Bubble-assisted Leptogenesis

Joint-KIAS Workshop - Nov 13th - Jeju

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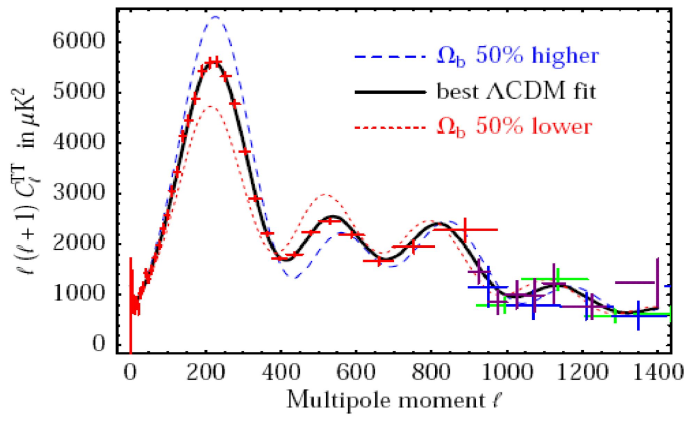
Eung Jin Chun, TPD, Tae Hyun Jung, Xander Nagels, Miguel Vanvlasselaer: 2305.10759, https://doi.org/10.1007/JHEP09(2023)164

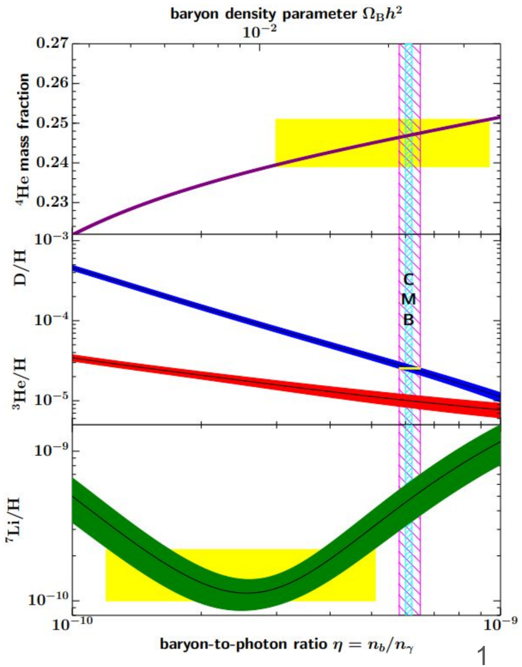
Observed asymmetry in baryons:

$$Y_B \equiv \left. \frac{n_B - n_{\overline{B}}}{s} \right|_0 \simeq 8.75 \times 10^{-11}$$

Dynamical explanation desired..



hep-ph/0608347



PDG 2020

Generating Baryon Asymmetry

Sakharov Conditions:

- **B violation**
- **C & CP violation**
- **Departure from thermal equilibrium**

Generating Baryon Asymmetry

Sakharov Conditions:

- B violation
- C & CP violation
- Departure from thermal equilibrium

The SM does not seem capable of satisfying these conditions..

(Thermal) Vanilla Leptogenesis

$$\mathcal{L} \supset \frac{1}{2}(M_N)_{ij}\overline{N_i^c}N_j + (Y_D)_{\alpha i}\,\bar{\ell}_\alpha H N_i + h.c.$$

Fukugita, Yanagida, 1986

Sakharov Conditions:

- **B violation**
- **C & CP violation**
- **Departure from thermal equilibrium**

$$Y_D \sim \mathcal{O}(1) \implies M_N \sim 10^{14}\ \text{GeV}$$

$$Y_D \sim \mathcal{O}(10^{-5}) \implies M_N \sim 10^{4}\ \text{GeV}$$

$$m_\nu \simeq \frac{Y_D^2 v_{\text{EW}}^2}{M_N}$$

(Thermal) Vanilla Leptogenesis

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Fukugita, Yanagida, 1986

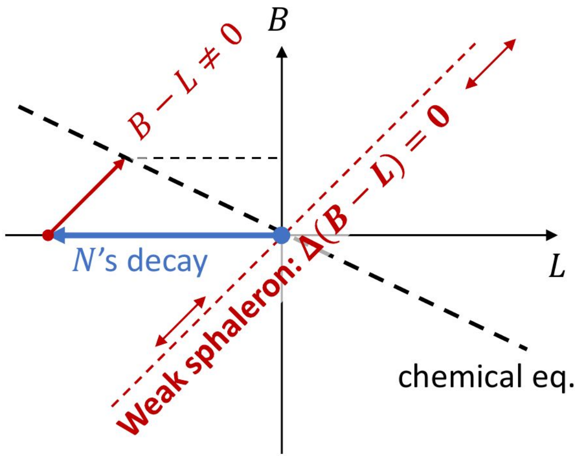
Sakharov Conditions:

- **B violation**

  $L(N, \ell) = 1$
  $L(H) = 0$

- **C & CP violation**

- **Departure from thermal equilibrium**



$$T_{\rm lepto} \gtrsim 140~\text{GeV}$$

$$\Longrightarrow\; M_N \gtrsim 140~\text{GeV}$$

(Thermal) Vanilla Leptogenesis

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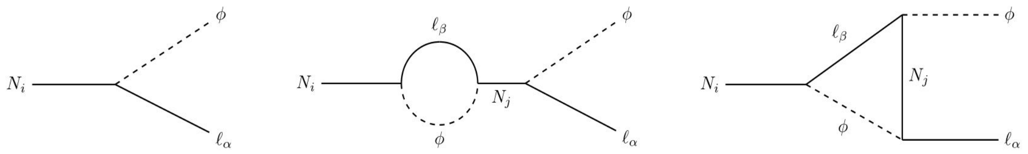


Can be easily included, need at least 2 RHNs (also for m_ν)

- **Departure from thermal equilibrium**

(Thermal) Vanilla Leptogenesis

$$\mathcal{L} \supset \frac{1}{2}(M_N)_{ij}\overline{N_i^c}N_j + (Y_D)_{\alpha i}\,\bar{\ell}_\alpha H N_i + h.c.$$

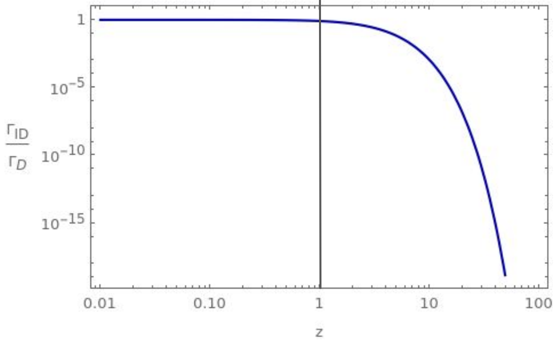
Fukugita, Yanagida, 1986

Sakharov Conditions:

- **B violation**
- **C & CP violation**
- **Departure from thermal equilibrium**

Induced by universe expansion **but** ‘smooth’ departure from equil.

$N \leftrightarrow LH$ | $N \rightarrow LH$



$T \to 0 \implies \frac{\Gamma_{\rm ID}(T)}{\Gamma_{\rm D}(T)} \to 0$ $\qquad \frac{\Gamma_{\rm ID}(z)}{\Gamma_{\rm D}(z)} = \frac{1}{2}z^2 K_2(z)$

Parameterisation

$$\overset{(1)}{} \quad \overset{(2)}{} \quad \overset{(3)}{} \quad \overset{(4)}{}$$

$$Y_B \simeq Y_{N_1}\, \epsilon_{\rm CP}^1\, \kappa_{\rm sph}\, \kappa_{\rm wash}$$

- **(1) - Number of RHNs available to decay**
- **(2) -** $\epsilon^i_{\rm CP} = \frac{\Gamma(N \to LH) - \Gamma(N \to L^c H^\dagger)}{\Gamma(N \to LH) + \Gamma(N \to L^c H^\dagger)} \sim \frac{1}{8\pi} \frac{\sum_j \mathrm{Im}\left[((Y_D)^\dagger Y_D)^2_{ij}\right]}{(Y_D^\dagger Y_D)_{11}}$
- **(3) - L to B conversion factor (sphalerons)**
- **(4) -** ***Washout*** **factor, smooth departure from equilibrium. Inverse processes do not instantly freeze out**

Parameterising Thermal Leptogenesis

(1) (2) (3) (4)

$$Y_B \simeq Y_{N_1}\, \epsilon^1_{\mathrm{CP}}\, \kappa_{\mathrm{sph}}\, \kappa_{\mathrm{wash}}$$

$$Y_B \simeq 8.75 \times 10^{-11} \underset{\epsilon_{\mathrm{CP}} \sim \frac{Y_D^2}{8\pi}}{\overset{M_{2,3} \gg M_1}{\Longrightarrow}} M_1 \simeq \frac{10^9\ \mathrm{GeV}}{\kappa_{\mathrm{wash}}}$$

hep-ph/0202239

Parameterising Thermal Leptogenesis

(1) (2) (3) (4)

$$Y_B \simeq Y_{N_1}\, \epsilon^1_{\rm CP}\, \kappa_{\rm sph}\, \kappa_{\rm wash}$$

$$Y_B \simeq 8.75 \times 10^{-11} \implies M_1 \simeq \frac{10^9\ \text{GeV}}{\kappa_{\rm wash}}$$

hep-ph/0202239

hep-ph/0401240

$\kappa_{\rm wash} \simeq 5 \times 10^{-3} \implies$ Scale is typically much higher, 'strong-washout leptogenesis', need assumptions to saturate bound. (Unless $m_{\nu_1} \lesssim 10^{-3}$ eV and/or quite specific couplings.)

$$\implies M_1 \gtrsim (\text{a few}) \times 10^{10} - 10^{11}\ \text{GeV}$$

No precise, model-independent, lower bound as serious treatment of flavour effects varies this bound.

Considering Alternatives

Why?

- **Testability - Seemingly no future prediction (e.g. GWs, CMB, BBN,...) from strong-washout regime...**
- Large RHN mass scales might be dangerous (Vissani bound, high-temperature relics? e.g. gravitinos..) hep-ph/9709409
- **Dynamics can simply be different if Type-I Seesaw + SM is not the full story.**

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- **Dynamics can simply be different if Type-I Seesaw + SM is not the full story..**

How? (examples)

- Increase the size of $\epsilon_{\rm CP}$: resonant mass splittings - $m_{N_2} - m_{N_1} \simeq \Gamma_{N_{1,2}}$ hep-ph/0309342
- Modify parameters in the seesaw relation: (i) 2HDM with sizable $\tan\beta$ 1505.05744
  (ii) $m_\nu^{\rm tree} \simeq -m_\nu^{1-{\rm loop}}$ 1809.08251
- **Decrease $\kappa_{\rm wash}$, leptogenesis @ Davidson-Ibarra bound without tuning(?)**

Bubble-assisted Leptogenesis

How can bubbles modify the dynamics of the vanilla leptogenesis scenario?

