One-loop expressions for $h \to \bar{l}l\gamma$ in Higgs extensions of the Standard Model

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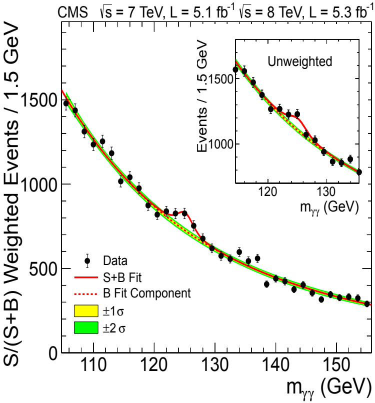
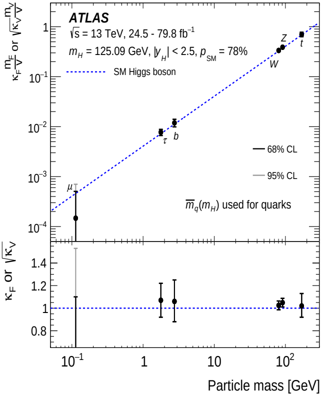
Outline

- Motivations for $h \to \bar{l}l\gamma$
- Higgs extensions of the Standard Models (HESM): Inert Doublet (IDM), Two Higgs Doublet Models, and Triplet-Higgs models (THM)
- One-loop expressions for $h \to \bar{l}l\gamma$ in Higgs extensions of the Standard Model
- Phenomenological results (THDM as example)
- Conclusion

The Higgs boson discovery

The discovery of SM-like Higgs boson at the LHC





[CMS], Phys. Lett. B 716 (2012) 30-61; [ATLAS], Phys. Lett. B 716 (2012) 1-29; [CMS], Phys. Rev. D 101 (2020) 1, 012002.

Questions for the Standard Model

- Experimental data on W-boson mass from CDF II, muon $(g-2)_{\mu,\, e}$ anomalies, the flavor experimental data, $\cdots$?
- Gauge hierarchy problem, neutrino masses?
- Unification of fundamental forces in the nature?
- Origin of matter-antimatter asymmetry?
- Observed dark matter and dark energy in the Universe?
- The nature of Higgs field and its potential is still unknown?

$$\mathcal{V}(\Phi) \quad ?=? \quad \mu^2\Phi^\dagger\Phi + \lambda(\Phi^\dagger\Phi)^2$$

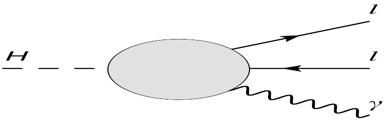
- More $\cdots$?

Beyond Standard Models

- Extended Higgs sectors: Higgs singlet extension, Two Higgs Doublet models, Higgs triplet extension, $\cdots$
- Extended gauge sectors: the left-right models (LR) constructed from the $SU(2)_L \times SU(2)_R \times U(1)_Y$, the 3-3-1 models ($SU(3)_L \times U(1)_X$), the 3-4-1 models ($SU(4)_L \times U(1)_X$), GUT, $\ldots$
- Supersymmetric theory: MSSM, NMSSM, $\cdots$
- Gauged Two Higgs Doublet Model (G2HDM, talked by Tzu-Chiang Yuan)
- More $\cdots$?

$\Rightarrow$ New particles: Extra charged (neutral) gauge bosons, charged (neutral) Higgs bosons and fermions, $\cdots$

- $h \to l\bar{l}\gamma$ have been greatly paid attention at the LHC $\Rightarrow$ test the SM at the high energy regions: *[CMS], JHEP 11 (2018) 152; Phys. Lett. B 753 (2016) 341; JHEP 09 (2018) 148; [ATLAS], Phys. Lett. B 819 (2021) 136412, etc*
- New particles (from many of BSMs) may propagate in the loop diagrams of the decay processes $\Rightarrow$ a useful tool for constraining new physic parameters.



- New neutral Higgs bosons in BSMs may be mixed with the SM-like one $\Rightarrow$ observed directly by measuring of the decay rates of $h \to l\bar{l}\gamma$, ...

K. H. Phan et al, Eur. Phys. J. C 82 (2022) 3, 277; PTEP 2023 (2023) 8, 083B06

- The renormalizable and gauge invariant Higgs potential is

$$\begin{aligned}\mathcal{V}(\Phi_1,\Phi_2) &= m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 - \left[m_{12}^2\Phi_1^\dagger\Phi_2 + \text{h.c.}\right] \\ &+\frac{\lambda_1}{2}(\Phi_1^\dagger\Phi_1)^2 + \frac{\lambda_2}{2}(\Phi_2^\dagger\Phi_2)^2 \\ &+\lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) \\ &+\frac{1}{2}[\lambda_5\,(\Phi_1^\dagger\Phi_2)^2 + \text{ h.c.}].\end{aligned}$$

- For the EWSB, two scalar doublets can be written as

$$\Phi_1 = \begin{bmatrix}\phi_1^+ \\ (v_1+\rho_1+i\eta_1)/\sqrt{2}\end{bmatrix} \quad \text{and} \quad \Phi_2 = \begin{bmatrix}\phi_2^+ \\ (v_2+\rho_2+i\eta_2)/\sqrt{2}\end{bmatrix},$$

where $v = \sqrt{v_1^2+v_2^2} = 246$ GeV.

