Interactions between several types of cosmic strings

The 3rd International Joint Workshop on the Standard Model and Beyond and the 11th KIAS Workshop on Particle Physics and Cosmology (2023/11/16)

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This work in collaboration with

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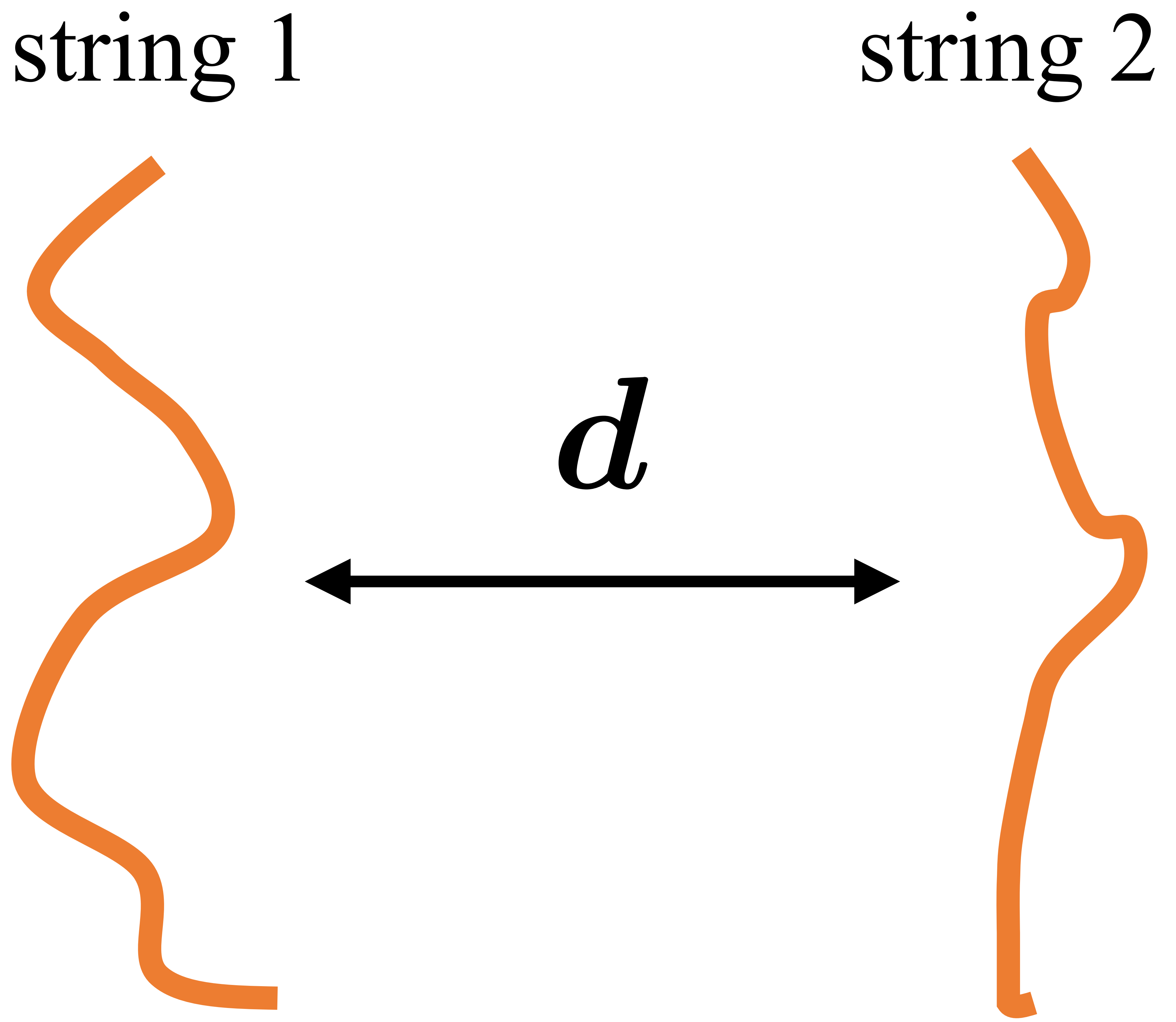
Masahide Yamaguchi (IBS)

Based on arXiv:2309.05515

The subject today

Suppose that two cosmic strings are parallel and separated by the fixed distance, d.



Can we estimate interactions between two strings?

Content

- Introduction

- Abrikosov-Nielsen-Olesen string (ANO string)

- Bosonic superconducting string

Cosmic Strings (Vortex lines)

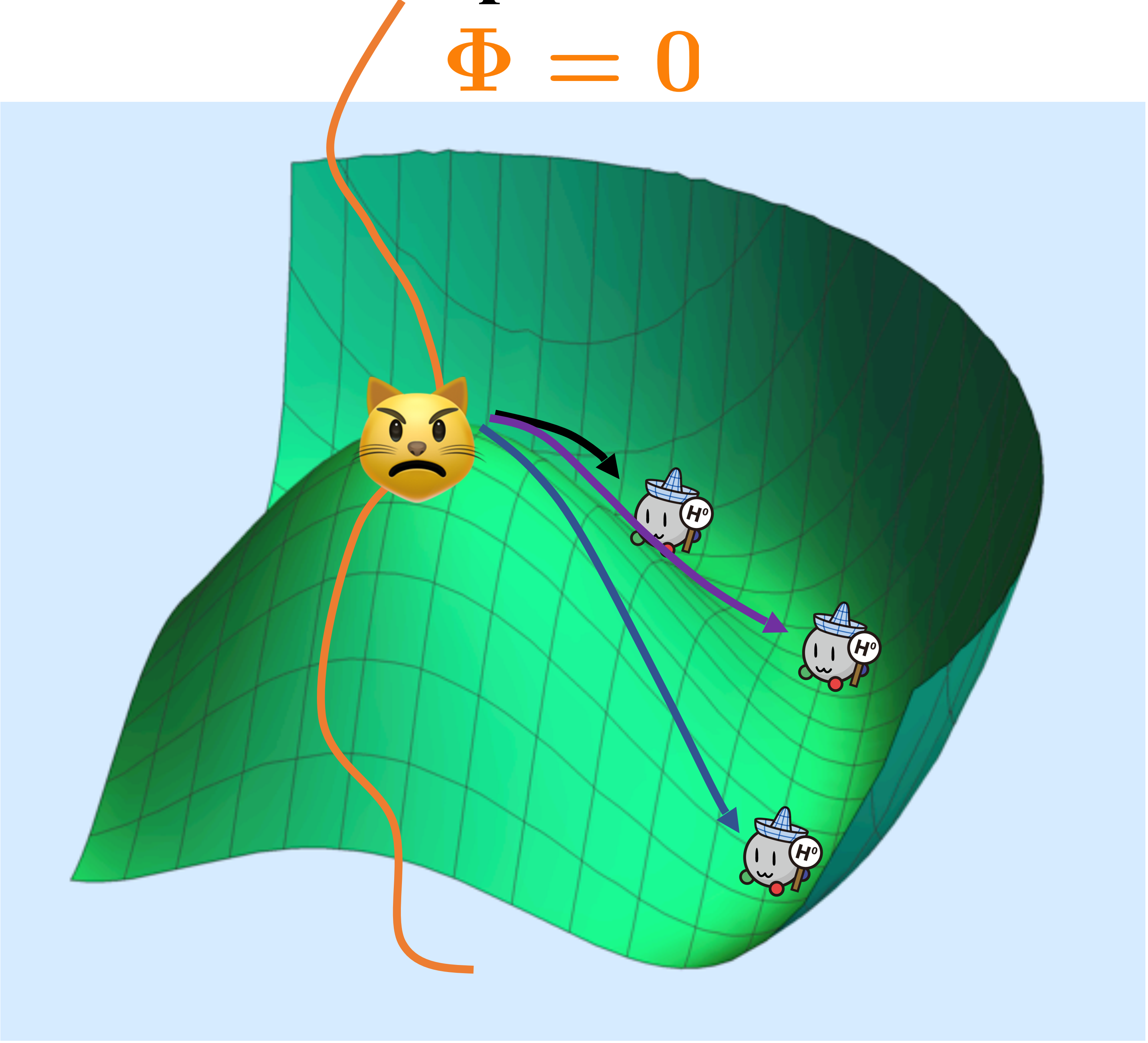
Cosmic Strings: A line-like topological defect which can be formed when a vacuum topology is $\mathbb{S}^1$.

