Gravitational waves from Gauss-Bonnet-corrected inflation

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November 16th, 2023

@ 3rd International Joint Workshop on BSM
/ 11th KIAS Workshop on Particle Physics and Cosmology,
Jeju, South Korea

Inflation with Gauss-Bonnet coupling

$$S = \int d^4x\sqrt{-g}\left[\frac{M_\mathrm{P}^2}{2}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - V(\varphi) - \frac{1}{16}\xi(\varphi)R_\mathrm{GB}^2\right]$$

$$R_\mathrm{GB}^2 \equiv R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$$

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- Higher-curvature correction [Weinberg (2008)]
- 1-loop correction to some superstring models [Antoniadis, Rizos, and Tamvakis (1994)], [Rizos and Tamvakis (1994)]
- Widely studied in the context of cosmology, including slow-roll inflation [Kawai, Sakagami, and Soda (1998)], [Kawai, Sakagami, and Soda (1999)], [Kawai and Soda (1999)], [Satoh and Soda (2008)], [Satoh (2010)], [Kawai and JK (2019)], ... [See also talk by Liliana Velasco-Sevilla]

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  - first part of the talk [2108.01340 (with Shinsuke Kawai)]

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- Potential term $\ll$ GB term:
  - GB dominance
  - second part of the talk [2308.13272 (with Shinsuke Kawai)]

Balance regime

Potential term $\approx$ GB term with $V_{,\varphi}\xi_{,\varphi} < 0$

$$0 = \ddot{\varphi} + 3H\dot{\varphi} + V_{,\varphi} + \frac{3}{2}H^2\left(\dot{H} + H^2\right)\xi_{,\varphi}$$

$$\implies \ddot{\varphi} + 3H\dot{\varphi} \approx 0\,.$$

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\begin{aligned}
0 &= \ddot{\varphi} + 3H\dot{\varphi} + V_{,\varphi} + \frac{3}{2}H^2\left(\dot{H} + H^2\right)\xi_{,\varphi} \\
&\Longrightarrow \ddot{\varphi} + 3H\dot{\varphi} \approx 0\,.
\end{aligned}
$$

- Ultra-slow-roll inflation

[Inoue and Yokoyama (2002)], [Kinney (2005)],... see also [Kawaguchi and Tsujikawa (2022)]

Slow-roll and ultra-slow-roll inflation

[Inoue and Yokoyama (2002)], [Kinney (2005)]

Ignoring the GB coupling for the moment,

$$\ddot{\varphi} + 3H\dot{\varphi} + V_{,\varphi} = 0\,.$$

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Standard slow-roll : $\ddot{\varphi} \approx 0$

$$\epsilon_1 \equiv -\frac{\dot{H}}{H^2} \ll 1$$

$$|\epsilon_2| \equiv \left|\frac{\dot{\epsilon}_1}{H\epsilon_1}\right| = \left|\frac{2\ddot{\varphi}}{H\dot{\varphi}} + 2\epsilon_1\right| \ll 1$$

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Standard slow-roll : $\ddot{\varphi} \approx 0$	Ultra-slow-roll : $V_{,\varphi} \approx 0$
$\epsilon_1 \equiv -\dfrac{\dot{H}}{H^2} \ll 1$	$\epsilon_1 \equiv -\dfrac{\dot{H}}{H^2} \ll 1$
$\lvert\epsilon_2\rvert \equiv \left\lvert\dfrac{\dot{\epsilon}_1}{H\epsilon_1}\right\rvert = \left\lvert\dfrac{2\ddot{\varphi}}{H\dot{\varphi}} + 2\epsilon_1\right\rvert \ll 1$	$\epsilon_2 \equiv \dfrac{\dot{\epsilon}_1}{H\epsilon_1} = -6 + 2\epsilon_1 \approx -6$

Ultra-slow-roll with Gauss-Bonnet coupling

$$\text{Potential term} \;\approx\; \text{GB term} \quad \text{with} \quad V_{,\varphi}\xi_{,\varphi} < 0$$

$$0 = \ddot{\varphi} + 3H\dot{\varphi} + V_{,\varphi} + \frac{3}{2}H^2\left(\dot{H} + H^2\right)\xi_{,\varphi}$$
$$\implies \ddot{\varphi} + 3H\dot{\varphi} \approx 0\,.$$

- Ultra-slow-roll inflation
- Without a near-inflection point in V itself

Ultra-slow-roll with Gauss-Bonnet coupling

For concreteness, let us consider

$$V = \Lambda^4 \left(1 + \cos\frac{\varphi}{f}\right) , \quad \xi = \xi_0 \tanh\left[\xi_1(\varphi - \varphi_c)\right] .$$

