# Scattering Amplitudes and Goldstone Boson Equivalence Theorems In Extra Dimensions

Dipan Sengupta

With

R. Sekhar Chivukula (UCSD), Dennis Foren (UCSD), Kirtimaan A. Mohan (MSU) , Elizabeth H. Simmons(UCSD), X. Wang (UCSD) and J. A. Gill (Adelaide)

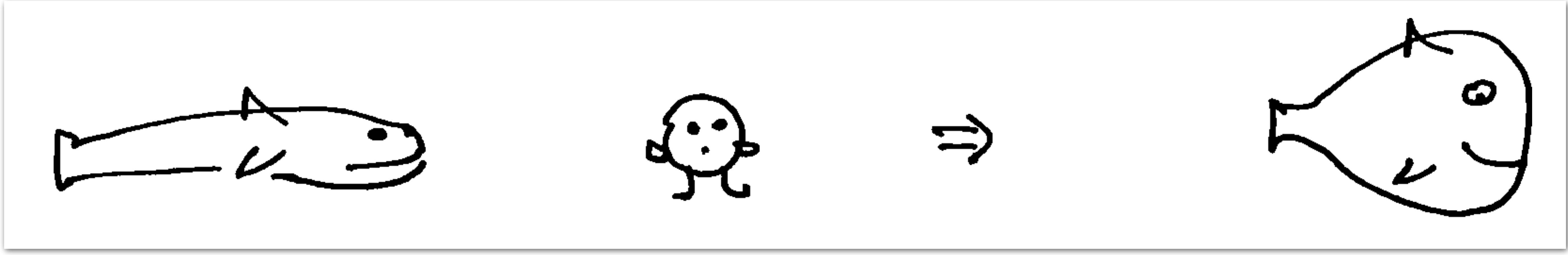
Phys.Rev.D 100 (2019) 11, 115033 Phys.Rev.D 101 (2020) 7, 075013 Phys.Rev.D 101 (2020) 5, 055013 Phys. Rev. D 103 (2022), 095024

Phys. Rev. D 107,(2023) 03505, PRD Letters L051702, arXiv:2311.00770, arXiv:2311.xxxxx

Adelaide -> UNSW, Sydney

ARC CENTRE OF EXCELLENCE FOR DARK MATTER PARTICLE PHYSICS

THE UNIVERSITY of ADELAIDE



# How calculations go wrong in DM

Brane Localized Scalar Dark Matter annihilating via a KK/massive spin-2 portal

# How calculations go wrong in DM

Brane Localized Scalar Dark Matter annihilating via a KK/massive spin-2 portal

**Gravity-mediated Scalar Dark Matter in Warped Extra-Dimensions**

**Miguel G. Folgado,[a] Andrea Donini,[a] Nuria Rius[a]**

Blatantly wrong-60 Citations

PRL 116, 101302 (2016) PHYSICAL REVIEW LETTERS week ending 11 MARCH 2016

**Planckian Interacting Massive Particles as Dark Matter**

Mathias Garny,[1,*] McCullen Sandora,[2,†] and Martin S. Sloth[2,‡]
[1]*CERN Theory Division, CH-1211 Geneva 23, Switzerland*
[2]*CP[3]-Origins, Center for Cosmology and Particle Physics Phenomenology, University of Southern Denmark, Campusvej 55, 5230 Odense M, Denmark*
(Received 20 November 2015; published 10 March 2016)

Partially wrong - 150 Citations

PHYSICAL REVIEW LETTERS 128, 081806 (2022)

**Massive Gravitons as Feebly Interacting Dark Matter Candidates**

Haiying Cai,[1,*] Giacomo Cacciapaglia,[2,3,†] and Seung J. Lee[1,‡]
[1]*Department of Physics, Korea University, Seoul 136-713, Korea*
[2]*University of Lyon, Université Claude Bernard Lyon 1, F-69001 Lyon, France*
[3]*Institut de Physique des 2 Infinis de Lyon (IP2I), UMR5822, CNRS/IN2P3, F-69622 Villeurbanne Cedex, France*

Blatantly wrong-10 citations

**Spin-2 portal dark matter**

Nicolás Bernal,[1,*] Maíra Dutra,[2,†] Yann Mambrini,[2,‡] Keith Olive,[3,4,§] Marco Peloso,[3,‖] and Mathias Pierre[2,¶]
[1]*Centro de Investigaciones, Universidad Antonio Nariño Carrera 3 Este # 47A-15, Bogotá, Colombia*
[2]*Laboratoire de Physique Théorique (UMR8627), CNRS, University of Paris-Sud, Université Paris-Saclay, 91405 Orsay, France*

Partially wrong-100 Citations

**Dark Matter Direct Detection from new interactions in models with spin-two mediators**

**A. Carrillo-Monteverde[1,♯], Yoo-Jin Kang[2,3,*], Hyun Min Lee[2,†], Myeonghun Park[4,♣], and Veronica Sanz[1,♠]**

[1]*Department of Physics and Astronomy, University of Sussex, Brighton BN1 9QH, UK.*
[2]*Department of Physics, Chung-Ang University, Seoul 06974, Korea.*
[3]*Center for Theoretical Physics of the Universe, Institute for Basic Science, Daejeon, 34051, Korea.*
[4]*Institute of Convergence Fundamental Studies and School of Liberal Arts, Seoul National University of Science and Technology, Seoul 01811, Korea.*

Blatantly wrong-75 Citations

# How calculations go wrong in DM

Brane Localized Scalar Dark Matter annihilating via a KK/massive spin-2 portal

**Gravity-mediated Scalar Dark Matter in Warped Extra-Dimensions**

**Miguel G. Folgado,[a] Andrea Donini,[a] Nuria Rius[a]**

Blatantly wrong-60 Citations

PRL **116,** 101302 (2016) PHYSICAL REVIEW LETTERS week ending 11 MARCH 2016

**Planckian Interacting Massive Particles as Dark Matter**

Mathias Garny,[1,*] McCullen Sandora,[2,†] and Martin S. Sloth[2,‡]
[1]*CERN Theory Division, CH-1211 Geneva 23, Switzerland*
[2]*CP³-Origins, Center for Cosmology and Particle Physics Phenomenology, University of Southern Denmark, Campusvej 55, 5230 Odense M, Denmark*
(Received 20 November 2015; published 10 March 2016)

Partially wrong - 150 Citations

PHYSICAL REVIEW LETTERS **128,** 081806 (2022)

**Massive Gravitons as Feebly Interacting Dark Matter Candidates**

Haiying Cai,[1,*] Giacomo Cacciapaglia,[2,3,†] and Seung J. Lee[1,‡]
[1]*Department of Physics, Korea University, Seoul 136-713, Korea*
[2]*University of Lyon, Université Claude Bernard Lyon 1, F-69001 Lyon, France*
[3]*Institut de Physique des 2 Infinis de Lyon (IP2I), UMR5822, CNRS/IN2P3, F-69622 Villeurbanne Cedex, France*

Blatantly wrong-10 citations

**Spin-2 portal dark matter**

Nicolás Bernal,[1,*] Maíra Dutra,[2,†] Yann Mambrini,[2,‡] Keith Olive,[3,4,§] Marco Peloso,[3,∥] and Mathias Pierre[2,¶]
[1]*Centro de Investigaciones, Universidad Antonio Nariño Carrera 3 Este # 47A-15, Bogotá, Colombia*
[2]*Laboratoire de Physique Théorique (UMR8627), CNRS, University of Paris-Sud, Université Paris-Saclay, 91405 Orsay, France*

Partially wrong-100 Citations

**Dark Matter Direct Detection from new interactions in models with spin-two mediators**

A. Carrillo-Monteverde[1,♯], Yoo-Jin Kang[2,3,*], Hyun Min Lee[2,†], Myeonghun Park[4,♣], and Veronica Sanz[1,♠]

[1]*Department of Physics and Astronomy, University of Sussex, Brighton BN1 9QH, UK.*
[2]*Department of Physics, Chung-Ang University, Seoul 06974, Korea.*
[3]*Center for Theoretical Physics of the Universe, Institute for Basic Science, Daejeon, 34051, Korea.*
[4]*Institute of Convergence Fundamental Studies and School of Liberal Arts, Seoul National University of Science and Technology, Seoul 01811, Korea.*

Blatantly wrong-75 Citations

Wrong Scaling

$$|\mathcal{M}|^2 \sim E^{12}/(m^8 m_p^4)$$

# How calculations go wrong in DM

Brane Localized Scalar Dark Matter annihilating via a KK/massive spin-2 portal

**Gravity-mediated Scalar Dark Matter in Warped Extra-Dimensions**

Miguel G. Folgado,[a] Andrea Donini,[a] Nuria Rius[a]

Blatantly wrong-60 Citations

PRL 116, 101302 (2016) PHYSICAL REVIEW LETTERS week ending 11 MARCH 2016

**Planckian Interacting Massive Particles as Dark Matter**

Mathias Garny,[1,*] McCullen Sandora,[2,†] and Martin S. Sloth[2,‡]
[1]*CERN Theory Division, CH-1211 Geneva 23, Switzerland*
[2]*$CP^3$-Origins, Center for Cosmology and Particle Physics Phenomenology, University of Southern Denmark, Campusvej 55, 5230 Odense M, Denmark*
(Received 20 November 2015; published 10 March 2016)

Partially wrong - 150 Citations

PHYSICAL REVIEW LETTERS 128, 081806 (2022)

**Massive Gravitons as Feebly Interacting Dark Matter Candidates**

Haiying Cai,[1,*] Giacomo Cacciapaglia,[2,3,†] and Seung J. Lee[1,‡]
[1]*Department of Physics, Korea University, Seoul 136-713, Korea*
[2]*University of Lyon, Université Claude Bernard Lyon 1, F-69001 Lyon, France*
[3]*Institut de Physique des 2 Infinis de Lyon (IP2I), UMR5822, CNRS/IN2P3, F-69622 Villeurbanne Cedex, France*

Blatantly wrong-10 citations

**Spin-2 portal dark matter**

Nicolás Bernal,[1,*] Maíra Dutra,[2,†] Yann Mambrini,[2,‡] Keith Olive,[3,4,§] Marco Peloso,[3,∥] and Mathias Pierre[2,¶]
[1]*Centro de Investigaciones, Universidad Antonio Nariño Carrera 3 Este # 47A-15, Bogotá, Colombia*
[2]*Laboratoire de Physique Théorique (UMR8627), CNRS, University of Paris-Sud, Université Paris-Saclay, 91405 Orsay, France*

Partially wrong-100 Citations

**Dark Matter Direct Detection from new interactions in models with spin-two mediators**

A. Carrillo-Monteverde[1,♯], Yoo-Jin Kang[2,3,*], Hyun Min Lee[2,†], Myeonghun Park[4,♣], and Veronica Sanz[1,♠]
[1]*Department of Physics and Astronomy, University of Sussex, Brighton BN1 9QH, UK.*
[2]*Department of Physics, Chung-Ang University, Seoul 06974, Korea.*
[3]*Center for Theoretical Physics of the Universe, Institute for Basic Science, Daejeon, 34051, Korea.*
[4]*Institute of Convergence Fundamental Studies and School of Liberal Arts, Seoul National University of Science and Technology, Seoul 01811, Korea.*

Blatantly wrong-75 Citations

Wrong Scaling

$$\boxed{|\mathcal{M}|^2 \sim E^{12}/(m^8 m_p^4)}$$

Issue: Breakdown of Diffeomorphism in naive massive gravity is restored in a full Kaluza-Klein theory

$$\boxed{|\mathcal{M}|^2 = \frac{E^2}{m_P^2}}$$

Phys.Rev.D 100 (2019) 11, 115033 Phys.Rev.D 101 (2020) 7, 075013 Phys.Rev.D 101 (2020) 5, 055013 **Phys. Rev. D 103 (2022), 095024**

**Phys. Rev. D 107,(2023) 03505, PRD Letters, ……**

**R. S. Chivukula, D. Foren, J.A.Gill, X. Wang, K. Mohan, E.H. Simmons, *DS***

# Massive Gauge Theories

# Massive Gauge Theories

Let's think of a Massive Photon

$$\boxed{\mathcal{L}_{\text{Proca}} = -\frac{1}{4}F_{\mu\nu}^2 - \frac{1}{2}m^2 A_\mu A^\mu}$$

Mass term explicitly breaks the gauge redundancy

Propagator $\frac{-i}{p^2+m^2}\left(\eta_{\mu\nu} + \frac{p_\mu p_\nu}{m^2}\right)$

**Stuckelberg Trick**

$$\boxed{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}m^2 A_\mu A^\mu - m A_\mu \partial^\mu \phi - \frac{1}{2}\partial_\mu \phi \partial^\mu \phi + A_\mu J^\mu - \frac{1}{m}\phi \partial_\mu J^\mu}$$

$$\delta A_\mu = \partial_\mu \Lambda$$

$$\delta \phi = -m\Lambda$$

Propagators $\quad \dfrac{-i\eta_{\mu\nu}}{p^2+m^2}, \quad \dfrac{-i}{p^2+m^2}$

# Massive Gauge Theories

Let's think of a Massive Photon

$$\boxed{\mathcal{L}_{\text{Proca}} = -\frac{1}{4}F_{\mu\nu}^2 - \frac{1}{2}m^2 A_\mu A^\mu}$$

Mass term explicitly breaks the gauge redundancy

Propagator $\frac{-i}{p^2+m^2}\left(\eta_{\mu\nu} + \frac{p_\mu p_\nu}{m^2}\right)$

Stuckelberg Trick $A_\mu \to A_\mu + \partial_\mu \phi$

$$\boxed{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}m^2 A_\mu A^\mu - mA_\mu \partial^\mu \phi - \frac{1}{2}\partial_\mu \phi \partial^\mu \phi + A_\mu J^\mu - \frac{1}{m}\phi \partial_\mu J^\mu}$$

$$\delta A_\mu = \partial_\mu \Lambda$$

$$\delta \phi = -m\Lambda$$

Propagators $\frac{-i\eta_{\mu\nu}}{p^2+m^2}, \quad \frac{-i}{p^2+m^2}$

Abelian Higgs Mechanism

$$\boxed{\mathcal{L}_{\text{AH}} = -\frac{1}{4}F_{\mu\nu}^2 - \frac{1}{2}\left(\mathcal{D}_\mu \phi\right)\left(\mathcal{D}^\mu \phi\right)^* - \lambda\left(\phi\phi^* - \Phi_0^2\right)^2}$$

$$\boxed{\phi = \varphi e^{i\chi}}$$

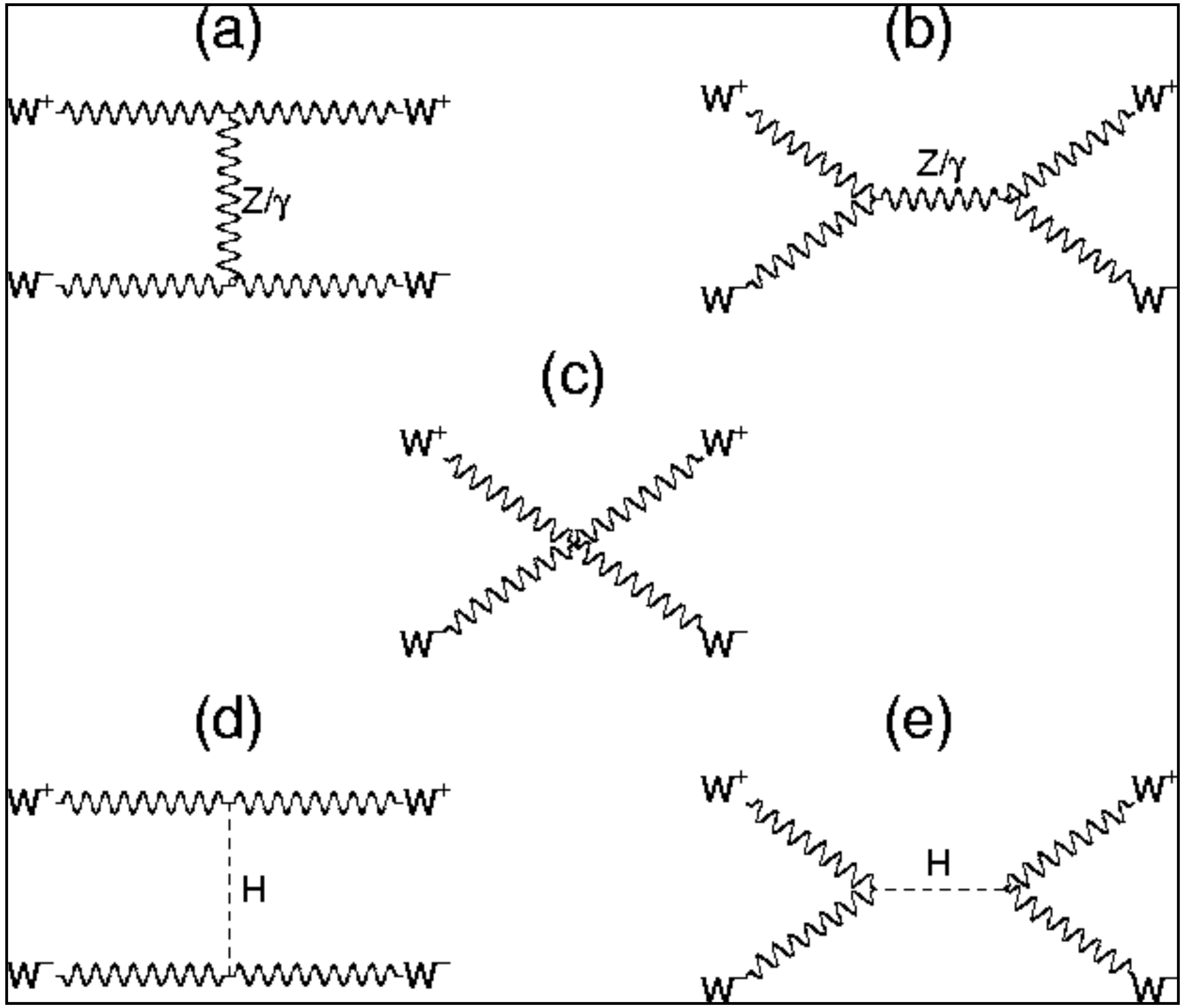
VEV of the Higgs

$$\boxed{\mathcal{L}_{\text{AH}} = -\frac{1}{4}F_{\mu\nu}^2 - \frac{1}{2}\varphi^2\left(qA_\mu - \partial_\mu \chi\right)^2 - \frac{1}{2}(\partial_\mu \varphi)^2 - \lambda\left(\varphi^2 - \Phi_0^2\right)^2}$$

$$\boxed{\begin{array}{ccc} A_\mu & \to & A_\mu + \partial_\mu \xi(x) \\ \chi & \to & \chi + q\,\xi(x)\,. \end{array}}$$

Restoration of U(1)

# Goldstone Boson Equivalence Theorems for the Standard Model W's

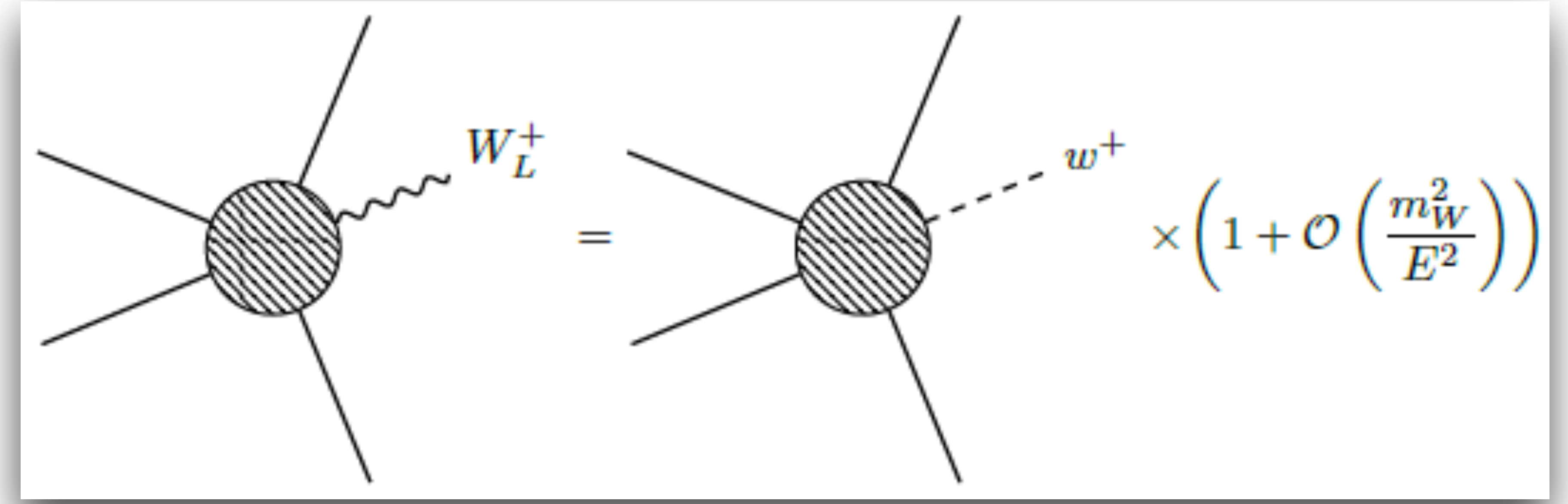


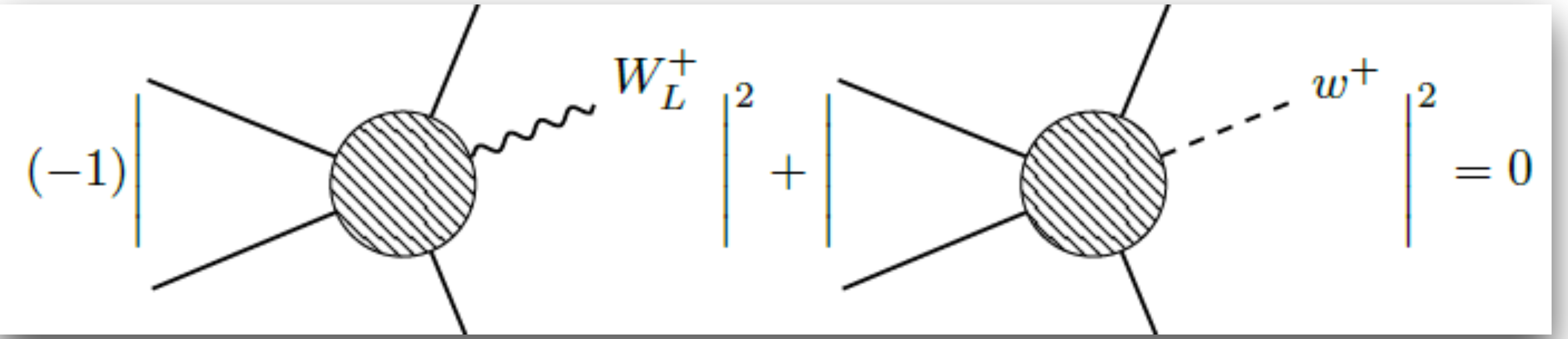

Polarization vectors of massive W's $\quad k^\mu = (E_k, 0, 0, k)$

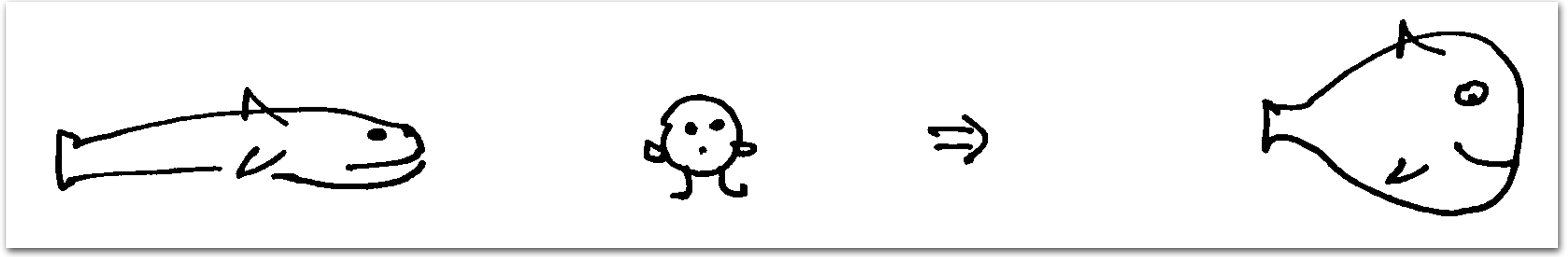
$$\epsilon^\mu_L(k) = \left(\frac{k}{m_W}, 0, 0, \frac{E_k}{m_W}\right) = \frac{k^\mu}{m_W} + \frac{1}{m_W}(-E_k + k, 0, 0, E_k - k)$$

$$\underset{E_k >> m_W}{\sim} \quad = \frac{k^\mu}{m_W} + \mathcal{O}\left(\frac{m_W}{E_k}\right)$$