$$V = \frac{\lambda}{4}\left(|\Phi|^2 - \eta^2\right)^2$$

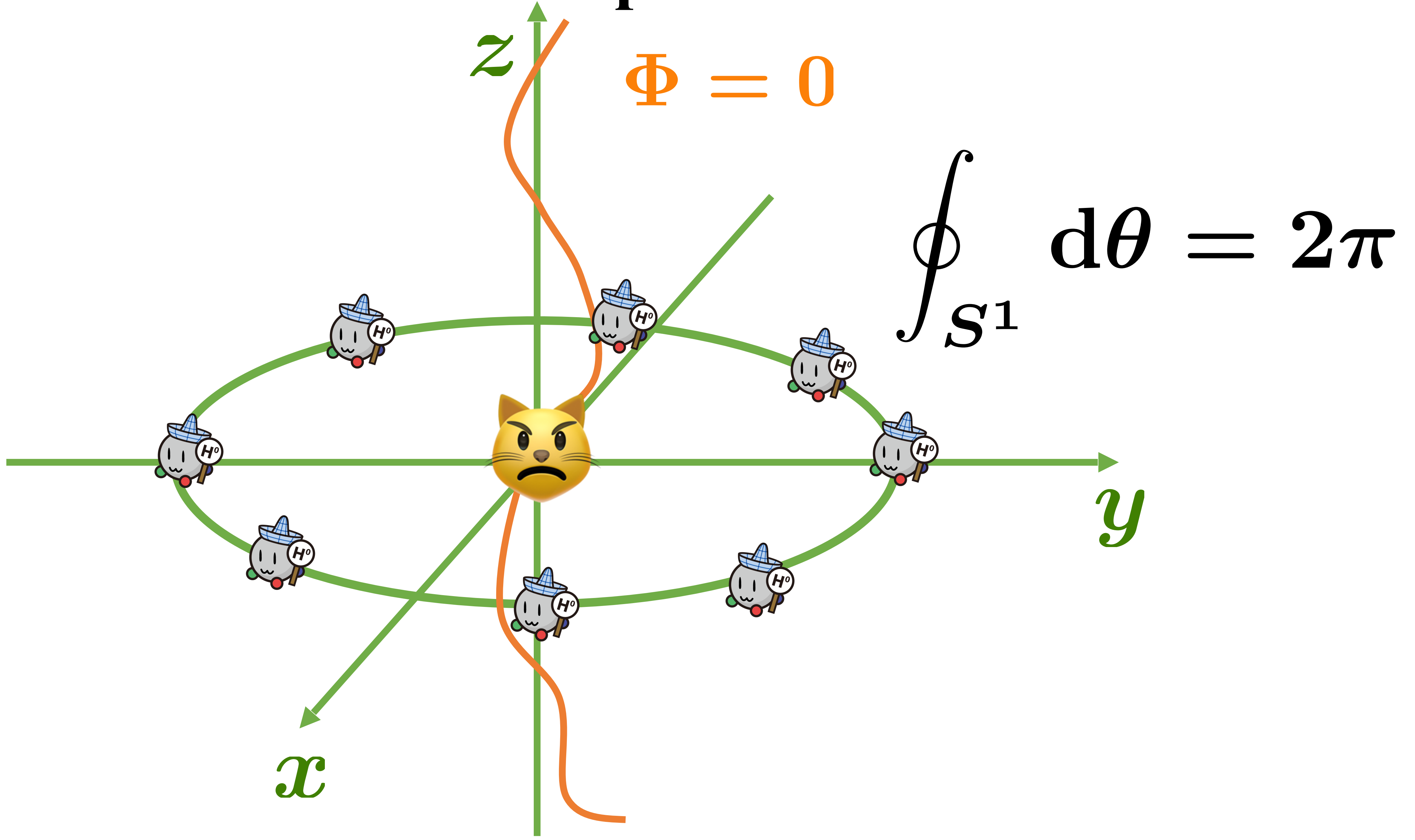
$$\Phi \to \eta\, e^{\pm i n \theta}$$

$n \in \mathbb{N}$: winding number

Field space



Position space

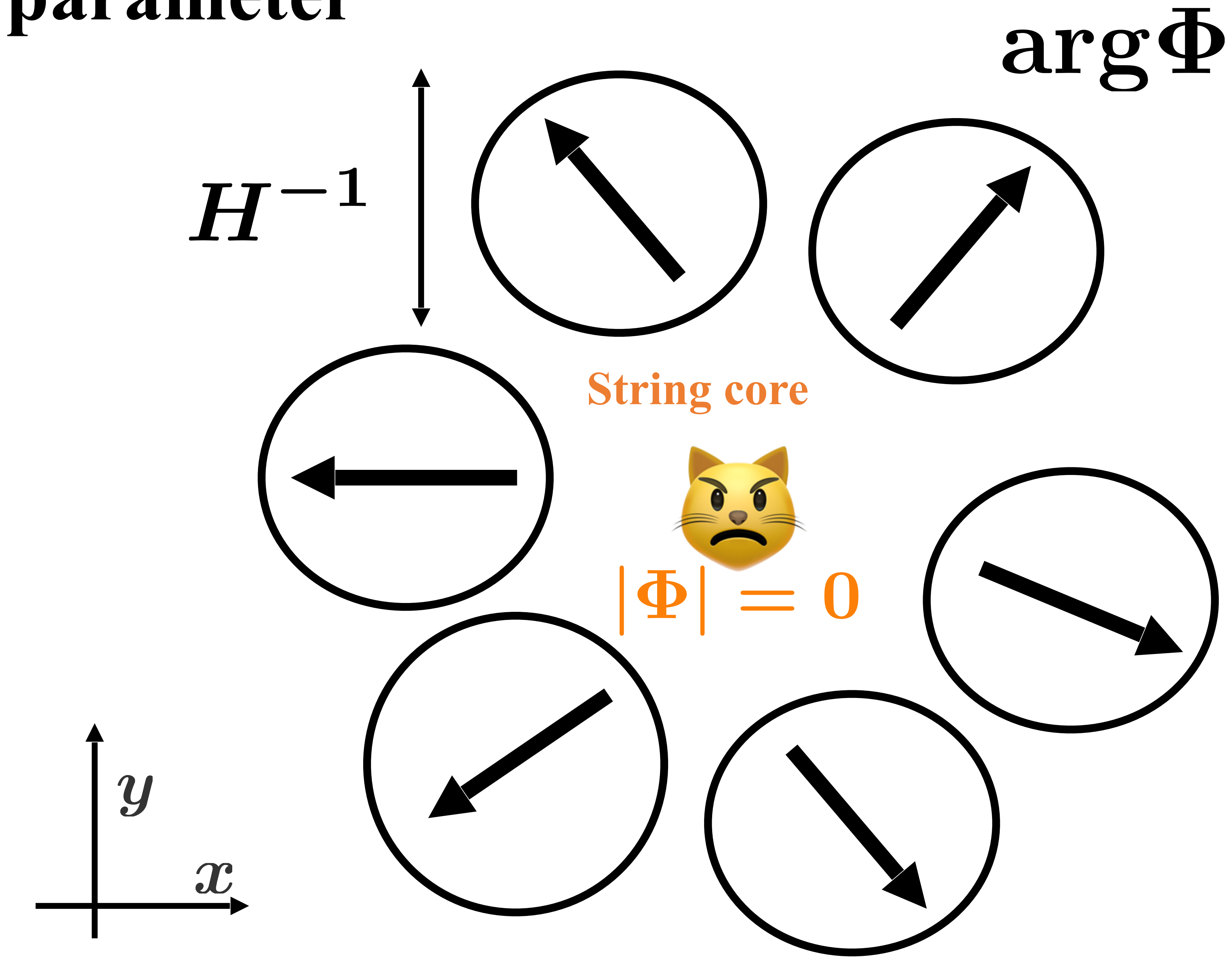


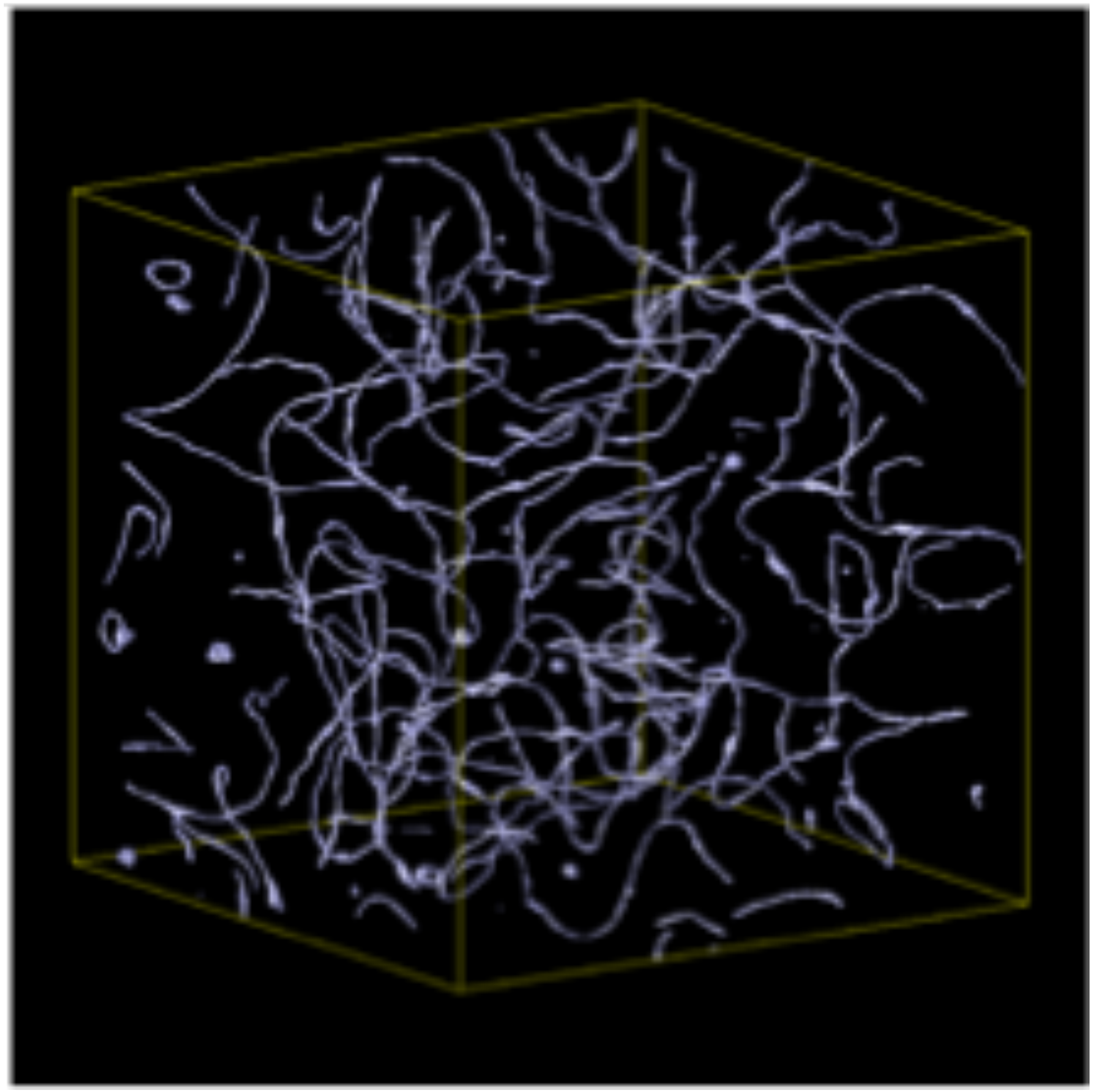
Production Mechanism

Randomness of the initial field configuration leads to the formation of topological soliton. (Kibble-Zurek mechanism)

[T. Kibble (1976)] [W.H. Zurek (1976)]

H:Hubble parameter





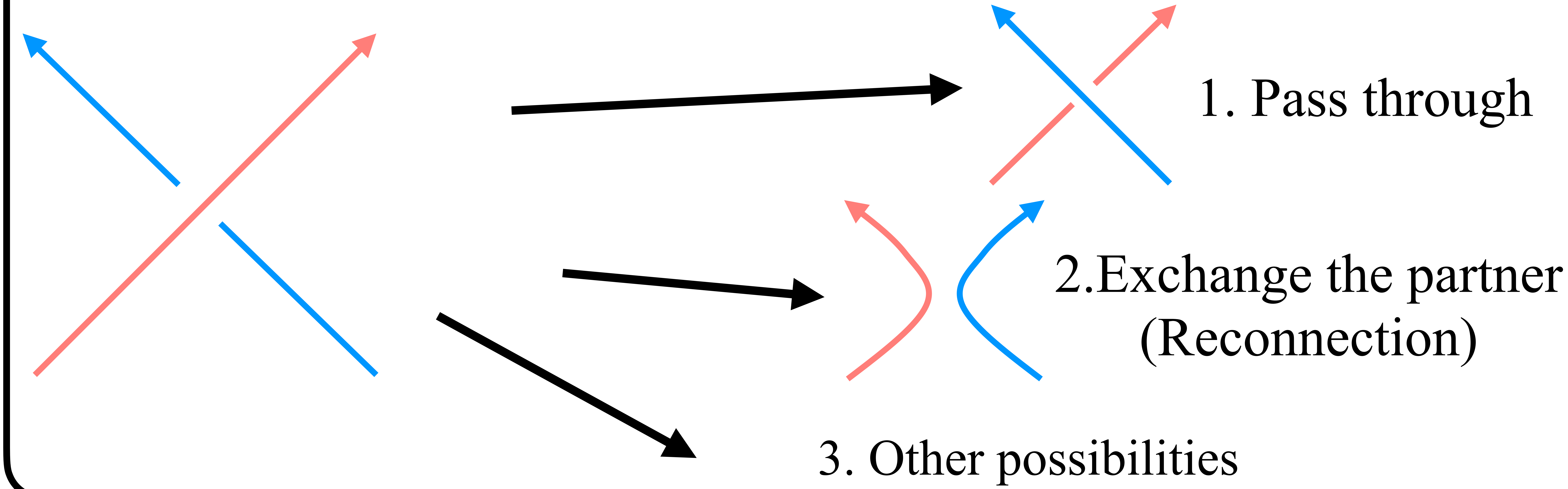
[T. Hiramatsu et al. (2013)]

After production of cosmic strings, they make up web-like structure. (String network)

Cosmic string collisions

Cosmic strings collide many times inside the string network.

Q: What happens when two strings collide?



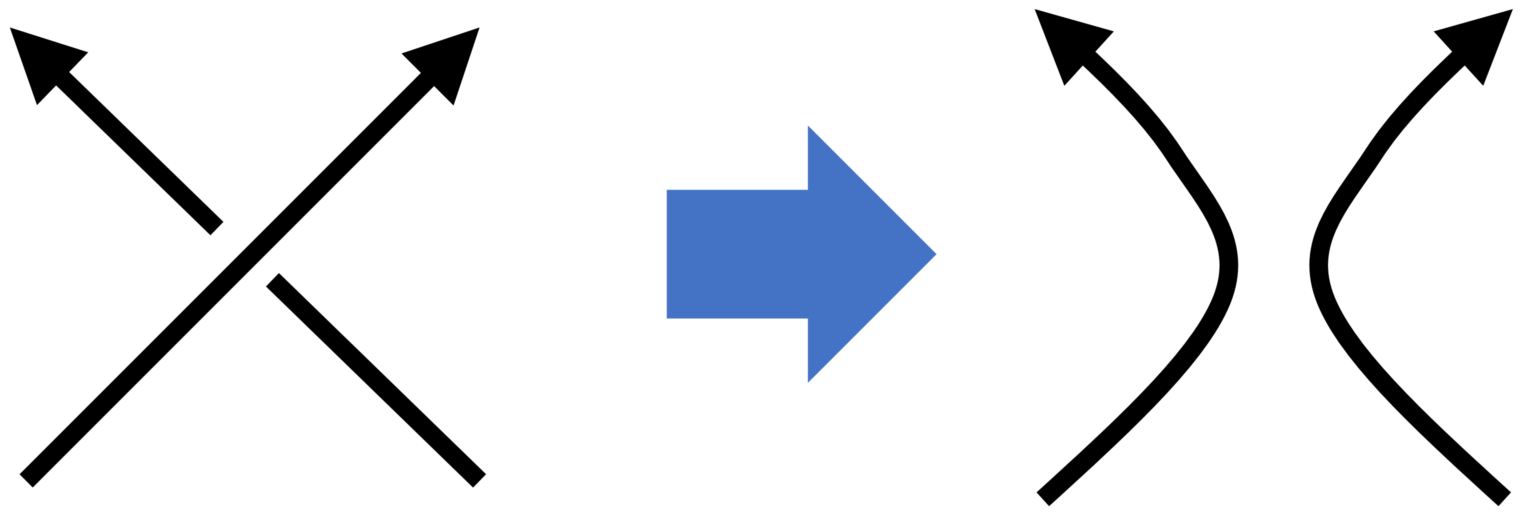
Reconnection

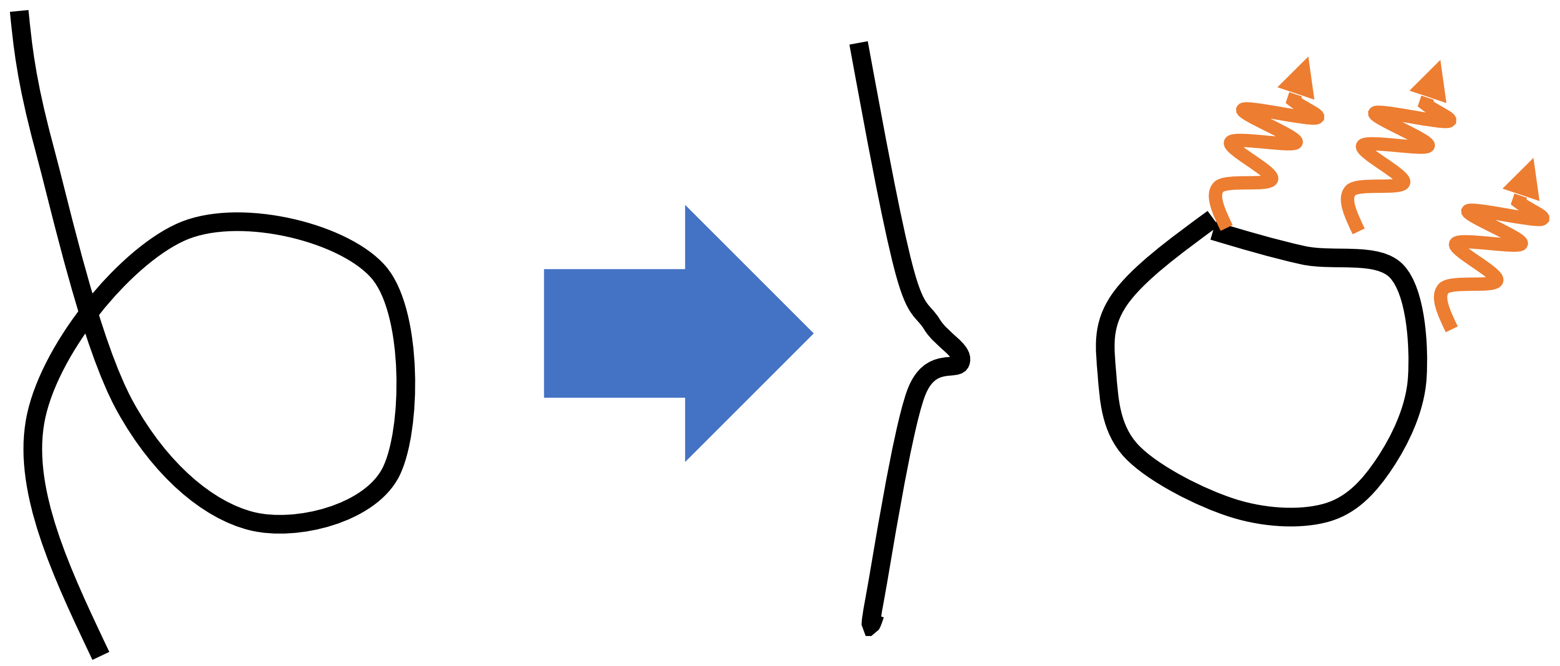
Conventional wisdom:
A reconnection would take place.

[E.P.S. Shellard] [R.A. Matzner]

Due to the reconnection, string loops are produced.

String loops loose their energy by emitting gravitational wave and/or NG-boson.

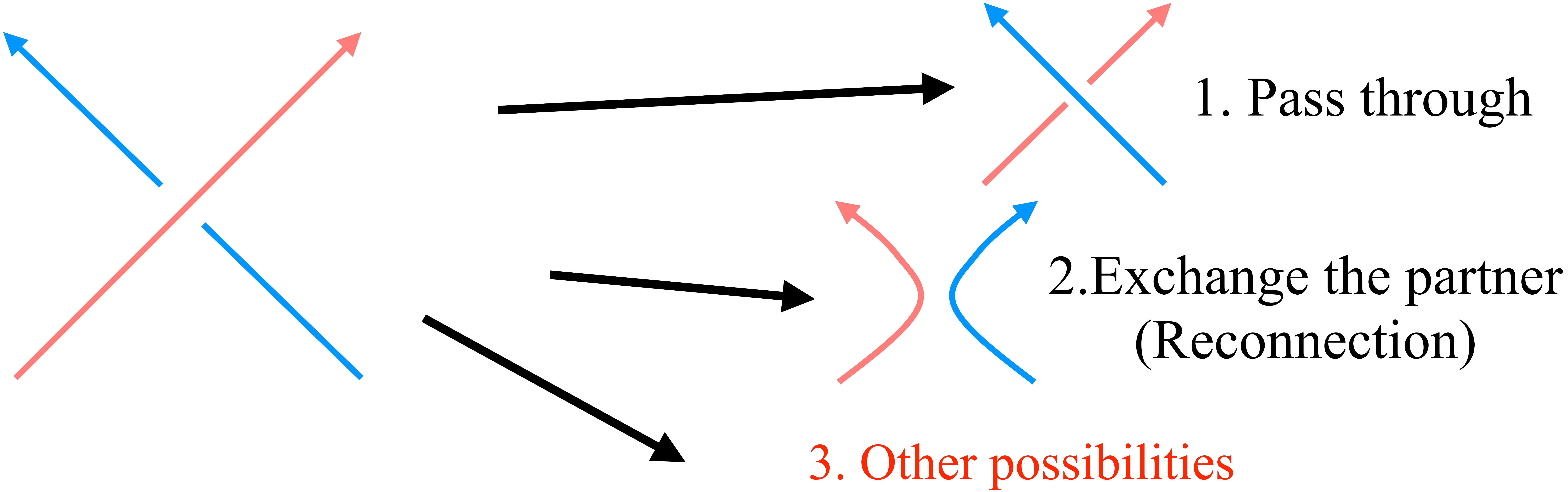




It is very widely believed that reconnection takes place in many field theoretical models (due to topological number conservation).

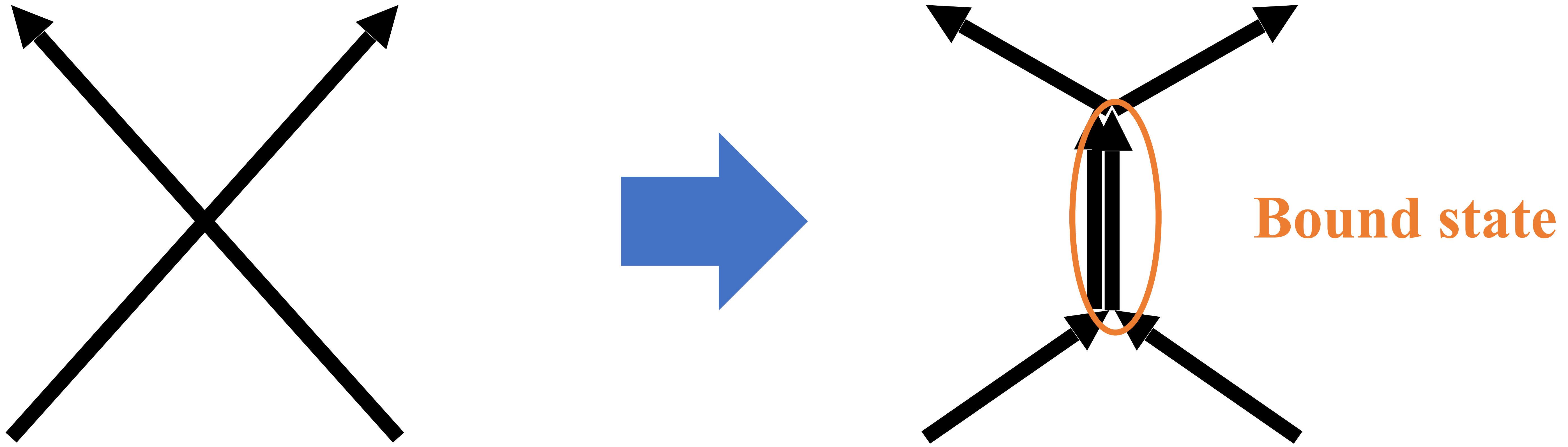
Other possibilities?

Q: What happens when two strings collide?



The third possibility

If interactions between two cosmic strings are attractive and strong enough, a string bound state would be formed after a collision. (Y-junction)



[J. Polchinski] [A. Bettencourt et al. (2006)] [E.J. Copeland et al. (2006)]

… and so many works.

Phenomenological consequences of Y-junction

Kinks and cusps generate gravitational wave bursts.

