# One-loop effect in Higgs inflation

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- Higgs inflation in Einstein-Cartan gravity
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# Introduction

- Cosmic inflation:
  - Exponential expansion of space

    ➡ homogeneous, isotropic, and flat observable Universe

  - Quantum fluctuations

    ➡ Set initial conditions for structure formation and GWs

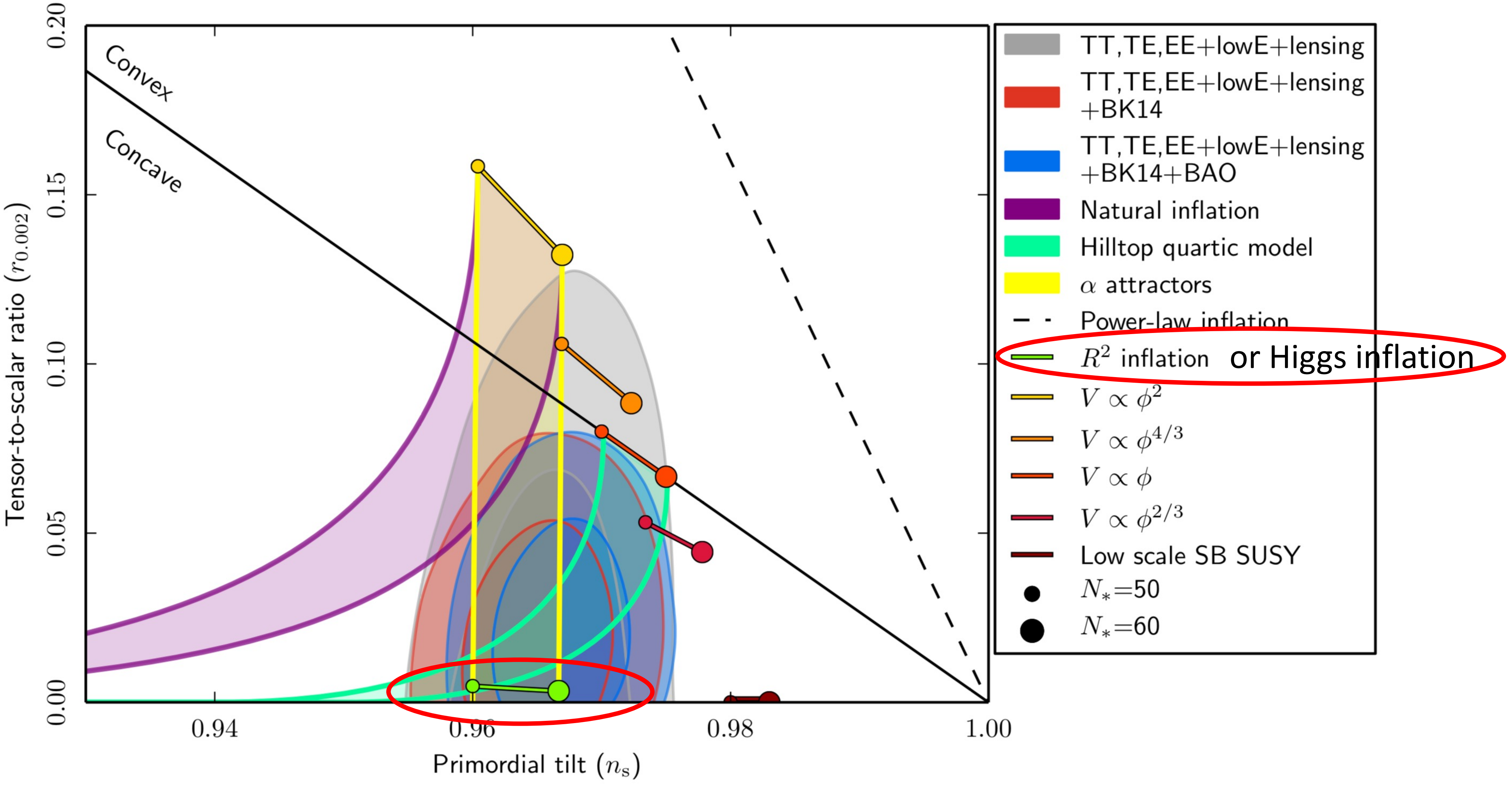
  - Reheating

    ➡ Entropy production

# Introduction

Starobinsky, 1980
Cervantes-Cota, Dehnen, 1995
Bezrukov, Shaposhnikov, 2007

- Co




Planck 2018

# Introduction

- Focus on Higgs inflation
  - In different formalisms of gravity, such as metric, Palatini, and Einstein-Cartan formalisms
- Strong motivations
  - SM Higgs field
  - Observationally favored predictions (closely related to Starobinsky model, but with different reheating)
- Doubts
  - Large non-minimal coupling (leading to low cutoff scale)
  - Strong coupling during preheating in metric formalism (but not in Palatini case)

Ema, Jinno, Mukaida, Nakayama, 2017

Rubio, Tomberg, 2019

# Higgs inflation in Einstein-Cartan gravity

- Action

$$S_{\mathrm{grav,J}} = \int \mathrm{d}^4 x \sqrt{-g_{\mathrm{J}}} F(\phi)\, g_{\mathrm{J}}^{\mu\nu} \bar{R}_{\mu\nu}(\bar{\Gamma})$$

$$S_{\mathrm{Higgs,J}} = \int \mathrm{d}^4 x \sqrt{-g_{\mathrm{J}}} \left[ -\frac{1}{2} g_{\mathrm{J}}^{\mu\nu} \partial_\mu \phi^i \partial_\nu \phi_i - V(\phi) \right]$$

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- Metric formalism of gravity ("usual case")
  - Fixing

$$\bar{\Gamma}^{\rho}{}_{\mu\nu} = \Gamma^{\rho}{}_{\mu\nu} \equiv \frac{1}{2} g^{\rho\sigma} \left( \partial_\mu g_{\nu\sigma} + \partial_\nu g_{\sigma\mu} - \partial_\sigma g_{\mu\nu} \right)$$

Cervantes-Cota, Dehnen, 1995
Bezrukov, Shaposhnikov, 2007

$\Longrightarrow$ Torsionless, metric compatible (or metricity) connection

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$\Longrightarrow$ Torsionless, metric compatible (or metricity) connection

- Alternative formalism of gravity such as Palatini and Einstein-Cartan
  - $g_{\mu\nu}$ and $\bar{\Gamma}^\rho_{\mu\nu}$ are *a priori* independent