G. C. Branco et al, Phys. Rept. 516 (2012) 1-102

- The mass and flavor base of all Higgs bosons relate to each other:

$$\begin{pmatrix} \phi_1^{\pm} \\ \phi_2^{\pm} \end{pmatrix} = \begin{pmatrix} c_\beta & -s_\beta \\ s_\beta & c_\beta \end{pmatrix} \begin{pmatrix} G^{\pm} \\ H^{\pm} \end{pmatrix},$$

$$\begin{pmatrix} \rho_1 \\ \rho_2 \end{pmatrix} = \begin{pmatrix} c_\alpha & -s_\alpha \\ s_\alpha & c_\alpha \end{pmatrix} \begin{pmatrix} H \\ h \end{pmatrix},$$

and

$$\begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} = \begin{pmatrix} c_\beta & -s_\beta \\ s_\beta & c_\beta \end{pmatrix} \begin{pmatrix} G^0 \\ A^0 \end{pmatrix}.$$

Where $t_\beta \equiv \tan\beta = v_2/v_1$.

$\Rightarrow$ New scalar particles: Neutral Higgses H, A^0, charged Higgs $H^{\pm}$.

G. C. Branco et al, Phys. Rept. 516 (2012) 1-102

Vertices	Notations	Couplings
$hW_\mu W_\nu$	g_{hWW}^{THDM}	$-i\dfrac{2M_W^2}{v}\, s_{\beta-\alpha}\, g_{\mu\nu}$
$hZ_\mu Z_\nu$	g_{hZZ}^{THDM}	$-i\dfrac{2M_Z^2}{v}\, s_{\beta-\alpha}\, g_{\mu\nu}$
$hH^\pm H^\mp$	$g_{hH^\pm H^\mp}^{\text{THDM}}$	$-\dfrac{i}{v}\Big[(2\mu^2 - 2M_{H^\pm}^2 - m_h^2)s_{\beta-\alpha}$ $+2\cot(2\beta)(\mu^2 - m_h^2)c_{\beta-\alpha}\Big]$
$Z_\mu H^\pm(p^+)H^\mp(p^-)$	$g_{ZH^\pm H^\mp}^{\text{THDM}}$	$\dfrac{M_Z}{v}\, c_{2W}(p^+ - p^-)_\mu$
$A_\mu H^\pm(p^+)H^\mp(p^-)$	$g_{AH^\pm H^\mp}^{\text{THDM}}$	$\dfrac{M_Z}{v}\, s_{2W}(p^+ - p^-)_\mu$

All the couplings involving the decay processes $h \to l\bar{l}\gamma$ in the THDM.

The Yukawa Lagrangian:

$$\mathcal{L}_{\text{Yukawa}} = -\sum_{f=u,d,l}\left(g_{hff}^{\text{THDM}}\bar{f}fh + g_{Hff}^{\text{THDM}}\bar{f}fH - ig_{A^0ff}^{\text{THDM}}\bar{f}\gamma_5 fA^0\right) + \cdots,$$

Type	g_{huu}^{THDM}	g_{hdd}^{THDM}	g_{hll}^{THDM}
I	$\frac{m_u}{\sqrt{2}v}\frac{c_\alpha}{s_\beta}$	$\frac{m_d}{\sqrt{2}v}\frac{c_\alpha}{s_\beta}$	$\frac{m_l}{\sqrt{2}v}\frac{c_\alpha}{s_\beta}$
II	$\frac{m_u}{\sqrt{2}v}\frac{c_\alpha}{s_\beta}$	$-\frac{m_d}{\sqrt{2}v}\frac{s_\alpha}{c_\beta}$	$-\frac{m_l}{\sqrt{2}v}\frac{s_\alpha}{c_\beta}$
X	$\frac{m_u}{\sqrt{2}v}\frac{c_\alpha}{s_\beta}$	$\frac{m_d}{\sqrt{2}v}\frac{c_\alpha}{s_\beta}$	$-\frac{m_l}{\sqrt{2}v}\frac{s_\alpha}{c_\beta}$
Y	$\frac{m_u}{\sqrt{2}v}\frac{c_\alpha}{s_\beta}$	$-\frac{m_d}{\sqrt{2}v}\frac{s_\alpha}{c_\beta}$	$\frac{m_l}{\sqrt{2}v}\frac{c_\alpha}{s_\beta}$

The Yukawa couplings in THDMs with type I,II, X, and Y respectively.

