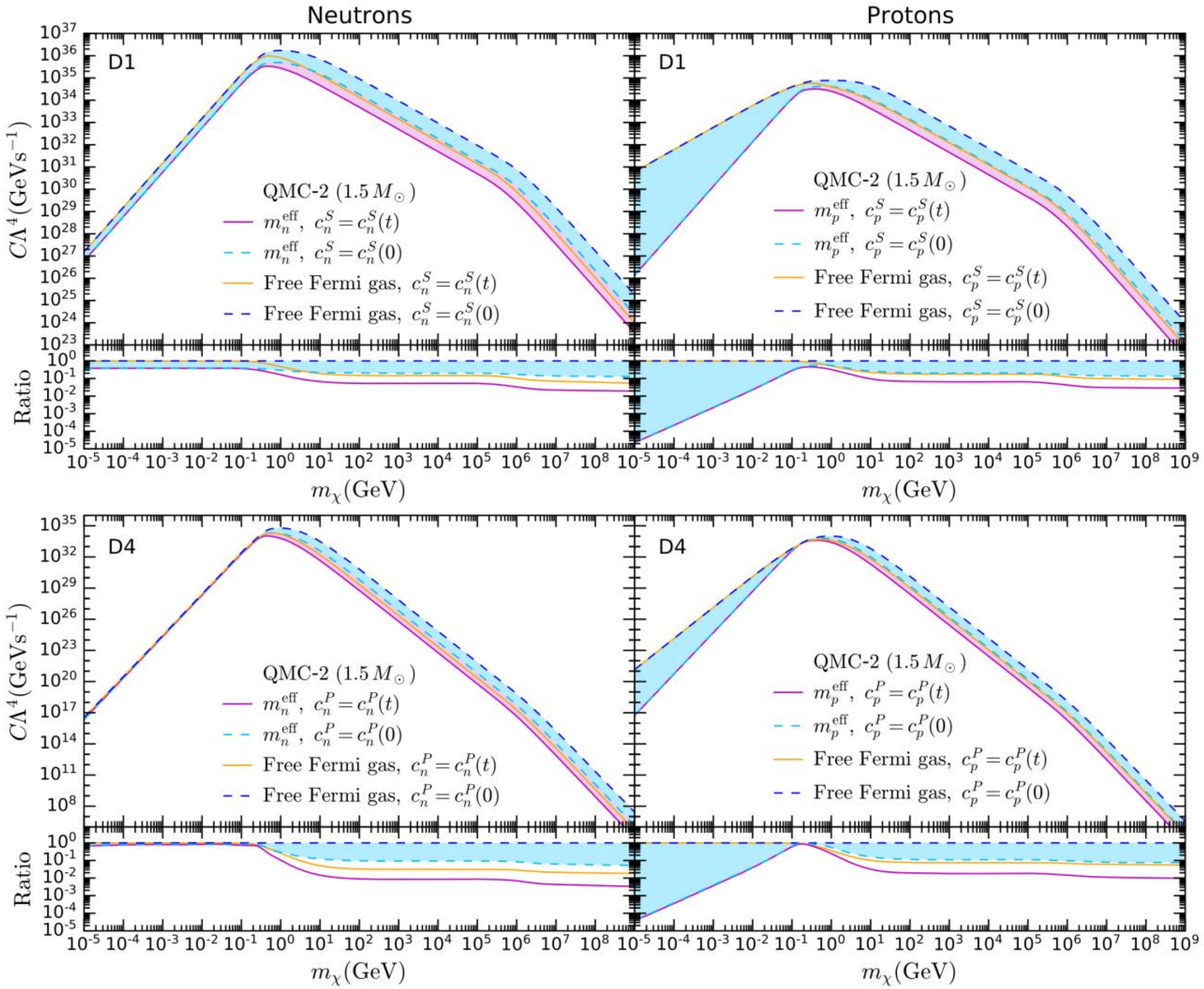
Density-dependent Target mass

- Effects of strong mean field in NS: mass of hadrons is different from the one in vacuum



Phys.Rev.Lett. 127 (2021) 11, 111803

Form Factors and effective mass



Phys.Rev.Lett. 127 (2021) 11, 111803

Calculating C in NS (degenerate targets)

Sun	**NS**
Newtonian gravity	GR
Sun structure from Standard Solar Model	NS structure from EOS
Non-relativistic kinematics	Relativistic kinematics
Atomic Nuclei Targets	Baryon and Lepton targets
Non-relativistic matrix element	Relativistic matrix element
MB distribution for targets	FD distribution for targets
Capture probability $\neq 1$	Capture probability = 1*
Star opacity	Star opacity
MS requires MC approach	MS can be treated (semi)analytically
Targets have FF	Targets have FF
Fixed Target mass	Density-dependent Target mass

Multiple Scattering

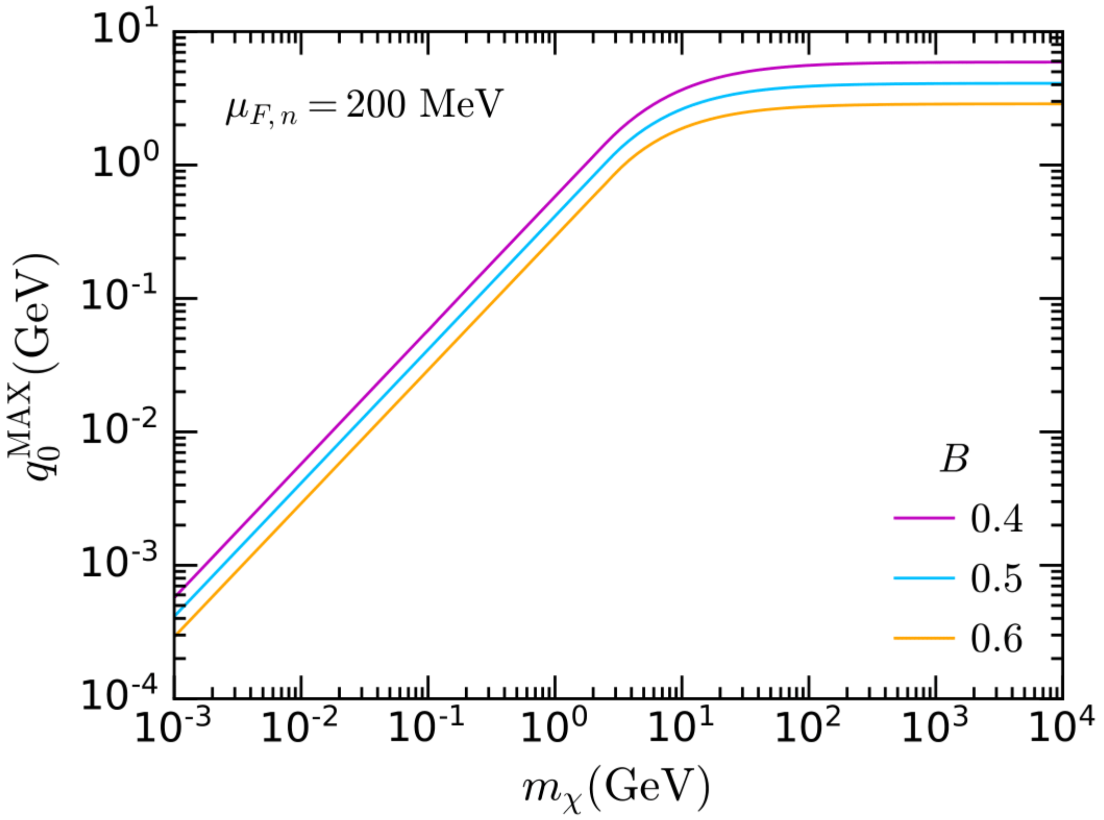
- At large m, q_0 reaches maximum value $\sim O(4GeV)$
- To get captured, DM needs to lose

$$K \sim \frac{1}{2} m u^2$$

With $u \sim 3 \cdot 10^{-3}$

- The number of scatterings required is

$$N = \frac{mu^2}{8GeV} \sim 10^{-6} \frac{m}{GeV}$$



Multiple Scattering

- So for $m < 10^6 GeV$ capture with single scattering
- For $m > 10^6 GeV$ requires

$$N = 10^{-6} \frac{m}{GeV}$$

Scatterings

- So the capture rate is diminished approximately by a factor

$$1/N \propto 1/m$$

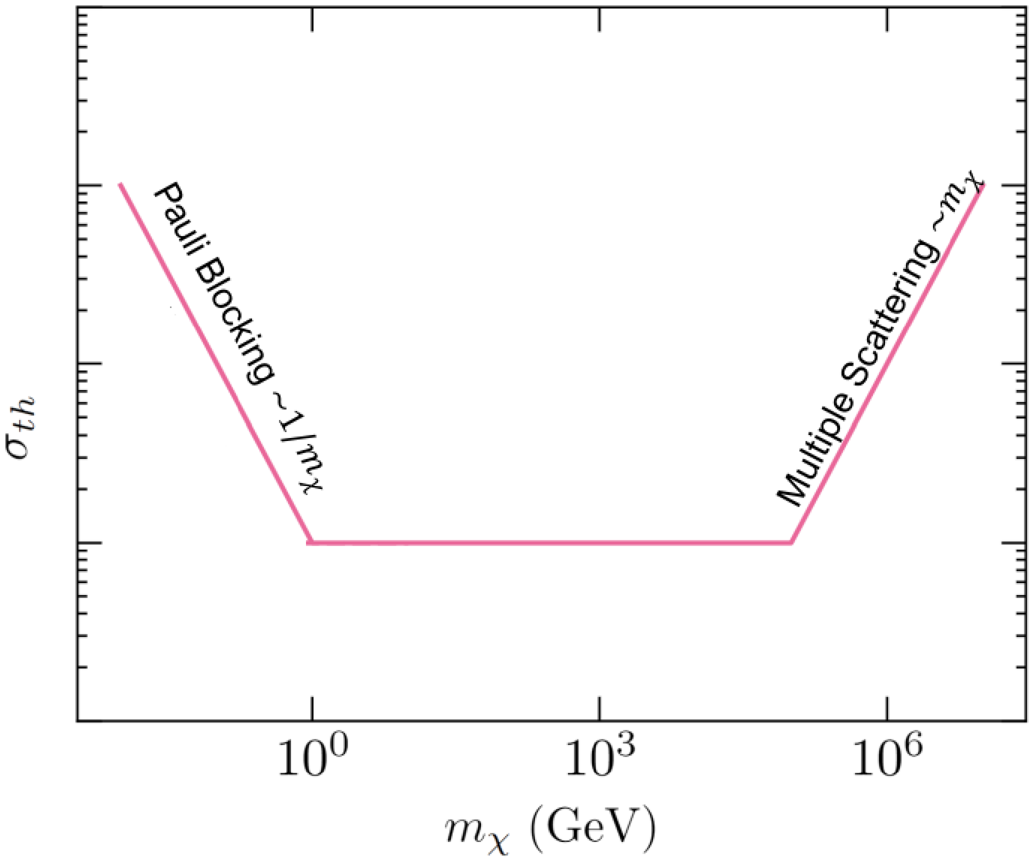
Multiple Scattering

$\sigma_{th} \propto 1/C$

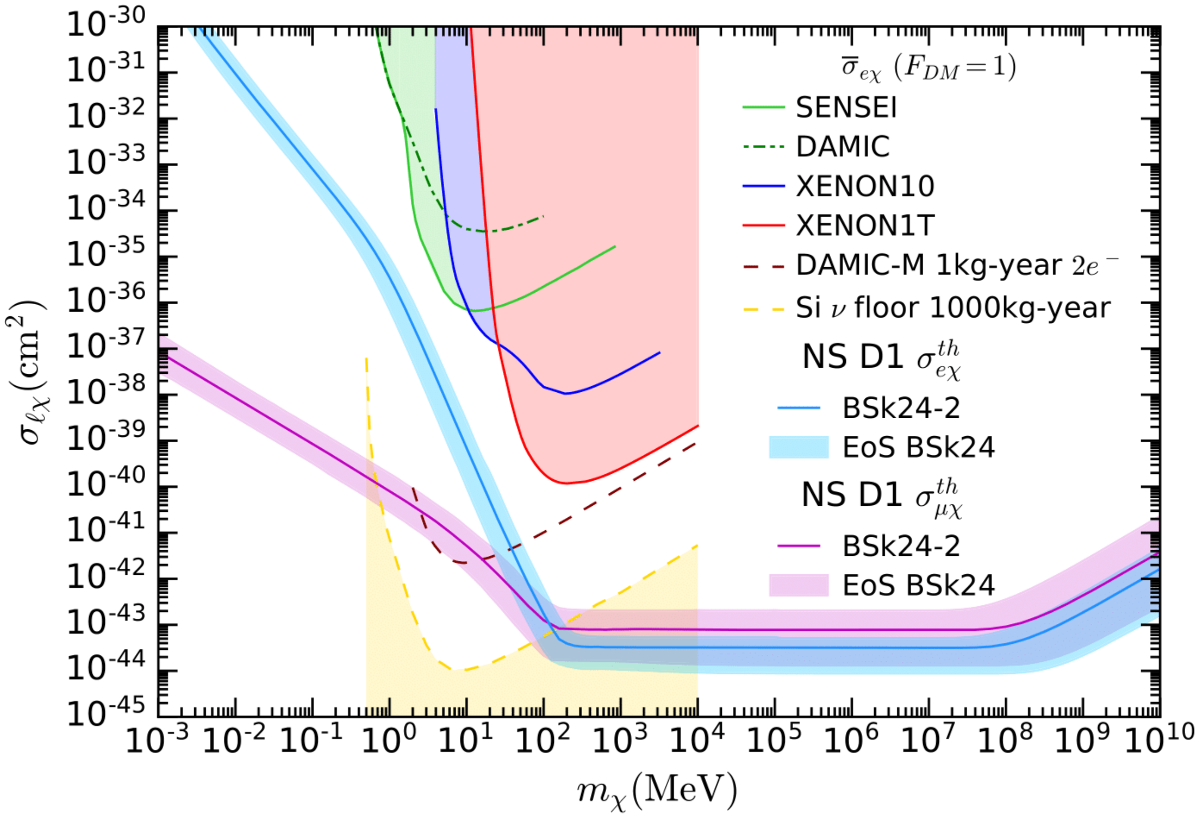
- So for $m < 10^6 GeV$ capture with single scattering
- For $m > 10^6 GeV$ requires
$$N = 10^{-6}\frac{m}{GeV}$$

