

The 3rd International Joint Workshop on the Standard Model and Beyond and  
the 11th KIAS Workshop on Particle Physics and Cosmology

# Complementarity of $\mu$ TRISTAN and Belle II in searches for CLFV.

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arXiv:2307.11369



November 12, 2023

# Overview

Flavour Triality

CLFV Tau decays at Belle II

Next-to-leading order Constraints

Phenomenology at  $\mu$ TRISTAN (arXiv:2307.11369)

## Flavour Triality

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# Flavour Physics

Why three families of quarks and leptons?

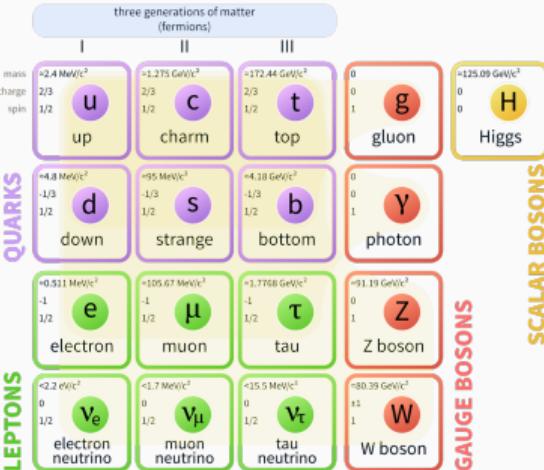
Explain the observed mixing patterns.

Fermion mass hierarchy.

Neutrino masses.

Charged Lepton Flavour Violation

**Standard Model of Elementary Particles**



## Charged Lepton Flavour Violation

Charged Lepton Flavour Violation (CLFV) has not yet been observed.

Neutrino oscillations show no individual lepton numbers  $L_e$ ,  $L_\mu$ , and  $L_\tau$  conservation.

Several BSM models predict CLFV.

Experimental bounds are stringent, especially on electron-muon CLFV.

- {  $\mu \rightarrow e\gamma$  at MEG,
- $\mu \rightarrow eee$  at Mu3e
- $\mu N \rightarrow \mu N$  at COMET, Mu2e and DeeMe

CLFV involving  $\tau$ :

- { Data are less constraining;
- Belle II future sensitivity will increase significantly.

Lepton Triality avoids  $\mu \leftrightarrow e$  conversions and allows CLFV tau decays.

# Lepton Triality

Motivated by flavour structure models

$A_4$  group can explain the quark and lepton mixing angles.

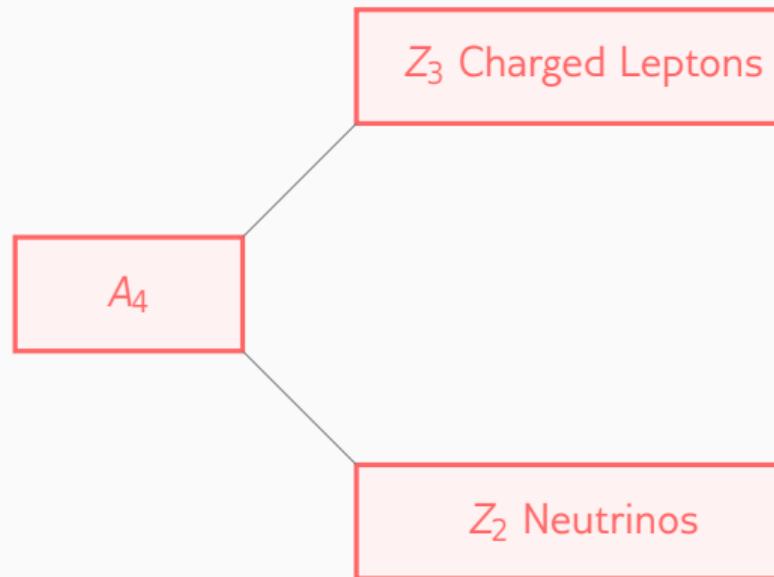
Tribimaximal mixing of neutrino flavours

Altarelli, Feruglio (2006)

He, Keum, Volkas (2006)

hep-ph/0601001

Ma,(2010) 1006.3524



# Lepton Triality

$A_4$  is generated by S and T Elements

$$T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}$$

$$S = \frac{1}{3} \begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}$$

$Z_3$  Lepton sector

$$L \rightarrow \omega^T L \text{ and } e_R \rightarrow \omega^T e_R,$$
$$\omega = e^{\frac{2\pi i}{3}}$$

