

Implication of High Quality QCD axion in Dual Description

Burgess, Choi, Quevedo [2301.00549]
Choi, Leedom[2307.08733]

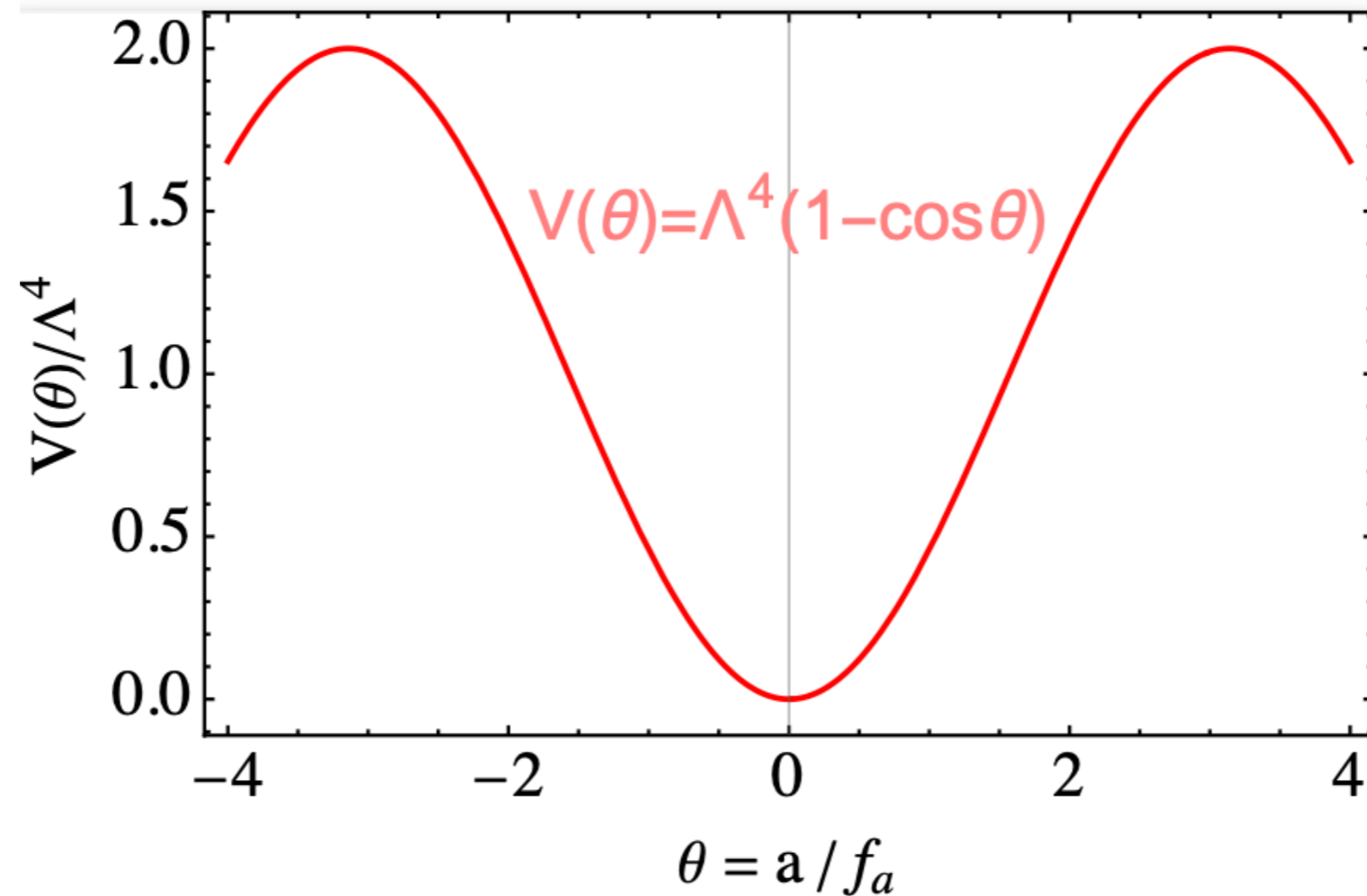
Gongjun Choi (U of Minnesota)

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Outline

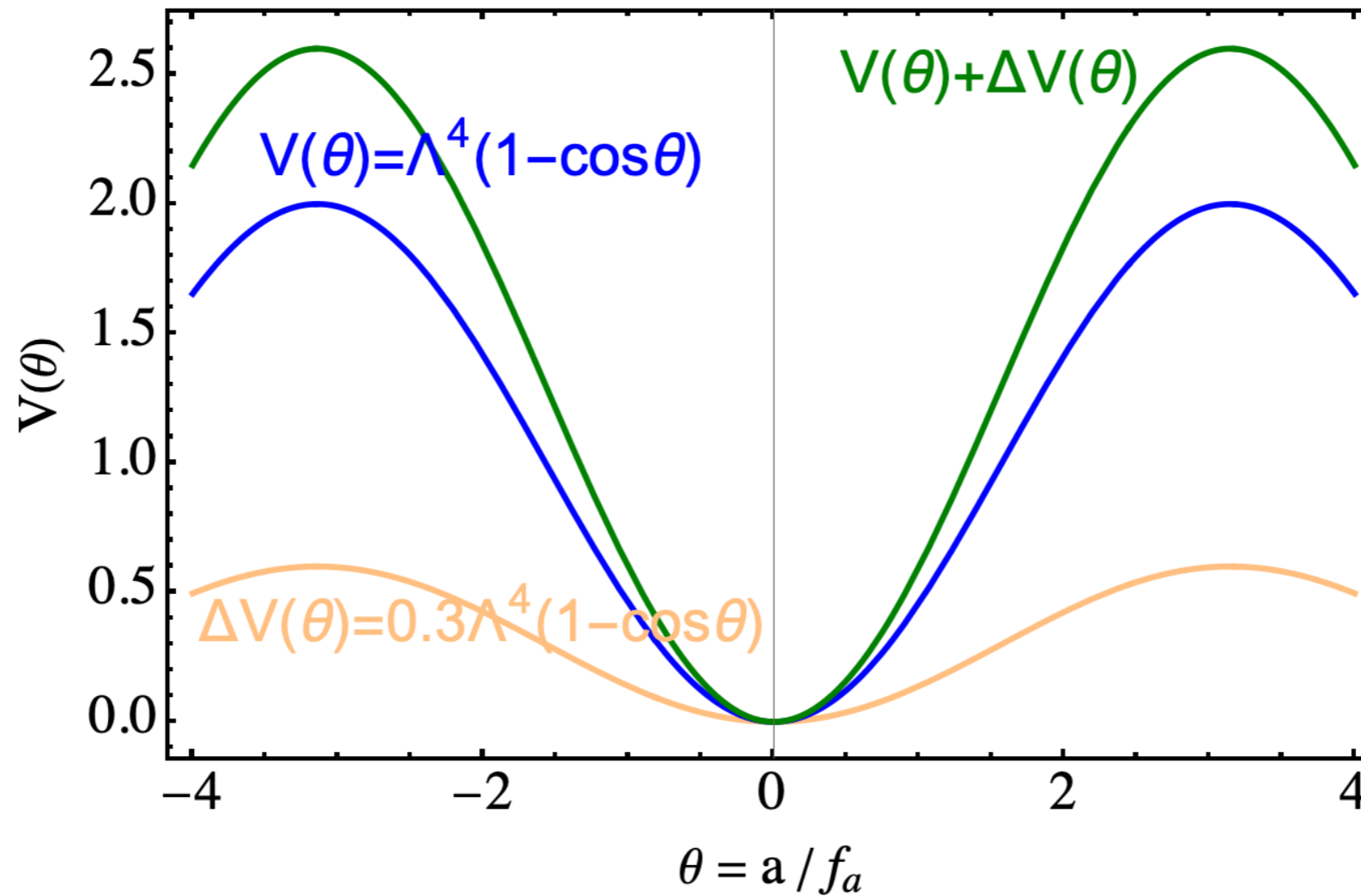
- Axion Quality Problem in PQ picture
- QCD axion in dual description
- QCD Axion Quality in dual description
- Implication of High Quality QCD axion in dual description

