Fermion Mass Hierarchy in a Multi-Higgs Model

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Based on
Seungwon Baek, JK, Pyungwon Ko, Liliana Velasco Sevilla, arXiv:2309.07788

Outline

Quark Masses and Mixing in the Standard Model

- Mass hierarchy

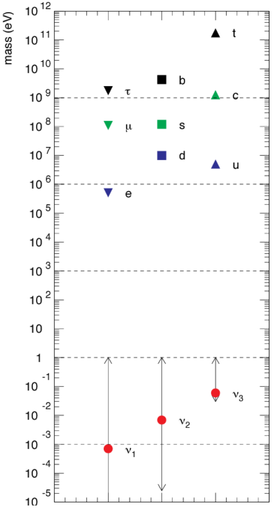
$$m_t = y_t v \simeq 173\,\text{GeV} \qquad \Rightarrow \qquad \frac{y_u}{y_t} \sim 10^{-5}$$
$$m_u = y_u v \simeq 2\,\text{MeV}$$

- Small but non-zero mixing

$$|V_{\text{CKM}}| \simeq \begin{pmatrix} 0.97 & 0.23 & 3.6\cdot 10^{-3} \\ 0.23 & 0.97 & 4.1\cdot 10^{-2} \\ 8.5\cdot 10^{-3} & 4.0\cdot 10^{-2} & 1.0 \end{pmatrix}$$

- Large CP phase

$$\delta \simeq 68^\circ$$



Why?

$\rightsquigarrow$ Froggatt-Nielsen mechanism, family symmetries, . . . $@\ M \gg v$

An Unfamiliar Way to Generate the Hierarchy

Ingredients

- 6 Higgs doublets: 1 for each quark mass

$$m_d \sim v_1 \ , \ m_u \sim v_2 \ , \ m_s \sim v_3 \ , \ m_c \sim v_4 \ , \ m_b \sim v_5 \ , \ m_t \sim v_6$$

$$v_1 \sim v_2 \ll v_3 \ll v_4 \ll v_5 \ll v_6$$

- Gauged family symmetry $U(1)_F \rightsquigarrow$ mixing pattern
- SM singlet scalars $\rightsquigarrow U(1)_F$ breaking
- Exotic fermions $\rightsquigarrow$ anomaly cancellation, dark matter?

Earlier work

Escudero, Muñoz, Teixeira, PRD **73** (2006), JHEP **07** (2006)
Porto, Zee, PLB **666** (2008), PRD **79** (2009)
BenTov, Zee, NPB **871** (2013), IJMPA **28** (20113)
Hill, Machado, Thomsen, Turner, PRD **100** (2019) (2×)
Altmannshofer, Gadam, Gori, Hamer, JHEP **07** (2021)

Particle Content for a Concrete Model Realization

- Quarks: $Q_{\mathrm{L}_i} = \begin{pmatrix} u_{\mathrm{L}_i} \\ d_{\mathrm{L}_i} \end{pmatrix}$, u_{R_i} , d_{R_i} , $i = 1, 2, 3$
- $SU(2)_\mathrm{L}$ doublet scalars: H_m , $m = 1 \ldots 6$

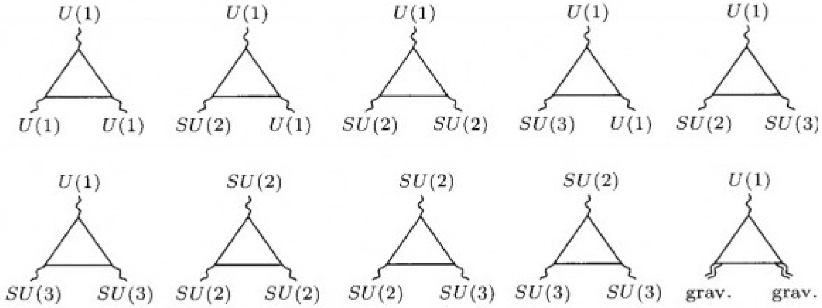
$$H_{1,3,5} = \begin{pmatrix} H^+_{1,3,5} \\ H^0_{1,3,5} \end{pmatrix} \quad (\text{hypercharge } Y = +\frac{1}{2})$$

$$H_{2,4,6} = \begin{pmatrix} H^0_{2,4,6} \\ H^-_{2,4,6} \end{pmatrix} \quad (Y = -\frac{1}{2})$$