H, quarks are singlets under triality

$$\mathcal{L}_Y = y_{e_i} \bar{L}_i e_{R_i} H + h.c.$$

$\mathcal{L}_Y$  is diagonal under  $Z_3$

# Lepton Triality

Triality sums modulo 3

$$\mu^- \rightarrow e^- \gamma \quad \Delta T \neq 0 \quad X$$

$T=2$        $T=1$

$$\tau^- \rightarrow \mu^+ e^- e^- \quad \Delta T = 0 \quad \checkmark$$

$T=3$        $T=-2$      $T=1$      $T=1$

$$e \rightarrow T = 1$$

$$\mu \rightarrow T = 2$$

$$\tau \rightarrow T = 3$$

Mediators of flavour triality  
transform under  $Z_3$  as:

$$k_1 \sim \omega,$$

$$k_2 \sim \omega^2,$$

$$k_3 \sim 1,$$

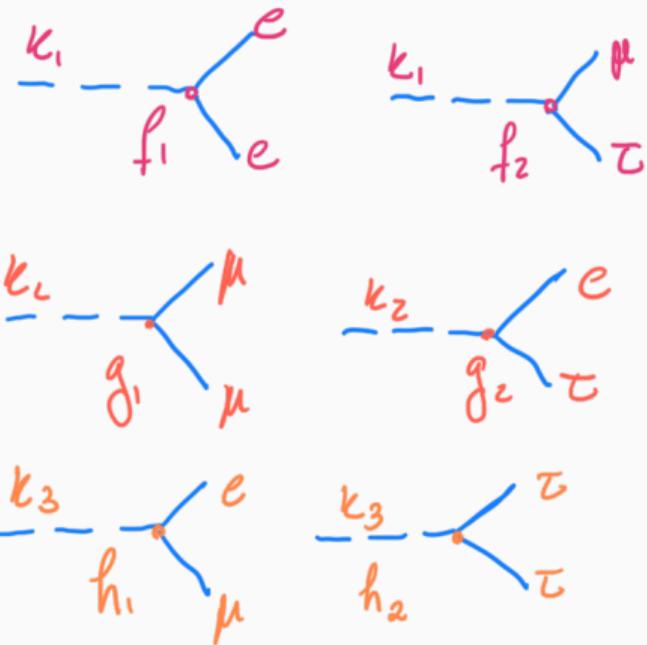
# Lepton Triality

Models T = 1, 2, 3 for the doubly charged singlet  $k_i$

$$\mathcal{L}_{k_1} = \frac{1}{2} \left( 2f_1 \overline{(\tau_R)^c} \mu_R + f_2 \overline{(e_R)^c} e_R \right) k_1 + \text{h.c.}$$

$$\mathcal{L}_{k_2} = \frac{1}{2} \left( 2g_1 \overline{(\tau_R)^c} e_R + g_2 \overline{(\mu_R)^c} \mu_R \right) k_2 + \text{h.c.}$$

$$\mathcal{L}_{k_3} = \frac{1}{2} \left( 2h_1 \overline{(\mu_R)^c} e_R + h_2 \overline{(\tau_R)^c} \tau_R \right) k_3 + \text{h.c..}$$



## CLFV Tau decays at Belle II

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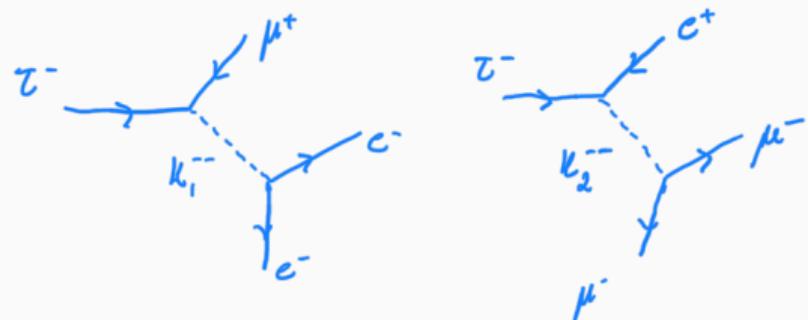
# Tau Decays

Bigaran, He, Schmidt, Valencia, Volkas, (2022) 2212.09760.

Tau CLFV decays:

$$\tau \rightarrow e/\mu + \gamma,$$

$$\tau \rightarrow e/\mu + l^+l^- \text{ where } l = e/\mu.$$



Observable	Present constraint	Projected sensitivity
$\text{BR}(\tau^- \rightarrow \mu^- \mu^- e^+)$	$1.7 \times 10^{-8} *$	$2.6 \times 10^{-10} **$
$\text{BR}(\tau^- \rightarrow \mu^+ e^- e^-)$	$1.5 \times 10^{-8} *$	$2.3 \times 10^{-10} **$

\* Belle Collaboration (2010) 1001.3221

\*\* Belle II (2022) 2203.14919

$$\mathcal{L}_{6,LFV} = C^{\prime\prime}(\bar{L}\gamma_\mu L)(\bar{L}\gamma^\mu L) + C^{ee}(\bar{e}_R\gamma_\mu e_R)(\bar{e}_R\gamma^\mu e_R) + C^{le}(\bar{L}\gamma_\mu L)(\bar{e}_R\gamma^\mu e_R)$$

$$C_{ee,1312}^{VRR} = \frac{f_1 f_2}{4m_{k_1^2}}$$

$$BR(\tau^\pm \rightarrow \mu^\mp e^\pm e^\pm) = \frac{f_1^2 f_2^2}{64 G_F^2 m_{k_1^2}^4} BR(\tau^- \rightarrow \mu^- \bar{\nu}_\mu \nu_\tau)$$

$$C_{ee,2321}^{VRR} = \frac{g_1 g_2}{4m_{k_2^2}}$$

$$BR(\tau^\pm \rightarrow \mu^\pm \mu^\pm e^\mp) = \frac{g_1^2 g_2^2}{64 G_F^2 m_{k_2^2}^4} \tilde{I}\left(\frac{m_\mu^2}{m_\tau^2}\right) BR(\tau^- \rightarrow \mu^- \bar{\nu}_\mu \nu_\tau)$$

