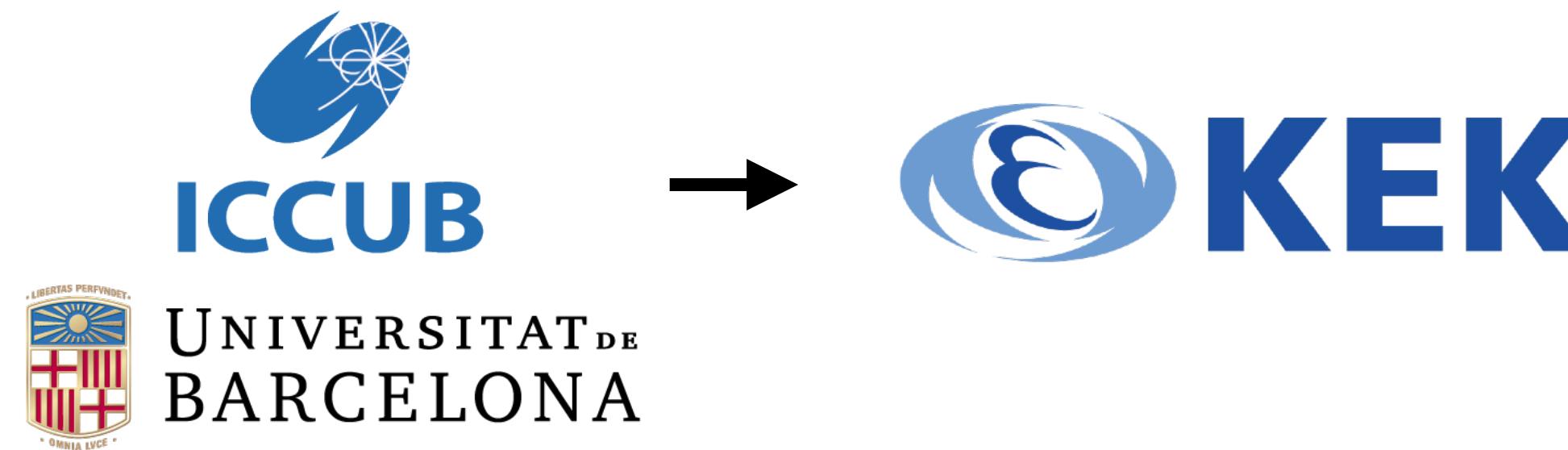


Renormalization group effects in QCD axion phenomenology



Shohei Okawa
(ICCUB, Barcelona → KEK)

Based on collaboration with
Luca Di Luzio, Federico Mescia, Enrico Nardi (2205.15326, PRD106 (2022) 5, 055016)
+ Maurizio Giannotti, Gioacchino Piazza (2305.11958, to appear in PRD)

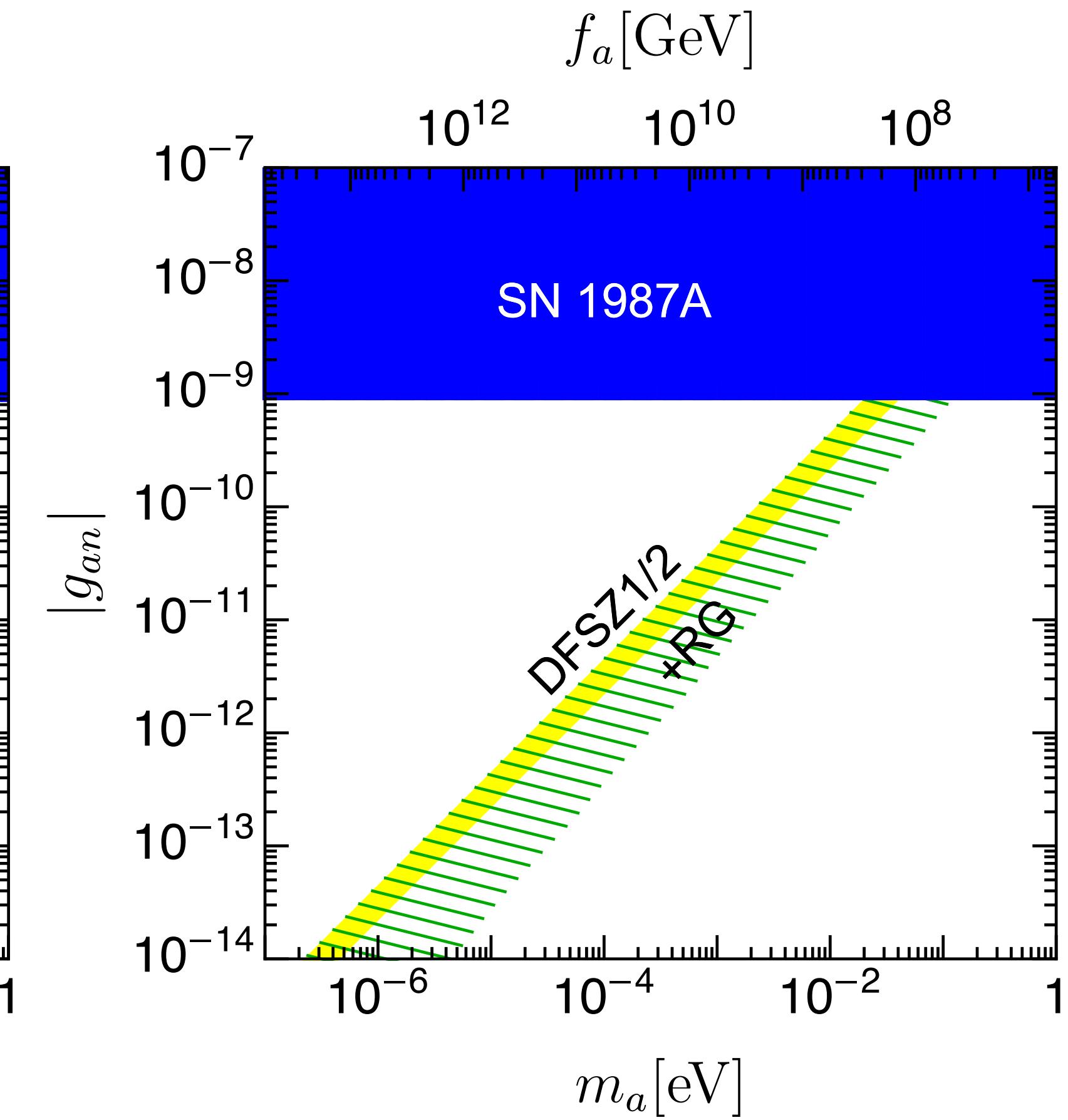
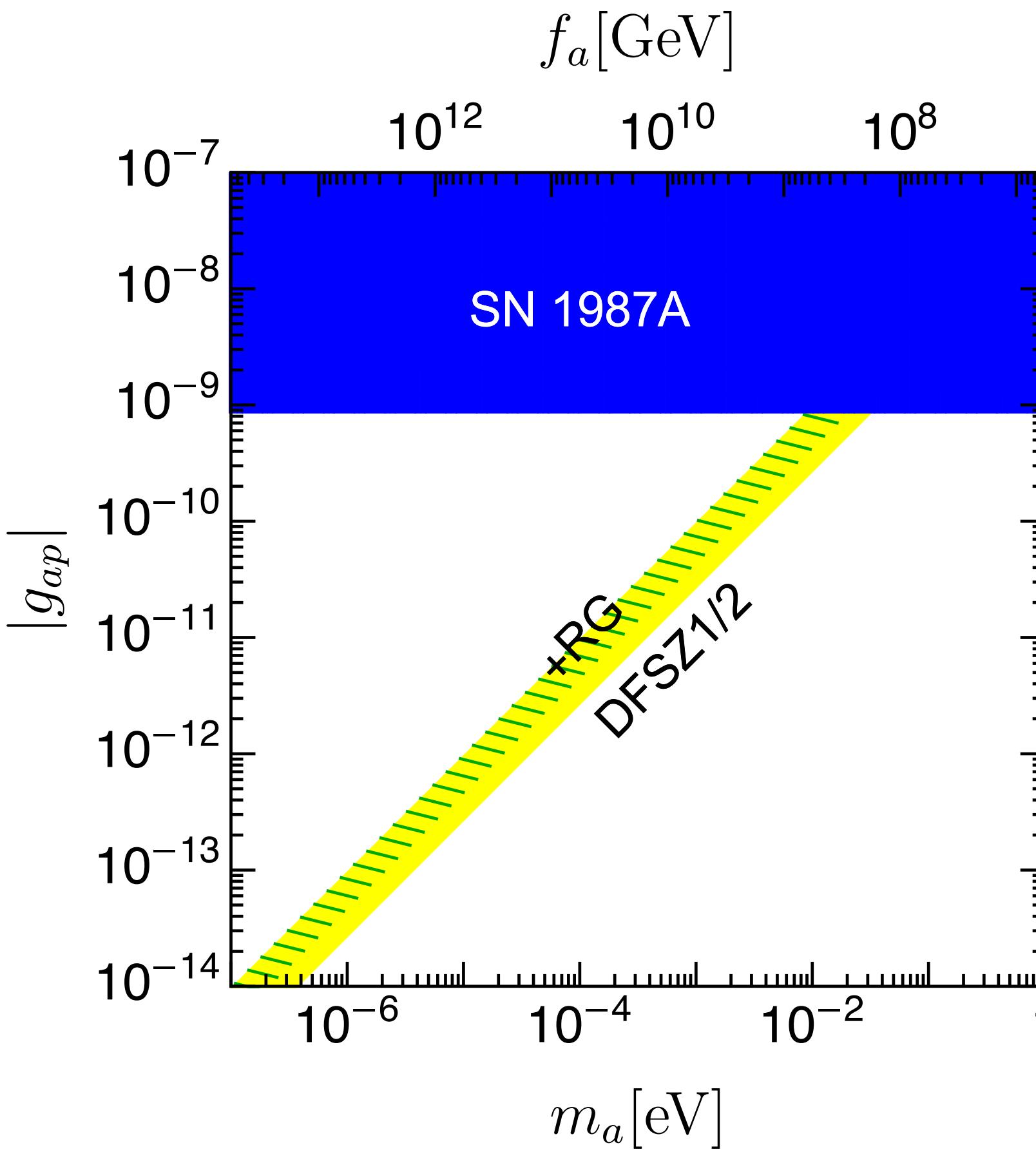
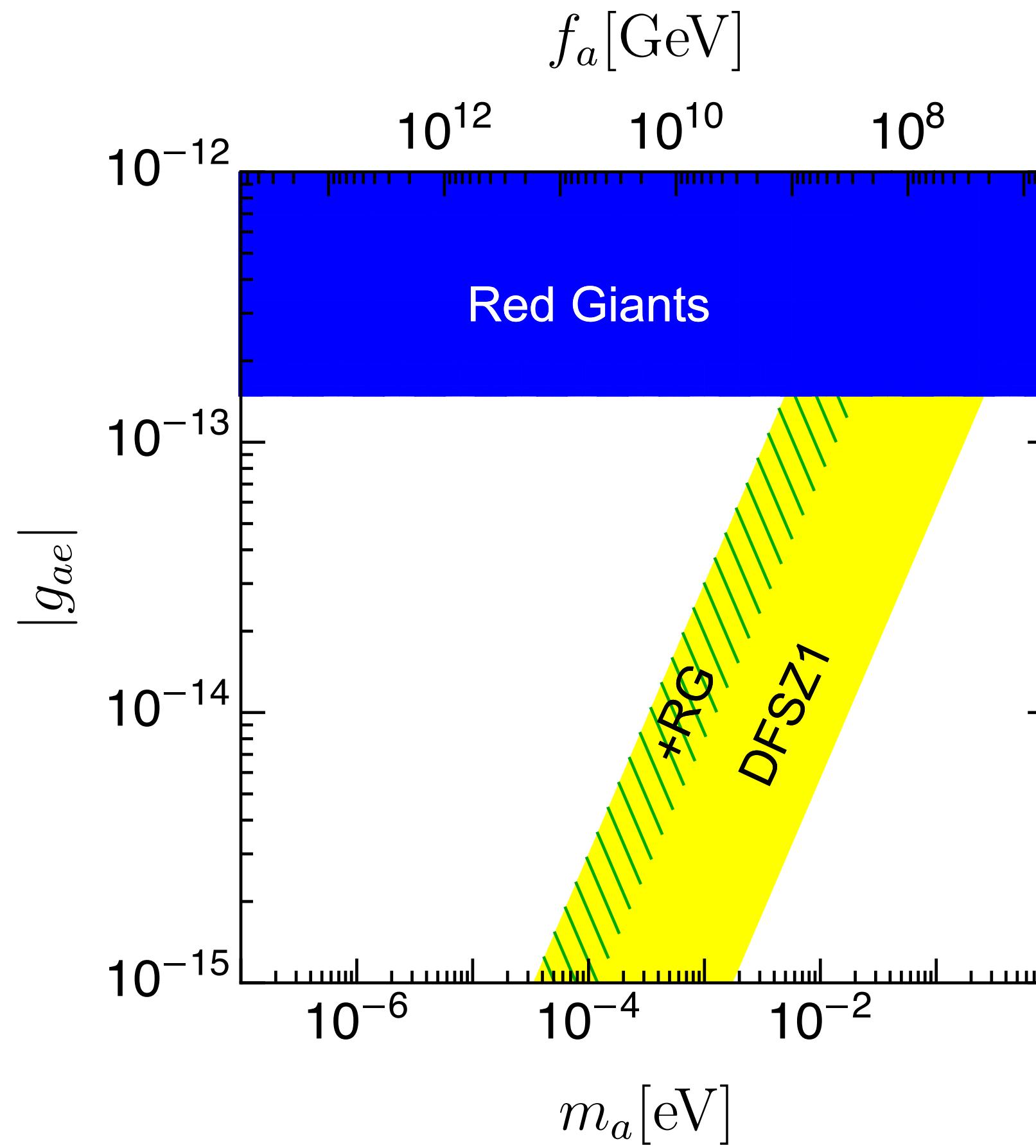
The 3rd International Joint Workshop on the Standard Model and Beyond

The 11th KIAS Workshop on Particle Physics and Cosmology

13 November 2023

Today's talk

Renormalization group corrections to axion couplings are **not negligible**



Yellow: Tree-level prediction in DFSZ models, Green hatch: +RG effects

A flash review of PQ mechanism and axion

- Axion is predicted in a solution to the Strong CP problem

$$\mathcal{L}_{\text{QCD}} \supset \theta \frac{\alpha_s}{8\pi} G_{\mu\nu} \tilde{G}^{\mu\nu} \rightarrow \frac{a(x)}{f_a} \frac{\alpha_s}{8\pi} G_{\mu\nu} \tilde{G}^{\mu\nu} \quad \rightarrow \quad \langle a/f_a \rangle = 0 \quad \text{in the QCD vacuum}$$

- ▶ $a(x)$ = NG boson from a chiral U(1) PQ symmetry breaking

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DFSZ axion: SM quarks and Higgses charged under PQ. [Zhitnitsky (1980), Dine, Fischler, Srednicki (1981)]

Minimally requires 2HDM + 1 scalar singlet. SM leptons are also PQ charged.