Bauer, Demir, 2008

Shaposhnikov, Shkerin, Timiryasov, Zell, 2020

$\Longrightarrow$ Allow connections with non-metricity and/or torsion

Non-dynamical in our setup

# Higgs inflation in Einstein-Cartan gravity

$$S_{\mathrm{grav,J}} = \int \mathrm{d}^4x\sqrt{-g_{\mathrm{J}}}F(\phi)g_{\mathrm{J}}^{\mu\nu}\bar{R}_{\mu\nu}(\bar{\Gamma})$$

$$S_{\mathrm{Higgs,J}} = \int \mathrm{d}^4x\sqrt{-g_{\mathrm{J}}}\left[-\frac{1}{2}g_{\mathrm{J}}^{\mu\nu}\partial_\mu\phi^i\partial_\nu\phi_i - V(\phi)\right]$$

- Comparison among these versions

| | metric | Palatini | Eintesin-Cartan |
|---|---|---|---|
| $g_{\mu\nu}$ and $\Gamma^{\rho}_{\mu\nu}$ | Levi-Civita connection | A priori independent | |
| $\Gamma^{\rho}_{\mu\nu} - \Gamma^{\rho}_{\nu\mu}$ | 0 | 0 | $T^{\rho}{}_{\mu\nu}$ |
| $\nabla_\mu g_{\nu\rho}$ | 0 | $\neq 0$ (non-metricity) | 0 |
| Kinetic term of Higgs in Einstein frame (single field case) $-\frac{1}{2}K(\phi)\partial_\mu\phi\partial^\mu\phi$ | $\dfrac{1+\xi\frac{\phi^2}{M_{\mathrm{pl}}^2}+6\xi^2\frac{\phi^2}{M_{\mathrm{pl}}^2}}{(1+\xi\frac{\phi^2}{M_{\mathrm{pl}}^2})^2}$ | $\dfrac{1}{1+\xi\frac{\phi^2}{M_{\mathrm{pl}}^2}}$ | |
| CMB normalization | $\xi/\sqrt{\lambda}\sim10^4$ | $\xi/\lambda\sim10^{10}$ | |
| Unitarity violation scale at $\phi = 0$ | $\sim M_{\mathrm{pl}}/\xi$ | $\sim M_{\mathrm{pl}}/\sqrt{\xi}$ | |

# Higgs inflation in Einstein-Cartan gravity

$$S_{\rm grav,J} = \int d^4x \sqrt{-g_J} F(\phi) g_J^{\mu\nu} \bar{R}_{\mu\nu}(\bar{\Gamma})$$

- Comparison among these versions

$$S_{\rm Higgs,J} = \int d^4x \sqrt{-g_J} \left[ -\frac{1}{2} g_J^{\mu\nu} \partial_\mu \phi^i \partial_\nu \phi_i - V(\phi) \right]$$

| | metric | Palatini | Eintesin-Cartan |
|---|---|---|---|
| $g_{\mu\nu}$ and $\Gamma^\rho_{\mu\nu}$ | Levi-Civita connection | | |
| $\Gamma^\rho_{\mu\nu} - \Gamma^\rho_{\nu\mu}$ | 0 | | |
| $\nabla_\mu g_{\nu\rho}$ | 0 | | |
| Kinetic term of Higgs in Einstein frame (single field case) $-\frac{1}{2} {\color{red}K(\phi)} \partial_\mu \phi \partial^\mu \phi$ | $\dfrac{1 + \xi \frac{\phi^2}{M_{\rm pl}^2} + 6\xi^2 \frac{\phi^2}{M_{\rm pl}^2}}{(1 + \xi \frac{\phi^2}{M_{\rm pl}^2})^2}$ | | |
| CMB normalization | $\xi/\sqrt{\lambda} \sim 10^4$ | | |
| Unitarity violation scale at $\phi = 0$ | $\sim M_{\rm pl}/\xi$ | $\sim M_{\rm pl}/\sqrt{\xi}$ | |

They are connected by projective symmetry

$$g_{\mu\nu} \to g_{\mu\nu}\,, \qquad \bar{\Gamma}^\rho{}_{\mu\nu} \to \bar{\Gamma}^\rho{}_{\mu\nu} + \delta^\rho_\nu U_\mu$$

# Higgs inflation in Einstein-Cartan gravity

Shaposhnikov, Shkerin, Timiryasov, Zell, 2020

- Action (Jordan frame)

$$S_{\mathrm{grav,J}} = \int \mathrm{d}^4x\sqrt{-g_\mathrm{J}}F(\phi)g_\mathrm{J}^{\mu\nu}\bar{R}_{\mu\nu}(\bar{\Gamma})$$

$$S_{\mathrm{NY,J}} = -\frac{\xi_\eta}{4}\int \mathrm{d}^4x\, G(\phi)\partial_\mu\left(\sqrt{-g_\mathrm{J}}E^{\mu\nu\rho\sigma}T_{\nu\rho\sigma}\right)$$

Nieh, Yan, 1982

Solely determined by torsion

$$S_{\mathrm{Higgs,J}} = \int \mathrm{d}^4x\sqrt{-g_\mathrm{J}}\left[-\frac{1}{2}g_\mathrm{J}^{\mu\nu}\partial_\mu\phi^i\partial_\nu\phi_i - V(\phi)\right]$$

$$V(\phi) = \Lambda + \frac{1}{2}m^2\phi^2 + \frac{1}{4}\lambda\phi^4 \qquad F(\phi) = \frac{M_{\mathrm{Pl}}^2}{2}\left(1+\xi\frac{\phi^2}{M_{\mathrm{Pl}}^2}\right) \qquad G(\phi) = \phi^2 \qquad r \equiv \frac{\xi_\eta}{\xi}$$

$\xi \gg 1$

Connect metric case ($r = 1$) and Palatini case ($r = 0$)!

# Higgs inflation in Einstein-Cartan gravity

Shaposhnikov, Shkerin, Timiryasov, Zell, 2020

- Action (Jordan frame)

All torsion considered here will be found non-dynamical, completely determined by $\phi$.
As a result, it changes the dynamics of $\phi$.

$$V(\phi) = \Lambda + \frac{1}{2}m^2\phi^2 + \frac{1}{4}\lambda\phi^4 \qquad F(\phi) = \frac{M_{Pl}^2}{2}\left(1 + \xi\frac{\phi^2}{M_{Pl}^2}\right) \qquad G(\phi) = \phi^2 \qquad r \equiv \frac{\xi_\eta}{\xi}$$

$\xi \gg 1$

Connect metric case ($r = 1$) and Palatini case ($r = 0$)!