Present bounds from Belle:

$$\sqrt{f_1 \times f_2} < 0.17 \frac{m_{k1}}{\text{TeV}}$$

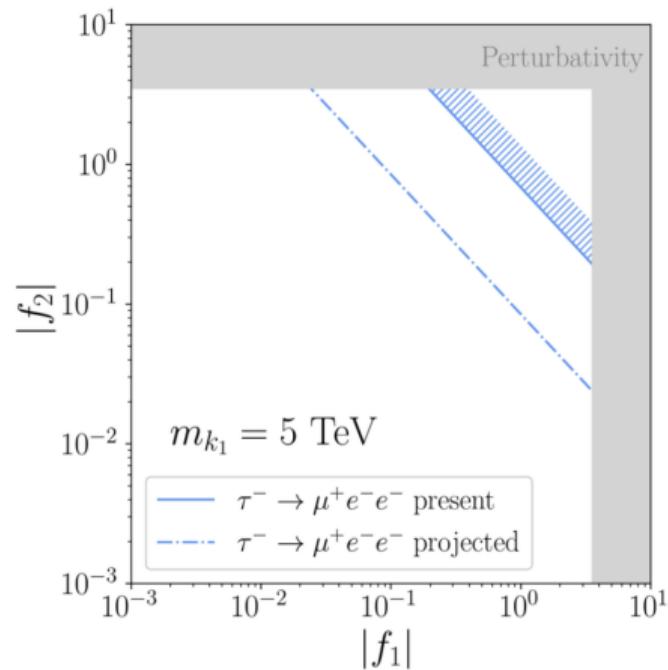
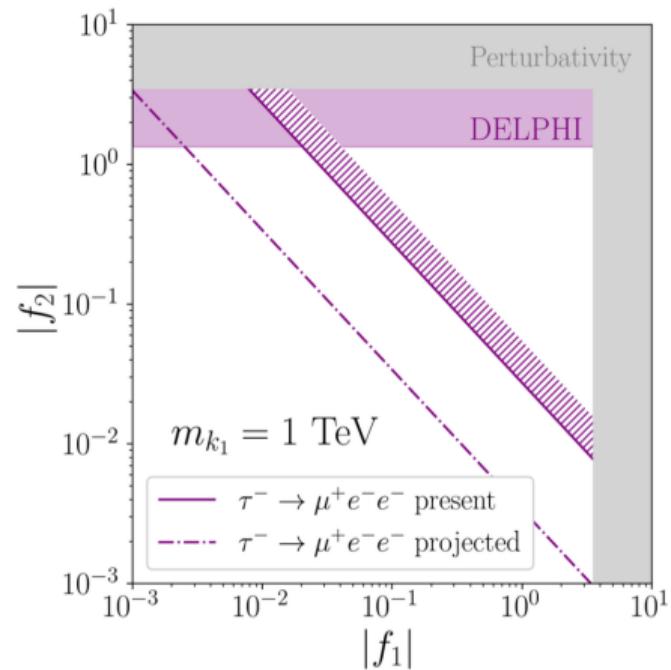
$$\sqrt{g_1 \times g_2} < 0.17 \frac{m_{k2}}{\text{TeV}}$$

Prediction for future sensitivity from Belle II:

$$\sqrt{f_1 \times f_2} < 0.06 \frac{m_{k1}}{\text{TeV}}$$

$$\sqrt{g_1 \times g_2} < 0.06 \frac{m_{k2}}{\text{TeV}}$$

# Belle II sensitivity on CLFV tau decays from Triality T=1



Bigaran, He, Schmidt, Valencia, Volkas, (2022) 2212.09760.

## Next-to-leading order Constraints

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# Next-to-leading order Constraints

$$\mathcal{L}_{k_1} = \frac{1}{2} \left( 2f_1 \overline{(\tau_R)^c} \mu_R + f_2 \overline{(e_R)^c} e_R \right) k_1 + \text{h.c.}$$

$$\mathcal{L}_{k_2} = \frac{1}{2} \left( 2g_1 \overline{(\tau_R)^c} e_R + g_2 \overline{(\mu_R)^c} \mu_R \right) k_2 + \text{h.c.}$$

$$\mathcal{L}_{k_3} = \frac{1}{2} \left( 2h_1 \overline{(\mu_R)^c} e_R + h_2 \overline{(\tau_R)^c} \tau_R \right) k_3 + \text{h.c..}$$

Potential:  $\mathcal{L} \supset |D_\mu k_i|^2 + \kappa_i \phi^\dagger \phi k_i^\dagger k_i$  where