Shuve, Tamarit: 1704.01979

Baldes, et al: 2106.15602

Huang, Xie: 2206.04691
Dasgupta, et al: 2206.07032

Chun, et al: 2305.10759

Setup

$$\mathcal{L} \supset \frac{1}{2}(Y_N)_{ij}\overline{N_i^c}\Phi N_j + (Y_D)_{\alpha i}\,\overline{\ell}_\alpha H N_i + h.c.$$

$$M_N = Y_N \langle \Phi \rangle$$

New scalar is assumed to undergo a **first-order phase transition**

Much stronger departure from thermal equilibrium compared to usual thermal leptogenesis

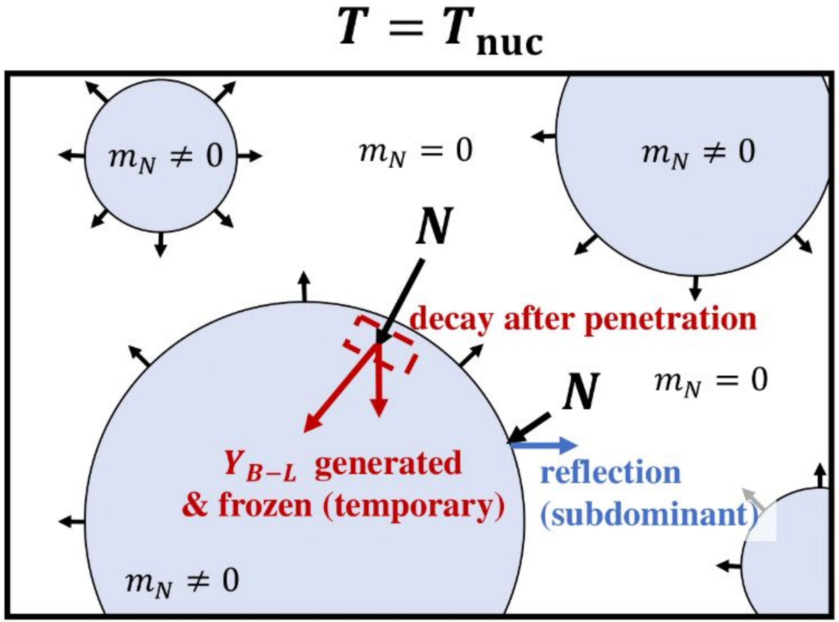
$$\langle \Phi \rangle \neq 0 \qquad M_N \neq 0 \qquad n_N^{\mathrm{eq}} \propto e^{-M_N/T}$$

$$\langle \Phi \rangle = 0 \qquad M_N = 0 \qquad n_N^{\mathrm{eq}} \propto T^3$$

Larger population of RHNs available to decay *if they can penetrate the bubbles* - $\gamma_w > 1$

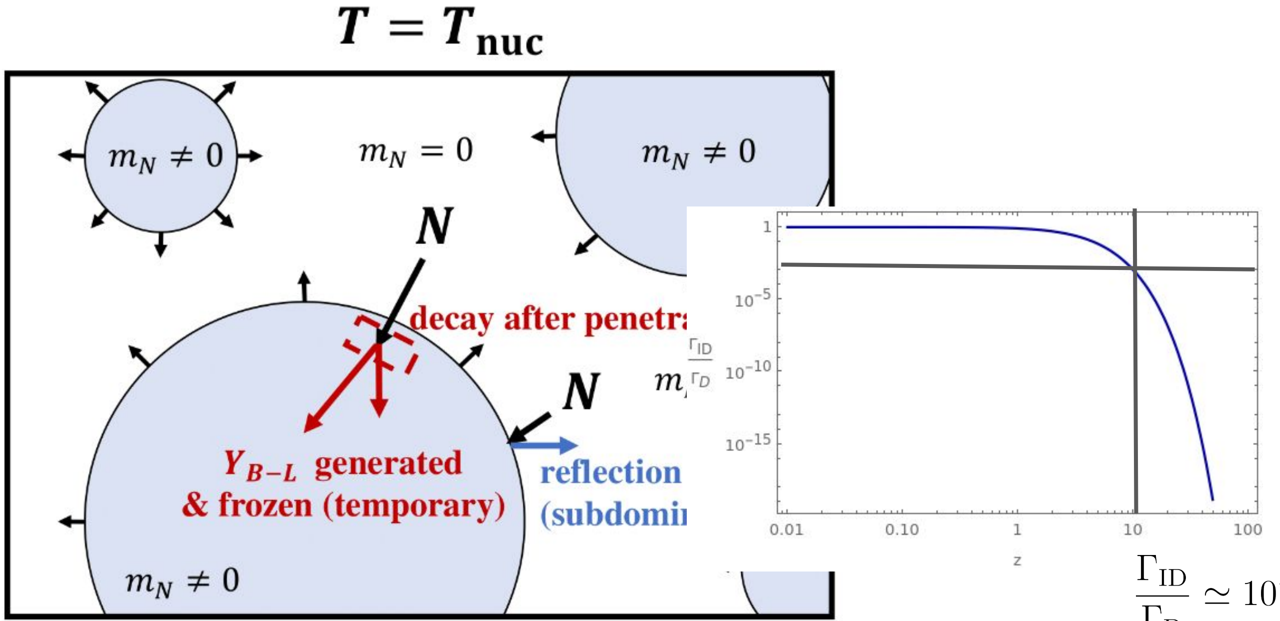
Bubbles begin to expand around the nucleation temperature

As bubbles expand



If $M_N \gg T_{\rm nuc}$ is satisfied (and RHNs penetrate) inverse decays could freeze out immediately after penetration $\kappa_{\rm wash} = 1$?

As bubbles expand



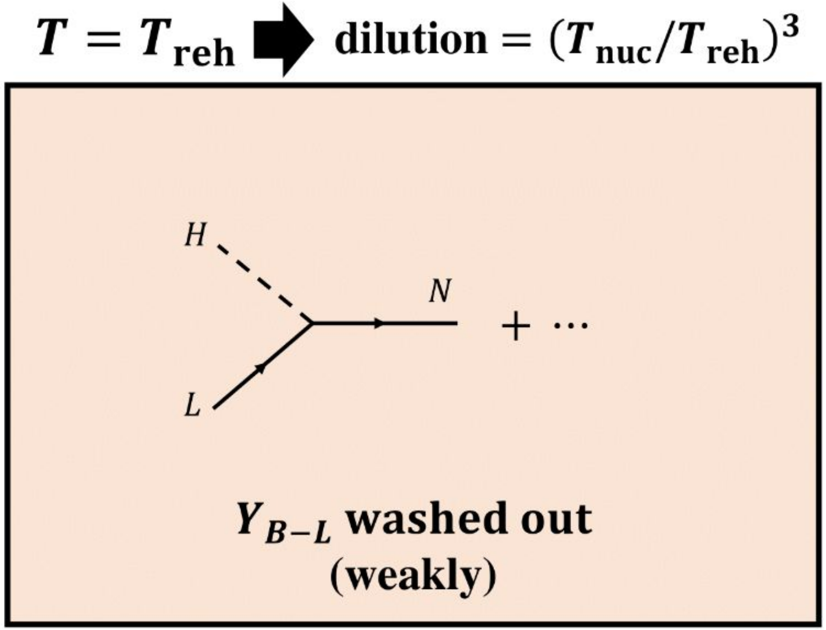
$$\frac{\Gamma_{\rm ID}}{\Gamma_{\rm D}} \simeq 10^{-3}$$

If $M_N \gg T_{\rm nuc}$ is satisfied (and RHNs penetrate) inverse decays could freeze out immediately after penetration $\kappa_{\rm wash} = 1?$ - fast bubbles req.

$$(\gamma_w T_{\rm nuc} > M_N > T_{\rm nuc})$$

At the end of the phase transition the bubbles collide and energy is released..

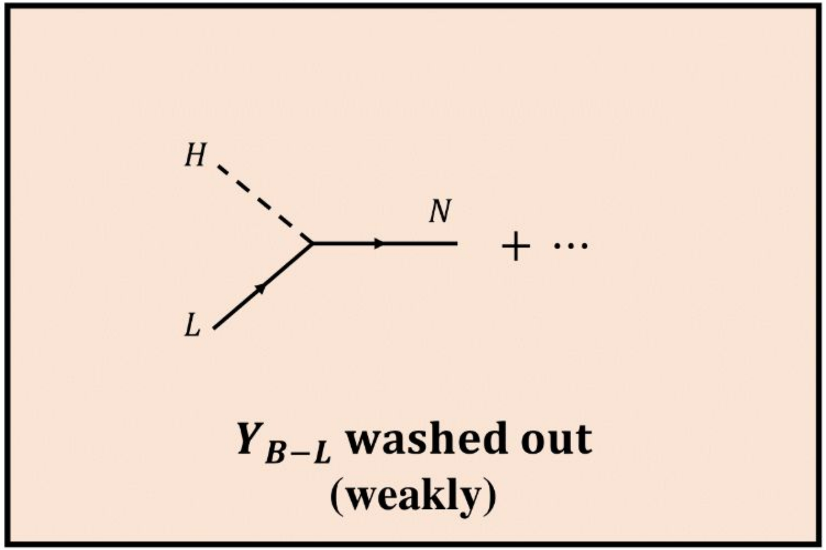
After bubbles collide



Universe reheats and the final asymmetry is diluted.
$M_N \gg T_{\text{reh}}$ also required to avoid washout

After bubbles collide

$T = T_{\text{reh}}$ ➡ dilution $= (T_{\text{nuc}}/T_{\text{reh}})^3$



The mechanism always seems to suffer from a mild tension between a fast bubble and a small reheating. (fast bubble -> large energy -> large reheating)

Setup details

$$\mathcal{L} \supset \frac{1}{2}(Y_N)_{ij}\overline{N_i^c}\Phi N_j + (Y_D)_{\alpha i}\,\bar{\ell}_\alpha H N_i + h.c.$$

Assume a classically scale-invariant potential

$$V^{\text{tree}}(\Phi, \dots) \supset \lambda\,|\Phi|^4$$

which develops a flat direction at some scale $\lambda(\mu_*) = 0 \quad (\langle\Phi\rangle \sim \mu_*)$

Scale-invariance radiatively broken, naturally can generate a strong phase transition.
Importantly, this **predicts a light scalar**.