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KSVZ axion: All SM fields are neutral under PQ. [Kim (1979), Shifman, Vainshtein, Zakharov (1980)]

QCD anomaly induced by new quarks that are vector-like under SM and chiral under PQ. Singlet scalar breaks PQ.

Axion couplings to matter and radiation

$$\mathcal{L}_a^{\text{eff}} = \frac{\partial_\mu a}{2f_a} \sum_{f=p,n,e} C_f \bar{f} \gamma^\mu \gamma_5 f + \frac{a}{f_a} \frac{e^2}{32\pi^2} \left(\frac{E}{N} - 1.92 \right) F \tilde{F}$$

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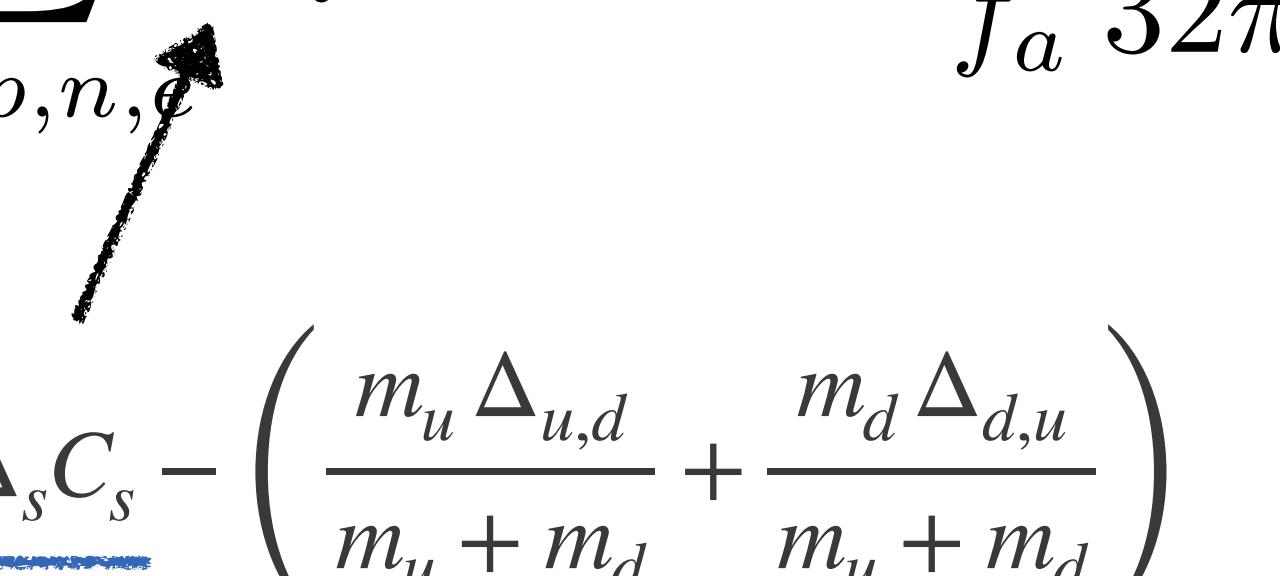
↑
PQ-EM anomaly

a- π^0 mixing (a $G\tilde{G}$ term)

Axion couplings to matter and radiation

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$C_{p,n} = \underline{\Delta_u C_{u,d} + \Delta_d C_{d,u} + \Delta_s C_s} - \underline{\left(\frac{m_u \Delta_{u,d}}{m_u + m_d} + \frac{m_d \Delta_{d,u}}{m_u + m_d} \right)}$
 $C_q = C_q(\mu_{\text{QCD}})$
 $s^\mu \Delta_{u,d,s} = \langle N | \bar{q} \gamma^\mu \gamma_5 q | N \rangle$
from $aG\tilde{G}$ term



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► completely fixed by $C_f(\mu_{\text{QCD}})$ and PQ charges

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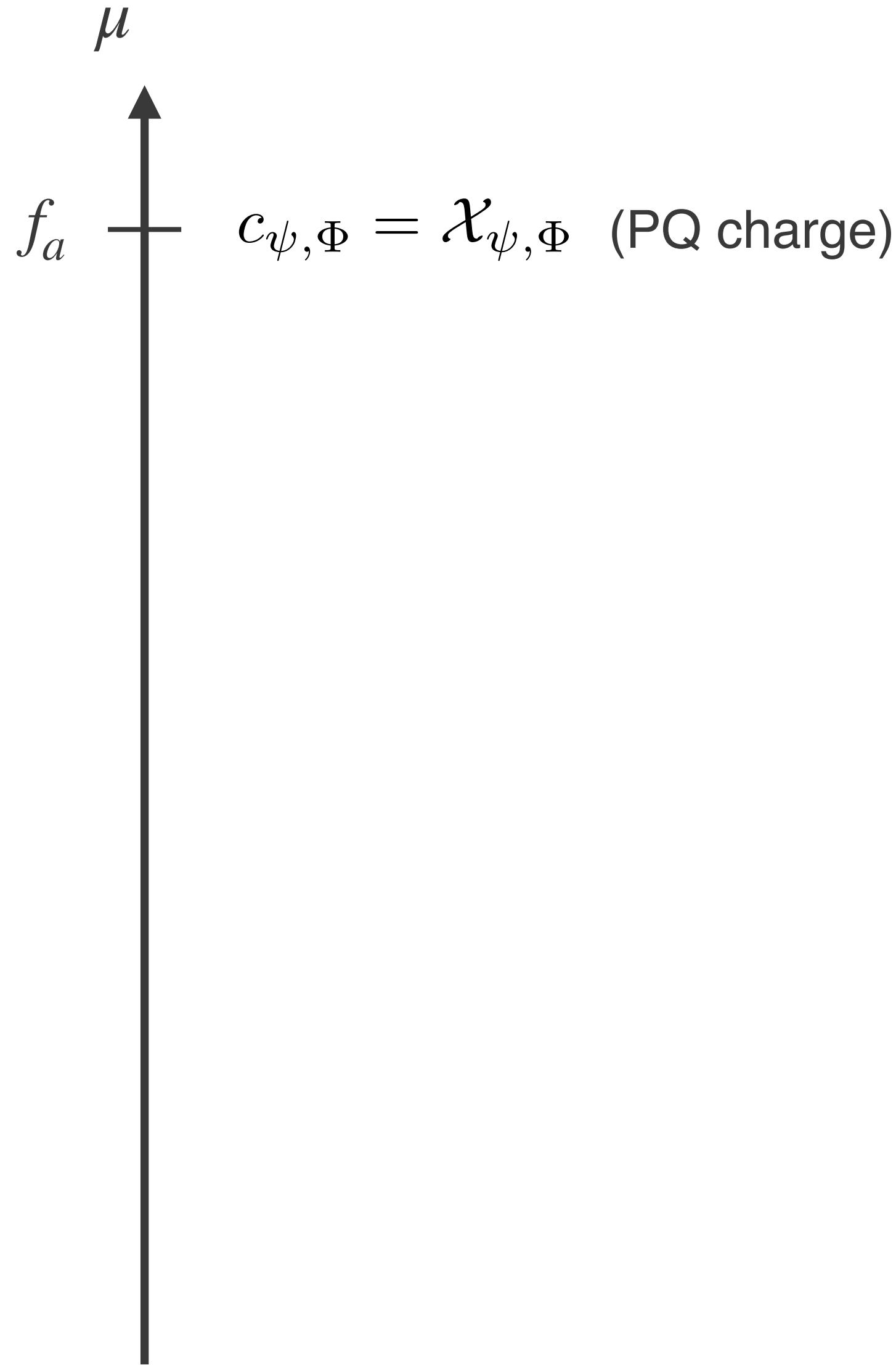
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- ▶ completely fixed by $C_f(\mu_{\text{QCD}})$ and **PQ charges**
- ▶ normally take $C_f(\mu_{\text{QCD}}) = C_f(f_a)$

but... since $\mu_{\text{QCD}} \ll f_a$, $C_f(\mu_{\text{QCD}}) = C_f(f_a) + \Delta C_f(\mu_{\text{QCD}}; f_a)$

*large log corrections from
RG evolution*

Running of DFSZ axion couplings

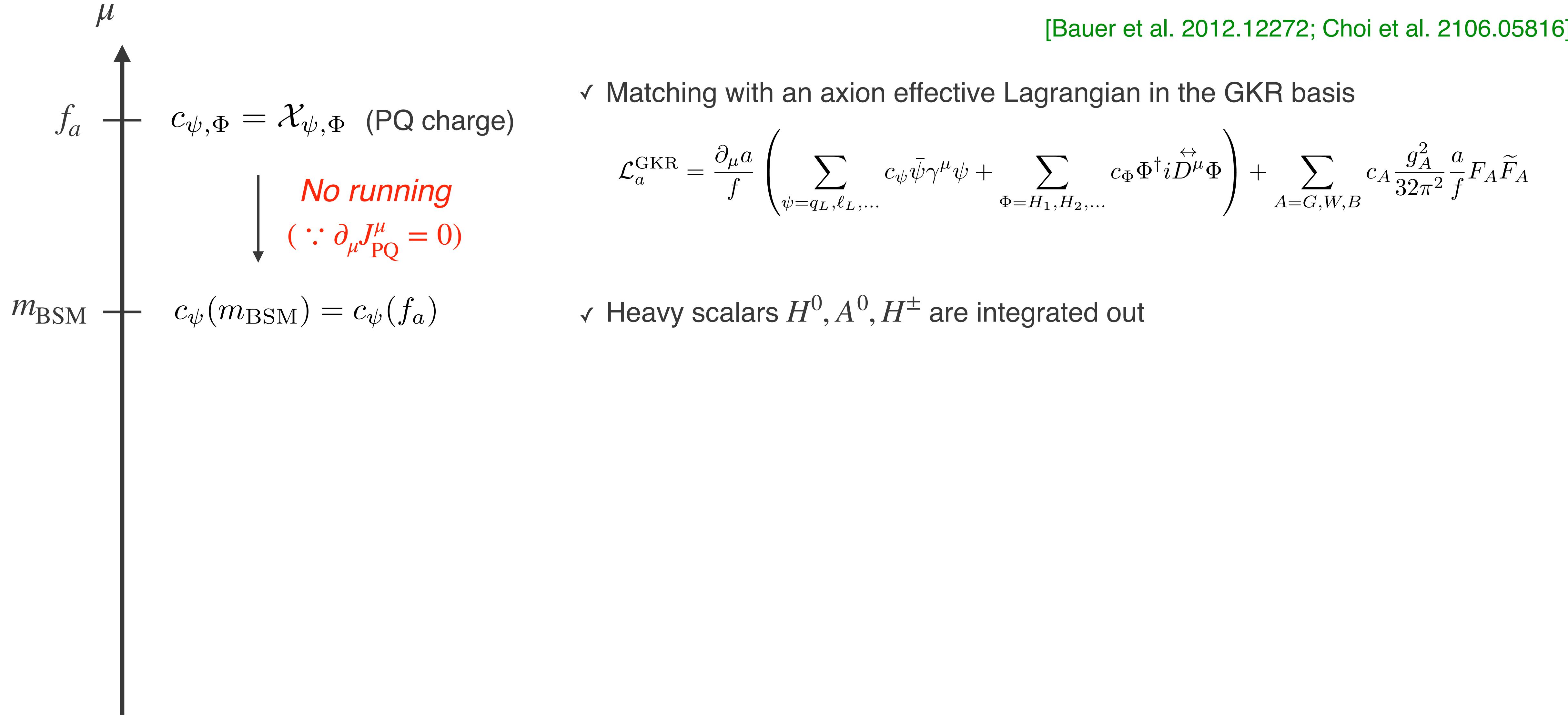


[Bauer et al. 2012.12272; Choi et al. 2106.05816]