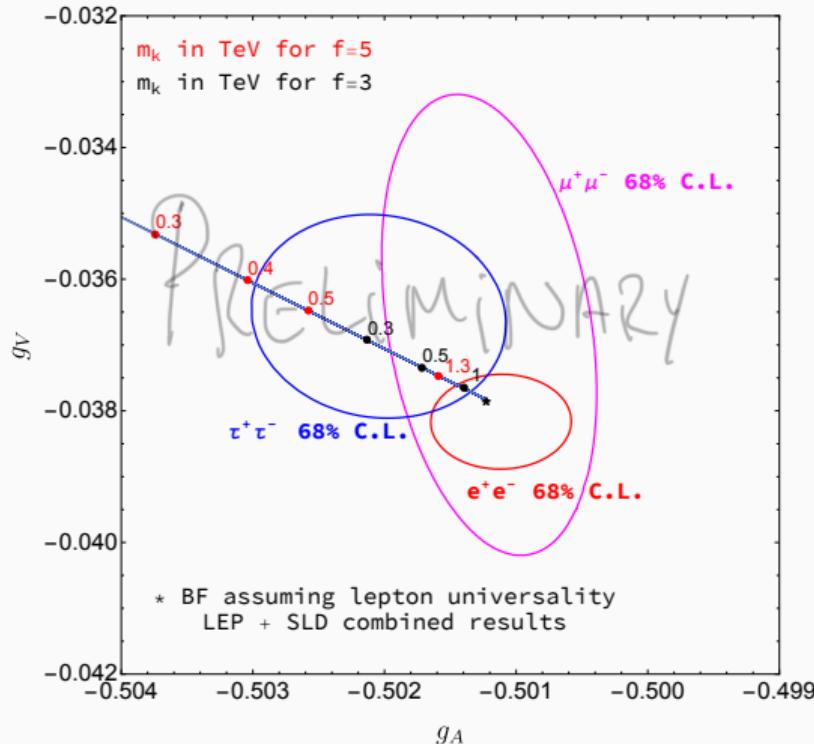
$$D_\mu = \partial_\mu + i2e(A_\mu - \tan\theta_W Z_\mu)$$

$Z \rightarrow l^+ l^-$ , with  $l = e, \mu, \tau$

$H \rightarrow \gamma\gamma$

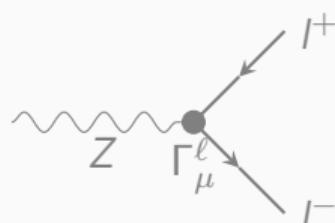
$H \rightarrow Z\gamma$

$Z \rightarrow l^+l^-$ , with  $l = e, \mu, \tau$



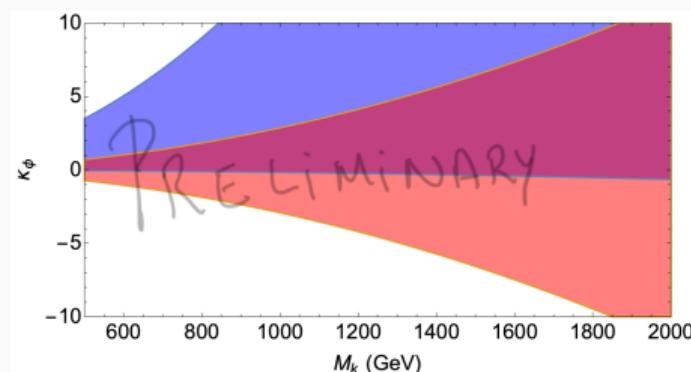
$$\Gamma_\mu^\ell = -2^{1/4} G_F^{1/2} M_Z \gamma_\mu [(r_\ell + \delta r_\ell) P_R + (I_\ell + \delta I_\ell) P_L],$$

$$g_A = (g_L - g_R)/2 \text{ and } g_V = (g_L + g_R)/2$$



PRELIMINARY

PRELIMINARY



$$R_{\gamma\gamma} = \frac{\Gamma(H \rightarrow \gamma\gamma)}{\Gamma(H \rightarrow \gamma\gamma)_{SM}}$$

Blue:  $R_{\gamma\gamma} = 1.088^{+0.095}_{-0.09}$

(ATLAS arXiv:2207.00092 [hep-ex])

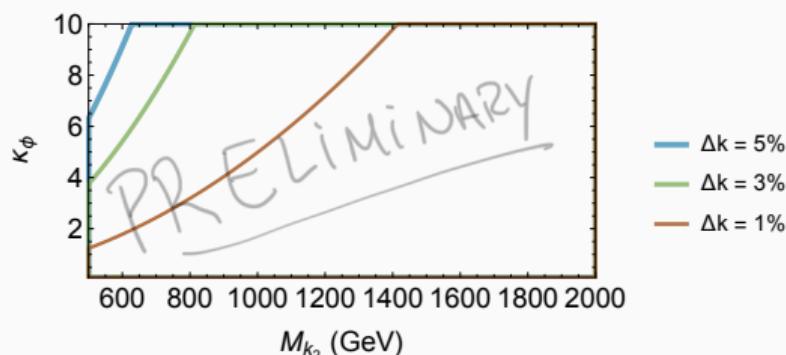
Red: Projected sensitivity with  
 $3000 \text{ fb}^{-1}$  at the HL-LHC,  
 $\Delta \kappa_\gamma = 1.8\%$

(Snowmass arXiv:1902.10229  
[hep-ex])

$$0.8 < R_{Z\gamma} < 1.15$$

$R_{Z\gamma} = 2.2 \pm 0.7$  (ATLAS and CMS arXiv:2309.03501 [hep-ex])

$\Delta\kappa_{Z\gamma} = 9.8\%$  (Snowmass ATL-PHYS-PUB-2022-018, 2022.)