Polarization vectors increasingly parellel to momentum vector



Courtesy Michael Peskin and Kurt Gottfried

# Goldstone Boson Equivalence Theorems for the Standard Model W's

$$\mathcal{M}_{4W} + \mathcal{M}^s_\gamma + \mathcal{M}^t_\gamma + \mathcal{M}^s_Z\mathcal{M}^t_Z = \frac{g^2(s+t)}{4m_W^2} - \frac{g^2\left(s^2+st-4t^2\cos^2\theta_W+t^2\right)}{2\left(st\cos^2\theta_W\right)}$$

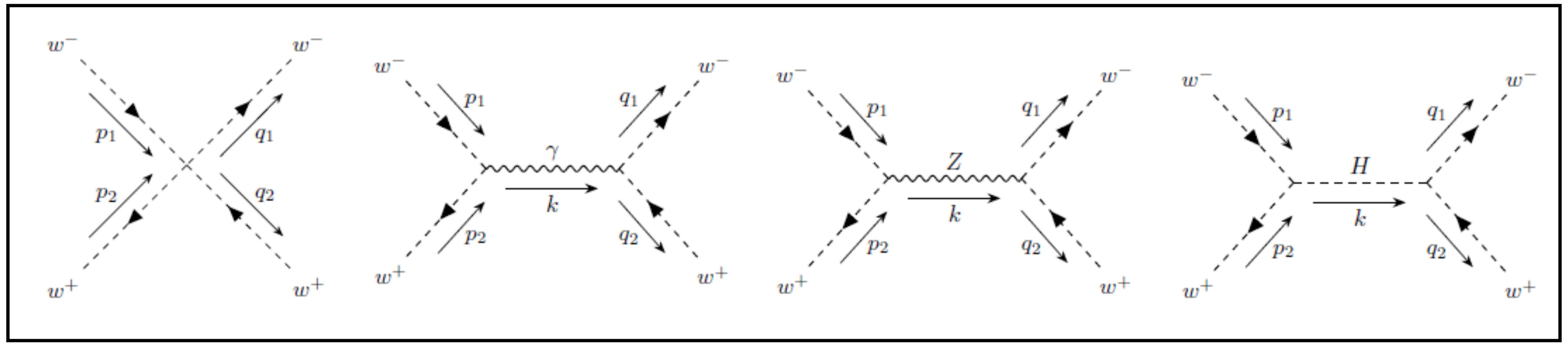
Improvement from E^4 to E^2

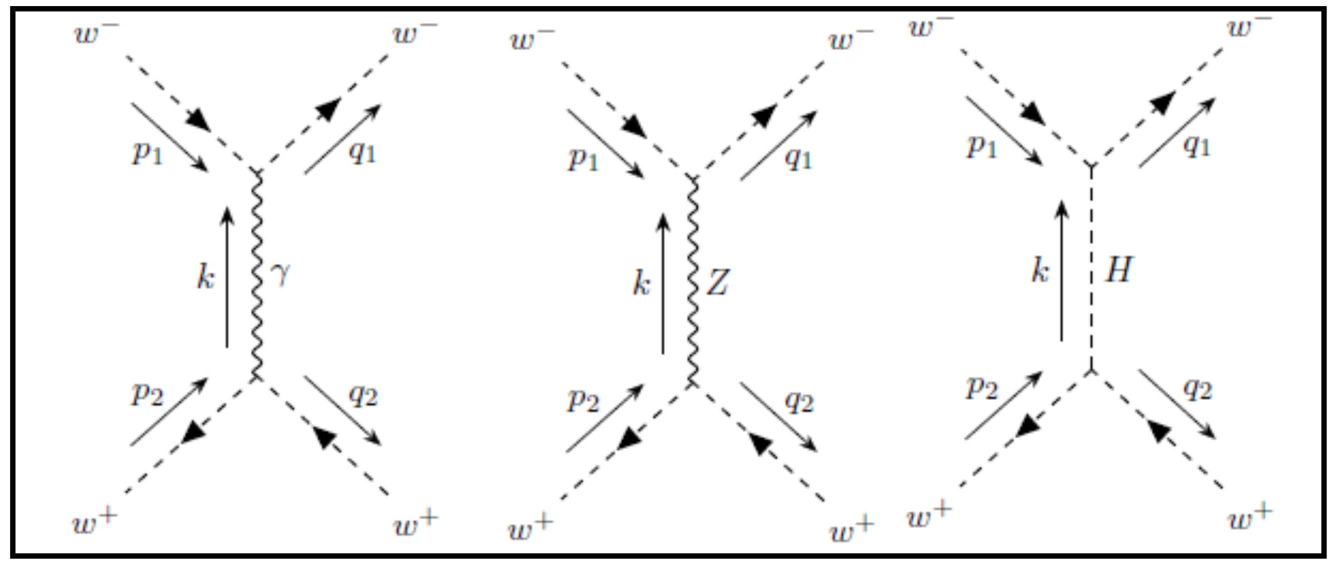
$$\mathcal{M}^t_H \quad \overset{HE}{\sim} \frac{1}{4}g^2\left(-\frac{m_H^2}{m_W^2} - \frac{8t}{s} - 4\right) - \frac{g^2 t}{4m_W^2}$$

Adding the two reduces the growth to a constant and restores unitarity

$$\boxed{\mathcal{M}_{(W_L^- W_L^+ \to W_L^- W_L^+)} = \frac{g^2}{2}\left(-\frac{m_H^2}{m_W^2} - \frac{s^2+st+t^2}{st\cos^2\theta_W}\right)}$$

Using the Goldstone Equivalence Theorem : In 't-Hooft Feynman Gauge replace longitudinal W's with Goldstones $w^\pm$




$$\boxed{\mathcal{M}_{(w^- w^+ \to w^- w^+)} \approx -\frac{g^2}{2}\left(\frac{m_H^2}{m_W^2} + \frac{s^2+st+t^2}{st\cos^2\theta_W}\right)}$$

No Large Cancellations that happened at low energies

# Gravity as an Effective Field Theory

# Gravity as an Effective Field Theory

## The Classical Action for Gravity

$$\boxed{S_g = \int d^4x \sqrt{g} \frac{2}{\kappa^2} R}$$

Einstein's Equation

$$\boxed{R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -8\pi G T_{\mu\nu}}$$

Conserved Source

$$\sqrt{g} T^{\mu\nu} \equiv -2 \frac{\partial}{\partial g_{\mu\nu}} \left(\sqrt{g} \mathcal{L}_m\right)$$

# Gravity as an Effective Field Theory

The Classical Action for Gravity

$$\boxed{S_g = \int d^4x \sqrt{g} \frac{2}{\kappa^2} R}$$

Einstein's Equation

$$\boxed{R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -8\pi G T_{\mu\nu}}$$

Conserved Source

$$\sqrt{g} T^{\mu\nu} \equiv -2 \frac{\partial}{\partial g_{\mu\nu}} (\sqrt{g} \mathcal{L}_m)$$

The Ricci is a two derivative object $\quad g_{\mu\nu} \to \eta_{\mu\nu} + \kappa h_{\mu\nu}$

$$\epsilon^{\mu\nu}_{\pm 2} = \epsilon^{\mu}_{\pm 1} \epsilon^{\nu}_{\pm 1}$$

$$\epsilon^{\mu}_{\pm 1} = \pm \frac{e^{\pm i\phi}}{\sqrt{2}} (0, -c_\theta c_\phi \pm i s_\phi, -c_\theta s_\phi \mp i c_\phi, s_\theta)$$

Einstein Hilbert Action is a dimension 6 operator with a cut-off $M_{Pl}$

## The Classical Action for Gravity

$$\boxed{S_g = \int d^4x \sqrt{g} \frac{2}{\kappa^2} R}$$

Einstein's Equation

$$\boxed{R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -8\pi G T_{\mu\nu}}$$

Conserved Source

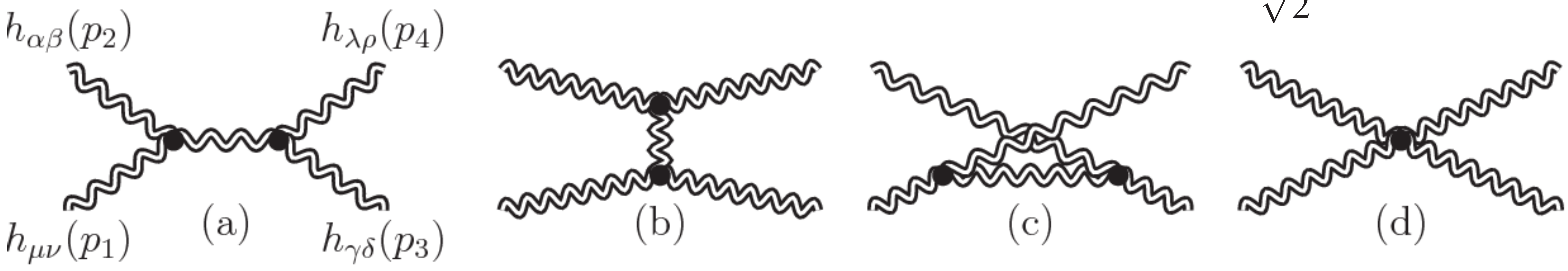
$$\sqrt{g} T^{\mu\nu} \equiv -2 \frac{\partial}{\partial g_{\mu\nu}} \left( \sqrt{g} \mathcal{L}_m \right)$$

The Ricci is a two derivative object $g_{\mu\nu} \to \eta_{\mu\nu} + \kappa h_{\mu\nu}$

Einstein Hilbert Action is a dimension 6 operator with a cut-off $M_{Pl}$

$$\epsilon^{\mu\nu}_{\pm 2} = \epsilon^{\mu}_{\pm 1} \epsilon^{\nu}_{\pm 1}$$

$$\epsilon^{\mu}_{\pm 1} = \pm \frac{e^{\pm i\phi}}{\sqrt{2}} (0, -c_\theta c_\phi \pm i s_\phi, -c_\theta s_\phi \mp i c_\phi, s_\theta)$$




The amplitude grows as $\boxed{s/M_{Pl}^2}$ Consistent with semi-classical gravity

Formal expansion of Massless Gravity

$$\boxed{S = \int d^4x \sqrt{g} \left\{ \Lambda + \frac{2}{\kappa^2} R + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} + \ldots + \mathcal{L}_{matter} \right\}}$$

# Gravity as an Effective Field Theory : Diffeomorphism and Mass Terms

**Diffeomorphism :** $h_{\mu\nu} \to h_{\mu\nu} + \partial_{(\mu}\xi_{\nu)}$

Equivalent to transformation for Gauge Theories $A_\mu \to A_\mu + \partial_\mu \xi$

**D.O.F counting in d dimensions for the massless graviton** $d(d+1)/2 - 2d = d(d-3)/2$

d=3, D.O.F =0
d=4, D.O.F=2
d=5, D.O.F=5

**D.O.F counting in d dimensions for the massless gauge boson** **d-2**

# Gravity as an Effective Field Theory : Diffeomorphism and Mass Terms

**Diffeomorphism :** $h_{\mu\nu} \to h_{\mu\nu} + \partial_{(\mu}\xi_{\nu)}$

Equivalent to transformation for Gauge Theories $\boxed{A_\mu \to A_\mu + \partial_\mu \xi}$

**D.O.F counting in d dimensions for the massless graviton** $d(d+1)/2 - 2d = d(d-3)/2$

d=3, D.O.F =0
d=4, D.O.F=2
d=5, D.O.F=5

**D.O.F counting in d dimensions for the massless gauge boson** $\boxed{\mathbf{d-2}}$

---

Let's think of a Massive Photon $\boxed{\mathcal{L}_{\text{Proca}} = -\frac{1}{4}F_{\mu\nu}^2 - \frac{1}{2}m^2 A_\mu A^\mu}$ Mass term explicitly breaks the gauge redundancy

Propagator $\frac{-i}{p^2+m^2}\left(\eta_{\mu\nu} + \frac{p_\mu p_\nu}{m^2}\right)$

Stuckelberg Trick $A_\mu \to A_\mu + \partial_\mu \phi$

$$\boxed{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}m^2 A_\mu A^\mu - mA_\mu \partial^\mu \phi - \frac{1}{2}\partial_\mu \phi \partial^\mu \phi + A_\mu J^\mu - \frac{1}{m}\phi \partial_\mu J^\mu}$$

$$\delta A_\mu = \partial_\mu \Lambda$$

$$\delta \phi = -m\Lambda$$

Propagators $\frac{-i\eta_{\mu\nu}}{p^2+m^2}, \quad \frac{-i}{p^2+m^2}$

# Gravity as an Effective Field Theory : Diffeomorphism and Mass Terms

Let's think of a Massive Graviton $$\boxed{S_G = \int d^4x \sqrt{g} R + m^2((h_{\mu\nu})^2 - h^2)} + \kappa h_{\mu\nu} T^{\mu\nu}$$ Fierz-Pauli Theory: 1940

# Gravity as an Effective Field Theory : Diffeomorphism and Mass Terms

Let's think of a Massive Graviton $\boxed{S_G = \int d^4x \sqrt{g} R + m^2((h_{\mu\nu})^2 - h^2)} + \kappa h_{\mu\nu} T^{\mu\nu}$ Fierz-Pauli Theory: 1940

Stuckelberg $\boxed{h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu A_\nu + \partial_\nu A_\mu}$ $\boxed{A_\mu \to A_\mu + \partial_\mu \phi}$

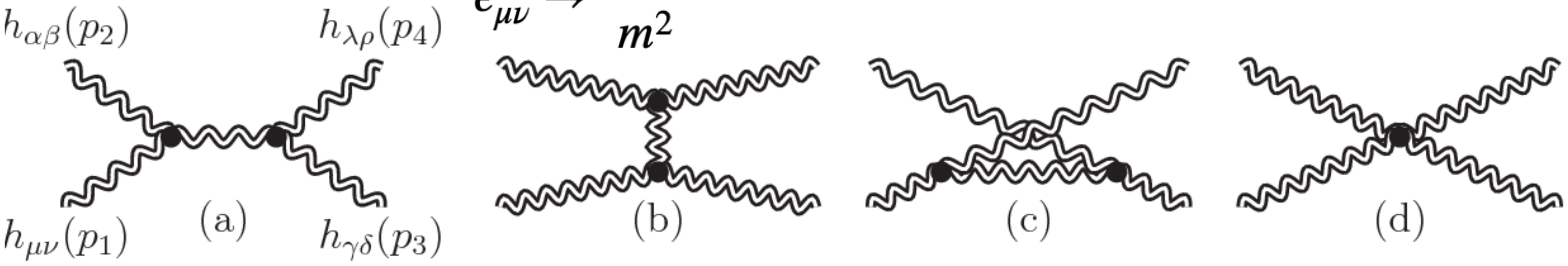
5 propagating degrees of freedom
2 transverse + 3 longitudinal

$\text{rescale } A_\mu \to \frac{1}{m} A_\mu, \ \phi \to \frac{1}{m^2}\phi$ Assume source is conserved , vanishing $\partial_\nu T^{\mu\nu}$

$$\boxed{S = \int d^D x \ \mathcal{L}_{m=0}(h') - \frac{1}{2} F_{\mu\nu} F^{\mu\nu} - 2\frac{D-1}{D-2} \partial_\mu \phi \partial^\mu \phi + \kappa h'_{\mu\nu} T^{\mu\nu} + \frac{2}{D-2} \kappa \phi T}$$

1. Scalar couples to the trace of the stress energy tensor and does not decouple.
2. Behaves like a Scalar-Tensor/Brans-Dicke Theory. Affects the Newtonian Potential
3. vanDam-Veltman-Zakharov Discontinuity. M-> 0 limit not smooth under Stuckleberg

# Gravity as an Effective Field Theory : Diffeomorphism and Mass Terms




$$\epsilon^0_{\mu\nu} \to \frac{k_\mu k_\nu}{m^2}$$

$$\epsilon^{\mu\nu}_{\pm 2} = \epsilon^\mu_{\pm 1}\epsilon^\nu_{\pm 1},$$

$$\epsilon^{\mu\nu}_{\pm 1} = \frac{1}{\sqrt{2}}[\epsilon^\mu_{\pm 1}\epsilon^\nu_0 + \epsilon^\mu_0\epsilon^\nu_{\pm 1}],$$

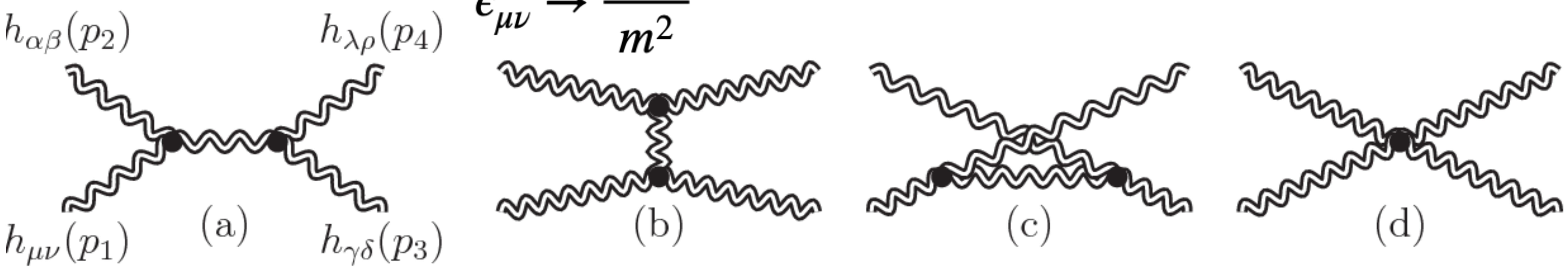
$$\epsilon^{\mu\nu}_0 = \frac{1}{\sqrt{6}}[\epsilon^\mu_{+1}\epsilon^\nu_{-1} + \epsilon^\mu_{-1}\epsilon^\nu_{+1} + 2\epsilon^\mu_0\epsilon^\nu_0]$$

$$\epsilon^\mu_{\pm 1} = \pm\frac{e^{\pm i\phi}}{\sqrt{2}}(0, -c_\theta c_\phi \pm i s_\phi, -c_\theta s_\phi \mp$$

$$\epsilon^\mu_0 = \frac{E}{m}\left(\sqrt{1-\frac{m^2}{E^2}}, \hat{p}\right),$$

# Gravity as an Effective Field Theory : Diffeomorphism and Mass Terms

$$\epsilon^0_{\mu\nu} \to \frac{k_\mu k_\nu}{m^2}$$




$$\epsilon^{\mu\nu}_{\pm 2} = \epsilon^{\mu}_{\pm 1}\epsilon^{\nu}_{\pm 1},$$

$$\epsilon^{\mu\nu}_{\pm 1} = \frac{1}{\sqrt{2}}[\epsilon^{\mu}_{\pm 1}\epsilon^{\nu}_{0} + \epsilon^{\mu}_{0}\epsilon^{\nu}_{\pm 1}],$$

$$\epsilon^{\mu\nu}_{0} = \frac{1}{\sqrt{6}}[\epsilon^{\mu}_{+1}\epsilon^{\nu}_{-1} + \epsilon^{\mu}_{-1}\epsilon^{\nu}_{+1} + 2\epsilon^{\mu}_{0}\epsilon^{\nu}_{0}]$$

$$\epsilon^{\mu}_{\pm 1} = \pm\frac{e^{\pm i\phi}}{\sqrt{2}}(0, -c_\theta c_\phi \pm i s_\phi, -c_\theta s_\phi \mp$$