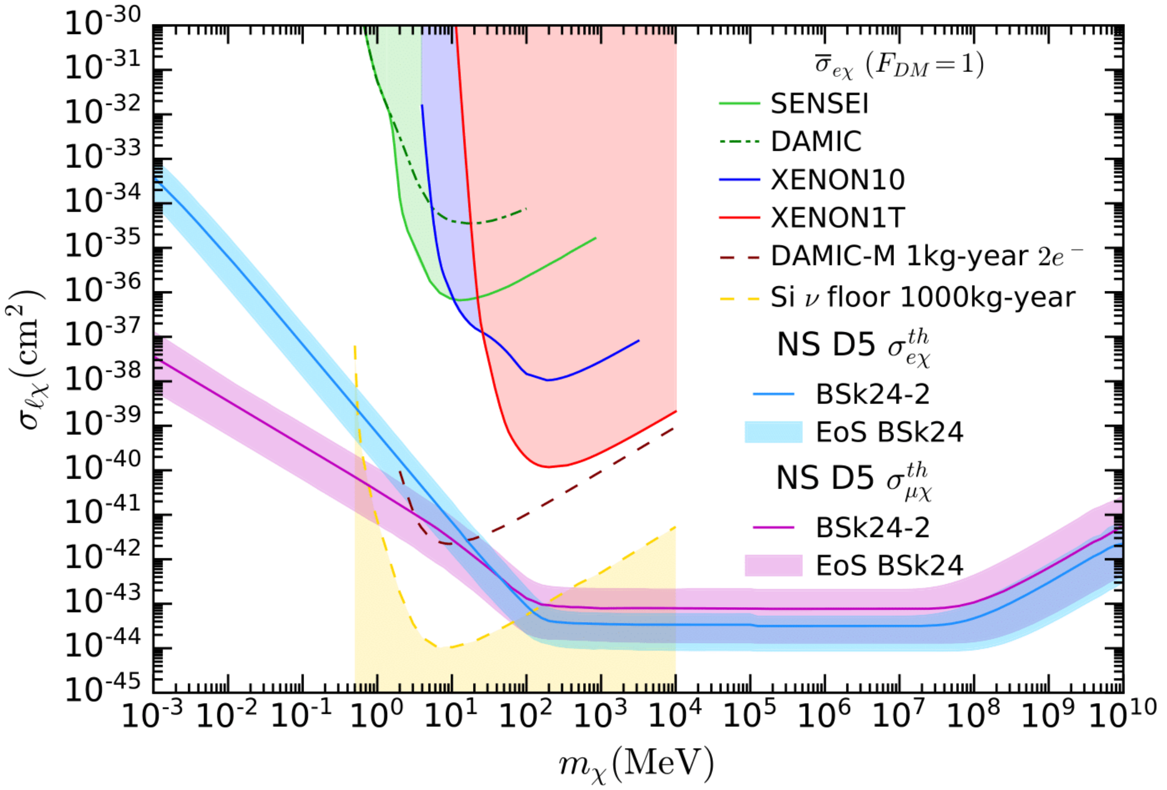
Scatterings

- So the capture rate is diminished approximately by a factor
$$1/N \propto 1/m$$

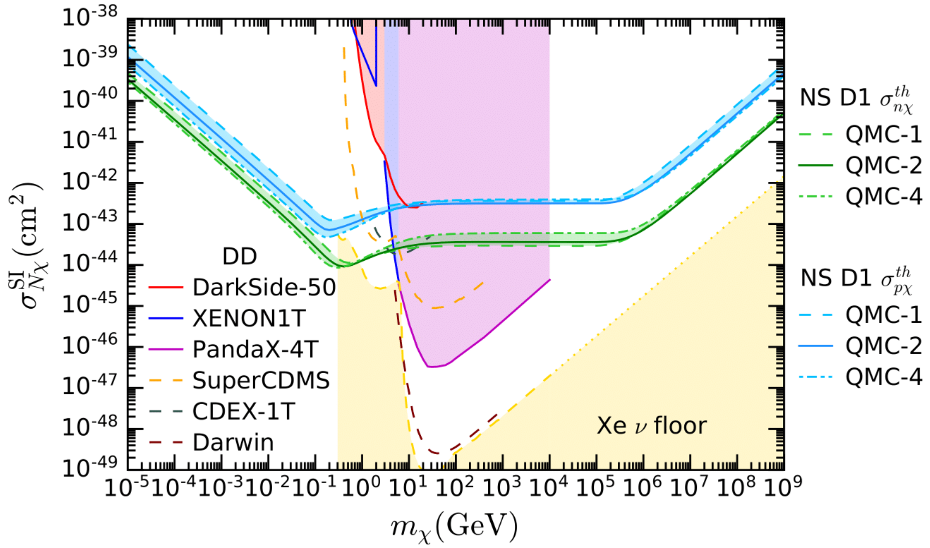


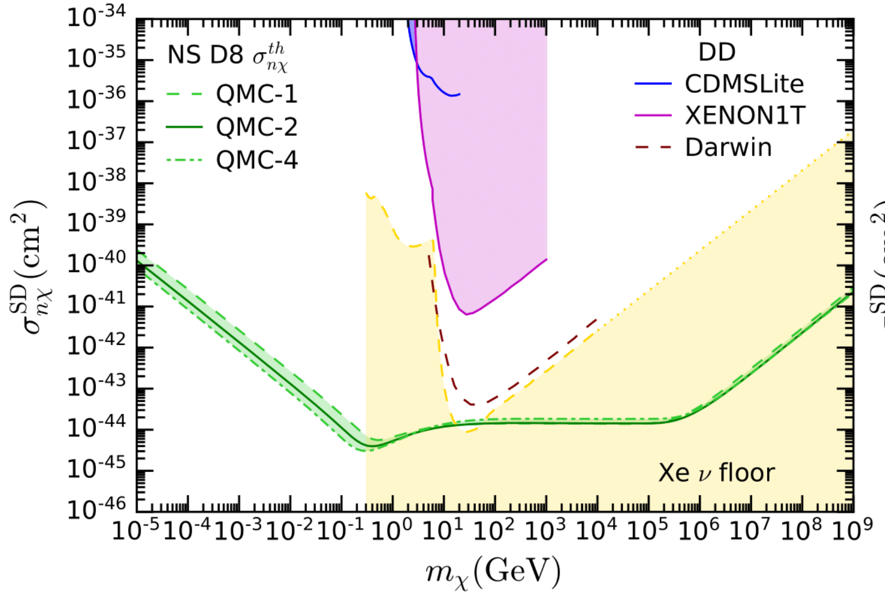
Capture Rate in NS: Leptons

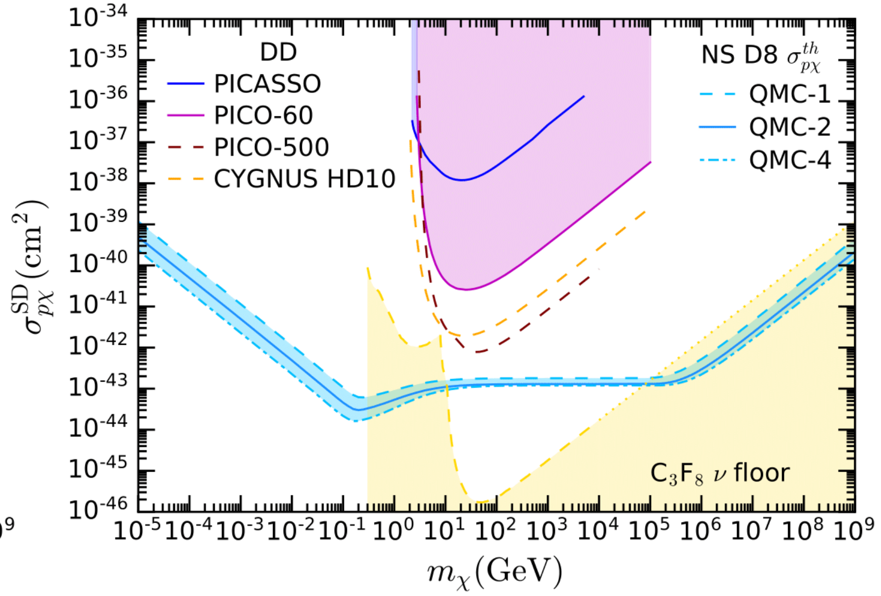




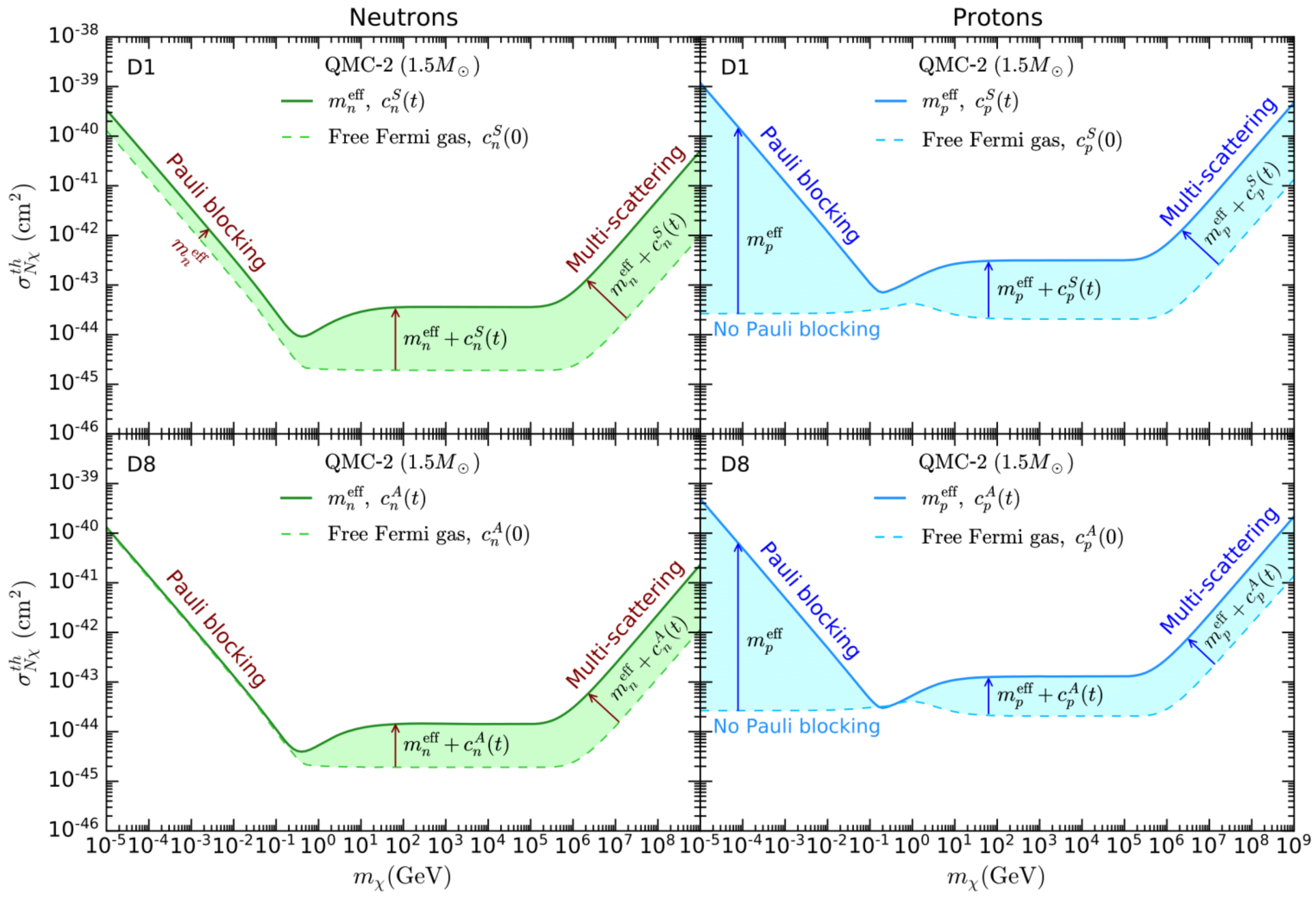
Capture Rate in NS: Hadrons







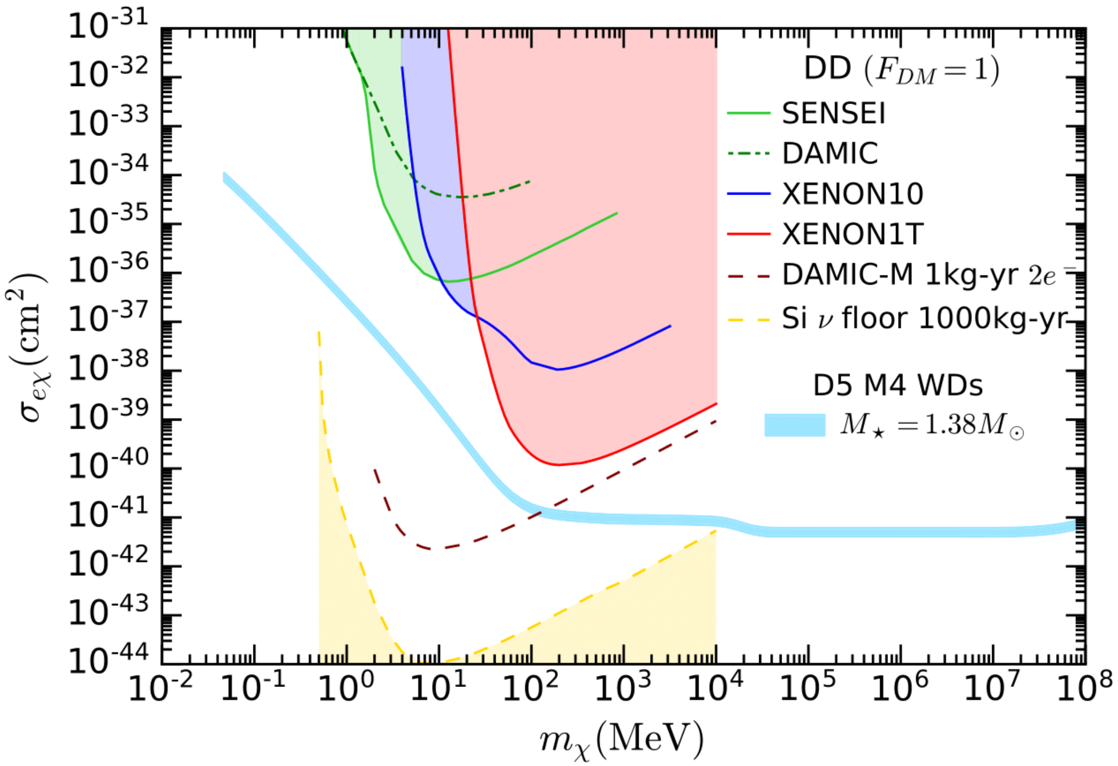
Capture Rate in NS: importance of FF and effective mass



White Dwarfs

- 2 kind of targets:
- Ions: non-degenerate. Gould Formalism can be applied (gravity is large but not too large in WD)
- Electrons: heavily degenerate, NS formalism applies
- No kinetic heating as gravity is "weak"
- Annihilation heating can increase luminosity