# Phenomenology at $\mu$ TRISTAN (arXiv:2307.11369)

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# $\mu$ TRISTAN

Hamada, Kitano, Matsudo, Takaura and Yoshida, (2022) 2201.06664

Ultracold muon technology from g-2 at J-PARC

$\mu^+ \mu^+$  proposal  $\sqrt{s} = 2$  TeV;

1 TeV  $\mu^+$  beams;

expected luminosity of  $12 \text{ fb}^{-1}$  per year.

$\mu^+ e^-$  proposal with asymmetric beam energies;

$\mu^+$  beams up to 1 to 3 TeV;

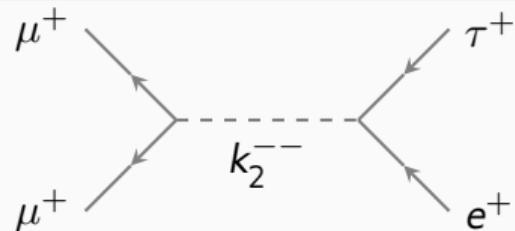
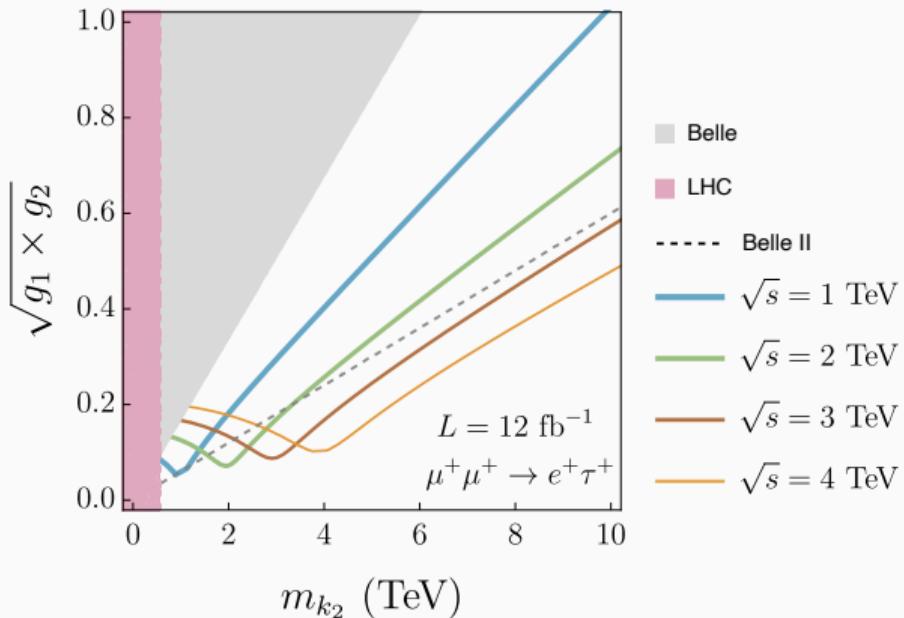
$e^-$  beams from Tristan at 30 to 50 GeV;

expected luminosity of  $100 \text{ fb}^{-1}$  per year.

# Future lepton Colliders

Model	Process	Lepton Collider
T=1	$\mu^+ e^- \rightarrow e^+ \tau^-$	$\mu$ TRISTAN
T=1	$e^+ e^- \rightarrow e^+ e^-$	$e^+ e^-$
T=1	$e^- e^- \rightarrow e^- e^-$	-
T=1	$e^- e^- \rightarrow \tau^- \mu^-$	-
T=2	$\mu^+ \mu^+ \rightarrow \tau^+ e^+$	$\mu$ TRISTAN
T=2	$\mu^+ \mu^+ \rightarrow \mu^+ \mu^+$	$\mu$ TRISTAN
T=2	$\mu^+ e^- \rightarrow \tau^+ \mu^-$	$\mu$ TRISTAN
T=2	$\mu^+ \mu^- \rightarrow \mu^+ \mu^-$	$\mu^+ \mu^-$
T=3	$\mu^+ e^- \rightarrow \mu^+ e^-$	$\mu$ TRISTAN
T=3	$\mu^+ e^+ \rightarrow \tau^+ \tau^+$	-

