

Dirac-Majorana neutrino type oscillation induced by a wave dark matter

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based on [[arXiv: 2305.16900](#)]

with YeolLin ChoeJo and Hye-Sung Lee

Unknown in Neutrino Physics

Neutrino oscillation parameters $\theta_{23} > \pi/4$, or $< \pi/4$
 $\Delta m_{13} > 0$, or < 0

CP violation in PMNS δ_{CP}

Neutrino absolute mass m_ν

Sterile neutrino ν_4

...

Neutrino is

Dirac type or **Majorana type** ?

$$\nu \neq \bar{\nu}$$

$$\nu = \bar{\nu}$$

Dirac mass **Majorana mass**

$$\mathcal{L} = -m_D \bar{\nu}_D \nu_D - \frac{1}{2} m_R \bar{\nu}_M \nu_M$$

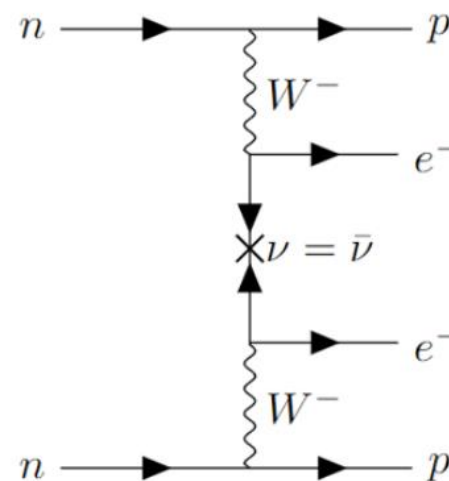
$\nu_D = \nu_L + \nu_R$ $\nu_M = \nu_R + \nu_R^c$
 ($\nu_M = \nu_M^c$)

Majorana type $\nu = \bar{\nu}$
 ($m_D \ll m_R$)

Seesaw mechanism
 (for small m_ν)

$$m_\nu = -\frac{m_D^2}{m_R}$$

Neutrinoless double beta decay
 (as LNV process)



Leptogenesis
 (for BAU)

$$\Gamma(N_i \rightarrow \ell \Phi) - \Gamma(N_i \rightarrow \bar{\ell} \Phi^\dagger)$$

(N_i is mass eigenstate)

$$\mathcal{L} = \underbrace{-m_D \bar{\nu}_D \nu_D}_{\text{Dirac mass}} - \underbrace{\frac{1}{2} m_R \bar{\nu}_M \nu_M}_{\text{Majorana mass}}$$

$\nu_D = \nu_L + \nu_R$
 $\nu_M = \nu_R + \nu_R^c$
($\nu_M = \nu_M^c$)



Quasi-Dirac type
($m_D \gg m_R$)

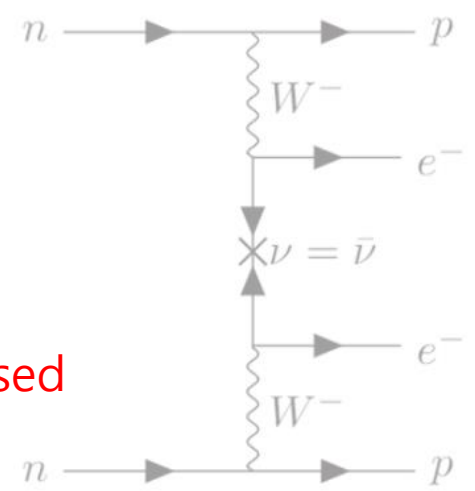
Neutrino is still Majorana spinor $\nu = \bar{\nu}$
But it behaves like Dirac spinor

Seesaw mechanism
(for small m_ν)

$$m_\nu = -\frac{m_D^2}{m_R}$$

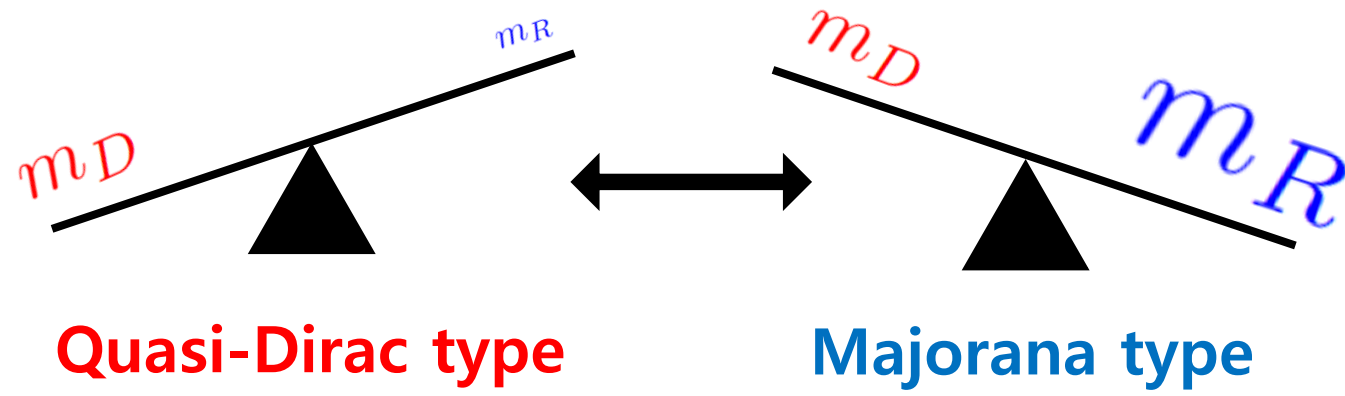
Neutrinoless double beta decay

Also, LNV is suppressed



Leptogenesis
(for BAU)

$$\Gamma(N_i \rightarrow \ell \Phi) - \Gamma(N_i \rightarrow \bar{\ell} \Phi^\dagger)$$



**“Dirac-Majorana
neutrino type oscillation”**

Wave dark matter ϕ has an oscillating scalar behavior.

$$10^{-22} \text{ eV} < m_\phi < 30 \text{ eV}$$

$$T = 2\pi/m_\phi = \mathcal{O}(\text{ps}) \sim \mathcal{O}(\text{year})$$

$$\text{EoM: } \ddot{\phi} + \cancel{3H\dot{\phi}} + m_\phi^2\phi = 0$$

$$\text{Sol: } \phi(t) = \frac{\sqrt{2\rho_\phi}}{m_\phi} \cos m_\phi t$$



Neutrino mass variation

$$\mathcal{L} \supset -g\phi\bar{\nu}\nu + h.c.$$

$$\rightarrow m_\nu(t) = g \frac{\sqrt{2\rho_\phi}}{m_\phi} \cos m_\phi t$$

[A. Berlin (2016),
Y. Zhao (2017),
G. Krnjaic et al (2017),
V. Brdar et al (2017) ...]