[P. Binetruy et al. (2009)] [P. Binetruy et al. (2010)] [Y. Matsui et al. (2020)]

Distinctive gravitational lensing effects

[B. Shlaer, M. Wyman (2005)] [R. Brandenberger et al. (2007)]

The subject today

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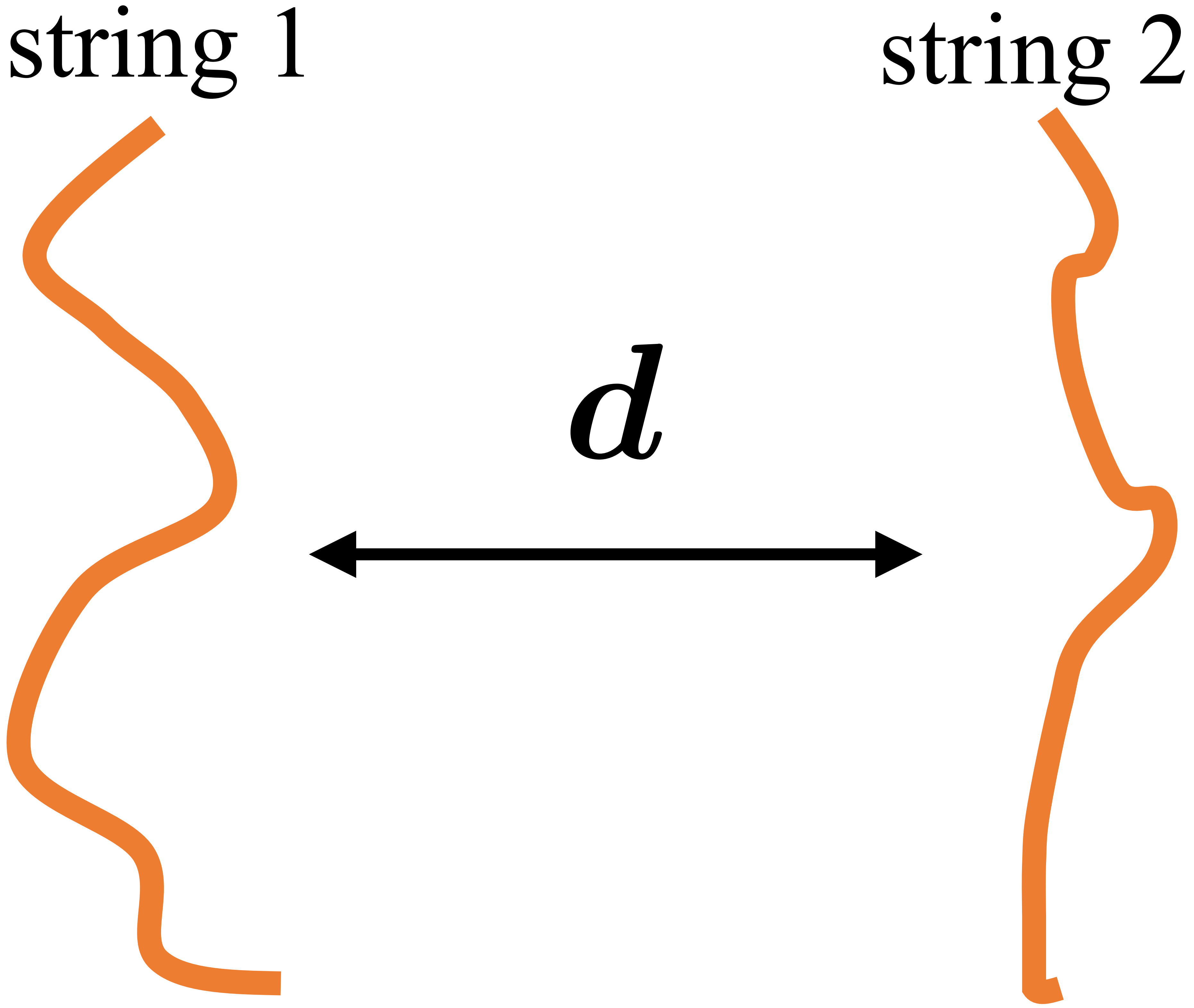


I would like to know interactions between two cosmic strings.
Attraction, or repulsion?

What I tell you today

1. Abrikosov-Nielsen-Olesen vortex string.

Nothing new, but I would like to briefly review the result.

[L. Jacobs and C. Rebbi (1979)]

[M. Eto, Y. Hamada, R. Jinno, M. Nitta, M. Yamada (2022)]

2. Bosonic superconducting string

New results (I will explain you our results.)

(We also analyze the global string, but I would like to omit it today.)

Content

- Introduction

- Abrikosov-Nielsen-Olesen string (ANO string)

- Bosonic superconducting string

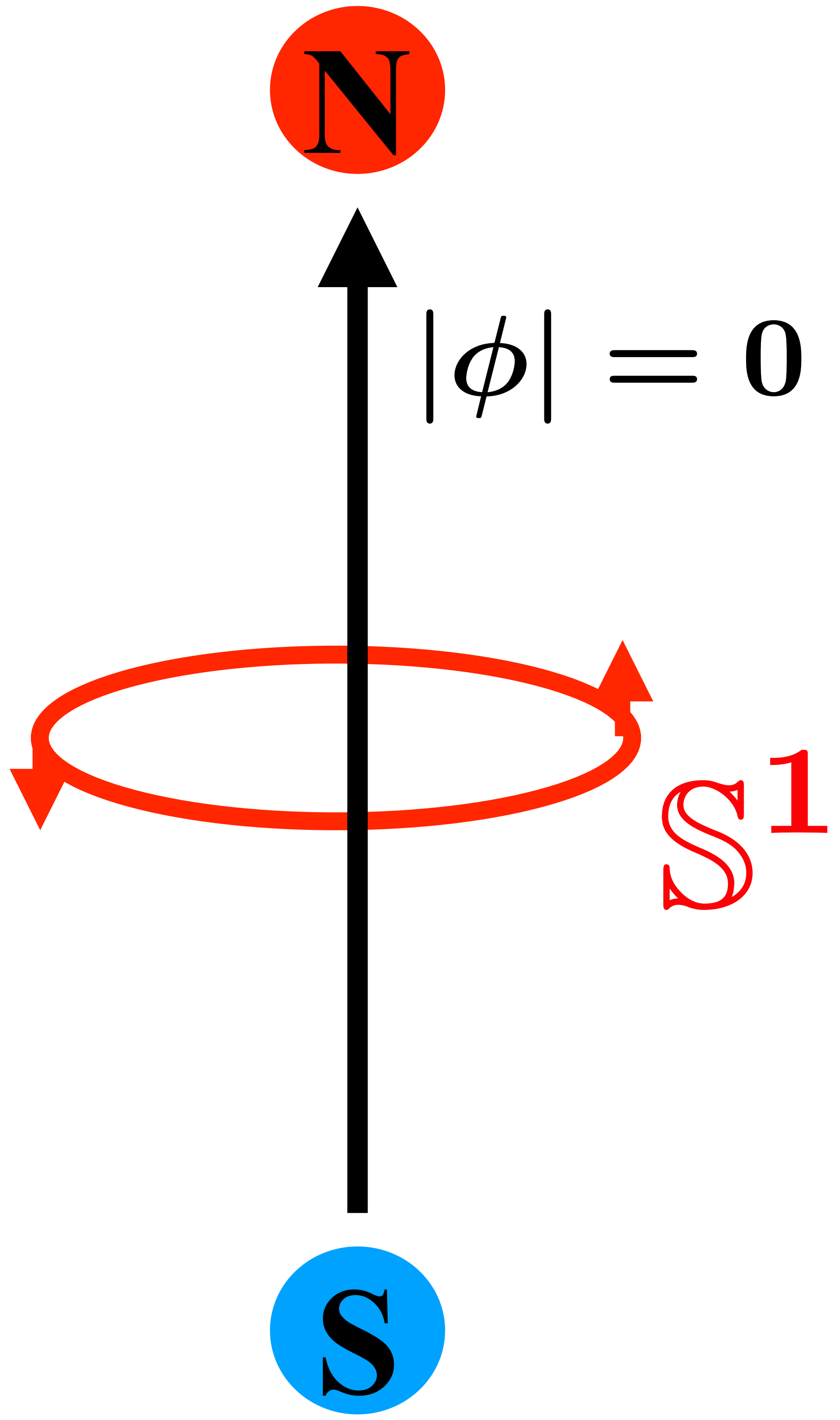
Abrikosov-Nielsen-Olesen vortex (1)

[A. A. Abrikosov (1957)] [H. B. Nielsen, P. Olesen (1973)]

- Abelian-Higgs model (The easiest model of vortex for human):

$$\mathcal{L}_{\mathrm{AH}} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + |D_\mu\phi|^2 - \frac{\lambda}{4}\left(|\phi|^2 - \eta^2\right)$$

$$D_\mu \equiv \partial_\mu - ieA_\mu \qquad F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$$



- Cylindrically symmetric configuration:

$$\phi = \phi_r(r)e^{in\theta}, \;\; A_\mu = A_\mu(r)\delta^\mu_\theta$$

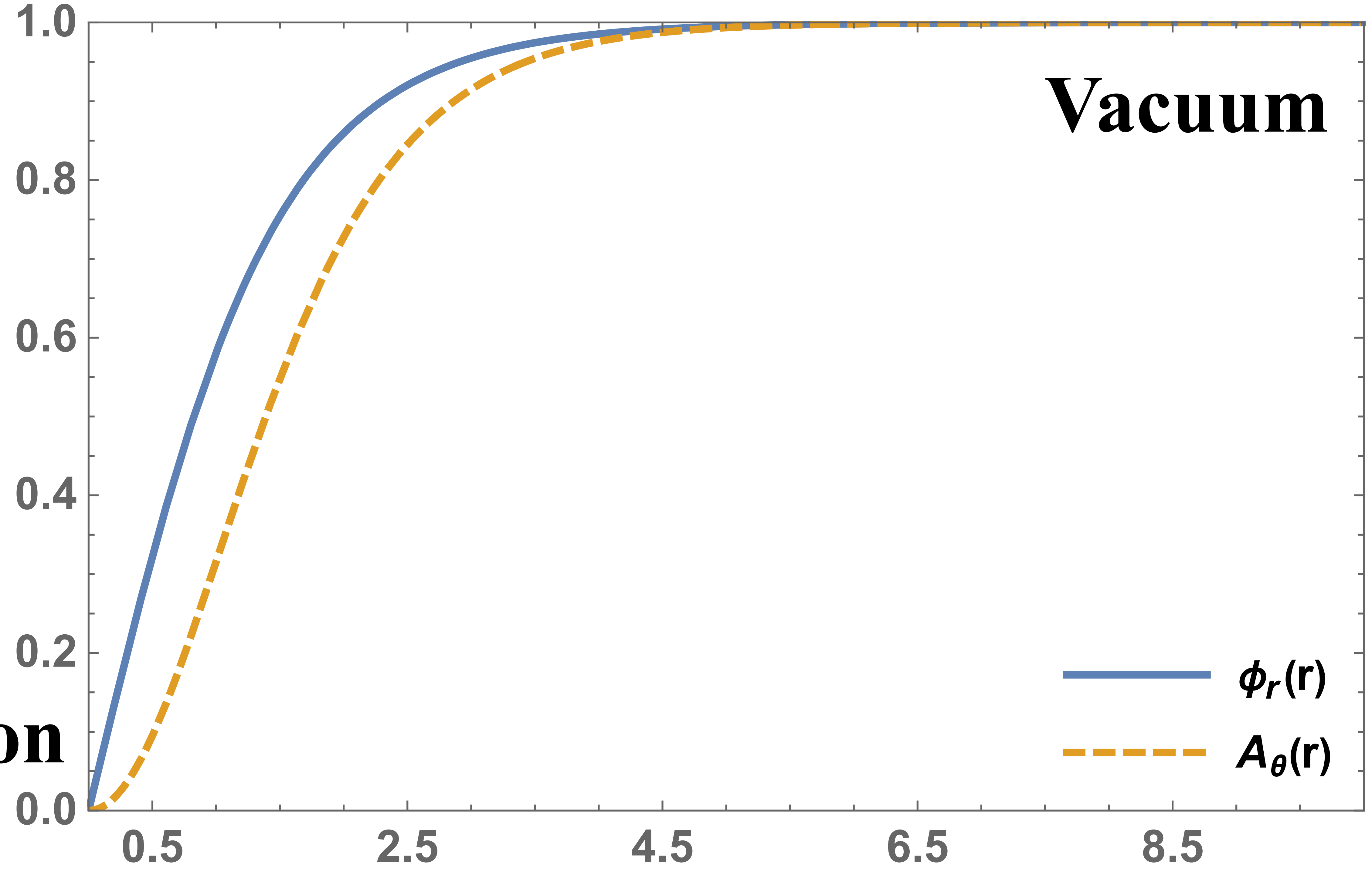
n: winding number

- **A quantization of the magnetic flux:** $\oint_{\mathbb{S}^1} A \sim \frac{1}{e}\oint_{\mathbb{S}^1} \mathrm{d}\theta \sim \frac{2\pi n}{e}$

Abrikosov-Nielsen-Olesen vortex (2)

Numerical solution

$\eta = 1$(unit)

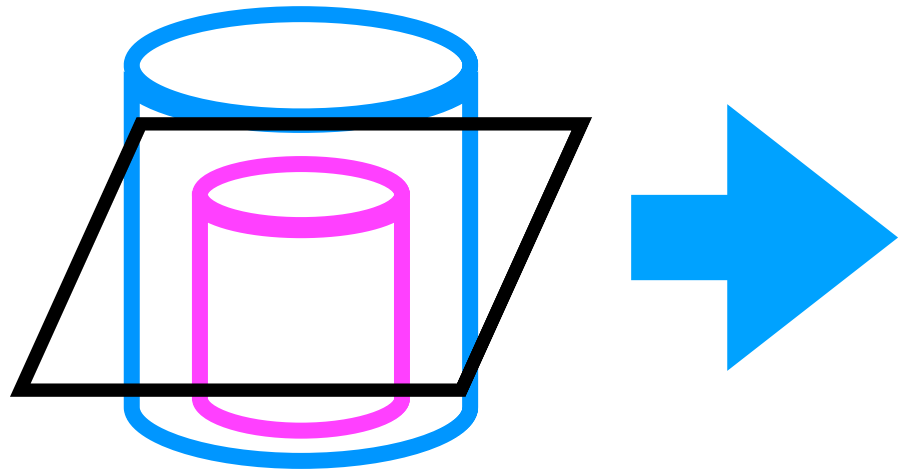


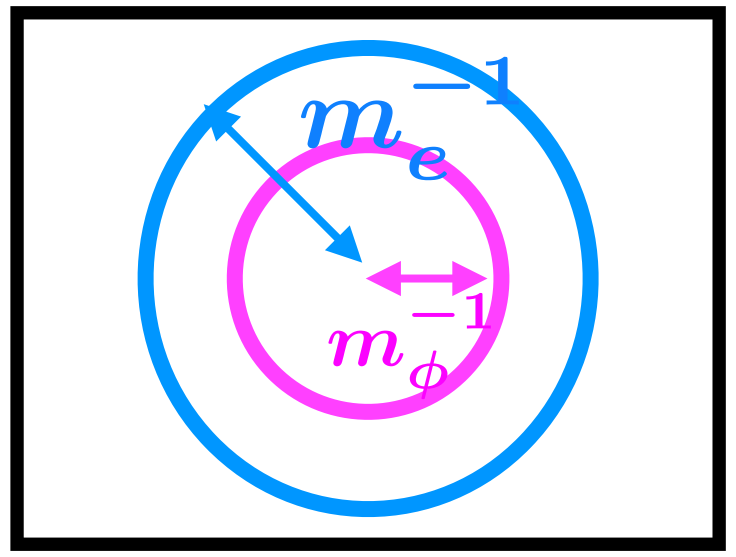
High energy region

$e\eta r$

$\lambda = 1,\ e = 1$

Thickness of the string: $R \sim \mathrm{Max}\{m_\phi^{-1}, m_e^{-1}\}$





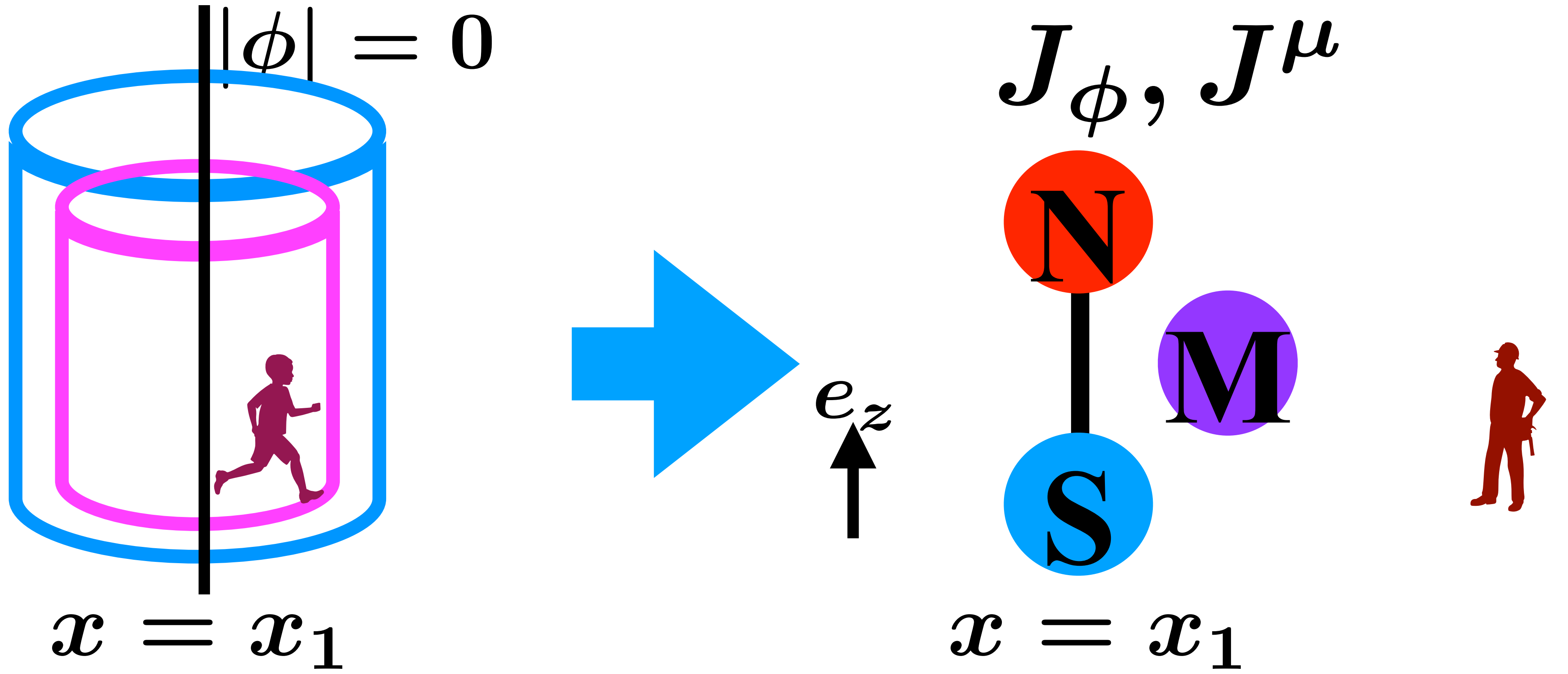
$m_\phi \sim \sqrt{\lambda}\eta \sim$ mass of the scalar field

$m_e \sim e\eta \sim$ mass of the gauge field

Effective interactions

At infinity, a straight cosmic string looks line-like field configuration which possesses scalar monopole charge and magnetic flux.