# CLFV s-channel at $\mu^+\mu^+$



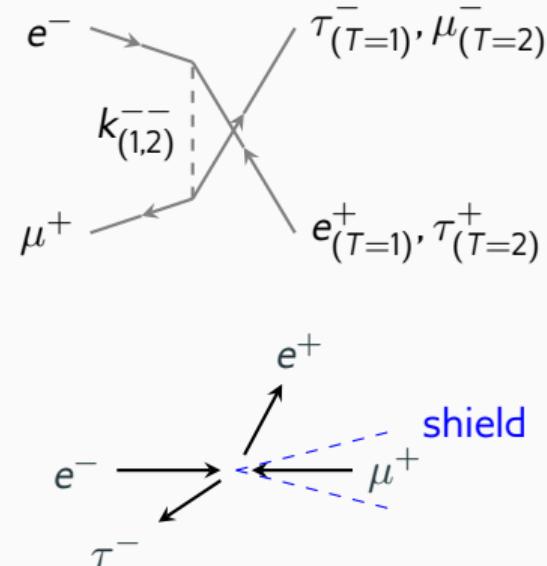
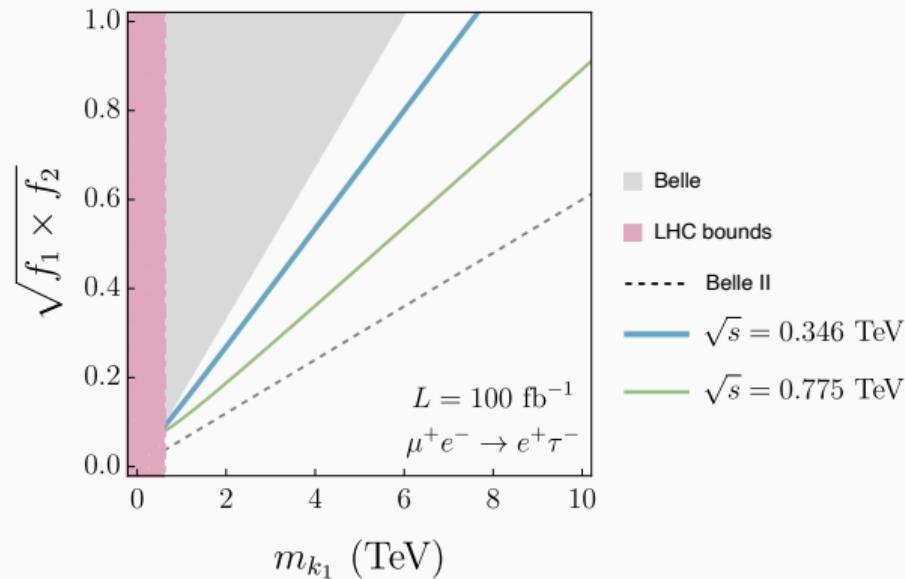
90% C.L. contour  
assuming no background  
 $N = 2.44;$

$$\sqrt{g_1 g_2} \lesssim 0.15 \left( \frac{N}{L_s} \right)^{\frac{1}{4}} \frac{m_{k_2}}{\text{TeV}}$$

For  $\sqrt{s} = 2$  TeV:

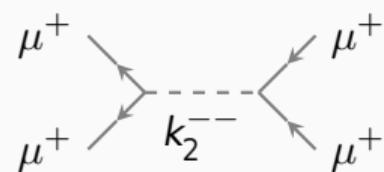
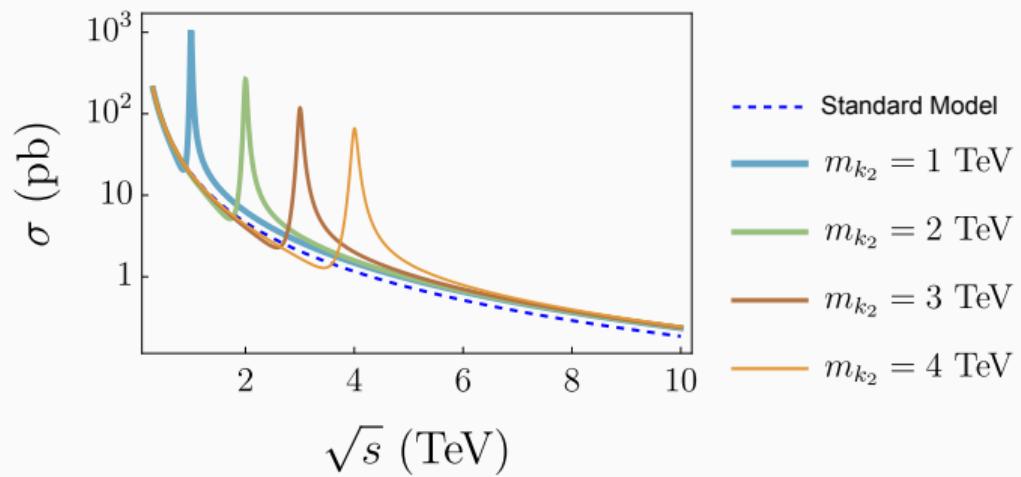
$$\sqrt{g_1 g_2} \lesssim 0.17 \frac{m_{k_2}}{\text{TeV}} .$$