Aside: this setup requires , $M_1 \sim M_2 \sim M_3$ but we assume no resonance

Otherwise: $T_{\text{reh}} \propto \max(M_i) > \min(M_i)$ and recover hierarchical thermal leptogenesis

Setup details

RHNs (fermions) destabilise the CW potential and are necessary for leptogenesis

Required to introduce new bosons for stability:

Consider two cases:

- **Gauged** $U(1)_{B-L}$ **(GC):**

$$m_A(\phi) = 2g_{B-L}\phi$$

- **Introduce additional singlet scalar (SC):**

$$\mathcal{L} \supset \frac{1}{2}\lambda_{s\phi}s^2\left|\Phi\right|^2 \qquad m_s^2(\phi) = \lambda_{s\phi}\phi^2$$

Steps to evaluate final asymmetry

- **(i) For given Lagrangian parameters evaluate phase transition dynamics**

Bubble-wall velocity, penetration rate of RHNs, FOPT properties

$\gamma_w \gg 1$ (ideally)

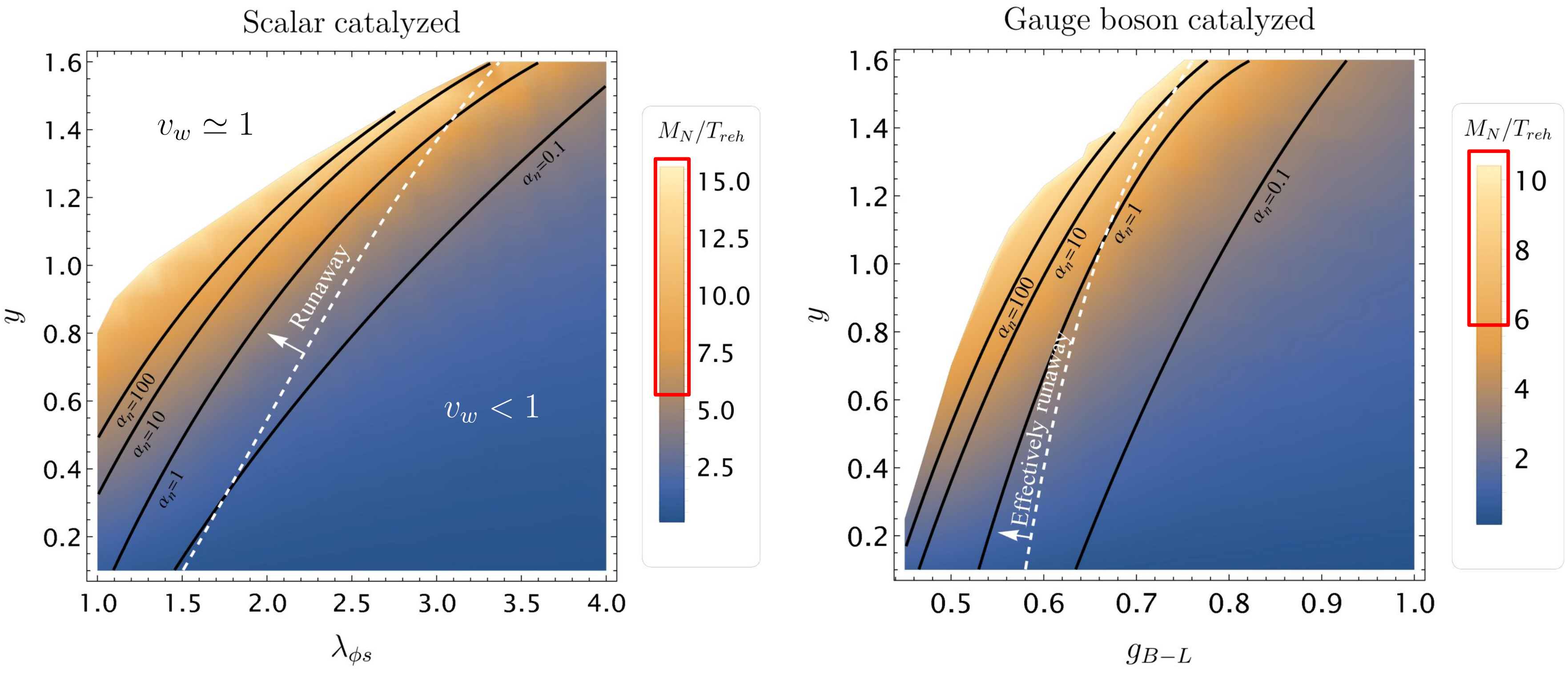
$\kappa_{\rm pen} \simeq 1$ (ideally)

$T_{\rm nuc},\, T_{\rm reh},\, \alpha_n,\, \beta_{\rm PT}$

$$\gamma_w T_{\rm nuc} > M_N > T_{\rm nuc}$$

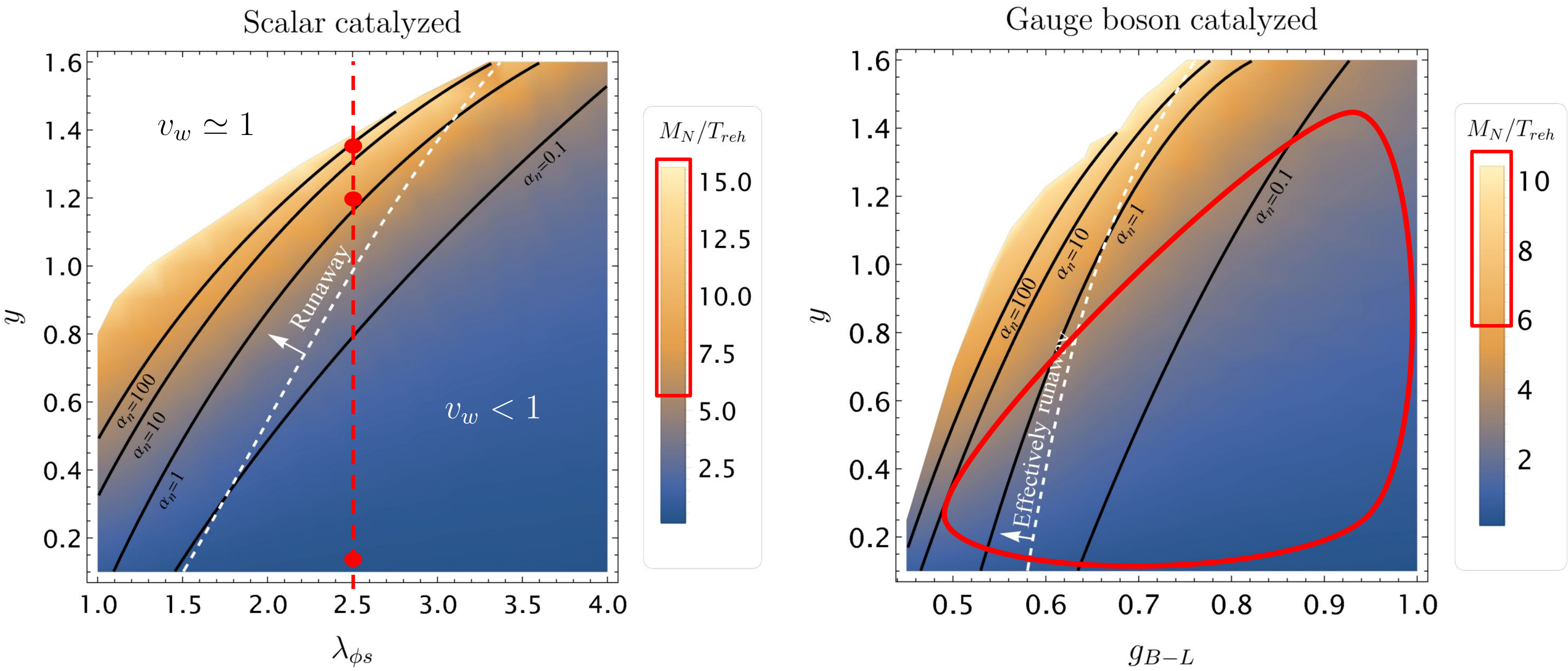
We **don't** want reflections along the bubble wall ideally.

Step (i)



Numerically find $M_N/T_{\rm reh} \gtrsim \mathcal{O}(5)$ required to avoid *conventional* washout

Step (i)



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Steps to evaluate final asymmetry

- **(i) For given Lagrangian parameters evaluate phase transition dynamics**

  Bubble-wall velocity, penetration rate of RHNs, FOPT properties

  $$\gamma_w \gg 1 \qquad \kappa_{\rm pen} \simeq 1 \qquad T_{\rm nuc},\, T_{\rm reh},\, \alpha_n,\, \beta_{\rm PT}$$

- **(ii) Within the bubbles, RHNs decay and generate an asymmetry**

  Solve the Boltzmann equations *including any unavoidable washout channels*

  $$\mathcal{L} \supset Y_N NN\phi + g_{B-L}\,(NN + ff)\,A_\mu$$

  $$NN \to \phi\phi, ff$$

  $$\left(\frac{T_{\rm reh}}{M_N} \propto \frac{\Delta V}{v_\phi^4} < 1\right) \implies m_\phi \ll M_N?$$

  Interactions which remove RHNs without generating asymmetry: depletions

Steps to evaluate final asymmetry

- **(i) For given Lagrangian parameters evaluate phase transition dynamics**

  Bubble-wall velocity, penetration rate of RHNs, FOPT properties

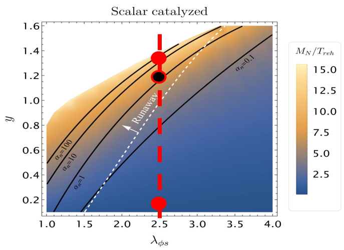
  $\gamma_w \gg 1$ $\kappa_{\rm pen} \simeq 1$ $T_{\rm nuc},\, T_{\rm reh},\, \alpha_n,\, \beta_{\rm PT}$

- **(ii) Within the bubbles, RHNs decay and generate an asymmetry**

  Solve the Boltzmann equations *including any unavoidable washout channels*

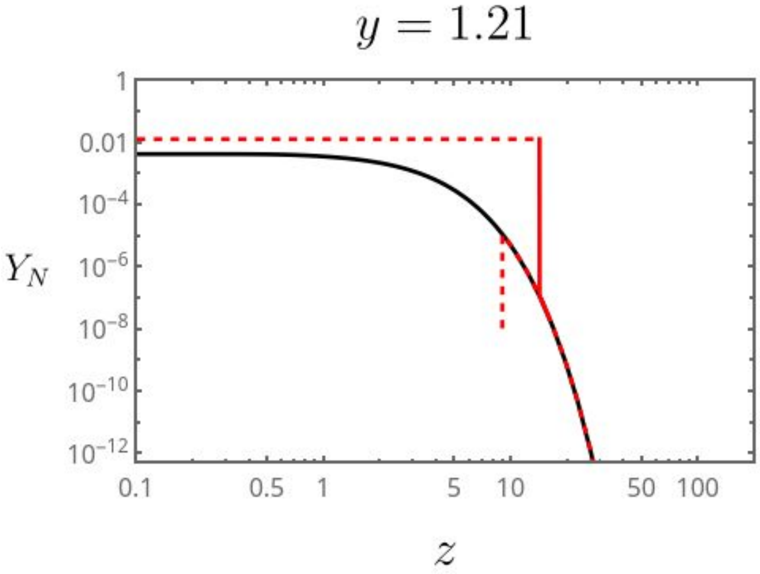
- **(iii) After bubbles collide the universe reheats, dilution + washout may occur**

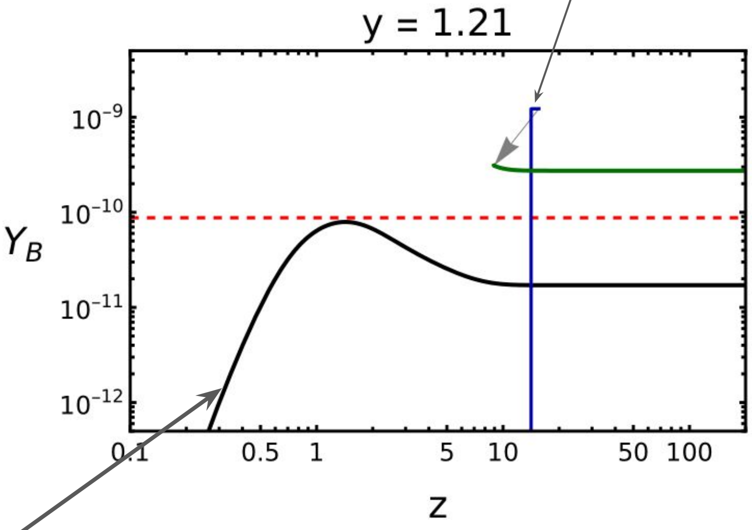
$$T_{\rm reh} \simeq T_{\rm nuc}\,(1+\alpha)^{1/4}$$