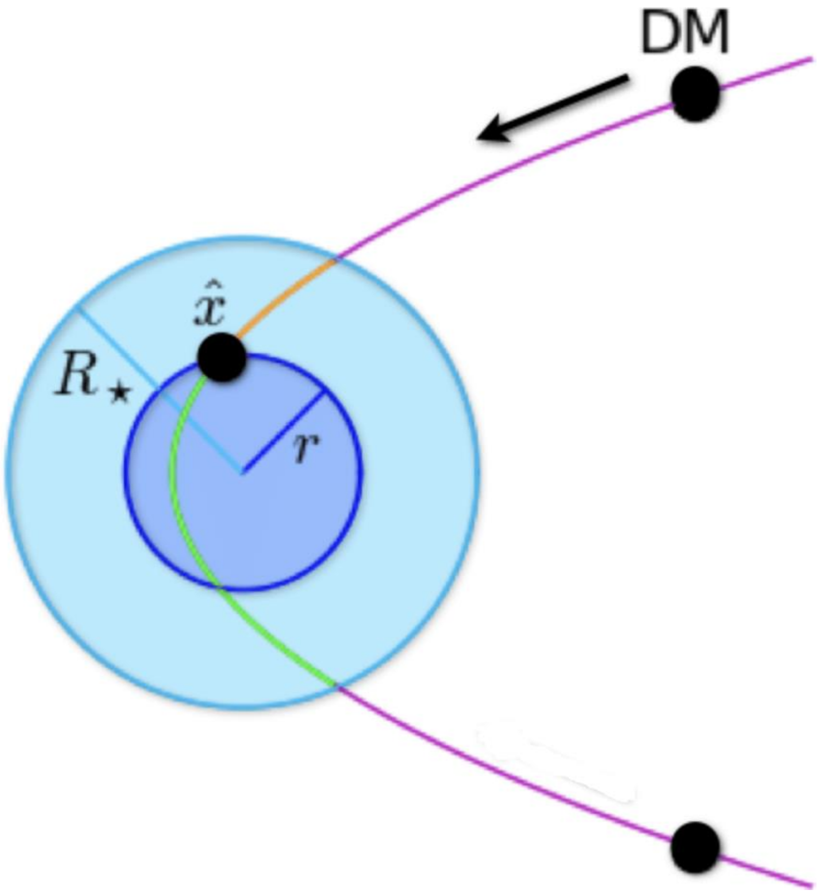
$$L = mC$$



3. Multiple Scattering

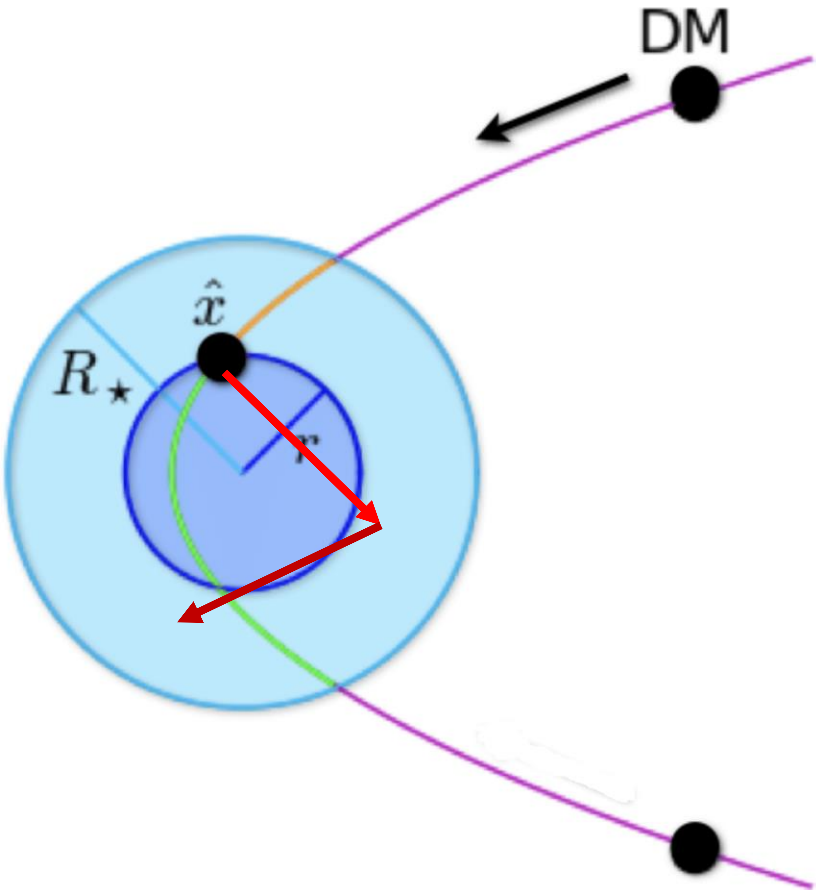
MS in the Sun

- In the sun, after each scattering before capture, the velocity of DM changes significantly



MS in the Sun

- In the sun, after each scattering before capture, the velocity of DM changes significantly
- This leads to some “random walk” inside the star
- Each part of the trajectory depends on previous scattering
- This happens because $u \sim v_{esc}$
- Requires MC simulation



MS in a compact star

- Before Capture, the variation of momentum in a single scattering for an heavy DM is, at most

$$\frac{\delta p}{p} \approx \frac{2GeV}{O(1) \times m} \ll 1$$

- Even after summing up over all scatterings required for capture, one gets

$$\frac{\delta p}{p} \approx N \frac{2GeV}{O(1) \times m} \frac{mu^2}{8GeV} \approx \frac{u^2}{O(2)} \ll 1$$

- This means one can assume the trajectory of DM is **unchanged** after each scattering. This is thanks to $v_{esc} \gg u$.

MS in a compact star

- We can use the normalized interaction rate as a PDF:

$$\xi(q_0, E_\chi, \mu_{F,n}) = \frac{1}{\Gamma(E_\chi)} \frac{d\Gamma}{dq_0}(q_0, E_\chi, \mu_{F,n}), \tag{4.4}$$

$$P_1(\delta q_0) = \int_{\delta q_0}^{\infty} dx \xi(x). \tag{4.5}$$

$$P_2(\delta q_0) = P_1(\delta q_0) + \int_{\delta q_0}^{\infty} dy \int_0^y dx \xi(x)\xi(y-x) = P_1(\delta q_0) + \int_0^{\delta q_0} dz P_1(\delta q_0 - z)\xi(z). \tag{4.6}$$

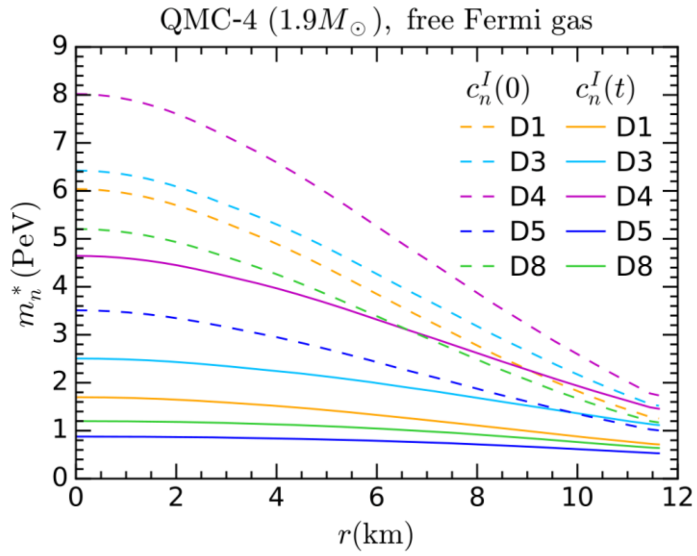
$$P_{N+1}(\delta q_0) = P_N(\delta q_0) + \int_0^{\delta q_0} dz P_N(\delta q_0 - z)\xi(z). \tag{4.7}$$

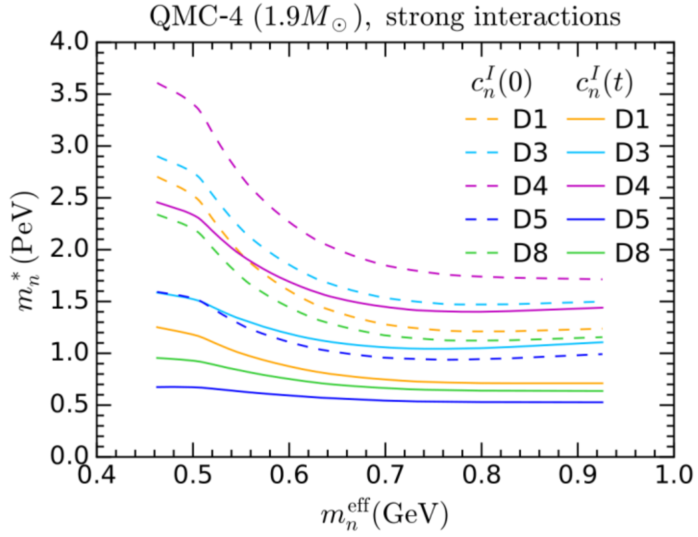
$P_N(x)$: probability to lose the energy x after at most N scatterings

NS: semi-analytical approach

- Interaction rates calculated analytically
- Functions P_N computed numerically for $N \leq 20$
- Fitting function (1 parameter m^*) to extrapolate above $N > 20$
- Simple result: Capture rate suppressed by

$$\eta = \frac{m^*}{m} e^{-\frac{\langle m^* \tau \rangle}{m}}$$





WD (ion targets): analytical approach (preliminary)

- Until few years ago, only Gould formalism for Multiple Scattering
- Formalism was developed for the Earth, and assumed:
- (i) DM trajectories are unaffected by collisions,
- (ii) constant escape velocity in the Earth's core;
- (iii) constant iron (matter) density,
- (iv) given that $v_{esc} \ll u_{\chi}$, DM follows linear trajectories outside and inside the Earth's core, hence gravitational focusing is neglected

WD (ion targets): analytical approach (preliminary)

- Until few years ago, only Gould formalism for Multiple Scattering
- Formalism was developed for the Earth, and assumed:
- (i) DM trajectories are unaffected by collisions,
- (ii) constant escape velocity in the Earth's core;
- (iii) constant iron (matter) density,
- (iv) given that $v_{esc} \ll u_\chi$, DM follows linear trajectories outside and inside the Earth's core, hence gravitational focusing is neglected

WD (ion targets): analytical approach (preliminary)

- Until few years ago, only Gould formalism for Multiple Scattering
- Formalism was developed for the Earth, and assumed:
- (i) DM trajectories are unaffected by collisions, ✓
- (ii) constant escape velocity in the Earth's core; ✗
- (iii) constant iron (matter) density,
- (iv) given that $v_{esc} \ll u_\chi$, DM follows linear trajectories outside and inside the Earth's core, hence gravitational focusing is neglected

This would mean neglecting the WD gravitational field!!!!!

WD (ion targets): analytical approach (preliminary)

- Until few years ago, only Gould formalism for Multiple Scattering
- Formalism was developed for the Earth, and assumed:
- (i) DM trajectories are unaffected by collisions,
- (ii) constant escape velocity in the Earth's core;
- (iii) constant iron (matter) density,
- (iv) given that $v_{esc} \ll u_\chi$, DM follows linear trajectories outside and inside the Earth's core, hence gravitational focusing is neglected

The WD density profiles are flat, but still not the best approximation

WD (ion targets): analytical approach (preliminary)

- Until few years ago, only Gould formalism for Multiple Scattering
- Formalism was developed for the Earth, and assumed:
- (i) DM trajectories are unaffected by collisions,
- (ii) constant escape velocity in the Earth's core;
- (iii) constant iron (matter) density,
- (iv) given that $v_{esc} \ll u_\chi$, DM follows linear trajectories outside and inside the Earth's core, hence gravitational focusing is neglected

This would mean, again, neglecting the WD gravitational field!!!!

WD (ion targets): analytical approach (preliminary)

- We need to calculate $P_N(x)$
- Assuming a constant interaction rate, like usually assumed in the literature, gets the problem very complicated…
- Solutions present in literature usually require a series of approximations, and sometimes require to sum C_N(=Capture rate for exactly N scatterings) over all values of the number of scatterings N
- This means that every time you increase your cross section by a factor of 10, you need to calculate 10 times more capture rates
- This can inflate your computation times very quickly!

WD (ion targets): analytical approach (preliminary)

- However, the rates are usually incorporating the Ion Nuclear FF, that has exponential form

$$F(E_R) \propto e^{-E_R/E_0}$$

- Using interaction rates proportional to this simple exponential law, it is possible to solve the problem exactly!

WD (ion targets): analytical approach (preliminary)

- The problem can be solved using Laplace transforms
- The normalised interaction rate can be Laplace-transformed

$$e^{-E_R/E_0} \longrightarrow \frac{1}{1+s}$$

- We can Laplace-transform the equations for P_1, P_{N+1}, we get

$$P_1 \longrightarrow \frac{1}{1+s}$$

$$P_{N+1} \longrightarrow \frac{1}{1+s} P_N$$

WD (ion targets): analytical approach (preliminary)

- Exact solution

$$P_N \longrightarrow \left(\frac{1}{1+s}\right)^N$$

- By inverse-transformation, one can get P_N

$$P_N(x) = \frac{e^{-x}x^{N-1}}{N-1!}$$

- Putting this result together with Poisson distribution for the probability to interact N times at an optical depth τ, we can get an exact result for the MS Capture rate

WD (ion targets): analytical approach (preliminary)

$$\mathcal{F}_N(\delta) = \frac{e^{-\delta}\delta^{N-1}}{N-1!}. \tag{3.29}$$

$$p_N(\tau_\chi) = e^{-\tau_\chi}\frac{\tau_\chi^N}{N!}. \tag{3.30}$$

$$C_1 = \frac{\rho_\chi}{m_\chi}\int_0^{R_*} dr 4\pi r^2 n_T(r)\sigma_{T\chi}(v_{esc}(r))v_{esc}^2(r)\int_0^1 \frac{ydy}{\sqrt{1-y^2}}\int_0^\infty du_\chi \frac{f_{\rm MB}(u_\chi)}{u_\chi}p_0(\tau_\chi)\mathcal{F}_1(\delta), \tag{3.31}$$

$$C_N = \frac{\rho_\chi}{m_\chi}\int_0^{R_*} dr 4\pi r^2 n_T(r)\sigma_{T\chi}(v_{esc}(r))v_{esc}^2(r)\int_0^1 \frac{ydy}{\sqrt{1-y^2}}\int_0^\infty du_\chi \frac{f_{\rm MB}(u_\chi)}{u_\chi}p_{N-1}(\tau_\chi)\mathcal{F}_N(\delta), \tag{3.32}$$

and the total capture rate is given by the sum over all N

$$C = \sum_N C_N. \tag{3.33}$$

Next, instead of evaluating the integrals in Eq. 3.32 and sum up later over all possible N scatterings, we sum the series first by introducing the response function, $G(\tau_\chi, \delta)$

$$\begin{aligned} G(\tau_\chi,\delta) &\equiv \sum_{N=1}^{\infty} p_{N-1}(\tau_\chi)\mathcal{F}_N(\delta) = \sum_{N=1}^{\infty}\frac{e^{-\tau_\chi}\tau_\chi^{N-1}}{(N-1)!}\frac{e^{-\delta}\delta^{N-1}}{(N-1)!} \\ &= e^{-\tau_\chi-\delta}I_0\left(2\sqrt{\tau_\chi\delta}\right), \end{aligned} \tag{3.34}$$

$$C = \frac{\rho_\chi}{m_\chi}\int_0^{R_*} dr 4\pi r^2 n_T(r)\sigma_{T\chi}(v_{esc}(r))v_{esc}^2(r)\int_0^\infty du_\chi \frac{f_{\rm MB}(u_\chi)}{u_\chi}\tilde{G}\left(r, \frac{m_\chi u_\chi^2}{2E_0}\right), \tag{3.36}$$