# CLFV u-channel at $\mu^+ e^-$

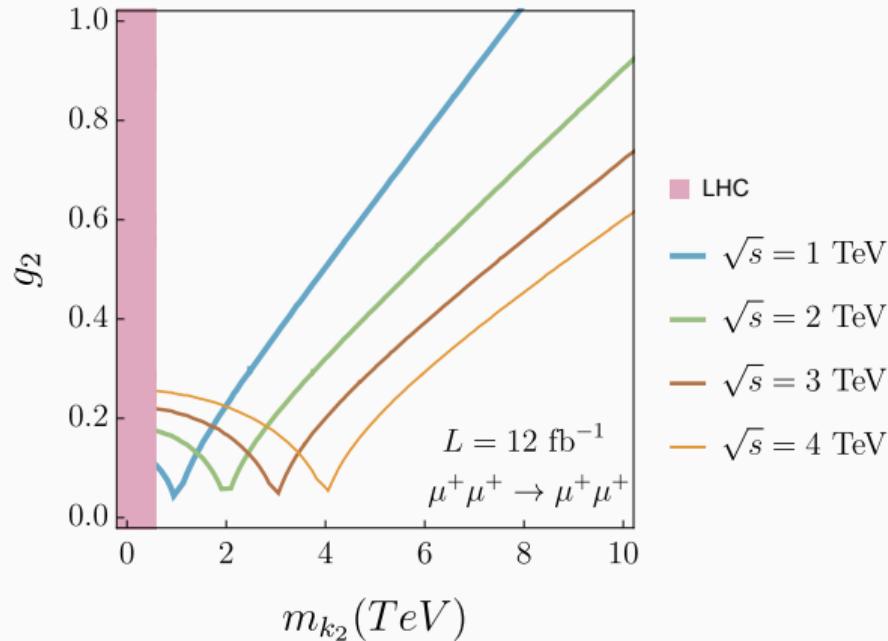


$$\sqrt{f_1 f_2} \lesssim 0.13 \frac{m_{k_1}}{\text{TeV}}$$

# Resonances in elastic scattering $\mu^+\mu^+ \rightarrow \mu^+\mu^+$



# Resonances in elastic scattering $\mu^+\mu^+ \rightarrow \mu^+\mu^+$

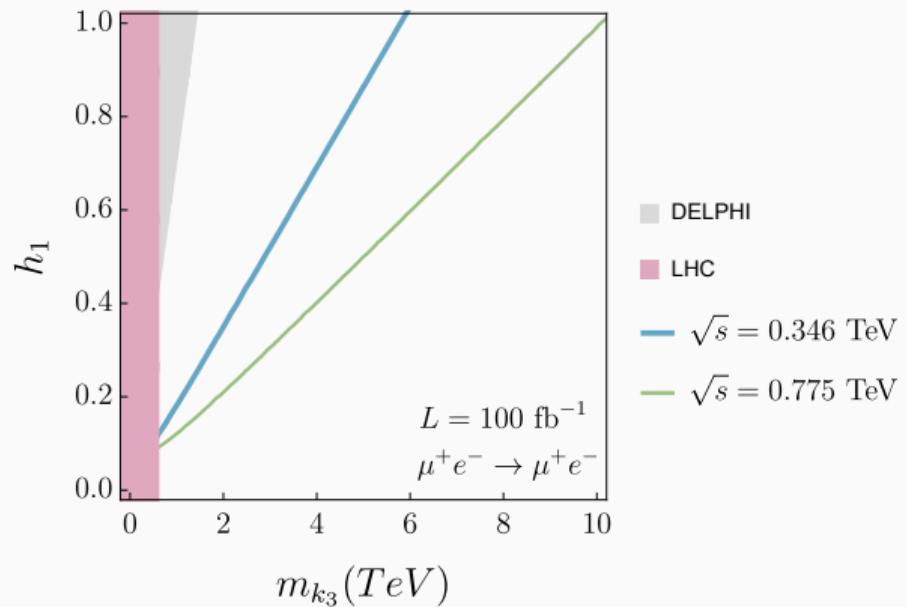


90% C.L. contour;  
SM contributions as  
background;  
 $S = 1.64$

$$S = \frac{|\sigma - \sigma_{SM}|}{\sqrt{\sigma_{SM}}} \sqrt{L};$$

$$g_2 \lesssim 0.18 \left( \frac{S^2}{Ls} \right)^{1/4} m_k;$$
$$g_2 \lesssim 0.09 \frac{m_{k_2}}{\text{TeV}}.$$