QCD axion quality problem in PQ picture



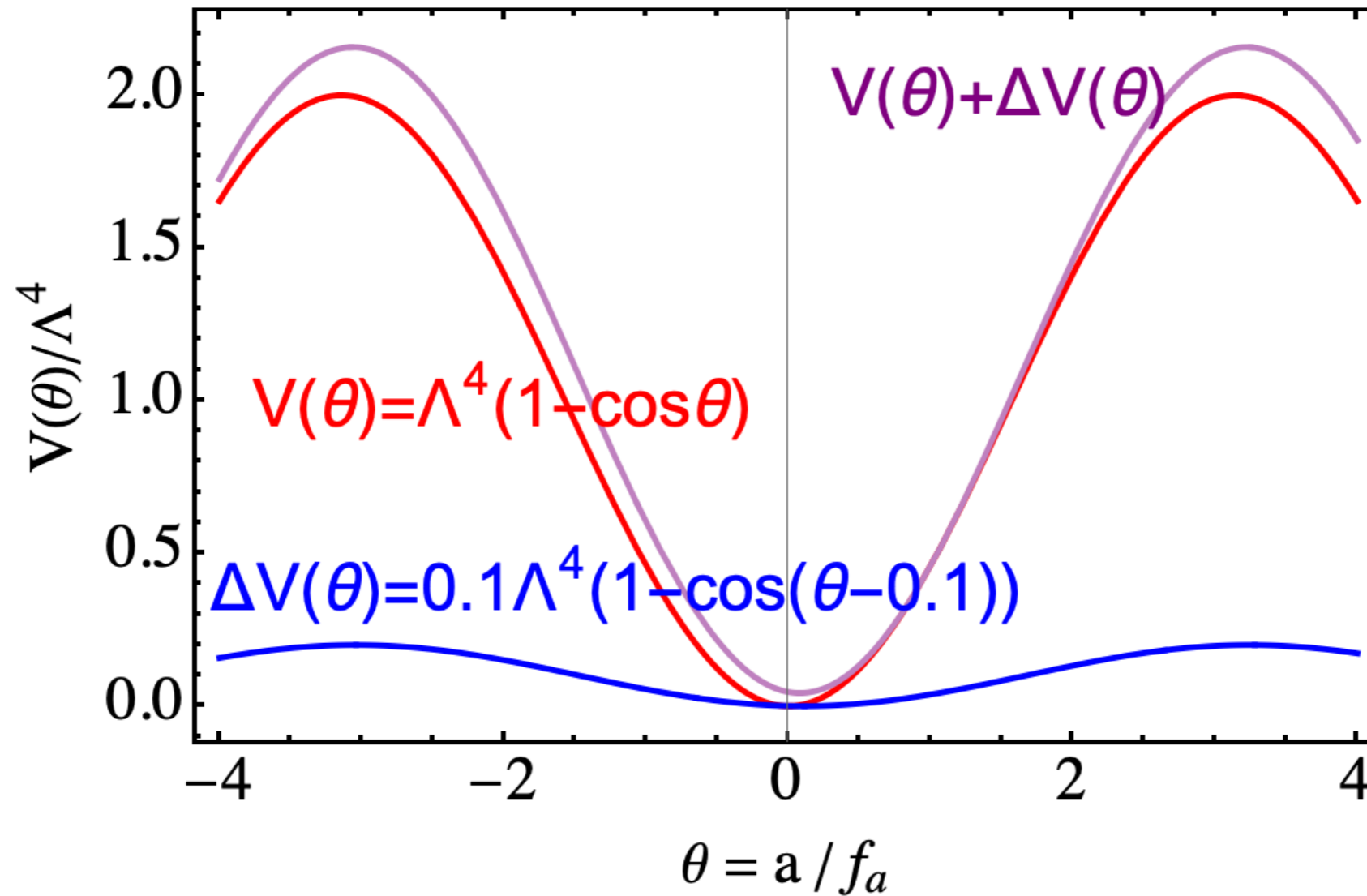
Good sol to strong CP problem iff $\theta_{\min} < 10^{-10}$

