Probing Parity-Violation in the Stochastic Gravitational Wave Background in Astrometry

—— 3rd International Joint Workshop & 11th KIAS workshop on BSM and cosmology

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Content

- Brief review of nano Hertz Stochastic gravitational wave background
    - Pulsar Timing Array system
    - Astrometry
- Parity-Violation signal in astrometry system

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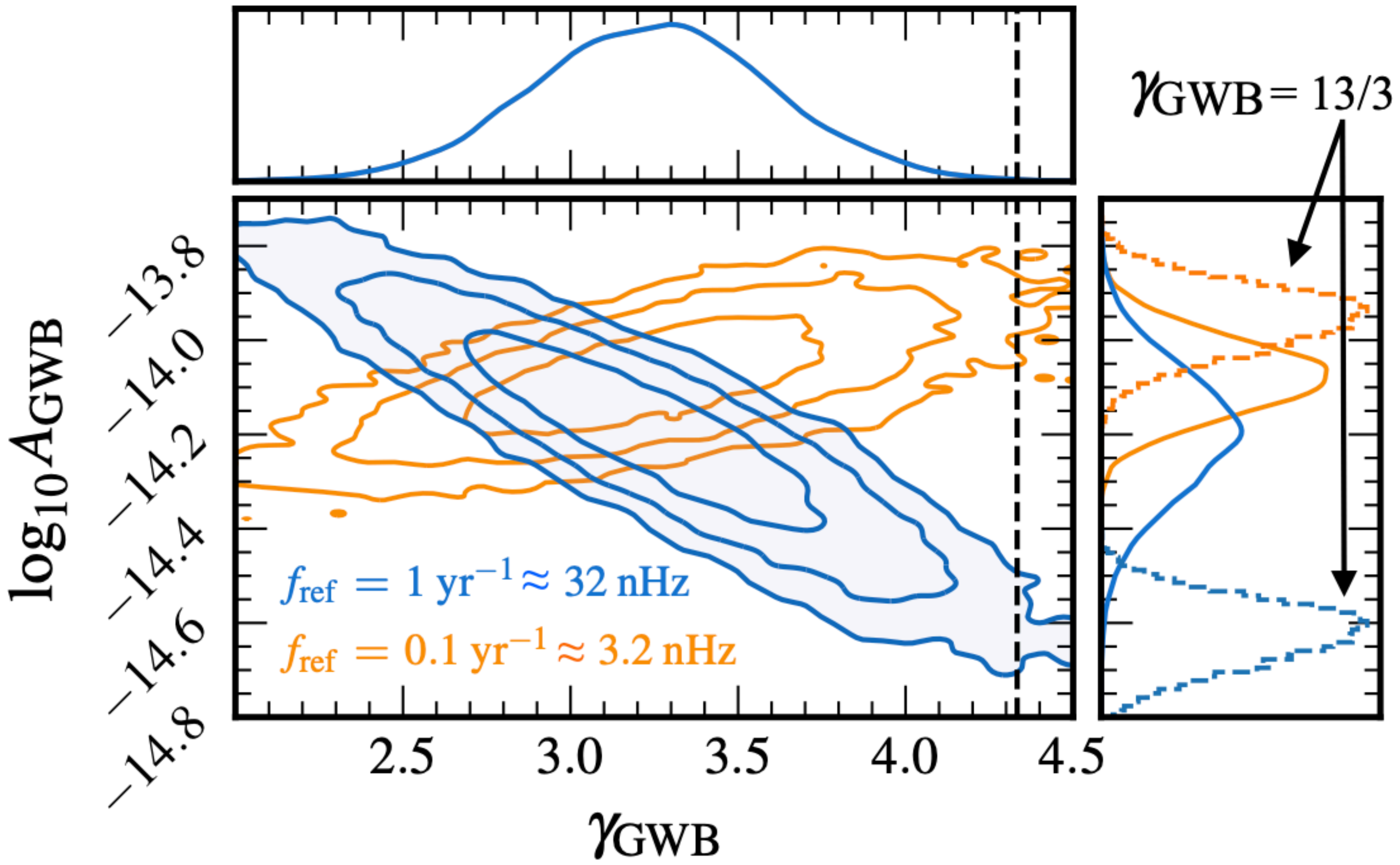
- Discussion