[J.M. Speight (1996))]



$J_\phi \propto |n|\delta^{(2)}(\mathbf{x} - \mathbf{x}_1)$:scalar monopole

$\boldsymbol{J} \propto n\mathbf{e}_z \times \nabla\delta^{(2)}(\mathbf{x} - \mathbf{x}_1)$:magnetic dipole moment

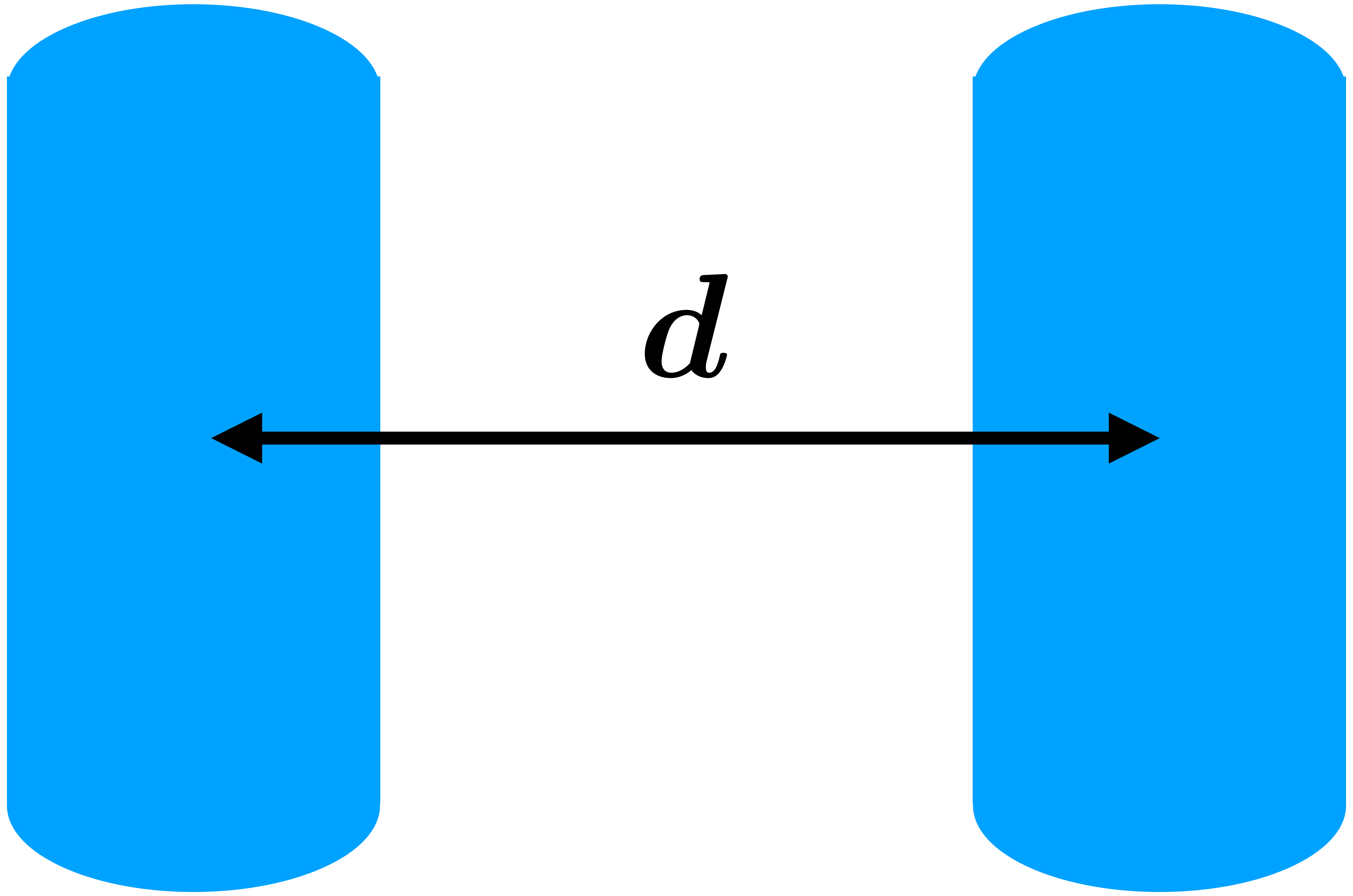
$\boldsymbol{n}$: winding number

Effective interactions:

$$\mathcal{L}_{\rm int} = -\delta\phi_r J_\phi - \delta A_\mu J^\mu$$

$\boldsymbol{\delta\phi, \delta A_\mu}$: field fluctuations from the vacuum state

A simplified version of the question:



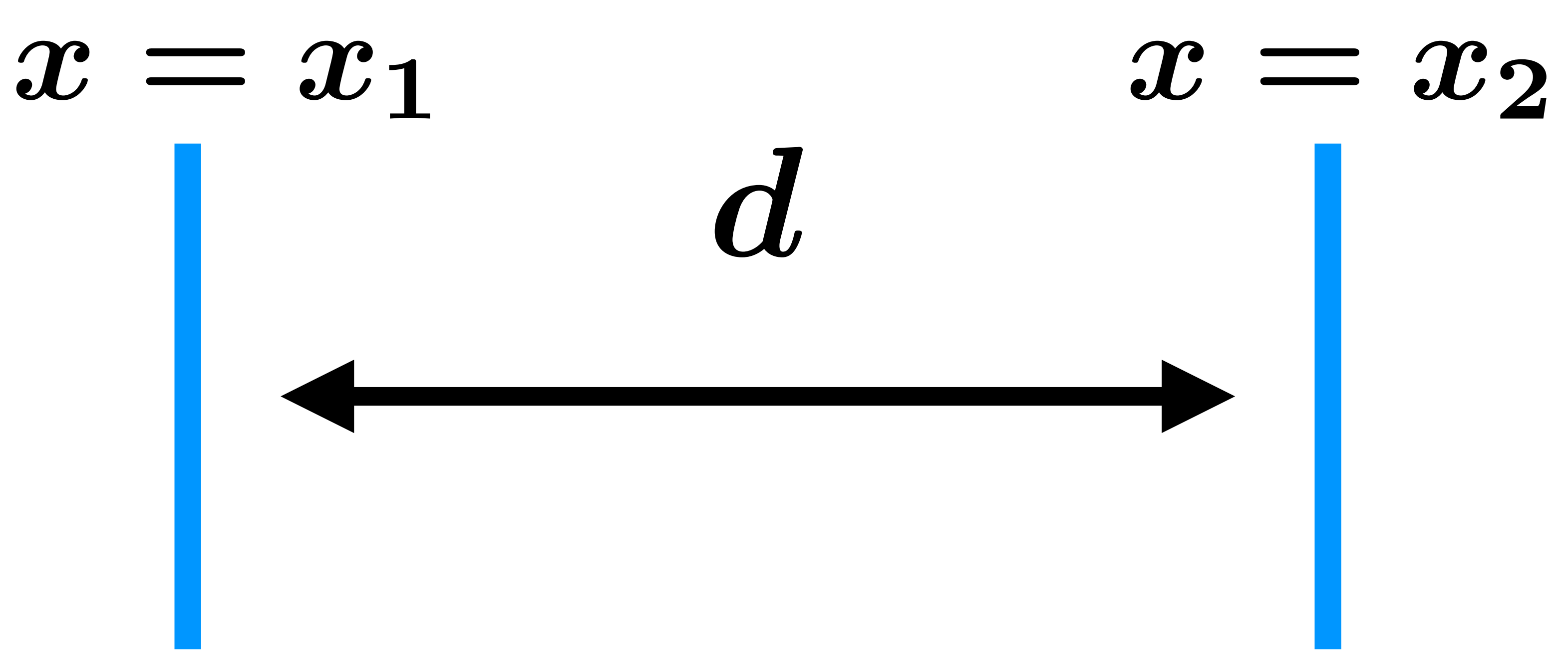
Q. Can we estimate interaction energy for $d \gg R$?

A. Yes (because we know the effective description.)

Point source formalism

Field configurations may be approximated by superposition of each strings for $\boldsymbol{d \gg R}$. (Superposition ansatz)

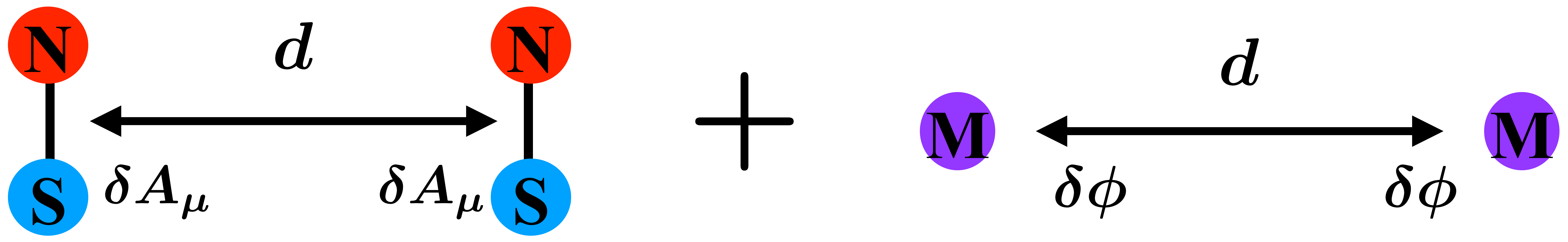
[J.M. Speight (1996))]



$$J_\phi = J_{\phi 1} + J_{\phi 2}, J^\mu = J_1^\mu + J_2^\mu$$

$$\mathcal{L}_{\rm int} = -\delta\phi_r J_\phi - \delta A_\mu J^\mu$$

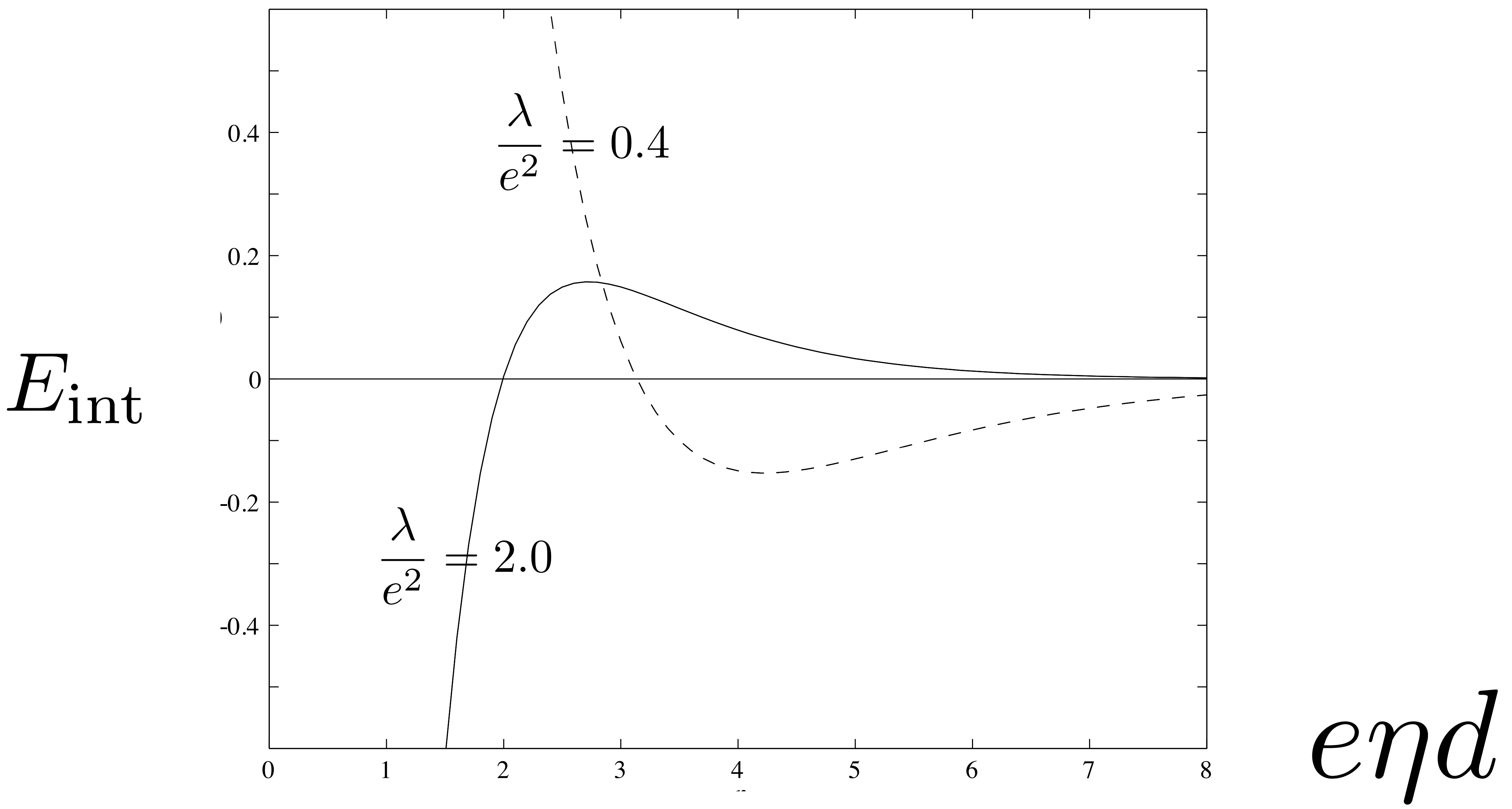
Naive interpretation: Interaction energies of two scalar monopole charges and two magnetic dipole moments.