QCD axion quality problem in PQ picture



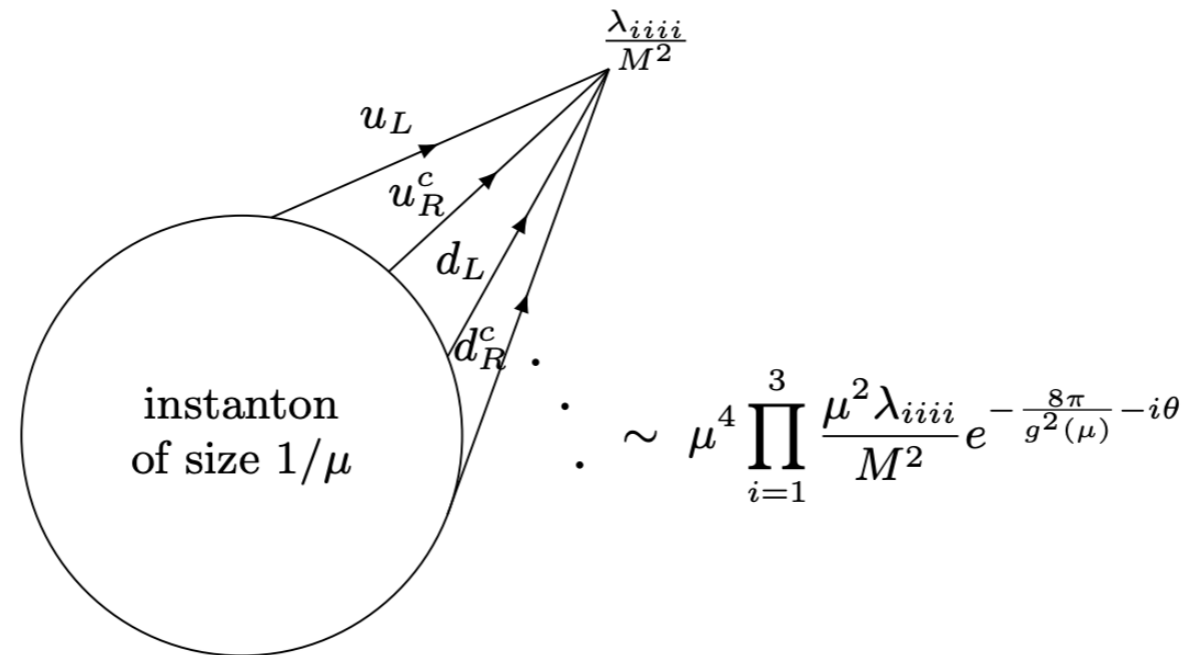
A height of ΔV does not matter if $\Delta\theta_{\min} < 10^{-10}$

QCD axion quality problem in PQ picture



If $\theta_{\min} \sim \Delta V / V > 10^{-10}$, no more meaning in axion

- Coefficient of an operator closing fermion zero modes is generally complex



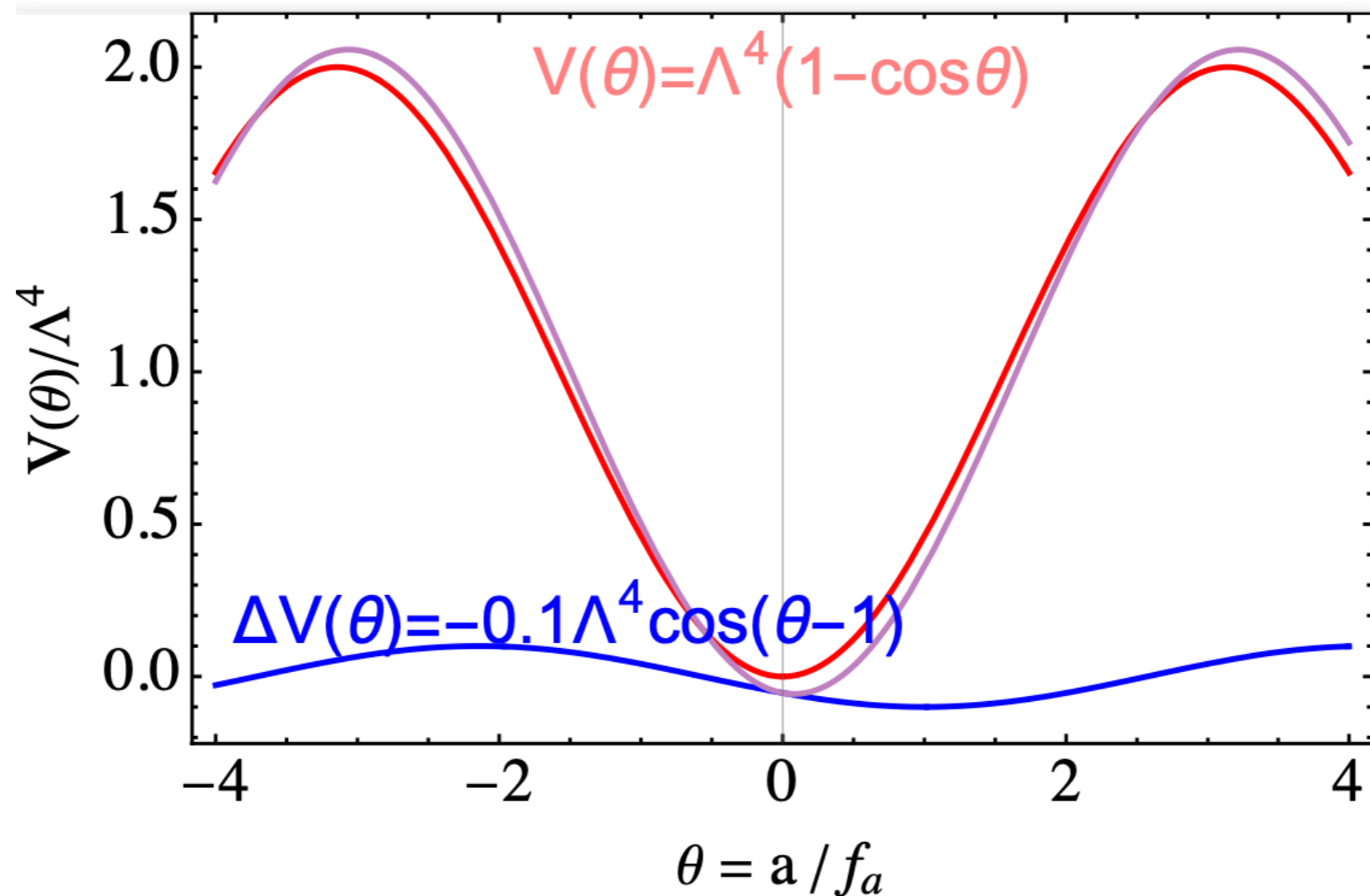
- Coefficient of a global symmetry breaking non-renormalizable operator is complex

Kamionkowski, March-Russell 92

Barr, Seckel 92

$$c \frac{\Phi^5}{M_P} + c^* \frac{\Phi^\dagger{}^5}{M_P}$$

QCD axion quality problem in PQ picture



How to suppress $\theta_{\min} \sim \Delta V / V$?