Nano Hertz SGWB

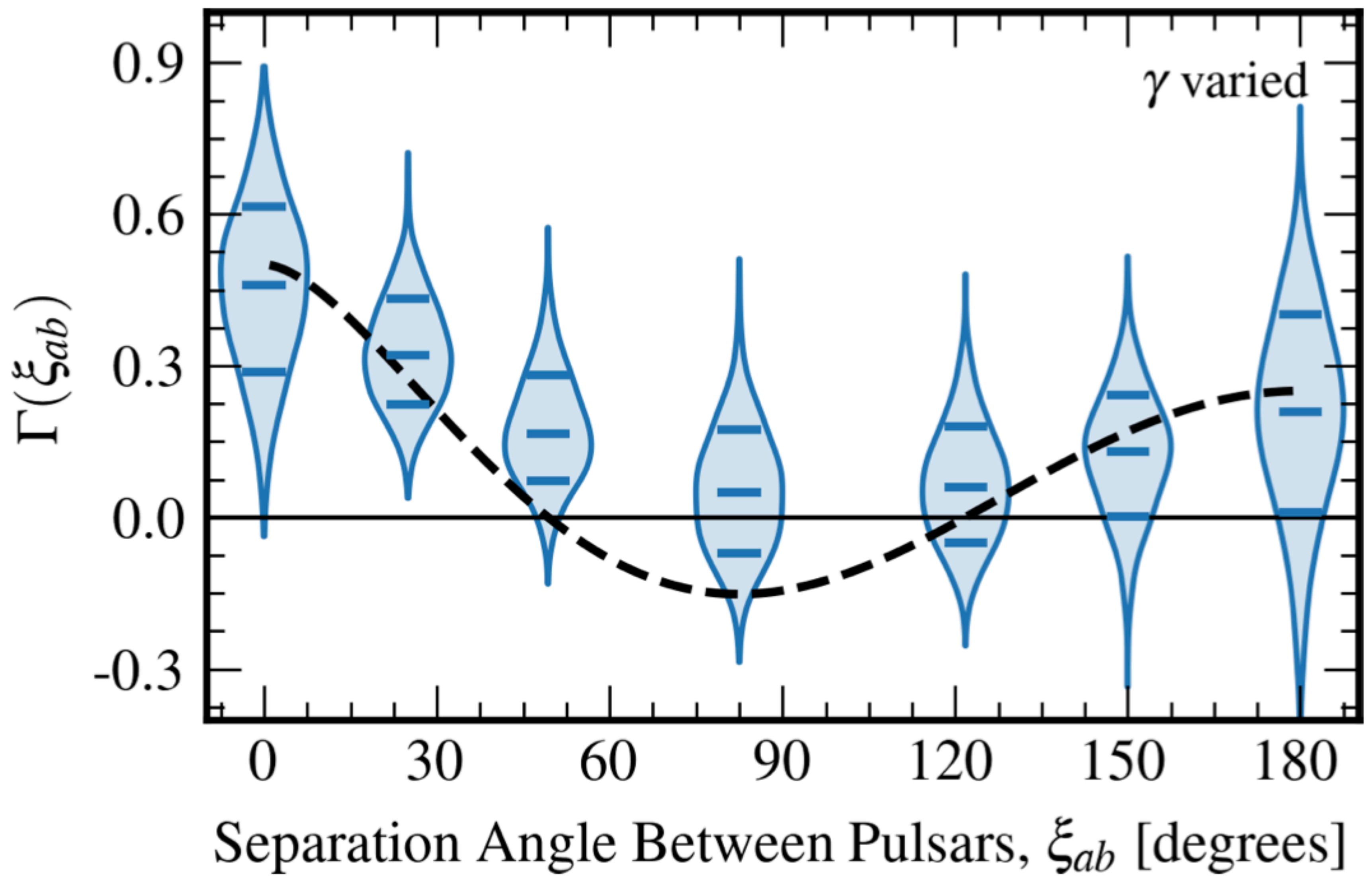
- Astrophysical Source: Supermassive Black Hole Binary;
- Cosmological Source: Primordial Gravitational Wave; Phase transition;

Nano Hertz SGWB

- Astrophysical Source: Supermassive Black Hole Binary;
- Cosmological Source: Primordial Gravitational Wave; Phase transition;
- PTA collaboration just claimed a detection in July!



Power spectrum



Angular correlation

Credit: NANOGrav 15 yr result

What can we learn from this SGWB?

- Astro: origin of supermassive black hole formation & population rate… 2306.17021,2306.16222 2305.05955
- Early universe: different inflation scenarios, primordial gravitational waves… 2212.05594 2311.03391 2311.02065 2311.00741
- Defect: cosmic string, phase transition,…2306.17205 2304.04793 2304.02636
- Beyond Standard Model physics: dark matter, baryon number violation, string compactification … 2306.05389 2305.11775 2304.10084 …
- Modified Gravity: 2304.02640 Q.Liang, M-X, Lin, M. Trodden 2108.05344 Q.Liang, M. Trodden