Result of the point source formalism

$$E_{\text{int}} = \#_1 n_1 n_2 K_0(m_e d) - \#_2 K_0(m_\phi d)$$

Repulsion of gauge field **Attraction of scalar field**



$\#_{1,2} > 0$: Charges of cosmic string

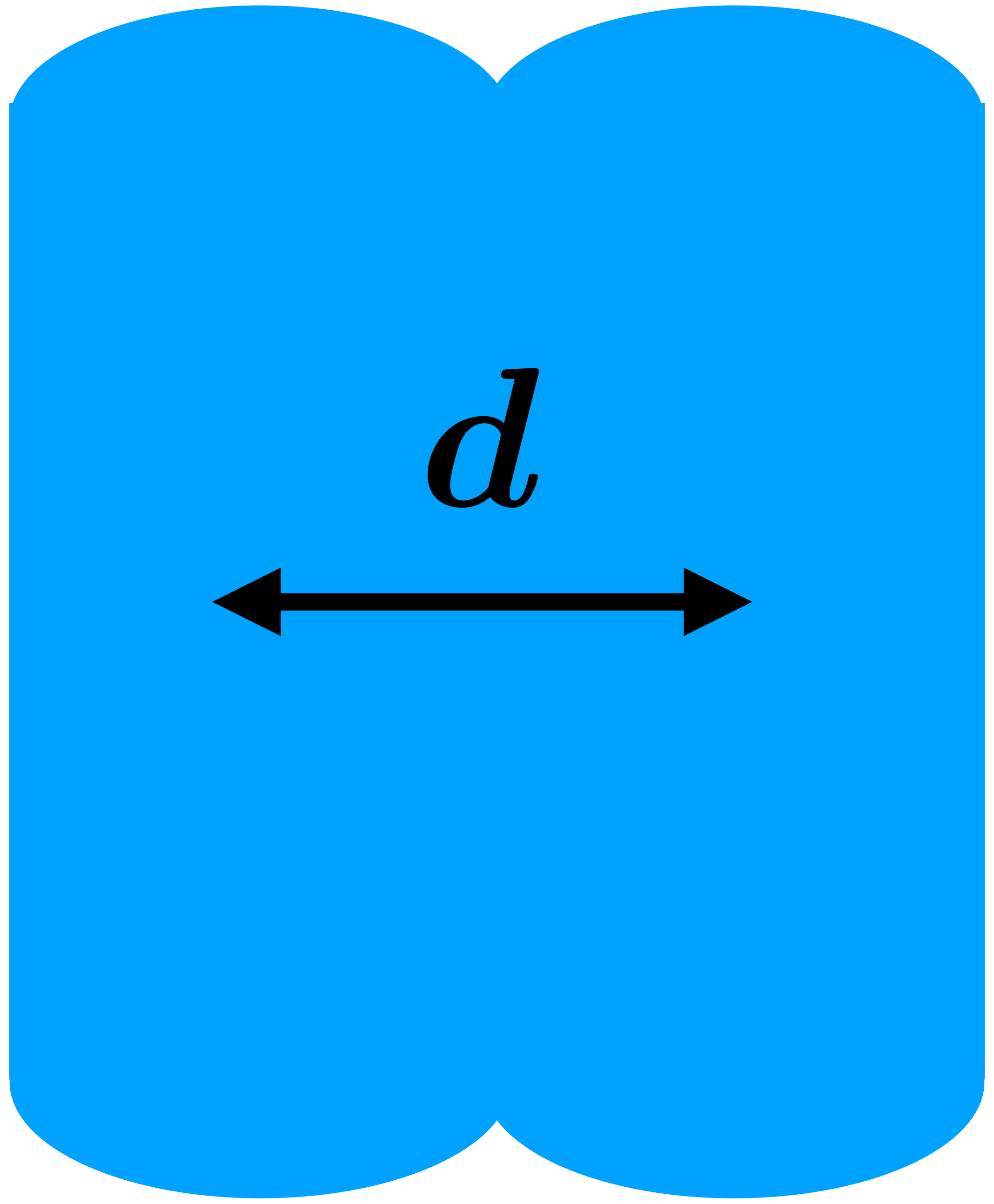
$K_n(x)$: Modified Bessel function of the second kind

[J.M. Speight (1996))]

Essentially, we obtain two-dimensional Yukawa potentials.

$m_\phi < m_e$ ($\lambda < 2e^2$) $\Rightarrow$ Attraction is dominated at infinity.

Non-trivial question:



Q.Can we estimate interaction energy for an arbitrarily d?

A. Numerical calculation is required. （I couldn’t estimate it analytically.）

Gradient Flow Method

Gradient flow (relaxation) method

Find the minimum energy configuration from the random field configuration.

$\mathcal{O}(x) \to \mathcal{O}(t,x)$ **t: fictitious time** $\quad (\mathcal{O} = \phi$ or $A_\mu)$

Heat equation: $$\frac{\partial \mathcal{O}}{\partial t} = -\frac{\delta E}{\delta \mathcal{O}}$$ **($E[\phi, A_\mu]$: The (2D) energy)**

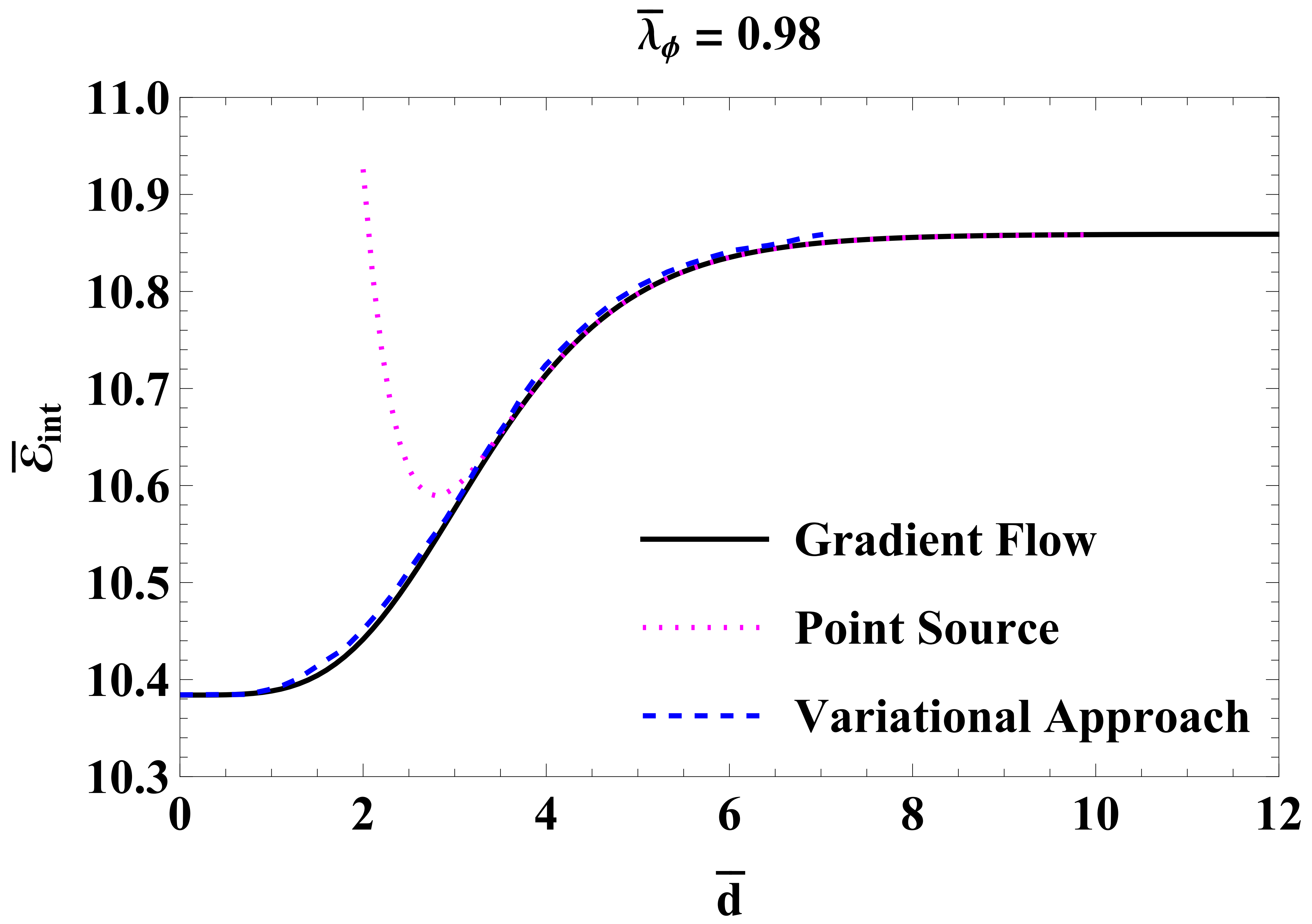
$$\frac{\mathrm{d}E}{\mathrm{d}t} = \frac{\delta E}{\delta \mathcal{O}}\frac{\partial \mathcal{O}}{\partial t} \propto -\left(\frac{\delta E}{\delta \mathcal{O}}\right)^2 < 0$$

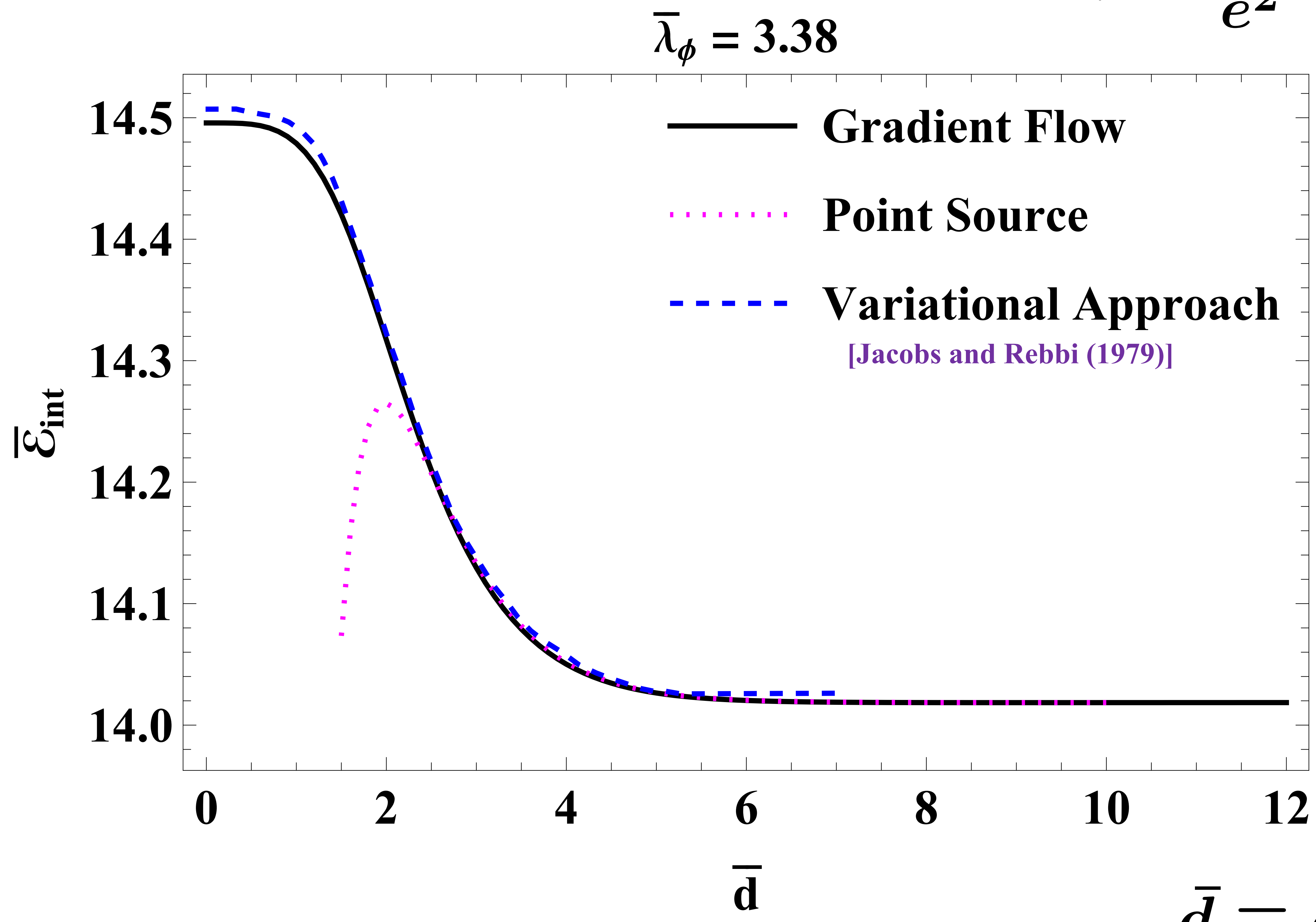
➡ Energy always decreases by the time development

$\frac{\partial \mathcal{O}}{\partial t} = 0 \Rightarrow \frac{\delta E}{\delta \mathcal{O}} = 0$: If time evolution is converged, field configurations satisfy original equation of motions.

Abrikosov-Nielsen-Olesen strings (2)

$$\bar{\lambda}_\phi \equiv \frac{\lambda}{e^2}$$





$$\bar{d} \equiv e\eta_\phi d$$

At long distance, numerical results agree with analytic estimate.

A phase structure of the ANO string is very simple.
When $e^2 > \lambda$, attraction always dominates for $\forall d$.

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- Introduction
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- Bosonic superconducting string

Bosonic superconducting string

- $U(1) \times \tilde{U}(1)$ **model:** [E. Witten (1985))]

$$\mathcal{L} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + |D_\mu \phi|^2 - \frac{1}{4} \tilde{F}^{\mu\nu} \tilde{F}_{\mu\nu} + |\tilde{D}_\mu \tilde{\phi}|^2 - V(\phi, \tilde{\phi})$$

$$V(\phi, \tilde{\phi}) = \frac{\lambda_\phi}{4} \left(|\phi|^2 - \eta_\phi^2 \right)^2 + \frac{\lambda_{\tilde{\phi}}}{4} \left(|\tilde{\phi}|^2 - \eta_{\tilde{\phi}}^2 \right)^2 + \beta |\phi|^2 |\tilde{\phi}|^2$$

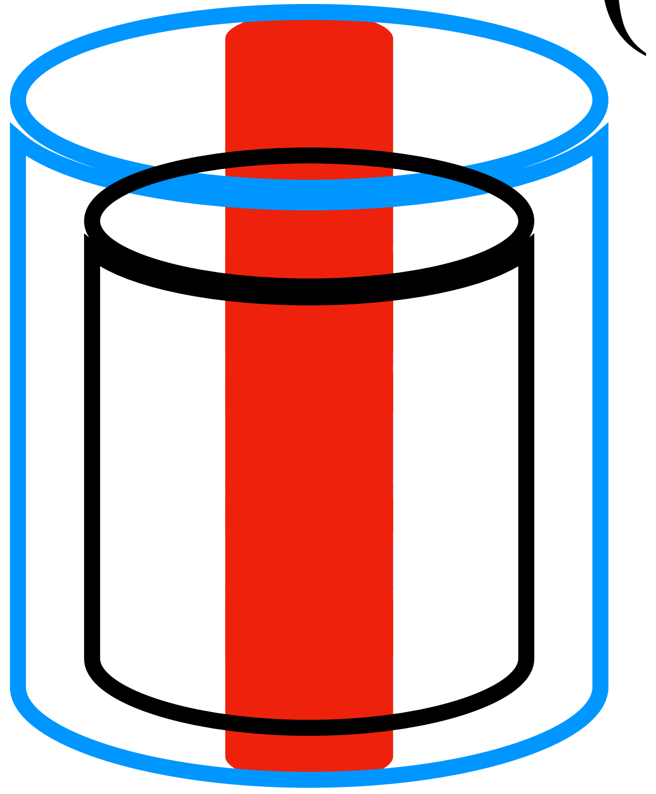
Bosonic superconducting string

- $U(1) \times \tilde{U}(1)$ **model:** [E. Witten (1985))]

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + |D_\mu \phi|^2 - \frac{1}{4}\tilde{F}^{\mu\nu}\tilde{F}_{\mu\nu} + |\tilde{D}_\mu \tilde{\phi}|^2 - V(\phi, \tilde{\phi})$$

$$V(\phi, \tilde{\phi}) = \frac{\lambda_\phi}{4}\left(|\phi|^2 - \eta_\phi^2\right)^2 + \frac{\lambda_{\tilde{\phi}}}{4}\left(|\tilde{\phi}|^2 - \eta_{\tilde{\phi}}^2\right)^2 + \beta|\phi|^2|\tilde{\phi}|^2$$

- There exists the case where $\tilde{U}(1)$ is only broken inside the string. ($U(1)$ plays a role of an ordinary string.)