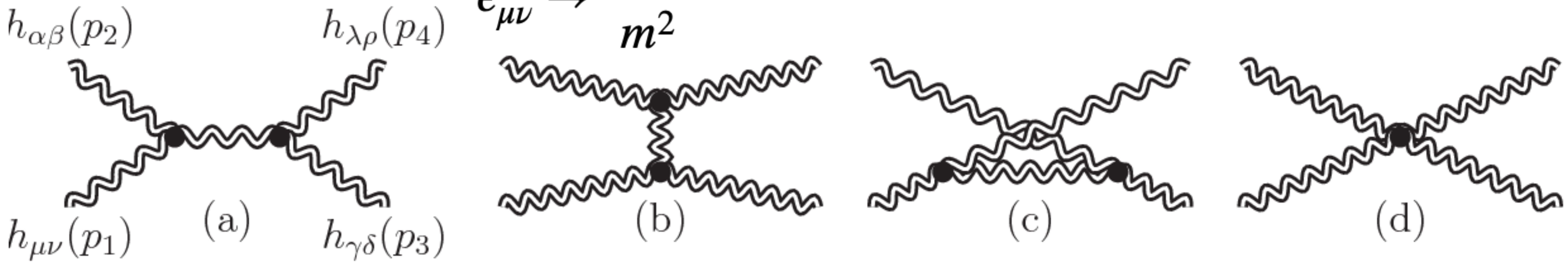
$$\epsilon^{\mu}_{0} = \frac{E}{m}\left(\sqrt{1-\frac{m^2}{E^2}}, \hat{p}\right),$$

Power Counting

1. Each external polarization grows as $s/m^2$
2. Each vertex grows as $s$
3. The propagator grows as $1/s$

# Gravity as an Effective Field Theory : Diffeomorphism and Mass Terms

$$\epsilon^0_{\mu\nu} \to \frac{k_\mu k_\nu}{m^2}$$




$$\epsilon^{\mu\nu}_{\pm 2} = \epsilon^{\mu}_{\pm 1}\epsilon^{\nu}_{\pm 1},$$

$$\epsilon^{\mu\nu}_{\pm 1} = \frac{1}{\sqrt{2}}[\epsilon^{\mu}_{\pm 1}\epsilon^{\nu}_{0} + \epsilon^{\mu}_{0}\epsilon^{\nu}_{\pm 1}],$$

$$\epsilon^{\mu\nu}_{0} = \frac{1}{\sqrt{6}}[\epsilon^{\mu}_{+1}\epsilon^{\nu}_{-1} + \epsilon^{\mu}_{-1}\epsilon^{\nu}_{+1} + 2\epsilon^{\mu}_{0}\epsilon^{\nu}_{0}]$$

$$\epsilon^{\mu}_{\pm 1} = \pm\frac{e^{\pm i\phi}}{\sqrt{2}}(0, -c_\theta c_\phi \pm i s_\phi, -c_\theta s_\phi \mp$$

$$\epsilon^{\mu}_{0} = \frac{E}{m}\left(\sqrt{1-\frac{m^2}{E^2}}, \hat{p}\right),$$

Power Counting

1. Each external polarization grows as $s/m^2$
2. Each vertex grows as s
3. The propagator grows as 1/s

Amplitude grows as $\dfrac{s^5}{m^8 M_{pl}^2}$ Discontinuity as m-> 0. Does not reduce to Einstein-Hilbert action

**Unitarity is violated at a scale** $\Lambda_5 = (M_{pl} m^4)^{1/5} \ll M_{pl}$


***De-Rham, Gabadadze, Tolley (2010)***
***Cheung and Remen (2017)***
***Bonifacio, Rosen, Hinterbichler (2019)***
***Georgi, Arkani-Hamed, Schwartz (2001)***
***Schwartz (2003)***

# Non-linear massive gravity

# Non-linear massive gravity

**Most general potential**

$$S = \frac{1}{2\kappa^2}\int d^D x \ \left[(\sqrt{-g}R) - \sqrt{-g}\frac{1}{4}m^2 V(g,h)\right]$$

# Non-linear massive gravity

**Most general potential**

$$S=\frac{1}{2\kappa^2}\int d^Dx\left[(\sqrt{-g}R)-\sqrt{-g}\frac{1}{4}m^2V(g,h)\right]$$

$$\boxed{\begin{aligned}
V_2(g,h) &= \langle h^2\rangle-\langle h\rangle^2,\\
V_3(g,h) &= +c_1\langle h^3\rangle+c_2\langle h^2\rangle\langle h\rangle+c_3\langle h\rangle^3,\\
V_4(g,h) &= +d_1\langle h^4\rangle+d_2\langle h^3\rangle\langle h\rangle+d_3\langle h^2\rangle^2+d_4\langle h^2\rangle\langle h\rangle^2+d_5\langle h\rangle^4,\\
V_5(g,h) &= +f_1\langle h^5\rangle+f_2\langle h^4\rangle\langle h\rangle+f_3\langle h^3\rangle\langle h\rangle^2+f_4\langle h^3\rangle\langle h^2\rangle+f_5\langle h^2\rangle^2\langle h\rangle\\
&\quad +f_6\langle h^2\rangle\langle h\rangle^3+f_7\langle h\rangle^5,
\end{aligned}}$$

**Tune coefficients to raise the scale, avoid ghosts**

# Non-linear massive gravity

**Most general potential**

$$S=\frac{1}{2\kappa^2}\int d^Dx\,\left[(\sqrt{-g}R)-\sqrt{-g}\frac{1}{4}m^2V(g,h)\right]$$

$$\begin{aligned}
V_2(g,h) &= \langle h^2\rangle-\langle h\rangle^2,\\
V_3(g,h) &= +c_1\langle h^3\rangle+c_2\langle h^2\rangle\langle h\rangle+c_3\langle h\rangle^3,\\
V_4(g,h) &= +d_1\langle h^4\rangle+d_2\langle h^3\rangle\langle h\rangle+d_3\langle h^2\rangle^2+d_4\langle h^2\rangle\langle h\rangle^2+d_5\langle h\rangle^4,\\
V_5(g,h) &= +f_1\langle h^5\rangle+f_2\langle h^4\rangle\langle h\rangle+f_3\langle h^3\rangle\langle h\rangle^2+f_4\langle h^3\rangle\langle h^2\rangle+f_5\langle h^2\rangle^2\langle h\rangle\\
&\quad +f_6\langle h^2\rangle\langle h\rangle^3+f_7\langle h\rangle^5,
\end{aligned}$$

**Tune coefficients to raise the scale, avoid ghosts**

$$\begin{aligned}
c_1 &= 2c_3+\frac{1}{2}, & c_2 &= -3c_3-\frac{1}{2},\\
d_1 &= -6d_5+\frac{3}{2}c_3+\frac{5}{16}, & d_2 &= 8d_5-\frac{3}{2}c_3-\frac{1}{4},\\
d_3 &= 3d_5-\frac{3}{4}c_3-\frac{1}{16}, & d_4 &= -6d_5+\frac{3}{4}c_3,
\end{aligned}$$

**De-Rahm, Gabadadze, Tolley**
**Cheung and Remen**
**Bonifiacio, Rosen, Hinterbichler**

# Non-linear massive gravity

**Most general potential**

$$S=\frac{1}{2\kappa^2}\int d^Dx\,\left[(\sqrt{-g}R)-\sqrt{-g}\frac{1}{4}m^2V(g,h)\right]$$

$$\boxed{\begin{aligned}
V_2(g,h) &= \langle h^2\rangle-\langle h\rangle^2,\\
V_3(g,h) &= +c_1\langle h^3\rangle+c_2\langle h^2\rangle\langle h\rangle+c_3\langle h\rangle^3,\\
V_4(g,h) &= +d_1\langle h^4\rangle+d_2\langle h^3\rangle\langle h\rangle+d_3\langle h^2\rangle^2+d_4\langle h^2\rangle\langle h\rangle^2+d_5\langle h\rangle^4,\\
V_5(g,h) &= +f_1\langle h^5\rangle+f_2\langle h^4\rangle\langle h\rangle+f_3\langle h^3\rangle\langle h\rangle^2+f_4\langle h^3\rangle\langle h^2\rangle+f_5\langle h^2\rangle^2\langle h\rangle\\
&\quad +f_6\langle h^2\rangle\langle h\rangle^3+f_7\langle h\rangle^5,
\end{aligned}}$$

**Tune coefficients to raise the scale, avoid ghosts**

$$c_1=2c_3+\frac{1}{2},\qquad c_2=-3c_3-\frac{1}{2},$$

$$d_1=-6d_5+\frac{3}{2}c_3+\frac{5}{16},\qquad d_2=8d_5-\frac{3}{2}c_3-\frac{1}{4},$$

$$d_3=3d_5-\frac{3}{4}c_3-\frac{1}{16},\qquad d_4=-6d_5+\frac{3}{4}c_3,$$

**De-Rahm, Gabadadze, Tolley**
**Cheung and Remen**
**Bonifiacio, Rosen, Hinterbichler**

**Cut-off scale raised to** $\Lambda_3$

**Can show that cut-off can't be raised above** $\Lambda_3$

**Realizations of this set up : dRGT gravity, Bi/Multigravity**

# Compactified 5D theory

$$\mathcal{S}_{5D}^{EH} \propto \frac{1}{M_5^3}\int d^5x\sqrt{-g}R_{5D}$$

***5D diffeomorphism with a 5D Planck mass***

***Compactification (IR phenomenon) should not change the high energy (UV) behavior,***

***High energy growth*** $\boxed{s/M_{Pl}^2}$

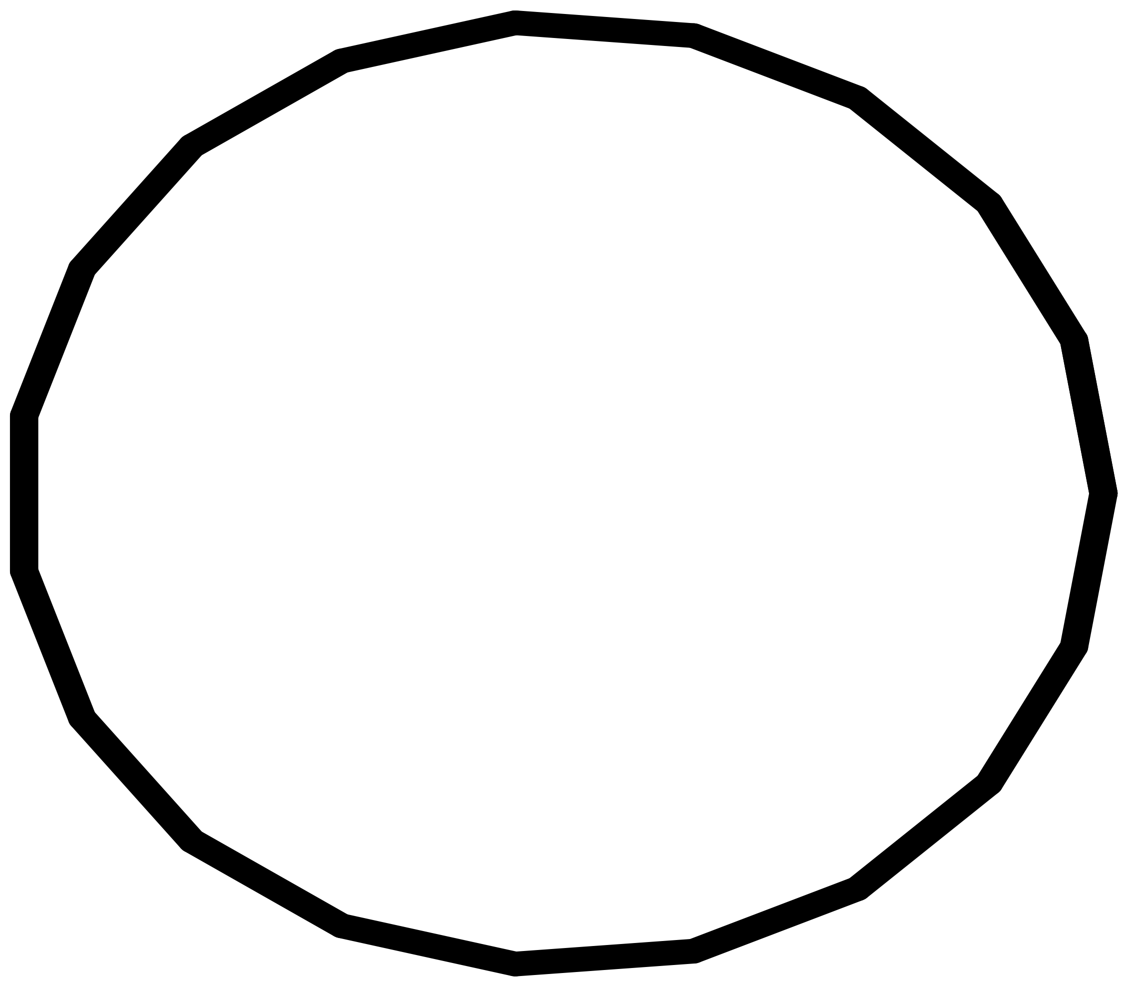
***Coupled channel analysis*** $\boxed{s^{3/2}/M_5^3}$

# Examine Strong coupling scale

A. Flat Extra dimension compactified on a torus
B. The Randall Sundrum Model (ADS)

# Understanding the problem : Geometrical Deconstruction of Dimensions

# Understanding the problem : Geometrical Deconstruction of Dimensions



minimal discretization

Discretize a dimension with nearest neighbour interaction

$$\boxed{\mathcal{L}_{\rm min} = \sum_j M^2\sqrt{g^j}R[g^j] + M^2m^2\sqrt{g^j}(g^j_{\mu\nu} - g^{j+1}_{\mu\nu})(g_j^{\mu\rho}g_j^{\nu\sigma} - g_j^{\mu\nu}g_j^{\rho\sigma})(g^j_{\rho\sigma} - g^{j+1}_{\rho\sigma})}$$

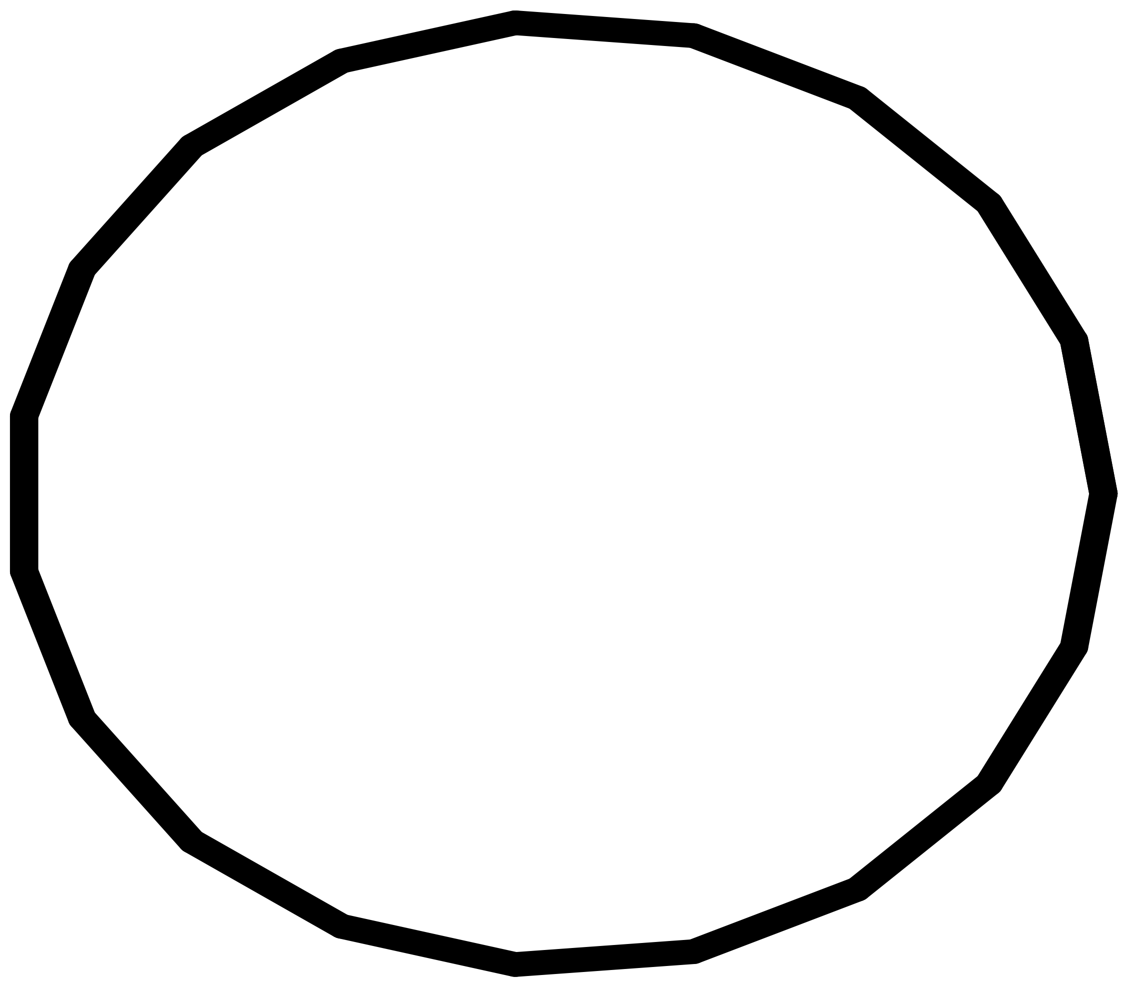
The mass terms correspond to broken diffeormorphisms, which can be replaced by Stuckelberg/Goldstone fields.

$$\Lambda_{\rm min} = (Nm_1^4 M_{\rm Pl})^{1/5}$$

$$\boxed{\Lambda_{\rm min} < \Lambda_{\rm max} = M_{\rm 5D}(RM_{\rm 5D})^{-5/8}}$$

Formally never recovers the full 5D

# Understanding the problem : Geometrical Deconstruction of Dimensions



**minimal discretization**

Discretize a dimension with nearest neighbour interaction

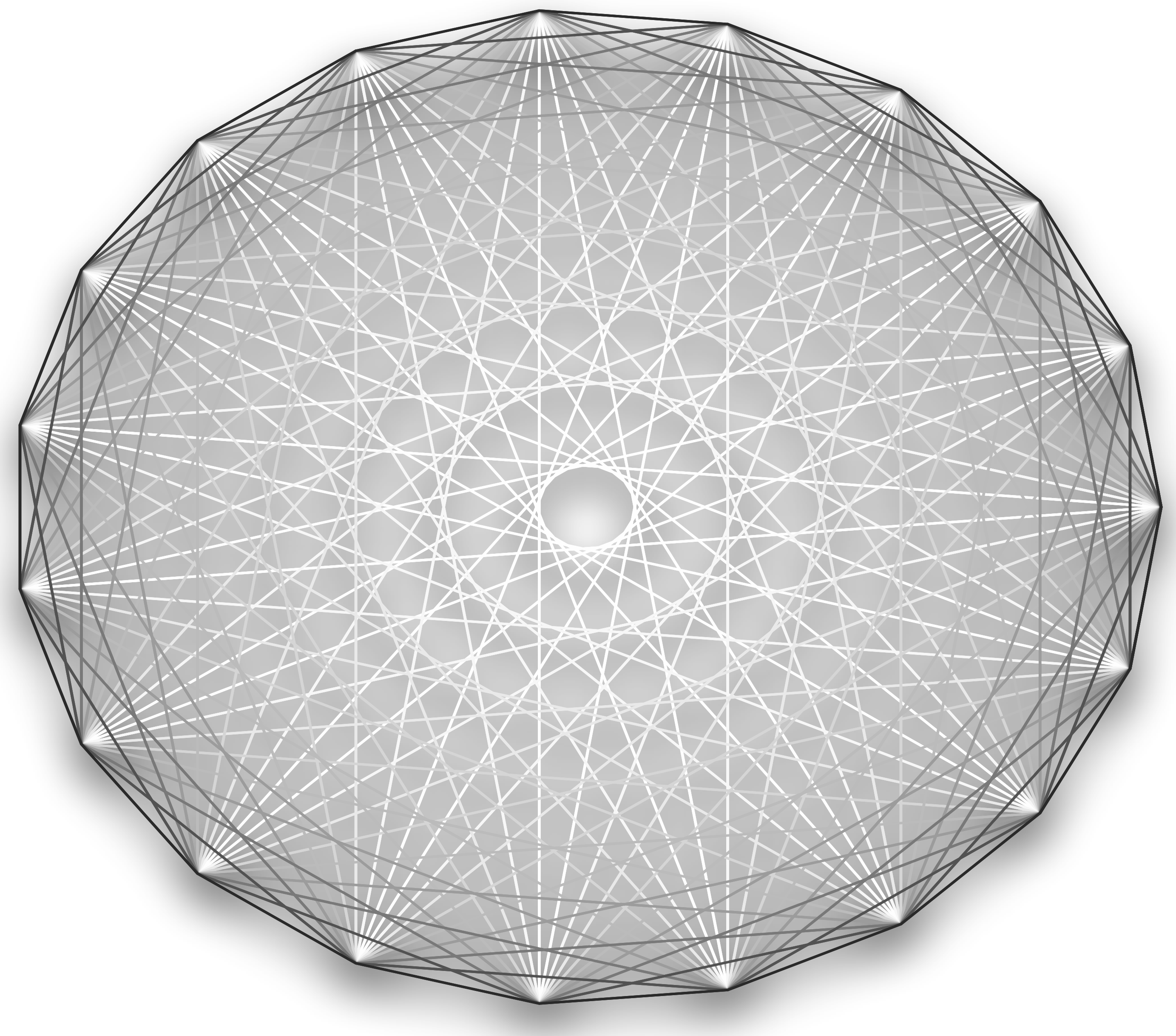
$$\boxed{\mathcal{L}_{\rm min} = \sum_j M^2 \sqrt{g^j} R[g^j] + M^2 m^2 \sqrt{g^j} (g^j_{\mu\nu} - g^{j+1}_{\mu\nu})(g_j^{\mu\rho} g_j^{\nu\sigma} - g_j^{\mu\nu} g_j^{\rho\sigma})(g^j_{\rho\sigma} - g^{j+1}_{\rho\sigma})}$$

The mass terms correspond to broken diffeormorphisms,
which can be replaced by Stuckelberg/Goldstone fields.

$$\Lambda_{\rm min} = (N m_1^4 M_{\rm Pl})^{1/5}$$

$$\boxed{\Lambda_{\rm min} < \Lambda_{\rm max} = M_{\rm 5D} (R M_{\rm 5D})^{-5/8}}$$