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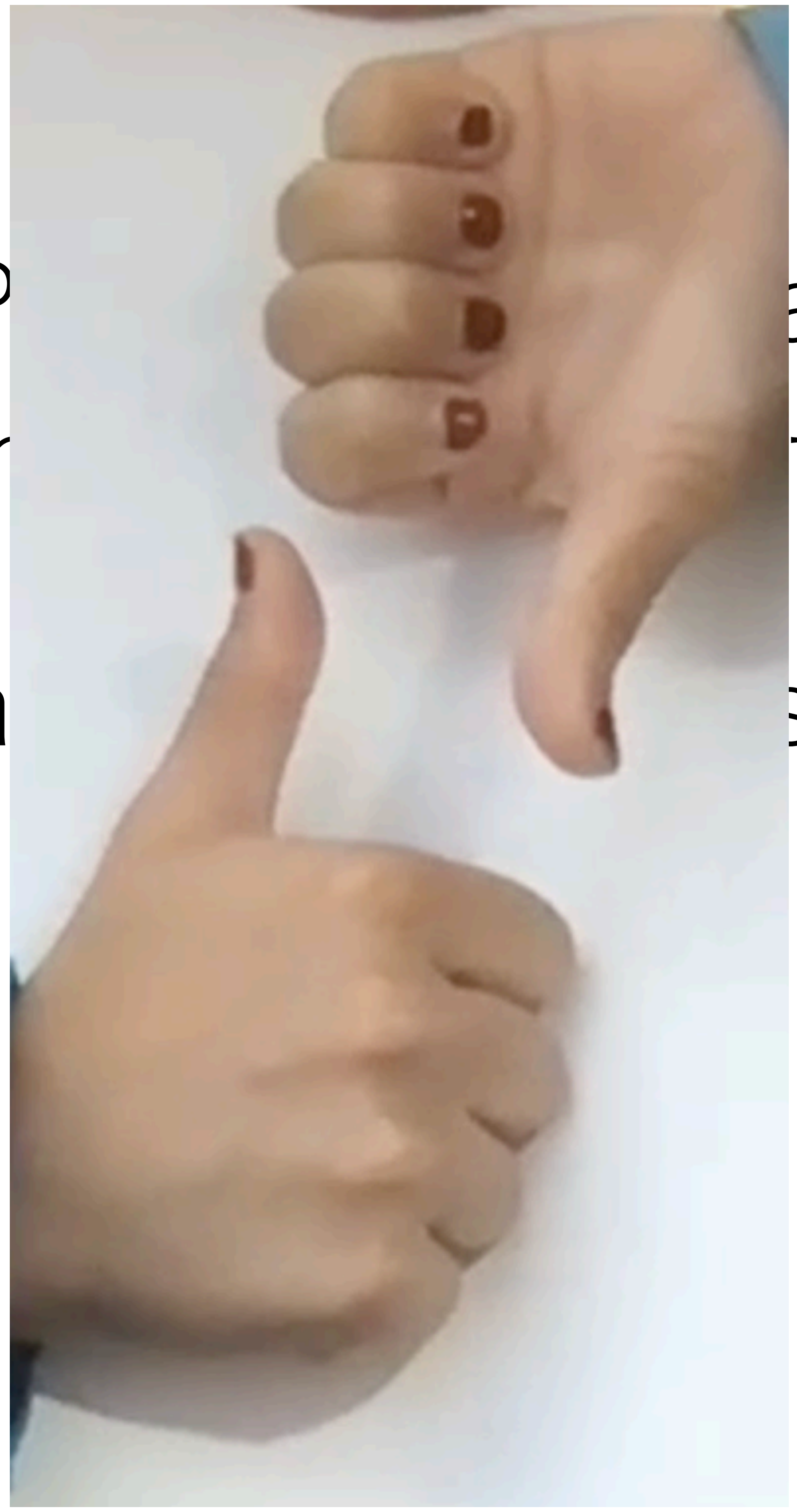

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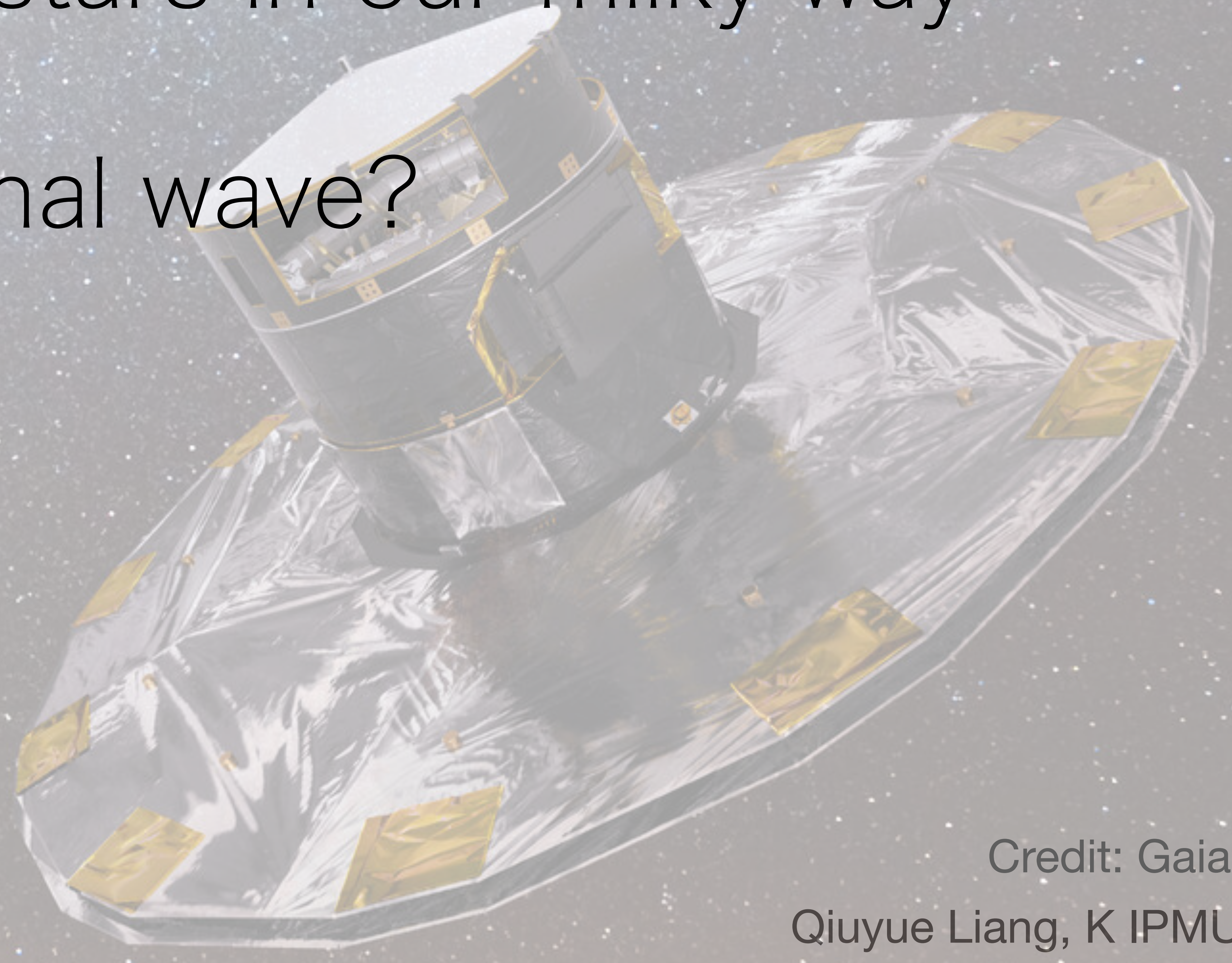
Astrometry!