At the string axis, $\phi \sim 0$: $A_\mu \sim 0$ and $\tilde{\phi} \sim \eta_{\tilde{\phi}}$

Far from the string, $\phi \sim \eta_\phi$: $A_\mu \sim (i/e)\partial_\mu \theta$ and $\tilde{\phi} \sim 0$

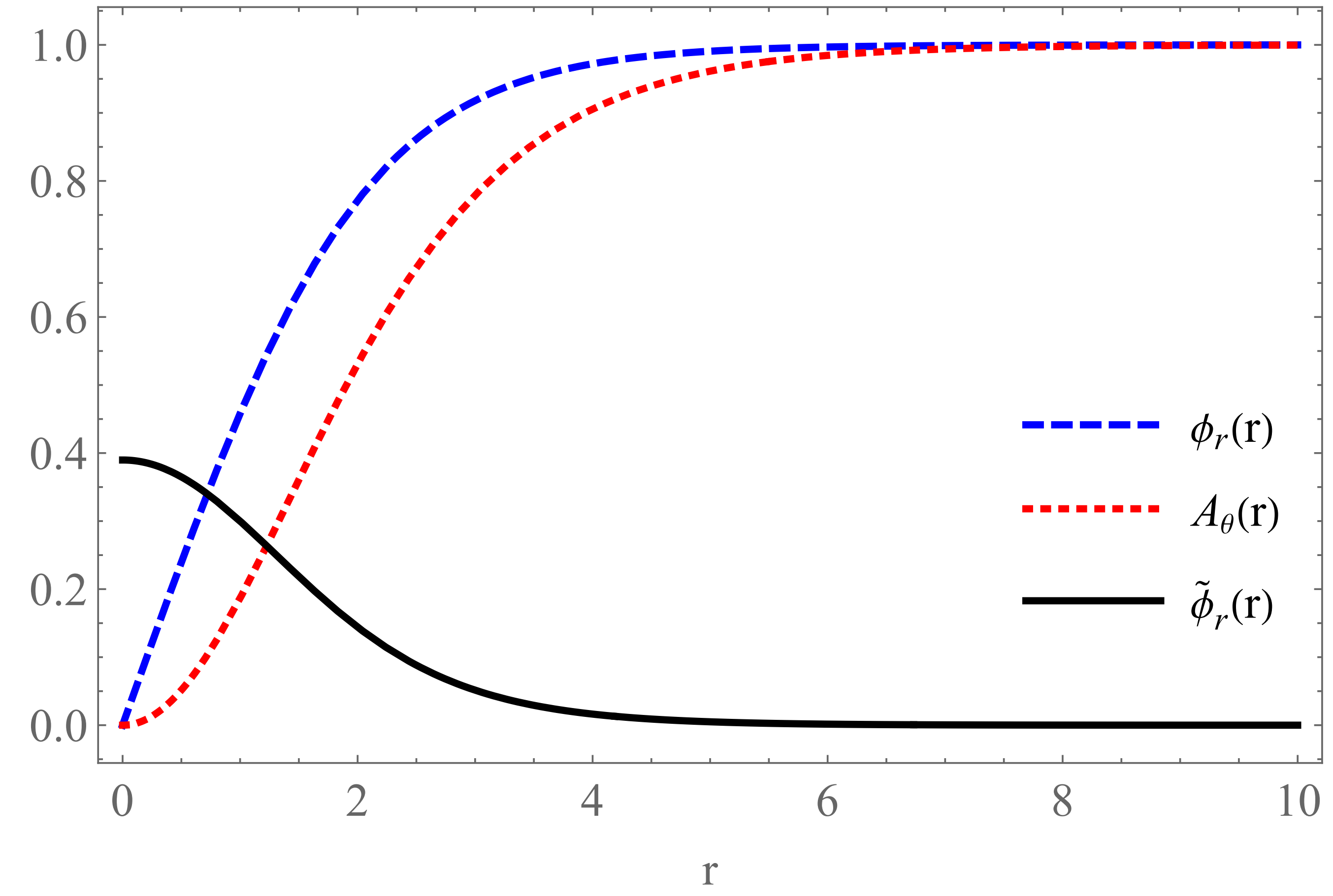
- Due to the time limitation, I do not include $\widetilde{U}(1)$ gauge field.

Bosonic superconducting string

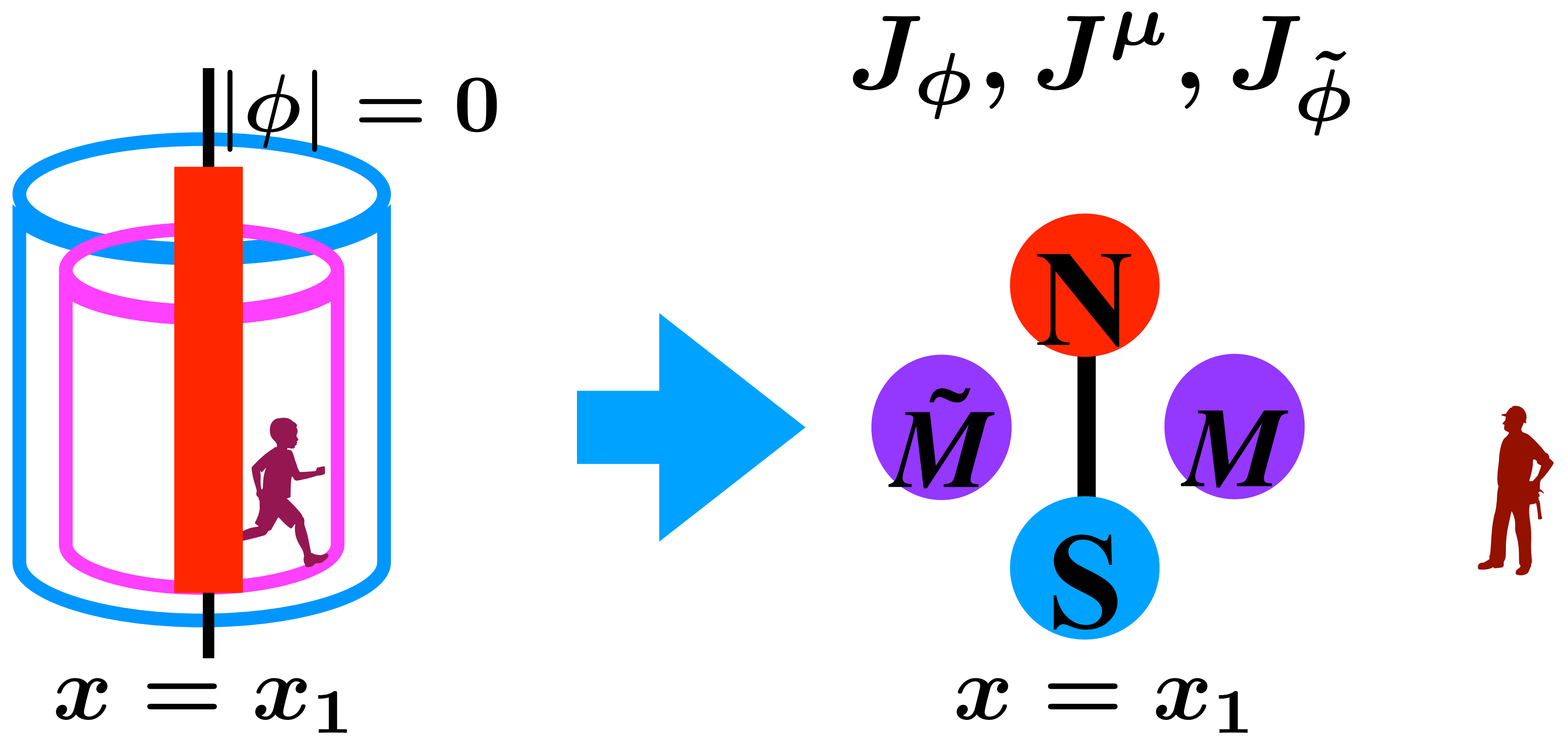
- A localized condensate of $\tilde{\phi}$ takes place when

$$\frac{\lambda_{\tilde{\phi}}}{2}\frac{\eta_{\tilde{\phi}}^2}{\eta_{\phi}^2} \lesssim \beta \lesssim \frac{1}{4}\frac{\lambda_{\tilde{\phi}}^2}{\lambda_{\phi}}\frac{\eta_{\tilde{\phi}}^4}{\eta_{\phi}^4}$$

[E. Witten (1985))]



- Effective interactions: magnetic dipole moment + "two" scalar charges



$\delta\phi, \delta A_\mu, \delta\tilde{\phi}$: Field fluctuations from the ground state

$$\mathcal{L}_{\rm int} \supset -\delta\phi_r J_\phi - \delta A_\mu J^\mu - \delta\tilde{\phi} J_{\tilde{\phi}}$$

$J_\phi \propto |n|\delta^{(2)}(\mathbf{x}-\mathbf{x}_1)$:**scalar monopole** $U(1)$

$\mathbf{J} \propto n\mathbf{e}_z \times \nabla\delta^{(2)}(\mathbf{x}-\mathbf{x}_1)$:**magnetic dipole**

$J_{\tilde{\phi}} \propto \delta^{(2)}(\mathbf{x}-\mathbf{x}_1)$:**scalar monopole** $\tilde{U}(1)$

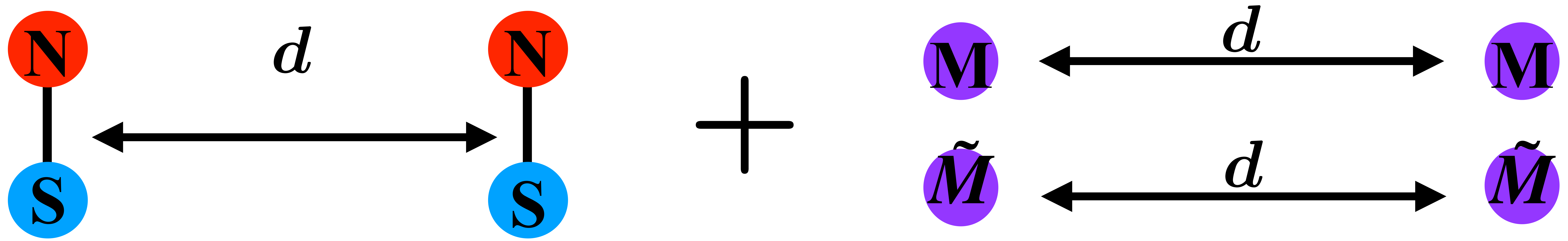
Results in linear analysis

$$E_{\rm int} = \#_1 n_1 n_2 K_0(m_e d) - \#_2 K_0(m_\phi d) - \#_3 K_0(m_{\tilde{\phi}} d))$$

$\#_{1,2,3} > 0$: numerical constants

$$m_e^2 \sim e^2 \eta_\phi^2,\ m_\phi^2 \sim \lambda_\phi \eta_\phi^2,\ m_{\tilde{\phi}}^2 \sim \beta \eta_\phi^2 - \frac{\lambda_{\tilde{\phi}}}{2} \eta_{\tilde{\phi}}^2$$

Naive interpretation: Interaction energies of two scalar charges with a magnetic dipole moment

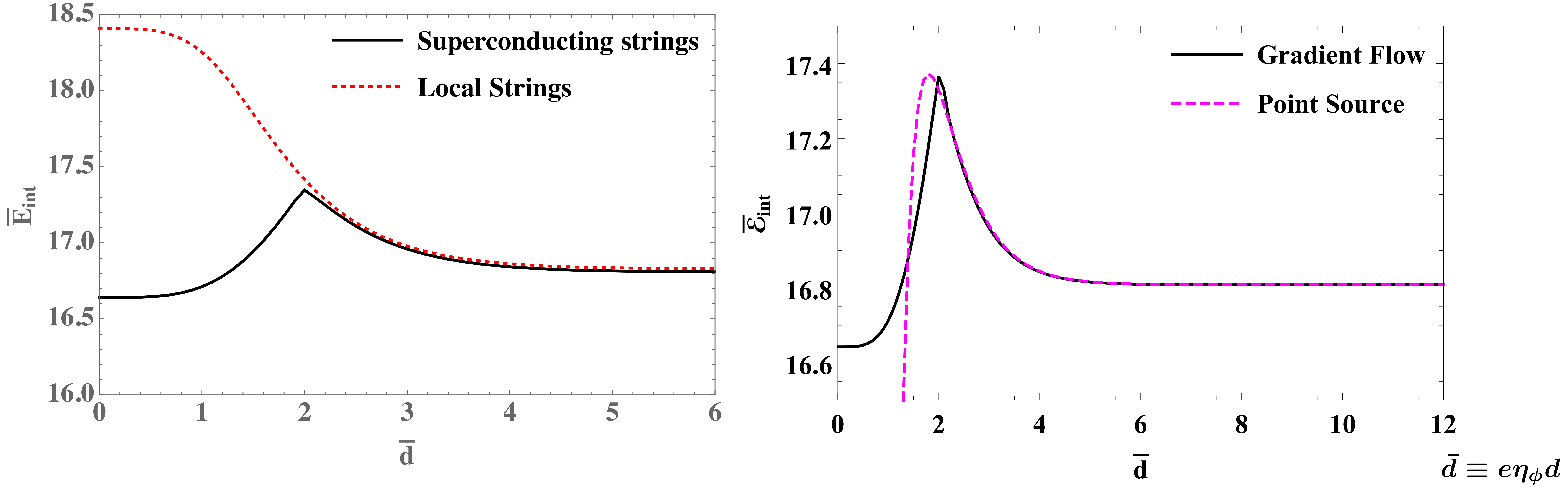


A point source formalism suggests an additional source of attraction.

I am going to show a numerical result.

Bosonic Superconducting String (2)

$$\frac{\lambda_\phi}{e^2} = 8,\ \frac{\lambda_{\tilde{\phi}}}{e^2} = 80,\ \frac{\beta}{e^2} = 24,\ \frac{\eta_{\tilde{\phi}}}{\eta_\phi} = 0.55$$

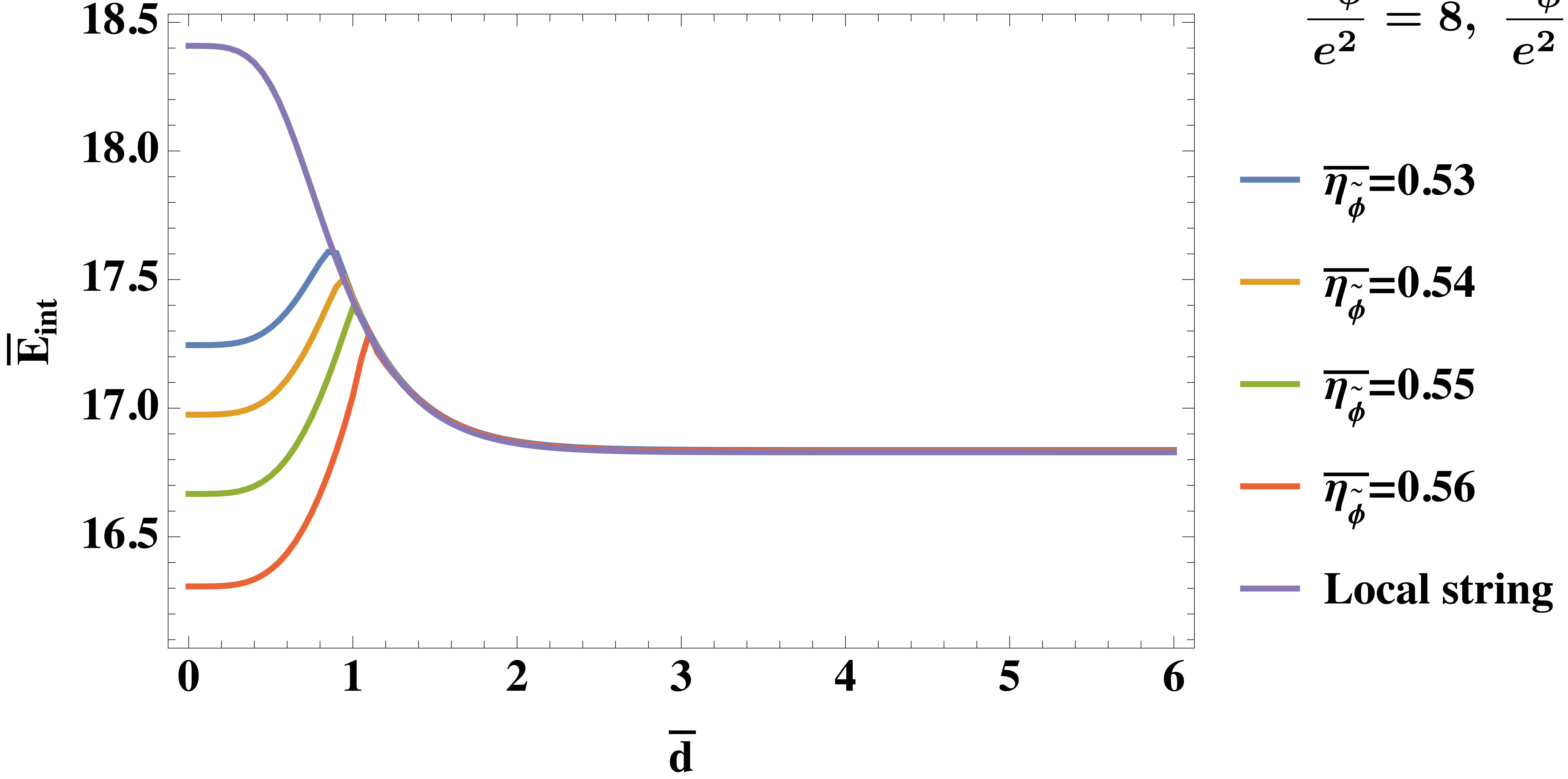


An additional attraction appears.

(Caution: This result cannot be observed in the point source formalism!)