Step (ii + iii)

Bubbles nucleate, leptogenesis begins - little washout

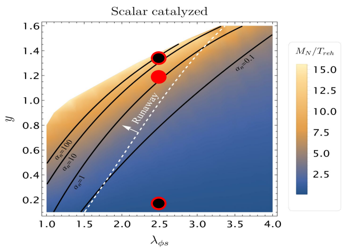




$$\left(\frac{T_{\rm nuc}}{T_{\rm reh}}\right)^3$$

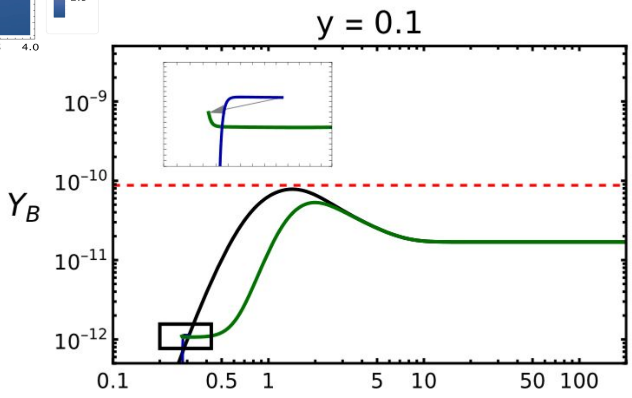
Comparison to conventional leptogenesis

reheating after collisions



Step (ii + iii)

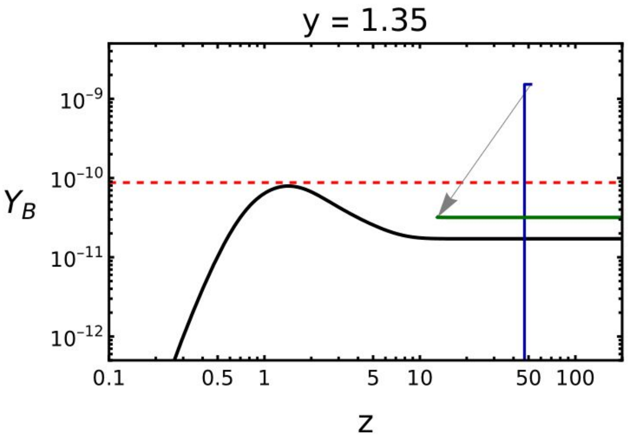
$\alpha < 1$



Phase transition too weak, no dilution from reheating but RHNs remain in equilibrium:

$$M_N < T_{\text{nuc}}, T_{\text{reh}}$$

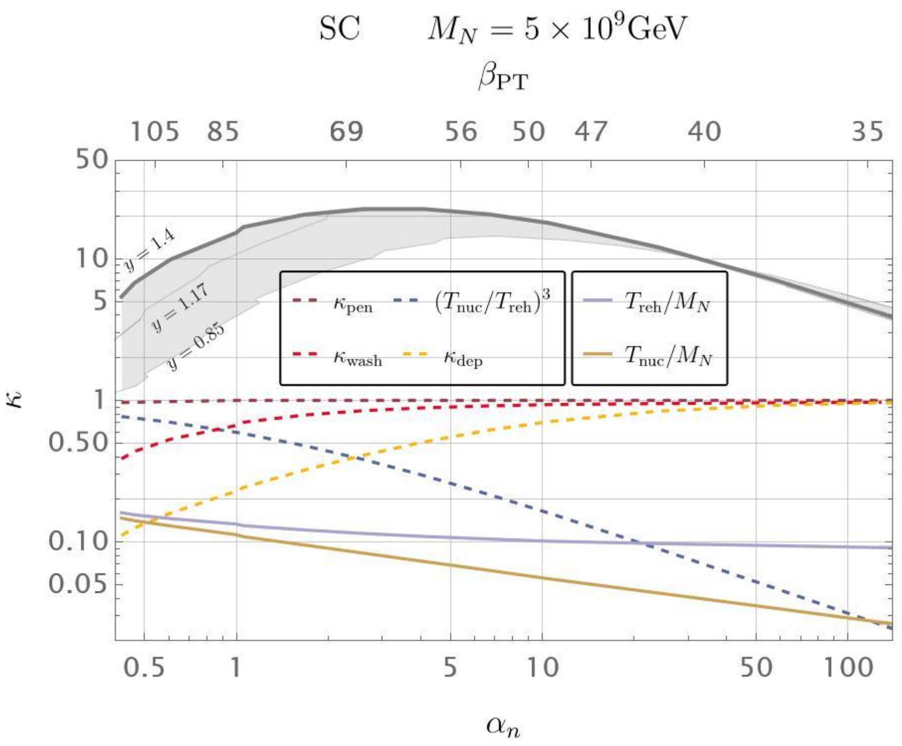
$\alpha \gg 1$

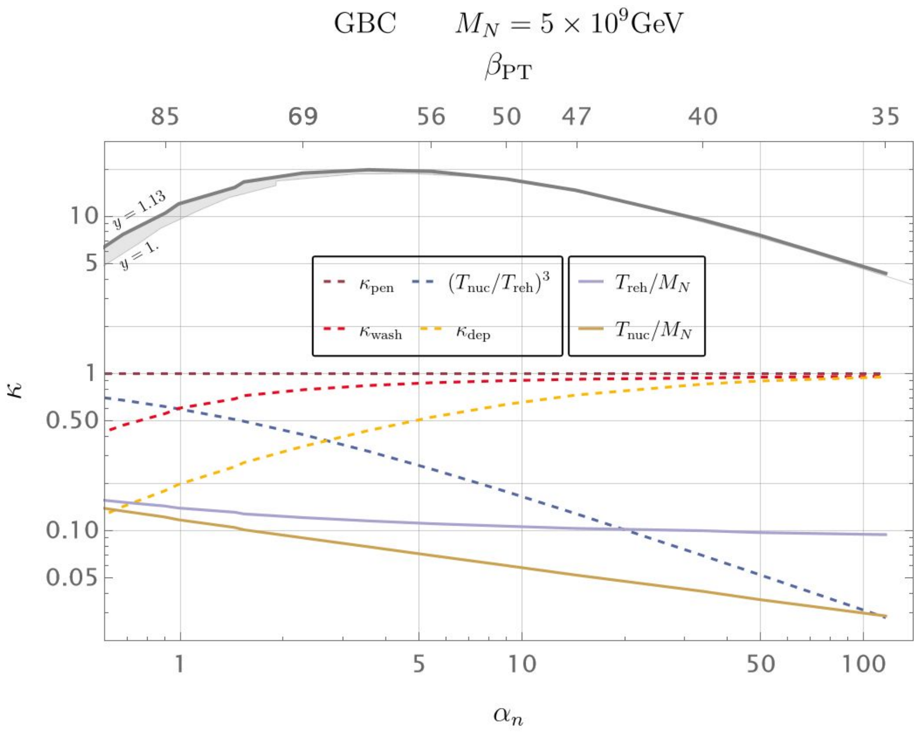


Phase transition too strong, extremely large reheating - final asymmetry is strongly diluted from bubble collisions

$$\left(\frac{T_{\text{nuc}}}{T_{\text{reh}}}\right)^3$$

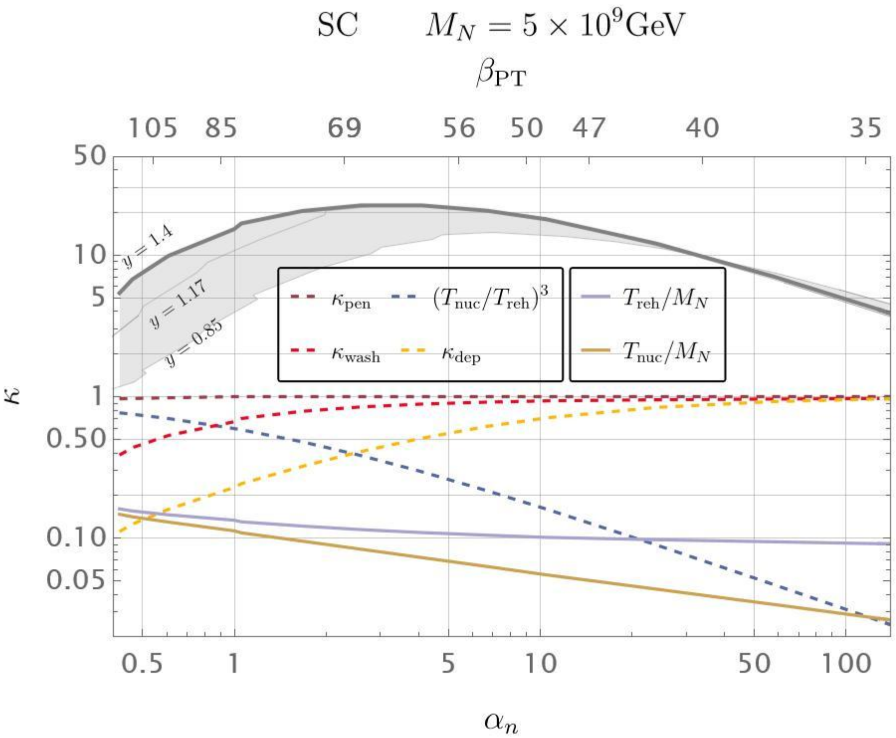
Results

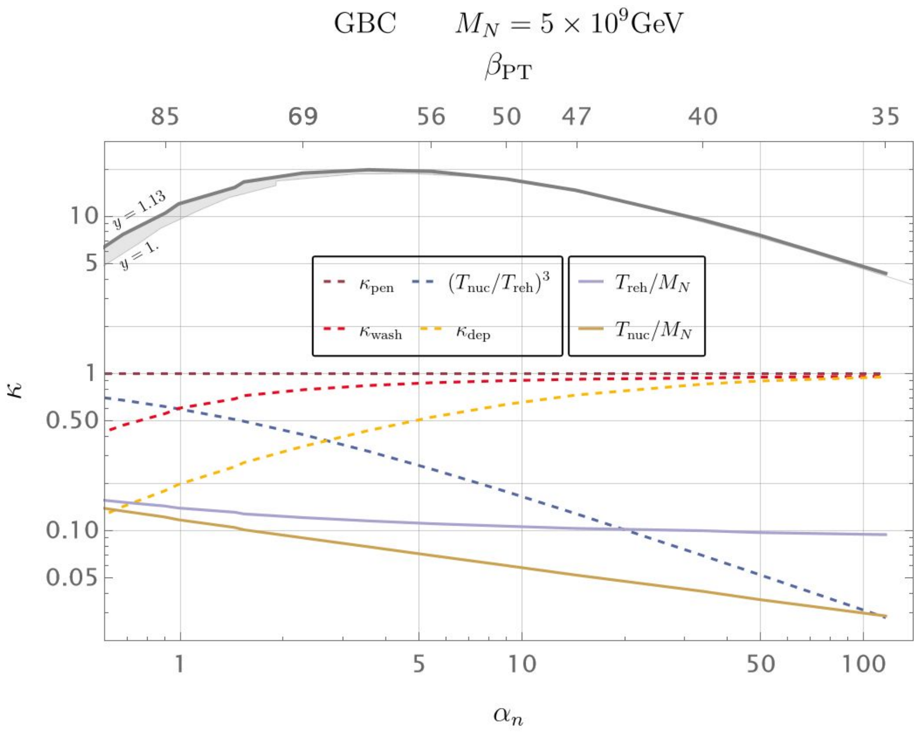




$$Y_B \sim Y_N^{\rm eq} \epsilon_{\rm CP}\, \kappa_{\rm sph}\, \kappa_{\rm wash}\, \kappa_{\rm pen}\, \kappa_{\rm dep} \left(\frac{T_{\rm nuc}}{T_{\rm reh}}\right)^3$$