- $SU(2)_\mathrm{L}$ singlet scalars: ϕ_n , $n = 1 \ldots 3$
- $SU(2)_\mathrm{L}$ singlet fermions (right-handed): χ_1 , χ_2
- $U(1)_F$ gauge boson: Z'_μ
- Other SM particles as usual

Anomaly Cancellation

Gauged family symmetry
$\rightsquigarrow$ triangle anomalies with external gauge bosons (incl. Z') must vanish



Anomaly Cancellation

$$0 = \frac{1}{6} \sum_{i=1}^{3} \left[c_{Q_L i} - 8\, c_{u_R\, i} - 2\, c_{d_R\, i} + 3\, c_{L_L\, i} - 6\, c_{e_R\, i} \right] + X_1$$

c_{ψ_i} : $U(1)_F$ charges

X_n : contributions from exotic fermions

Anomaly Cancellation

$$0 = \frac{1}{6} \sum_{i=1}^{3} \left[c_{Q_L i} - 8\, c_{u_R\, i} - 2\, c_{d_R\, i} + 3\, c_{L_L\, i} - \; 6\, c_{e_R\, i} \right] + X_1$$

$$0 = \frac{1}{2} \sum_{i=1}^{3} \left[3 \times c_{Q_L\, i} + c_{L_L\, i} \right] + X_2$$

$$0 = \frac{1}{2} \sum_{i=1}^{3} \left[2 \times c_{Q_L\, i} - c_{u_R\, i} - c_{d_R\, i} \right] + X_3$$

$$0 = \sum_{i=1}^{3} \left[c_{Q_L\, i}^2 - 2\, c_{u_R\, i}^2 + c_{d_R\, i}^2 - c_{L_L\, i}^2 + c_{e_R\, i}^2 \right] + X_F$$

$$0 = \sum_{i=1}^{3} \left[6\, c_{Q_L\, i}^3 - 3 \left(c_{u_R\, i}^3 + c_{d_R\, i}^3 \right) + 2\, c_{L_L\, i}^3 - c_{e_R\, i}^3 \right] + X_F^3$$

$$0 = \sum_{i=1}^{3} \left[6\, c_{Q_L\, i} - 3 \left(c_{u_R\, i} + c_{d_R\, i} \right) + 2\, c_{L_L\, i} - c_{e_R\, i} \right] + X_{\mathrm{GG}}$$

Yukawa Couplings

- Desired Yukawa couplings

$$\mathcal{L}_{\text{Yuk}} = -\,\overline{Q}_{Li}\left[(Y_1^d)_{ij}\,H_1 + (Y_3^d)_{ij}\,H_3 + (Y_5^d)_{ij}\,H_5\right] d_{Rj}$$
$$-\,\overline{Q}_{Li}\left[(Y_2^u)_{ij}\,H_2 + (Y_4^u)_{ij}\,H_4 + (Y_6^u)_{ij}\,H_6\right] u_{Rj}$$
$$-\,\overline{L}_{Li}\left[(Y_1^e)_{ij}\,H_1 + (Y_3^e)_{ij}\,H_3 + (Y_5^e)_{ij}\,H_5\right] e_{Rj} + \text{h.c.}$$

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- Undesired couplings

$$\mathcal{L}_{\text{Yuk}} \not\supset \overline{Q}_{L\,i}\,(Y_6^d)_{ij}\,\epsilon H_6^* d_{R\,j} + \overline{Q}_{L\,i}\,(Y_5^u)_{ij}\,\epsilon H_5^* u_{R\,j} \quad \text{etc.}$$

$\rightsquigarrow$ Forbid by $U(1)_F$ charge assignment

Yukawa Matrices

$$Y_1^d = \begin{pmatrix} 0 & \mathcal{O}(1) & \mathcal{O}(1) \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad Y_3^d = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \mathcal{O}(1) & \mathcal{O}(1) \\ \mathcal{O}(1) & 0 & 0 \end{pmatrix}$$

$$Y_5^d = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \mathcal{O}(1) & \mathcal{O}(1) \end{pmatrix}$$

$$Y_2^u = \begin{pmatrix} \mathcal{O}(1) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad Y_4^u = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \mathcal{O}(1) & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$Y_6^u = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \mathcal{O}(1) \end{pmatrix}$$