- New scalar particles: Neutral Higgses H, A^0, charged Higgs $H^{\pm}$
- The set $\mathcal{P}_{\rm THDM}$ of scanning parameters used for our numerical investigation is chosen as follows:

$$\mathcal{P}_{\rm THDM} = \{m_h^2, M_H^2, M_{A^0}^2, M_{H^\pm}^2, m_{12}^2, t_\beta, s_{\beta-\alpha}\}$$

- SM-like Higgs limit: $s_{\beta-\alpha} \to 1$.

Method

- Implementing the HESM into `FeynArt`,
- Using `FormCalc/FeynCalc` for generating one-loop amplitude,
- Colecting one-loop form factors which are expressed in term of scalar one-loop Feynman integrals,
- Using `LoopTools` for numerical evaluations for scalar one-loop Feynman integrals $\rightarrow$ decay rates.

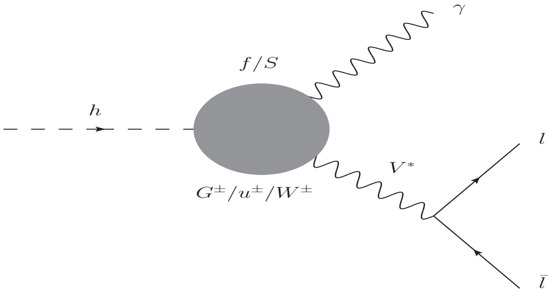


Figure: V^*-pole contributions: $V^* \equiv \gamma^*, Z^*$ and H, A^0 (can be inorged)

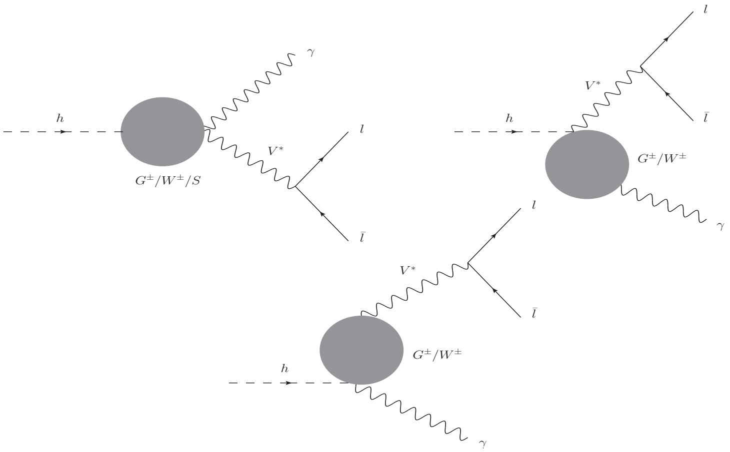


Figure: V^*-pole contributions: $V^* \equiv \gamma^*, Z^*$ and H, A^0 (can be inorged)

Decay $h \to l\bar{l}\gamma$: V^*-pole contributions

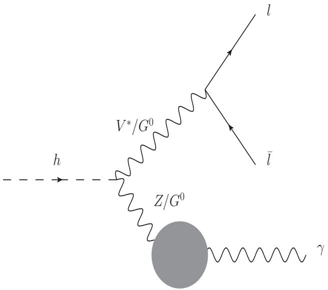


Figure: V^*-pole contributions: $V^* \equiv \gamma^*, Z^*$ and H, A^0 (can be inorged)

Decay $h \to l\bar{l}\gamma$: non-pole V-contributions

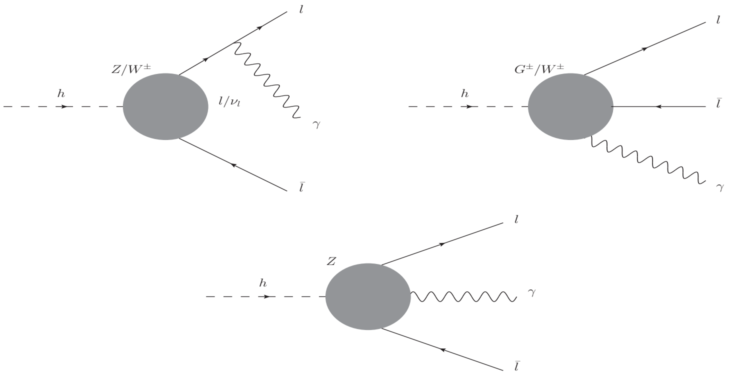
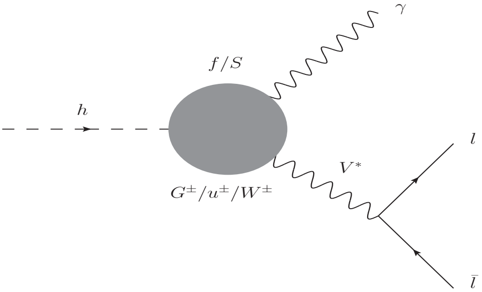