Formally never recovers the full 5D



**truncated KK theory**

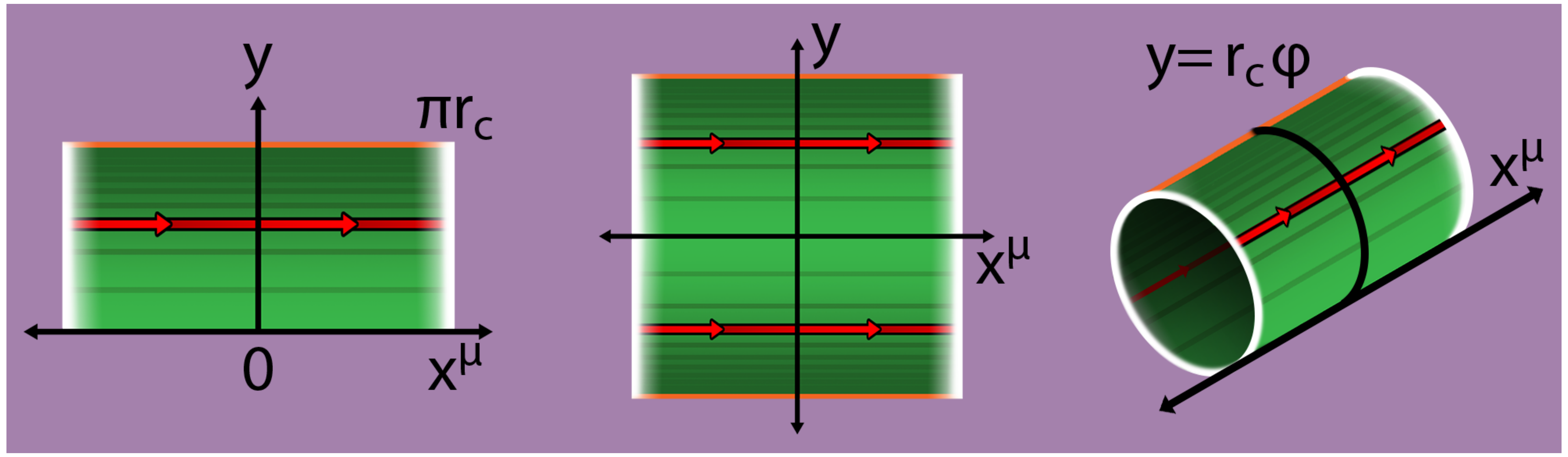
In a full/truncated KK theory,
we need all interactions and not just nearest neighbour ones

Arkani-Hamed, Georgi, Schwartz 2002
Arkani-Hamed, Schwartz 2003
Schwartz 2003

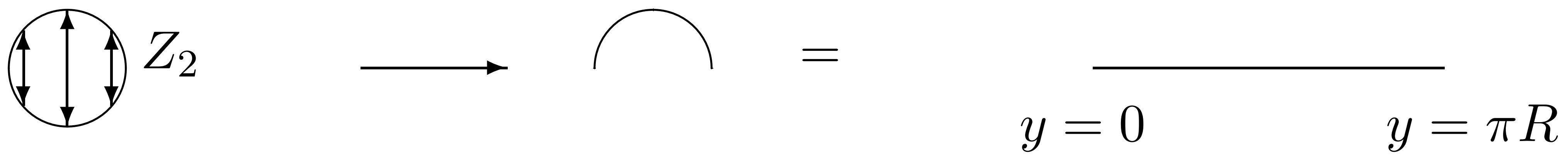
# Compact Extra Dimensions : A primer

# Compact Extra Dimensions : A primer

## Orbifolded Torus

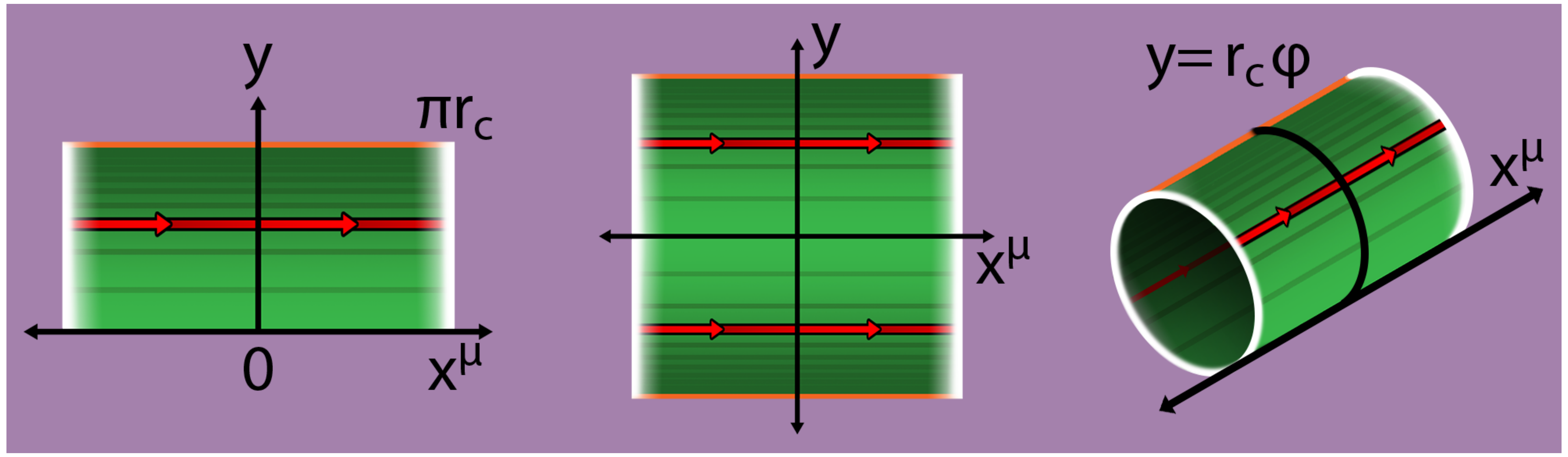



$$G^{(5\mathrm{DOT})}_{MN} = \begin{pmatrix} e^{\frac{-\kappa \hat{r}}{\sqrt{6}}} \left(\eta_{\mu\nu} + \kappa \hat{h}_{\mu\nu}\right) & 0 \\ 0 & -\left(1 + \frac{\kappa \hat{r}}{\sqrt{6}}\right)^2 \end{pmatrix}$$

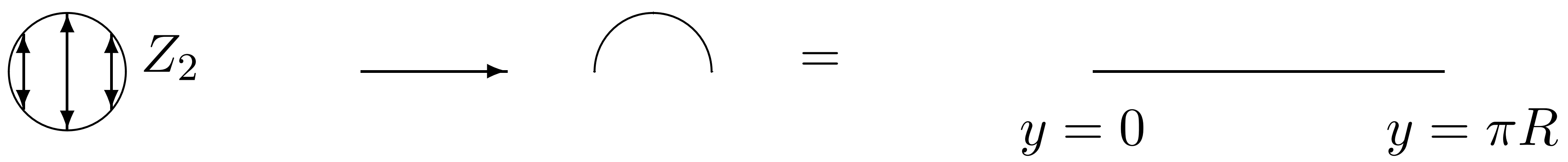


# Compact Extra Dimensions : A primer

## Orbifolded Torus




$$G^{(5DOT)}_{MN} = \begin{pmatrix} e^{\frac{-\kappa\hat{r}}{\sqrt{6}}}(\eta_{\mu\nu} + \kappa\hat{h}_{\mu\nu}) & 0 \\ 0 & -\left(1+\frac{\kappa\hat{r}}{\sqrt{6}}\right)^2 \end{pmatrix}$$




$$\boxed{S = M_5^3 \int d^4x dy \sqrt{G} R^{(5)}}$$

**Integrate over extra dimension: 4D EFT with a cut-off**

**KK Mode Expansions**

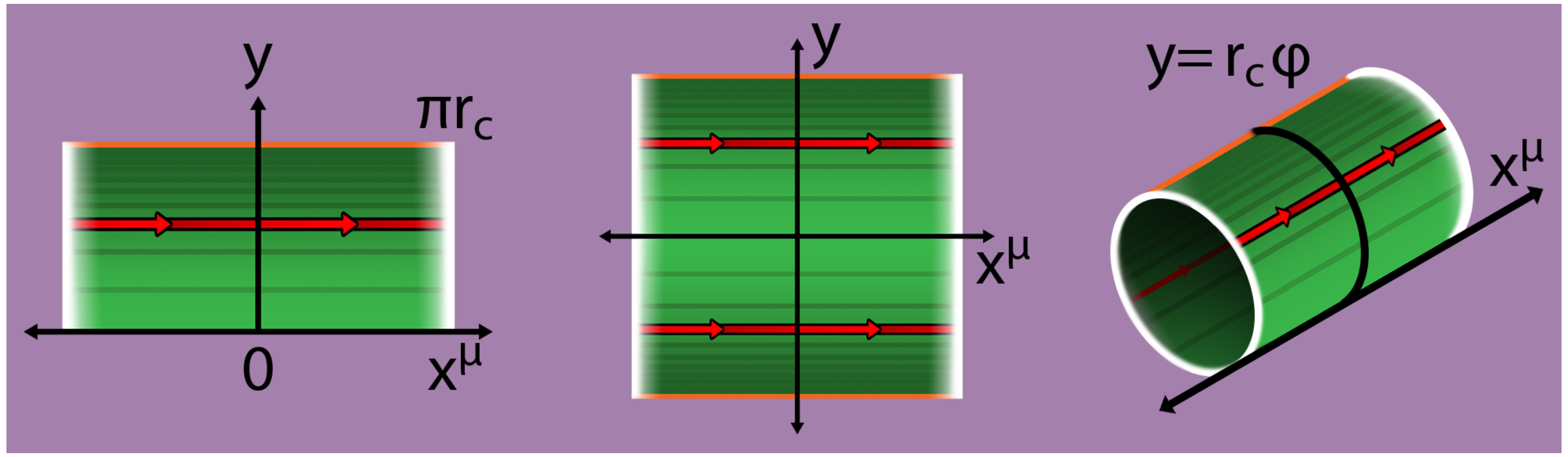
$$h_{\mu\nu}(x,y) = \sum_{n=-\infty} h_{\mu\nu,n}(x) e^{i\omega_n y}$$

$$h_{\mu 5}(x,y) = \sum_{n=-\infty}^{\infty} \rho_\mu(x,n) e^{i\omega_n y}$$

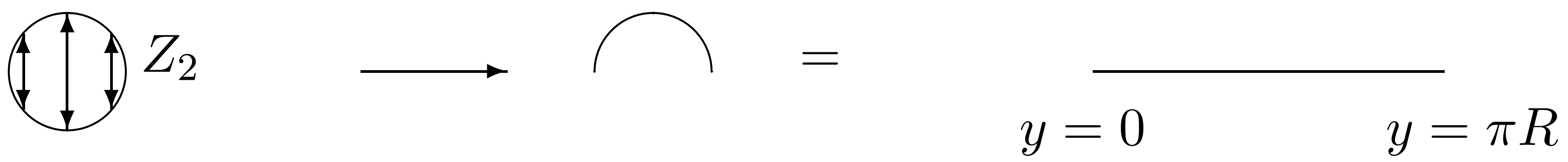
$$h_{55}(x,y) = \sum_{n=-\infty}^{\infty} r(x,n) e^{i\omega_n y}$$

# Compact Extra Dimensions : A primer

## Orbifolded Torus




$$G^{(5\mathrm{DOT})}_{MN} = \begin{pmatrix} e^{\frac{-\kappa\hat{r}}{\sqrt{6}}}\left(\eta_{\mu\nu} + \kappa\hat{h}_{\mu\nu}\right) & 0 \\ 0 & -\left(1 + \frac{\kappa\hat{r}}{\sqrt{6}}\right)^2 \end{pmatrix}$$




$$\boxed{S = M_5^3 \int d^4x dy \sqrt{G} R^{(5)}}$$

**Integrate over extra dimension: 4D EFT with a cut-off**

## KK Mode Expansions

$$h_{\mu\nu}(x,y) = \sum_{n=-\infty} h_{\mu\nu,n}(x) e^{i\omega_n y}$$

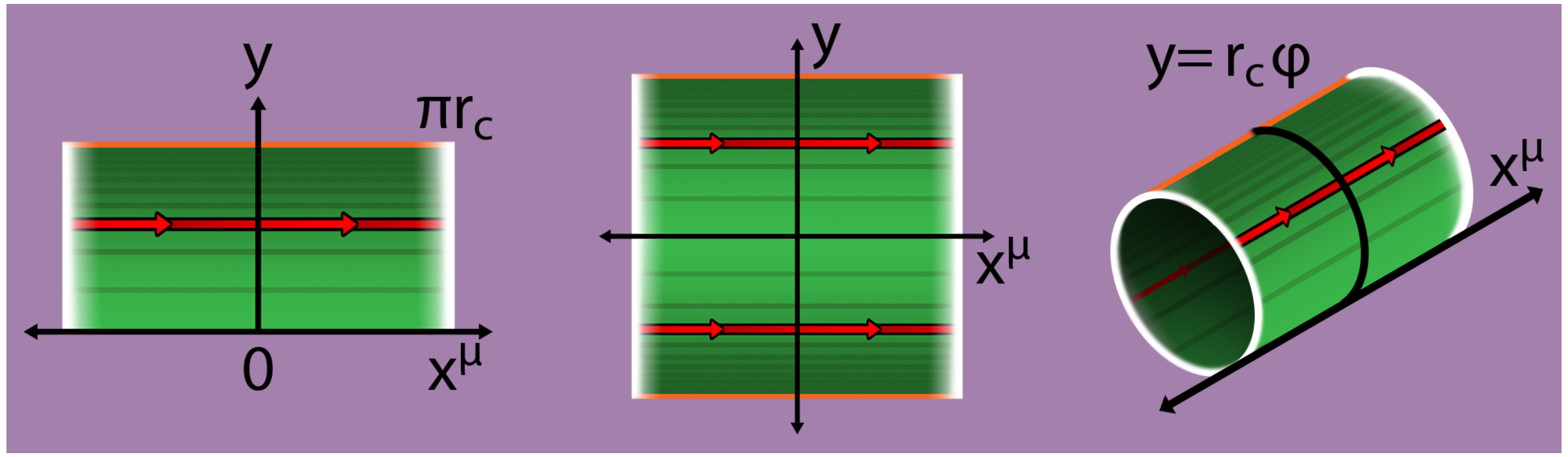
$$h_{\mu 5}(x,y) \cancel{=} \sum_{n=-\infty}^{\infty} \rho_\mu(x,n) e^{i\omega_n y}$$

$$h_{55}(x,y) = \sum_{n=-\infty}^{\infty} r(x,n) e^{i\omega_n y}$$

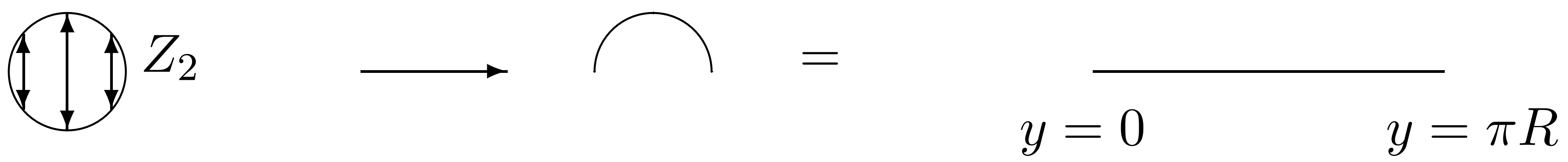
$$\begin{aligned} S = \int d^4x \quad & \frac{1}{2} h_{\mu\nu,0} \mathcal{E}^{\mu\nu,\alpha\beta} h_{\alpha\beta,0} - \frac{1}{2} F^2_{\mu\nu,0} + h_0^{\mu\nu}\left(\partial_\mu\partial_\nu\phi_0 - \eta_{\mu\nu}\phi_0\right) \\ & + \frac{1}{M_4} h_{\mu\nu,0} t_0^{\mu\nu} + \frac{2}{M_4} A_{\mu,0} j_0^\mu + \frac{1}{M_4}\phi_0 j_0 \\ & + \sum_{n=1}^{\infty} h'^{*}_{\mu\nu,n} \mathcal{E}^{\mu\nu,\alpha\beta} h'_{\alpha\beta,n} - \omega_n^2 \left(\left|h'_{\mu\nu,n}\right|^2 - \left|h'_n\right|^2\right) + \left[\frac{1}{M_4} h'_{\mu\nu,n} t_n^{\mu\nu *} + c.c.\right] \end{aligned}$$

# Compact Extra Dimensions : A primer

## Orbifolded Torus




$$G_{MN}^{(5\mathrm{DOT})} = \begin{pmatrix} e^{\frac{-\kappa\hat{r}}{\sqrt{6}}}\left(\eta_{\mu\nu} + \kappa\hat{h}_{\mu\nu}\right) & 0 \\ 0 & -\left(1 + \frac{\kappa\hat{r}}{\sqrt{6}}\right)^2 \end{pmatrix}$$




$$\boxed{S = M_5^3 \int d^4x dy \sqrt{G} R^{(5)}}$$

**Integrate over extra dimension: 4D EFT with a cut-off**

## KK Mode Expansions

$$h_{\mu\nu}(x,y) = \sum_{n=-\infty} h_{\mu\nu,n}(x) e^{i\omega_n y}$$

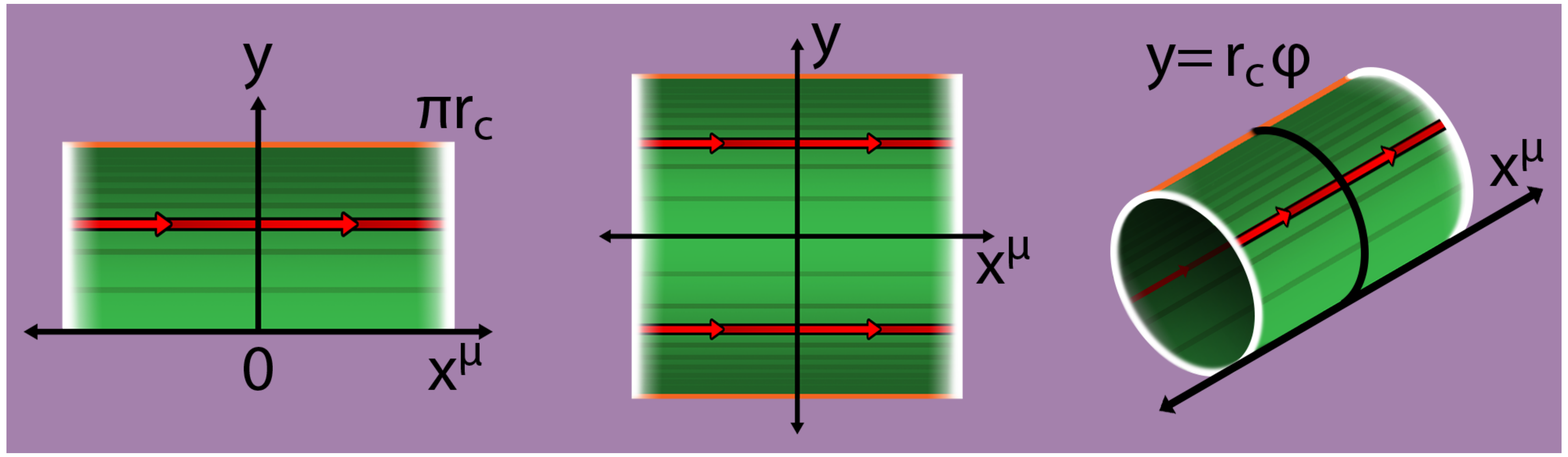
$$h_{\mu 5}(x,y) \cancel{=} \sum_{n=-\infty}^{\infty} \rho_\mu(x,n) e^{i\omega_n y}$$

$$h_{55}(x,y) = \sum_{n=-\infty}^{\infty} r(x,n) e^{i\omega_n y}$$

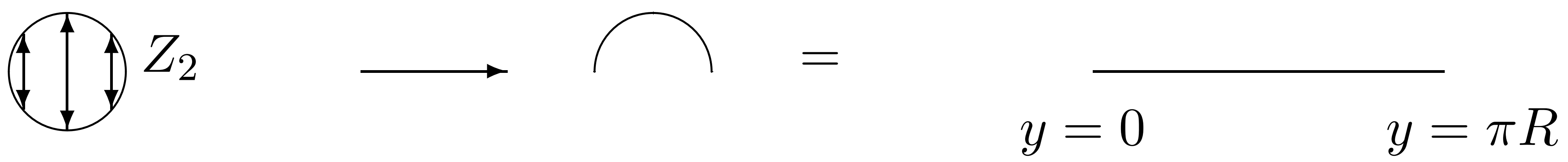
$$\begin{aligned} S = \int d^4x \quad & \frac{1}{2} h_{\mu\nu,0} \mathcal{E}^{\mu\nu,\alpha\beta} h_{\alpha\beta,0} - \cancel{\frac{1}{2} F^2_{\mu\nu,0}} + h_0^{\mu\nu}\left(\partial_\mu\partial_\nu\phi_0 - \eta_{\mu\nu}\phi_0\right) \\ & + \frac{1}{M_4} h_{\mu\nu,0} t_0^{\mu\nu} + \frac{2}{M_4} A_{\mu,0} j_0^\mu + \frac{1}{M_4}\phi_0 j_0 \\ & + \sum_{n=1}^{\infty} h'^{*}_{\mu\nu,n} \mathcal{E}^{\mu\nu,\alpha\beta} h'_{\alpha\beta,n} - \omega_n^2 \left(\left|h'_{\mu\nu,n}\right|^2 - \left|h'_n\right|^2\right) + \left[\frac{1}{M_4} h'_{\mu\nu,n} t_n^{\mu\nu *} + c.c.\right] \end{aligned}$$