Results

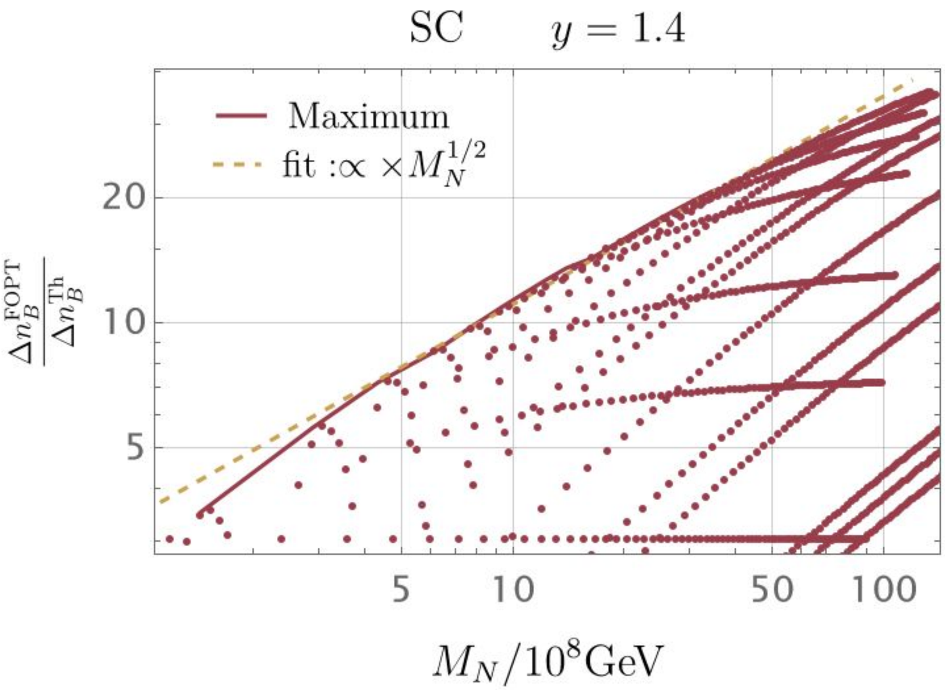


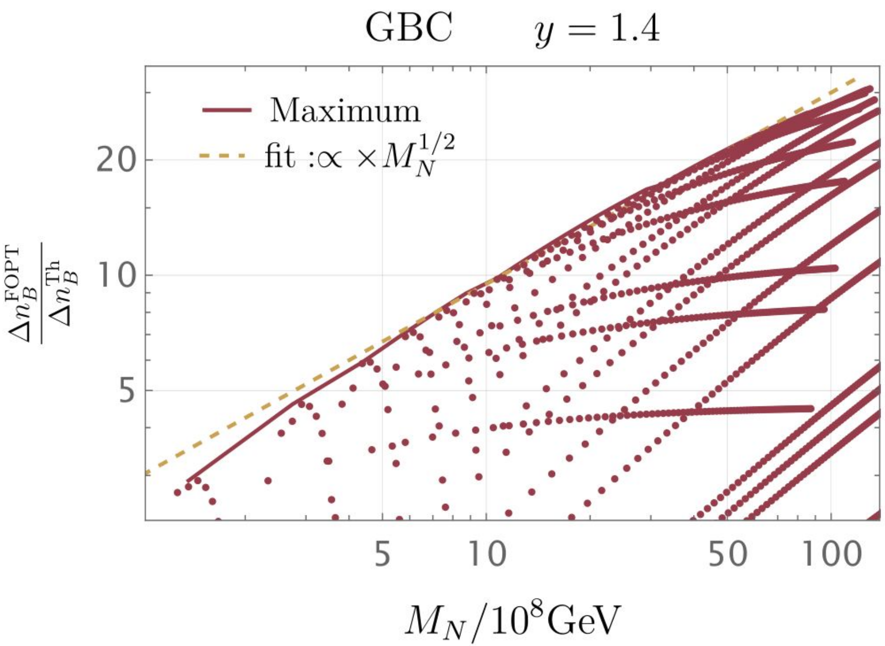


Significant enhancement in generated asymmetry compared to conventional scenario.

Strong-washout leptogenesis close to the Davidson-Ibarra bound

Bounded from below?



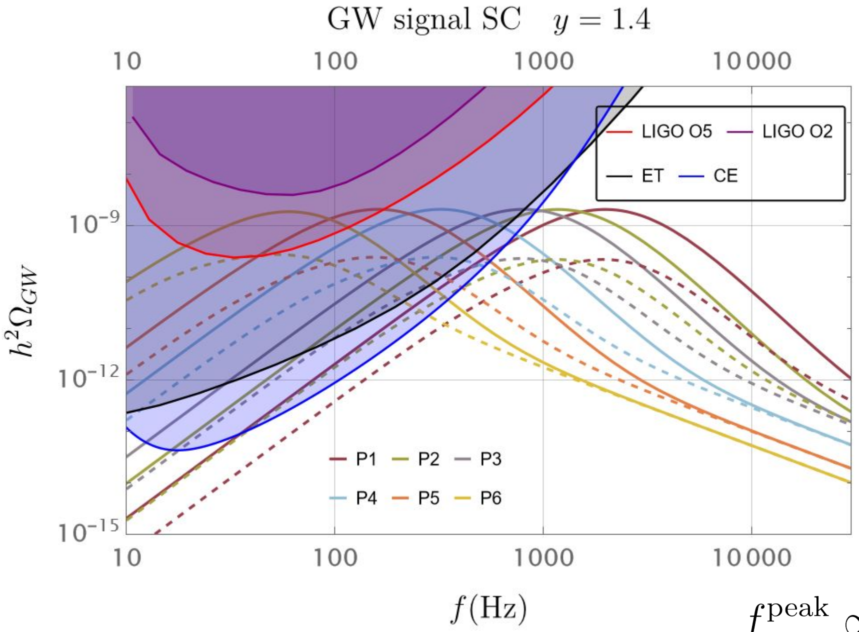


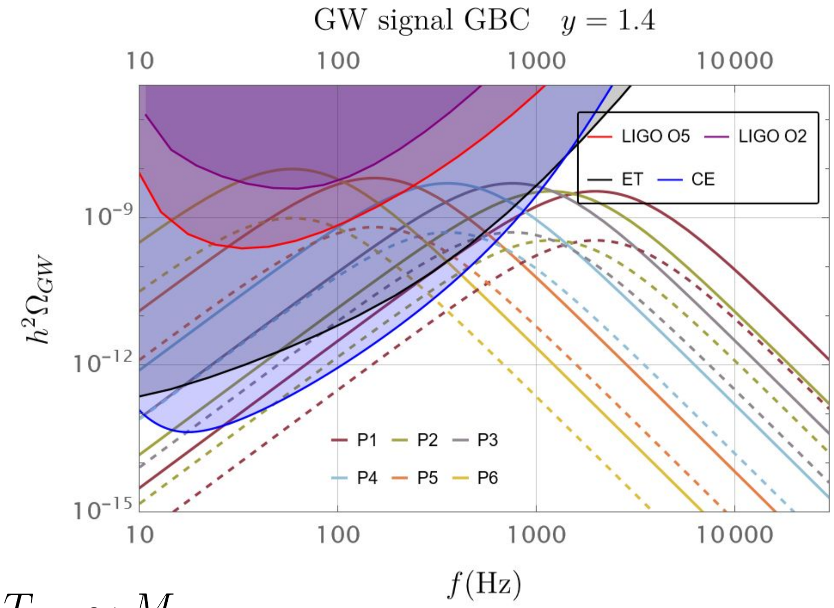
$$\frac{n_B^{\rm FOPT,max}}{n_B^{\rm thermal}} \sim \left(\frac{M_N}{10^7 \rm GeV}\right)^{1/2}$$

Lower bound below which bubble-assisted dynamics generate a *suppression* of the final asymmetry

$NN \to \phi\phi$

Gravitational waves



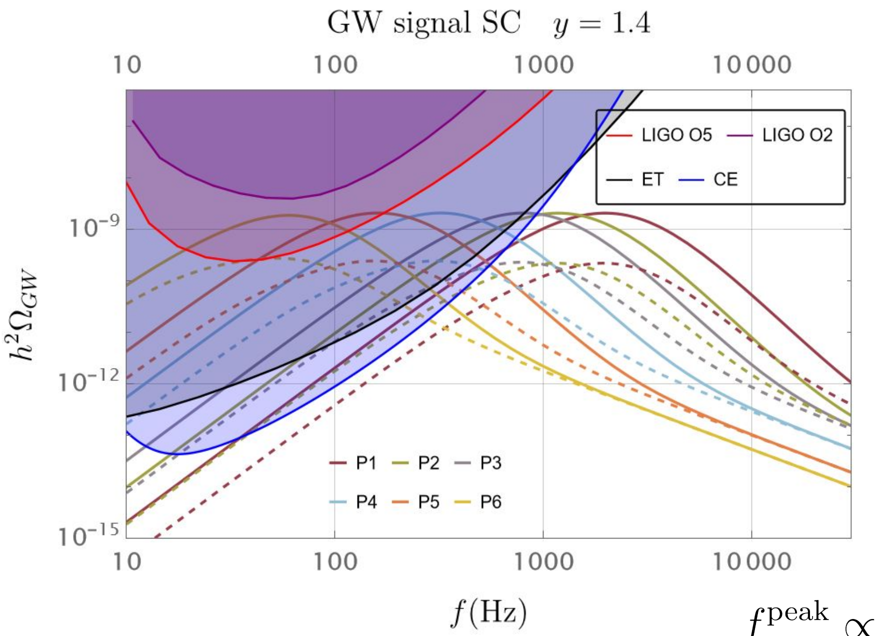


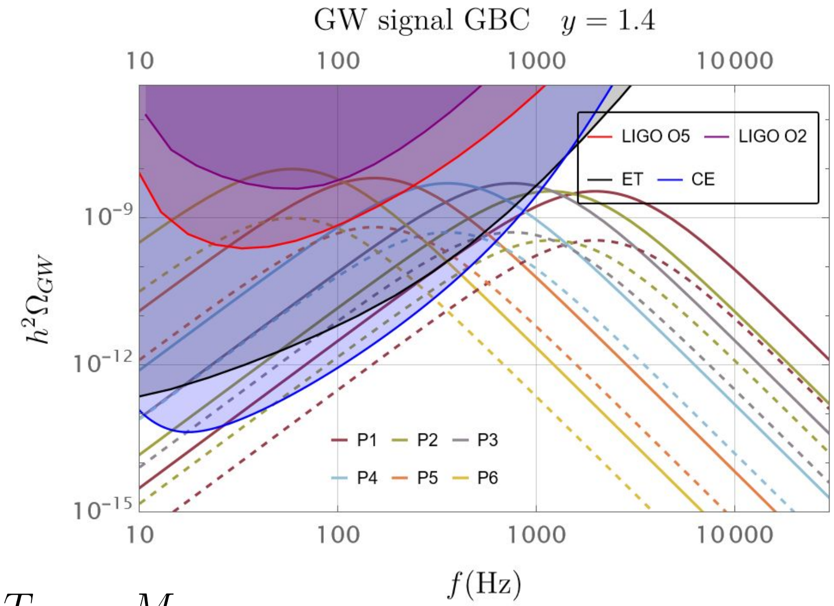
$$f^{\text{peak}} \propto T_{\text{reh}} \sim M_N$$

Considered production of GWs *during* the FOPT, e.g. bubble wall collision and sound waves for future GW detectors.