What is astrometry?

- Astrometry is a precise measurement of the positions and movements of stars and other celestial bodies
- Gaia is currently measuring 1800 million stars in our milky way
- Exoplanet; Dark Matter profile; Gravitational wave?



Credit: Gaia

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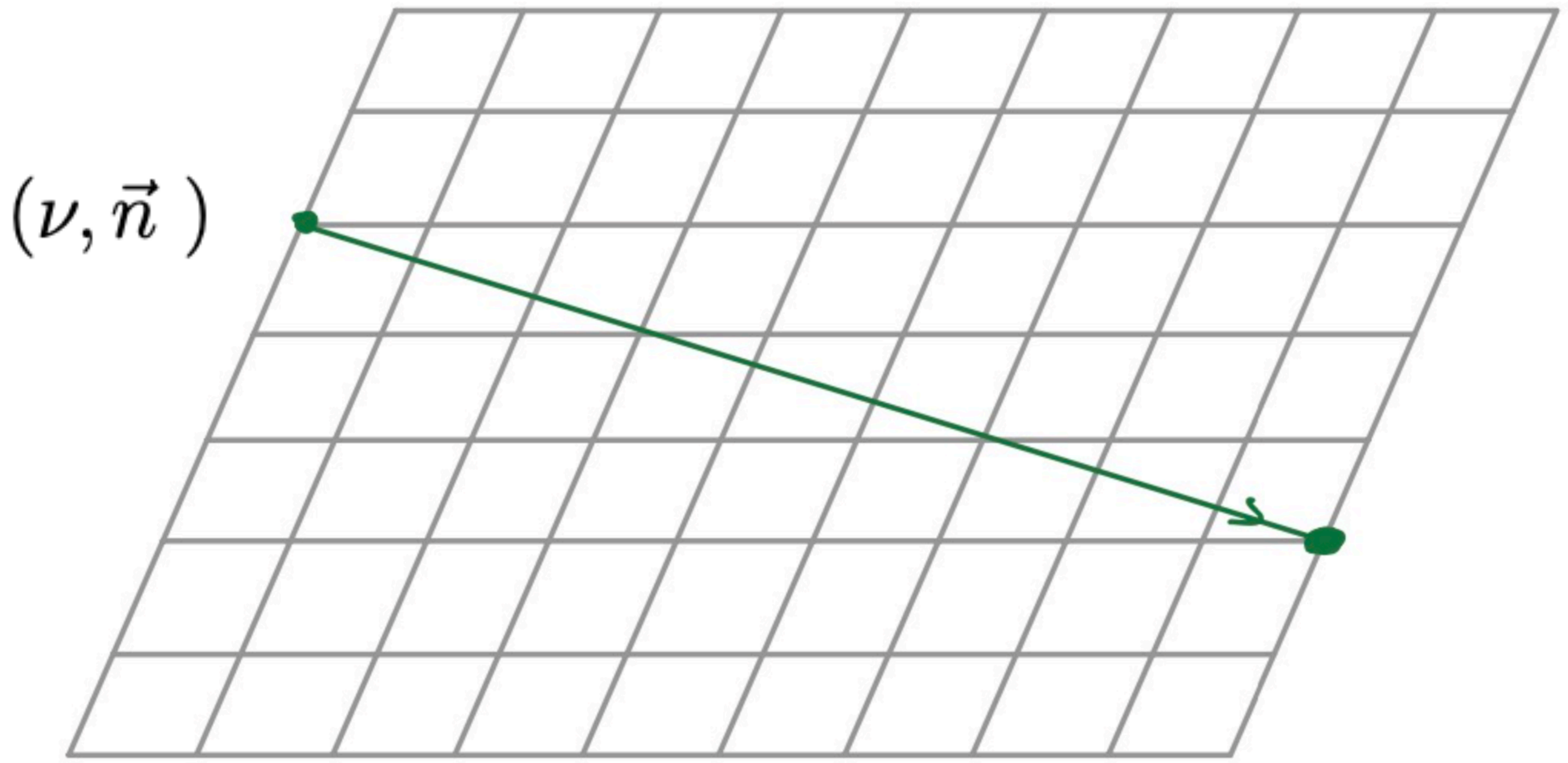
Using Astrometry to detect SGWB