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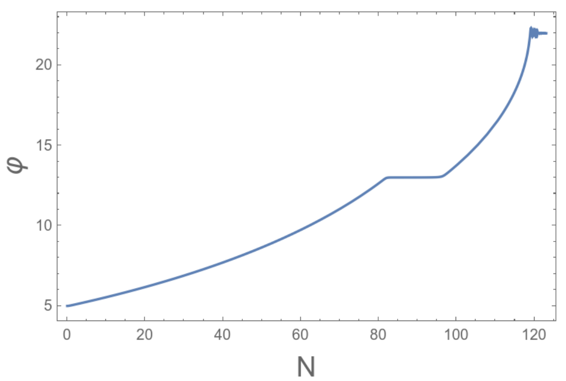
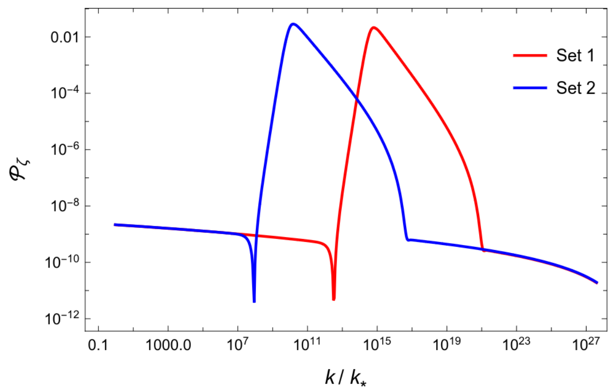


$$\{f[M_{\mathrm{P}}], \varphi_c[M_{\mathrm{P}}], \xi_0, \xi_1[M_{\mathrm{P}}^{-1}]\}$$
$$= \{7, 13.0, 6.044 \times 10^7, 15.0\}$$

Curvature power spectrum



Curvature perturbation enhanced

- $A_s \sim 1/\epsilon_1$ or $\sim 1/\dot{\varphi}$
- production of primordial black holes
- scalar-induced gravitational waves

[See also Kingman Cheung's talk]

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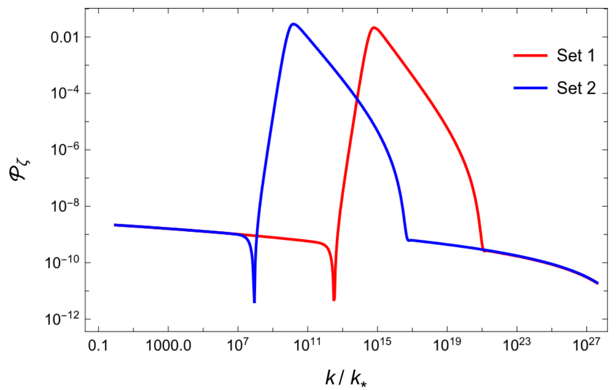

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3 free parameters for a given f:

- φ_c : peak position
- ξ_1 : width of the peak
- ξ_0 : magnitude of the peak

Primordial black holes

When very large density fluctuations re-enter the horizon, primordial black holes may form due to the gravitational collapse. [Zel'dovich and Novikov (1967)], [Hawking (1971)], [Carr and Hawking (1974)], [Polnarev and Khlopov (1985)], ...

Abundance of the PBHs: [See also Kingman Cheung's talk]

$$f_{\rm PBH} \equiv \frac{\Omega_{\rm PBH,0}}{\Omega_{\rm DM,0}} \approx \left(\frac{\beta(M)}{3.27\times 10^{-8}}\right)\left(\frac{0.2}{\gamma}\right)^{3/2}\left(\frac{106.75}{g_{*,f}}\right)^{1/4}\left(\frac{0.12}{\Omega_{\rm DM,0}h^2}\right)\left(\frac{M}{M_\odot}\right)^{-1/2}$$

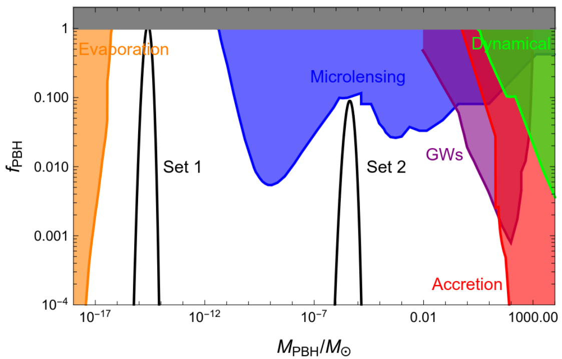
$$\beta = \int_{\delta_c} d\delta \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{\delta^2}{2\sigma^2}}\,, \quad \sigma^2 = \frac{16}{81}\int_0^\infty \frac{dq}{q}\left(\frac{q}{k}\right)^4 W^2\left(\frac{q}{k}\right)\mathcal{P}_\zeta(q)$$

Assumptions:

- Density fluctuation follows a Gaussian distribution.
- PBH formation occurs during radiation-dominated era.