(Effects on neutrino flavor oscillation, etc)

$$\mathcal{L} = -yHLN - g\phi NN + h.c.$$



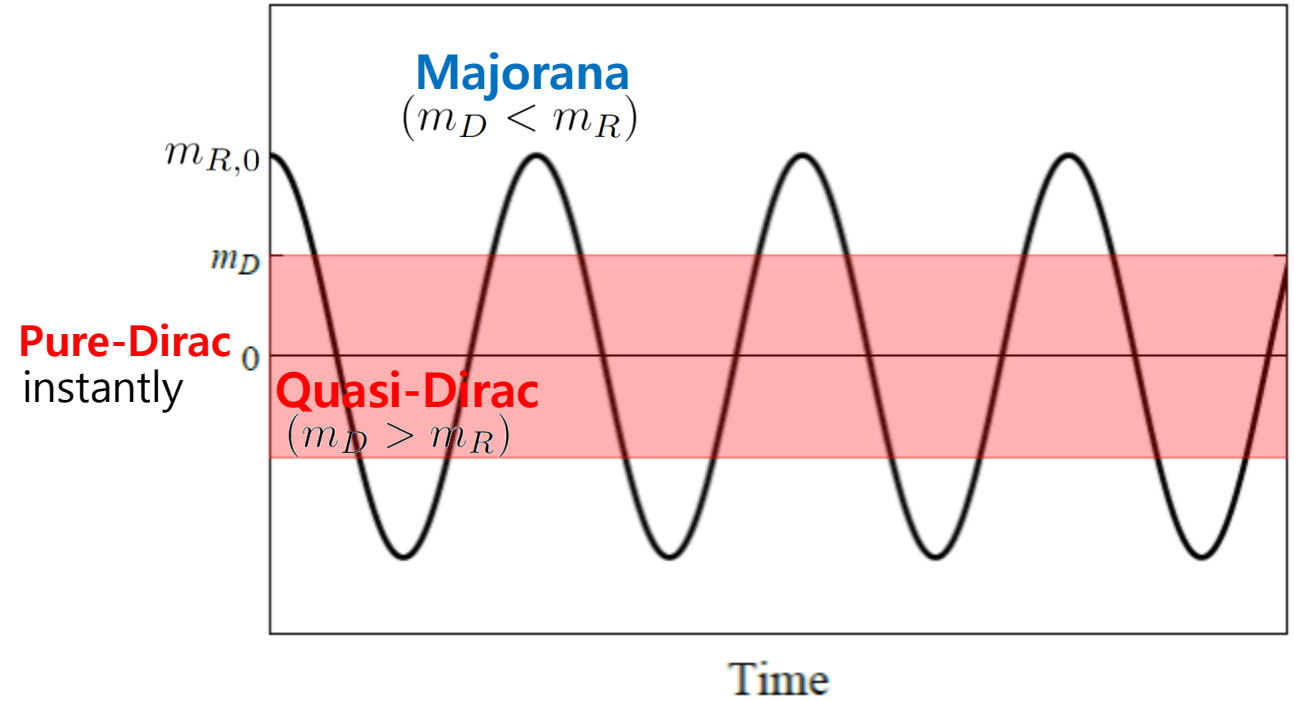
$$m_D = \frac{yv}{\sqrt{2}}$$



$$m_R(t) = m_{R,0} \cos m_\phi t$$

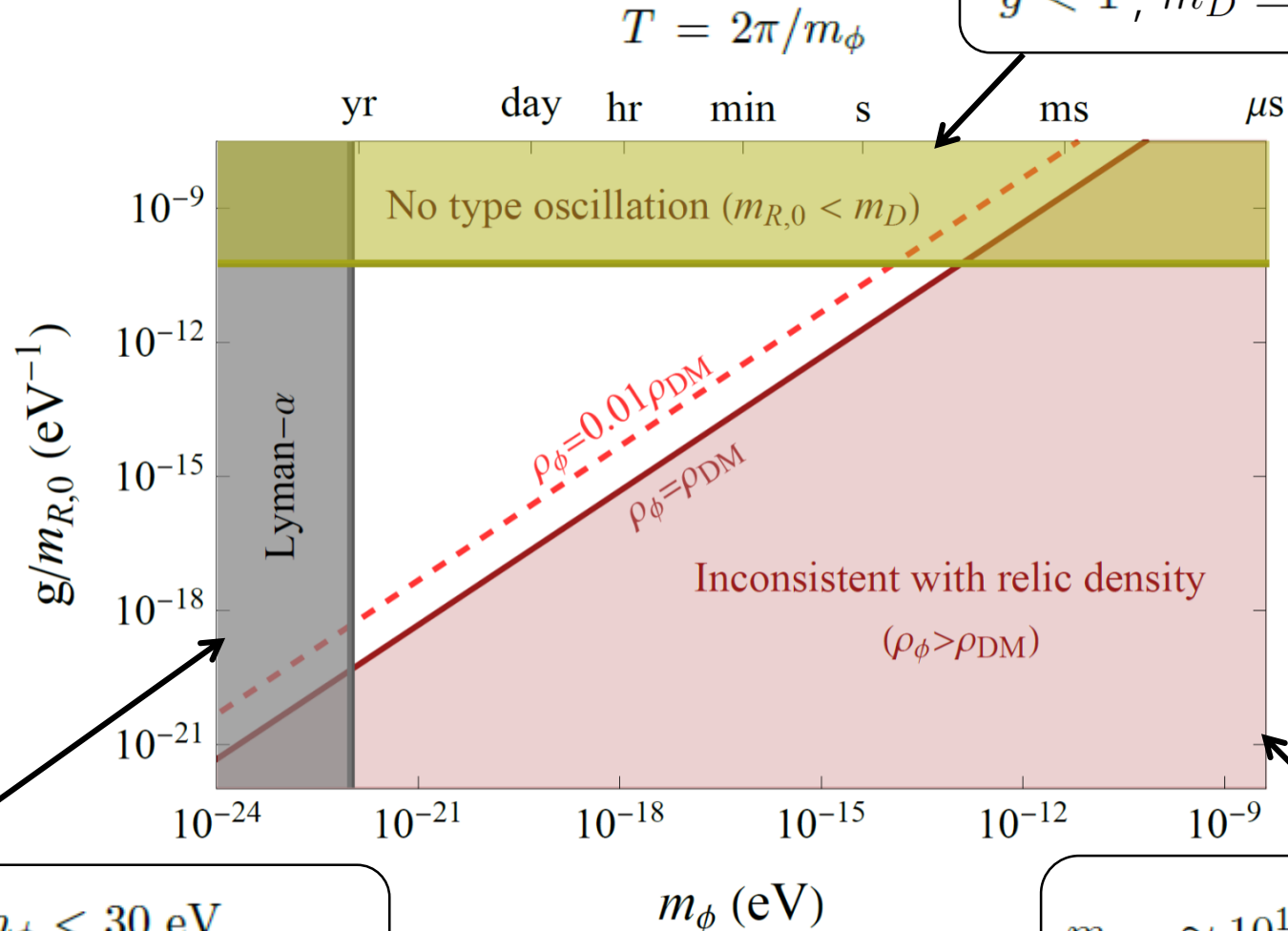
where $m_{R,0} = g \frac{\sqrt{2\rho_\phi}}{m_\phi}$