$$\delta\nu(t,\hat{n}) = h_{IJ}(\vec{k})F^{IJ}(\hat{n},\hat{k})e^{-ikt}$$

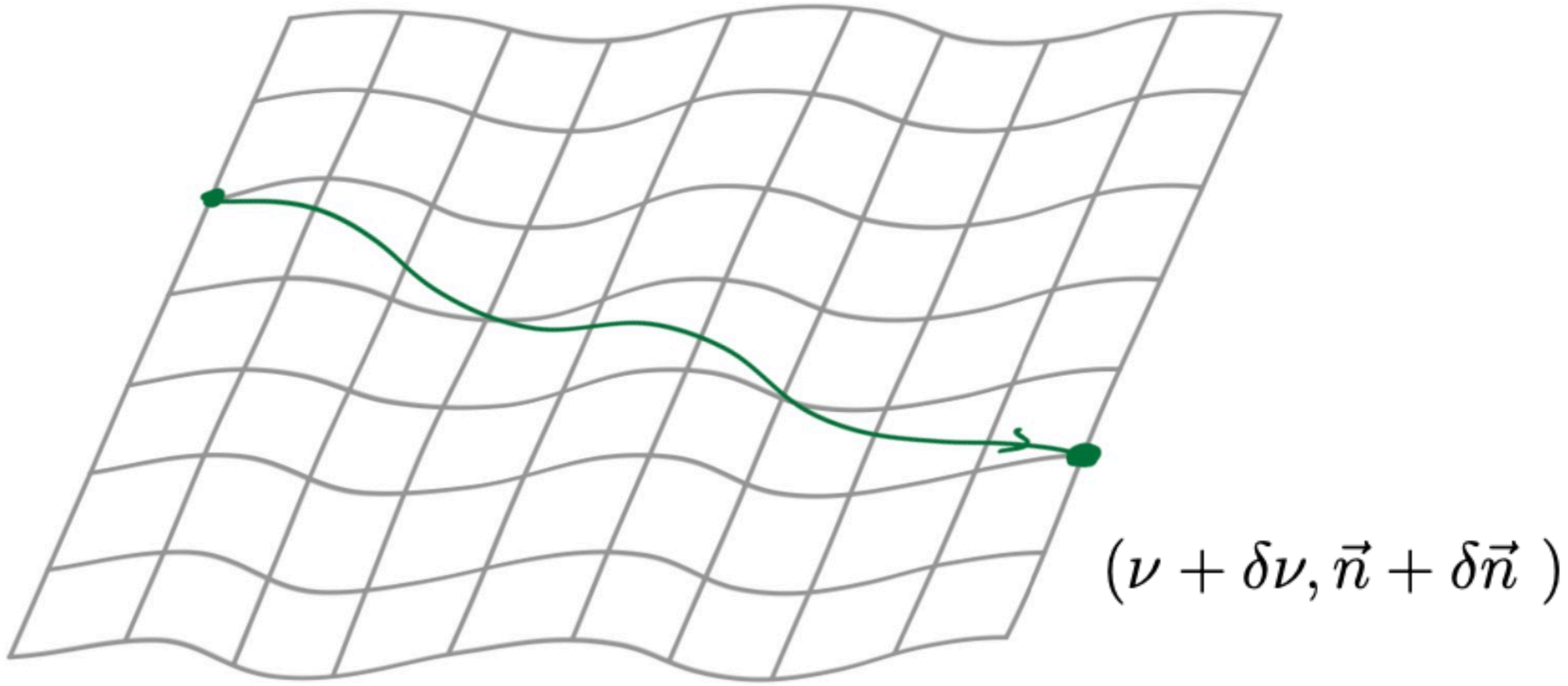
$$F^{IJ}(\hat{n},\hat{k}) = \frac{\hat{n}^I\hat{n}^J}{2(1+\hat{k}\cdot\hat{n})}$$

$$\delta n^I(t,\hat{n}) = e^I{}_\mu \delta n^\mu(t,\hat{n}) = \mathcal{R}^{IJK}(\hat{n},\hat{k})h_{JK}(\vec{k})e^{-ikt}$$

$$\mathcal{R}^{IJK}(\hat{n},\hat{k}) = \frac{\hat{n}^I+\hat{k}^I}{2(1+\hat{k}\cdot\hat{n})}\hat{n}^J\hat{n}^K - \frac{1}{2}\delta^{IJ}\hat{n}^K$$



Light path in flat space



Light path in perturbed space

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Correlation functions

- Assuming the isotropic background, one can separate the two-point correlation function in power spectrum and the overlap reduction function

$$\langle \delta\nu(t,\hat{n})\delta\nu\left(t,\hat{n}'\right)\rangle = \sum_{S,S'}\int \frac{\mathrm{d}^3\vec{k}}{(2\pi)^3} F^{KL}(\hat{n},\hat{k})F^{MN}\left(\hat{n}',\hat{k}\right) \times e^{S}_{KL}(\hat{k})e^{S'}_{MN}(-\hat{k})\left\langle h_S(t,\vec{k})h_{S'}\left(t,\vec{k}'\right)\right\rangle$$