$\rightsquigarrow$ Correct quark masses and mixings with $\mathcal{O}(1)$ couplings

Family Symmetry Charges

Quarks & leptons	Q_L	u_R	d_R	L_L	e_R	
1st generation	7	0	-2	-10	-6	
2nd generation	-2	-1	2	4	4	
3rd generation	-6	-3	2	9	8	
Scalar doublets	H_1	H_2	H_3	H_4	H_5	H_6
	5	7	-4	-1	-8	-3
Scalars breaking $U(1)_F$	ϕ_1	ϕ_2	ϕ_3			
	-2	7	3			
Exotic fermions	χ_1	χ_2				
	-1	1				

Challenges

- 6 $SU(2)_\mathrm{L}$ doublets, 3 singlets
  $\rightsquigarrow$ very complicated scalar potential
  $\rightsquigarrow$ comprehensive exploration of parameter space hard
  $\rightsquigarrow$ consider benchmark point
- Origin of vev hiearchy
- Radiative stability
- Constraints from gauge symmetries
  $\rightsquigarrow$ accidental global symmetries
  $\rightsquigarrow$ Nambu-Goldstone bosons
  $\rightsquigarrow$ 3 singlets with suitable $U(1)_F$ charges

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Doublet	Vev [GeV]	Doublet	Vev [GeV]	Singlet	Vev [GeV]
H_1	0.005	H_2	0.005	ϕ_1	1.1×10^8
H_3	0.05	H_4	0.05	ϕ_2	200
H_5	1	H_6	246	ϕ_3	1.5×10^3

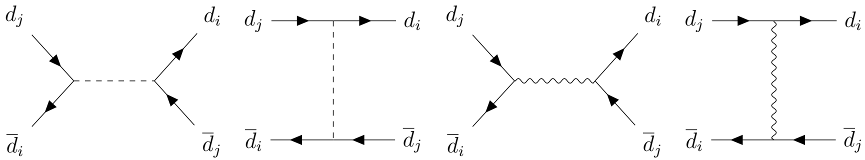
Scalar Spectrum

- 6 $SU(2)_L$ doublets, 3 complex singlets
  $\rightsquigarrow$ 9 neutral scalars, 7 neutral pseudoscalars, 5 charged scalars

Masses [GeV]		
Neutral Scalars	Pseudoscalars	Charged Scalars
125.2		
3.15×10^3	14.7	
1.56×10^4	1.56×10^4	1.56×10^4
6.02×10^6	6.02×10^6	6.02×10^6
9.11×10^6	9.11×10^6	9.11×10^6
1.36×10^8	1.36×10^8	2.94×10^8
1.69×10^8	2.94×10^8	6.03×10^8
2.94×10^8	6.03×10^8	
6.03×10^8		

- Light pseudoscalar is mostly singlet $\rightsquigarrow$ phenomenologically viable

Flavor-Changing Neutral Currents



- Down-type mass matrix $M_d = Y_1^d \langle H_1 \rangle + Y_3^d \langle H_3 \rangle + Y_5^d \langle H_5 \rangle$
  $\rightsquigarrow$ Yukawa couplings not diagonal in mass eigenstate basis
  $\rightsquigarrow$ Tree-level FCNC by scalar and pseudoscalar exchange
- Generation-dependent $U(1)_F$ charges
  $\rightsquigarrow$ Tree-level FCNC by Z' exchange
- Kinetic mixing between $U(1)$ gauge bosons (assumed small)
  $\rightsquigarrow$ Tree-level FCNC by Z exchange
- 1-loop SM contribution

FCNC Results

Observable	Model Contribution	Limit
Δm_{B_d}	4×10^{-16} GeV	$\sim 10^{-14}$ GeV
Δm_{B_s}	7×10^{-12} GeV	$\sim 10^{-14}$ GeV
$\lvert\epsilon_K\rvert$	4×10^{-4}	$\sim 10^{-4}$

- $2 \ldots 3\,\sigma$ tension for ϵ_K
- Δm_K not considered (unknown long-distance, dim-8 contributions) but likely in tension
- Dominant contribution from Z' exchange despite $m_{Z'} \sim 10^8$ GeV