$$m_R(t) = m_{R,0} \cos m_\phi t$$



Constraints & Disfavored region

Disfavored region of type oscillation
 $g < 1, m_D \simeq 246 \text{ GeV}$

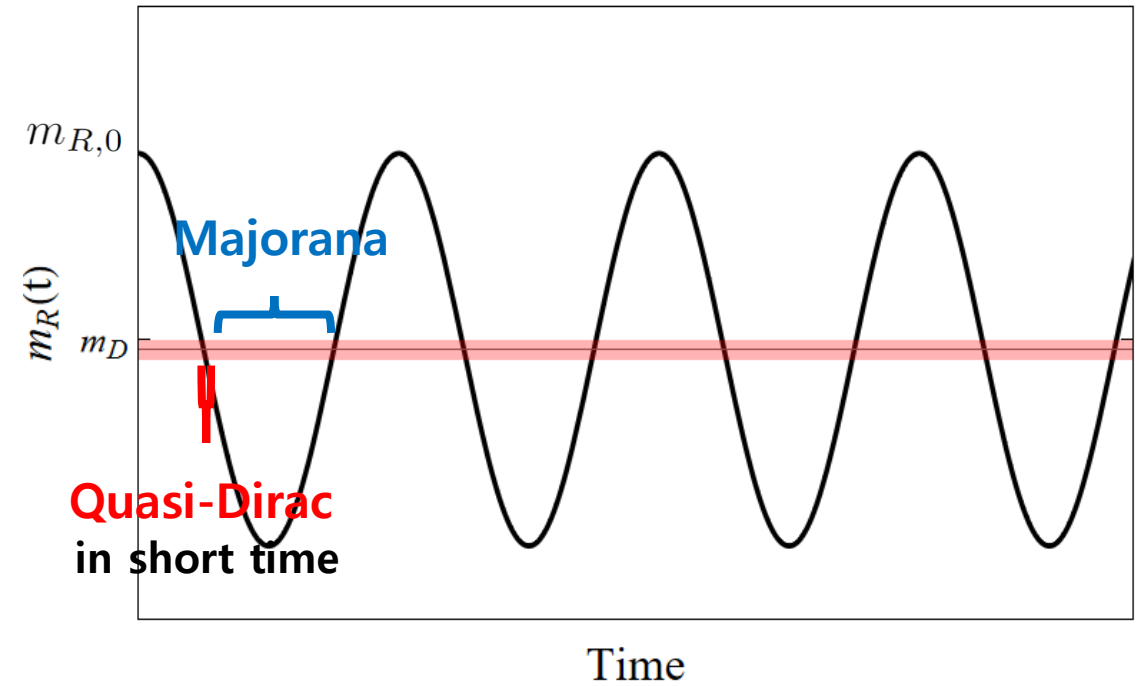
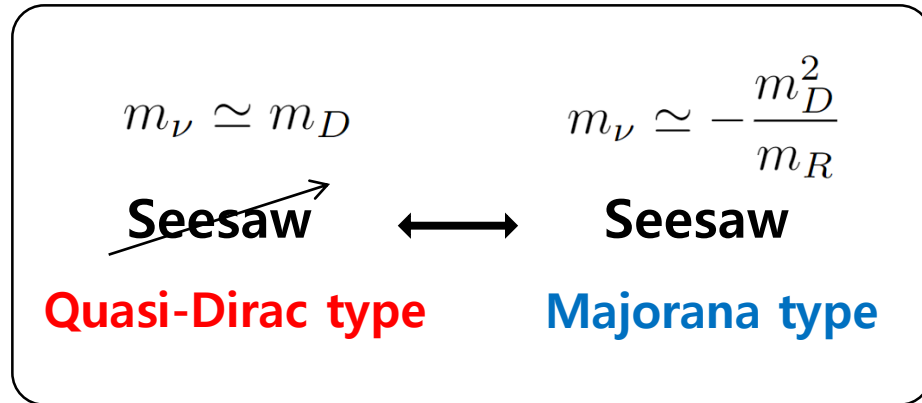


$10^{-22} \text{ eV} < m_{\phi} < 30 \text{ eV}$
 Constraint on wave DM mass

$m_{R,0} \simeq 10^{19} \text{ eV} \left(\frac{g}{1}\right) \left(\frac{\rho_{\phi}}{\rho_{\text{DM}}}\right)^{1/2} \left(\frac{10^{-22} \text{ eV}}{m_{\phi}}\right)$
 Constraint on DM relic density

Neutrino mass variation \rightarrow Neutrino-related physics (flavor oscillation, cosmological observables, etc)

Ex) Active neutrino mass



Time average

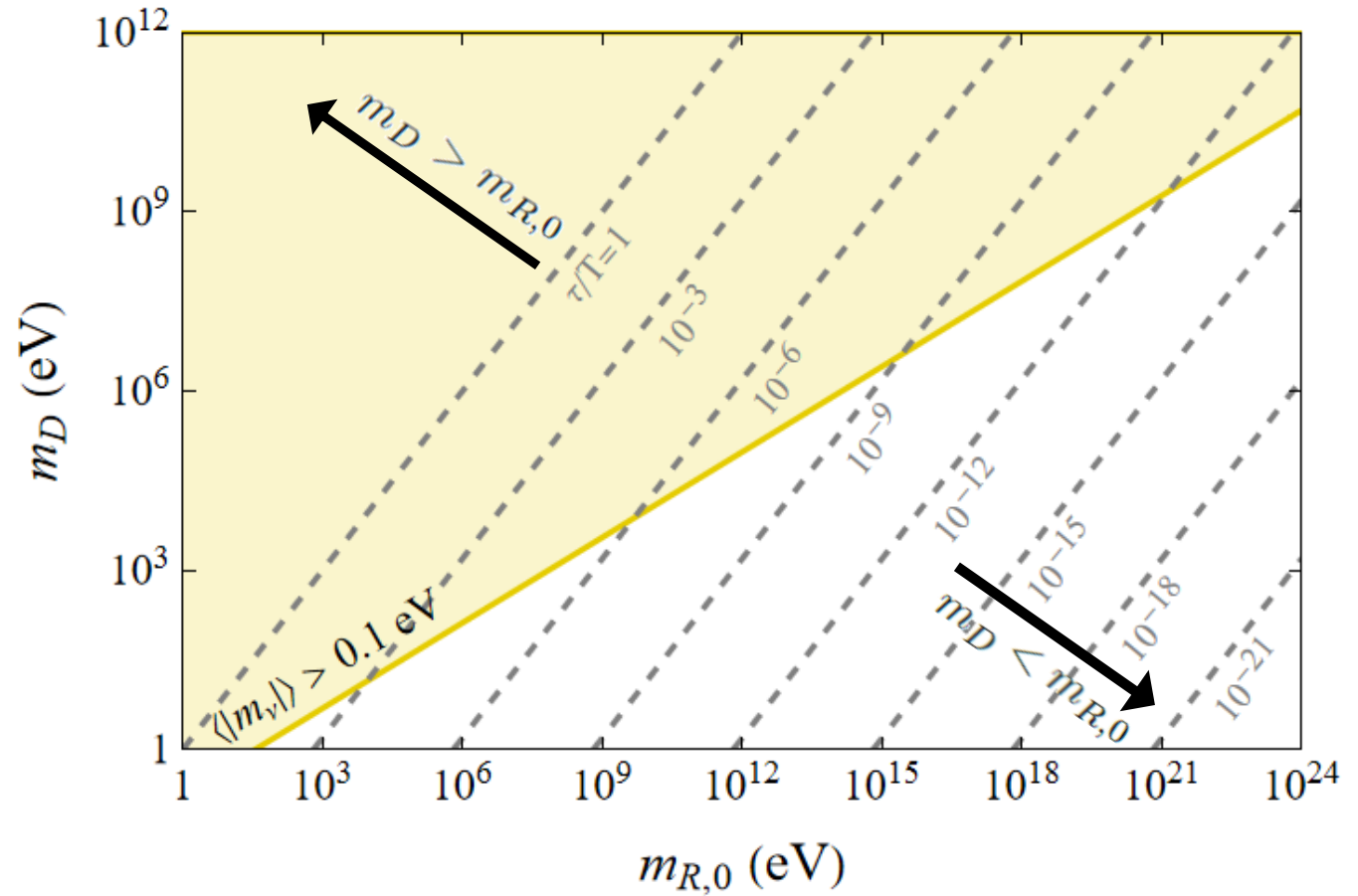
$$\langle |m_\nu| \rangle = \frac{1}{T} \int_0^T |m_\nu(t)| dt < 0.1 \text{ eV}$$