PTA Overlap reduction function Power spectrum Pss’

$$\langle \delta n^I(t,\hat{n})\ \delta n^J\left(t,\hat{n}'\right)\rangle = \sum_{S,S'}\int \frac{\mathrm{d}^3\vec{k}}{(2\pi)^3} \mathcal{R}^{IKL}(\hat{n},\hat{k})\mathcal{R}^{JMN}\left(\hat{n}',\hat{k}\right) \times e^{S}_{KL}(\hat{k})e^{S'}_{MN}(-\hat{k})\left\langle h_S(t,\vec{k})h_{S'}\left(t,\vec{k}'\right)\right\rangle$$

Astrometry Overlap reduction tensor H^{IJ}_{SS’} Power spectrum Pss’

Correlation functions

- For parity-even power spectrum, $P_{++} = P_{\times\times}$, $P_{+\times} = P_{\times +} = 0$, we have
- PTA: Hellings-Downs curve $\Gamma_{++}(\xi) = \frac{1}{8}\left(3 + \cos\xi + 6(1-\cos\xi)\log\frac{1-\cos\xi}{2}\right)$
- Astrometry: $H^{IJ}_{++}(\vec{n},\vec{n}') = \alpha(\xi)\left(A_I A_J - B_I C_J\right)$ $\cos\xi = \hat{n}\cdot\hat{n}'$

$$\vec{A} = \vec{n}\times\vec{n}', \quad \vec{B} = \vec{n}\times\vec{A}, \quad \vec{C} = -\vec{n}'\times\vec{A}$$

suppose we choose $\vec{n} = \hat{z}$, then the non-vanishing component in the overlap reduction tensor is the 11 component, and 22 component.

Sensitive gravitational strain

- When a light passes through a SGWB, the deflection angle is proportional to the characteristic GW strain and therefore relates to the GW amplitude through $\delta_{\rm rms}(f) \sim h_{\rm rms}(f) \sim \frac{H_0}{f}\sqrt{\Omega_{\rm gw}(f)}$.

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- For N sources, the angular velocity has a correlated signal of order

$$f\delta_{\rm rms} \sim \omega_{\rm rms} \sim \frac{\Delta\theta}{T\sqrt{N}}$$

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Angular resolution 10 microarcssec

Frequency

$$\boxed{f}\delta_{\boxed{\rm rms}} \sim \boxed{\omega_{\rm rms}} \sim \frac{\boxed{\Delta\theta}}{\boxed{T}\sqrt{N}}$$

Root mean square

Number of sources ~ 10^6

angular velocity

Time interval ~ 3 yr

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- For N sources, the angular velocity has a correlated signal of order

$$f\delta_{\rm rms} \sim \omega_{\rm rms} \sim \frac{\Delta\theta}{T\sqrt{N}} \qquad \Omega_{\rm gw}(f) \lesssim \frac{\Delta\theta^2}{NT^2H_0^2} \qquad \Omega_{\rm gw} \sim 10^{-3} - 10^{-6}$$

- Future Theia telescope can give $h_{\rm rms} \sim 10^{-14}$ which is capable to reach the sensitivity of PTA detection!

Correlation functions

- Assuming the isotropic background, one can separate the two-point correlation function in power spectrum and the overlap reduction function

$$\langle \delta\nu(t,\hat{n})\delta\nu\,(t,\hat{n}')\rangle = \sum_{S,S'}\int \frac{\mathrm{d}^3\vec{k}}{(2\pi)^3} F^{KL}(\hat{n},\hat{k})F^{MN}\left(\hat{n}',\hat{k}\right) \times e^S_{KL}(\hat{k})e^{S'}_{MN}(-\hat{k}) \left\langle h_S(t,\vec{k})h_{S'}\left(t,\vec{k}'\right)\right\rangle$$

PTA　　Overlap reduction function　　Power spectrum Pss'

$$\langle \delta n^I(t,\hat{n})\ \delta n^J\,(t,\hat{n}')\rangle = \sum_{S,S'}\int \frac{\mathrm{d}^3\vec{k}}{(2\pi)^3} \mathcal{R}^{IKL}(\hat{n},\hat{k})\mathcal{R}^{JMN}\left(\hat{n}',\hat{k}\right) \times e^S_{KL}(\hat{k})e^{S'}_{MN}(-\hat{k}) \left\langle h_S(t,\vec{k})h_{S'}\left(t,\vec{k}'\right)\right\rangle$$

Astrometry　　Overlap reduction tensor H^{IJ}_{SS'}　　Power spectrum Pss'

Parity violation

- For non-vanishing $P_{+\times}, P_{\times+}$, PTA won't response! $\Gamma_{+\times}(\xi) = \Gamma_{\times+}(\xi) = 0$
- Astrometry, on the other hand, has non-vanishing overlap reduction tensor for this signal

$$H^{IJ}_{+\times}(\hat{n}, \hat{n}') = \alpha(\Theta) A^I B_2^J + \beta(\Theta) B_1^I A^J$$
$$H^{IJ}_{\times+}(\hat{n}, \hat{n}') = \alpha_2(\Theta) A^I B_2^J + \beta_2(\Theta) B_1^I A^J$$

$$\vec{A} = \hat{n} \times \hat{n}', \quad \vec{B}_1 = \hat{n} \times \vec{A}, \quad \vec{B}_2 = \hat{n}' \times \vec{A}.$$