WD (ion targets): analytical approach (preliminary)

$$\mathcal{F}_N(\delta) = \frac{e^{-\delta}\delta^{N-1}}{N-1!}. \tag{3.29}$$

$$p_N(\tau_\chi) = e^{-\tau_\chi}\frac{\tau_\chi^N}{N!}. \tag{3.30}$$

$$C_1 = \frac{\rho_\chi}{m_\chi}\int_0^{R_*} dr 4\pi r^2 n_T(r)\sigma_{T\chi}(v_{esc}(r))v_{esc}^2(r)\int_0^1 \frac{ydy}{\sqrt{1-y^2}}\int_0^\infty du_\chi \frac{f_{\rm MB}(u_\chi)}{u_\chi} p_0(\tau_\chi)\mathcal{F}_1(\delta), \tag{3.31}$$

$$C_N = \frac{\rho_\chi}{m_\chi}\int_0^{R_*} dr 4\pi r^2 n_T(r)\sigma_{T\chi}(v_{esc}(r))v_{esc}^2(r)\int_0^1 \frac{ydy}{\sqrt{1-y^2}}\int_0^\infty du_\chi \frac{f_{\rm MB}(u_\chi)}{u_\chi} p_{N-1}(\tau_\chi)\mathcal{F}_N(\delta), \tag{3.32}$$

and the total capture rate is given by the sum over all N

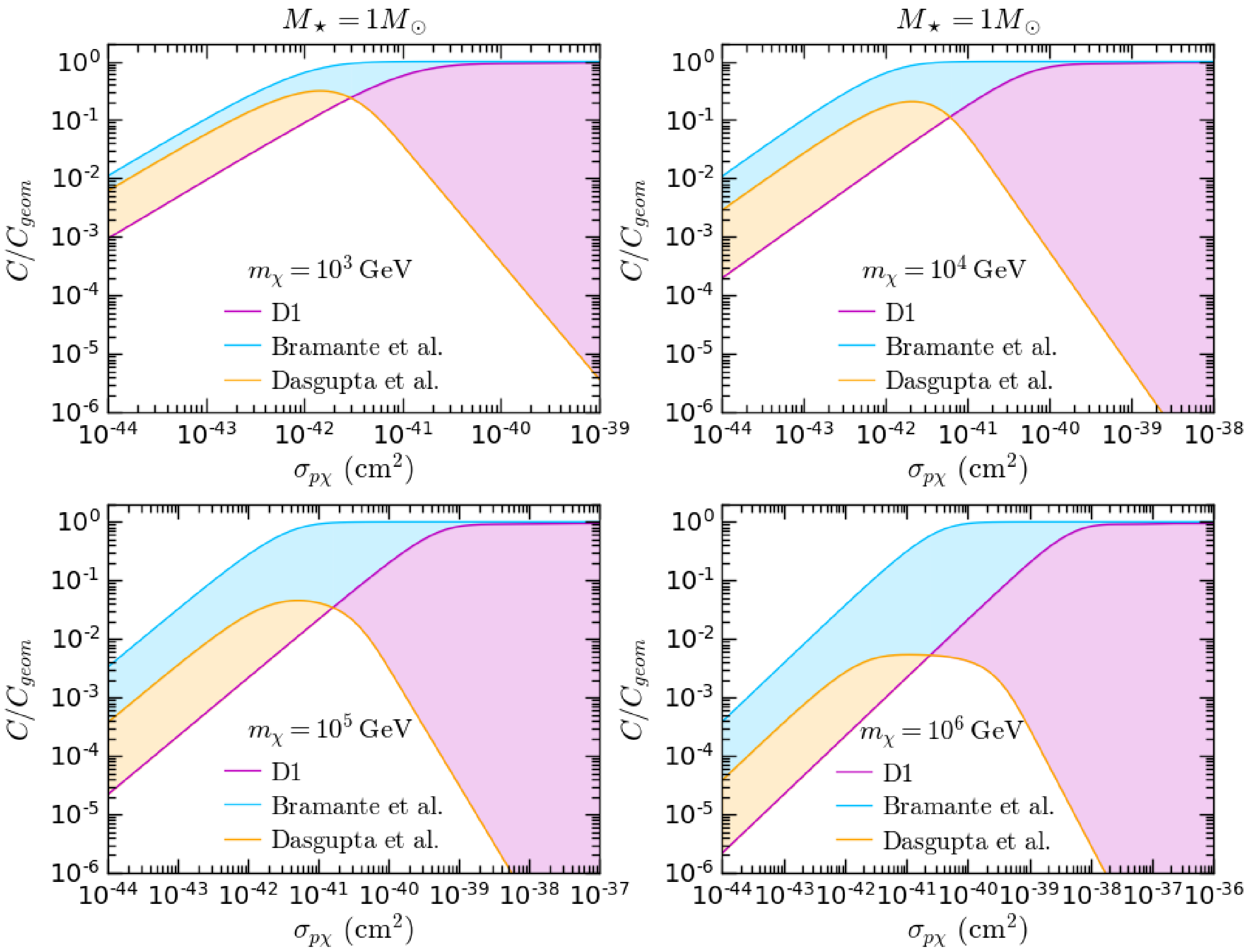
$$C = \sum_N C_N. \tag{3.33}$$

Next, instead of evaluating the integrals in Eq. 3.32 and sum up later over all possible N scatterings, we sum the series first by introducing the response function, $G(\tau_\chi, \delta)$

$$\begin{aligned} G(\tau_\chi,\delta) &\equiv \sum_{N=1}^{\infty} p_{N-1}(\tau_\chi)\mathcal{F}_N(\delta) = \sum_{N=1}^{\infty}\frac{e^{-\tau_\chi}\tau_\chi^{N-1}}{(N-1)!}\frac{e^{-\delta}\delta^{N-1}}{(N-1)!} \\ &= e^{-\tau_\chi-\delta}I_0\left(2\sqrt{\tau_\chi\delta}\right), \end{aligned} \tag{3.34}$$

$$C = \frac{\rho_\chi}{m_\chi}\int_0^{R_*} dr 4\pi r^2 n_T(r)\sigma_{T\chi}(v_{esc}(r))v_{esc}^2(r)\int_0^\infty du_\chi \frac{f_{\rm MB}(u_\chi)}{u_\chi}\bar{G}\left(r, \frac{m_\chi u_\chi^2}{2E_0}\right), \tag{3.36}$$

WD (ion targets): results (preliminary)



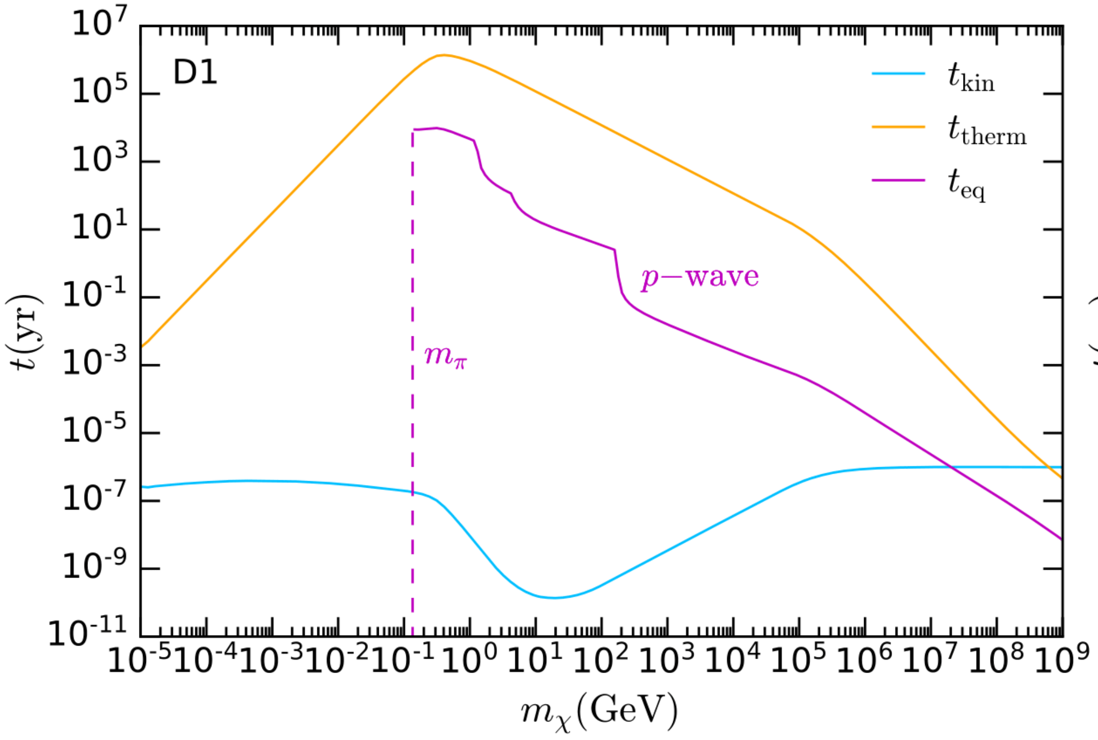
4. Thermalisation

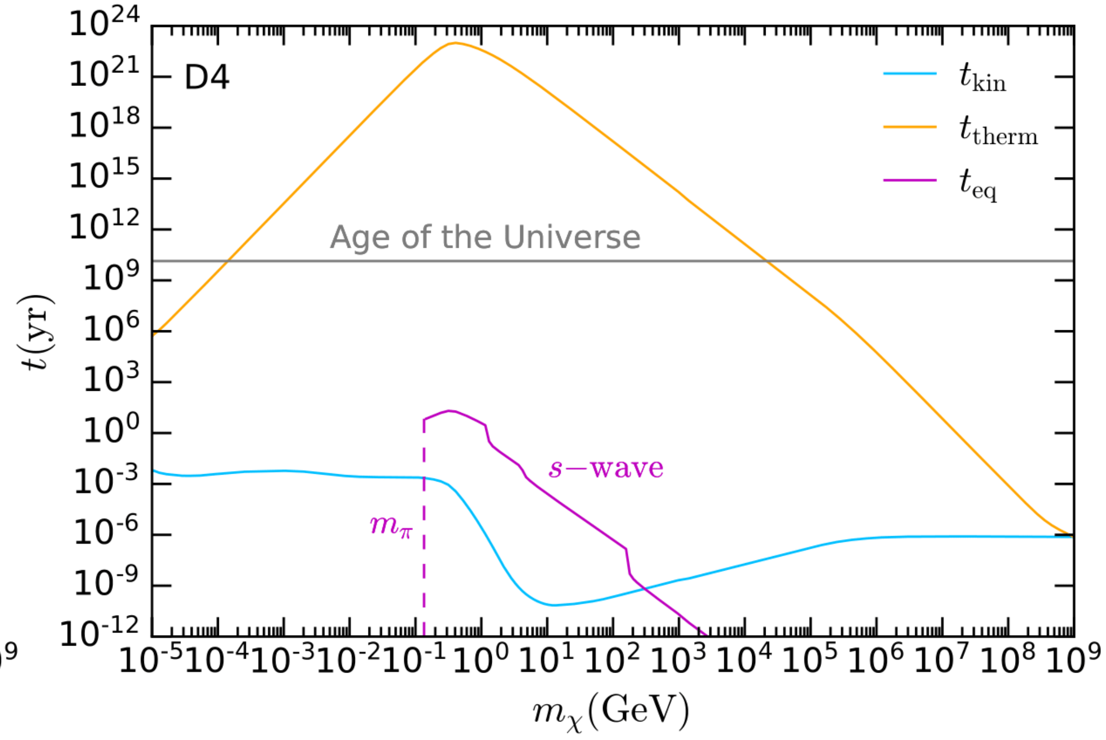
(preliminary)

After Capture

- What happens after DM capture?
- Timescale to transfer it kinetic energy to the star?
- Timescale to reach Capture-annihilation equilibrium (and therefore allow the annihilation heating)?
- Timescale for thermalization?

Thermalisation: NS (preliminary)





5. Conclusions

Summary

- Neutron Stars: cosmic laboratory to probe DM scattering interactions
- Completely different kinematic regime to direct detection experiments
- High energy scattering washes away momentum suppression
- Higher reach on inelastic scattering
- Can probe a very large mass range
- Very sensitive for all interactions, including momentum-suppressed and leptons
- Interesting complementary way to set bounds on DM

Backup

Pseudo-Dirac DM

The pseudo-Dirac Lagrangian reads

$$\mathcal{L} = \bar{\Psi}(i\partial\!\!\!/ - M_D)\Psi - \frac{1}{4}F^V_{\mu\nu}F^{\mu\nu}_V + \frac{1}{4}M^2 Z'_\mu Z'^\mu + Q_\Psi g \bar{\Psi}\gamma^\mu \Psi\, Z'_\mu + Q_q g \sum_q \bar{q}\gamma^\mu q\, Z'_\mu$$
$$-\frac{m_L}{2}\left(\overline{\Psi^c} P_L \Psi + \text{h.c.}\right) - \frac{m_R}{2}\left(\overline{\Psi^c} P_R \Psi + \text{h.c.}\right) , \qquad (2.22)$$

where Q_Ψ, Q_q are the DM and quark $U(1)$ charges. We shall set $Q_\Psi Q_q = 1$ throughout, as DD constraints do not depend on the individual charges, but only on their product. Taking $m_L = m_R = \frac{1}{2}\delta m \ll m_D$, the Majorana mass eigenstates become

$$\chi_1 = \frac{i}{\sqrt{2}}\left(\Psi - \Psi^c\right) , \qquad (2.23)$$
$$\chi_2 = \frac{1}{\sqrt{2}}\left(\Psi + \Psi^c\right) . \qquad (2.24)$$

Expressed in terms of the mass eigenstates, the DM Lagrangian effectively becomes

$$\mathcal{L} \supset \frac{1}{2}\,\bar{\chi}_1(i\partial\!\!\!/ - m_1)\chi_1 + \frac{1}{2}\,\bar{\chi}_2(i\partial\!\!\!/ - m_2)\chi_2 + iQ_\Psi g \bar{\chi}_2\gamma^\mu Z'_\mu\,\chi_1 + iQ_\Psi g \bar{\chi}_1\gamma^\mu Z'_\mu \chi_2, \qquad (2.25)$$

where $m_1 = m_D - \frac{1}{2}\delta m$ and $m_2 = m_D + \frac{1}{2}\delta m = m_1 + \delta m$. For more details about the model, refer to