# Higgs inflation in Einstein-Cartan gravity

Shaposhnikov, Shkerin, Timiryasov, Zell, 2020

- Einstein frame (after solving constraints for torsion and conformal transformation)

$$S = \int d^4x \sqrt{-g_{\mathrm{E}}} \left[ \frac{M_{\mathrm{pl}}^2}{2} R(g_{\mathrm{E}}) - \frac{1}{2} g_{\mathrm{E}}^{\mu\nu} K_{\mathrm{E}ij}(\phi) \partial_\mu \phi^i \partial_\nu \phi^j - \frac{M_{\mathrm{pl}}^4 V(\phi)}{4F^2} \right]$$

$$K_{\mathrm{E}ij} = \frac{M_{\mathrm{pl}}^2}{2F} \left( \delta_{ij} + \frac{3}{4F} r^2 \xi^2 \frac{\partial G}{\partial \phi^i} \frac{\partial G}{\partial \phi^j} \right)$$

$$= \frac{1}{1 + \xi \phi^2 / M_{\mathrm{pl}}^2} \left( \delta_{ij} + \frac{6}{M_{\mathrm{pl}}^2} \frac{r^2 \xi^2}{1 + \xi \phi^2 / M_{\mathrm{pl}}^2} \phi_i \phi_j \right)$$

$$F(\phi) = \frac{M_{\mathrm{Pl}}^2}{2} \left( 1 + \xi \frac{\phi^2}{M_{\mathrm{Pl}}^2} \right)$$

$$G(\phi) = \phi^2$$

Connect metric case ($r = 1$) and Palatini case ($r = 0$)!

# Higgs inflation in Einstein-Cartan gravity

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$$S = \int d^4x\sqrt{-g_{\mathrm{E}}}\left[\frac{M_{\mathrm{pl}}^2}{2}R(g_{\mathrm{E}}) - \frac{1}{2}g_{\mathrm{E}}^{\mu\nu}K_{\mathrm{E}ij}(\phi)\partial_\mu\phi^i\partial_\nu\phi^j - \frac{M_{\mathrm{pl}}^4 V(\phi)}{4F^2}\right]$$

- CMB normalization $\frac{2\lambda N_{\mathrm{e}}^2}{\xi + 6(r\xi)^2} \simeq 5.0 \times 10^{-7}$

- Predictions $n_s = 1 - \frac{2}{N_e} - \frac{3(\xi + 6r^2\xi^2)}{4N_e^2\xi^2}$ $r = \frac{2(\xi + 6r^2\xi^2)}{N_e^2\xi^2}$

Connect metric case ($r = 1$) and Palatini case ($r = 0$)!

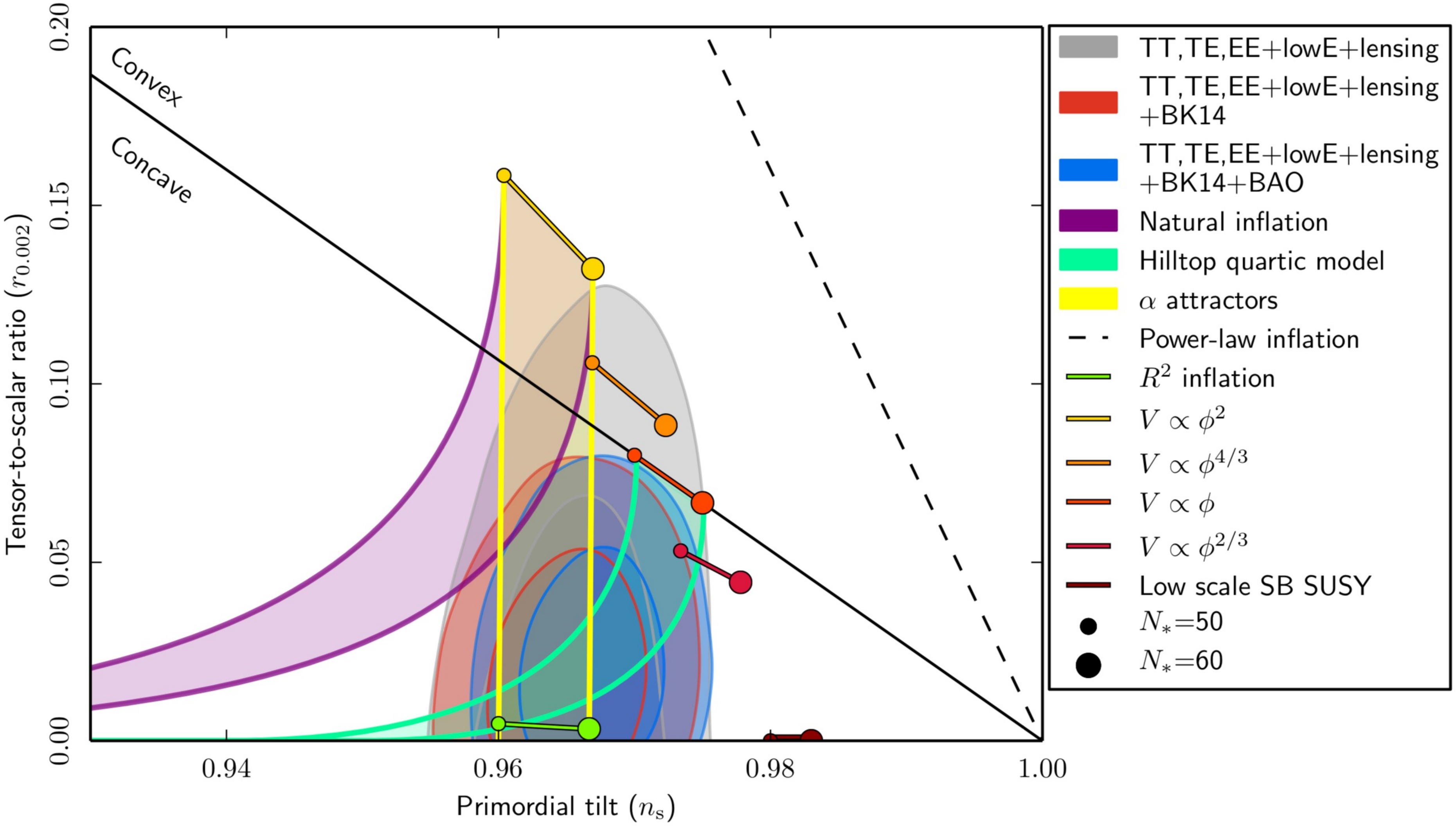
# Higgs inflation in Einstein-Cartan gravity

sov, Zell, 2020

- Ein tra

- CN

- Pre




Connect metric case ($r = 1$) and Palatini case ($r = 0$)!