Mass bound
from CMB/ 3H decay

Quasi-Dirac time ratio τ/T

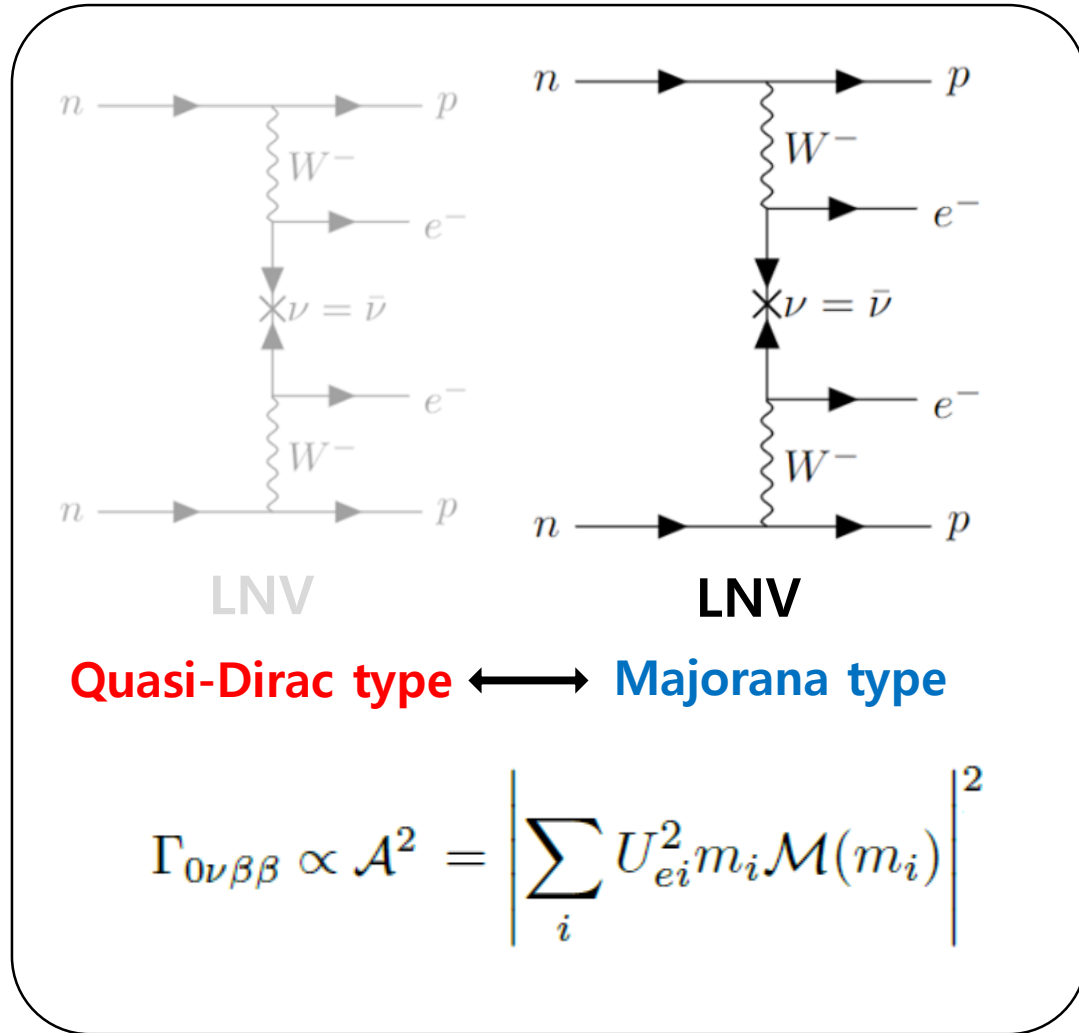
$$\frac{\tau}{T} = 1 \quad \text{for } m_D > m_{R,0}$$

$$\frac{m_D}{m_{R,0}} = \sin\left(\frac{\pi \tau}{2T}\right) \quad \text{for } m_D < m_{R,0}$$



Time-averaged quantity < Known bound \rightarrow Quasi-Dirac type in only short time

Neutrinoless double beta ($0\nu\beta\beta$) decay

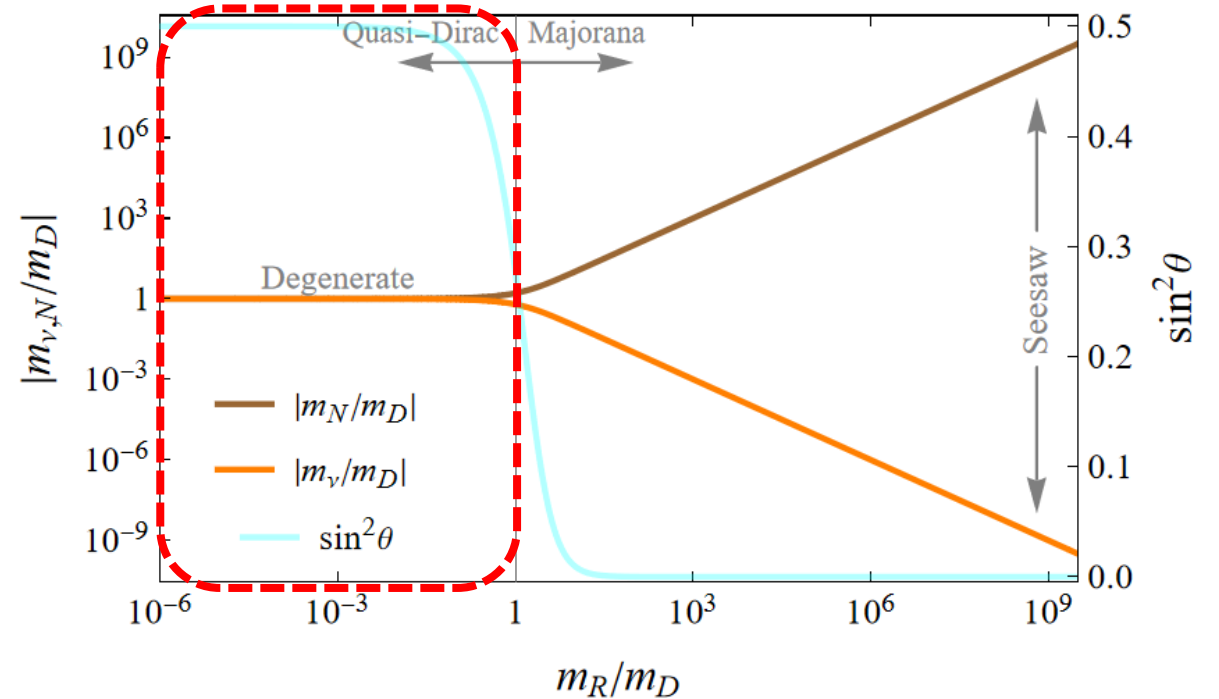


Ex) single-flavor case:

$$\mathcal{A} = \left| m_\nu \cos^2 \theta \mathcal{M}(m_\nu) + m_N \sin^2 \theta \mathcal{M}(m_N) \right|$$