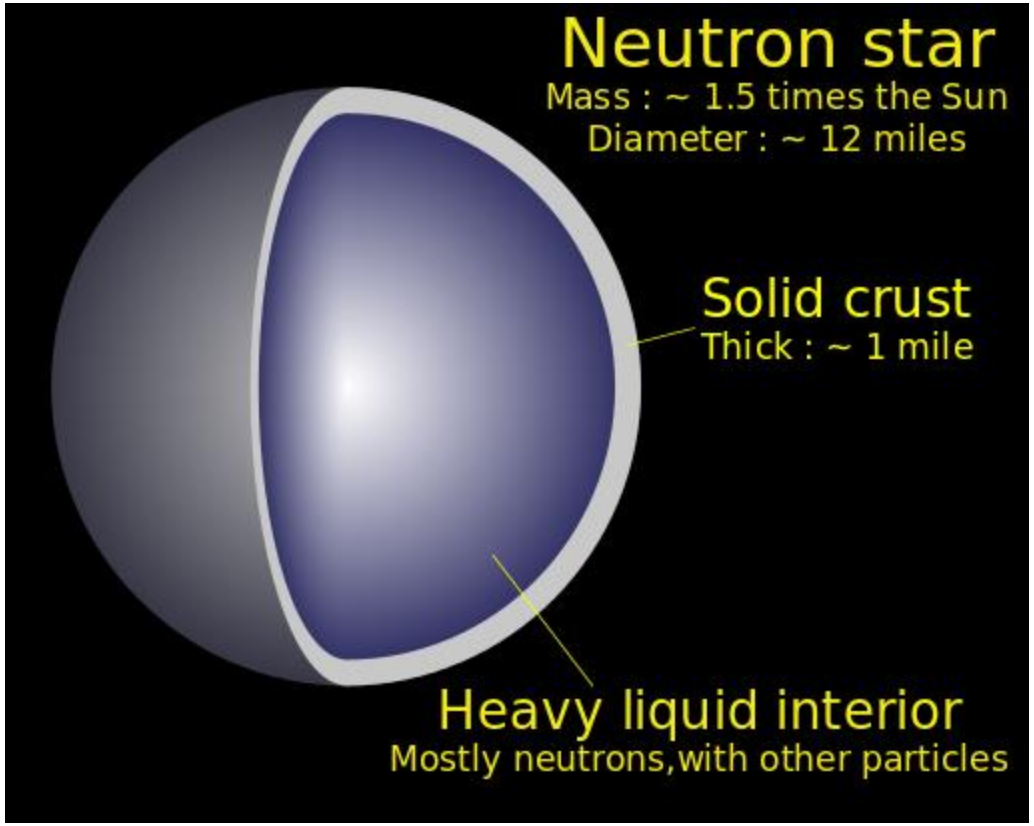
Operators

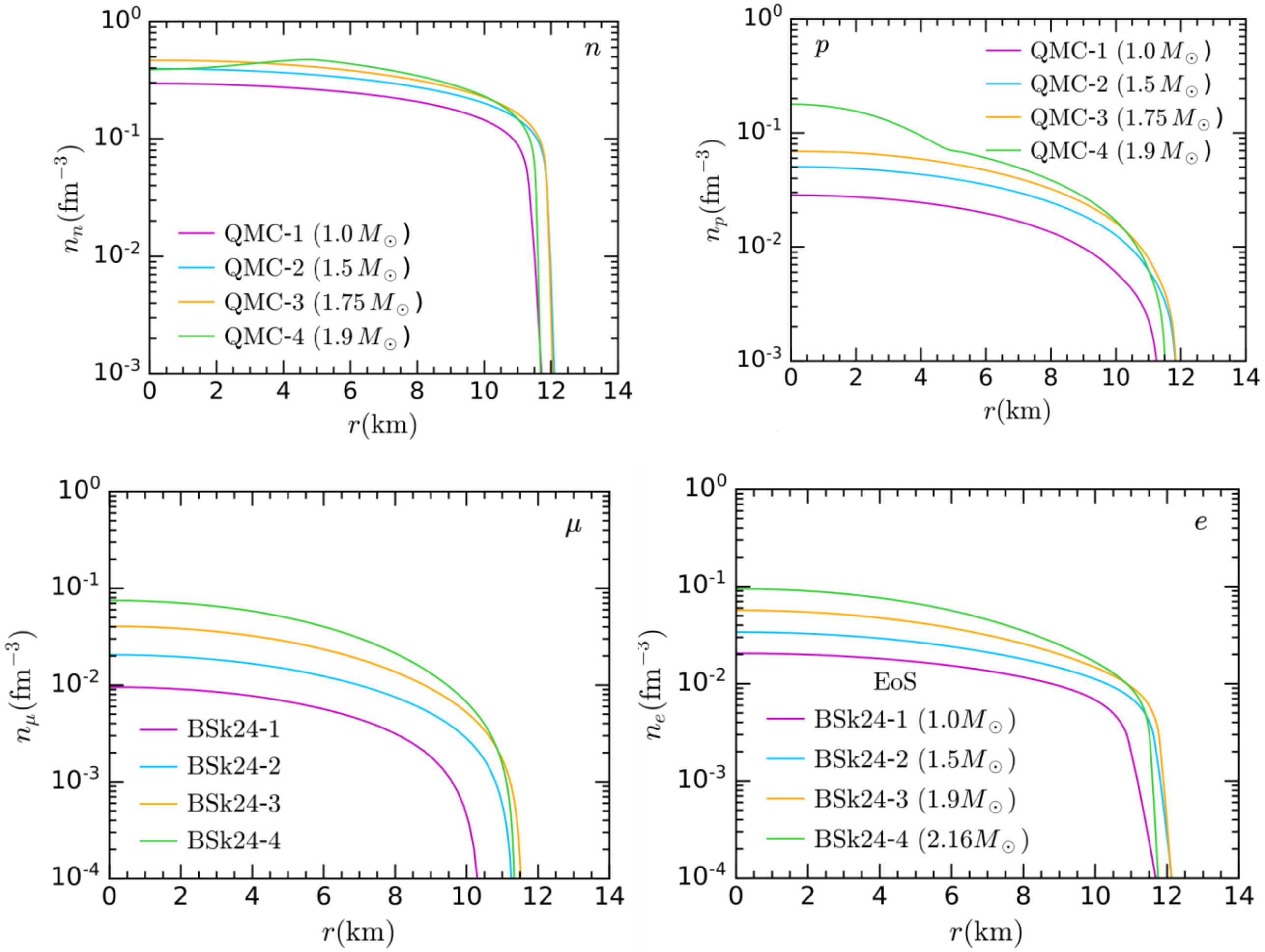
Name	Operator	Coupling	$\lvert\overline{M}\rvert^2(s,t)$
D1	$\bar{\chi}\chi\,\bar{\ell}\ell$	y_ℓ/Λ^2	$\frac{y_\ell^2}{\Lambda^4}\frac{\left(4m_\chi^2-t\right)\left(4m_\chi^2-\mu^2 t\right)}{\mu^2}$
D2	$\bar{\chi}\gamma^5\chi\,\bar{\ell}\ell$	iy_ℓ/Λ^2	$\frac{y_\ell^2}{\Lambda^4}\frac{t\left(\mu^2 t-4m_\chi^2\right)}{\mu^2}$
D3	$\bar{\chi}\chi\,\bar{\ell}\gamma^5\ell$	iy_ℓ/Λ^2	$\frac{y_\ell^2}{\Lambda^4}t\left(t-4m_\chi^2\right)$
D4	$\bar{\chi}\gamma^5\chi\,\bar{\ell}\gamma^5\ell$	y_ℓ/Λ^2	$\frac{y_\ell^2}{\Lambda^4}t^2$
D5	$\bar{\chi}\gamma_\mu\chi\,\bar{\ell}\gamma^\mu\ell$	$1/\Lambda^2$	$2\frac{1}{\Lambda^4}\frac{2\left(\mu^2+1\right)^2 m_\chi^4-4\left(\mu^2+1\right)\mu^2 s m_\chi^2+\mu^4\left(2s^2+2st+t^2\right)}{\mu^4}$
D6	$\bar{\chi}\gamma_\mu\gamma^5\chi\,\bar{\ell}\gamma^\mu\ell$	$1/\Lambda^2$	$2\frac{1}{\Lambda^4}\frac{2\left(\mu^2-1\right)^2 m_\chi^4-4\mu^2 m_\chi^2\left(\mu^2 s+s+\mu^2 t\right)+\mu^4\left(2s^2+2st+t^2\right)}{\mu^4}$
D7	$\bar{\chi}\gamma_\mu\chi\,\bar{\ell}\gamma^\mu\gamma^5\ell$	$1/\Lambda^2$	$2\frac{1}{\Lambda^4}\frac{2\left(\mu^2-1\right)^2 m_\chi^4-4\mu^2 m_\chi^2\left(\mu^2 s+s+t\right)+\mu^4\left(2s^2+2st+t^2\right)}{\mu^4}$
D8	$\bar{\chi}\gamma_\mu\gamma^5\chi\,\bar{\ell}\gamma^\mu\gamma^5\ell$	$1/\Lambda^2$	$2\frac{1}{\Lambda^4}\frac{2\left(\mu^4+10\mu^2+1\right)m_\chi^4-4\left(\mu^2+1\right)\mu^2 m_\chi^2(s+t)+\mu^4\left(2s^2+2st+t^2\right)}{\mu^4}$
D9	$\bar{\chi}\sigma_{\mu\nu}\chi\,\bar{\ell}\sigma^{\mu\nu}\ell$	$1/\Lambda^2$	$8\frac{1}{\Lambda^4}\frac{4\left(\mu^4+4\mu^2+1\right)m_\chi^4-2\left(\mu^2+1\right)\mu^2 m_\chi^2(4s+t)+\mu^4(2s+t)^2}{\mu^4}$
D10	$\bar{\chi}\sigma_{\mu\nu}\gamma^5\chi\,\bar{\ell}\sigma^{\mu\nu}\ell$	i/Λ^2	$8\frac{1}{\Lambda^4}\frac{4\left(\mu^2-1\right)^2 m_\chi^4-2\left(\mu^2+1\right)\mu^2 m_\chi^2(4s+t)+\mu^4(2s+t)^2}{\mu^4}$

Calculating C in NS (degenerate targets)

Sun	**NS**
Newtonian gravity	GR
Sun structure from Standard Solar Model	NS structure from EOS
Non-relativistic kinematics	Relativistic kinematics
Atomic Nuclei Targets	Baryon and Lepton targets
Non-relativistic matrix element	Relativistic matrix element
MB distribution for targets	FD distribution for targets
Capture probability $\neq 1$	Capture probability = $1*$
Star opacity	Star opacity
MS requires MC approach	MS can be treated (semi)analytically
Targets have FF	Targets have FF
Fixed Target mass	Density-dependent Target mass

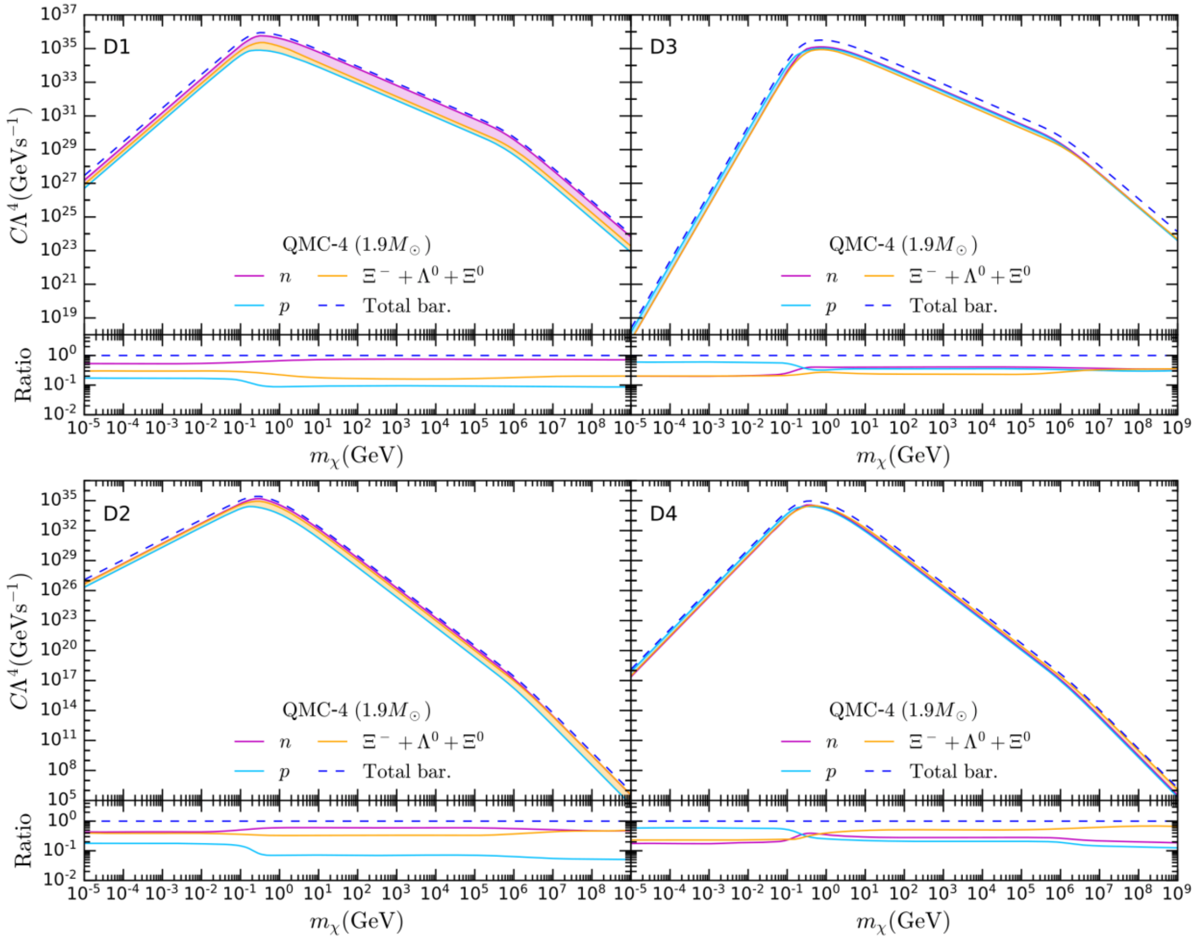
NS Structure





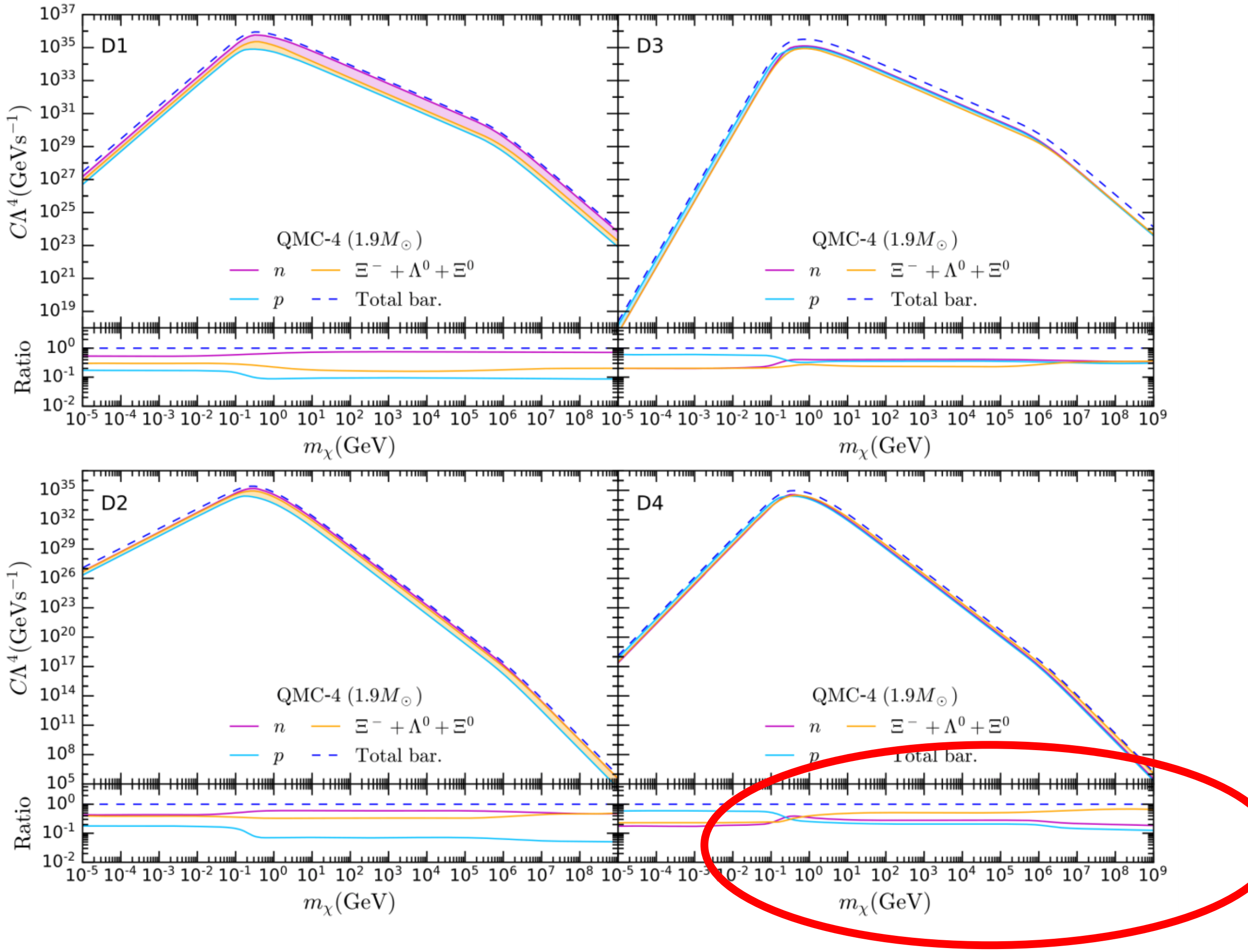
Exotic Hadrons

- Exotic hadrons, that can arise in heavy NS can give the leading contribution to Capture for some operators