Figure: Non-pole V-contributions



One-loop amplitude $h \to l(q_1)\bar{l}(q_2)\gamma(q_3)$ can be decomposed as:

$$\mathcal{A}_{V^*\text{-pole}} = \sum_{V^*=\{\gamma^*,Z^*\}} F_{\gamma V^*}^{V^*\text{-pole}} \Big[q^\mu q_3^\nu - (q \cdot q_3) g^{\mu\nu}\Big] \times \times \Big[\bar{u}(q_1)\gamma_\nu \Big(\sum_{j=\{L,R\}} g^j_{V^* l\bar{l}} P_j \Big) v(q_2)\Big] \varepsilon^*_\mu(q_3),$$

where $P_{L,R} = (1 \mp \gamma_5)/2$ and $q = q_1 + q_2$, $\varepsilon^*_\mu(q_3)$ is photon polarization.

One-loop form factors are colected as:

$$F_{\gamma V^*}^{V^*\text{-pole}} = \frac{e^2 g}{(4\pi)^2 M_W} \frac{1}{q_{12} - M_V^2 + iM_V\Gamma_V} \Big[F_{\gamma V^*}^{(f)} + F_{\gamma V^*}^{(W)} + F_{\gamma V^*}^{(S)} \Big],$$

where

$$F_{\gamma V^*}^{(f)} = \sum_f \frac{N_f^C m_f \; v}{(2\pi\alpha)(m_h^2 - q_{12})^2} \, g_{hf\bar{f}}^{\rm NP} g_{\gamma f\bar{f}}^{L} (g_{V^* f\bar{f}}^{L} + g_{V^* f\bar{f}}^{R}) \times$$
$$\times \Big\{ 2q_{12} \Big[B_0(m_h^2, m_f^2, m_f^2) - B_0(q_{12}, m_f^2, m_f^2) \Big]$$
$$+ (m_h^2 - q_{12}) \Big[2 + (q_{12} - m_h^2 + 4m_f^2) C_0(0, q_{12}, m_h^2, m_f^2, m_f^2, m_f^2) \Big] \Big\}.$$

One-loop form factors are colected as:

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where

$$\begin{aligned} F_{\gamma V^*}^{(W)} &= \frac{g_{hWW}^{\rm NP}\, v}{2M_W^2\,(m_h^2 - q_{12})^2} \times \\ &\times\Big\{2M_W^2(m_h^2 - q_{12})\Big[(\delta_{V^*G} - 5\delta_{V^*W})(m_h^2 - 2M_W^2) \\ &-2q_{12}(\delta_{V^*G} - 3\delta_{V^*W})\Big] C_0(0, q_{12}, m_h^2, M_W^2, M_W^2, M_W^2) \\ &\qquad - \Big[\delta_{V^*G}(m_h^2 + 2M_W^2) - \delta_{V^*W}(m_h^2 + 10M_W^2)\Big] \times \\ &\times\Big[q_{12}\Big(B_0(m_h^2, M_W^2, M_W^2) - B_0(q_{12}, M_W^2, M_W^2)\Big) + (m_h^2 - q_{12})\Big]\Big\}. \end{aligned}$$

One-loop form factors are colected as:

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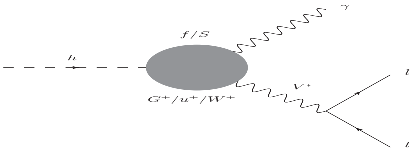
where

$$\begin{aligned} F_{\gamma V^*}^{(S)} &= \frac{4M_W}{g(m_h^2 - q_{12})^2} \sum_S Q_S^2 \, g_{hSS}^{(NP)} \, g_{ASS}^{(NP)} \, g_{VSS}^{(NP)} \times \\ &\times \Big\{ q_{12} \Big[B_0(m_h^2, M_S^2, M_S^2) - B_0(q_{12}, M_S^2, M_S^2) \Big] + \\ &+ (m_h^2 - q_{12}) \Big[2M_S^2 \, C_0(0, q_{12}, m_h^2, M_S^2, M_S^2, M_S^2) + 1 \Big] \Big\}, \end{aligned}$$

where Q_S is the electric charge of S.

Reduction to the decay $h \to \gamma\gamma$



$M_{V^*}, \Gamma_{V^*} \to 0$ and $q_{12} \to 0$

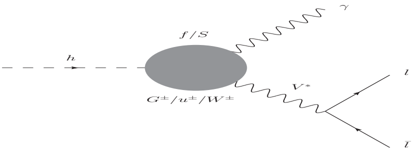
$$F^{(f,W)}_{\gamma\gamma^*} = \sum_f N_f^C Q_f^2 F^{(1/2)}_{\gamma\gamma^*}(\tau_f, \lambda_f) + F^{(1)}_{\gamma\gamma^*}(\tau_W, \lambda_W).$$

$$F^{(1)}_{\gamma\gamma^*}(\tau_W, \lambda_W) = 16\, I_2(\tau_W, \lambda_W) - \big(4/\tau_W + 6\big) I_1(\tau_W, \lambda_W),$$

$$F^{(1/2)}_{\gamma\gamma^*}(\tau_f, \lambda_f) = 4 I_1(\tau_f, \lambda_f) - 4 I_2(\tau_f, \lambda_f),$$

$$F^{(S)}_{\gamma\gamma}(\tau_S) = \frac{M_W}{g} \sum_S Q_S^2 \frac{g^{(NP)}_{hSS}\ [g^{(NP)}_{ASS}]^2}{M_S^2} \tau_S \Big[1 - \tau_S f(\tau_S)\Big].$$