Bosonic Superconducting String (3)

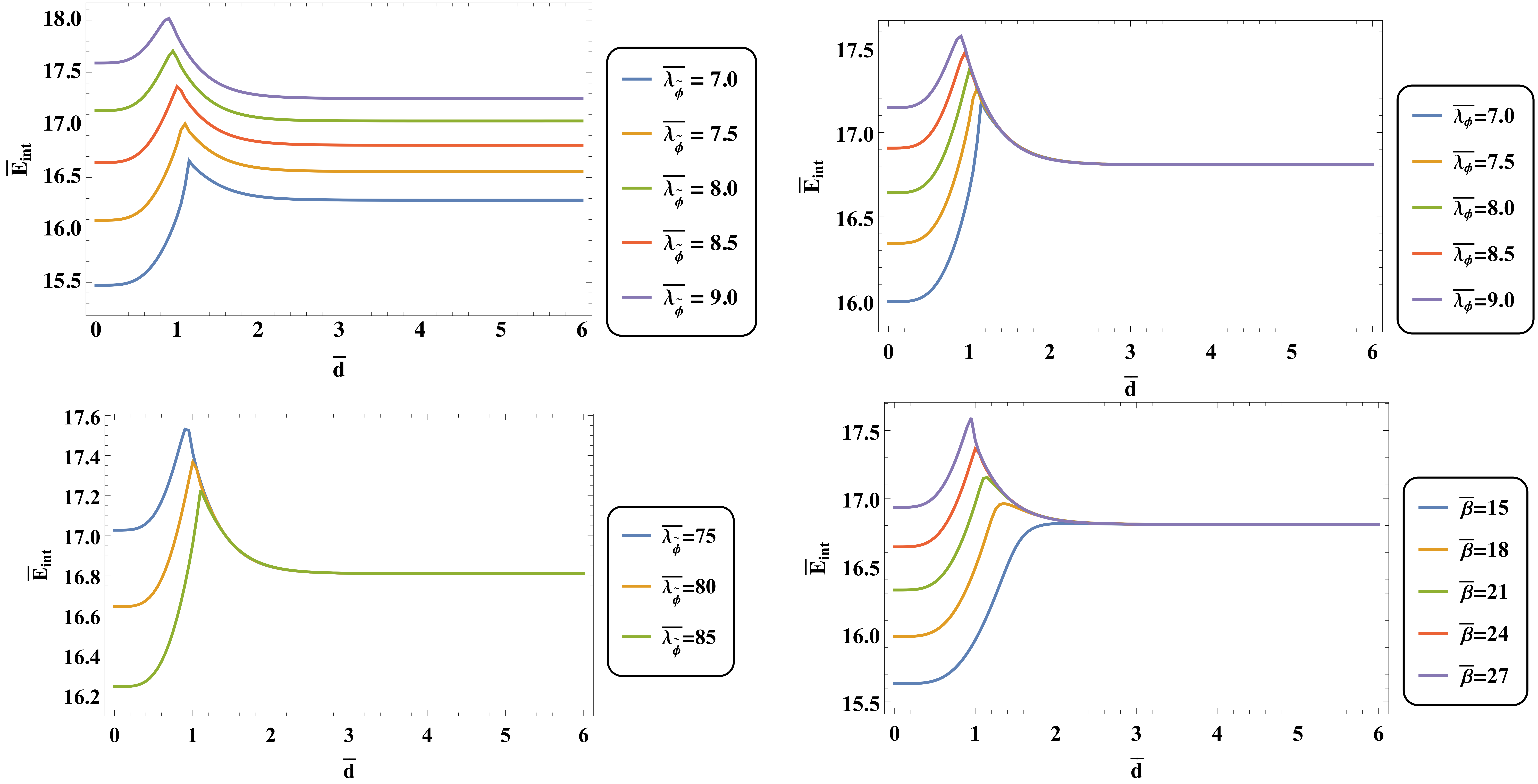
$$\frac{\lambda_\phi}{e^2} = 8, \ \frac{\lambda_{\tilde{\phi}}}{e^2} = 80, \ \frac{\beta}{e^2} = 24$$



$$\bar{\eta}_{\tilde{\phi}} \equiv \frac{\eta_\phi}{\eta_{\tilde{\phi}}}$$

A strength of attraction strongly depends on the $\widetilde{U}(1)$ breaking scale.

Various parameters…



Summary

We confirm that there exists the parameter region which leads to the attraction between two bosonic superconducting strings.

It is very interesting to investigate the Y-junction formation by studying dynamical collisions of bosonic superconuducting strings.

Conclusion

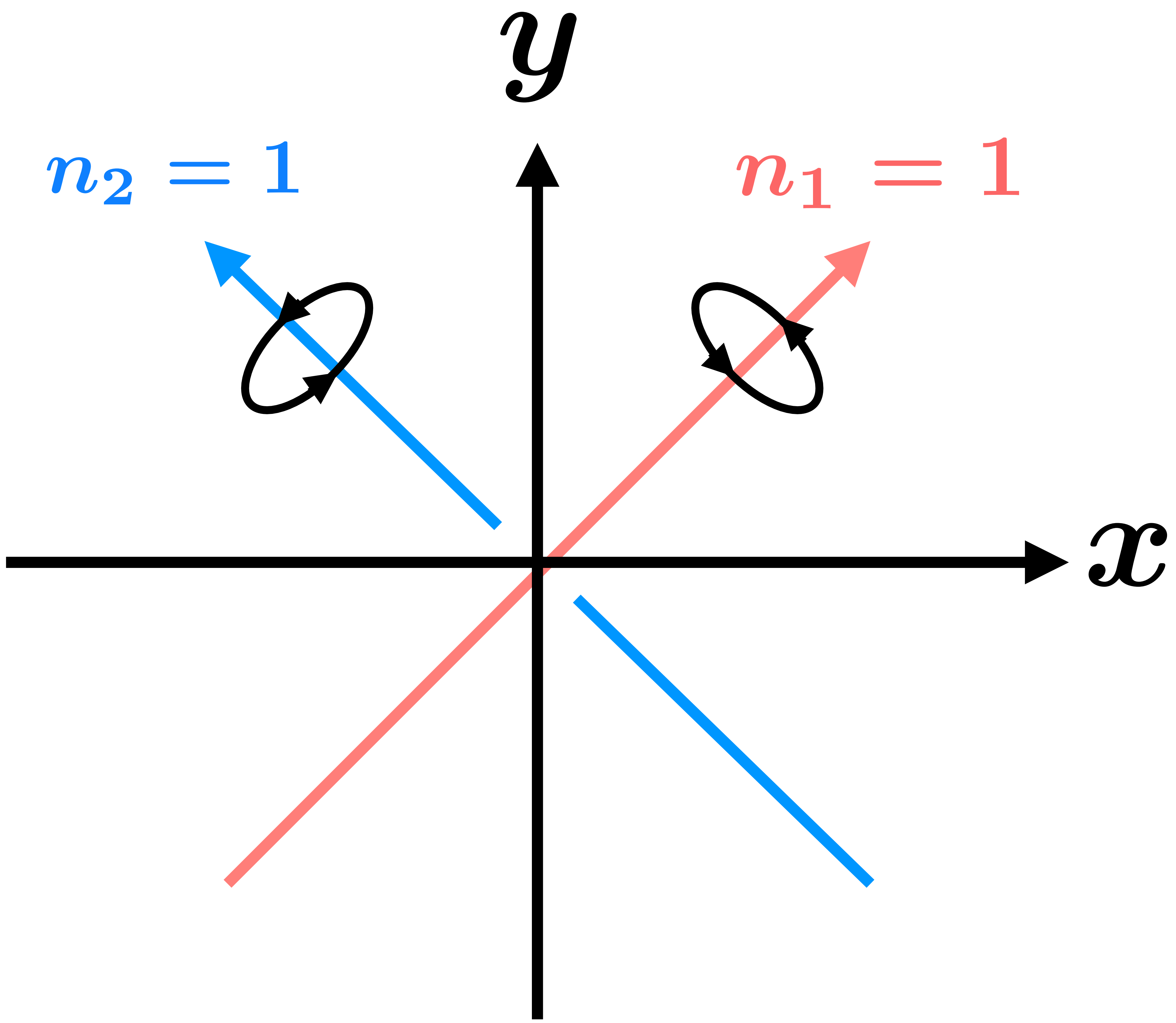
- Cosmic strings are line-like topological defects that may be formed in the early Universe. Interactions between cosmic strings may affect fate of the string network.

- For the bosonic superconducting string, we find that an additional attraction can dominate. This suggests the further analysis of the formation of Y-junctions.

- Non-linear physics is too difficult for me.

Reconnection (1)

Let us consider winding of cosmic strings.



x-direction: vortex anti-vortex pair

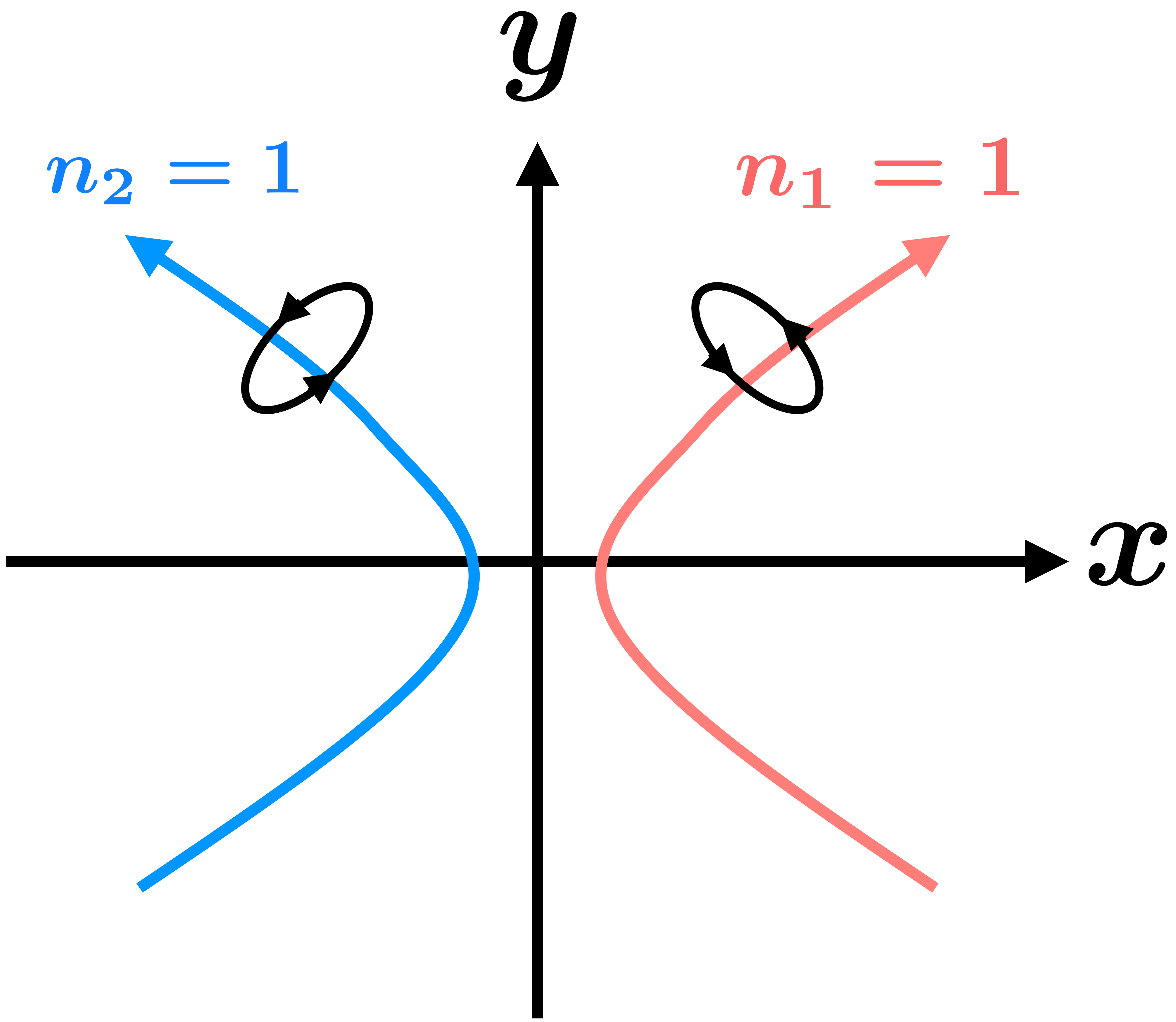
$$n_{1x} - n_{2x} = 0$$

y-direction: vortex vortex pair

$$n_{1y} + n_{2y} \neq 0$$

Reconnection (2)

Let us consider winding of cosmic strings.



x-direction: vortex anti-vortex pair

$$n_{1x} - n_{2x} = 0$$

A field configuration can be trivial

y-direction: vortex vortex pair

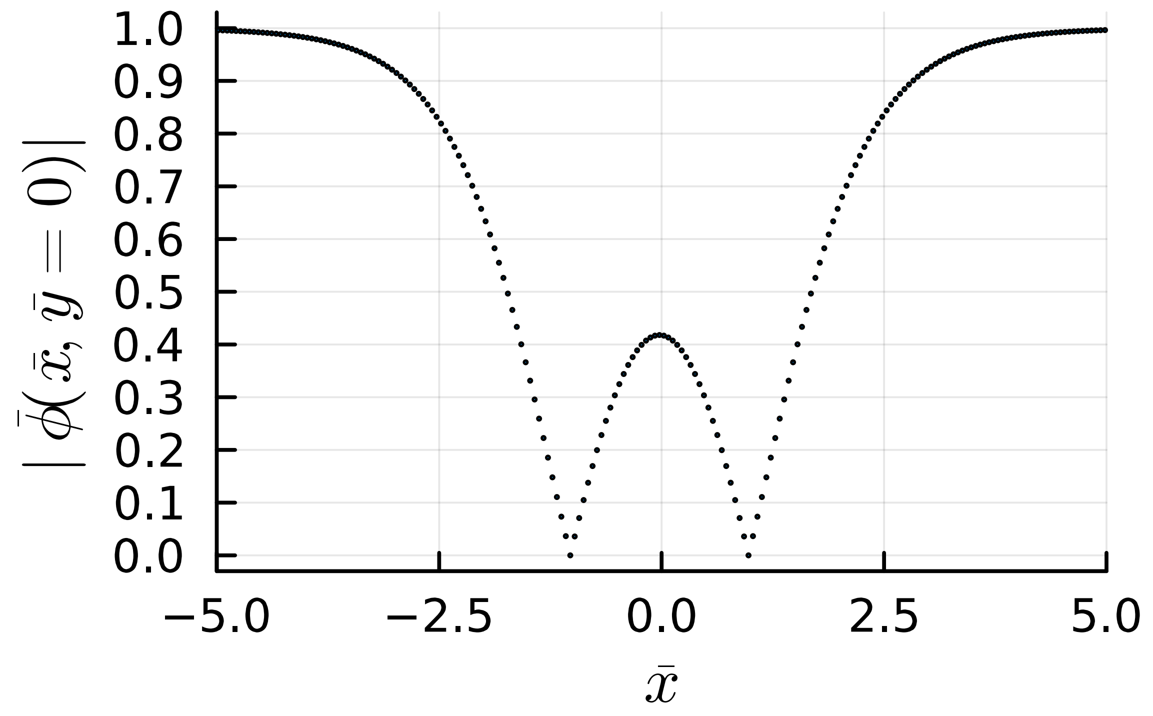
$$n_{1y} + n_{2y} \neq 0$$

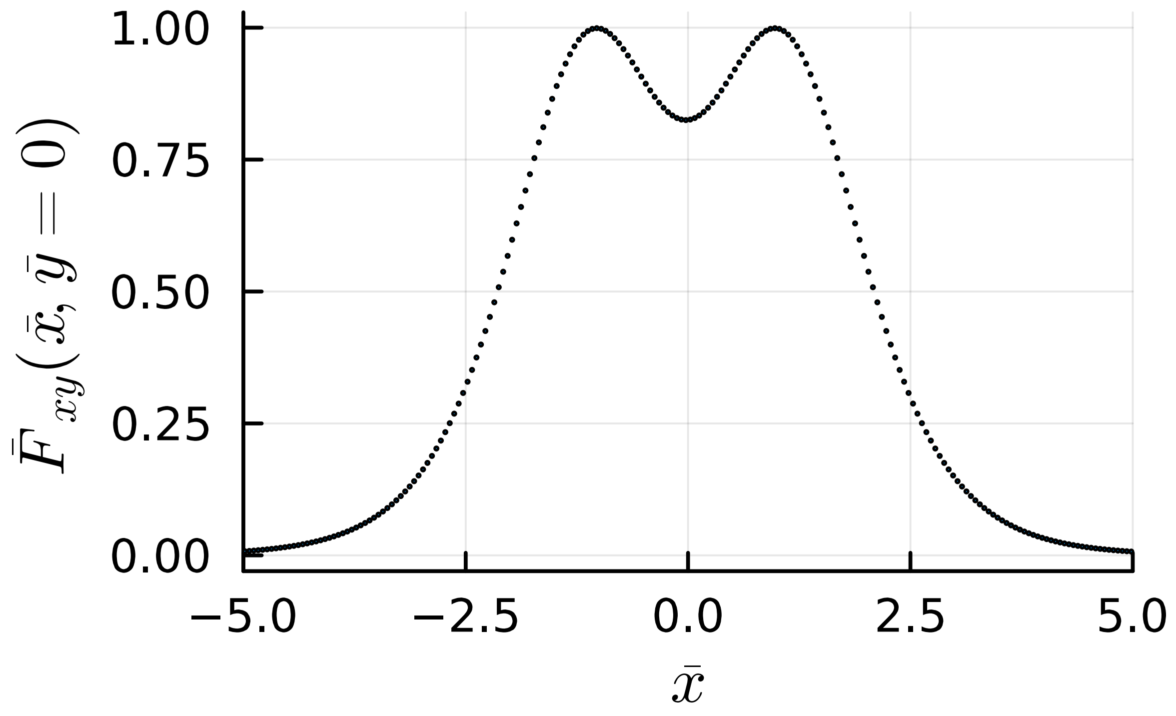
A field configuration cannot be trivial.