# Elastic scattering $\mu^+ e^- \rightarrow \mu^+ e^-$



$$S = \frac{|\sigma - \sigma_{SM}|}{\sqrt{\sigma_{SM}}} \sqrt{L} ;$$

$$h_1 \lesssim 0.17 \frac{m_{k3}}{\text{TeV}} .$$

# Summary Table

Experiment	Process	90% C.L. limit	Assumptions
Belle	$\tau^- \rightarrow \mu^+ e^- e^-$	$\sqrt{f_1 f_2} \lesssim 0.17 \frac{m_{k_1}}{\text{TeV}}$	$782 \text{ fb}^{-1}$
Belle	$\tau^- \rightarrow e^+ \mu^- \mu^-$	$\sqrt{g_1 g_2} \lesssim 0.17 \frac{m_{k_2}}{\text{TeV}}$	$782 \text{ fb}^{-1}$
Belle II	$\tau^- \rightarrow \mu^+ e^- e^-$	$\sqrt{f_1 f_2} \lesssim 0.06 \frac{m_{k_1}}{\text{TeV}}$	$50 \text{ ab}^{-1}$
Belle II	$\tau^- \rightarrow e^+ \mu^- \mu^-$	$\sqrt{g_1 g_2} \lesssim 0.06 \frac{m_{k_2}}{\text{TeV}}$	$50 \text{ ab}^{-1}$
DELPHI	$e^+ e^- \rightarrow e^+ e^-$	$f_2 \lesssim 1.4 \frac{m_{k_1}}{\text{TeV}}$	
DELPHI	$e^+ e^- \rightarrow \mu^+ \mu^-$	$h_1 \lesssim 0.72 \frac{m_{k_3}}{\text{TeV}}$	
DELPHI	$e^+ e^- \rightarrow \tau^+ \tau^-$	$g_1 \lesssim 0.66 \frac{m_{k_2}}{\text{TeV}}$	
$\mu^+ \mu^+$ collider	$\mu^+ \mu^+ \rightarrow \tau^+ e^+$	$\sqrt{g_1 g_2} \lesssim 0.07 \frac{m_{k_2}}{\text{TeV}}$	$12 \text{ fb}^{-1}, \sqrt{s} = 2 \text{ TeV}$
$\mu^+ \mu^+$ collider	$\mu^+ \mu^+ \rightarrow \mu^+ \mu^+$	$g_2 \lesssim 0.09 \frac{m_{k_2}}{\text{TeV}}$	$12 \text{ fb}^{-1}, \sqrt{s} = 2 \text{ TeV}$
$\mu^+ e^-$ collider	$\mu^+ e^- \rightarrow e^+ \tau^-$	$\sqrt{f_1 f_2} \lesssim 0.13 \frac{m_{k_1}}{\text{TeV}}$	$100 \text{ fb}^{-1}, (E_e, E_\mu) = (30, 1000) \text{ GeV}$
$\mu^+ e^-$ collider	$\mu^+ e^- \rightarrow \tau^+ \mu^-$	$\sqrt{g_1 g_2} \lesssim 0.13 \frac{m_{k_2}}{\text{TeV}}$	$100 \text{ fb}^{-1}, (E_e, E_\mu) = (30, 1000) \text{ GeV}$
$\mu^+ e^-$ collider	$\mu^+ e^- \rightarrow \mu^+ e^-$	$h_1 \lesssim 0.17 \frac{m_{k_3}}{\text{TeV}}$	$100 \text{ fb}^{-1}, (E_e, E_\mu) = (30, 1000) \text{ GeV}$

Tristan {

# Summary

Lepton Flavour Triality avoids CLFV bounds from muon decays while allowing tau CLFV interactions;

Belle II predictions of tau CLFV.

Next-to-leading order results from  $Z \rightarrow l^+l^-$ ,  $H \rightarrow \gamma\gamma$ ,  $H \rightarrow Z\gamma$

$\mu$ TRISTAN

$\mu^+\mu^+$  collider

Resonances searches

$\mu$ TRISTAN  $\mu^+e^-$  collider

# **Backup slides**

## Neutrino Masses

Include 3 RH sterile Neutrinos

T = 1, 2, 3 triality charges  $\nu_R \rightarrow \omega^T \nu_R$

$$-\mathcal{L} \supset y_{\nu i} \bar{L}_i \nu_{Ri} \tilde{H} + \frac{1}{2} M_{ij} (\bar{\nu}_{Ri})^c \nu_{Rj} + h.c.$$

$M_{ij}$  is a Majorana mass matrix

Incompatible with neutrino oscillations.

Break Triality with soft-breaking operators or introducing a Singlet complex scalar S (T=1), with non-zero VEV for S.

Type I see-saw, or Type III (triplet).

**Bigaran, He, Schmidt, Valencia, Volkas, (2022) 2212.09760.**

## Observables

Direct Searches of doubly charged scalar with CLFV:

$$m_{k_i} \geq 0.6 \text{ TeV (ATLAS).}$$

Lepton scattering (DELPHI):

$$\frac{m_{k_1}}{|f_2|} \geq 0.74 \text{ TeV;}$$

$$\frac{m_{k_2}}{|g_1|} \geq 1.5 \text{ TeV.}$$

Flavour-violating Z decays:

$BR(Z \rightarrow k_1 k_1 \rightarrow e^+ e^+ \mu^- \mu^-)$  is highly suppressed;

$BR(Z \rightarrow \tau^+ \tau^- \rightarrow e^+ e^+ \mu^- \mu^-)$ .

anomalous magnetic moment  $\rightarrow$  too small.

**Bigaran, He, Schmidt, Valencia, Volkas, (2022) 2212.09760.**