- For $\vec{n} = \hat{z}$, the non vanishing components would be $H^{12}_{+\times}$, $H^{21}_{+\times}$, $H^{23}_{+\times}$

Non-vanishing EB correlation

- Like CMB analysis, one can decompose the deflection vector to a spherical harmonic and work on the angular power spectrum

$$\delta n(t,\hat{n}) = \sum_{\ell m}\left[\delta n_{E\ell m}(t)\vec{Y}^{E}_{\ell m}(\hat{n}) + \delta n_{B\ell m}(t)\vec{Y}^{B}_{\ell m}(\hat{n})\right]$$

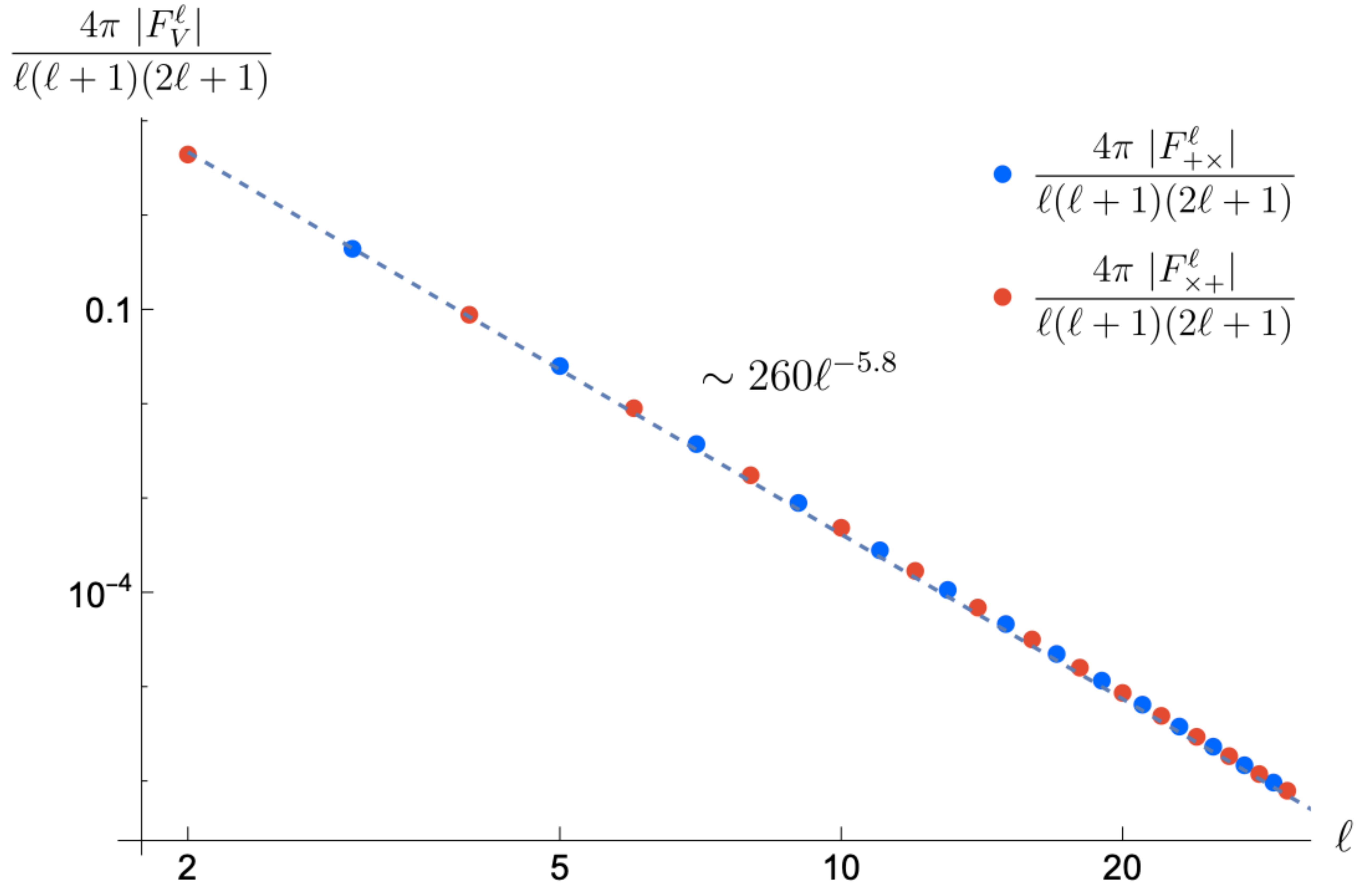
$$\langle \delta n_{E\ell m}(t)\delta n_{B\ell' m'}(t)^{*}\rangle = \int \mathrm{d}^2\Omega_{\hat{n}}\ \mathrm{d}^2\Omega_{\hat{n}'} Y^{E*}_{\ell m I}(\hat{n}) Y^{B}_{\ell' m' J}(\hat{n}') \left\langle \delta n^{I}(t,\hat{n})\delta n^{J}(t,\hat{n}')\right\rangle$$

$$= \frac{\delta_{\ell\ell'}\delta_{mm'}}{\ell(\ell+1)}\frac{4\pi}{2\ell+1}\left(\mathcal{A}_{+\times}F^{\ell}_{+\times} + \mathcal{A}_{\times +}F^{\ell}_{\times +}\right)$$