# Higgs inflation in Einstein-Cartan gravity

Hertzberg, 2010
Mikura, Tada, 2022
Many others…

- Unitarity violation scale
  - Target space (single-field case is trivial)
  - Higher dimensional operators in potential
- Example: metric Higgs inflation ($r = 1$)

$$S = \int d^4x\sqrt{-g_{\mathrm{E}}}\left[\frac{M_{\mathrm{pl}}^2}{2}R(g_{\mathrm{E}}) - \frac{1}{2}g_{\mathrm{E}}^{\mu\nu}K_{\mathrm{E}ij}(\phi)\partial_\mu\phi^i\partial_\nu\phi^j - \frac{M_{\mathrm{pl}}^4 V(\phi)}{4F^2}\right]$$

Higher dimensional operators

$$K_{\mathrm{E}ij} = \frac{1}{1+\xi\phi^2/M_{\mathrm{pl}}^2}\left(\delta_{ij} + \frac{6}{M_{\mathrm{pl}}^2}\frac{\xi^2}{1+\xi\phi^2/M_{\mathrm{pl}}^2}\phi_i\phi_j\right)$$

Curved!

e.g. $\Lambda \sim M_{\mathrm{pl}}/\xi$ at vacuum $\phi = 0$

# Higgs inflation in Einstein-Cartan gravity

- Unitarity violation scale
  - Target space (single-field case is trivial)
  - Higher dimensional operators in potential
- Example: metric Higgs inflation ($r = 1$) $\bar{\Gamma}^{\rho}{}_{\mu\nu} = \Gamma^{\rho}{}_{\mu\nu} \equiv \frac{1}{2} g^{\rho\sigma} \left( \partial_\mu g_{\nu\sigma} + \partial_\nu g_{\sigma\mu} - \partial_\sigma g_{\mu\nu} \right)$

$$S_{\text{grav,J}} = \int \mathrm{d}^4 x \sqrt{-g_{\text{J}}} F(\phi) g_{\text{J}}^{\mu\nu} \bar{R}_{\mu\nu}(\bar{\Gamma})$$

$$S_{\text{Higgs,J}} = \int \mathrm{d}^4 x \sqrt{-g_{\text{J}}} \left[ -\frac{1}{2} g_{\text{J}}^{\mu\nu} \partial_\mu \phi^i \partial_\nu \phi_i - V(\phi) \right]$$

  - Flat target space, up to quartic potential with small couplings

→ $\Lambda \sim M_{\text{pl}}$?

# Higgs inflation in Einstein-Cartan gravity

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  - Target space (single-field case is trivial)
  - Higher dimensional operators in potential
- Example: metric Higgs inflation ($r = 1$) $\bar{\Gamma}^{\rho}{}_{\mu\nu} = \Gamma^{\rho}{}_{\mu\nu} \equiv \frac{1}{2} g^{\rho\sigma} (\partial_\mu g_{\nu\sigma} + \partial_\nu g_{\sigma\mu} - \partial_\sigma g_{\mu\nu})$

$$S_{\mathrm{grav,J}} = \int \mathrm{d}^4 x \sqrt{-g_{\mathrm{J}}} F(\phi) g_{\mathrm{J}}^{\mu\nu} \bar{R}_{\mu\nu}(\bar{\Gamma})$$

Coupling with gravity

$$S_{\mathrm{Higgs,J}} = \int \mathrm{d}^4 x \sqrt{-g_{\mathrm{J}}} \left[ -\frac{1}{2} g_{\mathrm{J}}^{\mu\nu} \partial_\mu \phi^i \partial_\nu \phi_i - V(\phi) \right]$$

  - Flat target space, up to quartic potential with small couplings

→ $\Lambda \sim M_{\mathrm{pl}}$? No!

# Higgs inflation in Einstein-Cartan gravity

- Unitarity violation scale
  - Target space (single-field case is trivial)
  - Higher dimensional operators in potential
- Example: metric Higgs inflation ($r = 1$) $\Lambda \sim M_{\mathrm{pl}}/\xi$ at vacuum $\phi = 0$
  - Gravity inroduces ambiguity
  - More general case: $O(N)$ non-linear $\sigma$ model
  - Highly energetic gauge bosons are excited during preheating $p > \Lambda$

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Ema, Jinno, Mukaida, Nakayama, 2017

→ Strong coupling

Compare: Palatini Higgs inflation: low cutoff, no strong coupling during preheating

# Before proceeding…

- Metric Higgs inflation: cutoff $\sim M_{\mathrm{pl}}/\xi$, strong coupling in preheating
- Palatini Higgs inflation: cutoff $\sim M_{\mathrm{pl}}/\sqrt{\xi}$
- Einstein-Cartan Higgs inflation: unification of metric and Palatini
- Conformal mode can deal with the redundancy of frame choice

# Attempts of UV extension

- UV extension or tools for strong coupling are needed for metric Higgs inflation to have reliable predictions

  Ema 2017, Gorbunov, Tokareva 2018, MH, Kamada, Jinno, Park, Starobinsky, Yokoyama 2018
  - $R^2$ is one option to UV extend the model by inducing a new DoF, scalaron
  - $R^2$ can arise from the one-loop correction in large-$N$ limit

    Ema, 2019

- Palatini Higgs inflation has cutoff lower than Planck scale

  Enckell, Enqvist, Rasanen, Wahlman 2018, Mikura, Tada 2021, MH, Mikura, Tada 2022
  - $R^2$ cannot UV extend the model, no new DoF
  - Generally embedding the target space into 1-dim higher flat space cannot UV extend

- Einstein-Cartan Higgs inflation can unify metric and Palatini cases
  - Understand the role of $R^2$ (or something analogous) in a general setup
  - Maybe helpful to find the UV extension for Palatini case