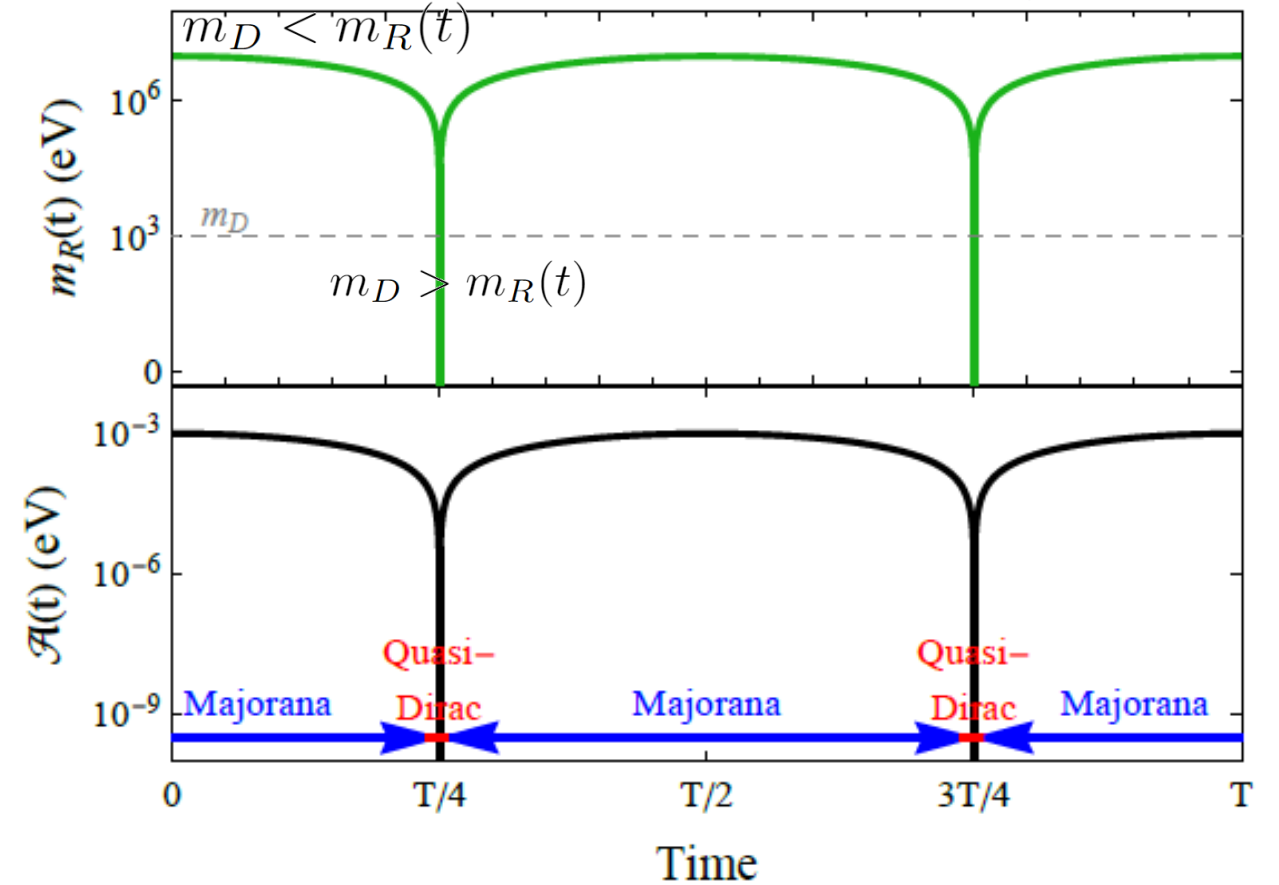
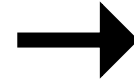
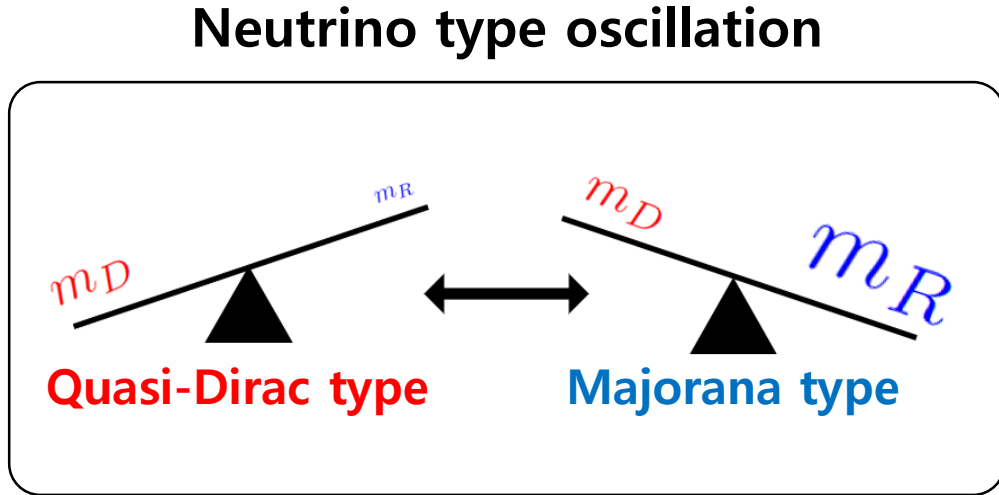
$|m_\nu| \simeq |m_N|$, $\cos^2 \theta \simeq \sin^2 \theta$ in **quasi-Dirac** limit

→ \mathcal{A} is strongly suppressed



Turn-on/off behavior of $0\nu\beta\beta$ decay

$$m_D = 10^3 \text{ eV}, m_{R,0} = 10^7 \text{ eV}$$



Neutrino type oscillation \rightarrow Modulation behavior

* Other process: Cosmic neutrino background (C ν B) $\Gamma_{C\nu B}^M = 2\Gamma_{C\nu B}^D$

Leptogenesis (LG)

Heavy Majorana neutrino decay (N_i is mass eigenstate)

$$N_i \rightarrow \ell\Phi, N_i \rightarrow \bar{\ell}\Phi^\dagger$$



Lepton asymmetry



Sphaleron process
($B-L$ is conserved)

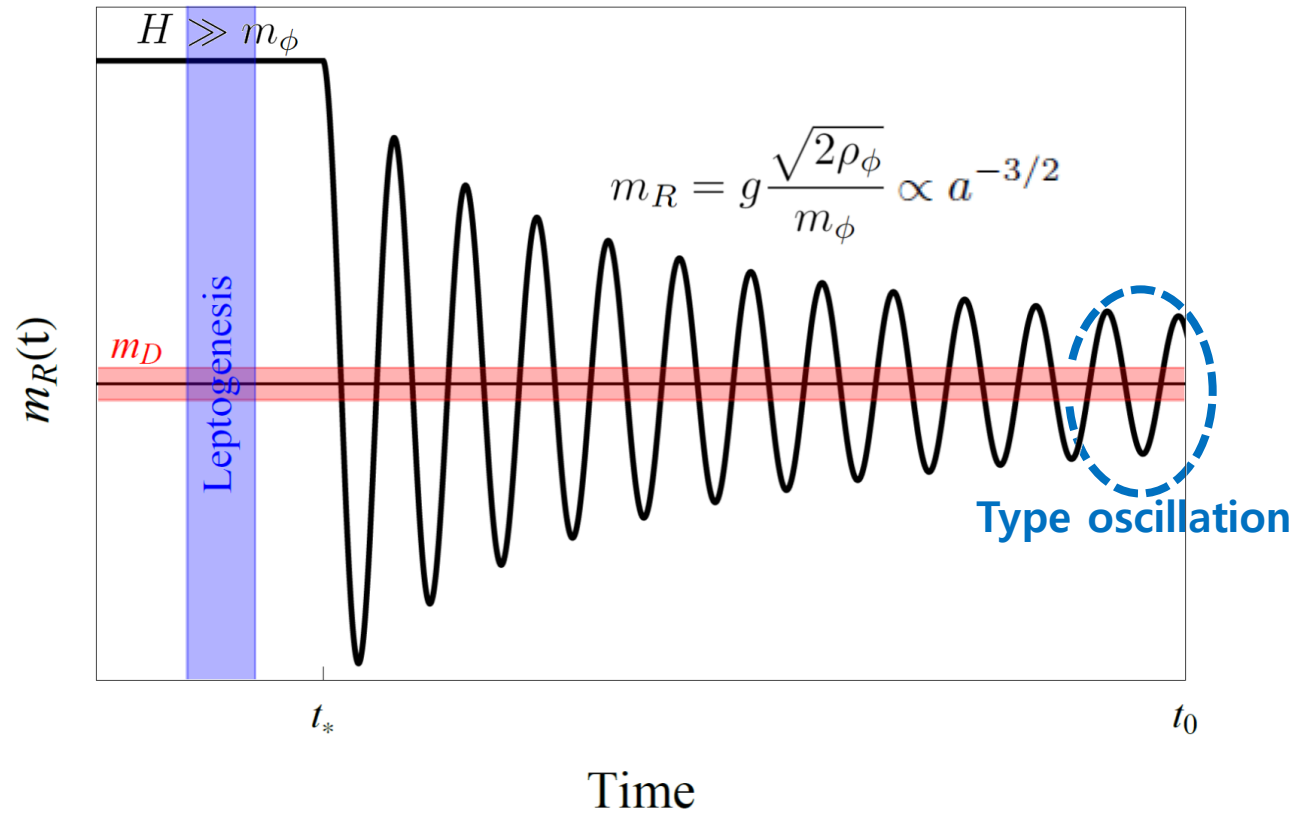
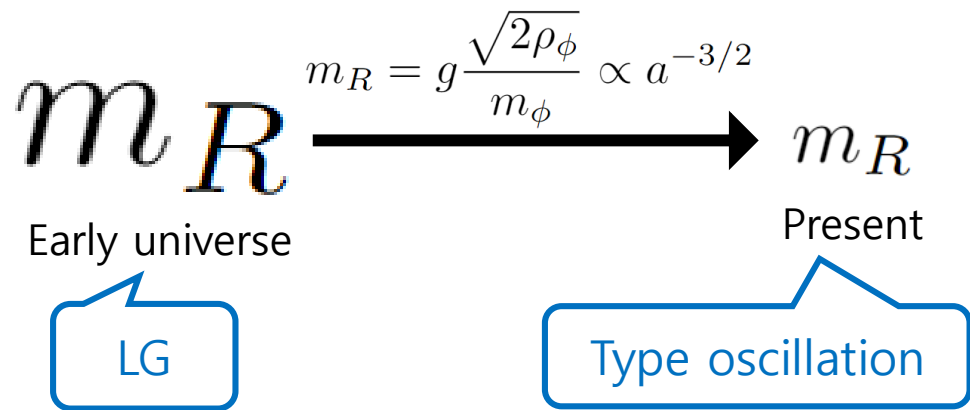
Baryon asymmetry of the universe (BAU)