Dual formulation of QCD axion

- Below Λ_{QCD} , there is a counterpart of CS 3-form, $C_{\mu\nu\lambda}$

$$\mathcal{L}(C) \supset -\frac{1}{2 \cdot 4!} H_{\mu\nu\lambda\rho} H^{\mu\nu\lambda\rho} - \frac{1}{4!} \epsilon^{\mu\nu\lambda\rho} \theta_{\text{QCD}} \Lambda_{\text{QCD}}^2 H_{\mu\nu\lambda\rho}$$

where $H = dC = (F \wedge F) / \Lambda_{\text{QCD}}^2$.

- Charge density

$$q(x) \equiv (1/16\pi^2) \text{Tr}[F_{\mu\nu} \tilde{F}^{\mu\nu}] \quad \langle q(x) \rangle \propto \sin \theta$$

- e.o.m of “massless” $C_{\mu\nu\lambda}$ gives
$$H_{\mu\nu\lambda\rho} = H_0 \epsilon_{\mu\nu\lambda\rho}$$
 - non-zero four form electric field
 - $\theta_{\text{QCD}} \neq 0$

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where $H = dC = (F \wedge F) / \Lambda_{\text{QCD}}^2$.

$$\langle q(x) \rangle = 0 \longrightarrow \theta = 0$$

- Charge density

$$q(x) \equiv (1/16\pi^2) \text{Tr}[F_{\mu\nu} \tilde{F}^{\mu\nu}]$$

$$\langle q(x) \rangle \propto \sin \theta$$

- e.o.m of “massless” $C_{\mu\nu\lambda}$ gives

$$H_{\mu\nu\lambda\rho} = H_0 \epsilon_{\mu\nu\lambda\rho}$$

→ non-zero four form electric field

→ $\theta_{\text{QCD}} \neq 0$

Dual formulation of QCD axion

- $\theta=0$ corresponds to Higgs phase of 3-form gauge theory
→ strong CP problem is translated to how to make CS 3-form massive

- Introduce 2-form field $B_{\mu\nu}$ giving the gauge invariant mass term $\propto (dB - mC)^2$

Dvali 05

- 2-form enjoys the gauged shift symmetry

$$B_{\mu\nu} \rightarrow B_{\mu\nu} + \Omega_{\mu\nu} \qquad \Omega_{\mu\nu} = A_{[\mu}^a \partial_{\nu]} \omega^a$$

- Dualizing 2-form gives the axion theory.

Dual formulation of QCD axion

- After introducing 2-form

$$\begin{aligned} \mathcal{L}(C, B) \supset & -\frac{1}{2 \cdot 4!} H_{\mu\nu\lambda\rho} H^{\mu\nu\lambda\rho} - \frac{1}{2 \cdot 3!} G_{\mu\nu\lambda} G^{\mu\nu\lambda} \\ & - \frac{1}{3!} \frac{\epsilon^{\mu\nu\lambda\rho}}{f_a} G_{\mu\nu\lambda} J_\rho - \frac{1}{4!} \epsilon^{\mu\nu\lambda\rho} \theta_{\text{QCD}} \Lambda_{\text{QCD}}^2 H_{\mu\nu\lambda\rho} \\ & - \frac{a}{3!} \epsilon^{\mu\nu\lambda\rho} \left(\partial_\mu G_{\nu\lambda\rho} - \frac{1}{4} \frac{\Lambda_{\text{QCD}}^2}{f_a} H_{\mu\nu\lambda\rho} \right) \end{aligned}$$

where $H = dC$, $G = dB + m_a C$.

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where $H = dC$, $G = dB + m_a C$.

Mixing of B and C

Dual formulation of QCD axion

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where $H = dC$, $G = dB + m_a C$.

- Impose Bianchi identity $dG - (\Lambda^2/f)X = 0$

$$X \equiv (1/4!) \epsilon^{\mu\nu\lambda\rho} H_{\mu\nu\lambda\rho}$$

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$$\mathcal{L}(C, B) \supset -\frac{1}{2 \cdot 4!} H_{\mu\nu\lambda\rho} H^{\mu\nu\lambda\rho} - \frac{1}{2 \cdot 3!} G_{\mu\nu\lambda} G^{\mu\nu\lambda} \\ - \frac{1}{3!} \frac{\epsilon^{\mu\nu\lambda\rho}}{f_a} G_{\mu\nu\lambda} J_\rho - \frac{1}{4!} \epsilon^{\mu\nu\lambda\rho} \theta_{\text{QCD}} \Lambda_{\text{QCD}}^2 H_{\mu\nu\lambda\rho} \\ - \frac{a}{3!} \epsilon^{\mu\nu\lambda\rho} \left(\partial_\mu G_{\nu\lambda\rho} - \frac{1}{4} \frac{\Lambda_{\text{QCD}}^2}{f_a} H_{\mu\nu\lambda\rho} \right)$$

where $H = dC$, $G = dB + m_a C$.

- Impose Bianchi identity $dG - (\Lambda^2/f)X = 0$
- “a” is a Lagrange multiplier
→ will be identified with the axion later.

Dual formulation of QCD axion

- Integrate out G (B) gives

$$\begin{aligned}\mathcal{L}(C, E, a) = & -\frac{1}{2}\partial_\mu a\partial^\mu a - \frac{1}{f_a}J^\mu\partial_\mu a - \frac{1}{2f_a^2}J_\mu J^\mu \\ & + (m_a a - \theta_{\text{QCD}}\Lambda_{\text{QCD}}^2)X \\ & - \frac{1}{2}X^2\end{aligned}$$

- From e.o.m of axion

$$\frac{\partial V(a)}{\partial a} = m_a X \quad X \equiv (1/4!)\epsilon^{\mu\nu\lambda\rho}H_{\mu\nu\lambda\rho}$$

- Vanishing X corresponds to extrema of V(a)
 - dynamically driven to minimum of V(a)
 - $X \sim \theta = 0$

QCD axion quality in dual description

- Another 3-form $E_{\mu\nu\lambda}$ coupled to axion

$$\begin{aligned}\mathcal{L}(C, E, B) \supset & -\frac{1}{2 \cdot 4!} \left(H_{\mu\nu\lambda\rho} H^{\mu\nu\lambda\rho} + K_{\mu\nu\lambda\rho} K^{\mu\nu\lambda\rho} \right) \\ & -\frac{1}{2 \cdot 3!} G_{\mu\nu\lambda} G^{\mu\nu\lambda} - \frac{1}{3!} \frac{\epsilon^{\mu\nu\lambda\rho}}{f_a} G_{\mu\nu\lambda} J_\rho \\ & -\frac{1}{4!} \epsilon^{\mu\nu\lambda\rho} \left(\theta_{\text{QCD}} \Lambda_{\text{QCD}}^2 H_{\mu\nu\lambda\rho} + \theta_h \Lambda_h^2 K_{\mu\nu\lambda\rho} \right)\end{aligned}$$

where $H = dC$, $K = dE$, $G = dB + m_a C + M_a E$.