Primordial black holes

constraints data : [Green and Kavanagh (2021)]



- Set 1 : $f_{\mathrm{PBH}}^{\mathrm{tot}} \approx 1$
- Set 2 : $f_{\mathrm{PBH}}^{\mathrm{tot}} \approx 0.087$

Scalar-induced gravitational waves

[Matarrese, Mollerach, and Bruni (1998)], [Mollerach, Harari, and Matarrese (2004)], [Ananda, Clarkson, and Wands (2007)], [Baumann, Steinhardt, Takahashi, and Ichiki (2007)], [Kohri and Terada (2018)], [Domènech (2020)]

Enhanced curvature perturbation : a source, $S_{\mathbf{k}}$, for the tensor perturbation

$$h''_{\mathbf{k}} + 2\mathcal{H}h'_{\mathbf{k}} + k^2 h_{\mathbf{k}} = S_{\mathbf{k}}$$

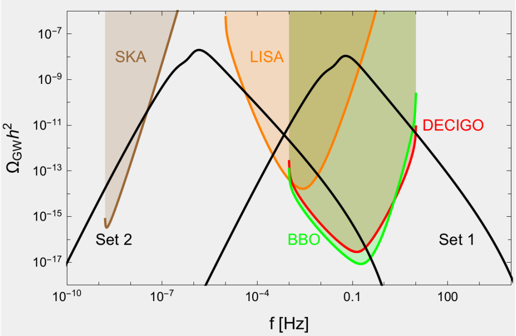
Scalar-induced second-order gravitational waves

Scalar-induced gravitational waves

[Kohri and Terada (2018)]

$$\Omega_{\mathrm{GW,f}}(k) = \frac{1}{12}\int_0^\infty dv \int_{|1-v|}^{1+v} du$$
$$\times \left(\frac{4v^2-(1+v^2-u^2)^2}{4uv}\right)^2$$
$$\times \mathcal{P}_\zeta(kv)\mathcal{P}_\zeta(ku)\left(\frac{3(u^2+v^2-3)}{4u^3v^3}\right)^2$$
$$\times \left[\left(-4uv+(u^2+v^2-3)\log\left|\frac{3-(u+v)^2}{3-(u-v)^2}\right|\right)^2\right.$$
$$\left.+\pi^2(u^2+v^2-3)^2\theta(v+u-\sqrt{3})\right].$$

sensitivity curves : [Schmitz (2021)]



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$$0 = \ddot{\varphi} + 3H\dot{\varphi} + V_{,\varphi} + \frac{3}{2}H^2\left(\dot{H} + H^2\right)\xi_{,\varphi}$$

- Potential term $\approx$ GB term:
  - Balance; cancellation when $V_{,\varphi}\xi_{,\varphi} < 0$
  - SR $\rightarrow$ USR $\rightarrow$ SR
  - primordial black hole formation
  - scalar-induced gravitational waves

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  - SR $\rightarrow$ GB domination $\rightarrow$ SR

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    - with $V_{,\varphi}\xi_{,\varphi} > 0$

Gauss-Bonnet domination regime

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We again consider

$$\xi = \xi_0 \tanh\left[\xi_1(\varphi - \varphi_c)\right] \, .$$

As $\xi_{,\varphi} \sim \mathrm{sech}^2[\xi_1(\varphi - \varphi_c)]$, the GB term may become dominant.

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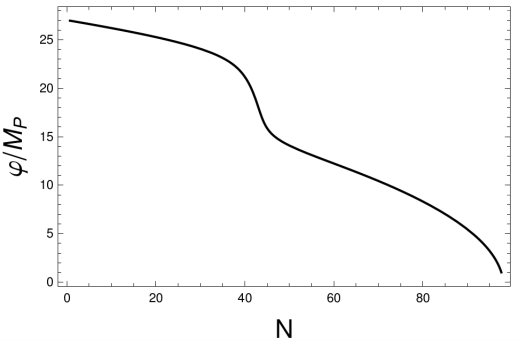
$\underline{\varphi \text{ away from } \varphi_c}$	$\underline{\varphi \text{ close to } \varphi_c}$	$\underline{\varphi \text{ away from } \varphi_c}$
SR	GB domination	SR

Gauss-Bonnet domination regime

For completeness, let us consider $V = m^2\varphi^2/2$.



$$\{\xi_0, \xi_1, \varphi_c\} = \{0.196 M_{\rm P}^2/m^2, 0.5/M_{\rm P}, 18.5 M_{\rm P}\}$$

- sudden acceleration $\rightarrow$ deceleration
- opposite to the USR case
- surge of gravitational wave