Exotic Hadrons

- Exotic hadrons, that can arise in heavy NS can give the leading contribution to Capture for some operators

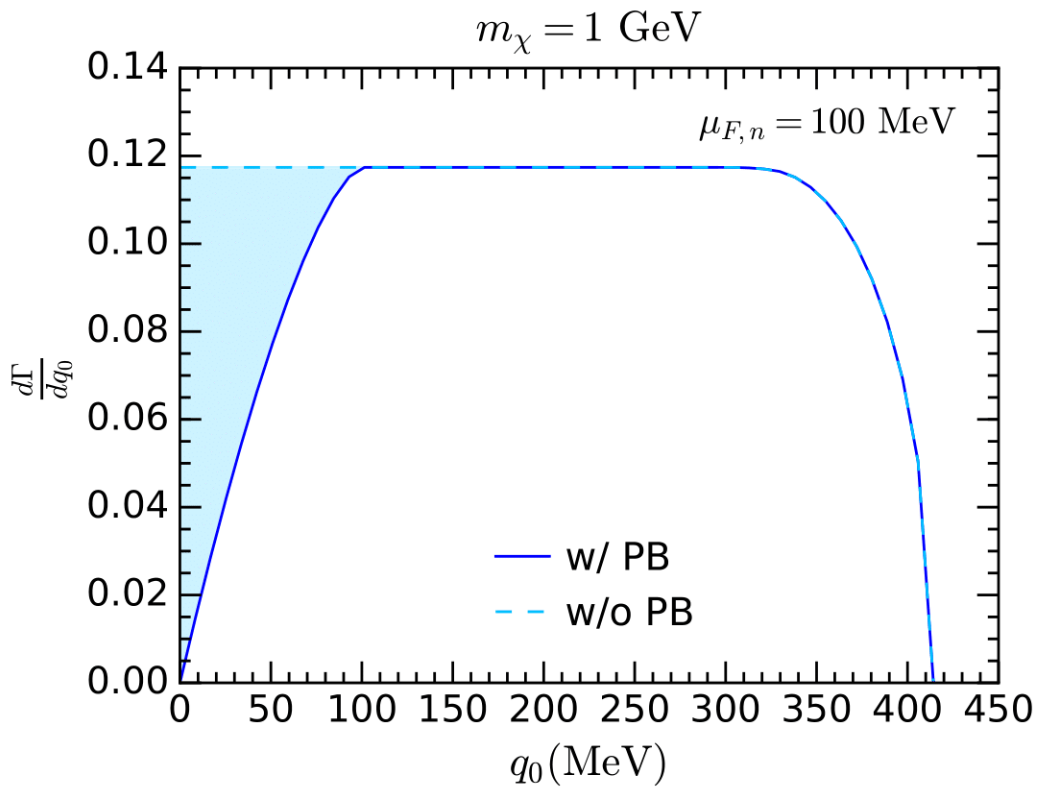


Calculating C in NS (degenerate targets)

Sun	**NS**
Newtonian gravity	GR
Sun structure from Standard Solar Model	NS structure from EOS
Non-relativistic kinematics	Relativistic kinematics
Atomic Nuclei Targets	Baryon and Lepton targets
Non-relativistic matrix element	Relativistic matrix element
MB distribution for targets	FD distribution for targets
Capture probability $\neq 1$	Capture probability = $1*$
Star opacity	Star opacity
MS requires MC approach	MS can be treated (semi)analytically
Targets have FF	Targets have FF
Fixed Target mass	Density-dependent Target mass

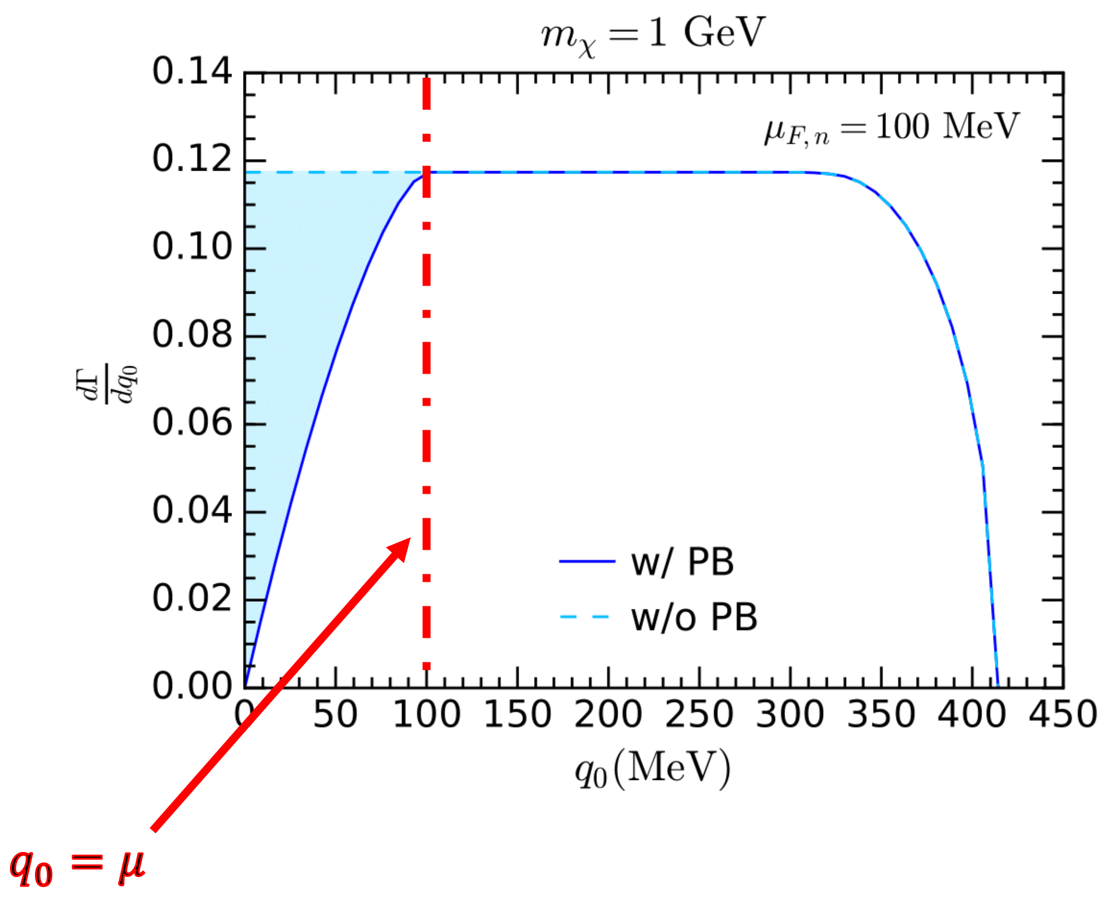
FD Distribution – Pauli Blocking

- Low temperature: distribution similar to theta function
- Levels with $E < \mu$ are populated, $E > \mu$ are empty



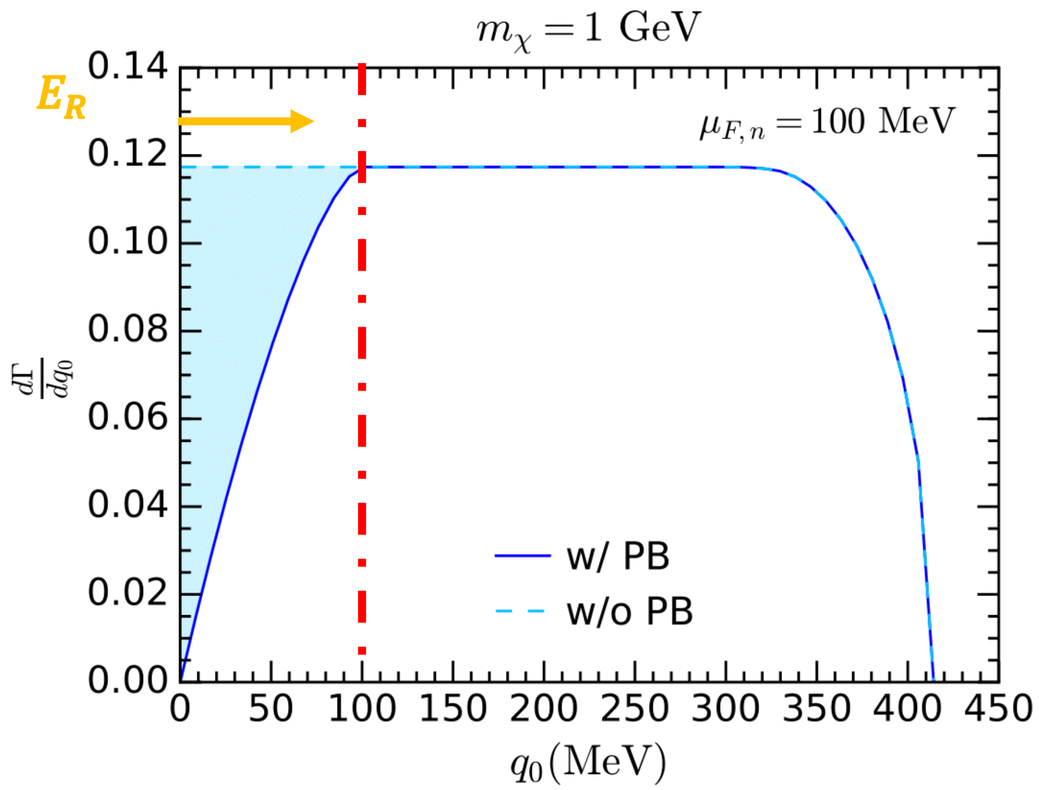
FD Distribution – Pauli Blocking

- Low temperature: distribution similar to theta function
- Levels with $E < \mu$ are populated, $E > \mu$ are empty



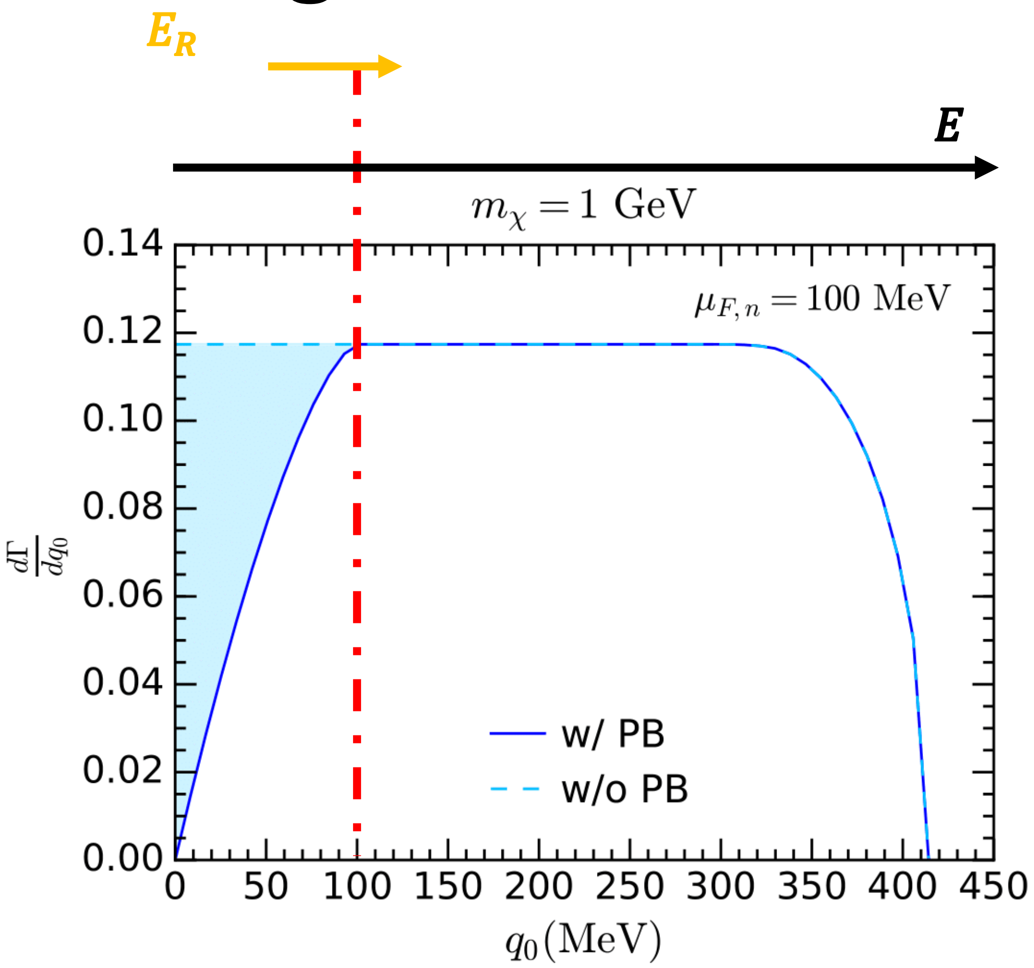
FD Distribution – Pauli Blocking

- Low temperature: distribution similar to theta function
- Levels with $E < \mu$ are populated, $E > \mu$ are empty