- Dualizing the theory gives

$$\frac{\partial V(a)}{\partial a} = m_a X + M_a Y$$

$$X \equiv (1/4!) \epsilon^{\mu\nu\lambda\rho} H_{\mu\nu\lambda\rho}$$

$$Y \equiv (1/4!) \epsilon^{\mu\nu\lambda\rho} K_{\mu\nu\lambda\rho}$$

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$$\frac{\partial V(a)}{\partial a} = m_a X - M_a Y$$

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$$Y \equiv (1/4!) \epsilon^{\mu\nu\lambda\rho} K_{\mu\nu\lambda\rho}$$

a_{\min} **does not** correspond to $\theta \sim X=0$!

QCD axion quality in dual description

- From e.o.m of C and E

$$X = m_a a - \theta_{\text{QCD}} \Lambda_{\text{QCD}}^2, \quad Y = M_a a - \theta_h \Lambda_h^2$$

- Global min of $V(a)$

$$a_{\text{min},X} = \frac{\theta_{\text{QCD}} \Lambda_{\text{QCD}}^2}{m_a} \quad a_{\text{min},X,Y} = \frac{m_a \theta_{\text{QCD}} \Lambda_{\text{QCD}}^2 + M_a \theta_h \Lambda_h^2}{m_a^2 + M_a^2}$$

- Shift in a_{min}

$$\begin{aligned} \Delta\theta_{\text{min}} &= \frac{a_{\text{min},X,Y} - a_{\text{min},X}}{f_a} \\ &= \begin{cases} \theta_h \frac{M_a}{m_a} \left(\frac{\Lambda_h}{\Lambda_{\text{QCD}}} \right)^2, & \text{for } m_a > M_a \\ \theta_h - \theta_{\text{QCD}}, & \text{for } m_a < M_a \end{cases} \end{aligned}$$

Higgsing all the 3-forms

- How to switch-off contribution to $V(a)$ from other 3-forms than QCD one?
→ # of 2-forms = # of 3-forms
- Consider second 2-form \tilde{B} with $\tilde{G} = d\tilde{B} + M_b E$ and Bianchi identity

$$-\frac{b}{3!} \epsilon^{\mu\nu\lambda\rho} \left(\partial_\mu \tilde{G}_{\nu\lambda\rho} - \frac{1}{4} \frac{\Lambda_h^2}{f_b} K_{\mu\nu\lambda\rho} \right)$$

- Integrating out G,C and E gives

$$\frac{\partial V(a)}{\partial a} = m_a X + M_a Y \qquad \frac{\partial V(b)}{\partial b} = M_b Y$$

Higgsing all the 3-forms

- Move to new basis

$$\bar{m}_a \bar{X} = m_a X + M_a Y, \quad \bar{m}_b \bar{Y} = M_b Y$$

which gives

$$\frac{\partial V(a)}{\partial a} = \bar{m}_a \bar{X} \quad , \quad \frac{\partial V(b)}{\partial b} = \bar{m}_b \bar{Y}$$

- The correspondence btw vanishing 4-form electric field ($\sim \theta$) and a_{\min} is restored!
- Can be readily generalized to multiple 3-form cases.
- # of 2-forms = # of 3-forms guarantees $\theta \sim X = 0$

Higgsing all the 3-forms

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Multiple axions enhancing the quality of QCD axion!
- # of 2-forms = # of 3-forms guarantees $\theta \sim X = 0$

Implication: QCD axion, sub-eV ALP

- Consider (1) QCD axion couples to C and E
(2) b-axion couples to E only

- After integrating out two 2-forms,

$$\mathcal{L}(X, Y) \supset -\frac{1}{2}X^2 - \frac{1}{2}Y^2 + am_a X + (aM_a + bM_b)Y$$

which gives (after integrating out X, Y)

$$V(a, b) = \frac{1}{2}m_a^2 a^2 + \frac{1}{2}(M_a a + M_b b)^2 \quad M_a = \frac{\Lambda_h^2}{f_a} \quad M_b = \frac{\Lambda_h^2}{f_b}$$

- In the mass eigenbasis ($r = M_b/M_a = f_a/f_b$)

$$\begin{aligned} a &= -\frac{r}{\sqrt{1+r^2}}a' + \frac{1}{\sqrt{1+r^2}}b', \\ b &= \frac{1}{\sqrt{1+r^2}}a' + \frac{r}{\sqrt{1+r^2}}b'. \end{aligned} \quad (m_{a'})^2 \simeq \left(\frac{r}{\sqrt{1+r^2}}\right)^2 m_a^2, \quad (m_{b'})^2 \simeq (1+r^{-2})M_b^2$$

Implication: QCD axion, sub-eV ALP

- From QCD axion- $U(1)_{\text{em}}$ coupling

$$\mathcal{L} \supset \left(\frac{r}{\sqrt{1+r^2}} a' - \frac{1}{\sqrt{1+r^2}} b' \right) \frac{g}{8} \epsilon^{\mu\nu\rho\lambda} F_{\mu\nu} F_{\rho\lambda}$$

- Define

$$g_{a\gamma\gamma} \equiv -\frac{gr}{\sqrt{1+r^2}}, \quad g_{b\gamma\gamma} \equiv \frac{g}{\sqrt{1+r^2}}$$

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$$g_{a\gamma\gamma} \equiv -\frac{gr}{\sqrt{1+r^2}}, \quad g_{b\gamma\gamma} \equiv \frac{g}{\sqrt{1+r^2}}$$

Identified with the known QCD axion-photon coupling

$$g_{a\gamma\gamma} \sim \alpha_{\text{em}} / (2\pi F_a)$$

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- Define

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Induced coupling of b-axion to photon

Implication: QCD axion, sub-eV ALP

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- Define

$$g_{a\gamma\gamma} \equiv -\frac{gr}{\sqrt{1+r^2}} \quad g_{b\gamma\gamma} \equiv \frac{g}{\sqrt{1+r^2}}$$