Tensor perturbation during the Gauss-Bonnet domination regime

$$h_{\mathbf{k}}'' + 2\frac{A_t'}{A_t}h_{\mathbf{k}}' + k^2 C_t^2 h_{\mathbf{k}} = 0 \Longleftrightarrow v_{\mathbf{k}}'' + \left(k^2 C_t^2 - \frac{A_t''}{A_t}\right) v_{\mathbf{k}} = 0$$

$$A_t^2 \equiv a^2\left(1 - \frac{\sigma_1}{2}\right), \quad C_t^2 \equiv 1 + \frac{a^2\sigma_1}{2A_t^2}\left(1 - \sigma_2 - \epsilon_1\right), \quad \sigma_1 \equiv \frac{H\dot{\xi}}{M_{\mathrm{P}}^2}, \quad \sigma_2 \equiv \frac{\dot{\sigma}_1}{H\sigma_1}$$

- Away from φ_c :

$$A_t \approx a\,, \quad C_t^2 \approx 1 \quad \Longrightarrow \quad \text{standard tensor perturbation}$$

- Near φ_c :

$$C_t^2 \ll 1$$

Primordial tensor power spectrum

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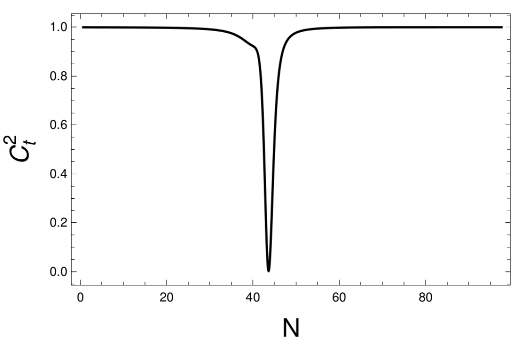
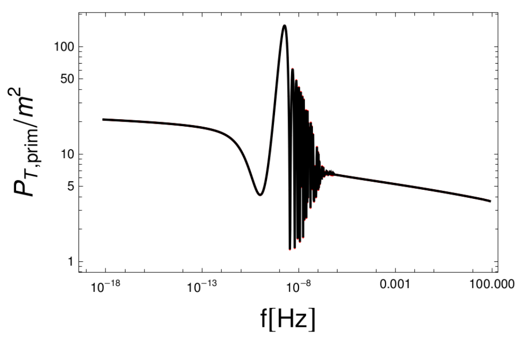


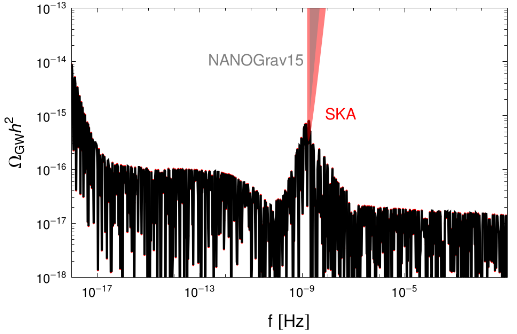


see also [Satoh, Kanno, and Soda (2008)], [Guo and Schwarz (2009)], [Cai, Wang, and Piao (2015, 2016)], [Cai and Piao (2022)]

Gravitational wave spectrum

[Guzzetti, Bartolo, Liguori, and Matarrese (2016)], [Kuroyanagi, Takahashi, and Yokoyama (2015, 2021)], [Boyle and Steinhardt (2008)], ...

$$\Omega_{\mathrm{GW}}(k) = \frac{1}{12}\left(\frac{k}{a_0 H_0}\right)^2 T^2(k)\mathcal{P}_{\mathrm{T,prim}}$$



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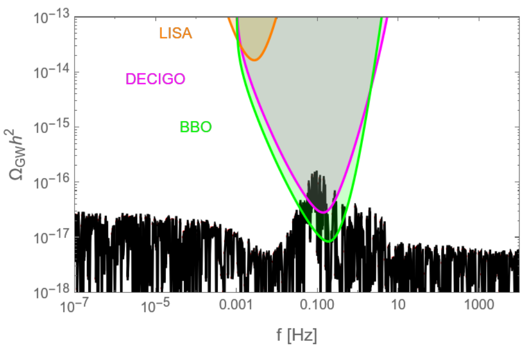

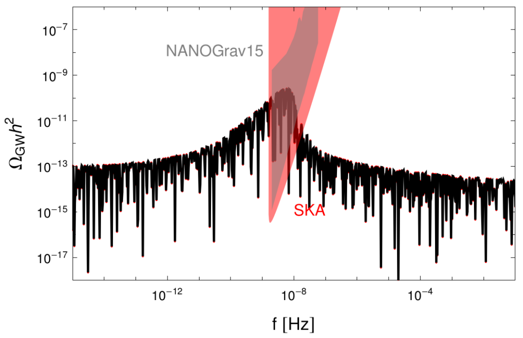

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