Peak frequency shifts with RHN mass: $\text{P1}: M_N = 6 \times 10^9\ \text{GeV} \ \rightarrow\ \text{P6}: M_N = 10^8\ \text{GeV}$

Gravitational waves





$$f^{\rm peak} \propto T_{\rm reh} \sim M_N$$

Maximal enhancement occurs at $\beta_{\rm PT} \sim \mathcal{O}(50),\ \alpha_n \sim \mathcal{O}(1-10)$

Lower mass region of bubble-assisted leptogenesis, $M_N \lesssim 5\times 10^9\ \mathrm{GeV}$, seems probable (not a smoking gun though)

Conclusions

- Bubble-assisted leptogenesis can allow for a strong departure from thermal equilibrium
  - Conventional washout can be fully suppressed
  - New channels ($NN \to \phi\phi$) become relevant
  - Dilution from reheating
    - Can enhance the final asymmetry sizeably (~20) for masses close to DI bound, but not maximally

- Enhancement cannot be arbitrary applied to other (low-scale) leptogenesis models
  - Strong annihilation for smaller masses
  - Enhancement disappears below $M_N \simeq 10^7 \text{ GeV}$

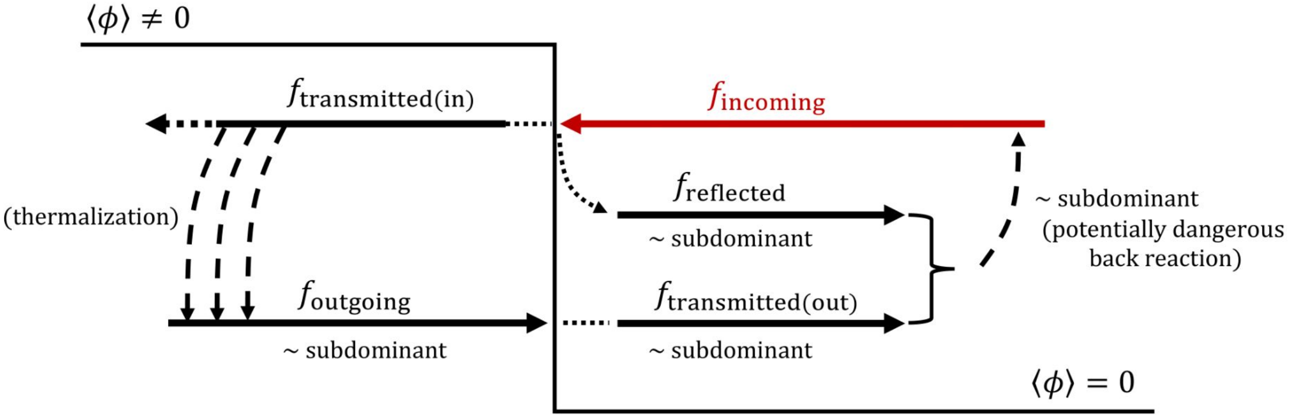
- GW signals seem to be possible from sound waves during the FOPT for lower mass-scale RHNs (partial testability)

- Are there different potentials beyond this toy model which can sizeably change these results?

Fin

Backup

RHN Penetration



$$\kappa_{\text{pen}} = \frac{\int_{p_z<-M_N} d^3p\, f_{\text{incoming}}}{\int_{p_z<0} d^3p\, f_{\text{incoming}}}$$

$$f_{\text{incoming,bubbleframe}} \simeq \frac{1}{e^{\gamma(E+vp_z)/T_{\text{nuc}}} \pm 1}$$

RHN Penetration

$$\mathcal{P}^i = \int \frac{d^3p}{(2\pi)^3} \, (\Delta p) f = \int \frac{dp_z \, dp_\perp \, 2\pi p_\perp}{(2\pi)^3} \, (\Delta p) f$$

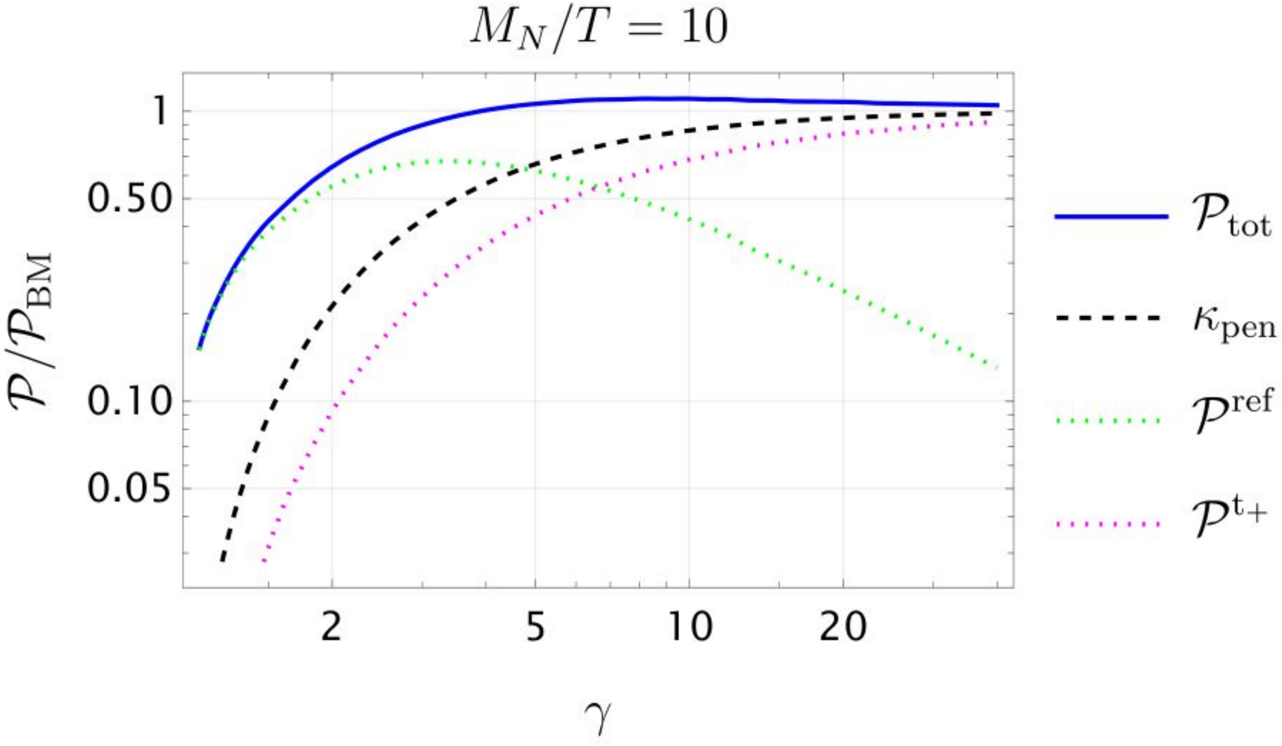
$$(\Delta p)_{\text{reflection}} = 2p_z$$

$$(\Delta p)_{\text{trans,incoming}} = p_z + \sqrt{p_z^2 - M_X^2}$$

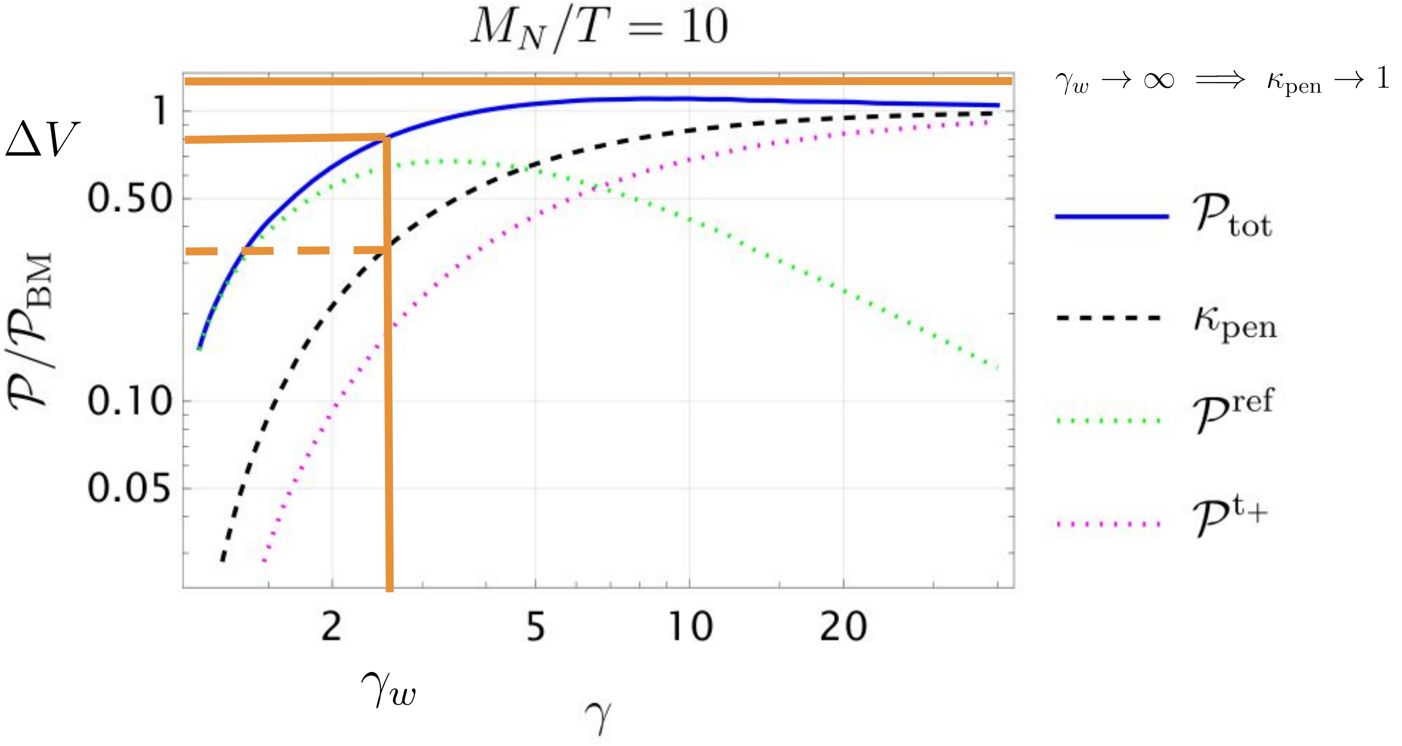
$$(\Delta p)_{\text{trans,outgoing}} = p_z - \sqrt{p_z^2 - M_X^2}$$

$$\Delta V + \mathcal{P}(v_w) \geq 0$$

RHN Penetration



RHN Penetration



Washout

$$Y_D = i\frac{\sqrt{2}}{v_{\mathrm{EW}}} U^* m_n^{1/2} R^T M^{1/2}$$

$$RR^T = 1$$

$$\kappa_{\mathrm{wash}}\left(K \equiv \frac{\Gamma_{N_1}}{H(M_1)}\right)$$

$$\Gamma_{N_I} = \frac{1}{8\pi}(Y_D^\dagger Y_D)_{II} M_{N_I} = \frac{1}{4\pi v_{\mathrm{EW}}^2} M_{N_I}^2 \left(m_{\nu_1}\left|R_{I1}\right|^2 + m_{\nu_2}\left|R_{I2}\right|^2 + m_{\nu_3}\left|R_{I3}\right|^2\right)$$