For $r \ll 1$, $g_{b\gamma\gamma} \gg g_{a\gamma\gamma}$

Implication: QCD axion, sub-eV ALP

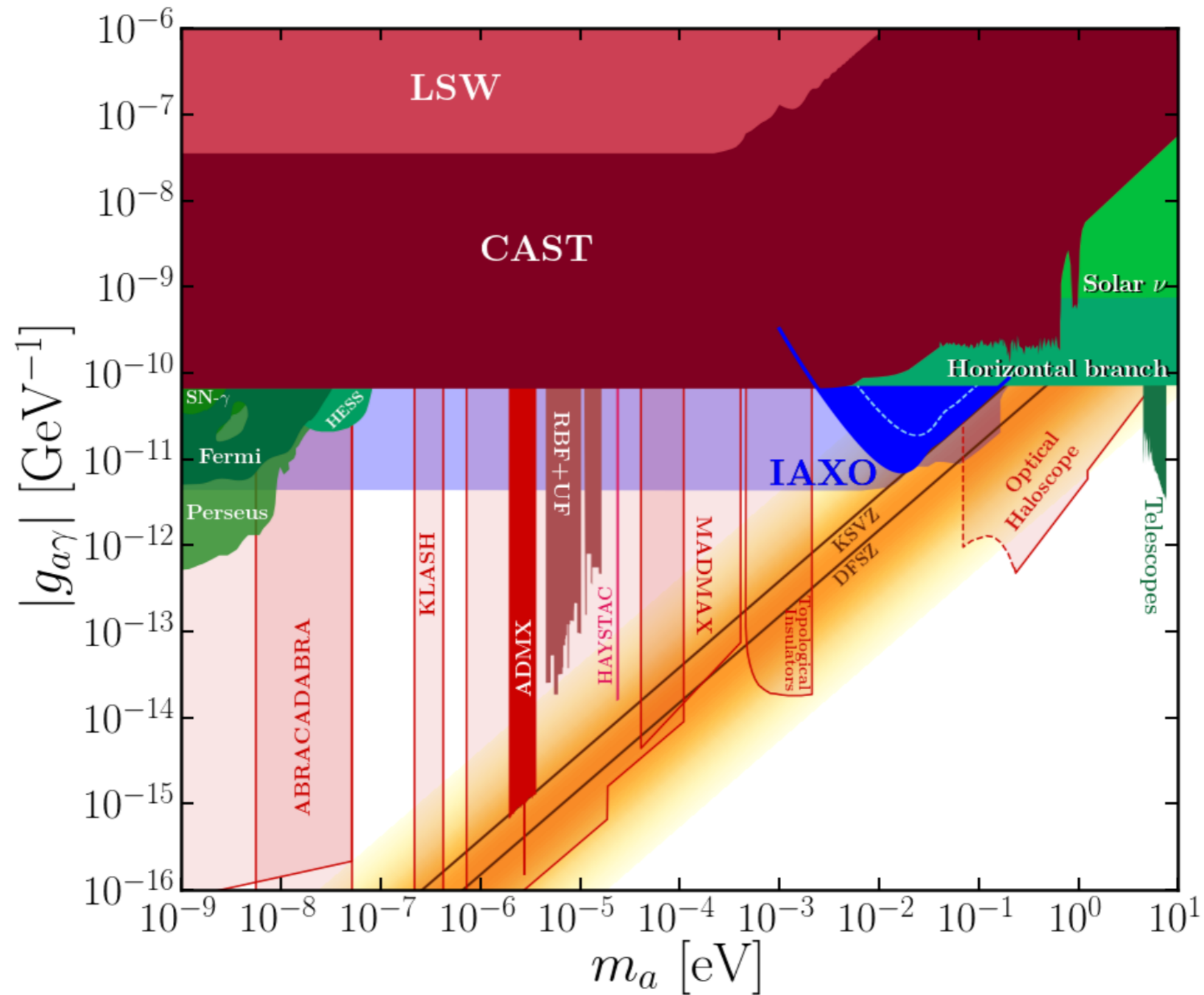
- For $r \ll 1$, $r = f_a/f_b = f_a/F_a$
- Mass and coupling

$$m_{b'}/m_{a'} \simeq r^{-1} (\Lambda_h/\Lambda_{\text{QCD}})^2 \text{ and } g_{b\gamma\gamma} = g_{a\gamma\gamma} r^{-1}$$

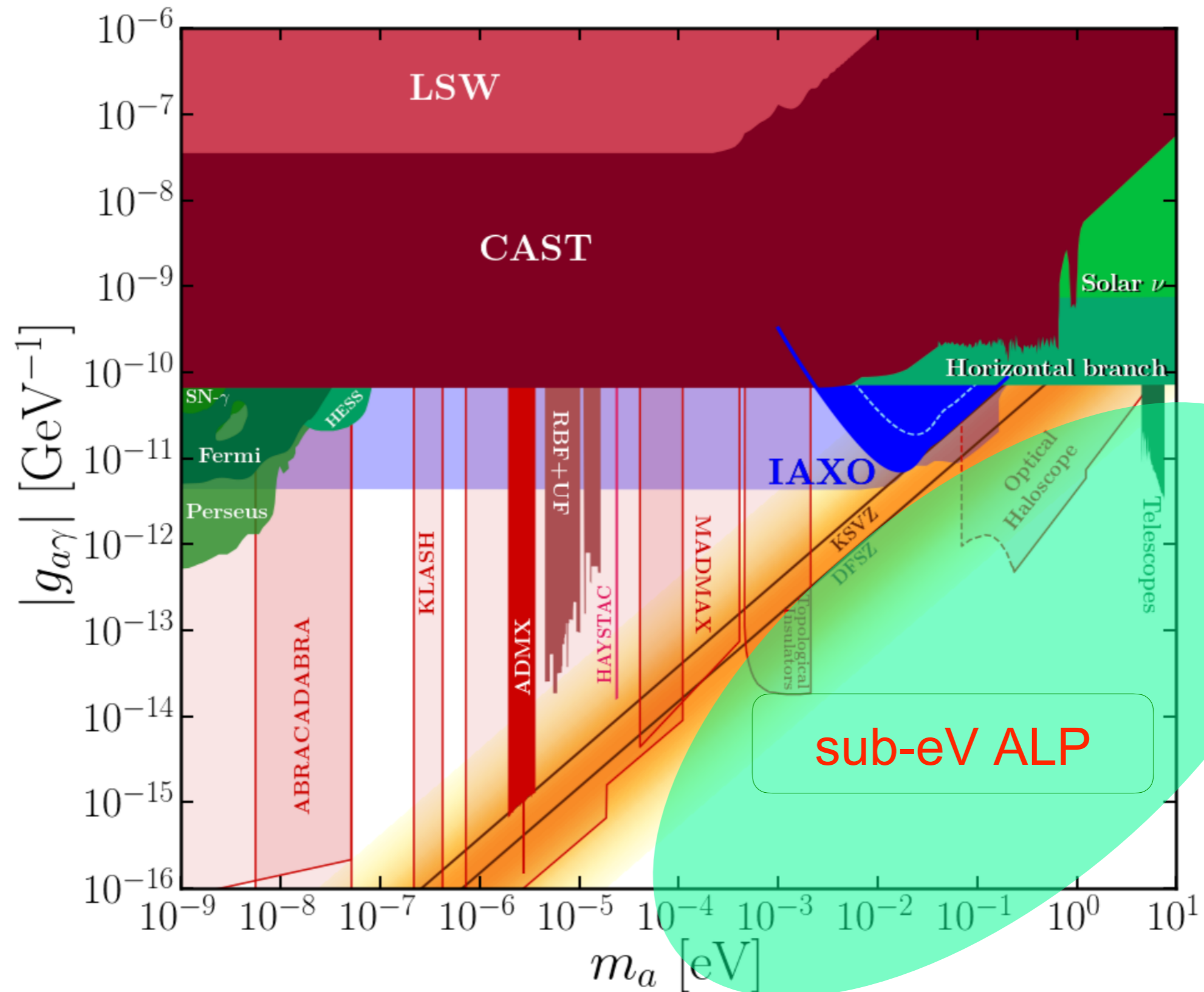
$$m_{a'} = \Lambda_{\text{QCD}}^2/F_a = (r/\sqrt{1+r^2})m_a.$$