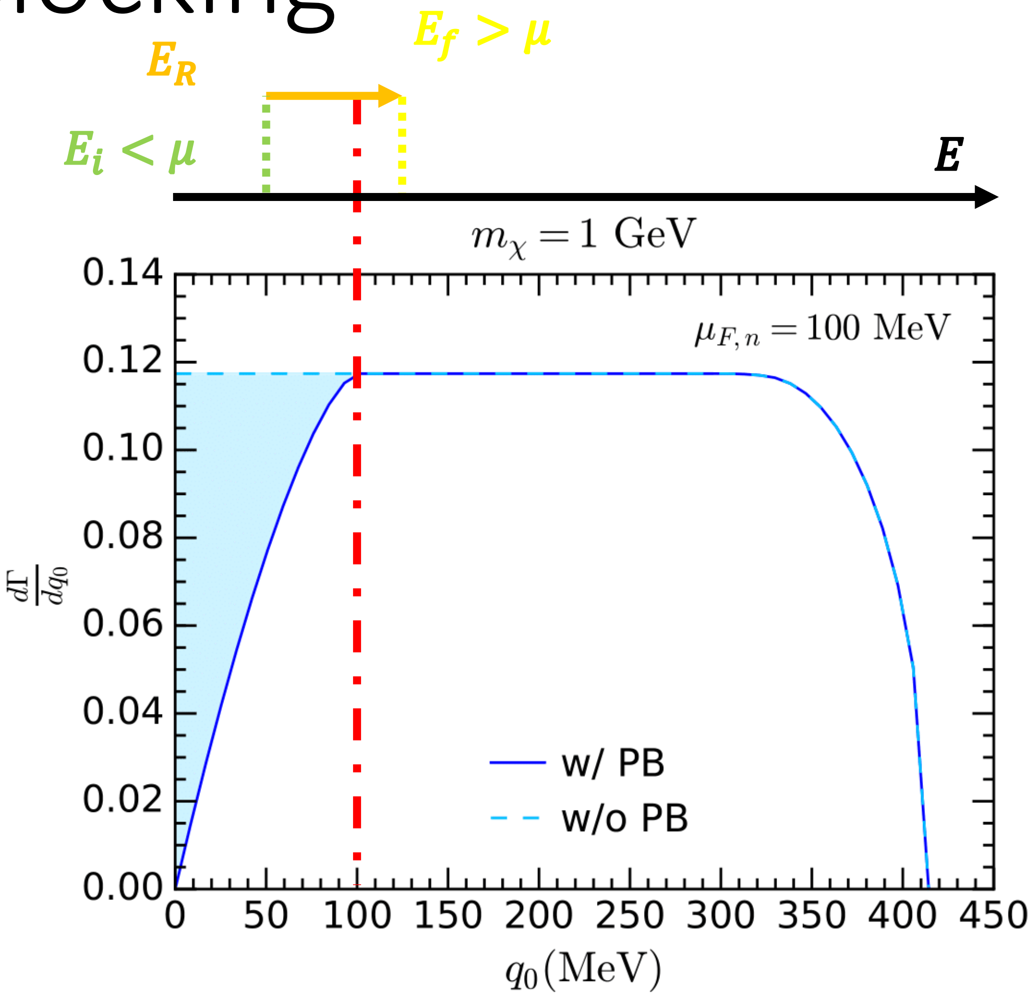
FD Distribution – Pauli Blocking

- Low temperature: distribution similar to theta function
- Levels with $E < \mu$ are populated, $E > \mu$ are empty



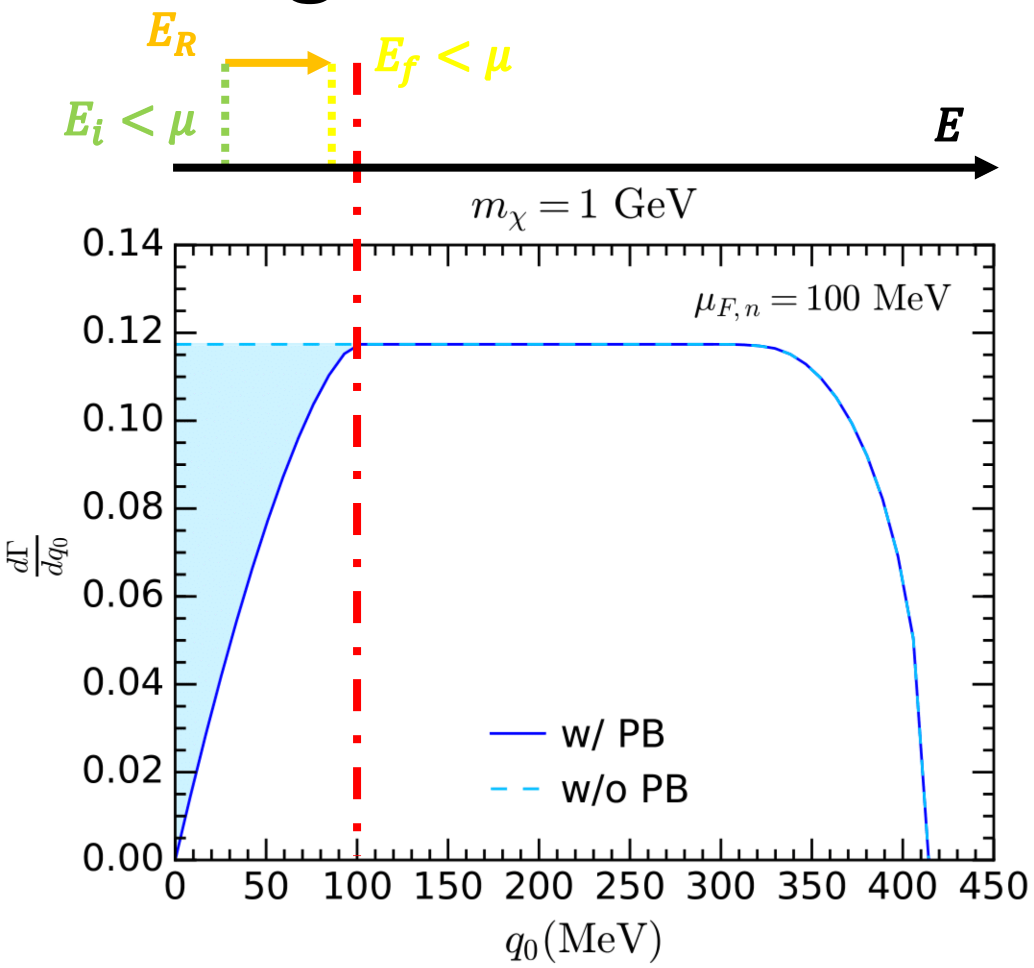
FD Distribution – Pauli Blocking

- Low temperature: distribution similar to theta function
- Levels with $E < \mu$ are populated, $E > \mu$ are empty
- Final state needs to be free, so $E_f > \mu$



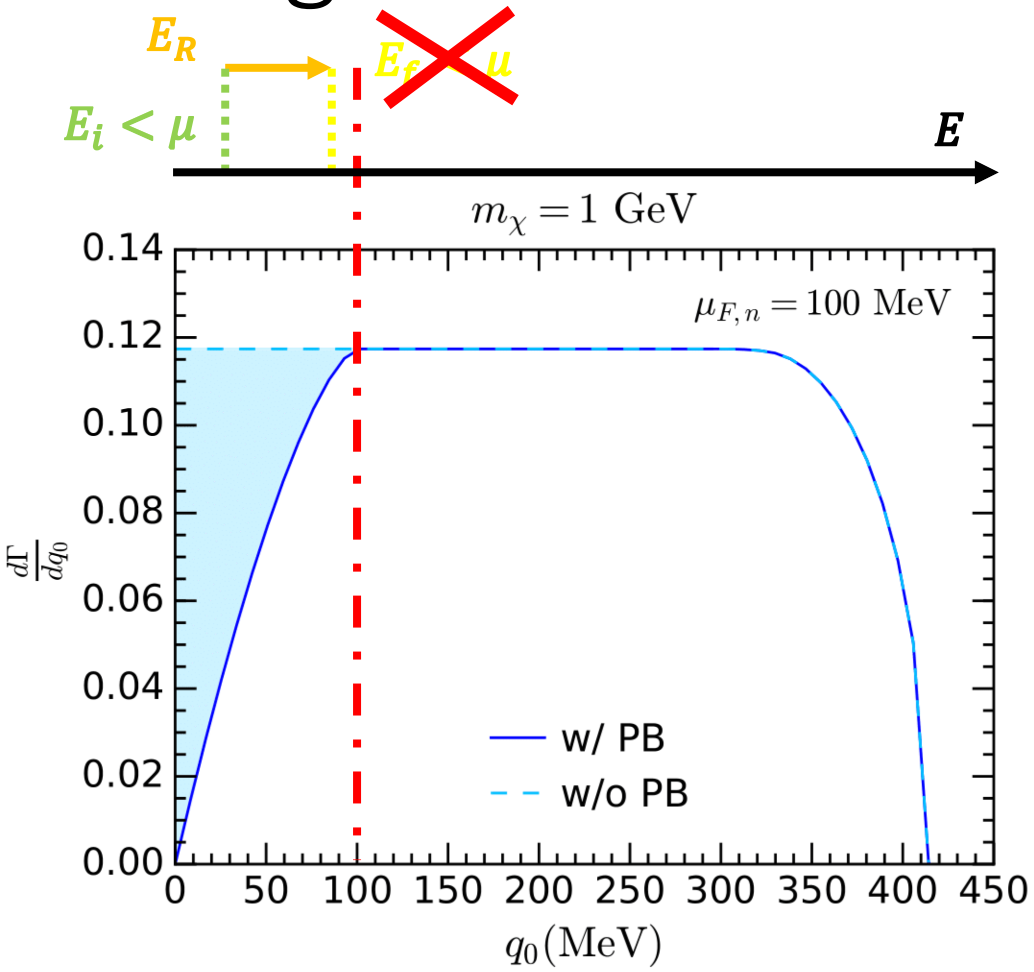
FD Distribution – Pauli Blocking

- Low temperature: distribution similar to theta function
- Levels with $E < \mu$ are populated, $E > \mu$ are empty
- Final state needs to be free, so $E_f > \mu$



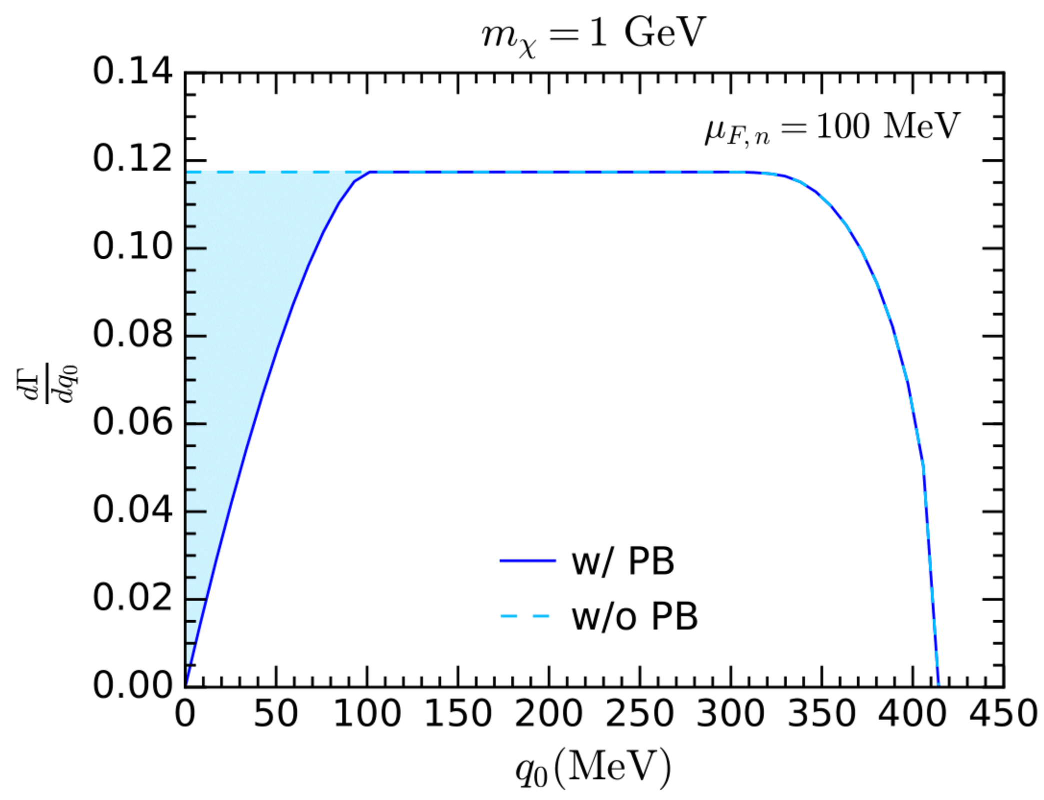
FD Distribution – Pauli Blocking

- Low temperature: distribution similar to theta function
- Levels with $E < \mu$ are populated, $E > \mu$ are empty
- Final state needs to be free, so $E_f > \mu$
- For each final state, not all possible kinematically allowed values of E_R are accessible



FD Distribution – Pauli Blocking

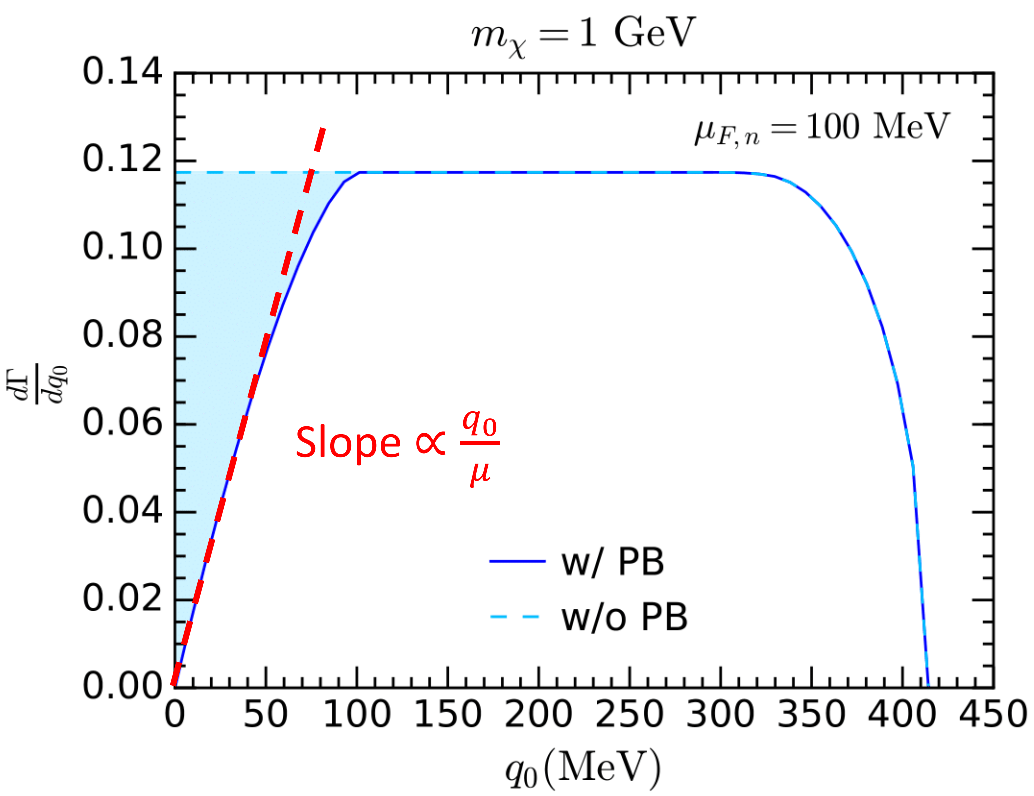
- For $E_R = q_0 \ll \mu$ one need the initial state to have energy
$$E_i \geq \mu - q_0$$
- So it needs to be very close to the Fermi surface. This is a tiny fraction of the total states,
$$\sim \frac{q_0}{\mu}$$