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$$\kappa_{\rm wash}\left(K \equiv \frac{\Gamma_{N_1}}{H(M_1)}\right)$$

$$\Gamma_{N_I} = \frac{1}{8\pi}(Y_D^\dagger Y_D)_{II} M_{N_I} = \frac{1}{4\pi v_{\rm EW}^2} M_{N_I}^2 \left(m_{\nu_1}\left|R_{I1}\right|^2 + m_{\nu_2}\left|R_{I2}\right|^2 + m_{\nu_3}\left|R_{I3}\right|^2\right)$$

$$\Gamma_{N_1} \sim \frac{2M_1 m_\nu}{8\pi v_{\rm EW}^2} \simeq 65\left(\frac{M_1}{10^9\ \rm GeV}\right)^2\ \rm GeV$$

$$(m_\nu = m_{\rm atm})$$

Strong/Weak Washout

$$Y_D = i\frac{\sqrt{2}}{v_{\rm EW}} U^* m_n^{1/2} R^T M^{1/2} \qquad RR^T = 1$$

$$\left((Y_D)^\dagger Y_D\right)_{IJ} = \frac{2}{v_{\rm EW}^2}\sqrt{M_{N_I}}\sqrt{M_{N_J}}\sum_k m_{\nu_k} R_{jk} R^*_{ik}$$

$$\Gamma_{N_I} = \frac{1}{8\pi}(Y_D^\dagger Y_D)_{II} M_{N_I} = \frac{1}{4\pi v_{\rm EW}^2} M_{N_I}^2 \left(m_{\nu_1}\left|R_{I1}\right|^2 + m_{\nu_2}\left|R_{I2}\right|^2 + m_{\nu_3}\left|R_{I3}\right|^2\right)$$

To achieve weak washout in Type-I:

$$R_{I2}, R_{I3} \ll R_{I1} \qquad m_{\nu_1} \lesssim 10^{-3}\ \text{eV}$$

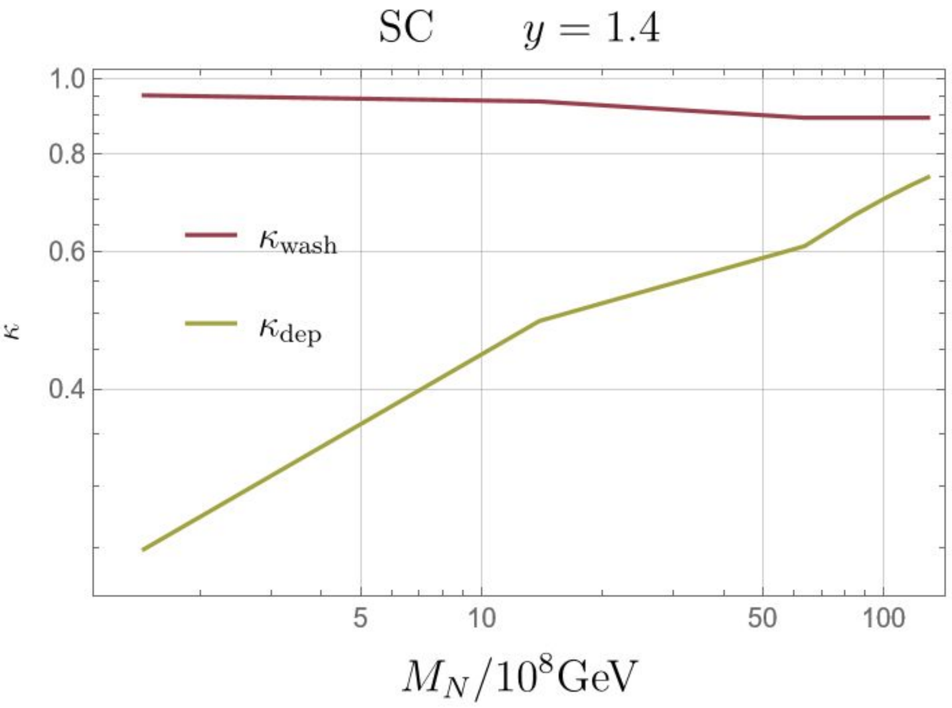
GW Benchmarks

	SC			GBC		
	$M_N/10^8\text{GeV}$	$\frac{n_B^{FOPT}}{n_B^{\text{thermal}}}$	α_n	$M_N/10^8\text{GeV}$	$\frac{n_B^{FOPT}}{n_B^{\text{thermal}}}$	α_n
P1	62	26	4.4	60	22	4.8
P2	34	21	6	37	17	5
P3	26	18	6.	25	15	9
P4	10	12	10	10	10	9.6
P5	4.2	8	25	4	5.6	15
P6	1.4	3.5	25	1.4	2.9	33

$$\Omega_{\text{GW}}^0 \sim 0.1 - 0.01$$

simulation of energy transmission during sound waves

Lower-bound for bubble enhancement



$M_N/T_{\mathrm{nuc}} = 5$

$$\Gamma_D \sim y_D^2 M_N \propto M_N^2 \qquad \Gamma_{NN\to\phi\phi} = \langle \sigma_{NN\to\phi\phi} v \rangle n_N \sim y^4 \left(\frac{T}{M_N}\right)^4 M_N \propto M_N$$

BE details

$$zHs\,Y_N'(z) = -\bar{\gamma}_D\left(\frac{Y_N}{Y_N^{(\mathrm{eq})}} - 1\right) - 2\gamma_{NN\to\phi\phi}\left(Y_N^2 - \left(Y_N^{(\mathrm{eq})}\right)^2\right) + [NN \to ff]$$

$$zHs\,Y_{B-L}'(z) = -\epsilon_{\mathrm{CP}}\bar{\gamma}_D\left(\frac{Y_N}{Y_N^{(\mathrm{eq})}} - 1\right) - \frac{1}{2}(c_L + c_H)\,\gamma_D\frac{Y_{B-L}}{Y^{(\mathrm{eq})}}$$

$$\bar{\gamma}_D \equiv \sum_I \gamma_D(N_I) = \sum_I n_{N_I}^{(\mathrm{eq})}\frac{K_I(z)}{K_2(z)}\Gamma_D(N_I)$$

$$\epsilon_{\mathrm{CP}}\,\bar{\gamma}_D \equiv \sum_I \epsilon_I\gamma_D(N_I)$$

$$\gamma_{NN\to\phi\phi} \equiv \frac{1}{9}s^2\sum\langle\sigma v\rangle_{N_I N_I\to\phi\phi}$$

Temperature (GeV)	c_l	c_H	$c_H + c_L$
10^{11-12}	$\frac{6}{35}$	$\frac{95}{460}$	~ 0.38
10^{8-11}	$\frac{5}{53}$	$\frac{47}{358}$	~ 0.22
$\ll 10^8$	$\frac{7}{79}$	$\frac{8}{79}$	~ 0.19

BE details

$$n_{N_I}^{(0)} = \kappa_{\rm pen} \, \frac{2 \cdot \frac{3}{4} \cdot \zeta(3)}{\pi^2} T_{\rm nuc}^3 \qquad\qquad z_{\rm col} \sim e^{H \Delta t_{\rm PT}} z_{\rm nuc} \sim 1.1 z_{\rm nuc}$$

Step (ii)

$$Y_N(z_{\rm nuc}) = \frac{3 n_{N_I}^{(0)}}{s(T_{\rm nuc})}, \quad Y_{B-L}(z_{\rm nuc}) = 0, \quad z_{\rm nuc} = \frac{M_N}{T_{\rm nuc}}$$

$$\tilde{Y}_N \equiv Y_N(z_{\rm col}), \quad \tilde{Y}_{B-L} \equiv Y_{B-L}(z_{\rm col})$$

Step (iii)

$$Y_N\left(z_{\rm reh}\right) = \tilde{Y}_N \left(\frac{T_{\rm nuc}}{T_{\rm reh}}\right)^3, \quad Y_{B-L}\left(z_{\rm reh}\right) = \tilde{Y}_{B-L} \left(\frac{T_{\rm nuc}}{T_{\rm reh}}\right)^3, \quad z_{\rm reh} = \frac{M_N}{T_{\rm reh}}$$

Thermal Potential

$$V(\phi,T) = V_0(\phi) + \sum_i V_{CW}(m_i^2(\phi)+\Pi_i) + \sum_i V_T(m_i^2(\phi)+\Pi_i)$$

$$V_{CW}(m_i^2(\phi)) = (-1)^{2s_i} g_i \frac{m_i^4(\phi)}{64\pi^2}\left[\log\left(\frac{m_i^2(\phi)}{\mu^2}\right) - c_i\right]$$

$$V_T(m_i^2(\phi)) = \pm\frac{g_i}{2\pi^2} T^4 J_{\rm B,F}\left(\frac{m_i^2(\phi)}{T^2}\right), \qquad J_{\rm B,F}(y^2) = \int_0^\infty dx\; x^2 \log\left[1 \mp \exp\left(-\sqrt{x^2+y^2}\right)\right],$$

$$(\Delta t)^{-1}_{\rm PT} \sim -\frac{d(S_3/T)}{dt}\bigg|_{T=T^{\rm nuc}} \equiv H_{\rm reh}\beta_{\rm PT}$$