# One-loop effect

- Action (Jordan frame)

$$S_{\mathrm{grav,J}} = \int \mathrm{d}^4x \sqrt{-g_{\mathrm{J}}} F(\phi) g_{\mathrm{J}}^{\mu\nu} \bar{R}_{\mu\nu}(\bar{\Gamma}) \qquad F(\phi) = \frac{M_{\mathrm{Pl}}^2}{2}\left(1 + \xi \frac{\phi^2}{M_{\mathrm{Pl}}^2}\right)$$

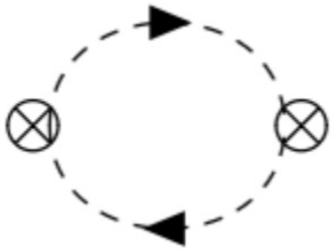
$$S_{\mathrm{NY,J}} = -\frac{\xi_\eta}{4} \int \mathrm{d}^4x \, G(\phi) \partial_\mu \left(\sqrt{-g_{\mathrm{J}}} E^{\mu\nu\rho\sigma} T_{\nu\rho\sigma}\right) \qquad G(\phi) = \phi^2$$

$$\supset \frac{\xi}{2} \boxed{\phi^2} \left(\bar{R} + \frac{r}{2} \nabla_\mu S^\mu\right) \qquad S^\mu = -E^{\mu\nu\rho\sigma} T_{\nu\rho\sigma}$$

*N* scalar fields

- Large-*N* limit

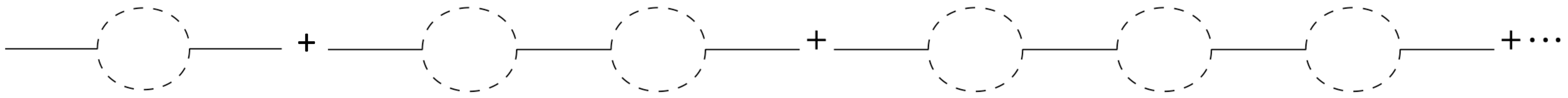
Counter term $\alpha \left(\bar{R} + \frac{r}{2} \nabla_\mu S^\mu\right)^2 \Longleftarrow$



closed Higgs loops enhanced by number of species $N$

# One-loop effect

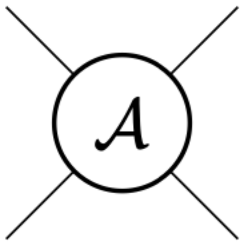
- Large-*N* limit
  - Counter term $$\alpha\left(\bar{R}+\frac{r}{2}\nabla_\mu S^\mu\right)^2$$




  - Resummation

Ema, 2019

➡ A new pole in the amplitude




➡ Mass of the pole $m_\sigma \propto M_{\rm pl}/\sqrt{\alpha}$

# One-loop effect

- Large-$N$ limit
  - Counter term $\alpha\left(\bar{R}+\frac{r}{2}\nabla_{\mu}S^{\mu}\right)^{2}$
- Large-$\xi$ limit
  - Other counter terms, for example, $\beta\bar{R}^{\mu\nu}\bar{R}_{\mu\nu}$

Ema, 2019

$$\frac{\mathrm{d}\alpha}{\mathrm{d}\ln\mu}\propto N\xi^{2} \gg \frac{\mathrm{d}\beta}{\mathrm{d}\ln\mu}\propto N$$

→ Keep only $\alpha{\sim}N\xi^{2}$

# One-loop effect

$$r \equiv \frac{\xi_\eta}{\xi}$$

- Einstein-Cartan Higgs inflation with one-loop correction in large-*N* and large-$\xi$ limit

$$S = \int d^4x\sqrt{-g_{\rm J}}\left[\frac{M_{\rm pl}^2}{2}\bar{R} - \mathcal{L}_{\rm Higgs} + \frac{\xi}{2}\phi^2\left(\bar{R} + \frac{r}{2}\nabla_\mu S^\mu\right) + \alpha\left(\bar{R} + \frac{r}{2}\nabla_\mu S^\mu\right)^2\right]$$

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Legendre transformation $\left(\bar{R} + \frac{r}{2}\nabla_\mu S^\mu - \gamma\right)L'(\gamma) + L(\gamma)$ $\quad L(\gamma) \equiv \frac{\xi}{2}\phi^2\gamma + \alpha\gamma^2$

$$\Rightarrow \quad S = \int \sqrt{-g_E} d^4x \left[\frac{M_{\rm Pl}^2}{2}R_E - \frac{1}{2}g_E^{\mu\nu}\left(\partial_\mu\sigma_E\partial_\nu\sigma_E + e^{-\sqrt{\frac{2}{3}}\frac{\sigma_E}{rM_{\rm Pl}}}\partial_\mu\phi_i\partial_\nu\phi^i\right) - U(\phi,\sigma_E)\right]$$

$$U(\phi,\sigma_E) \equiv e^{-2\sqrt{\frac{2}{3}}\frac{\sigma_E}{rM_{\rm Pl}}}\left[V(\phi) + \frac{M_{\rm Pl}^4}{16\alpha}\left(e^{\sqrt{\frac{2}{3}}\frac{\sigma_E}{rM_{\rm Pl}}} - 1 - \xi\frac{\phi^2}{M_{\rm Pl}^2}\right)^2\right]$$

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$$S = \int \sqrt{-g_{\mathrm{E}}}\mathrm{d}^4x \left[ \frac{M_{\mathrm{Pl}}^2}{2} R_{\mathrm{E}} - \frac{1}{2} g_{\mathrm{E}}^{\mu\nu} \left( \partial_\mu \sigma_{\mathrm{E}} \partial_\nu \sigma_{\mathrm{E}} + e^{-\sqrt{\frac{2}{3}} \boxed{\frac{\sigma_{\mathrm{E}}}{r M_{\mathrm{Pl}}}}} \partial_\mu \phi_i \partial_\nu \phi^i \right) - U(\phi, \sigma_{\mathrm{E}}) \right]$$

$$U(\phi, \sigma_{\mathrm{E}}) \equiv e^{-2\sqrt{\frac{2}{3}} \boxed{\frac{\sigma_{\mathrm{E}}}{r M_{\mathrm{Pl}}}}} \left[ V(\phi) + \frac{M_{\mathrm{Pl}}^4}{16\alpha} \left( e^{\sqrt{\frac{2}{3}} \boxed{\frac{\sigma_{\mathrm{E}}}{r M_{\mathrm{Pl}}}}} - 1 - \xi \frac{\phi^2}{M_{\mathrm{Pl}}^2} \right)^2 \right]$$