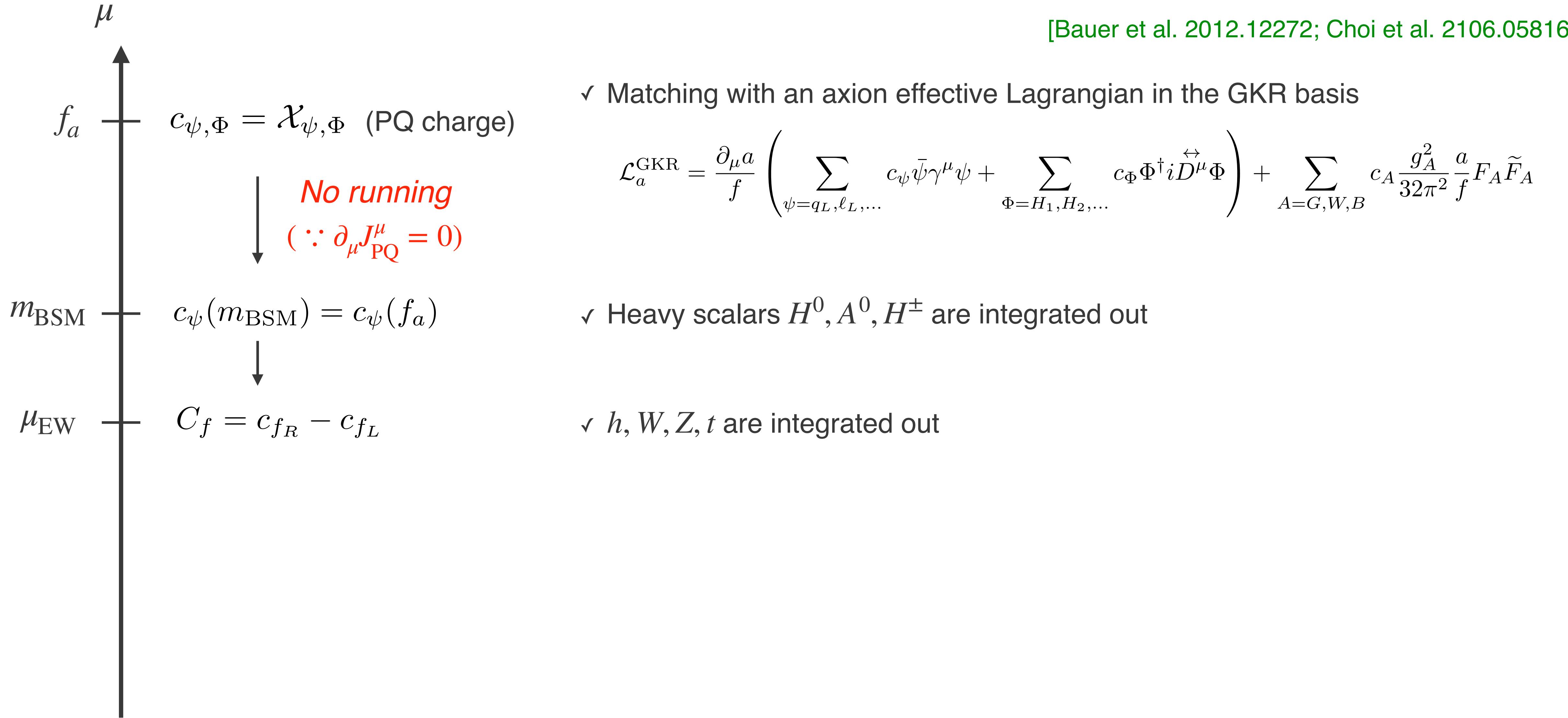
✓ Matching with an axion effective Lagrangian in the GKR basis

$$\mathcal{L}_a^{\text{GKR}} = \frac{\partial_\mu a}{f} \left(\sum_{\psi=q_L, \ell_L, \dots} c_\psi \bar{\psi} \gamma^\mu \psi + \sum_{\Phi=H_1, H_2, \dots} c_\Phi \Phi^\dagger i \overleftrightarrow{D}^\mu \Phi \right) + \sum_{A=G,W,B} c_A \frac{g_A^2}{32\pi^2} \frac{a}{f} F_A \tilde{F}_A$$

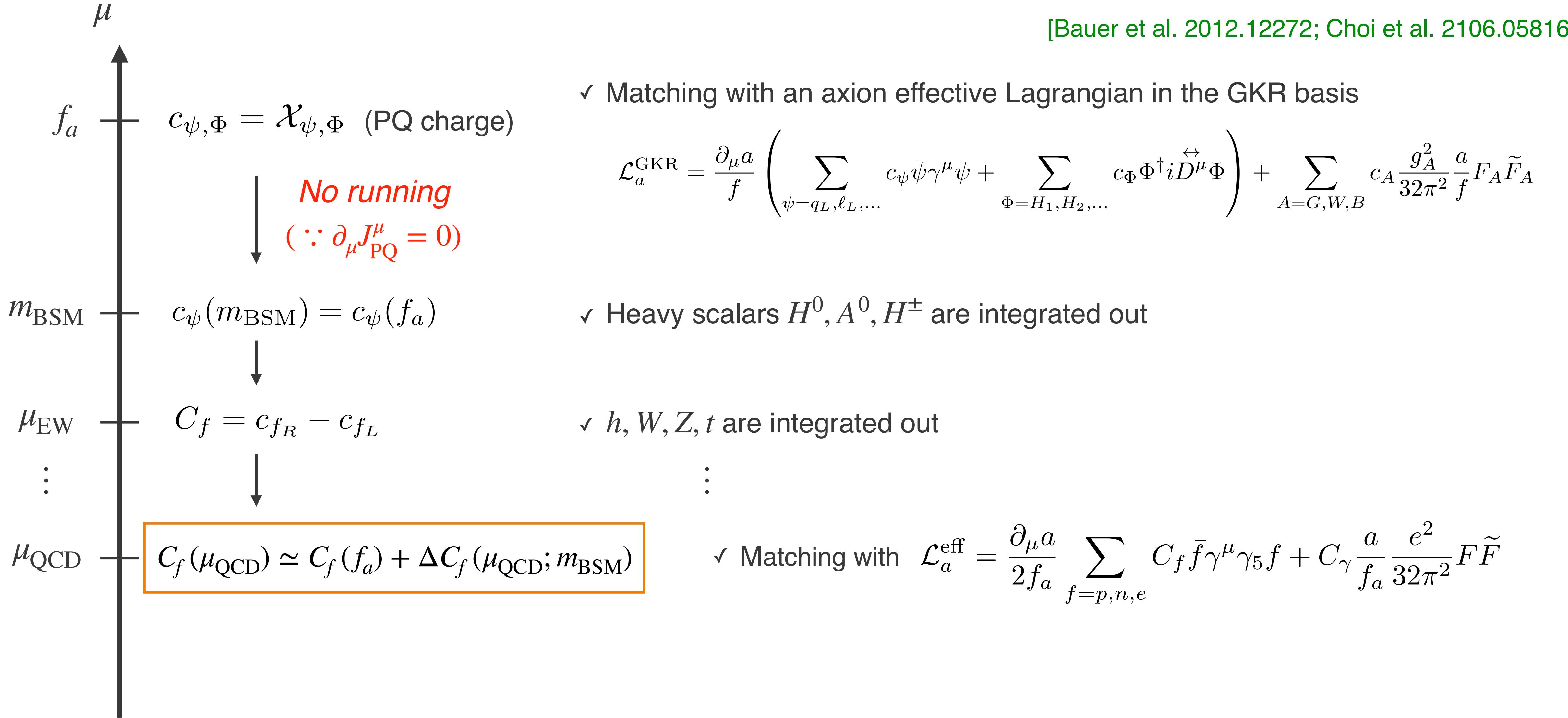
Running of DFSZ axion couplings



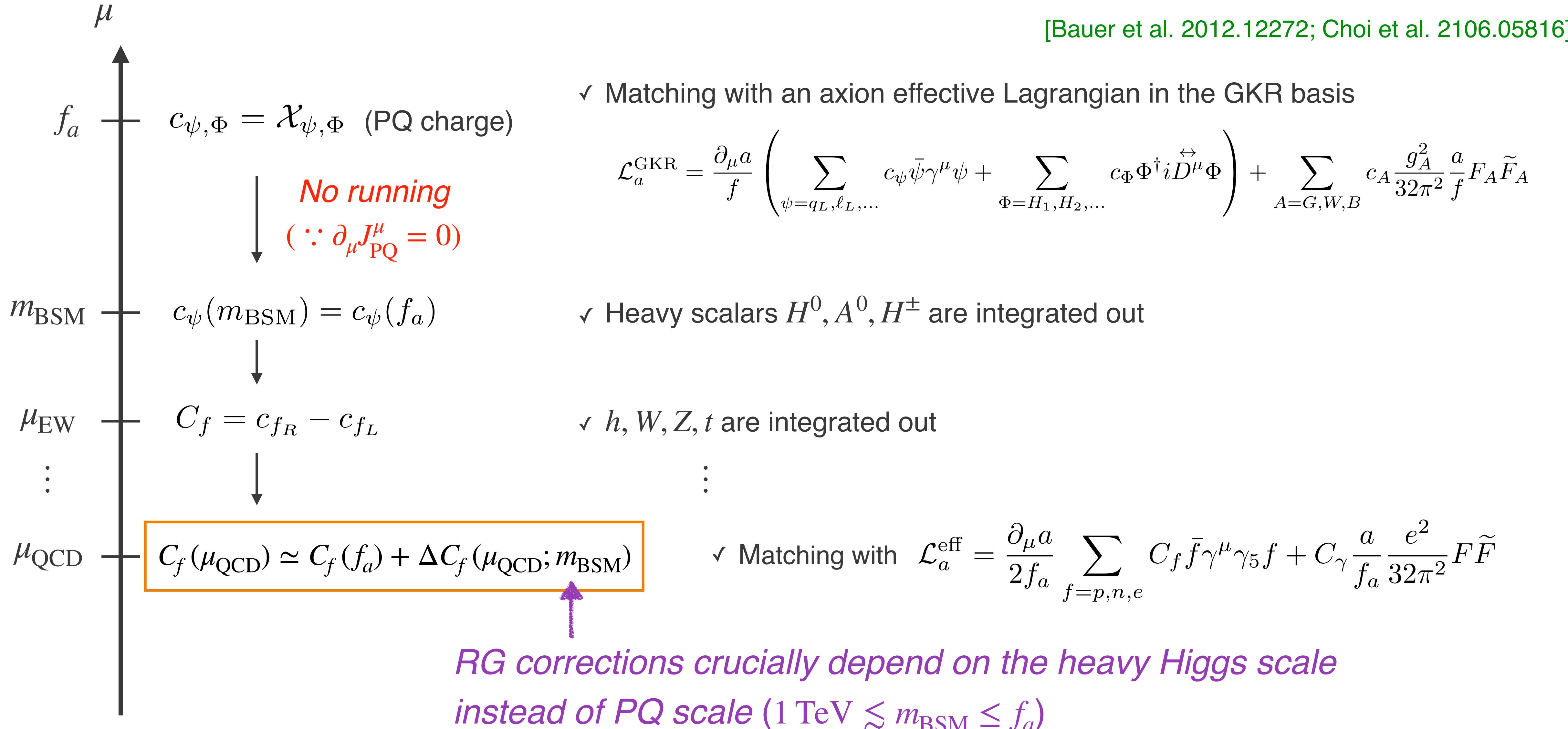
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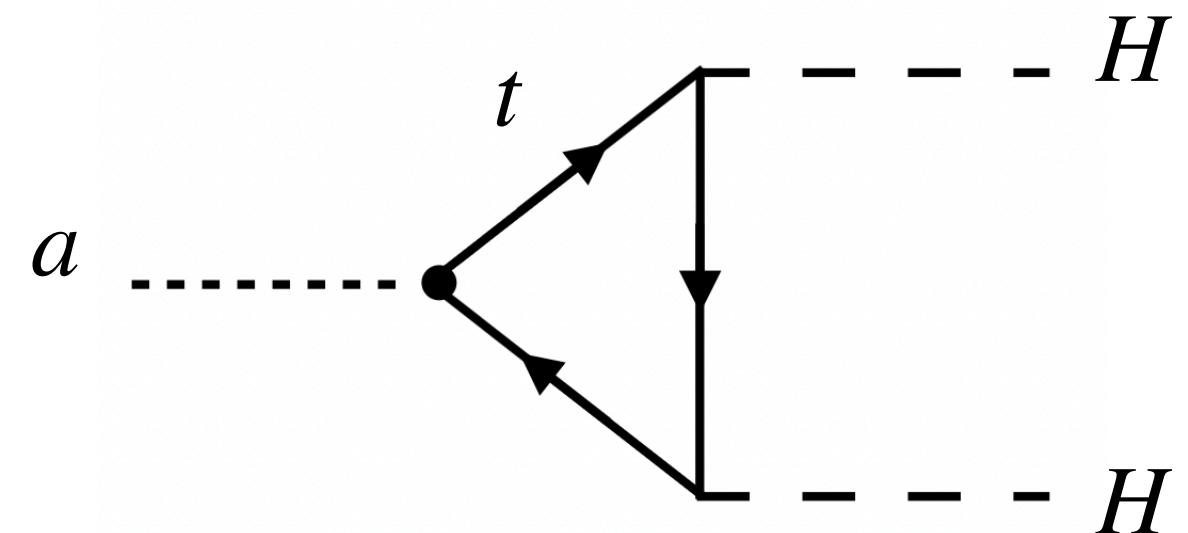
Running of DFSZ axion couplings



RG corrections in DFSZ axion models

In the DFSZ models, the leading contribution arises from **top loop diagrams** induced by **axion-top coupling C_t**

$$C_f(2\text{GeV}) \simeq C_f(f_a) + r_f^t(m_{\text{BSM}}) C_t(f_a)$$

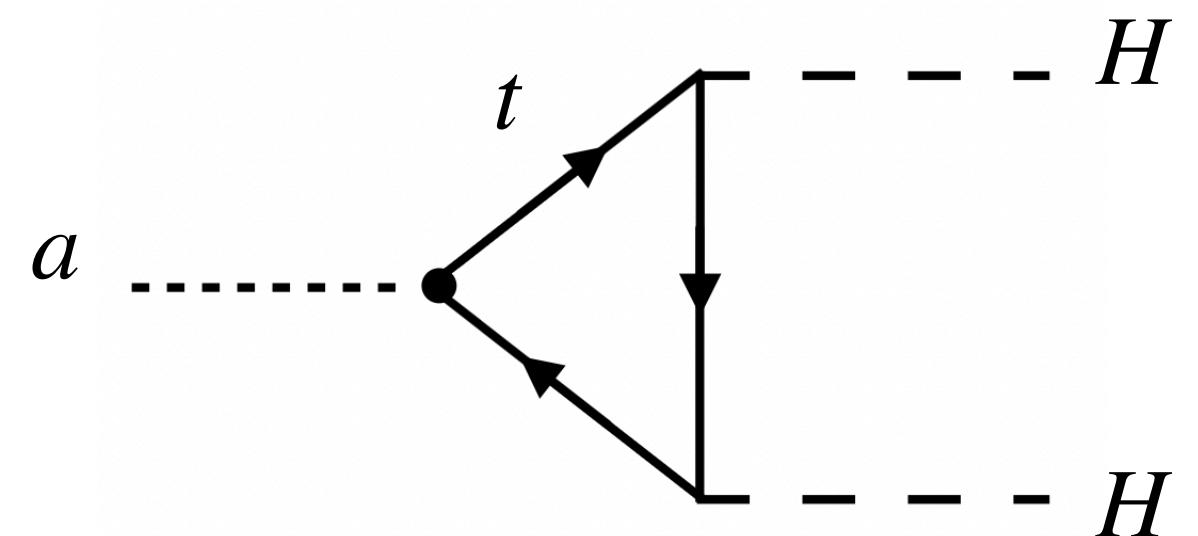


$$\frac{\partial_\mu a}{f} (H^\dagger i D_\mu H) \rightarrow \frac{\partial_\mu a}{f} \sum_{\psi=q_L, u_R, \dots} \beta_\psi \bar{\psi} \gamma_\mu \psi \quad (\beta_\psi = Y_\psi / Y_H)$$