Agreement with: *W. J. Marciano et al, Phys. Rev. D 85 (2012) 013002; R. Benbrik eta al, Nucl. Phys. B 990 (2023) 116154, etc*



$M_{V^*} \to M_Z, \Gamma_{V^*} \to \Gamma_Z$ and $q_{12} \to M_Z^2$ $[v_f = 2I_3^f - 4Q_f s_W^2]$

$$F_{\gamma Z^*}^{(f,W)} = \sum_f N_f^C \frac{Q_f v_f}{s_W c_W} F_{\gamma Z^*}^{(1/2)}(\tau_f, \lambda_f) + F_{\gamma Z^*}^{(1)}(\tau_W, \lambda_W),$$

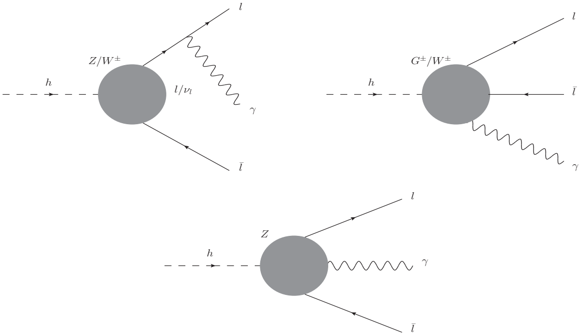
$$F_{\gamma Z^*}^{(1)}(\tau_W, \lambda_W) = \frac{c_W}{s_W}\Big\{\Big[\Big(\frac{2}{\tau_W}+1\Big)\frac{s_W^2}{c_W^2} - \Big(\frac{2}{\tau_W}+5\Big)\Big] I_1(\tau_W, \lambda_W) + 4\Big(3 - \frac{s_W^2}{c_W^2}\Big) I_2(\tau_W, \lambda_W)\Big\},$$

$$F_{\gamma Z^*}^{(1/2)}(\tau_f, \lambda_f) = I_1(\tau_f, \lambda_f) - I_2(\tau_f, \lambda_f)$$

$$F_{\gamma Z}^{(S)}(\tau_S, \rho_S) = -\frac{M_W}{2g}\sum_S Q_S^2 \frac{g_{hSS}^{(NP)}\, g_{ASS}^{(NP)}\, g_{ZSS}^{(NP)}}{M_S^2}\tau_S I_1(\tau_S, \rho_S).$$

Agreement with: *A. Djouadi et al, Eur. Phys. J. C 1 (1998) 163-175; R. Benbrik et al, Nucl. Phys. B 990 (2023) 116154, etc*

Non-pole V-contributions



One-loop amplitude is given

$$\mathcal{A}_{\text{Non-pole, }V} = \sum_{V=\{Z,W,S\}} \sum_{k=1}^{2} \Big\{ [q_3^{\mu} q_k^{\nu} - g^{\mu\nu} q_3 \cdot q_k] \bar{u}(q_1) \times \times \Big(\sum_{j=L,R} F_{k,j}^{\text{Non-pole, V}} \gamma_{\mu} P_j \Big) v(q_2) \Big\} \varepsilon_{\nu}^{*}(q_3).$$

Non-pole V-contributions

One-loop form factors for non-pole Z-contributions:

$$F_{1,L}^{\text{Non-pole, Z}} = \frac{\alpha\ v}{\pi} \frac{g_{hZZ}^{\text{NP}}\ (g_{Zl\bar{l}}^{L})^2}{M_Z\ s_{2W}} \Big[D_2 + D_{12} + D_{23}\Big](\cdots)$$

$$F_{1,R}^{\text{Non-pole, Z}} = F_{1,L}^{\text{Non-pole, Z}}\Big|_{g_{Zl\bar{l}}^{L} \to g_{Zl\bar{l}}^{R}},$$

$$F_{2,L}^{\text{Non-pole, Z}} = F_{1,L}^{\text{Non-pole, Z}}\Big|_{q_{13} \leftrightarrow q_{23}},$$

$$F_{2,R}^{\text{Non-pole, Z}} = F_{2,L}^{\text{Non-pole, Z}}\Big|_{g_{Zl\bar{l}}^{L} \to g_{Zl\bar{l}}^{R}},$$

where $q_{13} = (q_1 + q_3)^2, q_{23} = (q_2 + q_3)^2$.