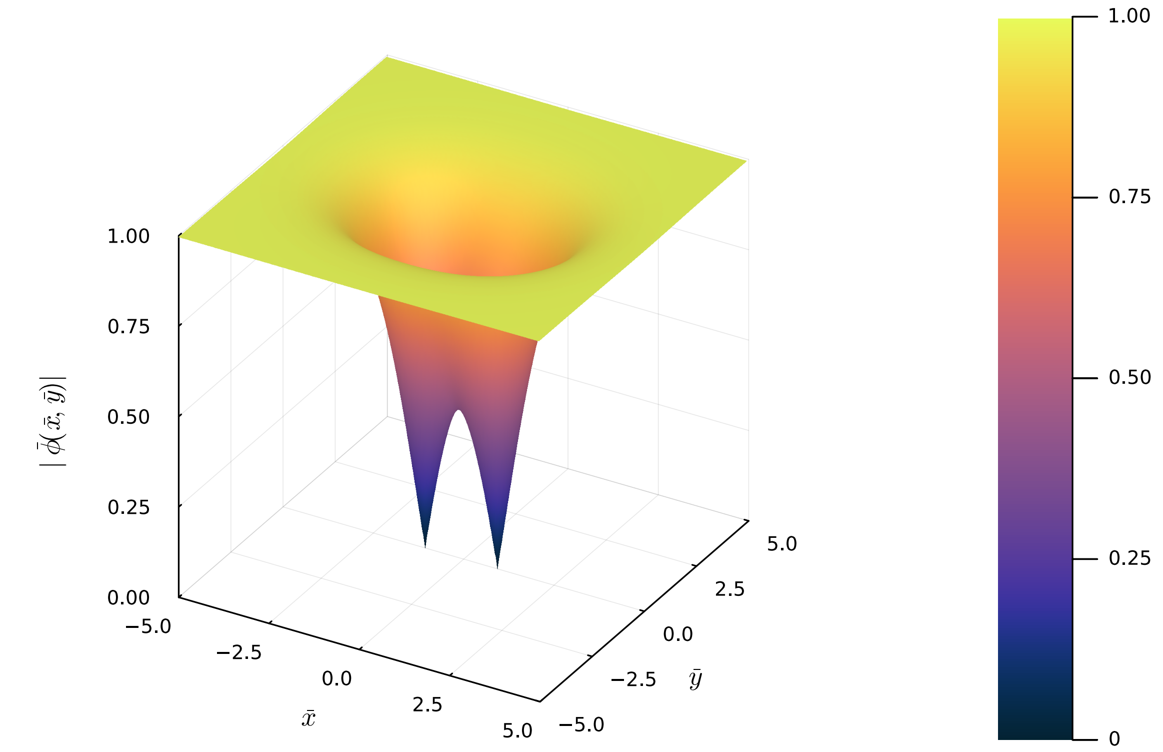
Abrikosov-Nielsen-Olesen strings (1)

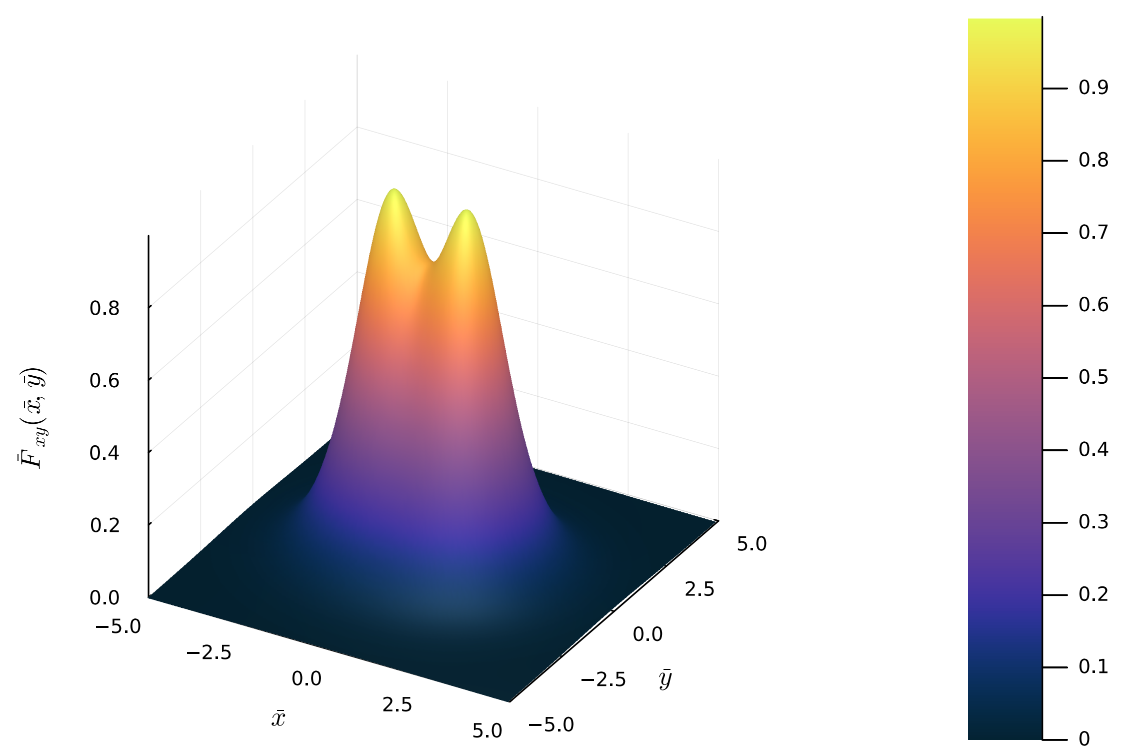
Field configurations of two string system

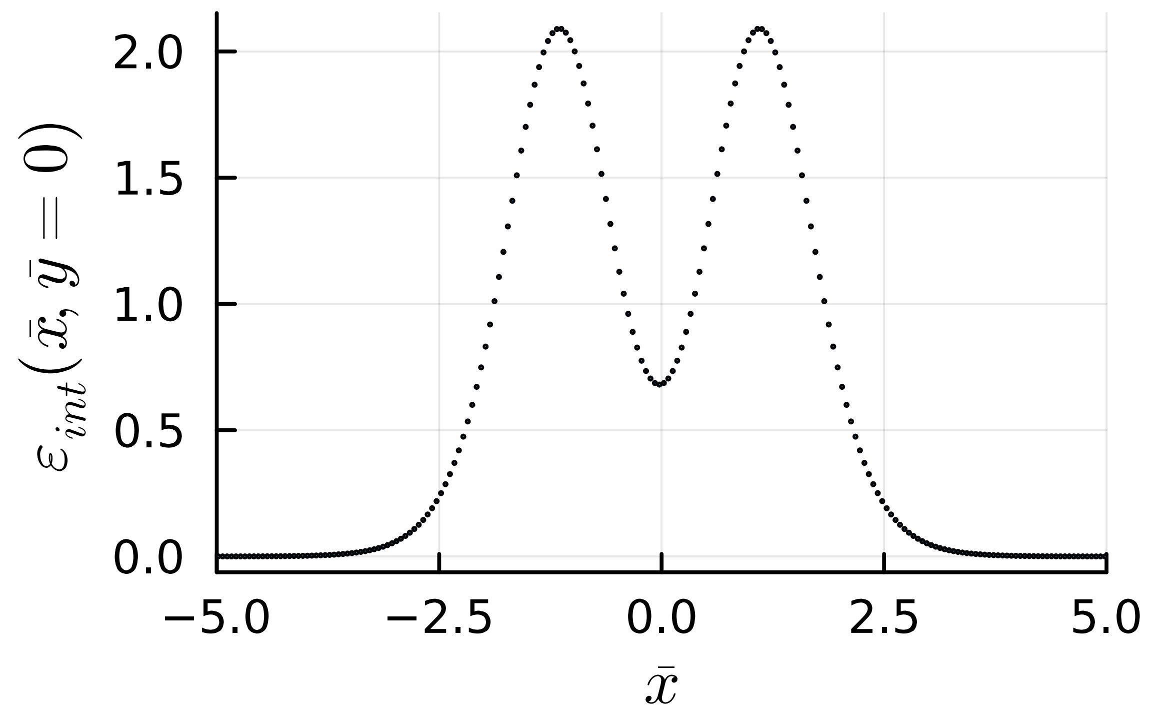
$$\bar{d} = 1, \ \ \frac{\lambda}{e^2} = 2$$

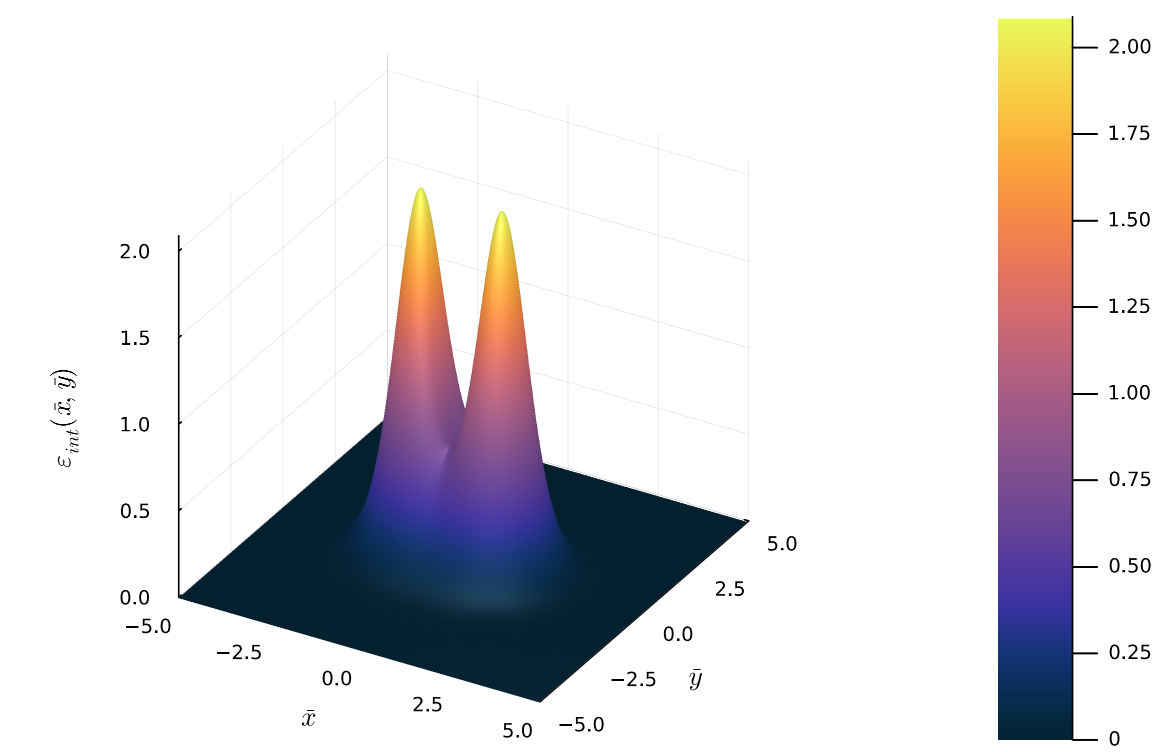










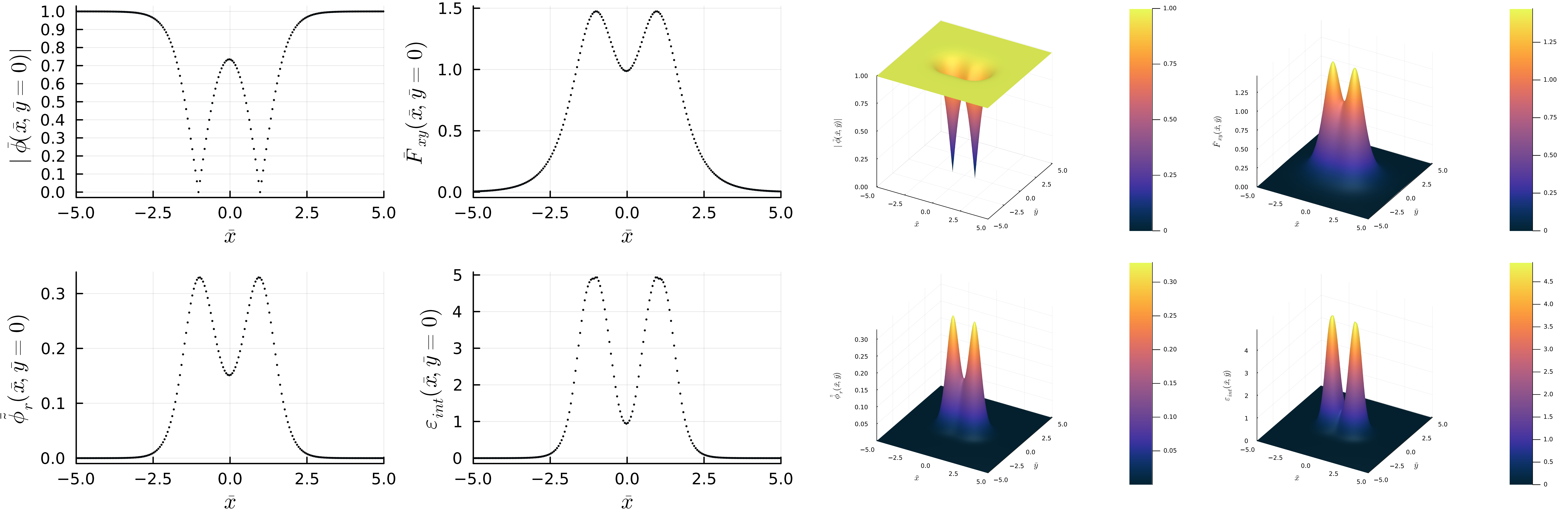


Length scale is given in the unit of $e\eta = 1$

Bosonic Superconducting String (1)

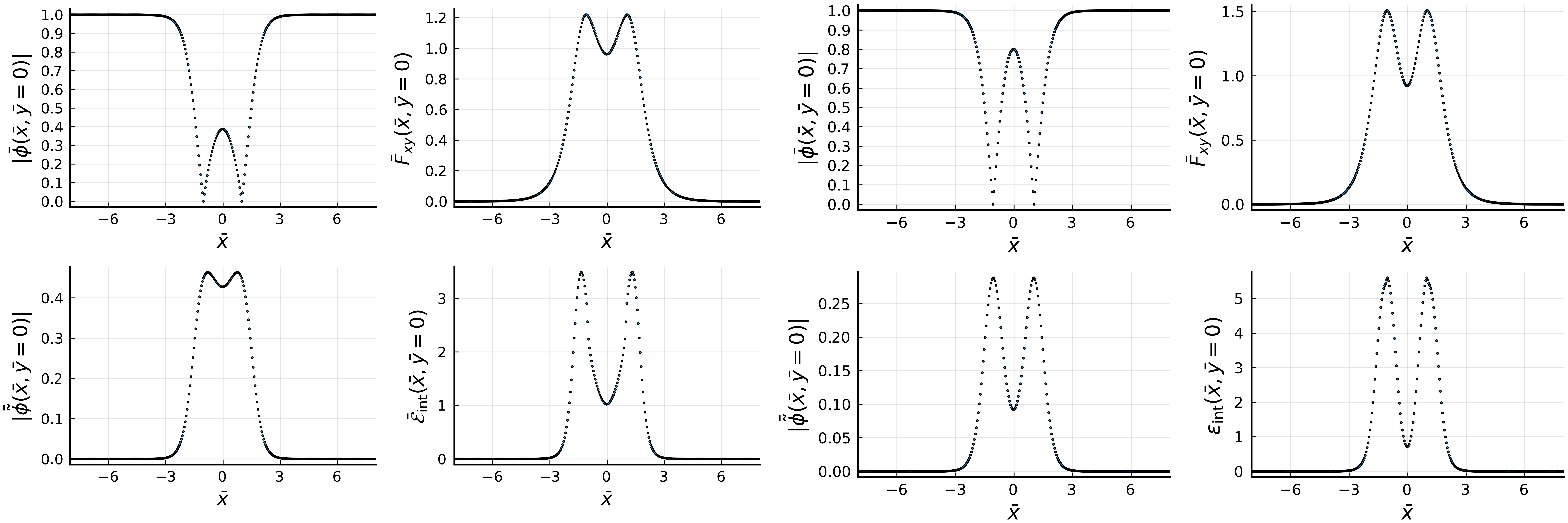
A result of gradient flow method

$$\frac{\lambda_\phi}{e^2} = 8, \ \frac{\lambda_{\tilde{\phi}}}{e^2} = 80, \ \frac{\beta}{e^2} = 24, \ \frac{\eta_{\tilde{\phi}}}{\eta_\phi} = 0.55$$



Length scale is given in the unit of $e\eta = 1$

Kink


$|\bar{\phi}(\bar{x}, \bar{y} = 0)|$
$\bar{F}_{xy}(\bar{x}, \bar{y} = 0)$
$|\tilde{\bar{\phi}}(\bar{x}, \bar{y} = 0)|$
$\bar{\mathcal{E}}_{\rm int}(\bar{x}, \bar{y} = 0)$
$\varepsilon_{\rm int}(\bar{x}, \bar{y} = 0)$
$\bar{x}$