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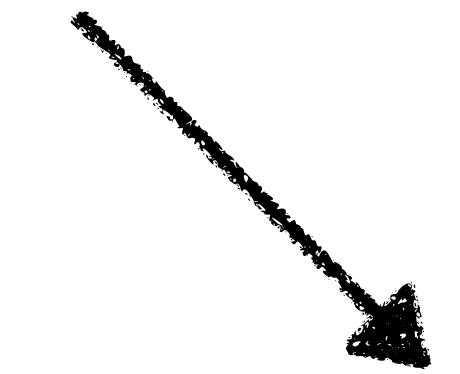


$$\begin{aligned} \frac{\partial_\mu a}{f} (H^\dagger i D_\mu H) &\rightarrow \frac{\partial_\mu a}{f} \sum_{\psi=q_L, u_R, \dots} \beta_\psi \bar{\psi} \gamma_\mu \psi \quad (\beta_\psi = Y_\psi / Y_H) \\ &\simeq \frac{\partial_\mu a}{2f} \sum_{f=u, d, \dots} \underline{(\beta_{f_R} - \beta_{f_L})} \bar{f} \gamma_\mu \gamma_5 f \quad \propto T_{3,f} \text{ weak isospin} \end{aligned}$$

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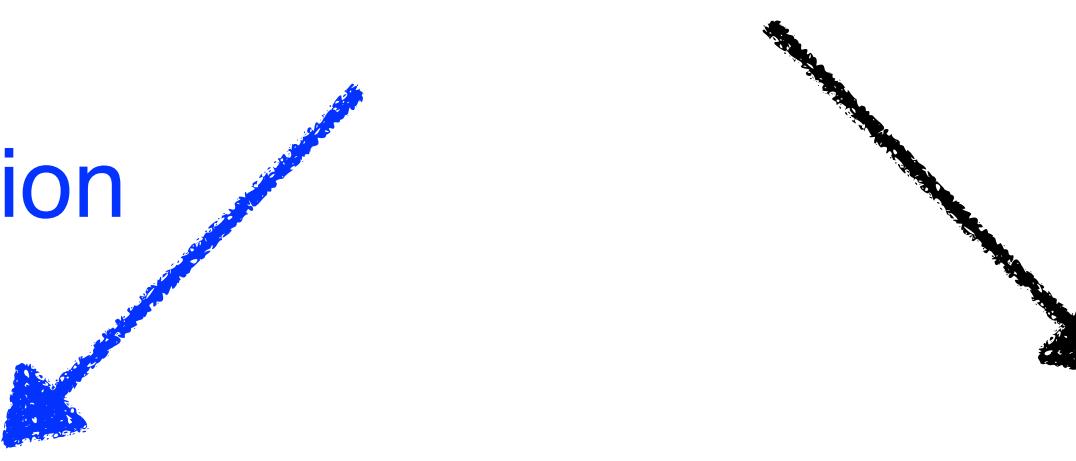
$$r_f^t \simeq T_{3,f} r_3^t \quad \text{independent of fermion species } f$$

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Analytical approximation
(~2% precision)



$$r_f^t \simeq T_{3,f} r_3^t$$

$$r_3^t = r_u^t - r_d^t \simeq -0.54 \ln(\sqrt{x} - 0.52)$$

$$r_0^t = r_u^t + r_d^t \simeq 3.8 \times 10^{-4} \ln^2(x - 1.25) \approx 0$$

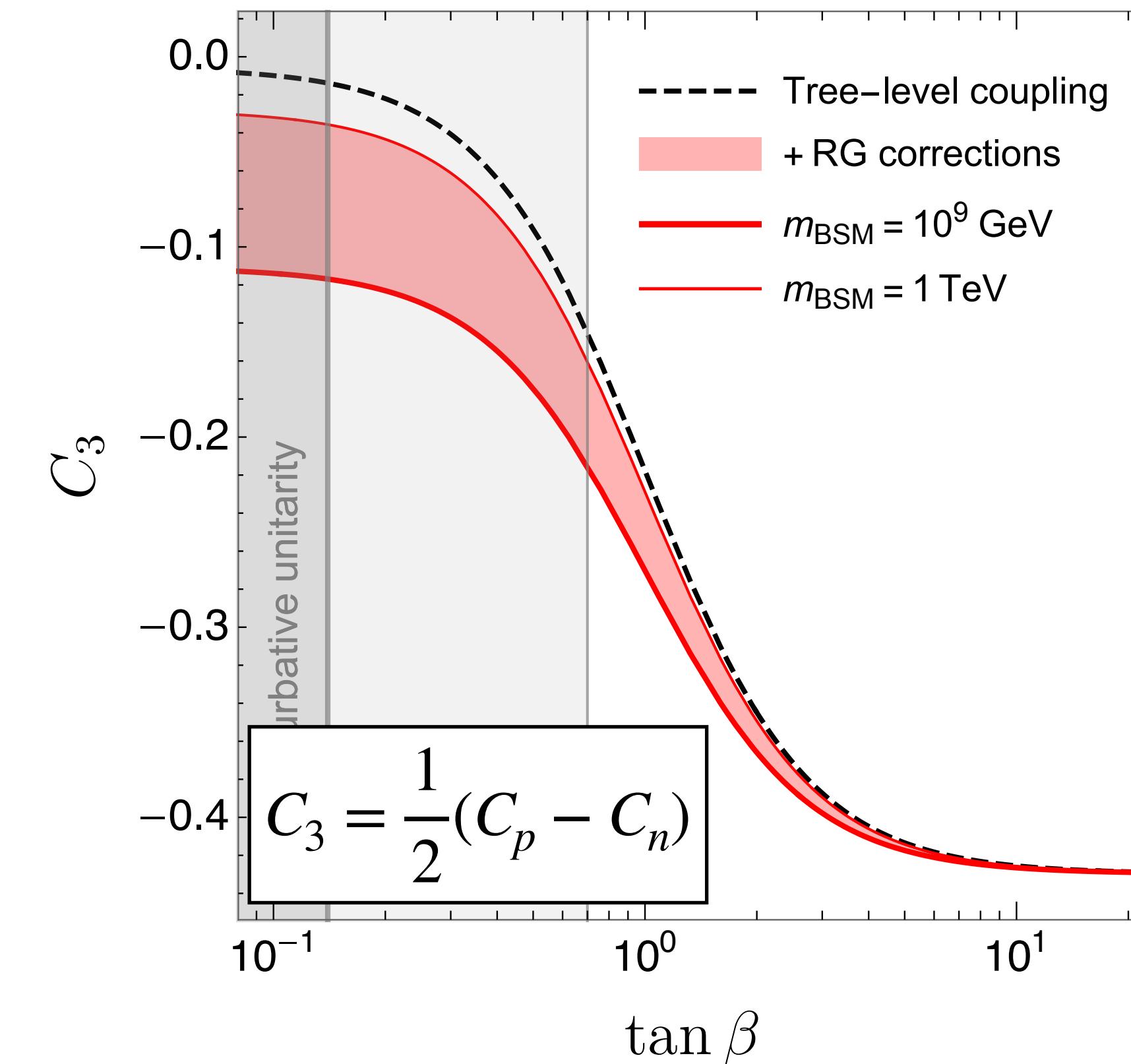
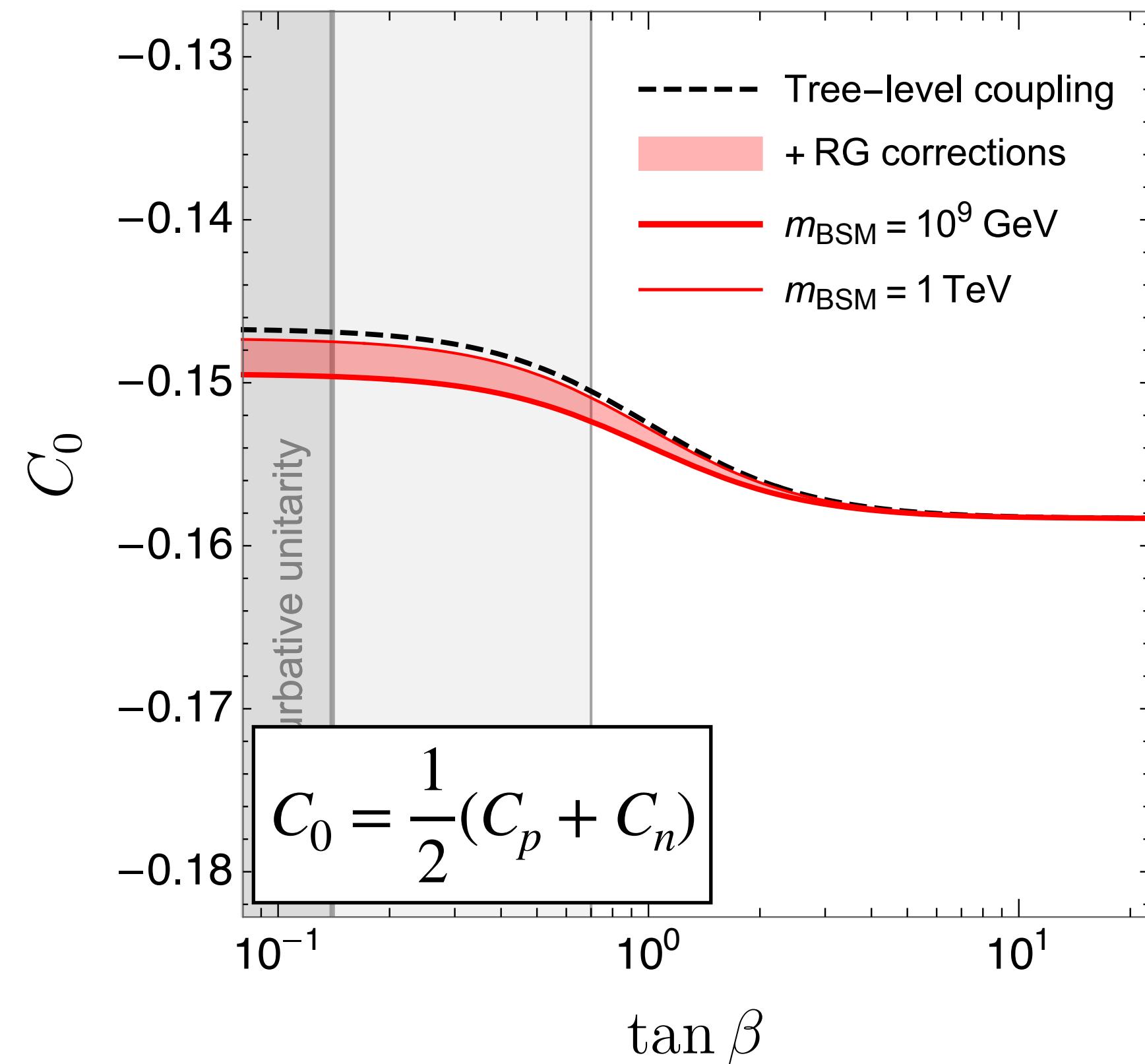
$$r_e^t \simeq -\frac{r_3^t}{2}$$

$$\left. \begin{array}{l} \\ \\ \end{array} \right\} \quad \text{with } x = \log_{10} \left(\frac{m_{\text{BSM}}}{\text{GeV}} \right)$$

RG effects on hadronic axion couplings

- DFSZ1: $\bar{q}_L H_1 u_R, \bar{q}_L H_2 d_R, \bar{l}_L H_2 e_R$
- DFSZ2: $\bar{q}_L H_1 u_R, \bar{q}_L H_2 d_R, \bar{l}_L \tilde{H}_1 e_R$

Coupling (DFSZ1)	Coupling (DFSZ2)	Approx. Correction
$C_0 \simeq -0.20$	$C_0 \simeq -0.20$	$\Delta C_0 \approx 0$
$C_3 \simeq -0.43 \sin^2 \beta$	$C_3 \simeq -0.43 \sin^2 \beta$	$\Delta C_3 \simeq -0.12 l(x) \cos^2 \beta$
$C_e = \frac{1}{3} \sin^2 \beta$	$C_e = -\frac{1}{3} \cos^2 \beta$	$\Delta C_e \simeq 0.094 l(x) \cos^2 \beta$
$C_\gamma = \frac{8}{3} - 1.92$	$C_\gamma = \frac{2}{3} - 1.92$	$\Delta C_\gamma = 0$