$$(\cdots) = (0, q_{13}, m_h^2, q_{23}, 0, 0, m_l^2, m_l^2, M_Z^2, M_Z^2)$$

One-loop form factors for non-pole W-contributions:

$$F_{1,L}^{\text{Non-pole, W}} = -\frac{\alpha^2\, v\, g_{hWW}^{\text{NP}}}{2\, M_W s_W^3} \times$$
$$\times \Big\{ \Big[D_1 + D_{13} \Big] (0, q_{12}, 0, q_{23}, 0, m_h^2, 0, M_W^2, M_W^2, M_W^2)$$
$$+ \Big[D_2 - D_{23} - D_{33} \Big] (0, q_{12}, 0, q_{13}, 0, m_h^2, 0, M_W^2, M_W^2, M_W^2) \Big\},$$

$$F_{1,R}^{\text{Non-pole, W}} = F_{2,R}^{\text{Non-pole, W}} = 0,$$
$$F_{2,L}^{\text{Non-pole, W}} = F_{1,L}^{\text{Non-pole, W}} \Big|_{q_{13} \leftrightarrow q_{23}}.$$

Phenomenological results

- The decay rate is given:

$$\frac{d\Gamma}{dq_{12}q_{13}} = \frac{q_{12}}{512\pi^3 m_h^3}\Big[q_{13}^2(|F_{1,R}|^2 + |F_{2,R}|^2) + q_{23}^2(|F_{1,L}|^2 + |F_{2,L}|^2)\Big].$$

Where $\left(m_{ll}^{\mathrm{cut}}\right)^2 \leq q_{12} \leq m_h^2$ and $0 \leq q_{13} \leq m_h^2 - q_{12}$.

- The enhancement factor of the decay rates as follows:

$$R_{\mathrm{NP}}(\mathcal{P}_{\mathrm{NP}}) = \frac{\Gamma^{\mathrm{NP}}_{h\to\bar{l}l\gamma}(\mathcal{P}_{\mathrm{NP}})}{\Gamma^{\mathrm{SM}}_{h\to\bar{l}l\gamma}}.$$

$\mathcal{P}_{\mathrm{NP}}$ is parameter space of new physics.

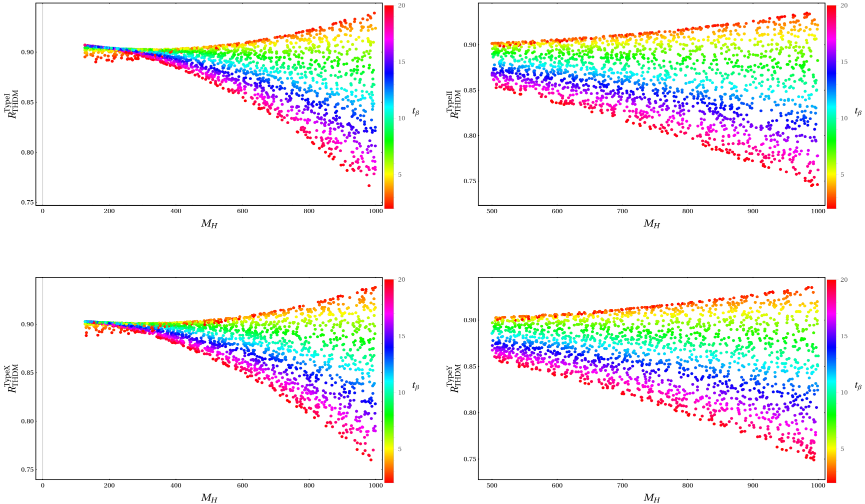
$$\mathcal{P}_{\mathrm{THDM}} = \{m_h^2, M_H^2, M_{A^0}^2, M_{H^\pm}^2, m_{12}^2, t_\beta, s_{\beta-\alpha}\}.$$

Phenomenological results

- **Theoretical constraints:** tree-level unitarity, vacuum stability, perturbativity regime. *S. Kanemura et al, Phys.Lett.B 471 (1999) 182-190; Phys.Lett.B 704 (2011) 303-307; etc*
- **The experimental constraints:** the EWPT of THDM, Higgs coupling measurements at the LHC, flavor experimental data, etc. *J. Haller et al, Eur.Phys.J.C 78 (2018) 8, 675; W. Xie et al, Phys.Rev.D 103 (2021) 9, 095030; etc*
- **In the type-I and X:** we take 126 GeV $\leq M_H \leq$ 1000 GeV, 60 GeV $\leq M_{A^0} \leq$ 1000 GeV and 80 GeV $\leq M_{H^\pm} \leq$ 1000 GeV.
- **In the Type-II and Y:** we scan logically the physical parameters as 500 GeV $\leq M_H \leq$ 1000 GeV, 500 GeV $\leq M_{A^0} \leq$ 1000 GeV and 580 GeV $\leq M_{H^\pm} \leq$ 1500 GeV.
- **In both types:** one takes $2 \leq t_\beta \leq 20$, $0.95 \leq s_{\beta-\alpha} \leq 1$ and $m_{12}^2 = M_H^2 s_\beta c_\beta$.