Non-vanishing EB correlation

$$\langle \delta n_{E\ell m}(t)\delta n_{B\ell' m'}(t)^* \rangle$$

$$= \frac{\delta_{\ell\ell'}\delta_{mm'}}{\ell(\ell+1)}\frac{4\pi}{2\ell+1}\left(\mathcal{A}_{+\times}F^{\ell}_{+\times}+\mathcal{A}_{\times+}F^{\ell}_{\times+}\right)$$



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Sensitivity for EB correlation

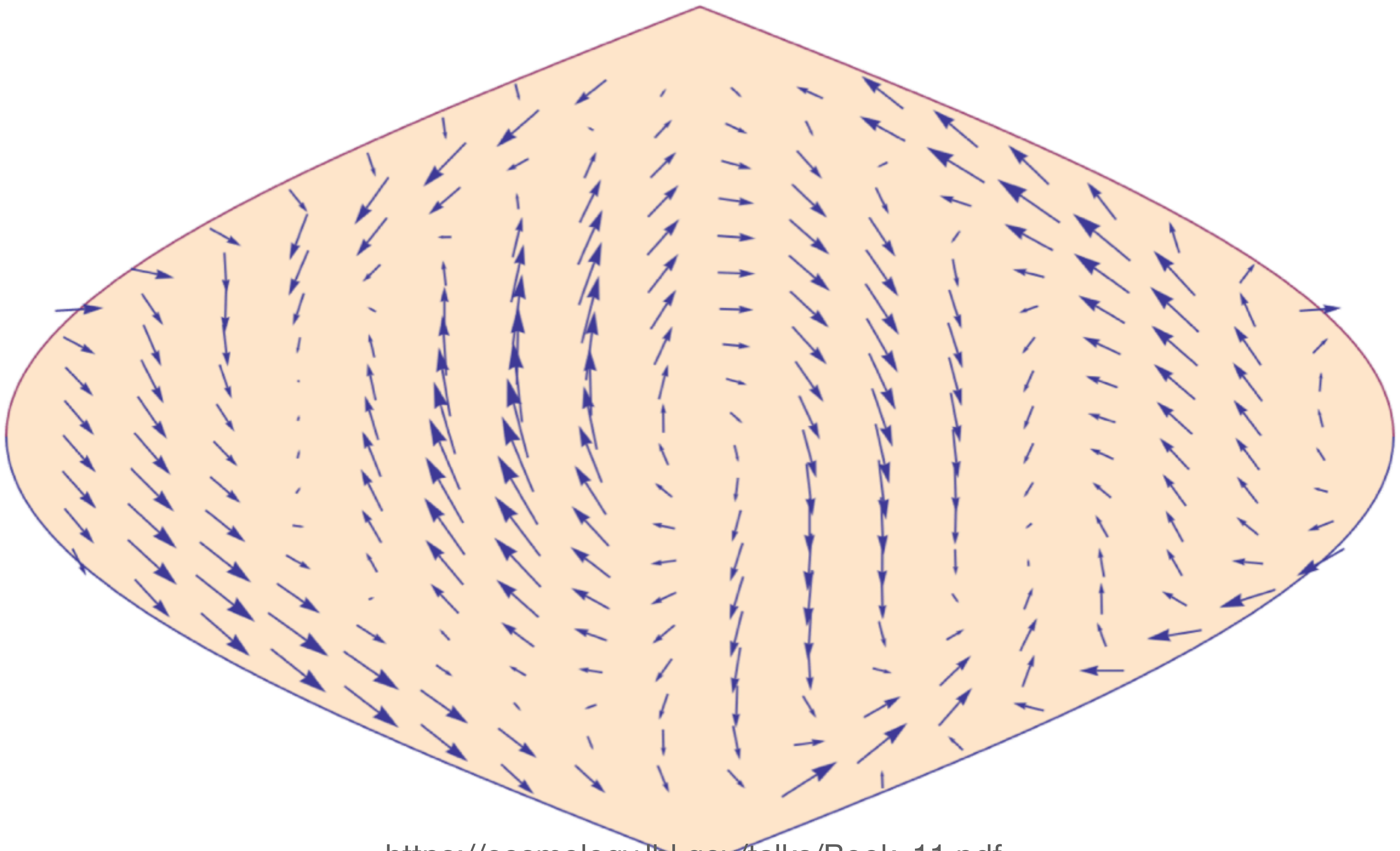
- Since in parity even theory, there is a vanishing EB correlation, the non-vanishing EB correlation is a null test.
- For stars in our galaxy, the sensitivity of EB correlation is again set by the angular resolution, the observation time, and the number of sources, since we have Milky way streams that breaks the parity symmetry.
- For quasars that are far away in other galaxies, we might assume a significantly smaller noise for EB correlation.

Conclusion and Discussion

- We compute the overlap reduction tensor for astrometry system, focusing on the parity-violation signal
- We further decompose it to spherical harmonics, and obtain the non-vanishing EB correlation
- Future work involving detectability of specific models need to be done!
- The current telescope might not be able to detect SGWB yet, but future galaxy survey should provide a parallel probe to PTA

Thanks for your attention!

Typical deflection pattern (would be cool if I have a deflection pattern with the parity violation signals …)



https://cosmology.lbl.gov/talks/Book_11.pdf