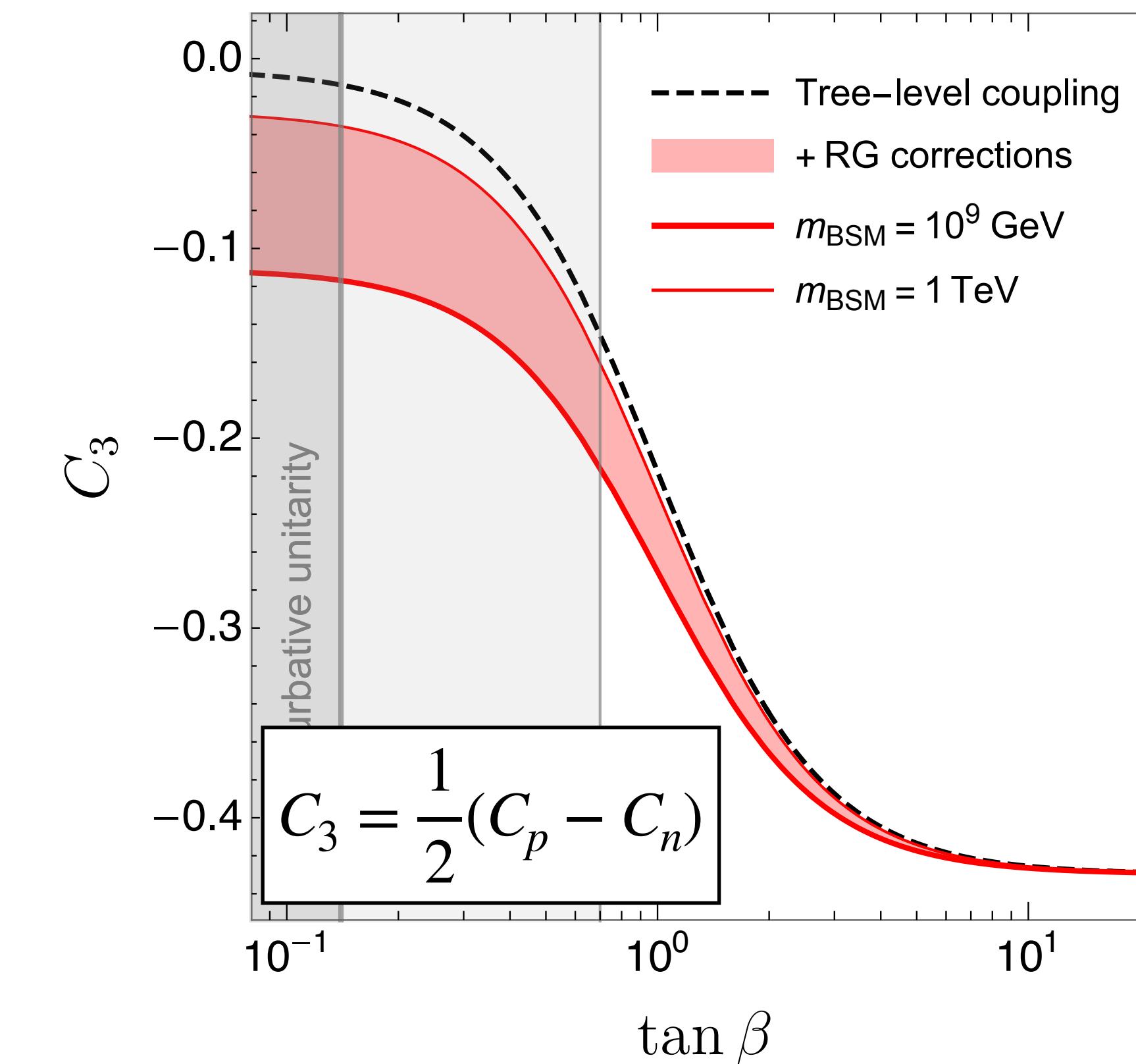
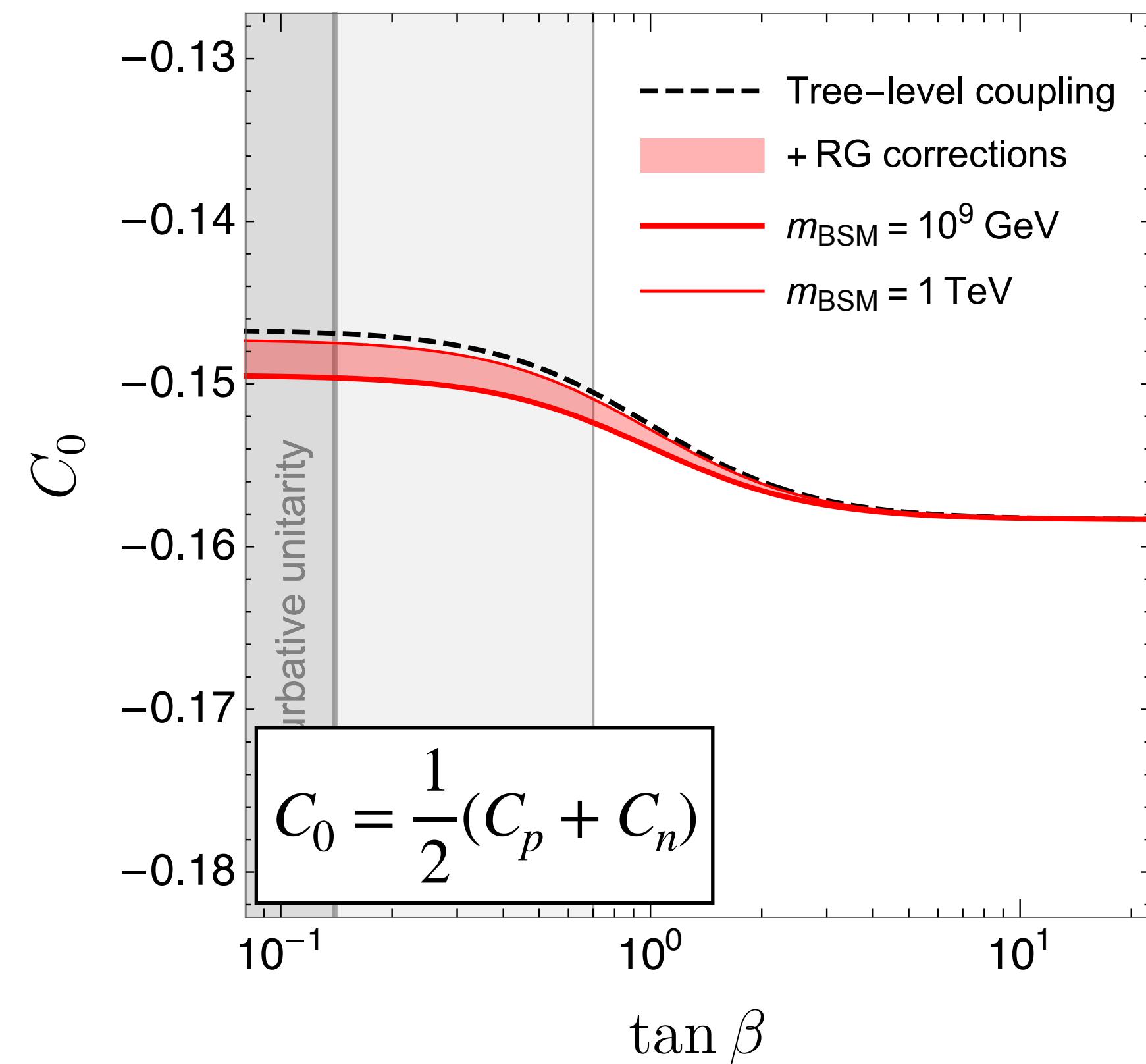


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tree couplings vanish at $\beta \rightarrow 0$

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RG corrections do not



$$l(x) = \ln(\sqrt{x} - 0.52)$$

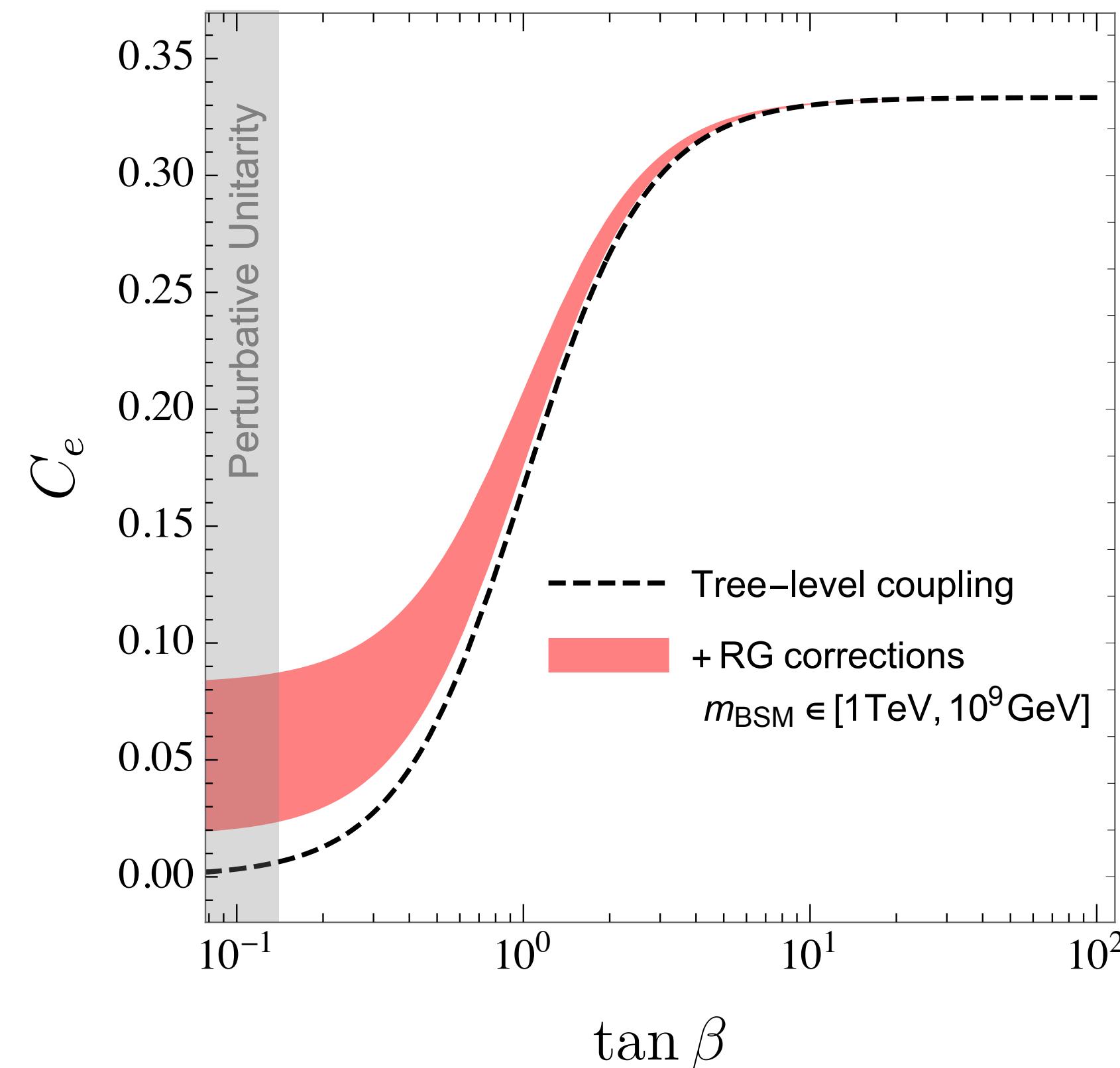
$$\tan \beta = v_1/v_2$$

RG effects on leptonic axion couplings

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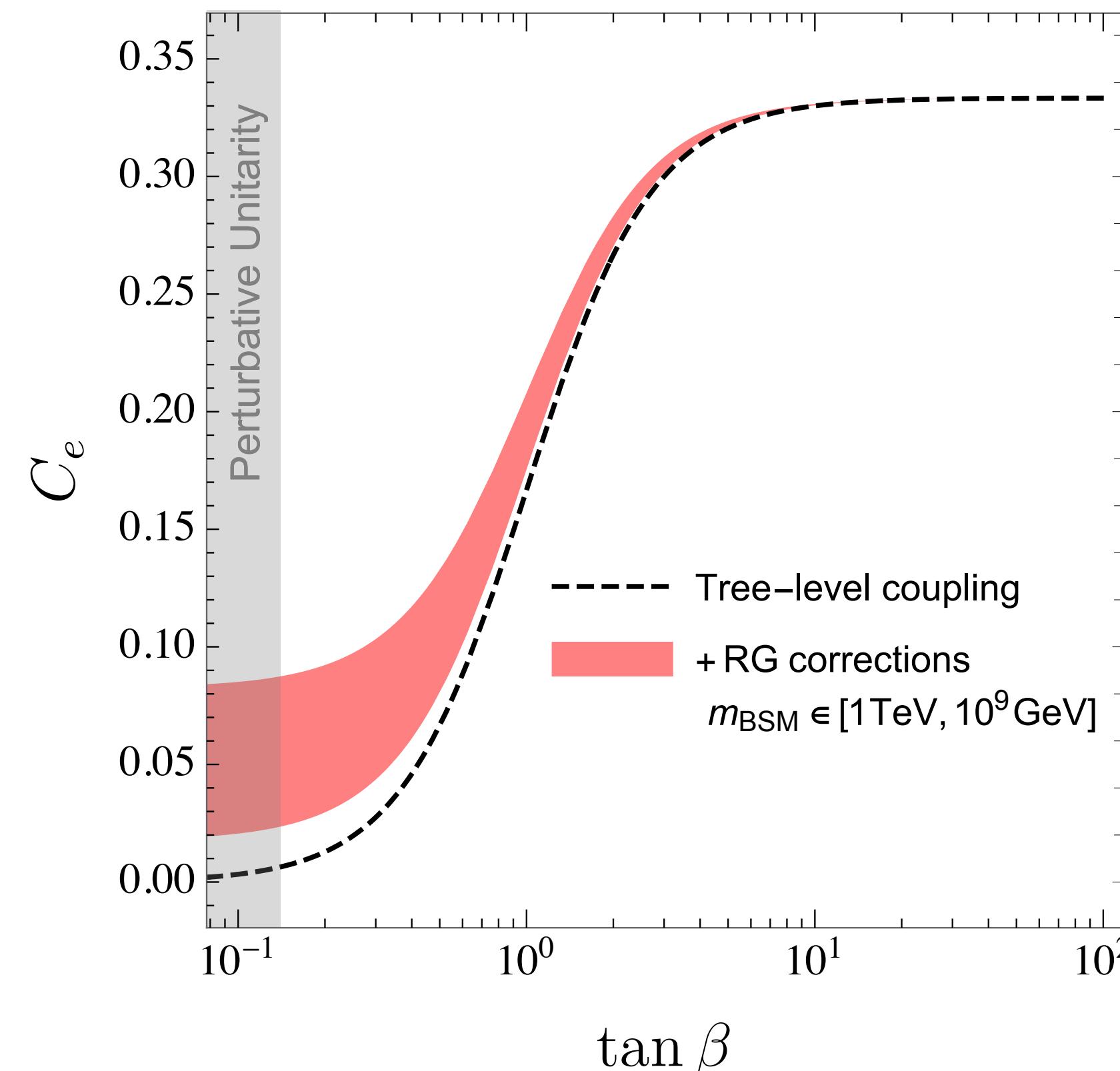
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$$l(x) = \ln(\sqrt{x} - 0.52)$$