FD Distribution – Pauli Blocking

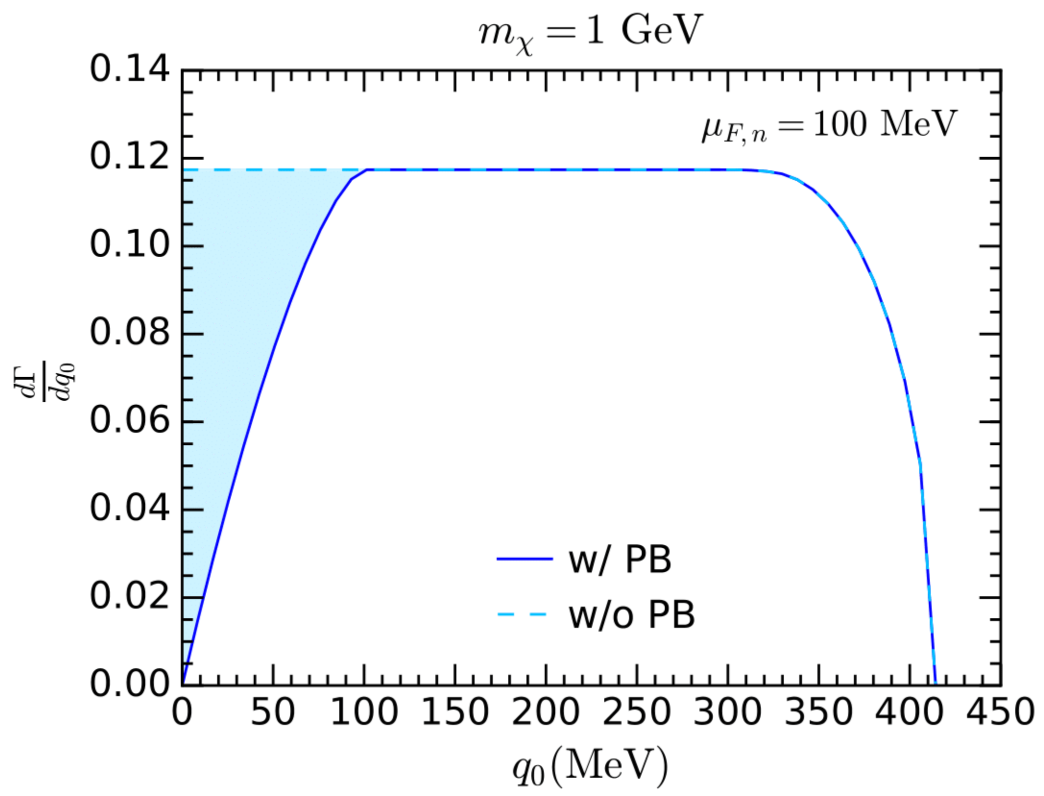
- For $E_R = q_0 \ll \mu$ one need the initial state to have energy
$$E_i = \mu - q_0$$
- So it needs to be very close to the Fermi surface. This is a tiny fraction of the total states,
$$\sim \frac{q_0}{\mu}$$
- At low q_0, rate gets suppressed by
$$\frac{q_0}{\mu}$$

Where $q_{0,MAX} \propto m_\chi$



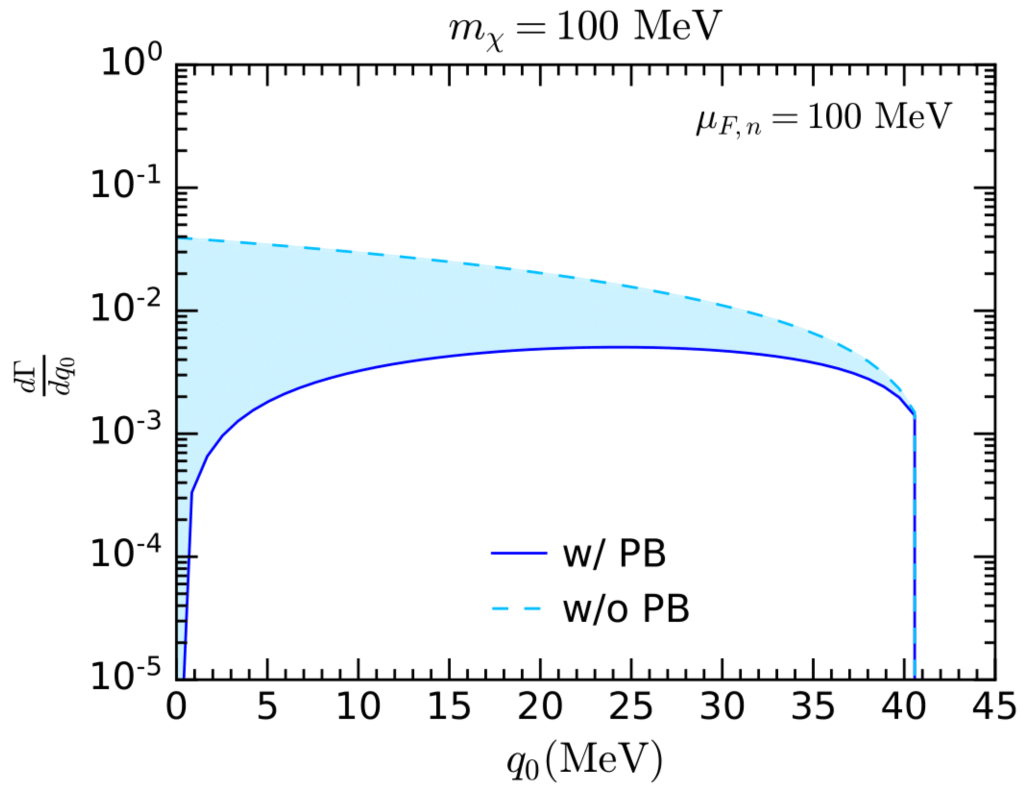
FD Distribution – Pauli Blocking

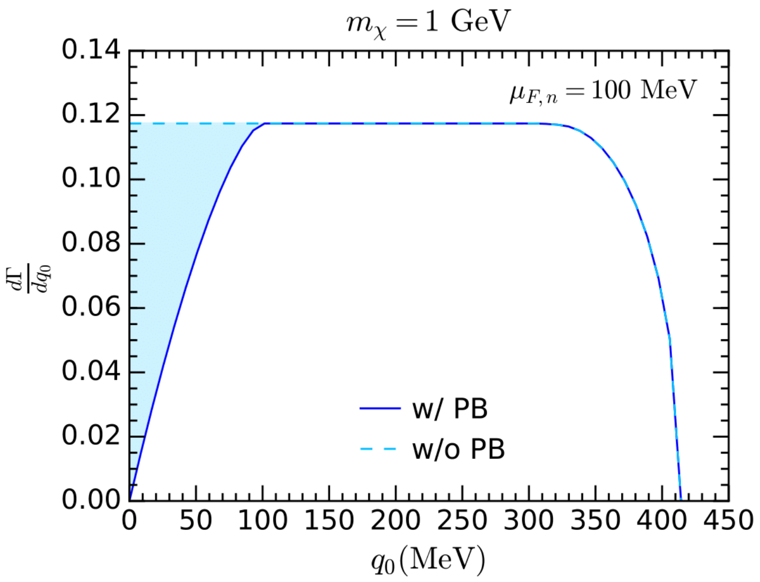
- Recoil energy grows with DM mass
- Lowering the DM mass lowers E_R
- Rates increasingly suppressed at low mass



FD Distribution – Pauli Blocking

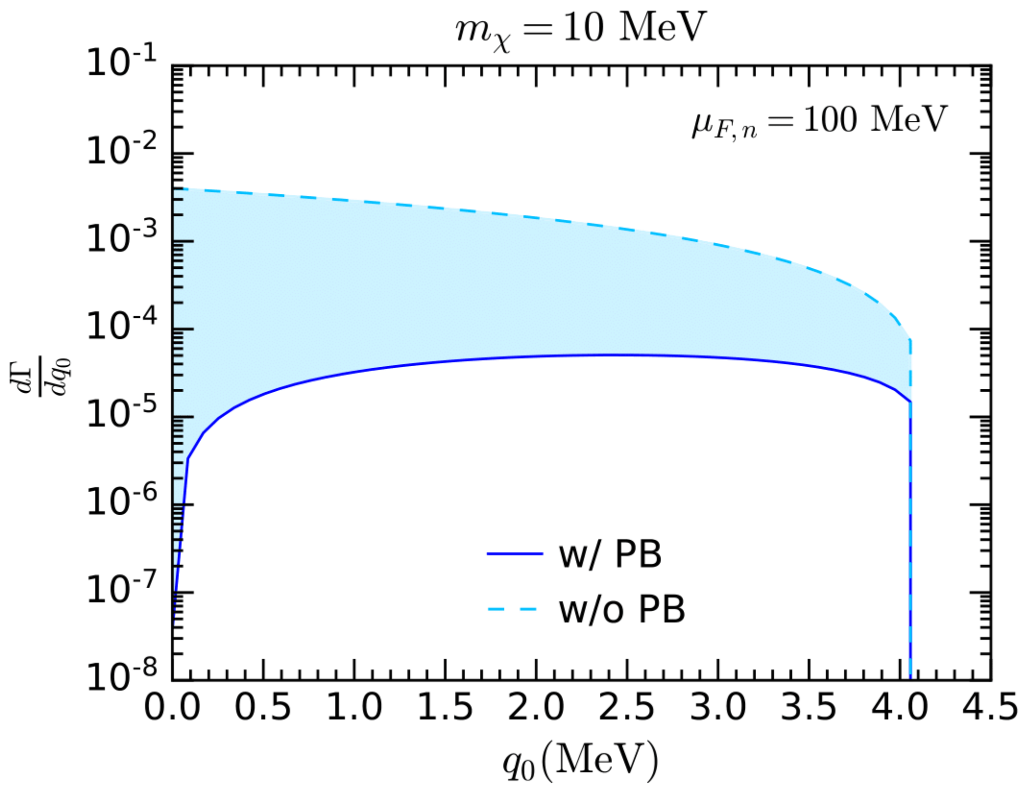
- Recoil energy grows with DM mass
- Lowering the DM mass lowers E_R
- Rates increasingly suppressed at low mass

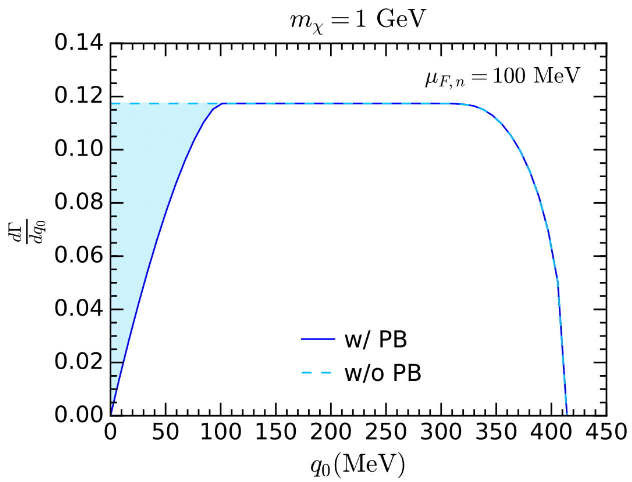


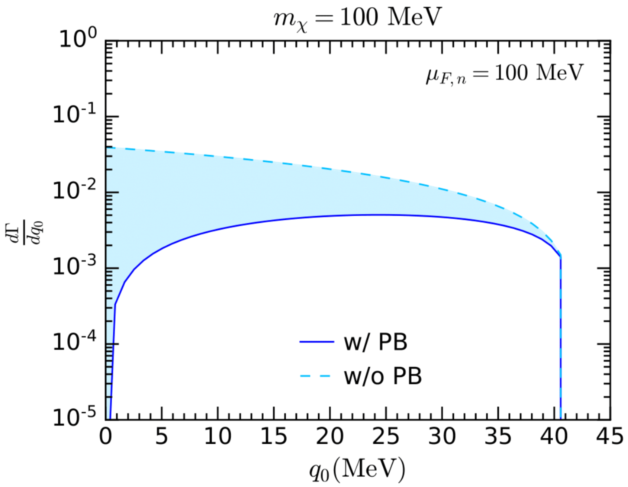


FD Distribution – Pauli Blocking

- Recoil energy grows with DM mass
- Lowering the DM mass lowers E_R
- Rates increasingly suppressed at low mass







Calculating C in NS (degenerate targets)

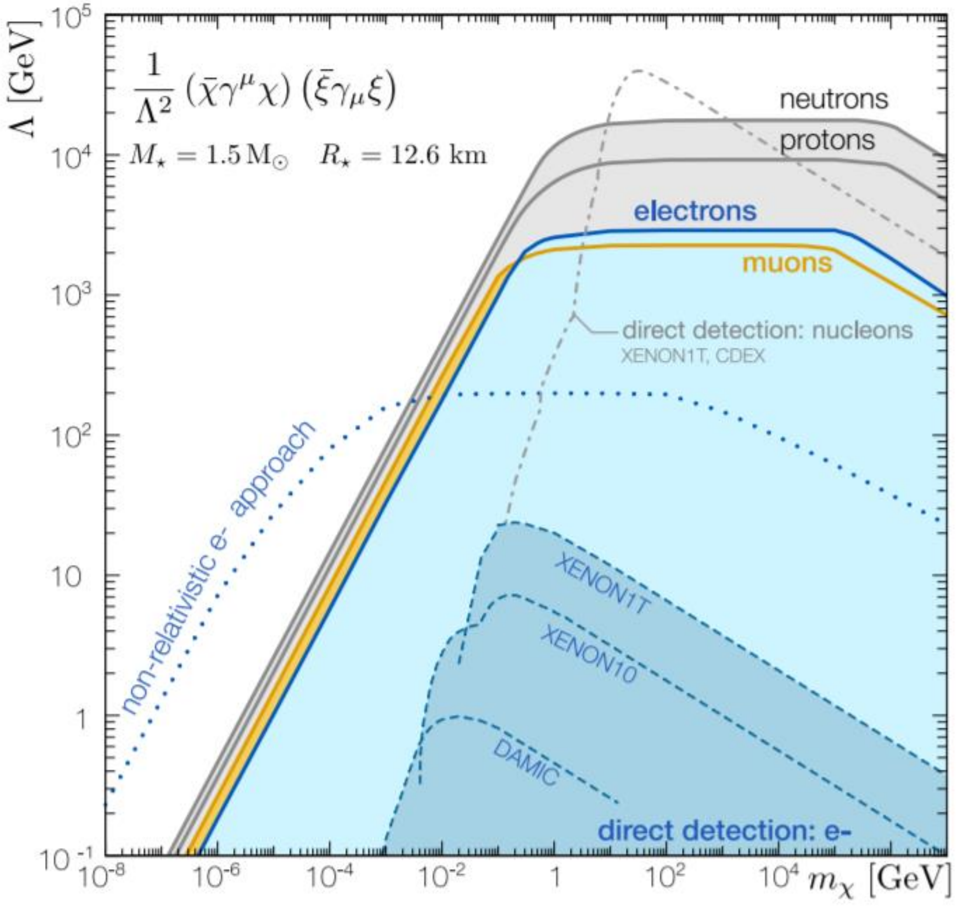
Sun	**NS**
Newtonian gravity	GR
Sun structure from Standard Solar Model	NS structure from EOS
Non-relativistic kinematics	Relativistic kinematics
Atomic Nuclei Targets	Baryon and Lepton targets
Non-relativistic matrix element	Relativistic matrix element
MB distribution for targets	FD distribution for targets
Capture probability $\neq 1$	Capture probability = $1*$
Star opacity	Star opacity
MS requires MC approach	MS can be treated (semi)analytically
Targets have FF	Targets have FF
Fixed Target mass	Density-dependent Target mass

Relativistic kinematics

- More important for more "relativistic" targets
- Larger μ, smaller mass make target more relativistic
- Largest effect for electrons
- Note:

$$\sigma \propto \frac{1}{\Lambda^4}$$

1911.13293



Relativistic kinematics

1911.13293

- More important for more "relativistic" targets
- Larger μ, smaller mass make target more relativistic
- Largest effect for electrons
- Note:

$$\sigma \propto \frac{1}{\Lambda^4}$$