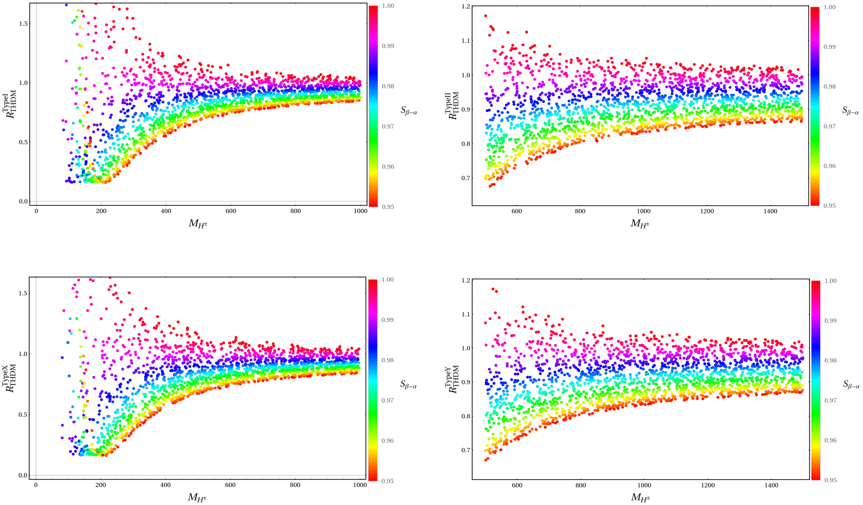
Phenomenological results

The enhancement factors as functions of t_β and M_H for four types of THDM, fixing $s_{\beta-\alpha} = 0.95$, and $M_{H^\pm} = 800$ GeV.



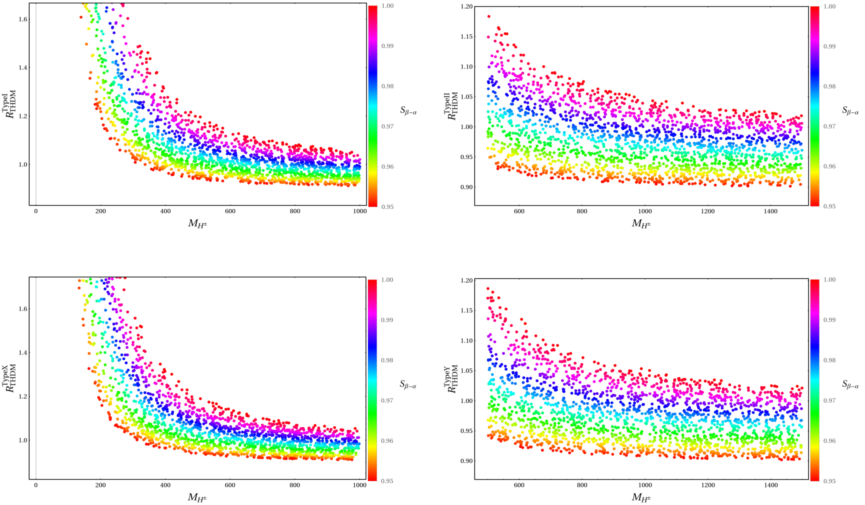
Phenomenological results

The enhancement factors as functions of $s_{\beta-\alpha}$ and $M_H^{\pm}$ for four types of THDM, fixing $t_\beta = 15$, and $M_H = 1000$ GeV.



Phenomenological results

The enhancement factors as functions of $s_{\beta-\alpha}$ and $M_H^{\pm}$ for four types of THDM, fixing $t_\beta = 5$, and $M_H = 1000$ GeV.



Lepton FB asymmetry in $h \to \bar{l}l\gamma$ is defined as:

$$\mathcal{A}_{\text{FB}}(\mathcal{P}_{\text{NP}}) =$$

$$= \int\limits_{4m_l^2}^{m_h/2} \left[\int_0^1 \frac{d\Gamma}{dE_\gamma d\cos\theta_l} d\cos\theta_l dE_\gamma - \int_{-1}^0 \frac{d\Gamma}{dE_\gamma d\cos\theta_l} d\cos\theta_l dE_\gamma \right] \times$$

$$\times \left\{ \int_{4m_l^2}^{m_h/2} \left[\int_0^1 \frac{d\Gamma}{dE_\gamma d\cos\theta_l} d\cos\theta_l dE_\gamma + \int_{-1}^0 \frac{d\Gamma}{dE_\gamma d\cos\theta_l} d\cos\theta_l dE_\gamma \right] \right\}^{-1}$$