New DoF — Scalaron $\sigma$

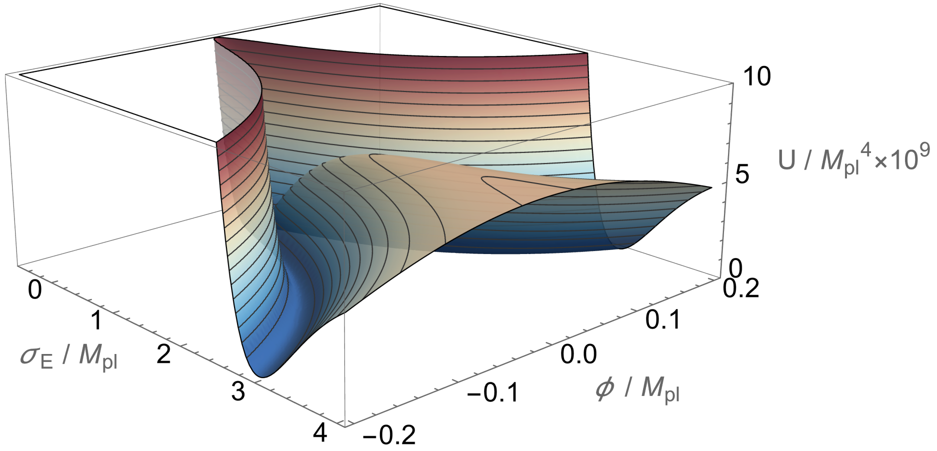
Scalaron mass — $m_\sigma^2 = \dfrac{M_{\mathrm{Pl}}^2}{12\alpha \boxed{r^2}}$

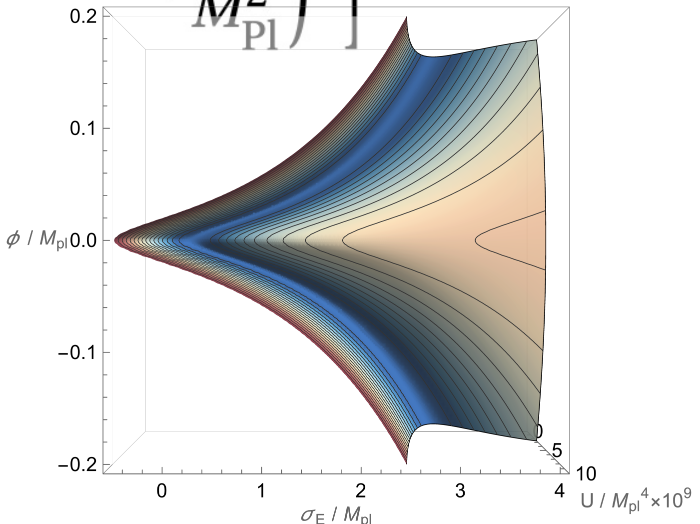
# One-loop effect

- Einstein-Cartan Higgs inflation with one-loop correction in large-$N$ and large-$\xi$ limit

$$S=\int\sqrt{-g_{\mathrm{E}}}\mathrm{d}^4x\left[\frac{M_{\mathrm{Pl}}^2}{2}R_{\mathrm{E}}-\frac{1}{2}g_{\mathrm{E}}^{\mu\nu}\left(\partial_\mu\sigma_{\mathrm{E}}\partial_\nu\sigma_{\mathrm{E}}+e^{-\sqrt{\frac{2}{3}}\frac{\sigma_{\mathrm{E}}}{rM_{\mathrm{Pl}}}}\partial_\mu\phi_i\partial_\nu\phi^i\right)-U(\phi,\sigma_{\mathrm{E}})\right]$$

$$U(\phi,\sigma_{\mathrm{E}})\equiv e^{-2\sqrt{\frac{2}{3}}\frac{\sigma_{\mathrm{E}}}{rM_{\mathrm{Pl}}}}\left[V(\phi)+\frac{M_{\mathrm{Pl}}^4}{16\alpha}\left(e^{\sqrt{\frac{2}{3}}\frac{\sigma_{\mathrm{E}}}{rM_{\mathrm{Pl}}}}-1-\xi\frac{\phi^2}{M_{\mathrm{Pl}}^2}\right)^2\right]$$



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$$U(\phi, \sigma_{\mathrm{E}}) \equiv e^{-2\sqrt{\frac{2}{3}} \frac{\sigma_{\mathrm{E}}}{r M_{\mathrm{Pl}}}} \left[ V(\phi) + \frac{M_{\mathrm{Pl}}^4}{16\alpha} \left( e^{\sqrt{\frac{2}{3}} \frac{\sigma_{\mathrm{E}}}{r M_{\mathrm{Pl}}}} - 1 - \xi \frac{\phi^2}{M_{\mathrm{Pl}}^2} \right)^2 \right]$$

- Metric limit ($r = 1$): mixed Higgs-$R^2$ model (in Higgs limit $\alpha \sim N\xi^2$)

  Ema, 2017, MH, Starobinsky, Yokoyama, 2018, Gundhi, Steinwachs, 2020, Enckell, Enqvist, Rasanen, Wahlman, 2020

  - No unitarity violation during preheating
  - High reheating temperature…

  Ema, 2017, Gorbunov, Tokareva, 2019, MH, Jinno, Kamada, Park, Starobinsky, Yokoyama, 2019

  Bezrukov, Gorbunov, Shepherd, Tokareva, 2019, MH, Jinno, Kamada, Starobinsky, Yokoyama, 2021, Bezrukov, Shepherd, 2020, MH, 2021

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- Einstein-Cartan Higgs inflation with one-loop correction in large-*N* and large-$\xi$ limit

$$S = \int \sqrt{-g_{\rm E}} d^4x \left[ \frac{M_{\rm Pl}^2}{2} R_{\rm E} - \frac{1}{2} g_{\rm E}^{\mu\nu} \left( \partial_\mu \sigma_{\rm E} \partial_\nu \sigma_{\rm E} + e^{-\sqrt{\frac{2}{3}} \frac{\sigma_{\rm E}}{r M_{\rm Pl}}} \partial_\mu \phi_i \partial_\nu \phi^i \right) - U(\phi, \sigma_{\rm E}) \right]$$

$$U(\phi, \sigma_{\rm E}) \equiv e^{-2\sqrt{\frac{2}{3}} \frac{\sigma_{\rm E}}{r M_{\rm Pl}}} \left[ V(\phi) + \frac{M_{\rm Pl}^4}{16\alpha} \left( e^{\sqrt{\frac{2}{3}} \frac{\sigma_{\rm E}}{r M_{\rm Pl}}} - 1 - \xi \frac{\phi^2}{M_{\rm Pl}^2} \right)^2 \right]$$