# Compact Extra Dimensions : A primer

## Orbifolded Torus




$$G_{MN}^{(5\mathrm{DOT})} = \begin{pmatrix} e^{\frac{-\kappa\hat{r}}{\sqrt{6}}}\left(\eta_{\mu\nu} + \kappa\hat{h}_{\mu\nu}\right) & 0 \\ 0 & -\left(1 + \frac{\kappa\hat{r}}{\sqrt{6}}\right)^2 \end{pmatrix}$$




$$\boxed{S = M_5^3 \int d^4x\, dy \sqrt{G} R^{(5)}}$$

**Integrate over extra dimension: 4D EFT with a cut-off**

## KK Mode Expansions

$$h_{\mu\nu}(x,y) = \sum_{n=-\infty} h_{\mu\nu,n}(x) e^{i\omega_n y}$$

$$h_{\mu 5}(x,y) \cancel{=} \sum_{n=-\infty}^{\infty} \rho_\mu(x,n) e^{i\omega_n y}$$

$$h_{55}(x,y) = \sum_{n=-\infty}^{\infty} r(x,n) e^{i\omega_n y}$$

$$\begin{aligned} S = \int d^4x \quad & \frac{1}{2} h_{\mu\nu,0} \mathcal{E}^{\mu\nu,\alpha\beta} h_{\alpha\beta,0} - \cancel{\frac{1}{2} F_{\mu\nu,0}^2} + h_0^{\mu\nu}\left(\partial_\mu\partial_\nu\phi_0 - \eta_{\mu\nu}\phi_0\right) \\ & + \frac{1}{M_4} h_{\mu\nu,0} t_0^{\mu\nu} + \cancel{\frac{2}{M_4} A_{\mu,0} j_0^\mu} + \frac{1}{M_4}\phi_0 j_0 \\ & + \sum_{n=1}^{\infty} h'^{*}_{\mu\nu,n} \mathcal{E}^{\mu\nu,\alpha\beta} h'_{\alpha\beta,n} - \omega_n^2 \left(\left|h'_{\mu\nu,n}\right|^2 - \left|h'_n\right|^2\right) + \left[\frac{1}{M_4} h'_{\mu\nu,n} t_n^{\mu\nu *} + c.c.\right] \end{aligned}$$

# Compact Extra Dimensions : A Primer

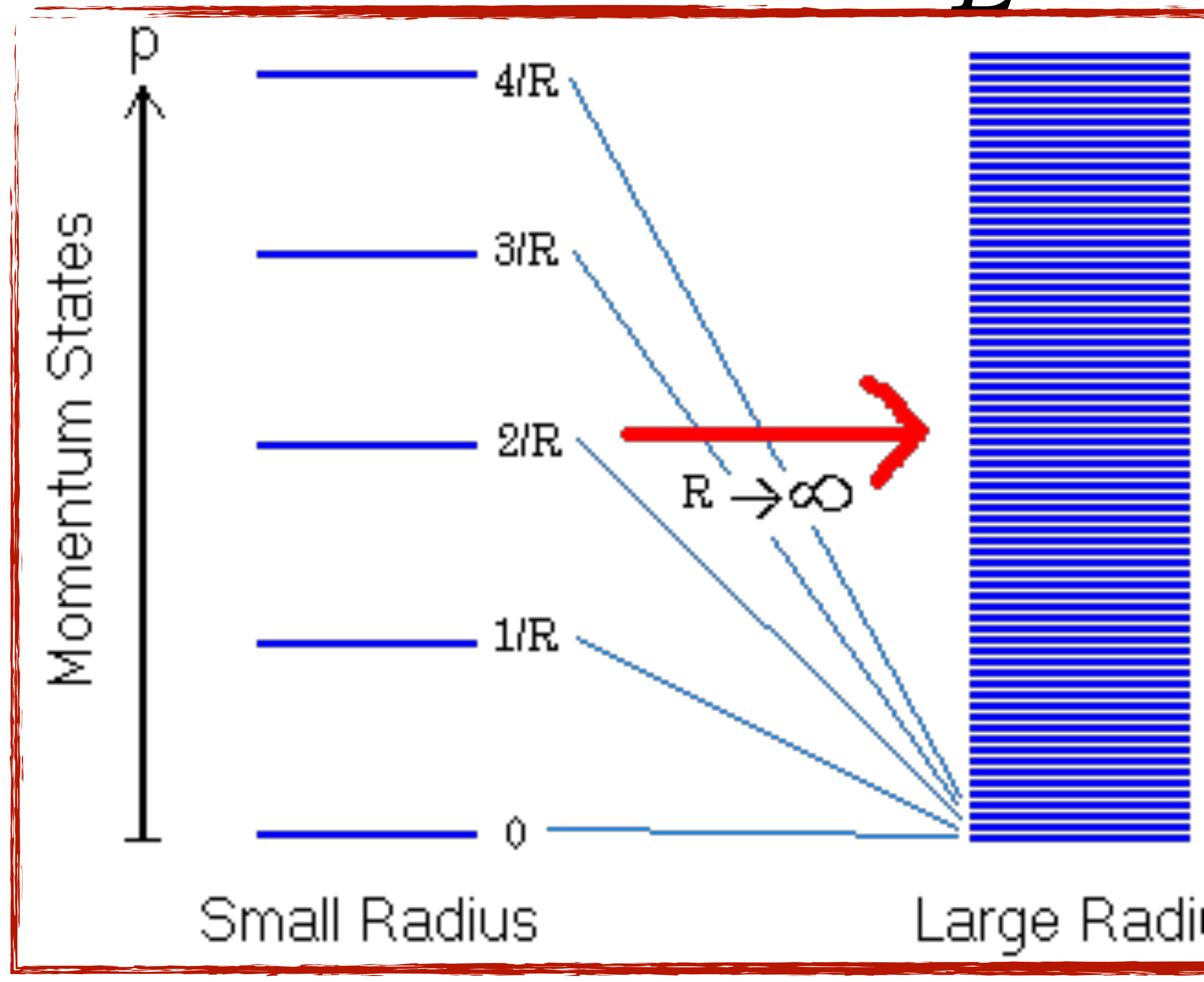
# Compact Extra Dimensions : A Primer

(i) A $5D$ field appears as a tower of KK modes from $4D$ point of view, with each mode having a profile in the extra dimension.

(ii) The profiles and the KK masses are obtained by solving an eigenvalue problem (or wave equations in $5D$ space-time).

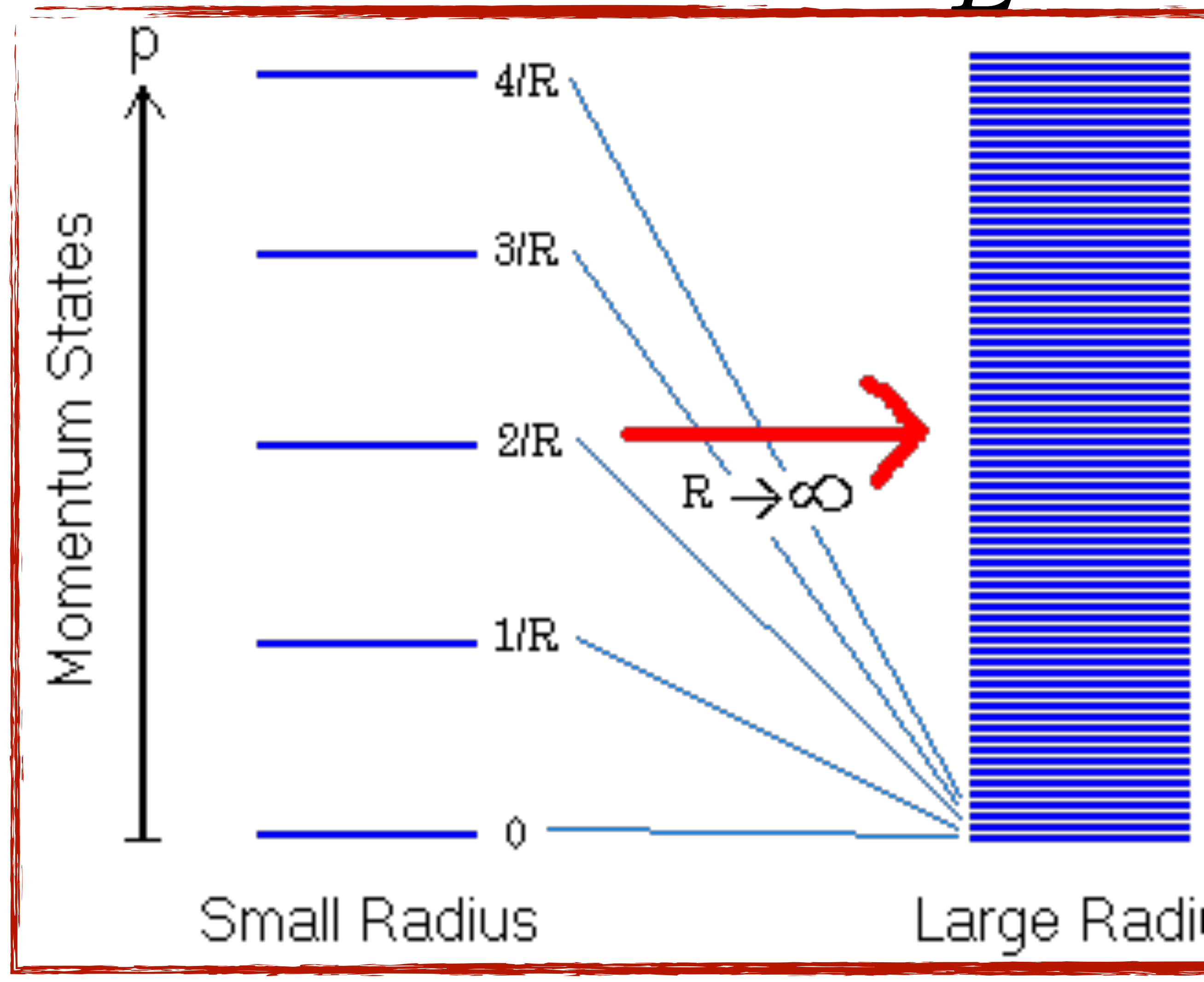
(iii) The coupling of particles (i.e., zero and KK modes) is proportional to the overlap of their profiles in the extra dimension.

$$m_n = \omega_n = \frac{2\pi n}{L}$$



# Compact Extra Dimensions : A Primer

$$m_n = \omega_n = \frac{2\pi n}{L}$$




(i) A $5D$ field appears as a tower of KK modes from $4D$ point of view, with each mode having a profile in the extra dimension.

(ii) The profiles and the KK masses are obtained by solving an eigenvalue problem (or wave equations in $5D$ space-time).

(iii) The coupling of particles (i.e., zero and KK modes) is proportional to the overlap of their profiles in the extra dimension.

- The 4D graviton and its KK modes $\left(\begin{array}{c|c} G^{\vec{k}}_{\mu\nu} & \\ \hline & \end{array}\right)$

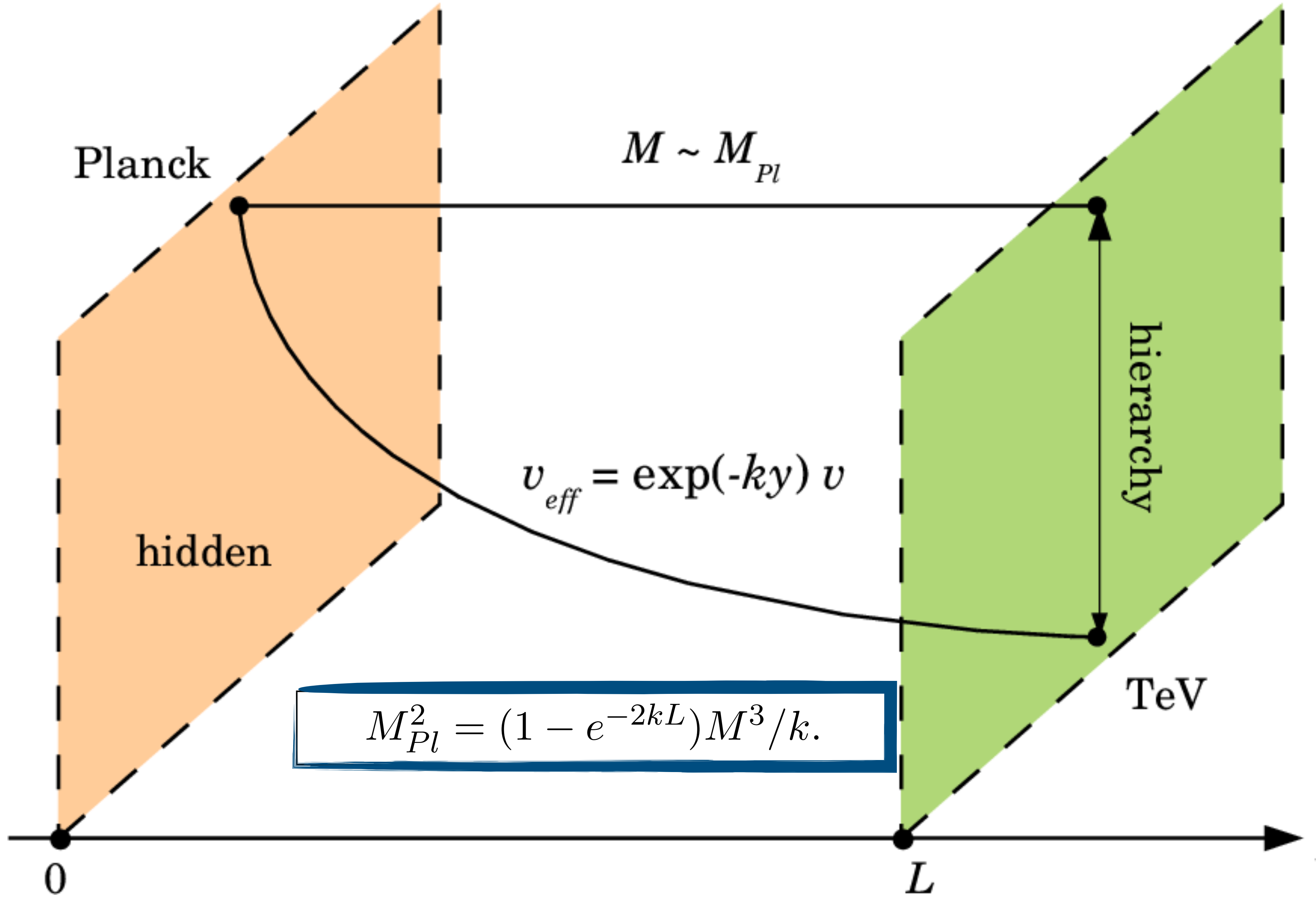
- 4D vectors and their KK modes $\left(\begin{array}{c|c} & V^{\vec{k}}_{\mu j} \\ \hline V^{\vec{k}}_{\mu j} & \end{array}\right)$

  Vanishes on Orbifold,
  Higher Modes get Higgsed

- 4D scalars and their KK modes $\left(\begin{array}{c|c} & \\ \hline & S^{\vec{k}}_{ij} \end{array}\right)$

  Higher modes of scalar get Higgsed,
  leaving behind a single field with a flat wave function
  (radion/dilaton/modulus/breathing mode)

# Compact Extra Dimensions : Randall-Sundrum Models



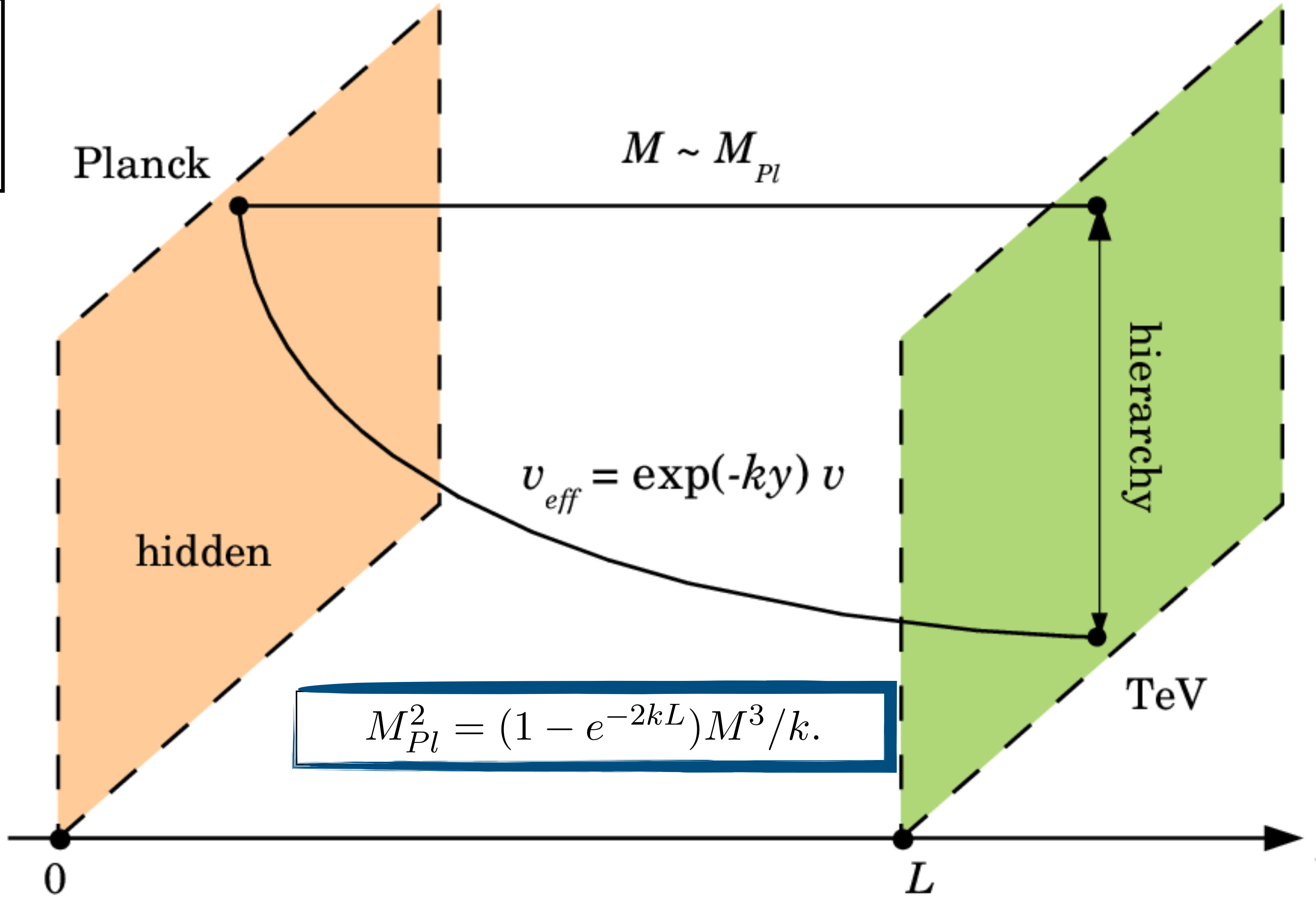
# Compact Extra Dimensions : Randall-Sundrum Models

$$\eta^{(\mathrm{RS})}_{MN} \equiv \begin{pmatrix} e^{-2k|y|}\eta_{\mu\nu} & 0 \\ 0 & -1 \end{pmatrix}$$

Einstein Frame

$$G^{(\mathrm{RS})}_{MN} = \begin{pmatrix} e^{-2(k|y|+\hat{u})}(\eta_{\mu\nu} + \kappa\hat{h}_{\mu\nu}) & 0 \\ 0 & -(1+2\hat{u})^2 \end{pmatrix}$$

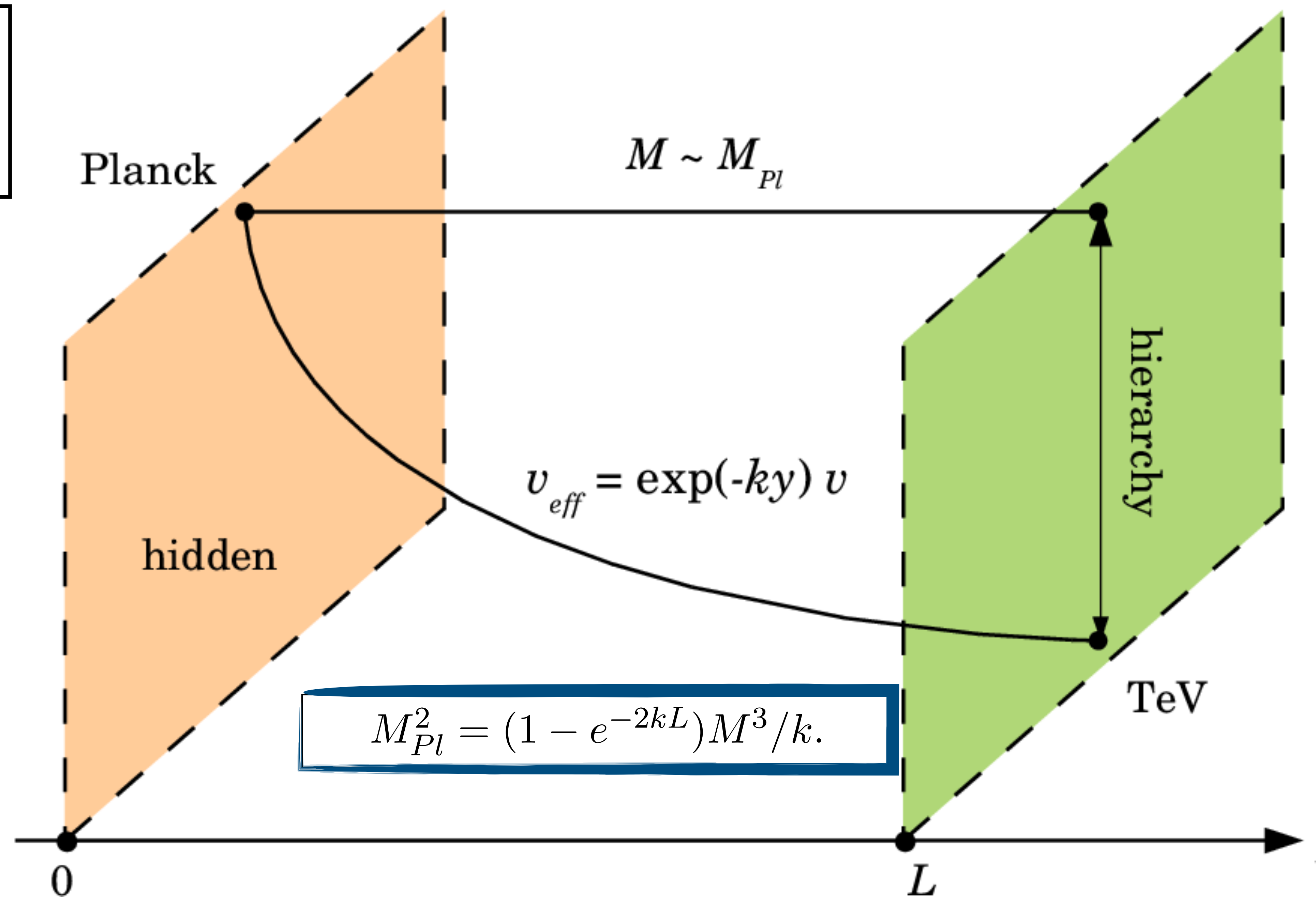
$$\hat{u}(x,y) \equiv \frac{\kappa\,\hat{r}}{2\sqrt{6}}\, e^{+k(2|y|-\pi r_c)}$$