$$\tan \beta = v_1/v_2$$

Large RG corrections for

- ▶ C_3, C_e (DFSZ1), C_3 (DFSZ2)
- ▶ *small* $\tan \beta$
- ▶ *large* m_{BSM}

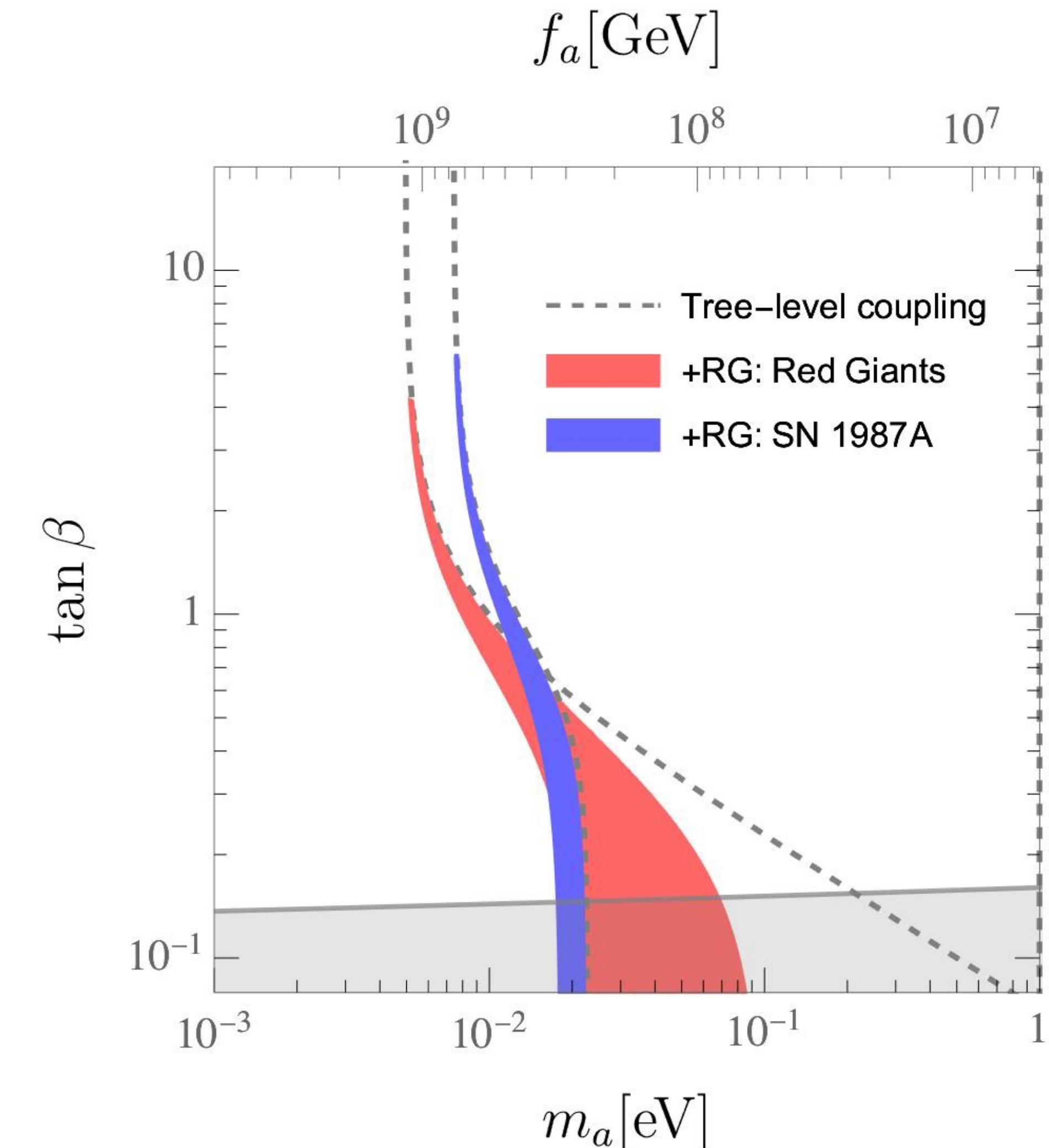
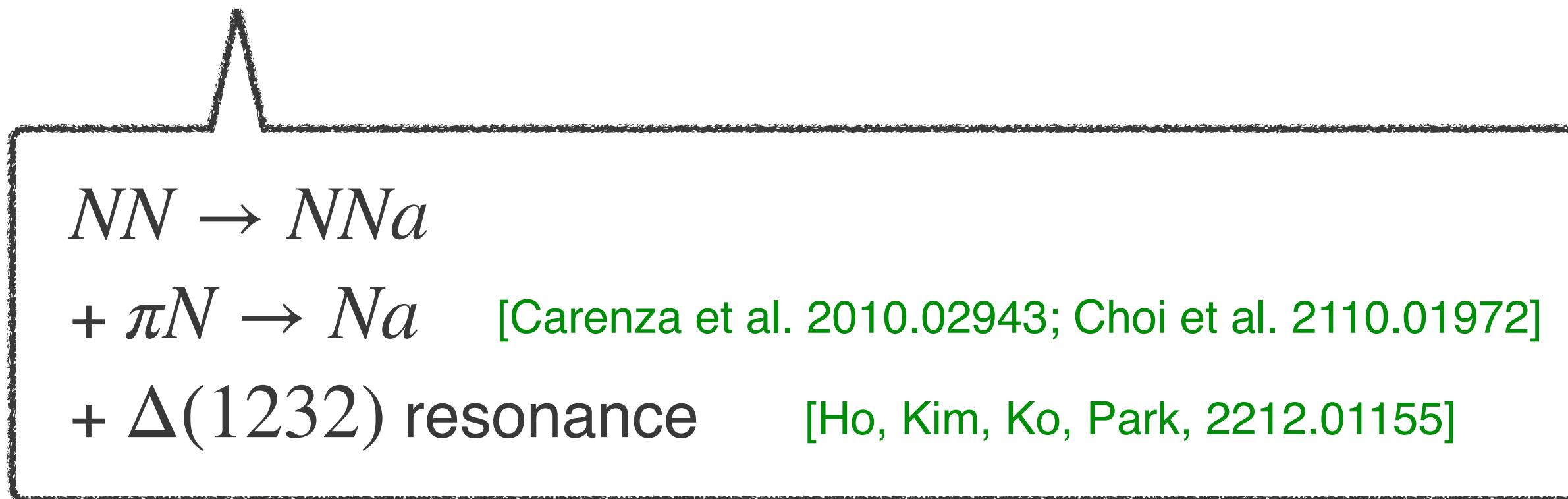
Impact on Axion Phenomenology

- **RGB bound:** $|C_e| \leq 1.65 \times 10^{-3} (m_a/\text{eV})^{-1}$

- **SN1987A:** $L_a \leq L_\nu = 3 \times 10^{52} \text{ erg/s}$

$$L_a = \epsilon_0 \left(\frac{m_N}{f_a} \right)^2 C_{\text{SN}}^2 \times 10^{70} \text{ erg/s} \quad \text{axion emission rate}$$

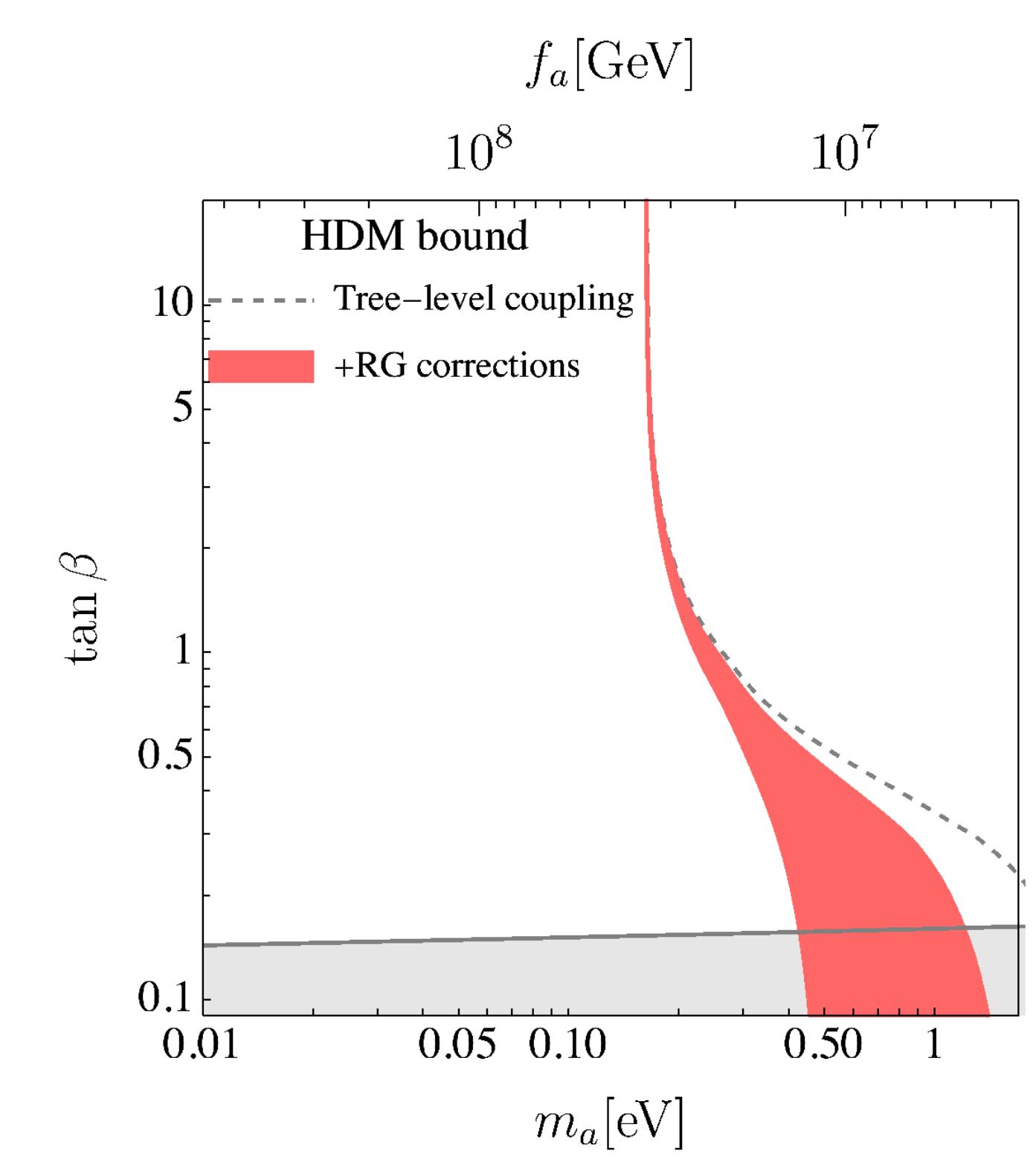
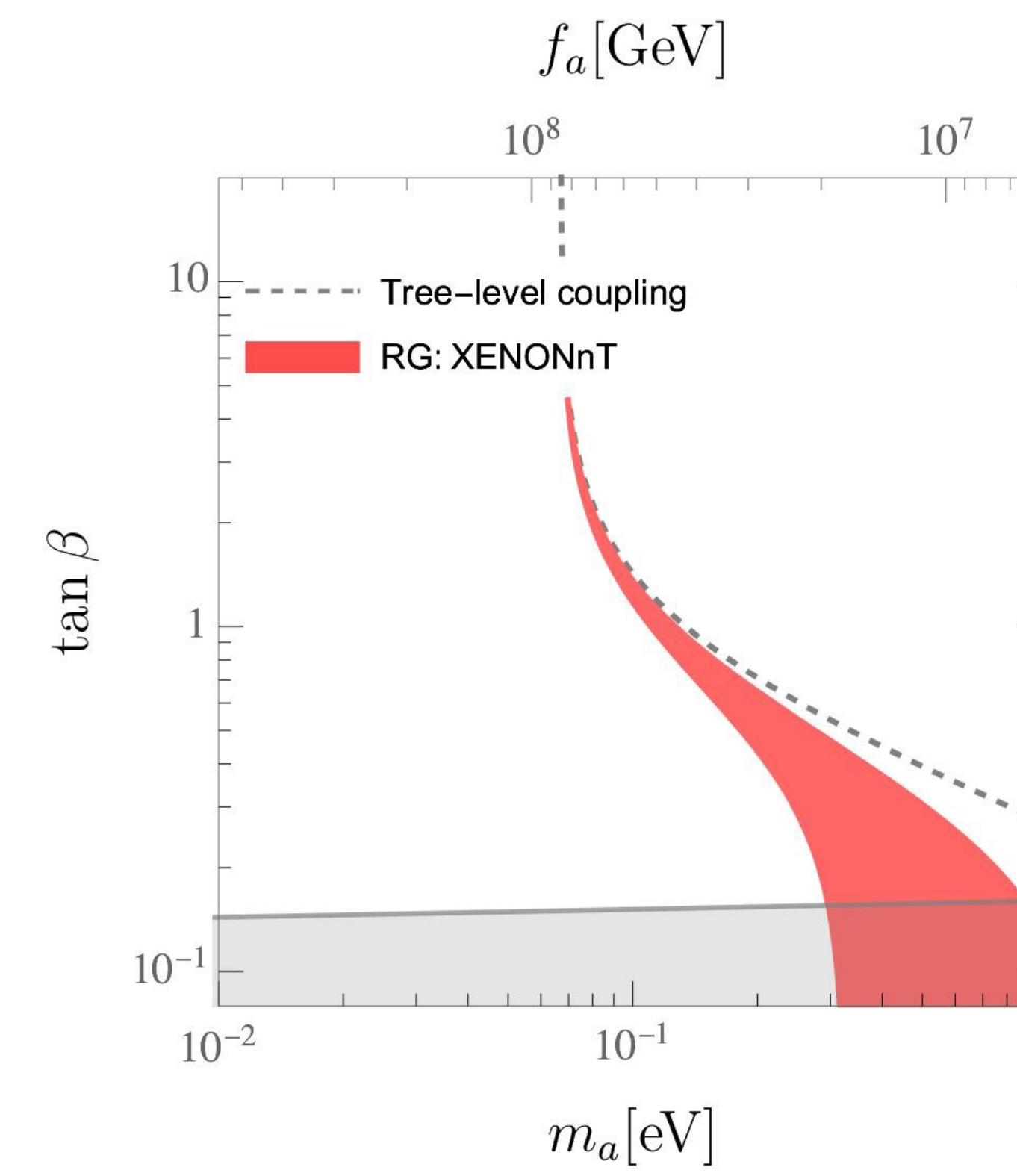
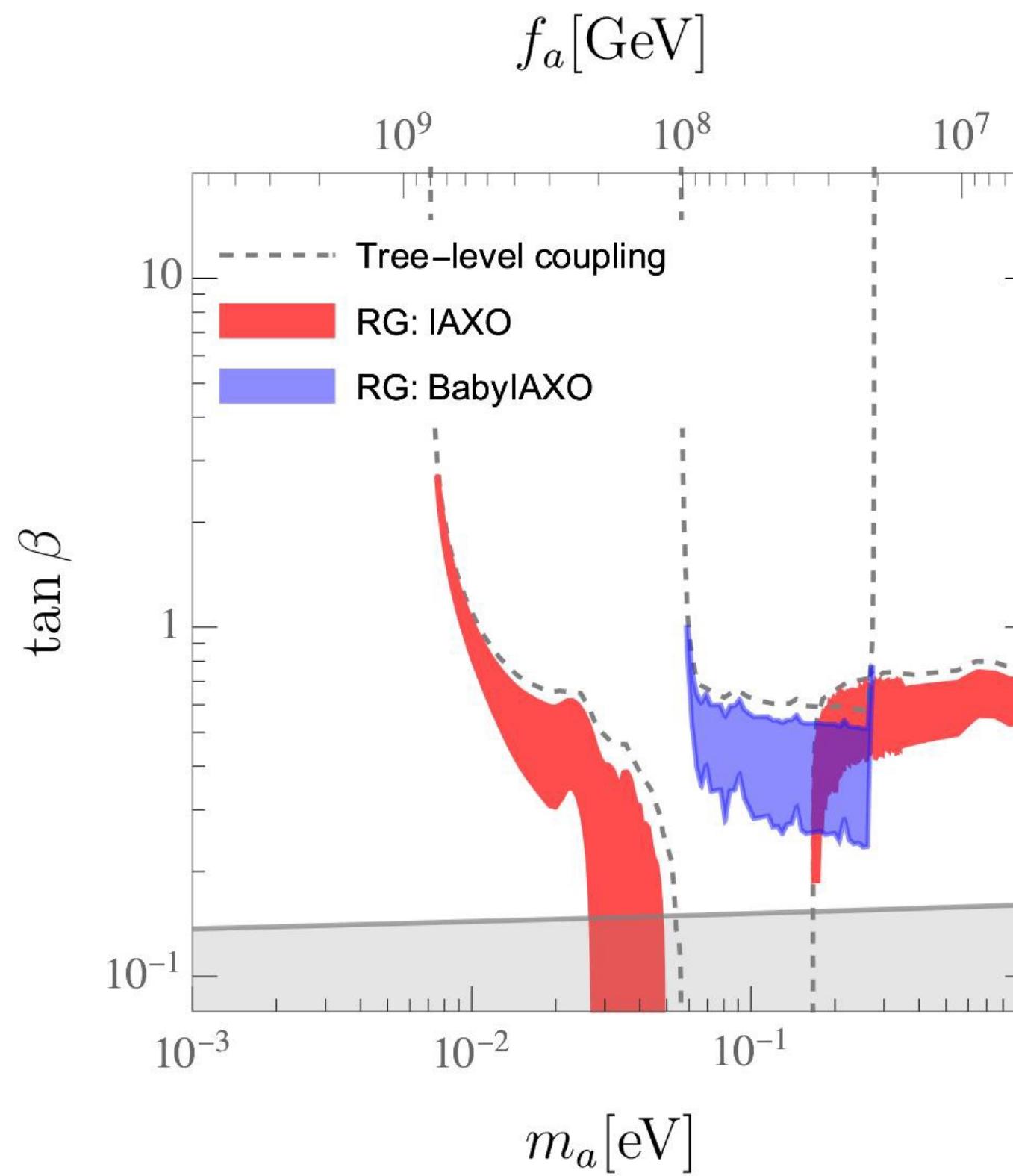
$$C_{\text{SN}} = 1.4 (C_0^2 + 1.3C_3^2 + 0.11C_0C_3) \quad [\text{Lella et al. 2211.13760}]$$