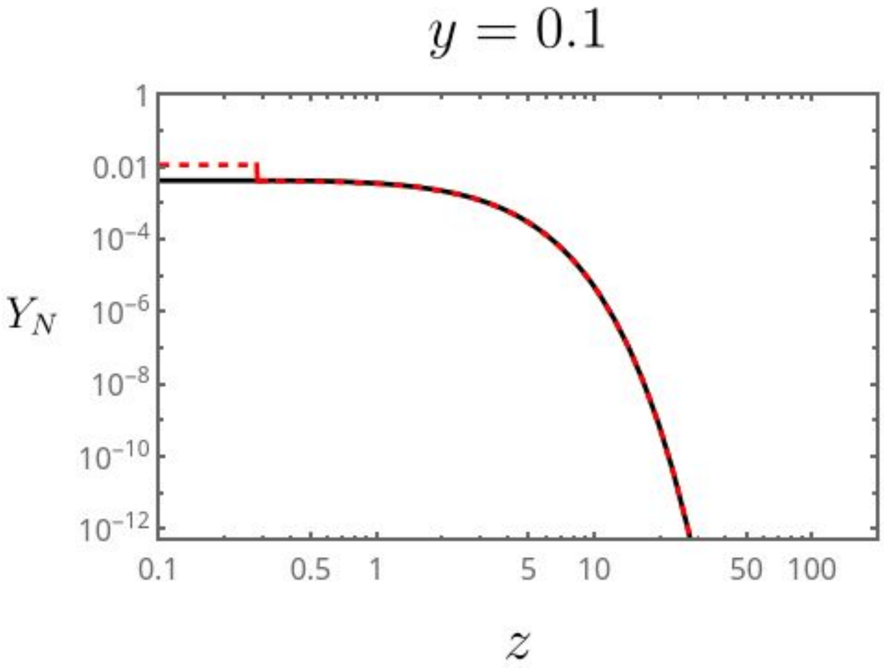
$$\rho(T_{\rm reh}) \simeq \rho(T_{\rm nuc}) + \Delta V$$

$$\Gamma_{\rm nuc.-rate}(T=T_{\rm nuc}) = H(T_{\rm nuc})^4$$

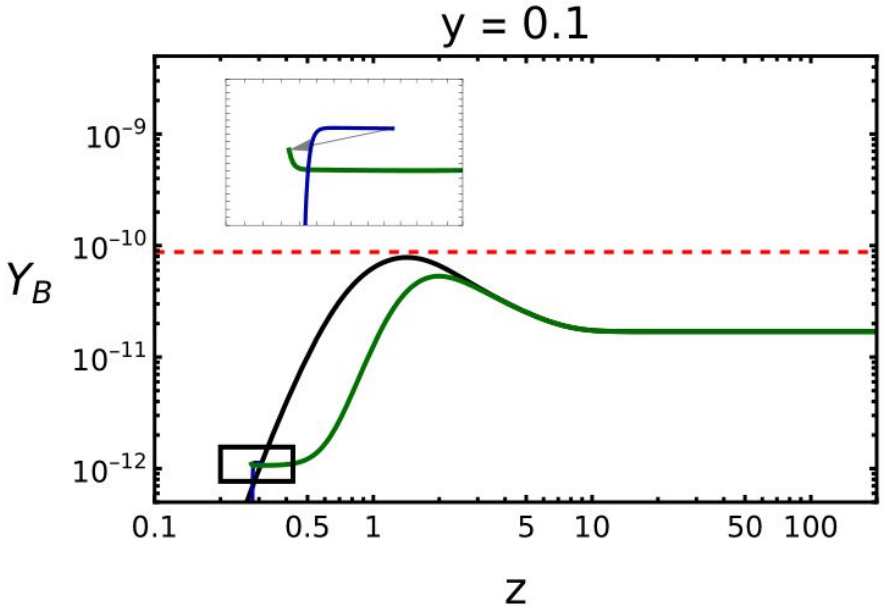
$$\alpha_n \equiv \frac{\Delta V}{\rho(T_{\rm nuc})}$$

$$\left(\frac{T_{\rm nuc}}{T_{\rm reh}}\right)^3 \simeq (1+\alpha_n)^{-3/4}$$

BEs

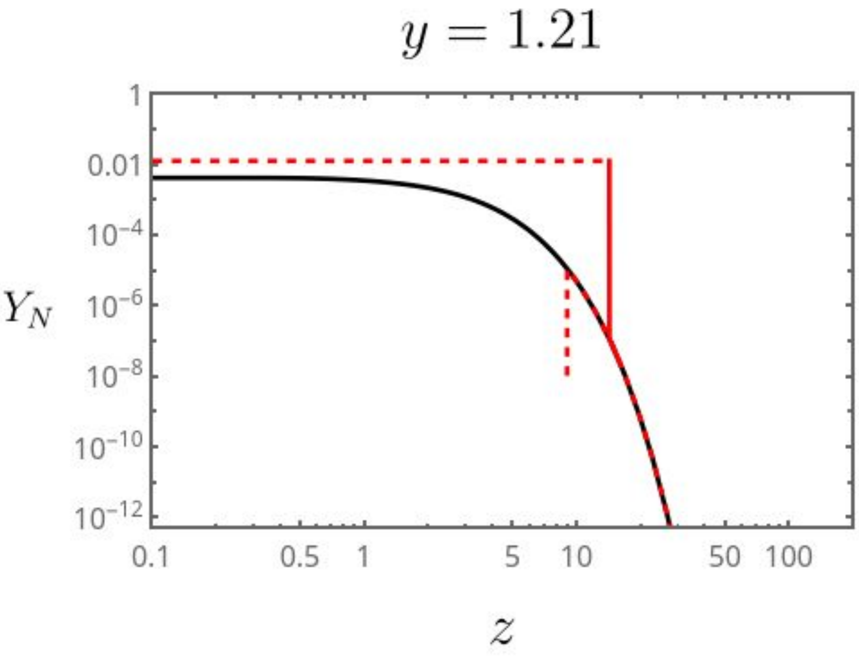


y = 0.1
1
0.01
10^{-4}
10^{-6}
10^{-8}
10^{-10}
10^{-12}
Y_N
0.1
0.5
1
5
10
50
100
z

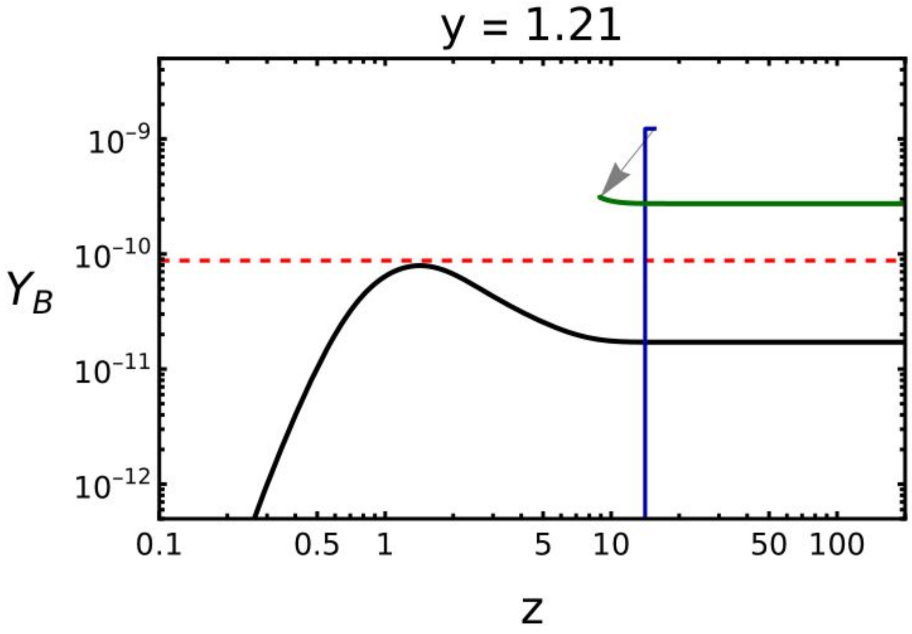


y = 0.1
10^{-9}
10^{-10}
10^{-11}
10^{-12}
Y_B
0.1
0.5
1
5
10
50
100
z

BEs

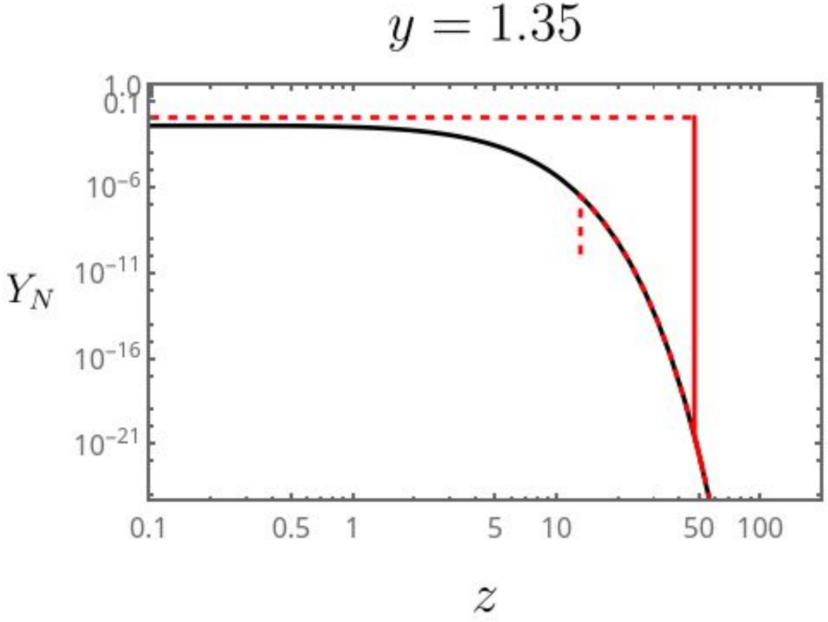


$y = 1.21$
1
0.01
10^{-4}
10^{-6}
10^{-8}
10^{-10}
10^{-12}
Y_N
0.1
0.5
1
5
10
50
100
z

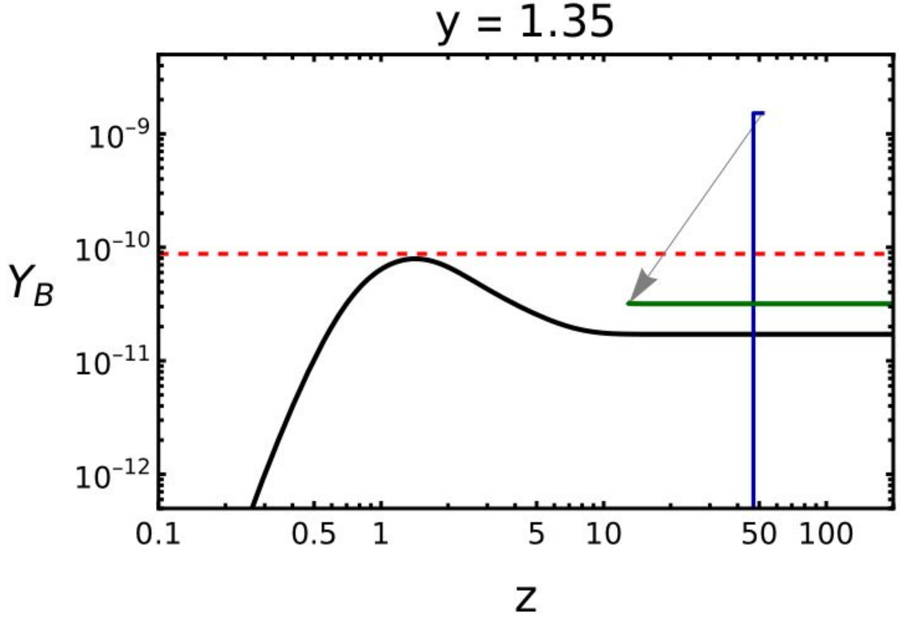


y = 1.21
10^{-9}
10^{-10}
10^{-11}
10^{-12}
Y_B
0.1
0.5
1
5
10
50
100
z

BEs



$y = 1.35$
1.0
0.1
10^{-6}
10^{-11}
10^{-16}
10^{-21}
Y_N
0.1
0.5
1
5
10
50
100
z



y = 1.35
10^{-9}
10^{-10}
10^{-11}
10^{-12}
Y_B
0.1
0.5
1
5
10
50
100
z