$$\eta_B = \frac{n_B - n_{\bar{B}}}{n_\gamma} \sim \mathcal{O}(10^{-10})$$

CP asymmetry

$$\epsilon_i = \frac{\Gamma(N_i \rightarrow \ell\Phi) - \Gamma(N_i \rightarrow \bar{\ell}\Phi^\dagger)}{\Gamma(N_i \rightarrow \ell\Phi) + \Gamma(N_i \rightarrow \bar{\ell}\Phi^\dagger)} \approx 10^{-6} \quad \text{for observed BAU}$$

**→ Majorana mass scale is
important parameter in leptogenesis**



➔ Link between early universe and present-time

$$\frac{m_{R,*}}{m_{R,0}} \simeq 10^{11} \left(\frac{m_\phi}{10^{-22} \text{ eV}} \right)^{3/4}$$

Ex) $m_\phi = 10^{-22} \text{ eV}$

$$m_{R,*} = 10^{15} \text{ GeV} \rightarrow m_{R,0} = 10 \text{ TeV}$$

(GUT) (Testable scale)

Casas-Ibarra parametrization

$$y = \sqrt{2} \hat{M}_N^{1/2} R \hat{M}_\nu^{1/2} U_{\text{PMNS}}^\dagger / v, \quad R = \begin{pmatrix} 0 & \cos z & \sin z \\ 0 & -\sin z & \cos z \end{pmatrix}$$



$$y \propto \sqrt{m_{R,0}}$$

Yukawa matrix is proportional to the current Majorana mass.

CP asymmetry

$$\epsilon_i \propto \sum_{k \neq i} \frac{\text{Im}(y^\dagger y)_{ik}^2}{(y^\dagger y)_{ii} (y^\dagger y)_{kk}} \frac{(M_i^2 - M_k^2) M_i \Gamma_k}{(M_i^2 - M_k^2)^2 + M_i^2 \Gamma_k^2}$$

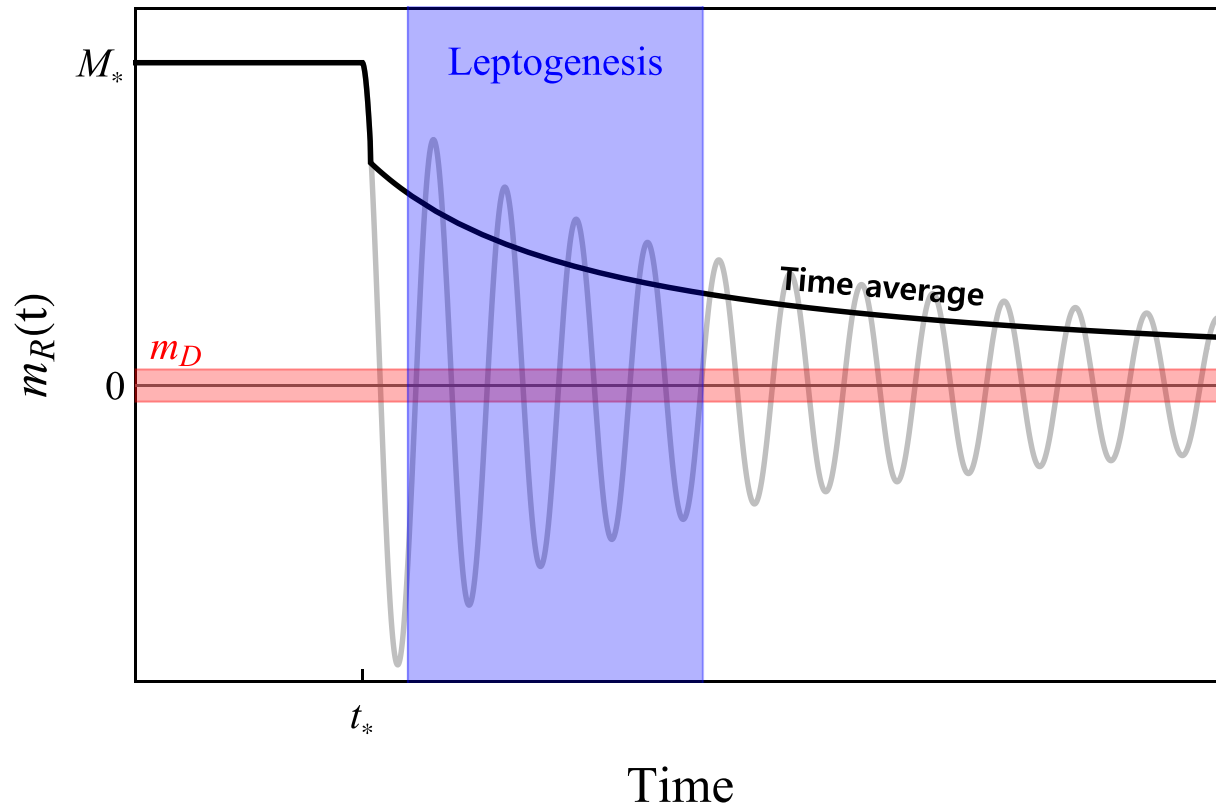
Resonance by the degenerate mass spectrum
 $(M \simeq m_R \text{ is mass eigenvalue})$

➔ The current Majorana mass scale is testable, but it requires the resonant leptogenesis.