Electroweak Precision Observables

- Loop corrections to W and Z propagators
  $\rightsquigarrow$ Changes of oblique parameters S, T, U
- T most important for multi-Higgs models
  Grimus, Lavoura, Øgreid, Osland, NPB **801** (2008)

	Model Value	Best Fit	CDF m_W
T	-0.21	0.03 ± 0.12	0.17

- Benchmark point allowed at $2\,\sigma$ level
- Wrong sign to explain m_W anomaly
- T increases if light pseudoscalar becomes lighter

Conclusions

- SM with a single Higgs doublet is the simplest choice
- Natural extension: additional scalar doublets
- Here: 6 doublets, gauged family symmetry $U(1)_F$
- Fermion mass hierarchy from vev hierarchy
- $U(1)_F$ charge assignments $\rightsquigarrow$ anomaly cancellation, CKM mixing

Conclusions

- SM with a single Higgs doublet is the simplest choice
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- Fermion mass hierarchy from vev hierarchy
- $U(1)_F$ charge assignments $\rightsquigarrow$ anomaly cancellation, CKM mixing
- Challenges / interesting lessons
  - Scalar potential
  - Difficult to avoid accidental symmetries
  - Light pseudoscalar
  - Problematic FCNC in kaon sector due to Z' exchange
  - T parameter linked to light pseudoscalar mass

$\rightsquigarrow$ Different parameters, charges, symmetries? No-go theorem?

FCNC Results

	SM / experimental values	Model contribution
$\left[\Delta m_{B_d}\right]_{\text{SM}}$	(0.543 ± 0.029) ps^{-1} $= (3.57 \pm 0.19) \times 10^{-13}$ GeV	$2\left\lvert\left[M_{12}^{db}\right]_{\text{BSM}}\right\rvert =$ 4×10^{-16} GeV
$(\Delta M_{db})_{\text{exp}}$	$(50.65 \pm 0.19) \times 10^{10}\ \hbar s^{-1}$ $= (3.33 \pm 0.013) \times 10^{-13}$ GeV	
$\left[\Delta m_{B_s}\right]_{\text{SM}}$	(18.77 ± 0.86) ps^{-1} $= (1.235 \pm 0.057) \times 10^{-11}$ GeV	$2\left\lvert\left[M_{12}^{sb}\right]_{\text{BSM}}\right\rvert =$ 7×10^{-16} GeV
$(\Delta M_{sb})_{\text{exp}}$	$(17.765 \pm 0.006) \times 10^{12}\ \hbar s^{-1}$ $= (1.169 \pm 0.0004) \times 10^{-11}$ GeV	
$\lvert\epsilon_K\rvert_{\text{SM}}$	$(2.170 \pm 0.065_{\text{pert.}} \pm 0.076_{\text{nonpert.}} \pm 0.153_{\text{param.}}) \times 10^{-3}$ $= (2.170 \pm 0.1828) \times 10^{-3}$	$\lvert\epsilon_K\rvert_{\text{BSM}} =$ 4×10^{-4}
$\lvert\epsilon_K\rvert_{\text{exp}}$	$(2.228 \pm 0.0011) \times 10^{-3}$	

Lepton Sector

- Couplings

$$\overline{L_3} H_1 e_2 \quad , \quad \overline{L_1} H_3 e_1 \quad , \quad \overline{L_2} H_3 e_3 \quad , \quad \overline{L_3}\, \epsilon H_4^* e_3$$

- Charged lepton mass matrix

$$M^e = \frac{1}{\sqrt{2}} \begin{pmatrix} Y^e_{1,1} v_3 & 0 & 0 \\ 0 & 0 & Y^e_{2,3} v_3 \\ 0 & Y^e_{3,2} v_1 & Y^e_{3,3} v_4 \end{pmatrix}$$

$\rightsquigarrow$ Correct charged lepton masses

- Neutrino mass generation and mixing not addressed

Advantages of Gauged Family Symmetry

- Anomaly-freedom required $\rightsquigarrow$ fewer free parameters
- $U(1)_F$ gauge boson as additional DM candidate
- DM stabilized by gauge symmetry $\rightsquigarrow$ safe from decay via Planck-suppressed operators (unlike in case of stabilization by global symmetry)