Phenomenological results

$(t_\beta, s_{\beta-\alpha}, M_{H^\pm}, M_H)$	$(R^{(e)}_{\rm THDM}\ ,\ \mathcal{A}^{(e)}_{\rm FB})$	$(R^{(\mu)}_{\rm THDM}\ ,\ \mathcal{A}^{(\mu)}_{\rm FB})$
$(5, 0.95, 200, 400)$	$(0.9516\ ,\ 0.3548)$	$(0.8754\ ,\ 0.3221)$
	$(0.9424\ ,\ 0.3543)$	$(0.8135\ ,\ 0.3415)$
$(5, 0.99, 400, 200)$	$(0.9946\ ,\ 0.3487)$	$(0.9119\ ,\ 0.3176)$
	$(0.9925\ ,\ 0.3499)$	$(0.8276\ ,\ 0.3483)$
$(10, 0.95, 400, 600)$	$(0.8439\ ,\ 0.3338)$	$(0.7809\ ,\ 0.3017)$
	$(0.8423\ ,\ 0.3431)$	$(1.1276\ ,\ 0.2448)$
$(10, 0.99, 600, 400)$	$(0.9968\ ,\ 0.3493)$	$(0.9112\ ,\ 0.3190)$
	$(0.9972\ ,\ 0.3512)$	$(0.8573\ ,\ 0.3478)$
$(15, 0.95, 800, 600)$	$(0.8721\ ,\ 0.3399)$	$(0.8025\ ,\ 0.3088)$
	$(0.8875\ ,\ 0.3469)$	$(1.9386\ ,\ 0.1499)$
$(15, 0.99, 600, 800)$	$(0.9851\ ,\ 0.3473)$	$(0.9007\ ,\ 0.3172)$
	$(0.9879\ ,\ 0.3558)$	$(0.9299\ ,\ 0.3154)$
$(20, 0.95, 500, 1000)$	$(0.5310\ ,\ 0.2421)$	$(0.5197\ ,\ 0.2097)$
	$(0.5469\ ,\ 0.2466)$	$(2.8866\ ,\ 0.0687)$
$(20, 0.99, 1000, 500)$	$(0.9857\ ,\ 0.3474)$	$(0.9008\ ,\ 0.3174)$
	$(0.9935\ ,\ 0.3557)$	$(1.1117\ ,\ 0.2737)$

$\mathcal{A}^{(e)}_{\bf FB}[SM] = 0.3673$; $\mathcal{A}^{(\mu)}_{\bf FB}[SM] = 0.2843$

Conclusions

- Overview of Higgs extensions of the Standard Model (THDM as example)
- One-loop contributing to the decay processes $h \to \bar{l}l\gamma$ in HESM have been calculated.
- Phenomenological results of the decay processes for HESM have studied in detail.
- Extended this work for other BSMs: G2HDM, LR model $SU(2)_L \times SU(2)_R \times U(1)_Y$, 3-3-1 model ($SU(3)_L \times U(1)_X$), 3-4-1 model $SU(4)_L \times U(1)_X$, $\cdots$

See our new paper *K. H. Phan, et al, arXiv:2311.02998* for more detail $\cdots$

Thank you very much for attention!

$$I_1(\tau,\lambda) = \frac{\tau\lambda}{2(\tau-\lambda)} + \frac{\tau^2\lambda^2}{2(\tau-\lambda)^2}\Big[f(\tau) - f(\lambda)\Big] + \frac{\tau^2\lambda}{(\tau-\lambda)^2}\Big[g(\tau) - g(\lambda)\Big],$$

$$I_2(\tau,\lambda) = -\frac{\tau\lambda}{2(\tau-\lambda)}\Big[f(\tau) - f(\lambda)\Big].$$

Two complex functions $f,\ g$ can be expressed as follows:

$$f(\tau) = \begin{cases} \arcsin^2\sqrt{\tau} & \text{for} \quad \tau \leq 1, \\ -\frac{1}{4}\left[\log\frac{1+\sqrt{1-\tau^{-1}}}{1-\sqrt{1-\tau^{-1}}} - i\pi\right]^2 & \text{for} \quad \tau > 1, \end{cases}$$

$$g(\tau) = \begin{cases} \sqrt{\tau^{-1}-1}\,\arcsin\sqrt{\tau} & \text{for} \quad \tau \geq 1, \\ \frac{\sqrt{1-\tau^{-1}}}{2}\left[\log\frac{1+\sqrt{1-\tau^{-1}}}{1-\sqrt{1-\tau^{-1}}} - i\pi\right] & \text{for} \quad \tau < 1. \end{cases}$$

The mentioned parameters are taken for Z^*-pole and γ^*-pole as follows:

$$\delta_{V^*W}\,;\,\delta_{V^*G} = \begin{cases} -1\,;\,1 & \text{if } V^* \equiv \gamma^*, \\ \dfrac{c_W}{s_W}\,;\,\dfrac{s_W}{c_W} & \text{if } V^* \equiv Z^*. \end{cases}$$