For RG effects, $m_{\text{BSM}} \in [1 \text{TeV}, f_a]$

Impact on Axion Phenomenology

- (Baby)IAXO ($g_{a\gamma}, g_{ae}$)
- XENON-nT (g_{ae})
- Hot Dark Matter ($g_{a\pi}$)
 $a\pi \leftrightarrow \pi\pi$



For RG effects, $m_{\text{BSM}} \in [1\text{TeV}, f_a]$

Application to other axion models

■ Nucleophobic axion

- ▶ Strong SN1987A bound is relaxed if $C_N \approx 0$
- ▶ At the tree level, this suppression is realized by **generation dependent PQ charge** for quarks such that

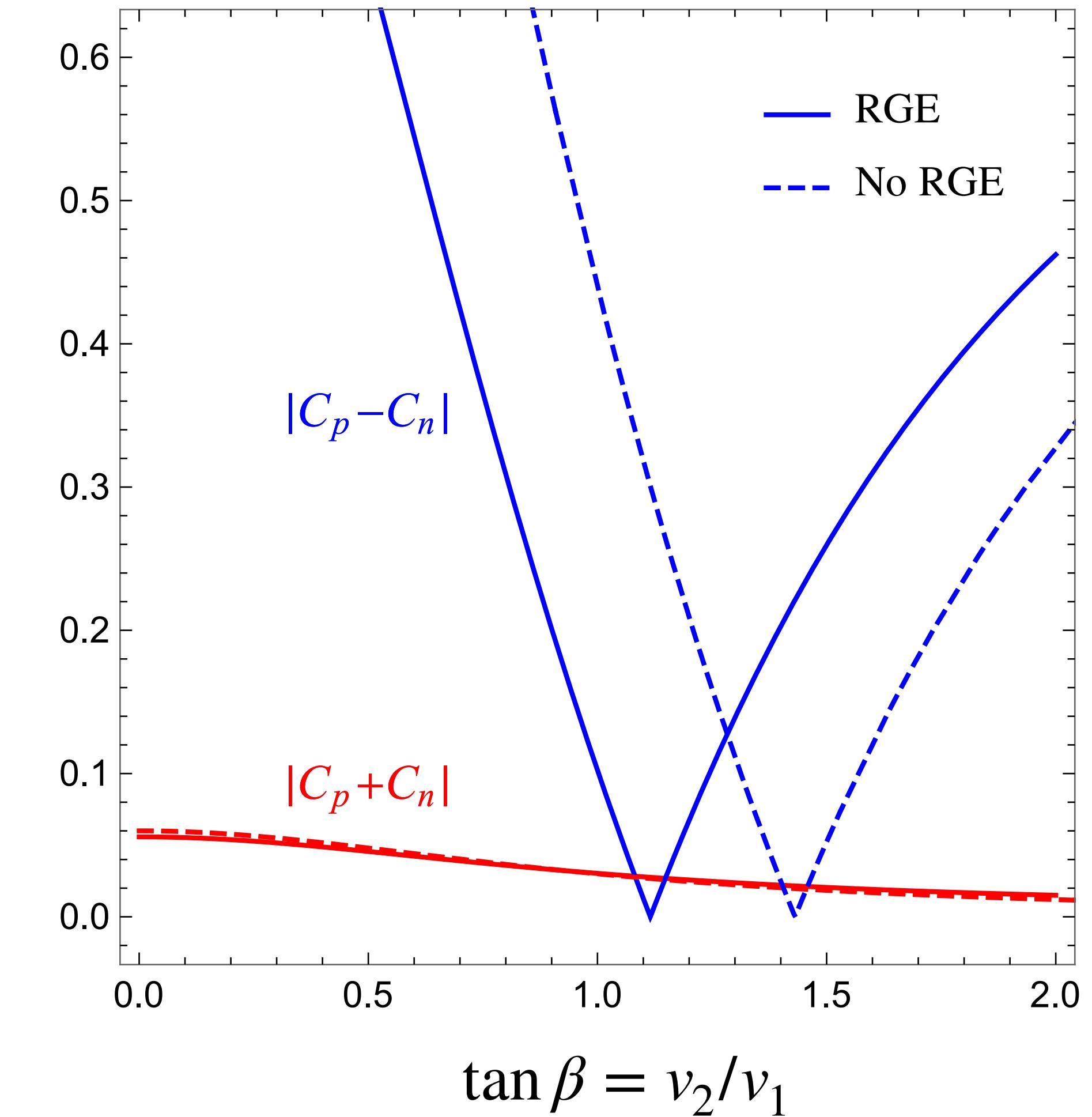
(i) $N = N_{1\text{st}}, N_{2\text{nd}} = -N_{3\text{rd}}$ (N: PQ-QCD anomaly)

(ii) $\nu_2^2/\nu_1^2 = \tan^2 \beta = 2$ [Alves, Weiner (2017), Alves (2020)]

→ These relations are modified at low energy scales by the RG corrections (see figure)

- ▶ *Axion nucleophobia keeps holding even after including the RG effects, albeit with a fairly different VEV ratio*

[Di Luzio, Mescia, Nardi, Panci, Ziegler, 1712.04940]
[Di Luzio, Mescia, Nardi, SO, 2205.15326]



Summary

- RG effects to DFSZ axion couplings have been assessed:

$$C_f(2\text{GeV}) \simeq C_f(f_a) + T_{3,f} \color{red}{r_3^t(m_{\text{BSM}})} C_t(f_a)$$
$$r_3^t = r_u^t - r_d^t \simeq -0.54 \ln(\sqrt{x} - 0.52) \quad \text{with } x = \log_{10} \left(\frac{m_{\text{BSM}}}{\text{GeV}} \right)$$

- You can apply those effects to any other (DFSZ-like) axion models!
 - ▶ your axion model predictions might be changed (e.g. axion nucleophobia)

Thanks for your attention

Back up

Axion interactions to matter and radiation

$$\begin{aligned}
 \mathcal{L}_a = & C_\gamma \frac{\alpha}{8\pi} \frac{a}{f_a} F^{\mu\nu} \tilde{F}_{\mu\nu} + \sum_{f=p,n,e} C_f \frac{\partial_\mu a}{2f_a} \bar{f} \gamma^\mu \gamma_5 f & \leftarrow \text{RGB bound } (C_e), \text{SN1987A } (NN \rightarrow NNa) \\
 & + C_\pi \frac{\partial_\mu a}{f_a f_\pi} (2\partial^\mu \pi^0 \pi^+ \pi^- - \pi^0 \partial^\mu \pi^+ \pi^- - \pi^0 \pi^+ \partial^\mu \pi^-) & \leftarrow \text{HDM bound } (a\pi \leftrightarrow \pi\pi) \\
 & + C_{\pi N} \frac{\partial_\mu a}{2f_a f_\pi} (i\pi^+ \bar{p} \gamma^\mu n - i\pi^- \bar{n} \gamma^\mu p) & \leftarrow \text{SN1987A } (\pi N \rightarrow Na) \\
 & + C_{N\Delta} \frac{\partial^\mu a}{2f_a} \left(\bar{p} \Delta_\mu^+ + \bar{\Delta}_\mu^+ p + \bar{n} \Delta_\mu^0 + \bar{\Delta}_\mu^0 n \right) + \dots, & \leftarrow \text{SN1987A } (\Delta\text{-resonance})
 \end{aligned}$$

$$\begin{aligned}
 C_\gamma &= \frac{E}{N} - 1.92; & \frac{E}{N} &= \frac{c_W + c_B}{c_G} \\
 C_\pi &= -\frac{2}{3} g_A^{-1} C_3; & C_{\pi N} &= \sqrt{2} g_A^{-1} C_3; & C_{N\Delta} &= -\sqrt{3} C_3 \\
 C_{0,3} &= \frac{1}{2}(C_p \pm C_n)
 \end{aligned}$$

Axion couplings to SM fields

- Axion effective Lagrangian in the Georgi-Kaplan-Randall basis:

$$\mathcal{L}_a^{\text{GKR}} = \frac{\partial_\mu a}{f} \left(\sum_{\psi=q_L, \ell_L, \dots} c_\psi \bar{\psi} \gamma^\mu \psi + \sum_{\Phi=H_1, H_2, \dots} c_\Phi \Phi^\dagger i \overset{\leftrightarrow}{D}{}^\mu \Phi \right) + \sum_{A=G,W,B} c_A \frac{g_A^2}{32\pi^2} \frac{a}{f} F_A \tilde{F}_A$$

- U(1)_{PQ} non-linearly realized: $a \rightarrow a + \alpha f$
- Heavy $O(f)$ radial mode ignored
- At $\mu = f$, one can take this basis by performing axion-dependent field redefinition $\psi \rightarrow e^{-i\chi_\psi a/f} \psi$
-> axion couplings correspond to **PQ charge** of fields and **PQ-(gauge)^2 anomaly coefficients**

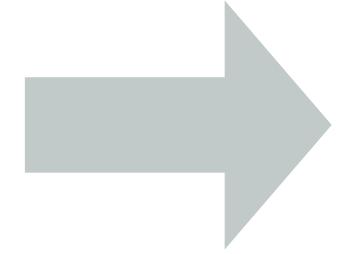
$$c_{\psi, \Phi} = \chi_{\psi, \Phi}$$

$$c_A = \sum_{\psi_R} 2\chi_{\psi_R} \text{Tr } T_A^2(\psi_R) - \sum_{\psi_L} 2\chi_{\psi_L} \text{Tr } T_A^2(\psi_L)$$

Axion couplings to nucleons

$$C_0 = \frac{1}{2}(C_p + C_n) = 0.22(C_u + C_d - 1) - 0.035C_s$$

$$C_3 = \frac{1}{2}(C_p - C_n) = 0.64 \left(C_u - C_d - \frac{m_d - m_u}{m_d + m_u} \right)$$



$$C_3 \simeq -0.43 \sin^2 \beta + 0.64 \left(\frac{1}{3} - \frac{m_d - m_u}{m_d + m_u} \right)$$

$$C_u = \cos^2 \beta / 3$$

$$C_d = \sin^2 \beta / 3$$

$$\simeq 0 \ (\because m_d/m_u \simeq 2)$$

Renormalization group equations (1/2)