Leptogenesis with the Neutrino mass variation *(ongoing project)*

with Yeollin ChoeJo, Kazuki Enomoto, and Hye-Sung Lee

If wave DM oscillation starts earlier than leptogenesis...



... the behavior of leptogenesis can be changed.

Ex) Production rate of heavy neutrino gets

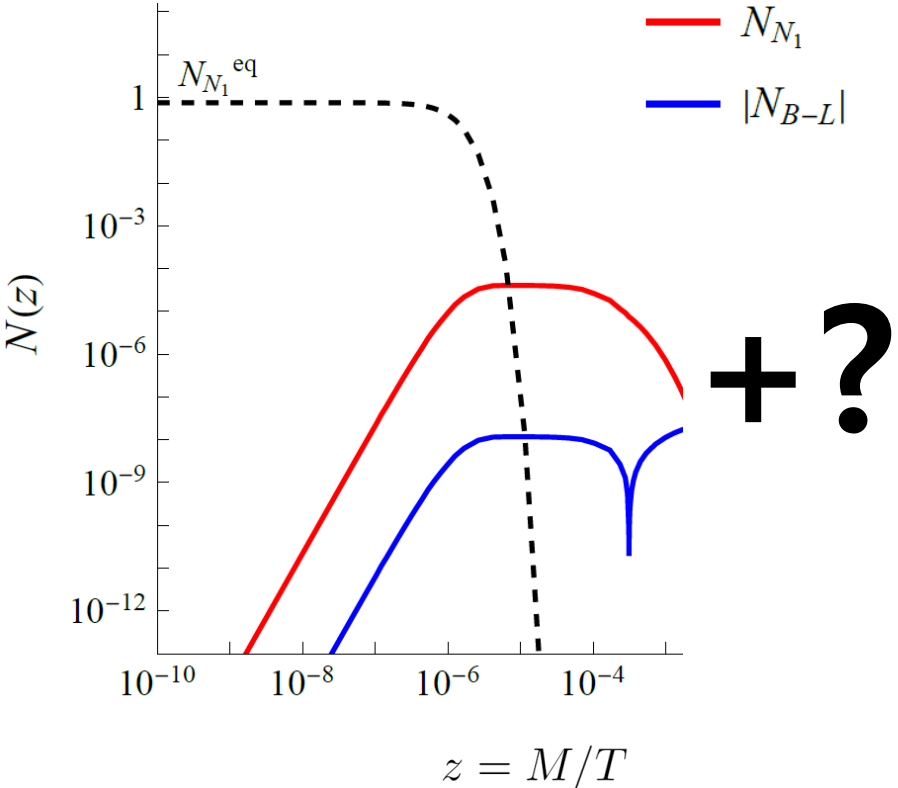
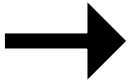
$$\Gamma_{\text{prod}} \propto e^{-M/T} \quad \text{with} \quad M \propto a^{-3/2}$$

(ongoing project)

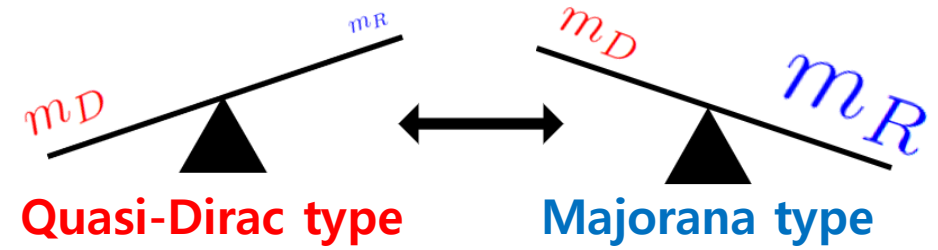
Boltzmann equations

$$\begin{aligned} \frac{dN_{N_1}}{dz} &= -D(N_{N_1} - N_{N_1}^{\text{eq}}), \\ \frac{dN_{B-L}}{dz} &= -\epsilon_1 D(N_{N_1} - N_{N_1}^{\text{eq}}) - W N_{B-L} \end{aligned}$$

Plug the mass varying effect



Summary



- The ratio of the Dirac/Majorana masses can change in time if they arise from the wave dark matter. It provides "**Dirac-Majorana neutrino type oscillation.**"
- Quasi-Dirac moment should be short to explain the small neutrino mass by the seesaw mechanism.
- The neutrino type oscillation predicts the modulation behavior of lepton number violation (LNV) process like as $0\nu\beta\beta$ decay.
- There is an interesting connection between early universe physics and present-time physics because the mass scale decreases over time.
- The heavy neutrino mass is an important parameter in leptogenesis, so the neutrino mass variation scenario may change the behavior of leptogenesis.

Backup Slides

Dirac/Majorana spinors formalism

$$\mathcal{L} = -yHLN - MNN + h.c.$$

Dirac neutrino $\nu_D = \nu_L + \nu_R$

Majorana neutrino $\nu_M = \nu_R + \nu_R^c$

Dirac mass term Majorana mass term

$$\begin{aligned} \mathcal{L} &= \boxed{-m_D \bar{\nu}_D \nu_D} - \boxed{\frac{1}{2} m_R \bar{\nu}_M \nu_M} \\ &= -\frac{1}{2} (\bar{\nu}_L \quad \bar{\nu}_R^c) \begin{pmatrix} 0 & m_D \\ m_D & m_R \end{pmatrix} \begin{pmatrix} \nu_L^c \\ \nu_R \end{pmatrix} + h.c. \end{aligned}$$

light/heavy mass eigenvalues:

$$m_{l,h} = \frac{1}{2} \left(m_R \mp \sqrt{m_R^2 + 4m_D^2} \right)$$

light/heavy mass eigenstates:

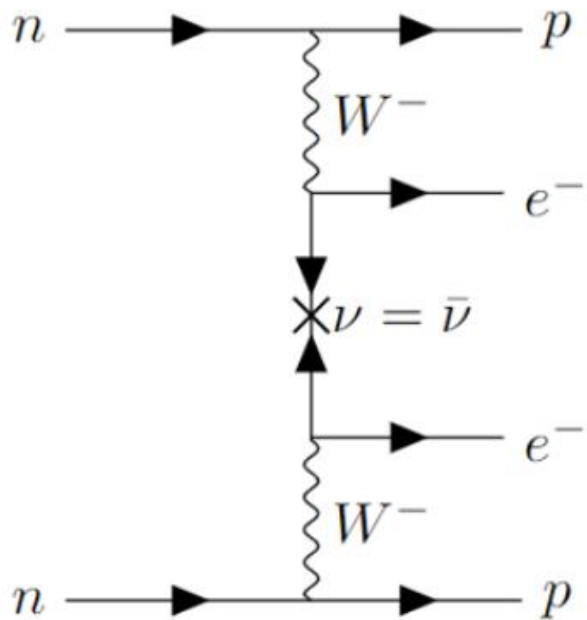
$$\begin{pmatrix} \nu_l \\ \nu_h \end{pmatrix} = \begin{pmatrix} \cos \theta_{LR} & \sin \theta_{LR} \\ -\sin \theta_{LR} & \cos \theta_{LR} \end{pmatrix} \begin{pmatrix} \nu_L^c \\ \nu_R \end{pmatrix} + h.c.$$

$$\text{where } \sin^2 \theta_{LR} = \frac{1}{2} \left(1 - \frac{m_R}{\sqrt{m_R^2 + 4m_D^2}} \right)$$