# Compact Extra Dimensions : Randall-Sundrum Models

$$\eta^{(\mathrm{RS})}_{MN} \equiv \begin{pmatrix} e^{-2k|y|}\eta_{\mu\nu} & 0 \\ 0 & -1 \end{pmatrix} \xrightarrow{\text{Einstein Frame}} G^{(\mathrm{RS})}_{MN} = \begin{pmatrix} e^{-2(k|y|+\hat{u})}(\eta_{\mu\nu}+\kappa\hat{h}_{\mu\nu}) & 0 \\ 0 & -(1+2\hat{u})^2 \end{pmatrix}$$

$$\hat{u}(x,y) \equiv \frac{\kappa\,\hat{r}}{2\sqrt{6}}\, e^{+k(2|y|-\pi r_c)}$$




## Conformal co-ordinates : patch of AdS

$$ds^2 = \frac{L^2}{z^2}(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2) \xrightarrow{\text{Invariance}} z \to \lambda z,\ x^\mu \to \lambda x^\mu$$

**Mass spectrum** $m_n = k x_n e^{-k r_c \pi}$ **TeV scale masses for $kr_c$ =11-12**

# Wave Functions on a Torus/ADS

# Wave Functions on a Torus/ADS

$$S = \int d^4x[dy\mathcal{L}_{5D}] \equiv \int d^4x\mathcal{L}_{4D}^{(\text{eff})}$$

$$\boxed{\hat{h}_{\mu\nu}(x,y) = \frac{1}{\sqrt{\pi r_c}}\sum_{n=0}^{+\infty}\hat{h}_{\mu\nu}^{(n)}(x)\psi_n(\varphi)}$$

$$\boxed{\underbrace{\hat{r}(x)}_{\text{5D field}} = \frac{1}{\sqrt{\pi r_c}}\underbrace{\hat{r}^{(0)}(x)}_{\text{4D fields}}\underbrace{\psi_0}_{\text{wfxn}}}$$

**Radion KK Decomposition**

# Wave Functions on a Torus/ADS

$$S = \int d^4x[dy\mathcal{L}_{5D}] \equiv \int d^4x\mathcal{L}_{4D}^{(\text{eff})}$$

$$\boxed{\hat{h}_{\mu\nu}(x,y) = \frac{1}{\sqrt{\pi r_c}}\sum_{n=0}^{+\infty}\hat{h}_{\mu\nu}^{(n)}(x)\psi_n(\varphi)}$$

$$\boxed{\underbrace{\hat{r}(x)}_{\text{5D field}} = \frac{1}{\sqrt{\pi r_c}}\underbrace{\hat{r}^{(0)}(x)}_{\text{4D fields}}\underbrace{\psi_0}_{\text{wfxn}}}$$

**Radion KK Decomposition**

- is **orthonormal+complete** with **discrete spectrum** $\mu_n$
- takes 5D graviton $\to$ 4D graviton + massive spin-2 tower

Describes a Hilbert space of wave Functions

# Wave Functions on a Torus/ADS

$$S = \int d^4x[dy\mathcal{L}_{5\text{D}}] \equiv \int d^4x\mathcal{L}_{4\text{D}}^{(\text{eff})}$$

$$\boxed{\hat{h}_{\mu\nu}(x,y) = \frac{1}{\sqrt{\pi r_c}}\sum_{n=0}^{+\infty}\hat{h}_{\mu\nu}^{(n)}(x)\psi_n(\varphi)}$$

$$\boxed{\underbrace{\hat{r}(x)}_{\text{5D field}} = \frac{1}{\sqrt{\pi r_c}}\underbrace{\hat{r}^{(0)}(x)}_{\text{4D fields}}\underbrace{\psi_0}_{\text{wfxn}}}$$

**Radion KK Decomposition**

- is **orthonormal+complete** with **discrete spectrum** $\mu_n$
- takes 5D graviton $\rightarrow$ 4D graviton + massive spin-2 tower

Describes a Hilbert space of wave Functions

Schrodinger Equation

$$\boxed{-\frac{d}{dy}\left[e^{-4k|y|}\frac{d\psi_n}{dy}\right] = m_n^2 e^{-2k|y|}\psi_n}$$

Orthonormality

$$\boxed{\frac{1}{\pi r_c}\int_{-\pi r_e}^{+\pi r_e} dy e^{-2k|y|}\psi_m(y)\psi_n(y) = \delta_{mn}}$$

Completeness

$$\boxed{\frac{1}{\pi r_c}e^{-2k|y|}\sum_j \psi_j(y)\psi_j(y') = \delta(y-y')}$$

# Wave Functions on a Torus/ADS

$$S = \int d^4x[dy\mathcal{L}_{5\mathrm{D}}] \equiv \int d^4x\mathcal{L}_{4\mathrm{D}}^{(\mathrm{eff})}$$

$$\boxed{\hat{h}_{\mu\nu}(x,y) = \frac{1}{\sqrt{\pi r_c}}\sum_{n=0}^{+\infty}\hat{h}_{\mu\nu}^{(n)}(x)\psi_n(\varphi)}$$

$$\boxed{\underbrace{\hat{r}(x)}_{\text{5D field}} = \frac{1}{\sqrt{\pi r_c}}\underbrace{\hat{r}^{(0)}(x)}_{\text{4D fields}}\underbrace{\psi_0}_{\text{wfxn}}}$$

**Radion KK Decomposition**

- is **orthonormal+complete** with **discrete spectrum** $\mu_n$
- takes 5D graviton $\rightarrow$ 4D graviton + massive spin-2 tower

Describes a Hilbert space of wave Functions

Schrodinger Equation

$$\boxed{-\frac{d}{dy}\left[e^{-4k|y|}\frac{d\psi_n}{dy}\right] = m_n^2 e^{-2k|y|}\psi_n}$$

Orthonormality

$$\boxed{\frac{1}{\pi r_c}\int_{-\pi r_e}^{+\pi r_e} dy e^{-2k|y|}\psi_m(y)\psi_n(y) = \delta_{mn}}$$

Completeness

$$\boxed{\frac{1}{\pi r_c}e^{-2k|y|}\sum_j \psi_j(y)\psi_j(y') = \delta(y-y')}$$

Solutions

Torus

$$\psi_n = \begin{cases}\psi_0 = \frac{1}{\sqrt{2}} \\ \psi_n = -\cos(n|\varphi|)\end{cases}$$

$$\mu_n = m_n r_c = n$$

ADS

$$\psi_0 = \sqrt{\frac{\pi k r_c}{1-e^{-2\pi k r_c}}}$$

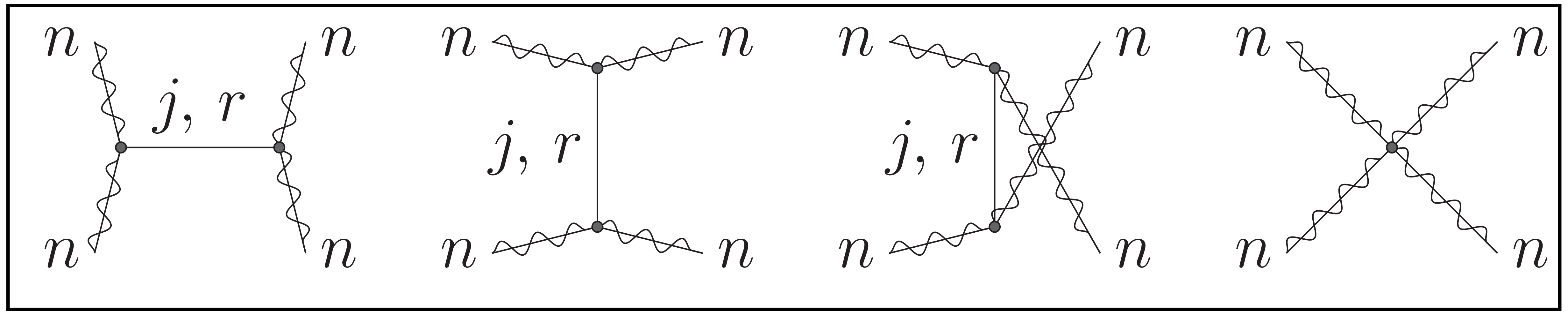
$$\psi_n(y) = \frac{e^{+2k|y|}}{N_n}\left[J_2\left(\frac{m_n}{k}e^{+k|y|}\right) + b_{n2}Y_2\left(\frac{m_n}{k}e^{+k|y|}\right)\right]$$

$$\boxed{m_n = kx_n e^{-kr_c\pi}}$$

# Amplitudes and Coupling Structures on a Torus/ADS

# Amplitudes and Coupling Structures on a Torus/ADS

An Elastic scattering process in compactified theories

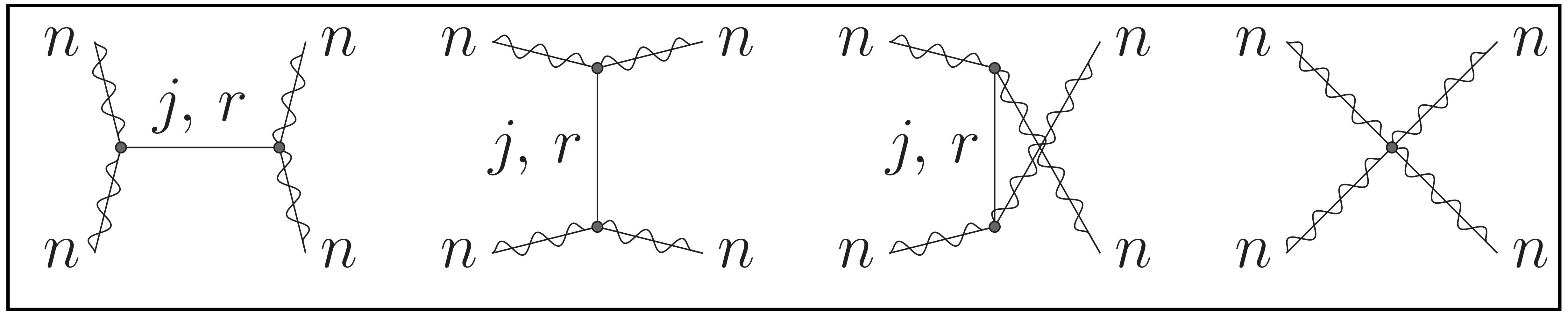



$$R_{5\mathrm{D}} = \tilde{G}^{MN} R_{MN}$$

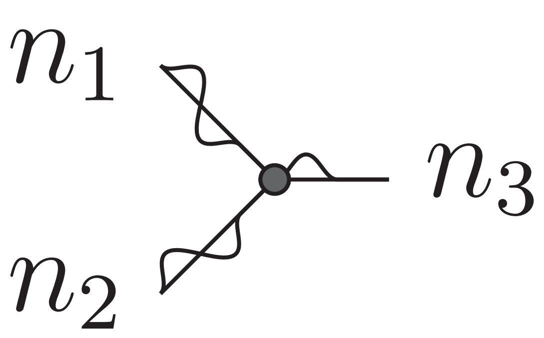
# Amplitudes and Coupling Structures on a Torus/ADS

An Elastic scattering process in compactified theories

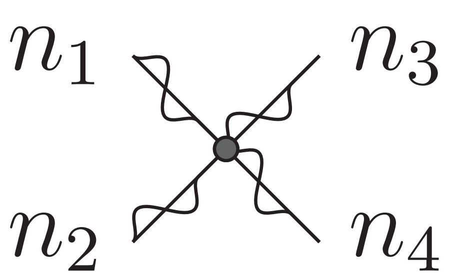
$$R_{5\mathrm{D}} = \tilde{G}^{MN} R_{MN}$$



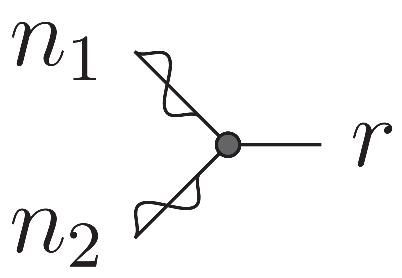

Expansion of Ricci gives two different types of coupling structures




$\supset \quad a_{n_1 n_2 n_3} \quad b_{\mathcal{P}[n_1, n_2, n_3]}$



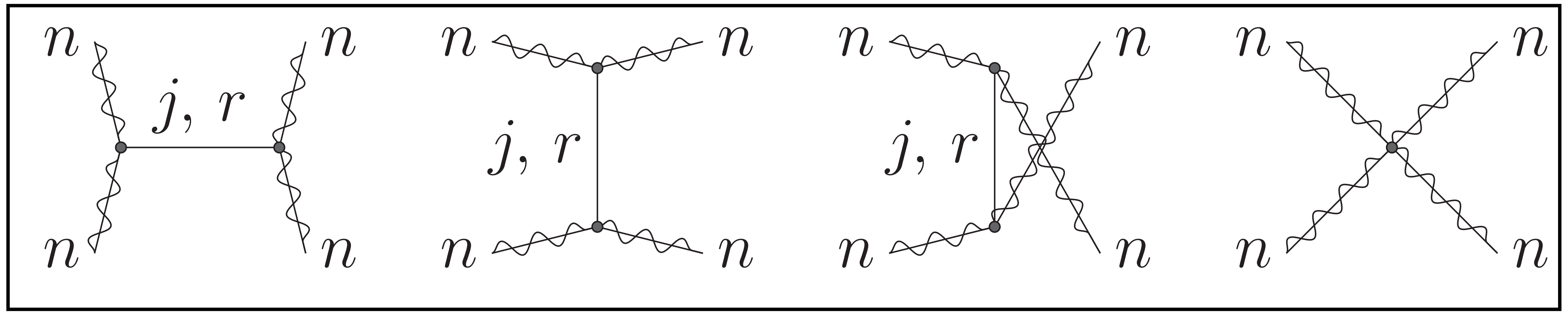

$\supset \quad a_{n_1 n_2 n_3 n_4} \quad b_{\mathcal{P}[n_1, n_2, n_3, n_4]}$




$\supset \quad b_{n_1 n_2 r}$

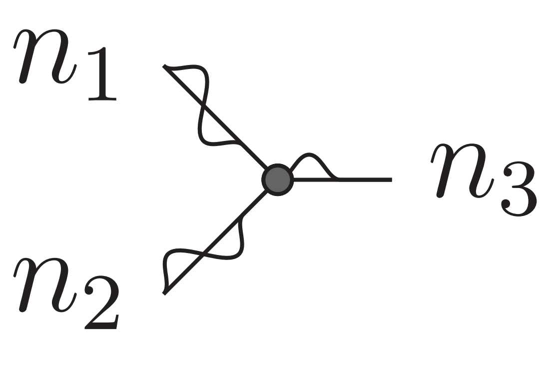
# Amplitudes and Coupling Structures on a Torus/ADS

## An Elastic scattering process in compactified theories

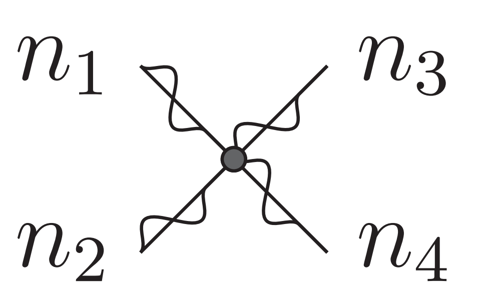



$$R_{5\text{D}} = \tilde{G}^{MN} R_{MN}$$

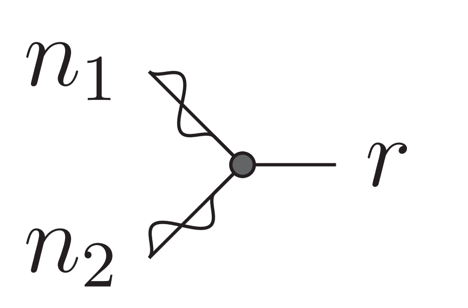
Expansion of Ricci gives two different types of coupling structures




$\supset \quad a_{n_1 n_2 n_3} \quad b_{\mathcal{P}[n_1,n_2,n_3]}$




$\supset \quad a_{n_1 n_2 n_3 n_4} \quad b_{\mathcal{P}[n_1,n_2,n_3,n_4]}$




$\supset \quad b_{n_1 n_2 r}$

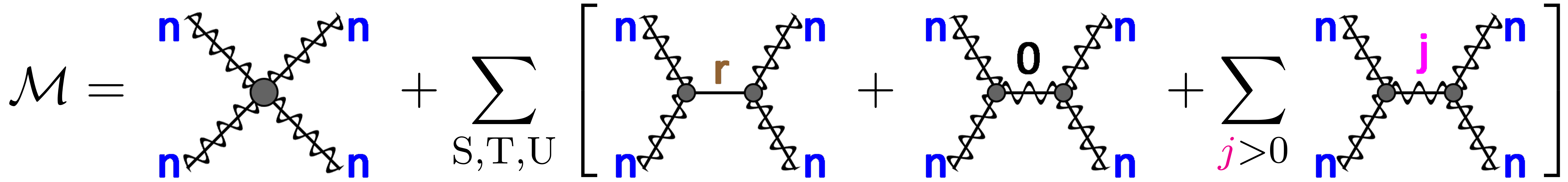
Define $\mathcal{L}^{(\text{RS})}_{h^H r^R} \equiv$ all terms in $\mathcal{L}^{(\text{RS})}_{5\text{D}}$ with $H$ graviton fields and $R$ radion fields. By construction, each term in this set is either

- **A-Type:** has two spatial derivatives $\partial_\mu \partial_\nu$, or
- **B-Type:** has two extra-dimensional derivatives $\partial_y^2$

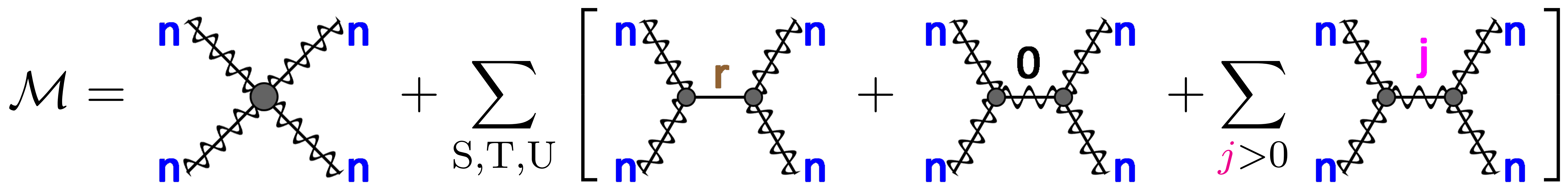
$$\mathcal{L}^{(\text{RS})}_{h^H r^R} = \mathcal{L}^{(\text{RS})}_{A:h^H r^R} + \mathcal{L}^{(\text{RS})}_{B:h^H r^R}$$
$$= \kappa^{(H+R-2)} \left[ e^{k[2(R-1)|y| - R\pi r_c]}\, \overline{\mathcal{L}}^{(\text{RS})}_{A:h^H r^R} + e^{k[2(R-2)|y| - R\pi r_c]}\, \overline{\mathcal{L}}^{(\text{RS})}_{B:h^H r^R} \right]$$

# Amplitudes and Coupling Structures on a Torus/ADS

# Amplitudes and Coupling Structures on a Torus/ADS



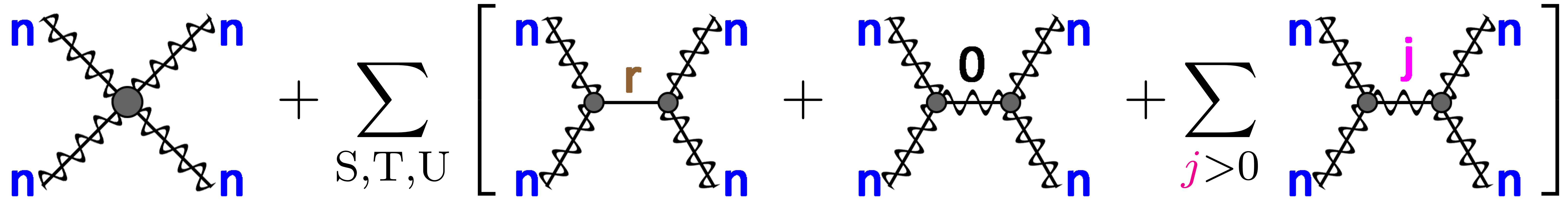
# Amplitudes and Coupling Structures on a Torus/ADS



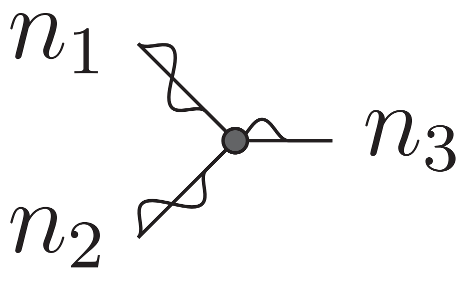

Couplings are proportional to products/overlaps of wave functions and derivatives of them

# Amplitudes and Coupling Structures on a Torus/ADS

$\mathcal{M} =$




Couplings are proportional to products/overlaps of wave functions and derivatives of them