- Metric limit ($r = 1$): mixed Higgs-$R^2$ inflation model (in Higgs limit)

- Palatini limit ($r \to 1/\sqrt{\xi}$): naively $m_\sigma^2 = \frac{M_{\rm Pl}^2}{12\alpha r^2} \to M_{\rm pl}^2/\xi \sim \Lambda_{\rm Palatini}^2$

Consistent with literature

Enckell, Enqvist, Rasanen, Wahlman, 2019, Antoniadis, Karam, Lykkas, Tamvakis, 2018

# One-loop effect

- Unitarity violation scale
  - Target space

$$\Lambda_{\text{E–C}} = |R_{N+s}|^{-1/2} = \frac{\sqrt{6}r}{\sqrt{|1-r^2|}} M_{\text{Pl}}$$

  - Higher dimensional operators in potential

$$\Lambda \sim r M_{\text{Pl}}$$

  - $r = 1,$ $\Lambda \sim M_{\text{pl}}/\sqrt{N}$
  - $r \to 1/\sqrt{\xi},$ $\Lambda \sim M_{\text{pl}}/\sqrt{\xi}$ as in Palatini case
  - $r \to 1/\xi,$ $\Lambda \sim M_{\text{pl}}/\xi$ even lower

# One-loop effect

- Unitarity violation scale
  - Target space

$$\Lambda_{\text{E–C}} = |R_{N+s}|^{-1/2} = \frac{\sqrt{6}r}{\sqrt{|1-r^2|}} M_{\text{Pl}}$$

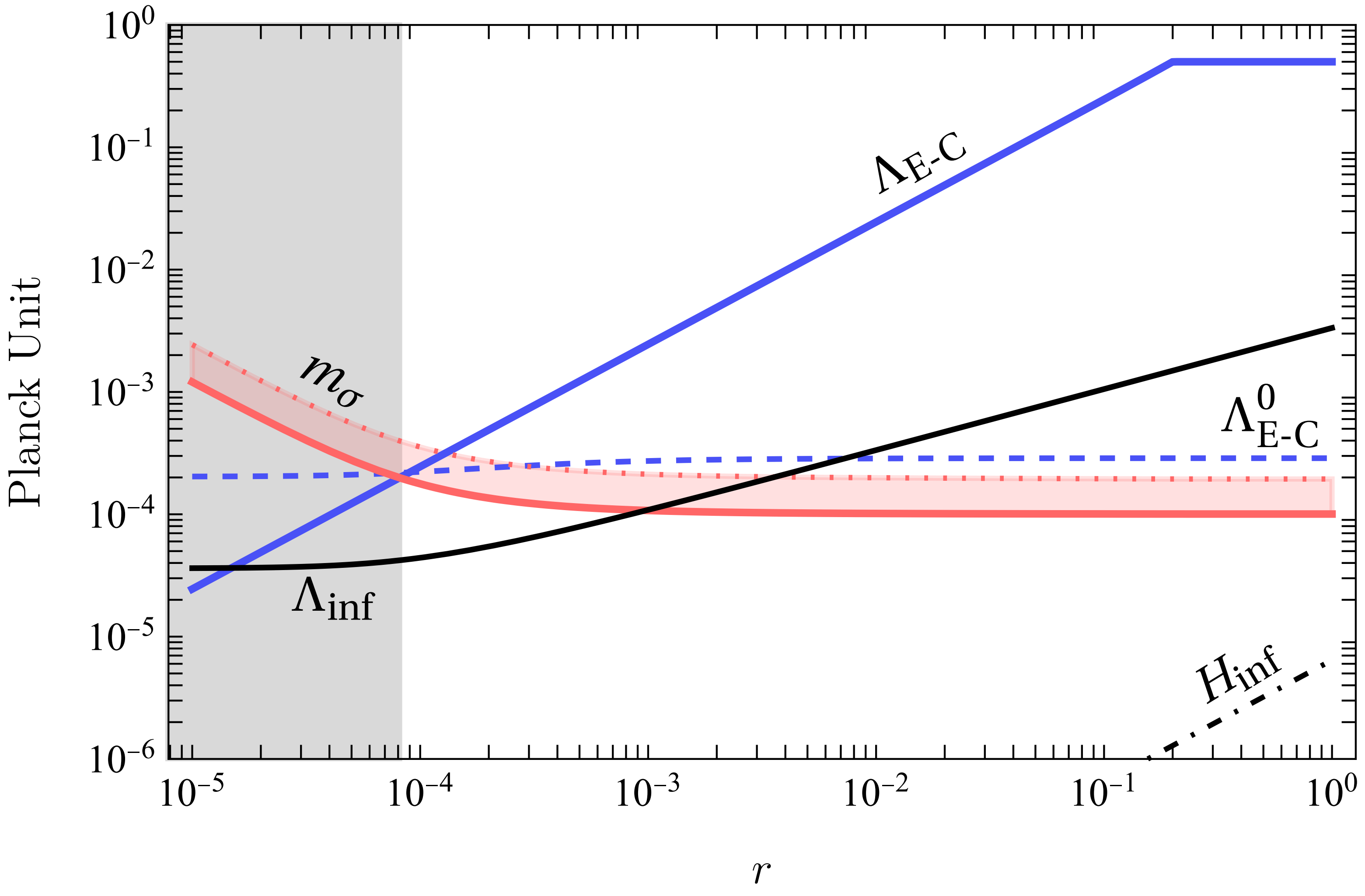
  - Higher dimensional operators in potential

$$\Lambda \sim r M_{\text{Pl}}$$

$\alpha \sim N\xi^2$

| | $\boldsymbol{r=1}$ | $\boldsymbol{r \lesssim 1}$ | $\boldsymbol{r \to 1/\sqrt{\xi}}$ | $\boldsymbol{r \to 1/\xi}$ |
|---|---|---|---|---|
| $\Lambda$ | $M_{\text{pl}}/\sqrt{N}$ | $r\, M_{\text{pl}}$ | $M_{\text{pl}}/\sqrt{\xi}$ | $M_{\text{pl}}/\xi$ |
| $m_\sigma$ | $M_{\text{pl}}/\xi$ | $M_{\text{pl}}/r\sqrt{\alpha}$ | $M_{\text{pl}}/\sqrt{\xi}$ | $M_{\text{pl}}$ |