■ $\mu_{EW} < \mu < m_{BSM}$

$$(4\pi)^2 \frac{dc'_{q_L}}{d\log\mu} = \frac{1}{2} \{c'_{q_L}, Y_u Y_u^\dagger + Y_d Y_d^\dagger\} - Y_u c'_{u_R} Y_u^\dagger - Y_d c'_{d_R} Y_d^\dagger \\ + \left(8\alpha_s^2 \tilde{c}_G + \frac{9}{2}\alpha_2^2 \tilde{c}_W + \frac{1}{6}\alpha_1^2 \tilde{c}_B\right) \mathbf{1} - \beta_q \gamma_H \mathbf{1},$$

$$(4\pi)^2 \frac{dc'_{u_R}}{d\log\mu} = \{c'_{u_R}, Y_u^\dagger Y_u\} - 2Y_u^\dagger c'_{q_L} Y_u - \left(8\alpha_s^2 \tilde{c}_G + \frac{8}{3}\alpha_1^2 \tilde{c}_B\right) \mathbf{1} \\ - \beta_u \gamma_H \mathbf{1},$$

$$(4\pi)^2 \frac{dc'_{d_R}}{d\log\mu} = \{c'_{d_R}, Y_d^\dagger Y_d\} - 2Y_d^\dagger c'_{q_L} Y_d - \left(8\alpha_s^2 \tilde{c}_G + \frac{2}{3}\alpha_1^2 \tilde{c}_B\right) \mathbf{1} \\ - \beta_d \gamma_H \mathbf{1},$$

$$(4\pi)^2 \frac{dc'_{\ell_L}}{d\log\mu} = \frac{1}{2} \{c'_{\ell_L}, Y_e Y_e^\dagger\} - Y_e c'_{e_R} Y_e^\dagger + \left(\frac{9}{2}\alpha_2^2 \tilde{c}_W + \frac{3}{2}\alpha_1^2 \tilde{c}_B\right) \mathbf{1} \\ - \beta_\ell \gamma_H \mathbf{1},$$

$$(4\pi)^2 \frac{dc'_{e_R}}{d\log\mu} = \{c'_{e_R}, Y_e^\dagger Y_e\} - 2Y_e^\dagger c'_{\ell_L} Y_e - 6\alpha_1^2 \tilde{c}_B \mathbf{1} - \beta_e \gamma_H \mathbf{1},$$

where

$$\gamma_H = -2 \operatorname{Tr} \left(3Y_u^\dagger c'_{q_L} Y_u - 3Y_d^\dagger c'_{q_L} Y_d - Y_e^\dagger c'_{\ell_L} Y_e \right) \\ + 2 \operatorname{Tr} \left(3Y_u c'_{u_R} Y_u^\dagger - 3Y_d c'_{d_R} Y_d^\dagger - Y_e c'_{e_R} Y_e^\dagger \right),$$

$$\tilde{c}_G = c_G - \operatorname{Tr} \left(c'_{u_R} + c'_{d_R} - 2c'_{q_L} \right),$$

$$\tilde{c}_W = c_W + \operatorname{Tr} \left(3c'_{q_L} + c'_{\ell_L} \right),$$

$$\tilde{c}_B = c_B - \operatorname{Tr} \left(\frac{1}{3}(8c'_{u_R} + 2c'_{d_R} - c'_{q_L}) + 2c'_{e_R} - c'_{\ell_L} \right).$$

► Axion-Higgs coupling c_H is removed at any scale by performing an axion-dependent hypercharge rotation:

$$\psi \rightarrow e^{-ic_H \beta_\psi a/f} \psi \quad (\beta_\psi = Y_\psi / Y_H)$$

► This redefines all axion-fermion couplings as $c_\psi \rightarrow c'_\psi = c_\psi - \beta_\psi c_H$, which you see in the RGEs

Renormalization group equations (2/2)

- $\mu_{QCD} < \mu < \mu_{EW}$

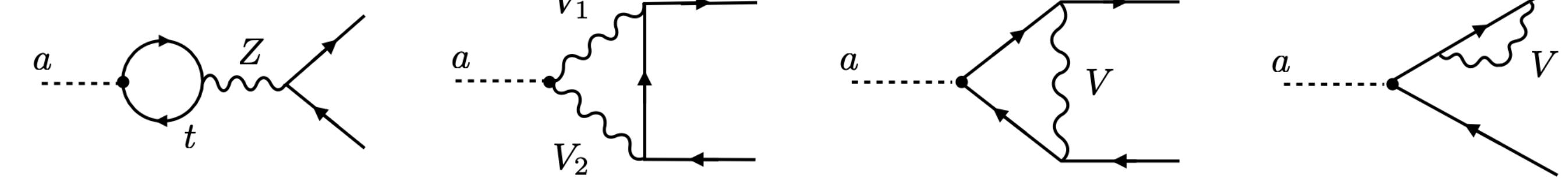
$$(4\pi)^2 \frac{d(C_u^A)_{ii}}{d \log \mu} = -16\alpha_s^2 \tilde{c}_G - \frac{8}{3}\alpha_{\text{em}}^2 \tilde{c}_\gamma,$$

► add **threshold corrections** at the EW scale

$$(4\pi)^2 \frac{d(C_d^A)_{ii}}{d \log \mu} = -16\alpha_s^2 \tilde{c}_G - \frac{2}{3}\alpha_{\text{em}}^2 \tilde{c}_\gamma,$$

$$C_f^A(\mu_{EW}) = c_{f_R}(\mu_{EW}) - c_{f_L}(\mu_{EW}) + \underline{\Delta c_{f_R} - \Delta c_{f_L}}$$

$$(4\pi)^2 \frac{d(C_e^A)_{ii}}{d \log \mu} = -6\alpha_{\text{em}}^2 \tilde{c}_\gamma,$$



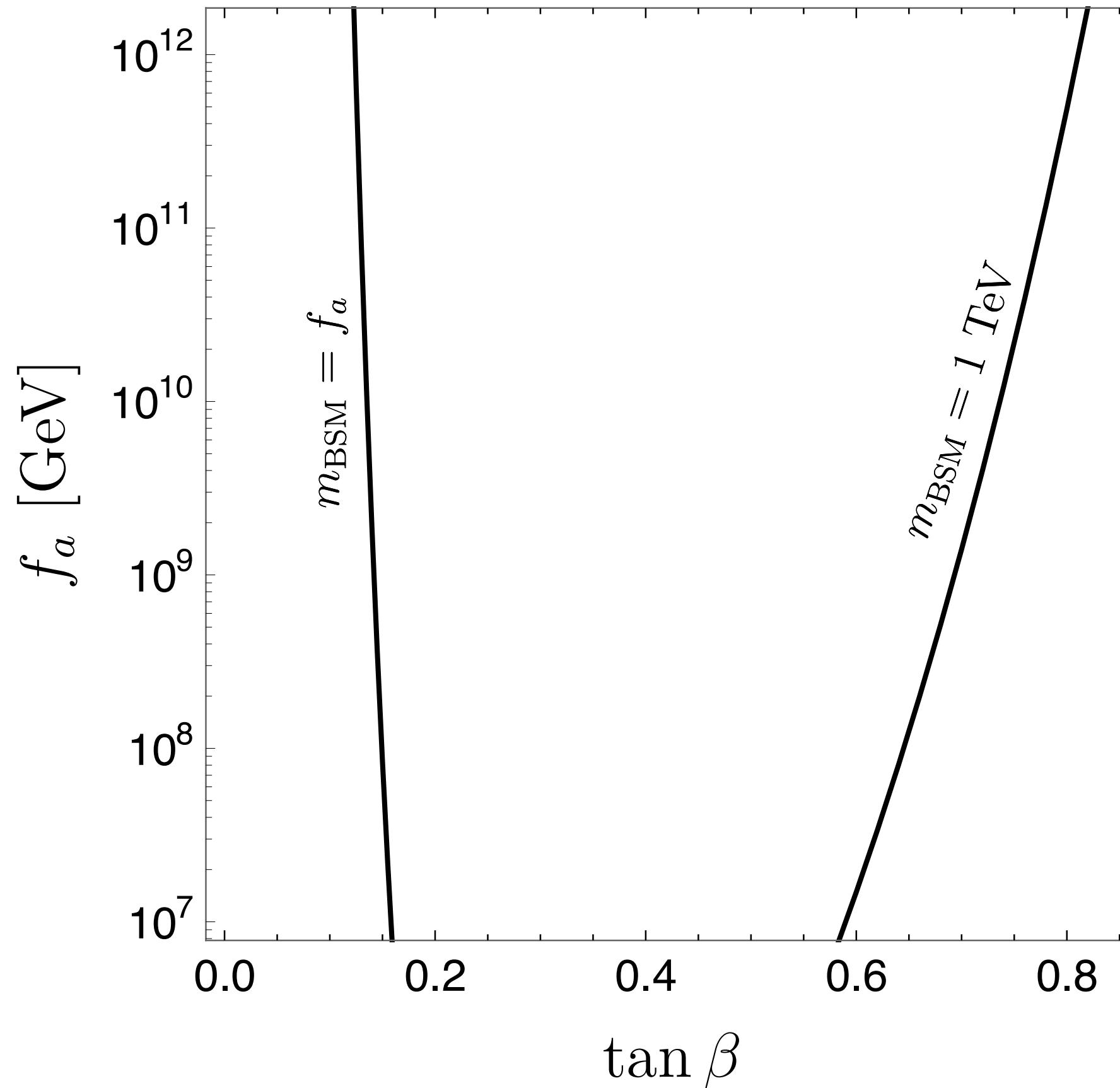
where

$$\tilde{c}_G(\mu) = 1 - \sum_q C_q^A(\mu) \Theta(\mu - m_q),$$

$$\tilde{c}_\gamma(\mu) = \frac{c_\gamma}{c_G} - 2 \sum_f N_c^f Q_f^2 C_f^A(\mu) \Theta(\mu - m_f),$$

[Bauer, Neubert, Renner, Schnabel, Thamm, 2012.12272]

Perturbative Unitarity Bounds

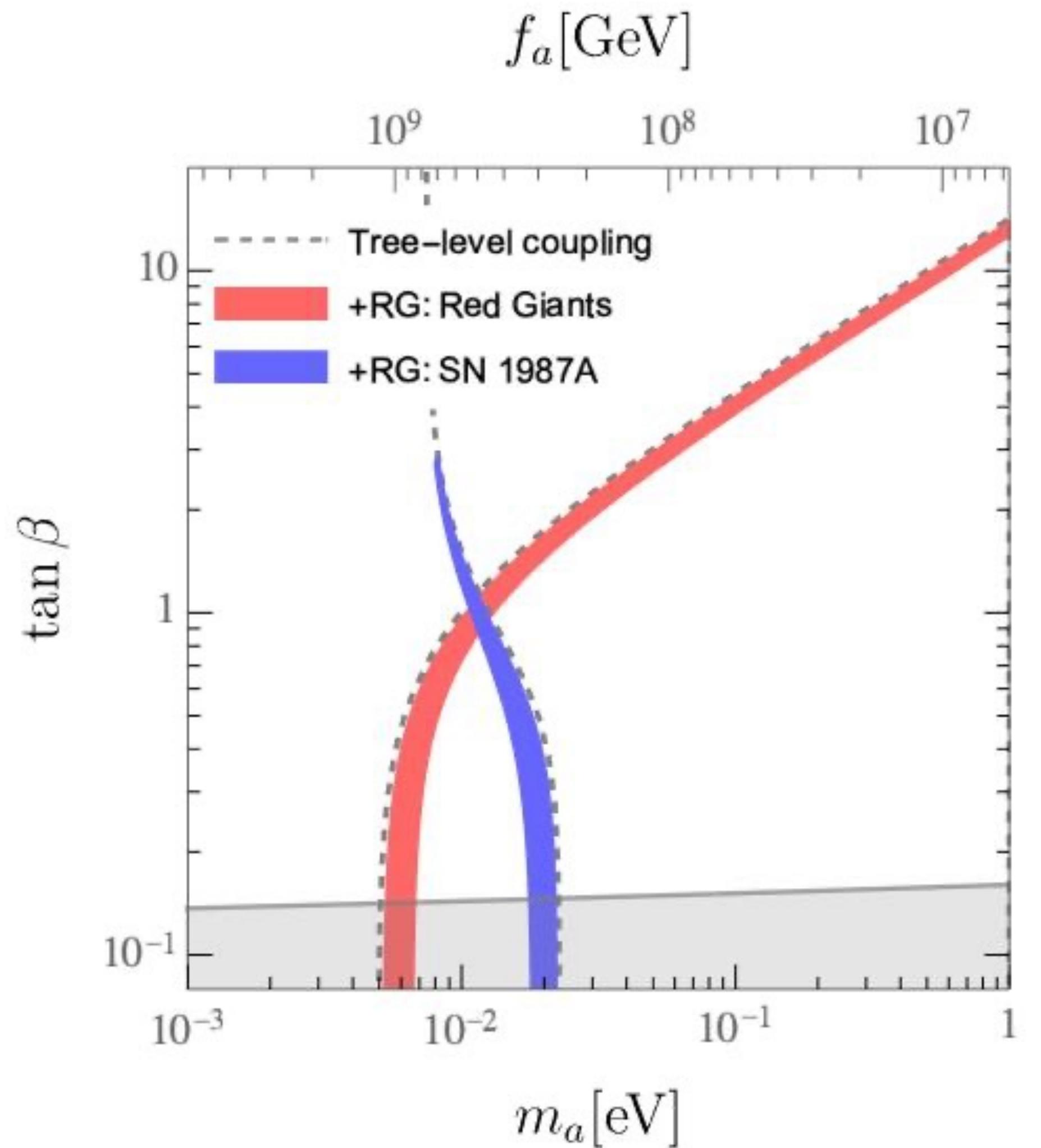


- Small $\tan\beta$ leads low Landau pole scales
 - ▷ impose perturbative unitarity on Higgs-mediated fermion $2 \rightarrow 2$ scattering up to $\mu = f_a$:
 $\Rightarrow Y_{t,b}^{2\text{HDM}} \leq \sqrt{16\pi/3}$ [Di Luzio, Kamenik, Nardecchia (2016)]
(region left to the black line is excluded)
 - ▷ Yukawa couplings are RG-evolved from m_Z to f_a , while appropriately matching with the 2HDM at $\mu = m_{\text{BSM}}$

Astrophysical bounds in DFSZ2 model

■ DFSZ2 model:

- ▶ axion-electron coupling is not suppressed in the small $\tan\beta$ region: $C_e(f_a) = -c_\beta^2/3$
⇒ RG effect **less important** for the **RGB** bound than in the DFSZ1 model
- ▶ Hadronic couplings are unchanged
⇒ RG effect is **unchanged** for the **SN1987A** bound



For RG effects, $m_{\text{BSM}} \in [1\text{TeV}, f_a]$