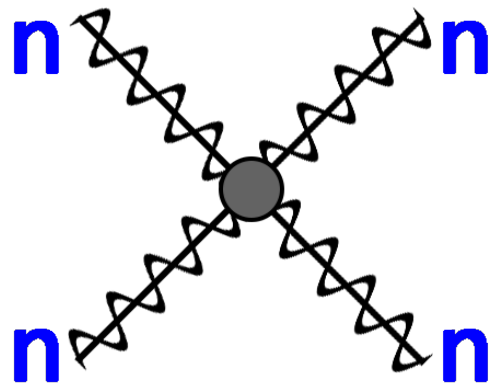
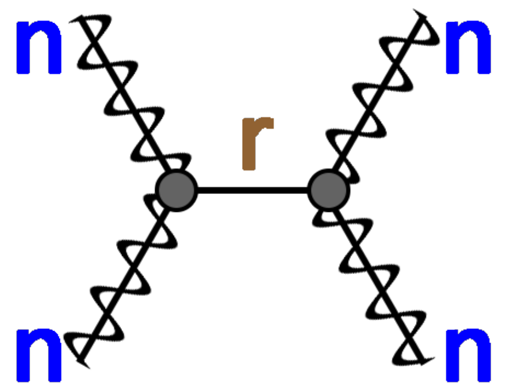
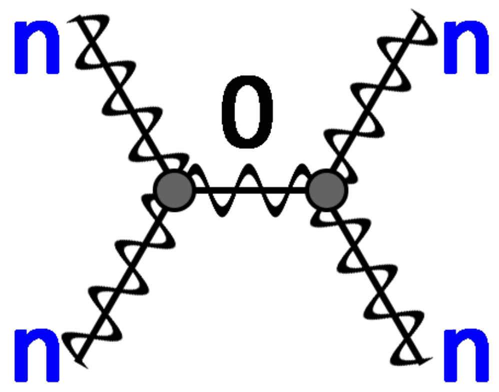
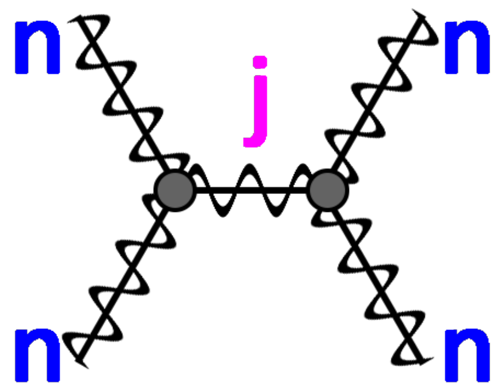



$$\supset \quad a_{n_1 n_2 n_3} \quad b_{\mathcal{P}[n_1,n_2,n_3]}$$

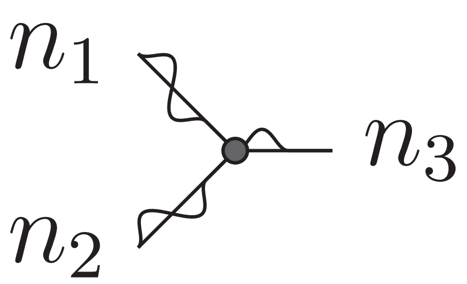
$$a_{nnj} = \frac{1}{\pi r_c}\int_{-\pi r_c}^{\pi r_c} dy e^{-2k|y|}\psi_n(y)\psi_n(y)\psi_j(y)$$

$$b_{nnj} = \frac{r_c}{\pi}\int_{-\pi r_c}^{\pi r_c} dy e^{-4k|y|}(\partial_y\psi_n(y)\partial_y\psi_n(y))\psi_j(y)$$

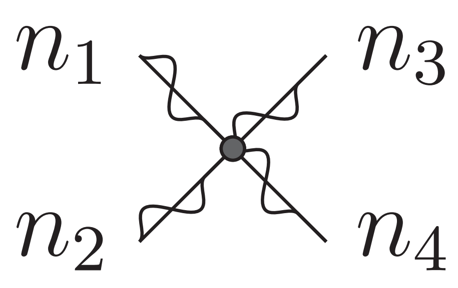
# Amplitudes and Coupling Structures on a Torus/ADS

$\mathcal{M} =$  $+ \sum_{S,T,U} \Bigg[$  $+$  $+ \sum_{j>0}$  $\Bigg]$

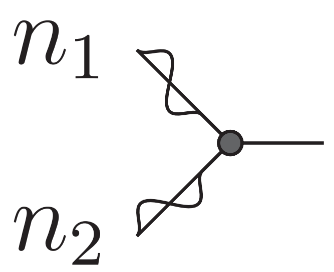

Couplings are proportional to products/overlaps of wave functions and derivatives of them



$\supset \quad a_{n_1 n_2 n_3} \quad b_{\mathcal{P}[n_1,n_2,n_3]}$



$\supset \quad a_{n_1 n_2 n_3 n_4} \quad b_{\mathcal{P}[n_1,n_2,n_3,n_4]}$



$\supset \quad b_{n_1 n_2 r}$

3 and 4 point non-derivative couplings

$$a_{nnj} = \frac{1}{\pi r_c} \int_{-\pi r_c}^{\pi r_c} dy e^{-2k|y|} \psi_n(y) \psi_n(y) \psi_j(y)$$

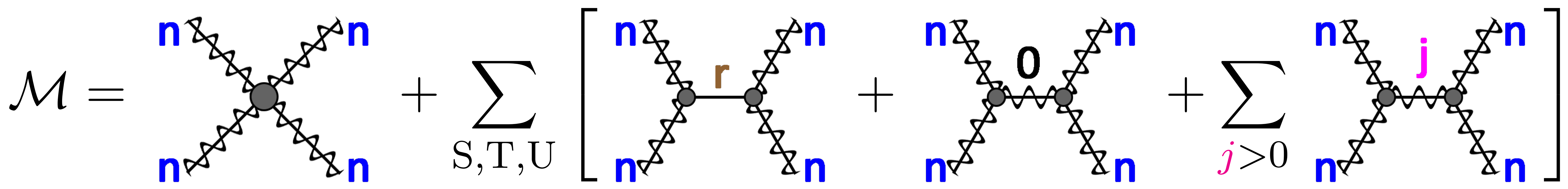
$$a_{nnnn} = \frac{1}{\pi r_c} \int_{-\pi r_c}^{\pi r_c} dy e^{-2k|y|} \psi_n(y) \psi_n(y) \psi_n(y) \psi_n(y)$$

3 and 4 point derivative couplings

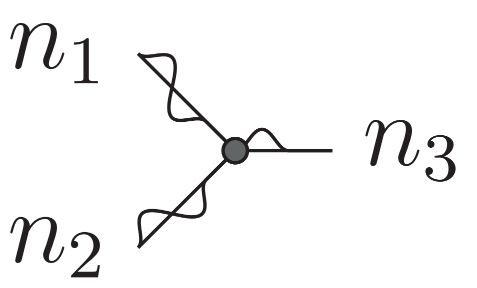
$$b_{nnj} = \frac{r_c}{\pi} \int_{-\pi r_c}^{\pi r_c} dy e^{-4k|y|} (\partial_y \psi_n(y) \partial_y \psi_n(y)) \psi_j(y)$$

$$b_{nnnn} = \frac{r_c}{\pi} \int_{-\pi r_c}^{\pi r_c} dy e^{-4k|y|} (\partial_y \psi_n(y) \partial_y \psi_n(y)) \psi_n(y) \psi_n(y)$$

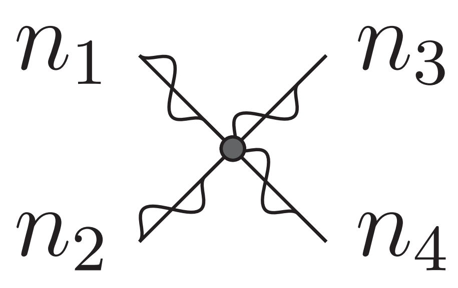
# Amplitudes and Coupling Structures on a Torus/ADS



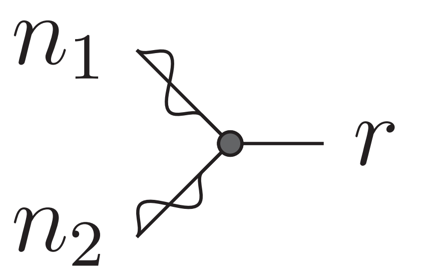

Couplings are proportional to products/overlaps of wave functions and derivatives of them




$\supset \quad a_{n_1 n_2 n_3} \quad b_{\mathcal{P}[n_1,n_2,n_3]}$




$\supset \quad a_{n_1 n_2 n_3 n_4} \quad b_{\mathcal{P}[n_1,n_2,n_3,n_4]}$




$\supset \quad b_{n_1 n_2 r}$

3 and 4 point non-derivative couplings

$$a_{nnj} = \frac{1}{\pi r_c}\int_{-\pi r_c}^{\pi r_c} dy e^{-2k|y|}\psi_n(y)\psi_n(y)\psi_j(y)$$

$$a_{nnnn} = \frac{1}{\pi r_c}\int_{-\pi r_c}^{\pi r_c} dy e^{-2k|y|}\psi_n(y)\psi_n(y)\psi_n(y)\psi_n(y)$$

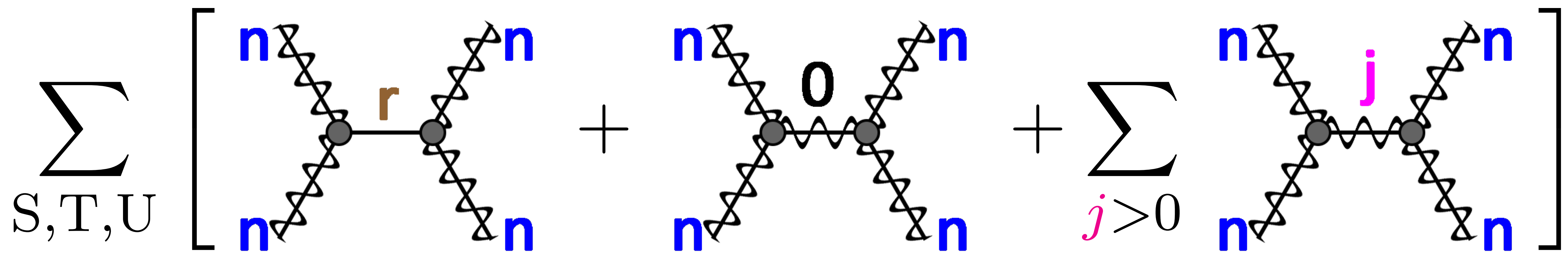
3 and 4 point derivative couplings

$$b_{nnj} = \frac{r_c}{\pi}\int_{-\pi r_c}^{\pi r_c} dy e^{-4k|y|}(\partial_y\psi_n(y)\partial_y\psi_n(y))\psi_j(y)$$

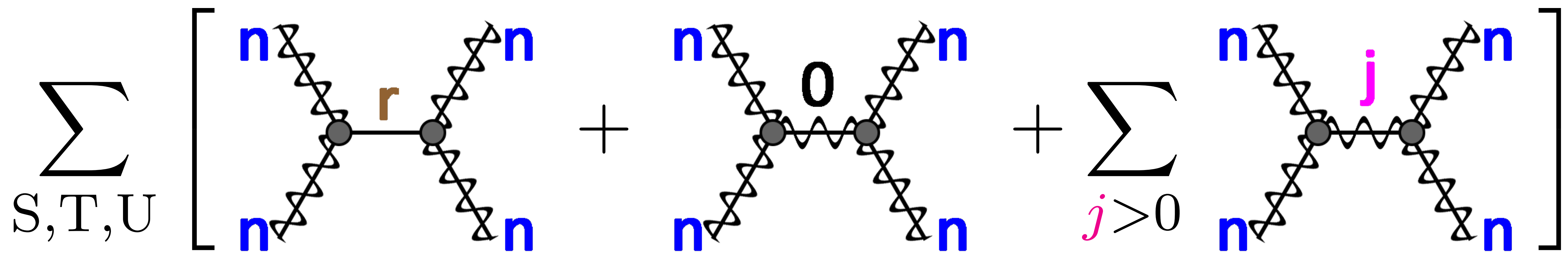
$$b_{nnnn} = \frac{r_c}{\pi}\int_{-\pi r_c}^{\pi r_c} dy e^{-4k|y|}(\partial_y\psi_n(y)\partial_y\psi_n(y))\psi_n(y)\psi_n(y)$$

$$b_{nnr} = \frac{r_c}{\pi}e^{-\pi k r_c}\int_{-\pi r_c}^{\pi r_c} dy e^{-2k|y|}(\partial_y\psi_n(y)\partial_y\psi_n(y))\psi_0$$

# Amplitudes and Coupling Structures on a Torus/ADS



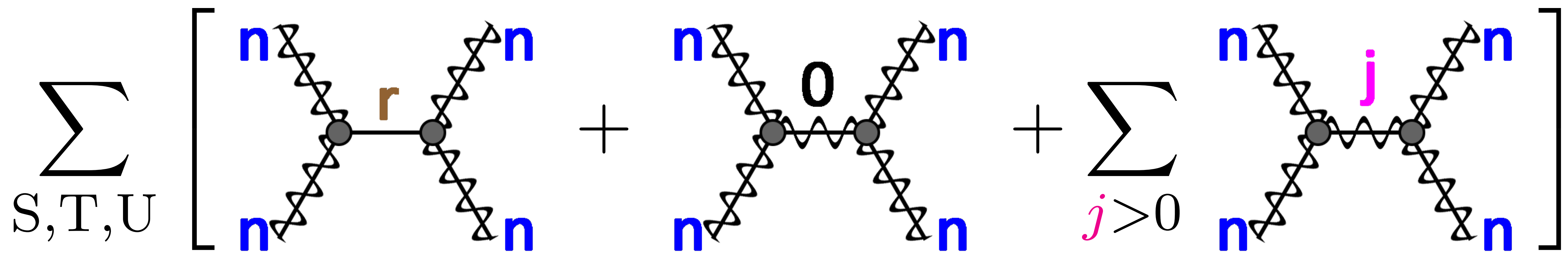
# Amplitudes and Coupling Structures on a Torus/ADS




$$\sum_j a_{nnj}^2 \qquad a_{nnj} = \frac{1}{\pi r_c}\int_{-\pi r_c}^{\pi r_c} dy e^{-2k|y|}\psi_n(y)\psi_n(y)\psi_j(y)$$

$$\frac{1}{\pi r_c} e^{-2k|y|}\sum_j \psi_j(y)\psi_j(y') = \delta(y-y')$$

# Amplitudes and Coupling Structures on a Torus/ADS



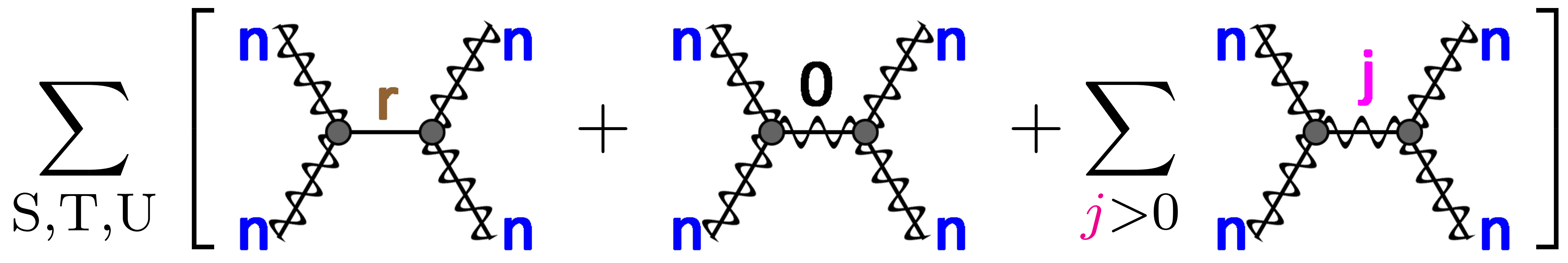

$$\sum_j a_{nnj}^2 \qquad a_{nnj} = \frac{1}{\pi r_c}\int_{-\pi r_c}^{\pi r_c} dy e^{-2k|y|}\psi_n(y)\psi_n(y)\psi_j(y)$$

$$\frac{1}{\pi r_c} e^{-2k|y|}\sum_j \psi_j(y)\psi_j(y') = \delta(y-y')$$

Product of two 3 point a couplings is a 4 point a-type coupling

$$\boxed{a_{nnnn} = \sum_j a_{nnj}^2}$$

# Amplitudes and Coupling Structures on a Torus/ADS




$$\sum_j a_{nnj}^2 \qquad a_{nnj} = \frac{1}{\pi r_c} \int_{-\pi r_c}^{\pi r_c} dy e^{-2k|y|} \psi_n(y) \psi_n(y) \psi_j(y)$$

$$\frac{1}{\pi r_c} e^{-2k|y|} \sum_j \psi_j(y) \psi_j(y') = \delta(y - y')$$

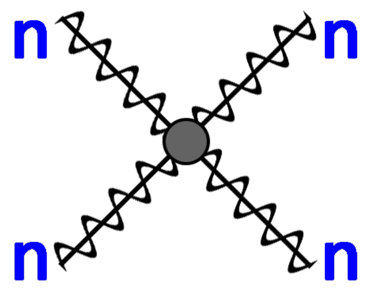
Product of two 3 point a couplings is a 4 point a-type coupling

$$\boxed{a_{nnnn} = \sum_j a_{nnj}^2}$$

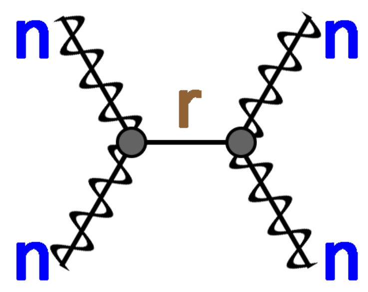
$$\boxed{m_n^2 a_{nnnn} = \frac{3}{4} \sum_j m_j^2 a_{nnj}^2} \qquad \boxed{b_{nnnn} = \frac{1}{3} m_n^2 r_c^2 a_{nnnn}}$$

# Amplitudes and Coupling Structures on a Torus/ADS

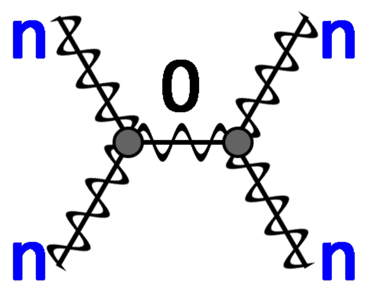
$$\mathcal{M} =$$



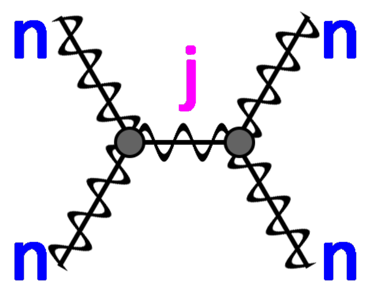

$$+ \sum_{\mathrm{S,T,U}} \Bigg[$$




$$+$$

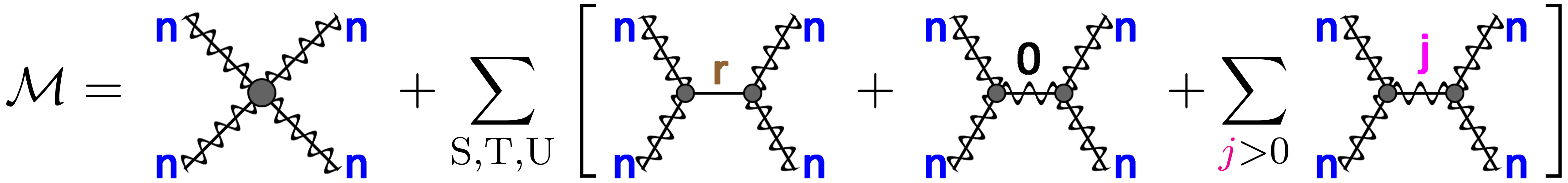



$$+ \sum_{j>0}$$




$$\Bigg]$$

# Amplitudes and Coupling Structures on a Torus/ADS




$$\mathcal{M}(s,\cos\theta) \equiv \sum_{k=-\infty}^{+5} \overline{\mathcal{M}}^{(k)}(\cos\theta)\cdot s^k$$

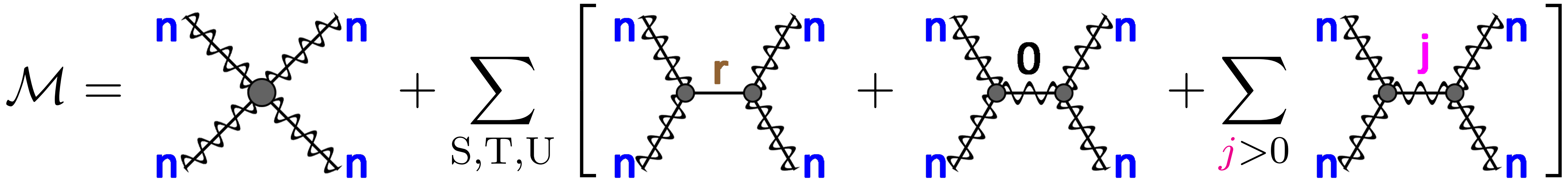
$$\epsilon^{\mu\nu}_{\pm 2} = \epsilon^{\mu}_{\pm 1}\epsilon^{\nu}_{\pm 1},$$

$$\epsilon^{\mu\nu}_{\pm 1} = \frac{1}{\sqrt{2}}[\epsilon^{\mu}_{\pm 1}\epsilon^{\nu}_{0} + \epsilon^{\mu}_{0}\epsilon^{\nu}_{\pm 1}],$$

$$\epsilon^{\mu\nu}_{0} = \frac{1}{\sqrt{6}}[\epsilon^{\mu}_{+1}\epsilon^{\nu}_{-1} + \epsilon^{\mu}_{-1}\epsilon^{\nu}_{+1} + 2\epsilon^{\mu}_{0}\epsilon^{\nu}_{0}]$$

$$\epsilon^{\mu}_{\pm 1} = \pm\frac{e^{\pm i\phi}}{\sqrt{2}}(0, -c_\theta c_\phi \pm i s_\phi, -c_\theta s_\phi$$

$$\epsilon^{\mu}_{0} = \frac{E}{m}\left(\sqrt{1-\frac{m^2}{E^2}}, \hat{p}\right),$$




$$\mathcal{M}(s,\cos\theta) \equiv \sum_{k=-\infty}^{+5} \overline{\mathcal{M}}^{(k)}(\cos\theta)\cdot s^k$$