- For $\Lambda_{\text{QCD}} < \Lambda_h \lesssim \sqrt{r}10^{4.5}\text{GeV}$, we can have sub-eV b-axion with $g_{b\gamma\gamma} \gg g_{a\gamma\gamma}$
- Motivates ALP search in $g_{a\gamma\gamma}$ below QCD axion band!

Implication: QCD axion, sub-eV ALP



Implication: QCD axion, sub-eV ALP



Implication: QCD axion, a heavy ALP

- $f_a \sim f_b \sim 10^{16} \text{GeV}$ (string axion) $\rightarrow r \sim 1$ Svrcek, Witten 06
- $\Lambda_h > 10^{10} \text{GeV} \rightarrow m_b' \sim \text{TeV scale}$
- Decay of heavy b-axion can save string QCD axion from two cosmological tensions
 1. Too much abundance

$$\Omega_a h^2 \simeq (2 \times 10^4) \times \theta_i^2 \times \left(\frac{F_a}{10^{16} \text{GeV}} \right)^{\frac{7}{6}}$$

2. High scale inflation inconsistent with $P_{\text{iso}} < 0.038 \times (2 \times 10^{-9})$

$$H_I \lesssim 2.7 \times 10^{10} \text{GeV} \left(\frac{\theta_i}{0.1} \right) \left(\frac{F_a}{10^{16} \text{GeV}} \right) \left(\frac{\Omega_{\text{CDM}} h^2}{\Omega_a h^2} \right)$$

Implication: Quintessence axion

- Recently, non-vanishing rotation angle of CMB linear polarization was reported (isotropic birefringence)
→ $\beta = (0.342 \pm 0.09)\text{deg}$ (3.6σ)

Minami, Komatsu 20

- A pseudo-scalar field A coupled to photon via

$$\mathcal{L}_{\text{eff}} \supset -c_\gamma \frac{g_{\text{em}}^2}{16\pi^2} \frac{A}{F_A} F^{\mu\nu} \tilde{F}_{\mu\nu} \qquad g_{\phi\gamma} = \frac{c_\gamma \alpha_{\text{em}}}{\pi F_A}$$

can induce $\beta \neq 0$

$$\beta = 0.42 \text{ deg} \times \frac{c_\gamma}{2\pi} \times \frac{A(t_0) - A(t_{\text{LSS}})}{F_A}$$

Implication: Quintessence axion

- Quintessence axion can explain the non-zero $\beta=O(0.1)$ (from an anomalous global $U(1)_X$)

$$\mathcal{L}_{\text{eff}} \supset -c_\gamma \frac{g_{\text{em}}^2}{16\pi^2} \frac{A}{F_A} F^{\mu\nu} \tilde{F}_{\mu\nu} + \frac{V_0}{2} \left[1 - \cos \left(\frac{A}{F_A} \right) \right]$$

$$\beta = 0.42 \text{ deg} \times \frac{c_\gamma}{2\pi} \times \frac{A(t_0) - A(t_{\text{LSS}})}{F_A}$$

- $m_q \sim H_0 \sim 10^{-33} \text{ eV}$ for the slow-roll today

$$\rightarrow f_q \sim M_P \text{ from } \Lambda_{\text{DE}}^2 = (2 \text{ meV})^2 \sim m_q f_q$$

- $\Delta q \sim O(0.1-1) f_q$ $\delta q / f_q \propto \exp(M_P^2 H_0 t / f_q^2)$

- With $c_\gamma \sim O(1-10)$, can explain $\beta=O(0.1)$!

Implication: Quintessence axion

- Challenges of quintessence from $U(1)_x$
 - (1) quality issue (why so small mass)
 - (2) Why does the quintessence couple to $U(1)_{em}$?

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Contrived set-up for the fermions...

Implication: Quintessence axion

- Challenges of quintessence from $U(1)_X$
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 - (2) Why does the quintessence couple to $U(1)_{em}$?
- 2-form and 3-form gauge theory may resolve these issues simultaneously?

Implication: Quintessence axion

- Challenges of quintessence from $U(1)_x$
 - (1) quality issue (why so small mass)
 - (2) Why does the quintessence couple to $U(1)_{em}$?
- 2-form and 3-form gauge theory may resolve these issues simultaneously?
- The mixing among axions resulting from the Higgsing all the 3-forms
 - may induce the high quality quintessence coupling to $U(1)_{em}$

Implication: Quintessence axion

- Quintessence model

$$\Lambda_{\text{QCD}} \leftrightarrow \Lambda_{\text{DE}} \simeq 2\text{meV}, \quad \theta_{\text{QCD}} \leftrightarrow \theta_{\text{DE}} \quad f_a \leftrightarrow f_q \simeq M_P$$

$$m_a \leftrightarrow m_q \simeq H_0 \simeq 10^{-33}\text{eV}$$

- Suppose both QCD axion and quintessence suffer from coupling to $P_{\mu\nu\lambda}$ ($\Lambda_h \gg \Lambda_{\text{QCD}}$) with $dP=S$, $Z \equiv (1/4!)\epsilon^{\mu\nu\lambda\rho} S_{\mu\nu\lambda\rho}$

$$\begin{aligned} \mathcal{L}(C, E, P, a, q, b) \supset & -\frac{1}{2}X^2 - \frac{1}{2}Y^2 - \frac{1}{2}Z^2 \\ & + am_a X + qm_q Y \\ & + (aM_a + qM_q + bM_b)Z \end{aligned}$$