# One-loop effect



# Summary

- Einstein-Cartan Higgs inflation is a general setup to study the UV behavior of metric and Palatini cases in a systematic way
- One-loop correction in large-$N$ limit generally induce a new scalar DoF, *i.e.* scalaron, whose mass depends on parameter $r$
- Cutoff scale with quantum correction is affected by the relation between scalaron mass and the model is UV-extended for *r* down to $\sim 10^{-4}$ (given $\lambda \sim 10^{-3}$)

# Thank you for your attention!

# Higgs inflation in Einstein-Cartan gravity

Shaposhnikov, Shkerin, Timiryasov, Zell, 2020

- Jordan frame (after solving constraints for torsion)

$$S = \int d^4x \sqrt{-g_{\mathrm{J}}} \left[ F(\phi) R(g_{\mathrm{J}}) - \frac{1}{2} g_{\mathrm{J}}^{\mu\nu} K_{\mathrm{J}ij}(\phi) \partial_\mu \phi^i \partial_\nu \phi^j - V(\phi) \right]$$

$$K_{\mathrm{J}ij} = \delta_{ij} - 3F \frac{\partial \ln F}{\partial \phi^i} \frac{\partial \ln F}{\partial \phi^j} + \frac{3}{4} \frac{r^2 \xi^2}{F} \frac{\partial G}{\partial \phi^i} \frac{\partial G}{\partial \phi^j}$$

$$= \delta_{ij} - (1 - r^2) \frac{6}{M_{\mathrm{pl}}^2} \frac{1}{1 + \xi \phi^2 / M_{\mathrm{pl}}^2} \xi^2 \phi_i \phi_j$$

# Higgs inflation in Einstein-Cartan gravity

Shaposhnikov, Shkerin, Timiryasov, Zell, 2020

- Einstein frame (after solving constraints for torsion and conformal transformation)

$$S = \int d^4x \sqrt{-g_{\mathrm{E}}} \left[ \frac{M_{\mathrm{pl}}^2}{2} R(g_{\mathrm{E}}) - \frac{1}{2} g_{\mathrm{E}}^{\mu\nu} K_{\mathrm{E}ij}(\phi) \partial_\mu \phi^i \partial_\nu \phi^j - \frac{M_{\mathrm{pl}}^4 V(\phi)}{4F^2} \right]$$

$$K_{\mathrm{E}ij} = \frac{M_{\mathrm{pl}}^2}{2F} \left( \delta_{ij} + \frac{3}{4F} r^2 \xi^2 \frac{\partial G}{\partial \phi^i} \frac{\partial G}{\partial \phi^j} \right)$$

$$= \frac{1}{1 + \xi \phi^2 / M_{\mathrm{pl}}^2} \left( \delta_{ij} + \frac{6}{M_{\mathrm{pl}}^2} \frac{r^2 \xi^2}{1 + \xi \phi^2 / M_{\mathrm{pl}}^2} \phi_i \phi_j \right)$$

# Higgs inflation in Einstein-Cartan gravity

- Unitarity violation scale
  - Target space (single-field case is trivial)
  - Higher dimensional operators in potential

Hertzberg, 2010
Mikura, Tada, 2022

- Example: metric Higgs inflation ($r = 1$)
- Conformal mode

Conformal transformation $g_{\mu\nu} \to \Omega^2 g_{\mu\nu}$ It is a field redefinition

Conformal mode $$g_{\mu\nu} = \frac{\Phi^2}{6M_{\rm pl}^2}\tilde{g}_{\mu\nu} \qquad \tilde{g} \equiv \det[\tilde{g}_{\mu\nu}] = 1$$

Ema, Mukaida, van de Vis, 2020

➡ Conformal transformation is redefining a scalar field, the conformal mode

# Higgs inflation in Einstein-Cartan gravity

Hertzberg, 2010
Mikura, Tada, 2022

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$$(\varphi_{\rm J}^a) = (\Phi_{\rm J}, \phi^i) \qquad S = \int d^4x \left( \frac{\Phi_{\rm J}^2}{12} \Omega^2 \tilde{R} - \frac{\tilde{g}^{\mu\nu}}{2} G_{ab}^{\rm J} \partial_\mu \varphi_{\rm J}^a \partial_\nu \varphi_{\rm J}^b - \frac{\Phi_{\rm J}^4 V}{36 M_{\rm Pl}^4} \right)$$

Extended target space $\left(G_{ab}^{\rm J}\right) \equiv \begin{pmatrix} -\Omega^2 & -\xi \Phi_{\rm J} \phi_j / M_{\rm Pl}^2 \\ -\xi \Phi_{\rm J} \phi_i / M_{\rm Pl}^2 & \frac{\Phi_{\rm J}^2}{6 M_{\rm Pl}^2} \delta_{ij} \end{pmatrix}$

Now conformal transf. is just changing the coordinate! Geometry is unaffected.

# Higgs inflation in Einstein-Cartan gravity

Hertzberg, 2010
Mikura, Tada, 2022

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- Example: metric Higgs inflation ($r = 1$)
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$$\left(G^{\mathrm{J}}_{ab}\right) \equiv \begin{pmatrix} -\Omega^2 & -\xi \Phi_{\mathrm{J}} \phi_j / M_{\mathrm{Pl}}^2 \\ -\xi \Phi_{\mathrm{J}} \phi_i / M_{\mathrm{Pl}}^2 & \frac{\Phi_{\mathrm{J}}^2}{6 M_{\mathrm{Pl}}^2} \delta_{ij} \end{pmatrix}$$

Geometric approach of scattering amplitude is already established in HEFT context.

$$\left(\prod_{i=1}^{4} \bar{G}^{1/2}_{a_i a_i}\right) \mathscr{M}_{a_1 a_2 \leftrightarrow a_3 a_4} = \frac{2}{3}\left[ s_{12} \bar{R}_{a_1 (a_3 a_4) a_2} + s_{13} \bar{R}_{a_1 (a_2 a_4) a_3} + s_{14} \bar{R}_{a_1 (a_2 a_3) a_4} \right]$$

Alonso+ 2016A,B, Nagai+ 2019, Cohen+ 2021, Cheung et al 2022