Quasi-Dirac & Majorana limits

	Pure Dirac case ($m_R = 0$)	Quasi-Dirac limit ($m_D \gg m_R$)	Majorana limit ($m_D \ll m_R$)
Mass eigenvalues	$ m_{l,h} = m_D$	$ m_{l,h} \simeq m_D(1 \mp 2\delta)$ where $\delta = m_R/4m_D$	$m_l \simeq -\frac{m_D^2}{m_R}$, $m_h \simeq m_R$ Seesaw mechanism
Mixing angle	$\sin^2 \theta_{LR} = \frac{1}{2}$	$\sin^2 \theta_{LR} \simeq \left(\frac{1-\delta}{\sqrt{2}}\right)^2$	$\sin^2 \theta_{LR} \simeq \left(\frac{m_D}{m_R}\right)$
Mass eigenstates	$\nu_{l,h} = \frac{1}{\sqrt{2}} \left((\nu_L + \nu_L^c) \pm (\nu_R + \nu_R^c) \right)$	$\nu_{l,h} \simeq \pm \frac{1 \pm \delta}{\sqrt{2}} (\nu_L + \nu_L^c) + \frac{1 \mp \delta}{\sqrt{2}} (\nu_R + \nu_R^c)$	$\nu_l \simeq \nu_L + \nu_L^c$, $\nu_h \simeq \nu_R + \nu_R^c$
Mass term	$\frac{1}{2}(m_l \bar{\nu}_l \nu_l + m_h \bar{\nu}_h \nu_h) = m_D \bar{\nu}_D \nu_D$	$\frac{1}{2}(m_l \bar{\nu}_l \nu_l + m_h \bar{\nu}_h \nu_h) \simeq m_D \bar{\nu}_D \nu_D + \mathcal{O}(\delta)$	$\frac{1}{2}(m_l \bar{\nu}_l \nu_l + m_h \bar{\nu}_h \nu_h)$

Neutrinoless double beta decay ($0\nu\beta\beta$): only for massive Majorana neutrinos



$$\Gamma_{0\nu\beta\beta} \propto \mathcal{A}^2$$

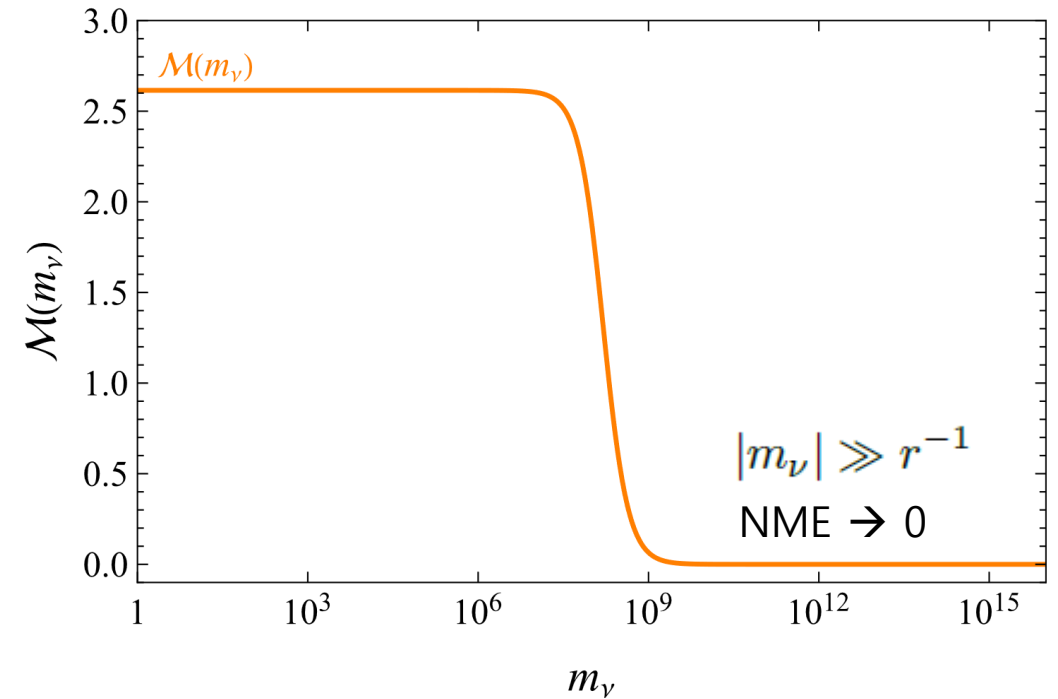
$$\mathcal{A} = \left| \sum_i U_{ei}^2 m_i \mathcal{M}(m_i) \right|$$

Extended PMNS matrix

Nuclear matrix element (NME)

$$\mathcal{M}(m_\nu) = m_p m_e \frac{|\mathcal{M}_N^{0\nu}|}{\langle p \rangle^2 + m_\nu^2}$$

$$\text{where } \langle p \rangle^2 = m_p m_e \left| \frac{\mathcal{M}_N^{0\nu}}{\mathcal{M}_\nu^{0\nu}} \right|$$



r : inter-nucleon distance (fm)

Single-flavor case:

$$\mathcal{A} = |m_l \cos^2 \theta_{LR} \mathcal{M}(m_l) + m_h \sin^2 \theta_{LR} \mathcal{M}(m_h)|$$

Majorana limit in the most of the time:

$$\sin^2 \theta_{LR} \simeq 0, \mathcal{M}(m_h) \simeq 0 \rightarrow 0\nu\beta\beta \text{ bound is reduced into } \langle |m_l| \rangle \text{ bound}$$

Three-flavors case: $\mathcal{A} = \left| \sum_i U_{ei}^2 m_i \mathcal{M}(m_i) \right|$

Mixing matrix element

$$U_{e1} = \cos \theta_{13} \cos \theta_{12} \cos \theta_{LR}^{11}$$

$$U_{e2} = \cos \theta_{13} \sin \theta_{12} \cos \theta_{LR}^{22}$$

$$U_{e3} = \sin \theta_{13} \cos \theta_{LR}^{33}$$

$$U_{e4} = \cos \theta_{13} \cos \theta_{12} \sin \theta_{LR}^{11}$$

$$U_{e5} = \cos \theta_{13} \sin \theta_{12} \sin \theta_{LR}^{22}$$

$$U_{e6} = \sin \theta_{13} \sin \theta_{LR}^{33}$$

Three pairs of
light/heavy mass term



\mathcal{A} is suppressed
in quasi-Dirac limit

Observed BAU

$$\eta = \frac{n_B - n_{\bar{B}}}{n_\gamma} = 6.21 \cdot 10^{-10}$$

Sphaleron process (above EWPT):

$B-L$ is conserved, but $B+L$ is violated.

Leptogenesis: Heavy Majorana neutrino decay \rightarrow lepton asymmetry \rightarrow B-L asymmetry

CP asymmetry

$$\epsilon_i = \frac{\Gamma(N_i \rightarrow \ell\Phi) - \Gamma(N_i \rightarrow \bar{\ell}\Phi^\dagger)}{\Gamma(N_i \rightarrow \ell\Phi) + \Gamma(N_i \rightarrow \bar{\ell}\Phi^\dagger)} \approx 10^{-6} \quad \text{for observed BAU}$$

notation

$$N \equiv \nu_h, \quad M \equiv m_h$$

$$\epsilon_i \propto \sum_{k \neq i} \frac{\text{Im}(y^\dagger y)_{ik}^2}{(y^\dagger y)_{ii}(y^\dagger y)_{kk}} \frac{(M_i^2 - M_k^2)M_i\Gamma_k}{(M_i^2 - M_k^2)^2 + M_i^2\Gamma_k^2}$$

1) Minimal leptogenesis 2) Resonant leptogenesis

$$M_1 \ll M_{2,3} \rightarrow \epsilon_1 \propto M_1$$

Nearly degenerate M_i 's

Lower bound: $M_1 > 10^9$ GeV

Mass bound can be lowered down to TeV