Implication: Quintessence axion

- Diagonalizing mass matrix gives

$$a = -\frac{(f_a/f_b)}{\sqrt{1+(f_a/f_b)^2}}a' - \frac{(f_a/f_q)}{\sqrt{1+(f_a/f_q)^2}}q' \\ + \frac{(f_b/f_a)}{\sqrt{1+(f_b/f_a)^2+(f_b/f_q)^2}}b'.$$

- QCD axion-SM photon coupling gives $-(g_{q\gamma\gamma}/2)q \wedge F^{\text{em}} \wedge F^{\text{em}}$
with $g_{q\gamma\gamma} = c_a \alpha_{\text{em}} / (2\pi f_q)$

- Cosmic birefringence due to quintessence

$$\beta = \frac{g_{q\gamma\gamma}}{2} \times [q(t_0) - q(t_{\text{LSS}})] \\ \simeq 0.42 \text{ deg} \times \frac{c_a}{4\pi} \times \frac{q(t_0) - q(t_{\text{LSS}})}{f_q}$$

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- QCD axion-SM photon coupling gives $-(g_{q\gamma\gamma}/2)q \wedge F^{\text{em}} \wedge F^{\text{em}}$ with $g_{q\gamma\gamma} = c_a \alpha_{\text{em}} / (2\pi f_q)$

- Cosmic birefringence due to quintessence

$$\beta = \frac{g_{q\gamma\gamma}}{2} \times [q(t_0) - q(t_{\text{LSS}})] \\ \simeq 0.42 \text{ deg} \times \frac{c_a}{4\pi} \times \frac{q(t_0) - q(t_{\text{LSS}})}{f_q}$$

$$c_a = \mathcal{O}(10), \Delta q/f_q = \mathcal{O}(1) \\ \rightarrow \beta = \mathcal{O}(0.1)$$

Conclusion

- Quality issue still remains in the dual formulation
 - unique source → unique solution (better control than PQ)
 - hint for multiple axion scenerio → axiverse
- Implication for Pheno

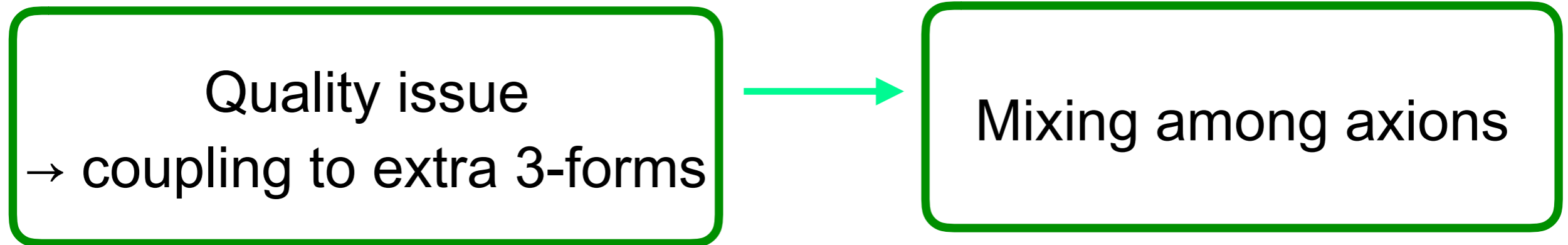
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Quality issue
→ coupling to extra 3-forms

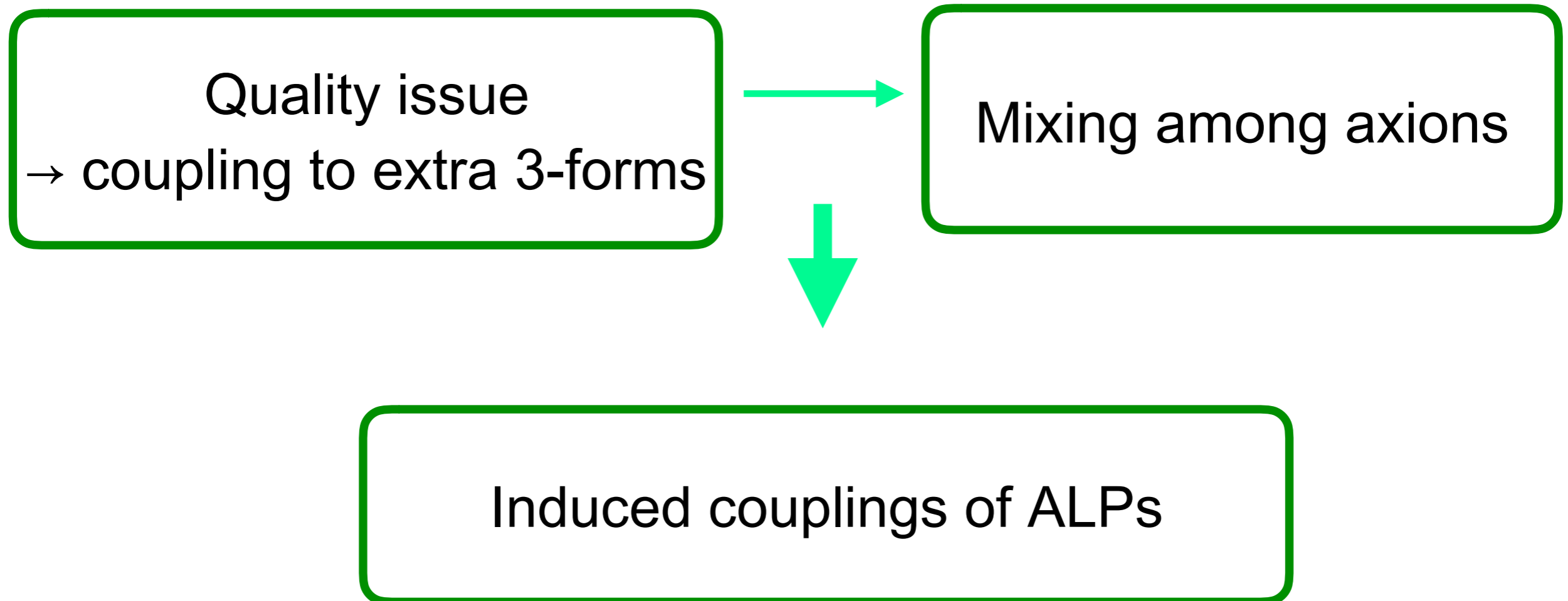
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