## O(s⁵) & O(s⁴) Sum Rules

$$\mathcal{M}^{(5)}(\cos\theta) = -\frac{\kappa^2}{\pi r_c}\frac{(7+\cos 2\theta)\sin^2\theta}{2304 m_n^8}\cdot\left(a_{nnnn} - \sum_j a_{nnj}^2\right)$$

$$m_n^2 a_{nnnn} = \frac{3}{4}\sum_j m_j^2 a_{nnj}^2$$

$$\mathcal{M}^{(4)}(\cos\theta) = \frac{\kappa^2}{\pi r_c}\frac{(7+\cos 2\theta)^2}{27648 m_n^8}\cdot\left(4m_n^2 a_{nnnn} - 3\sum_j m_j^2 a_{nnj}^2\right)$$

$$m_n^2 a_{nnnn} = \frac{3}{4}\sum_j m_j^2 a_{nnj}^2$$

$$\epsilon^{\mu\nu}_{\pm 2} = \epsilon^{\mu}_{\pm 1}\epsilon^{\nu}_{\pm 1},$$

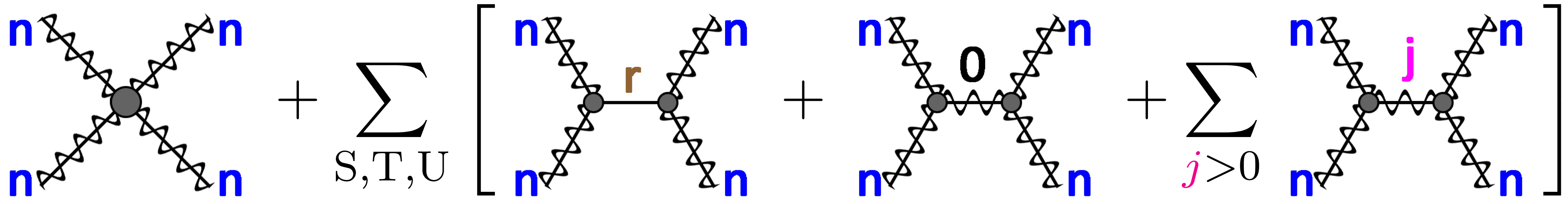
$$\epsilon^{\mu\nu}_{\pm 1} = \frac{1}{\sqrt{2}}[\epsilon^{\mu}_{\pm 1}\epsilon^{\nu}_{0} + \epsilon^{\mu}_{0}\epsilon^{\nu}_{\pm 1}],$$

$$\epsilon^{\mu\nu}_{0} = \frac{1}{\sqrt{6}}[\epsilon^{\mu}_{+1}\epsilon^{\nu}_{-1} + \epsilon^{\mu}_{-1}\epsilon^{\nu}_{+1} + 2\epsilon^{\mu}_{0}\epsilon^{\nu}_{0}]$$

$$\epsilon^{\mu}_{\pm 1} = \pm\frac{e^{\pm i\phi}}{\sqrt{2}}(0, -c_\theta c_\phi \pm i s_\phi, -c_\theta s_\phi$$

$$\epsilon^{\mu}_{0} = \frac{E}{m}\left(\sqrt{1-\frac{m^2}{E^2}}, \hat{p}\right),$$

# Amplitudes and Coupling Structures on a Torus/ADS

$\mathcal{M} =$ 


$$\mathcal{M}(s,\cos\theta) \equiv \sum_{k=-\infty}^{+5} \overline{\mathcal{M}}^{(k)}(\cos\theta)\cdot s^k$$

## O(s⁵) & O(s⁴) Sum Rules

$$\mathcal{M}^{(5)}(\cos\theta) = -\frac{\kappa^2}{\pi r_c}\frac{(7+\cos 2\theta)\sin^2\theta}{2304 m_n^8}\cdot\left(a_{nnnn} - \sum_j a_{nnj}^2\right)$$

$$m_n^2 a_{nnnn} = \frac{3}{4}\sum_j m_j^2 a_{nnj}^2$$

$$\mathcal{M}^{(4)}(\cos\theta) = \frac{\kappa^2}{\pi r_c}\frac{(7+\cos 2\theta)^2}{27648 m_n^8}\cdot\left(4m_n^2 a_{nnnn} - 3\sum_j m_j^2 a_{nnj}^2\right)$$

$$m_n^2 a_{nnnn} = \frac{3}{4}\sum_j m_j^2 a_{nnj}^2$$

$$\epsilon^{\mu\nu}_{\pm 2} = \epsilon^\mu_{\pm 1}\epsilon^\nu_{\pm 1},$$

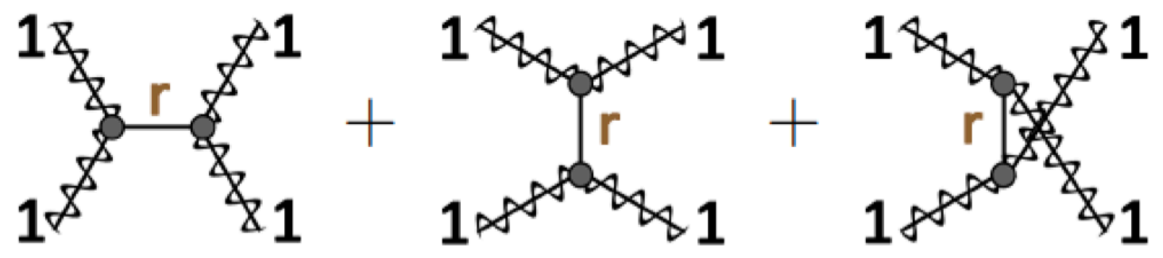
$$\epsilon^{\mu\nu}_{\pm 1} = \frac{1}{\sqrt{2}}[\epsilon^\mu_{\pm 1}\epsilon^\nu_0 + \epsilon^\mu_0\epsilon^\nu_{\pm 1}],$$

$$\epsilon^{\mu\nu}_0 = \frac{1}{\sqrt{6}}[\epsilon^\mu_{+1}\epsilon^\nu_{-1} + \epsilon^\mu_{-1}\epsilon^\nu_{+1} + 2\epsilon^\mu_0\epsilon^\nu_0]$$

$$\epsilon^\mu_{\pm 1} = \pm\frac{e^{\pm i\phi}}{\sqrt{2}}(0, -c_\theta c_\phi \pm i s_\phi, -c_\theta s_\phi$$

$$\epsilon^\mu_0 = \frac{E}{m}\left(\sqrt{1-\frac{m^2}{E^2}}, \hat{p}\right),$$

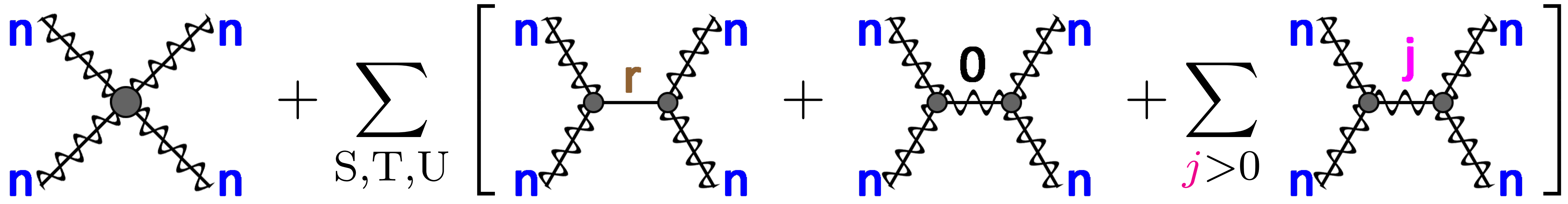
## O(s³): Radion Coupling

$\mathcal{M}_r =$ 


$$\mathcal{M}^{(3)}(\cos\theta) = \frac{\kappa^2}{\pi r_c}\frac{\sin^2\theta}{3456 m_n^8}\cdot\left(-108\frac{b_{nnr}^2}{r_c^4} + 12 m_n^4 a_{nn0}^2 - 16 m_n^4 a_{nnnn} + 15\sum_j m_j^4 a_{nnj}^2\right)$$

# Amplitudes and Coupling Structures on a Torus/ADS

$\mathcal{M} =$




$$\mathcal{M}(s,\cos\theta) \equiv \sum_{k=-\infty}^{+5} \overline{\mathcal{M}}^{(k)}(\cos\theta)\cdot s^k$$

## O(s⁵) & O(s⁴) Sum Rules

$$\mathcal{M}^{(5)}(\cos\theta) = -\frac{\kappa^2}{\pi r_c}\frac{(7+\cos 2\theta)\sin^2\theta}{2304 m_n^8}\cdot\left(a_{nnnn} - \sum_j a_{nnj}^2\right)$$

$$m_n^2 a_{nnnn} = \frac{3}{4}\sum_j m_j^2 a_{nnj}^2$$

$$\mathcal{M}^{(4)}(\cos\theta) = \frac{\kappa^2}{\pi r_c}\frac{(7+\cos 2\theta)^2}{27648 m_n^8}\cdot\left(4m_n^2 a_{nnnn} - 3\sum_j m_j^2 a_{nnj}^2\right)$$

$$m_n^2 a_{nnnn} = \frac{3}{4}\sum_j m_j^2 a_{nnj}^2$$

$$\epsilon^{\mu\nu}_{\pm 2} = \epsilon^{\mu}_{\pm 1}\epsilon^{\nu}_{\pm 1},$$

$$\epsilon^{\mu\nu}_{\pm 1} = \frac{1}{\sqrt{2}}[\epsilon^{\mu}_{\pm 1}\epsilon^{\nu}_{0} + \epsilon^{\mu}_{0}\epsilon^{\nu}_{\pm 1}],$$

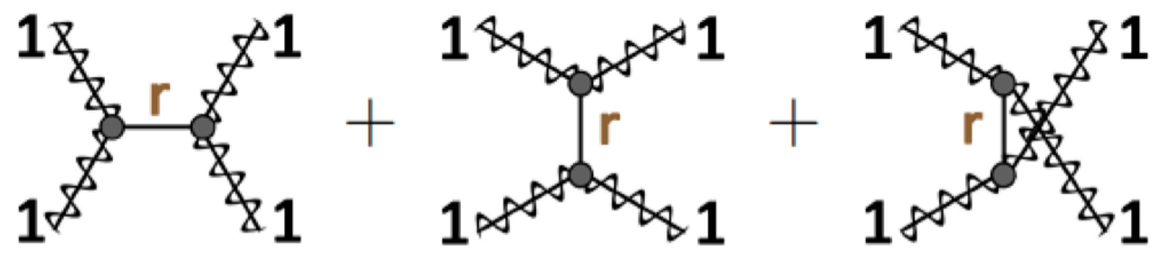
$$\epsilon^{\mu\nu}_{0} = \frac{1}{\sqrt{6}}[\epsilon^{\mu}_{+1}\epsilon^{\nu}_{-1} + \epsilon^{\mu}_{-1}\epsilon^{\nu}_{+1} + 2\epsilon^{\mu}_{0}\epsilon^{\nu}_{0}]$$

$$\epsilon^{\mu}_{\pm 1} = \pm\frac{e^{\pm i\phi}}{\sqrt{2}}(0, -c_\theta c_\phi \pm i s_\phi, -c_\theta s_\phi$$

$$\epsilon^{\mu}_{0} = \frac{E}{m}\left(\sqrt{1-\frac{m^2}{E^2}}, \hat{p}\right),$$

## O(s³): Radion Coupling

$\mathcal{M}_r =$




$$\mathcal{M}^{(3)}(\cos\theta) = \frac{\kappa^2}{\pi r_c}\frac{\sin^2\theta}{3456 m_n^8}\cdot\left(-108\frac{b_{nnr}^2}{r_c^4} + 12 m_n^4 a_{nn0}^2 - 16 m_n^4 a_{nnnn} + 15\sum_j m_j^4 a_{nnj}^2\right)$$

## O(s²)

$$\mathcal{M}^{(2)}(\cos\theta) = -\frac{\kappa^2}{\pi r_c}\frac{(7+\cos 2\theta)}{5184 m_n^8}\cdot\left(-108\frac{b_{nnr}^2 m_n^2}{r_c^4} + 12 m_n^6 a_{nn0}^2 + 16 m_n^6 a_{nnnn} - 15 m_n^2\sum_j m_j^4 a_{nnj}^2 + 6\sum_j m_j^6 a_{nnj}^2\right)$$

$$\sum_j m_j^6 a_{nnj}^2 = 5 m_n^2\sum_j m_j^4 a_{nnj}^2 - \frac{16}{3} m_n^6 a_{nnnn}$$

# Amplitudes and Coupling Structures on a Torus/ADS

## Elastic 2-to-2 KK Mode Scattering Matrix Elements in RS1
## Fastest Energy Growth per Helicity Combination: $(\lambda_1,\lambda_2) \rightarrow (\lambda_3,\lambda_4)$

**BEFORE SUM RULES**

| $\lambda_1$ \ $\lambda_2$ | -2 | | | | | -1 | | | | | 0 | | | | | +1 | | | | | +2 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 2 | 3 | 4 | 3 | 2 | 3 | 4 | 5 | 4 | 3 | 4 | 5 | 6 | 5 | 4 | 3 | 4 | 5 | 4 | 3 | 2 | 3 | 4 | 3 | 2 |
| | 3 | 4 | 5 | 4 | 3 | 4 | 5 | 6 | 5 | 4 | 5 | 6 | 7 | 6 | 5 | 4 | 5 | 6 | 5 | 4 | 3 | 4 | 5 | 4 | 3 |
| -2 | 4 | 5 | 6 | 5 | 4 | 5 | 6 | 7 | 6 | 5 | 6 | 7 | 8 | 7 | 6 | 5 | 6 | 7 | 6 | 5 | 4 | 5 | 6 | 5 | 4 |
| | 3 | 4 | 5 | 4 | 3 | 4 | 5 | 6 | 5 | 4 | 5 | 6 | 7 | 6 | 5 | 4 | 5 | 6 | 5 | 4 | 3 | 4 | 5 | 4 | 3 |
| | 2 | 3 | 4 | 3 | 2 | 3 | 4 | 5 | 4 | 3 | 4 | 5 | 6 | 5 | 4 | 3 | 4 | 5 | 4 | 3 | 2 | 3 | 4 | 3 | 2 |
| | 3 | 4 | 5 | 4 | 3 | 4 | 5 | 6 | 5 | 4 | 5 | 6 | 7 | 6 | 5 | 4 | 5 | 6 | 5 | 4 | 3 | 4 | 5 | 4 | 3 |
| | 4 | 5 | 6 | 5 | 4 | 5 | 6 | 7 | 6 | 5 | 6 | 7 | 8 | 7 | 6 | 5 | 6 | 7 | 6 | 5 | 4 | 5 | 6 | 5 | 4 |
| -1 | 5 | 6 | 7 | 6 | 5 | 6 | 7 | 8 | 7 | 6 | 7 | 8 | 9 | 8 | 7 | 6 | 7 | 8 | 7 | 6 | 5 | 6 | 7 | 6 | 5 |
| | 4 | 5 | 6 | 5 | 4 | 5 | 6 | 7 | 6 | 5 | 6 | 7 | 8 | 7 | 6 | 5 | 6 | 7 | 6 | 5 | 4 | 5 | 6 | 5 | 4 |
| | 3 | 4 | 5 | 4 | 3 | 4 | 5 | 6 | 5 | 4 | 5 | 6 | 7 | 6 | 5 | 4 | 5 | 6 | 5 | 4 | 3 | 4 | 5 | 4 | 3 |
| | 4 | 5 | 6 | 5 | 4 | 5 | 6 | 7 | 6 | 5 | 6 | 7 | 8 | 7 | 6 | 5 | 6 | 7 | 6 | 5 | 4 | 5 | 6 | 5 | 4 |
| | 5 | 6 | 7 | 6 | 5 | 6 | 7 | 8 | 7 | 6 | 7 | 8 | 9 | 8 | 7 | 6 | 7 | 8 | 7 | 6 | 5 | 6 | 7 | 6 | 5 |
| 0 | 6 | 7 | 8 | 7 | 6 | 7 | 8 | 9 | 8 | 7 | 8 | 9 | 10 | 9 | 8 | 7 | 8 | 9 | 8 | 7 | 6 | 7 | 8 | 7 | 6 |
| | 5 | 6 | 7 | 6 | 5 | 6 | 7 | 8 | 7 | 6 | 7 | 8 | 9 | 8 | 7 | 6 | 7 | 8 | 7 | 6 | 5 | 6 | 7 | 6 | 5 |
| | 4 | 5 | 6 | 5 | 4 | 5 | 6 | 7 | 6 | 5 | 6 | 7 | 8 | 7 | 6 | 5 | 6 | 7 | 6 | 5 | 4 | 5 | 6 | 5 | 4 |
| | 3 | 4 | 5 | 4 | 3 | 4 | 5 | 6 | 5 | 4 | 5 | 6 | 7 | 6 | 5 | 4 | 5 | 6 | 5 | 4 | 3 | 4 | 5 | 4 | 3 |
| | 4 | 5 | 6 | 5 | 4 | 5 | 6 | 7 | 6 | 5 | 6 | 7 | 8 | 7 | 6 | 5 | 6 | 7 | 6 | 5 | 4 | 5 | 6 | 5 | 4 |
| +1 | 5 | 6 | 7 | 6 | 5 | 6 | 7 | 8 | 7 | 6 | 7 | 8 | 9 | 8 | 7 | 6 | 7 | 8 | 7 | 6 | 5 | 6 | 7 | 6 | 5 |
| | 4 | 5 | 6 | 5 | 4 | 5 | 6 | 7 | 6 | 5 | 6 | 7 | 8 | 7 | 6 | 5 | 6 | 7 | 6 | 5 | 4 | 5 | 6 | 5 | 4 |
| | 3 | 4 | 5 | 4 | 3 | 4 | 5 | 6 | 5 | 4 | 5 | 6 | 7 | 6 | 5 | 4 | 5 | 6 | 5 | 4 | 3 | 4 | 5 | 4 | 3 |
| | 2 | 3 | 4 | 3 | 2 | 3 | 4 | 5 | 4 | 3 | 4 | 5 | 6 | 5 | 4 | 3 | 4 | 5 | 4 | 3 | 2 | 3 | 4 | 3 | 2 |
| | 3 | 4 | 5 | 4 | 3 | 4 | 5 | 6 | 5 | 4 | 5 | 6 | 7 | 6 | 5 | 4 | 5 | 6 | 5 | 4 | 3 | 4 | 5 | 4 | 3 |
| +2 | 4 | 5 | 6 | 5 | 4 | 5 | 6 | 7 | 6 | 5 | 6 | 7 | 8 | 7 | 6 | 5 | 6 | 7 | 6 | 5 | 4 | 5 | 6 | 5 | 4 |
| | 3 | 4 | 5 | 4 | 3 | 4 | 5 | 6 | 5 | 4 | 5 | 6 | 7 | 6 | 5 | 4 | 5 | 6 | 5 | 4 | 3 | 4 | 5 | 4 | 3 |
| | 2 | 3 | 4 | 3 | 2 | 3 | 4 | 5 | 4 | 3 | 4 | 5 | 6 | 5 | 4 | 3 | 4 | 5 | 4 | 3 | 2 | 3 | 4 | 3 | 2 |

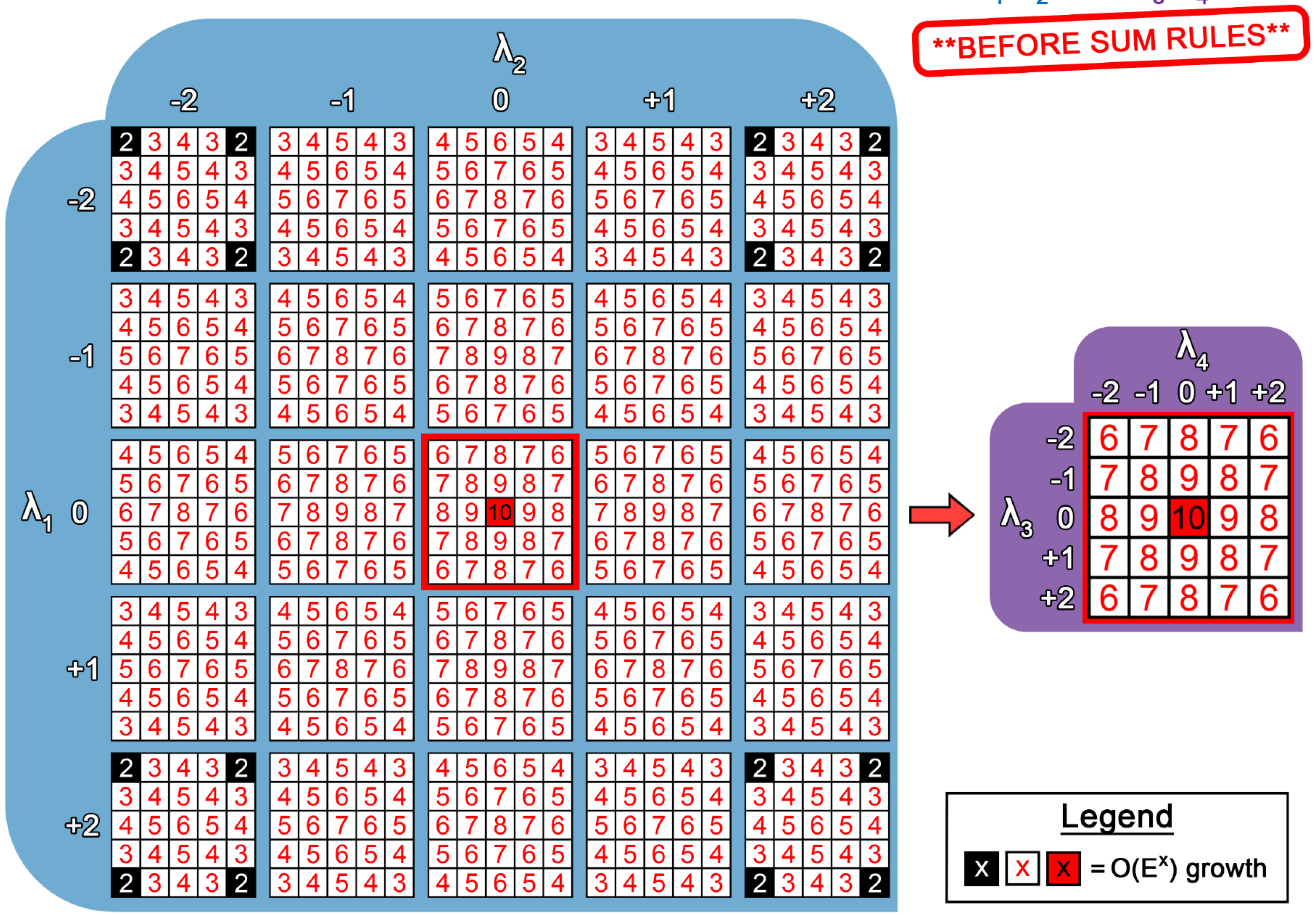
| $\lambda_3$ \ $\lambda_4$ | -2 | -1 | 0 | +1 | +2 |
|---|---|---|---|---|---|
| -2 | 6 | 7 | 8 | 7 | 6 |
| -1 | 7 | 8 | 9 | 8 | 7 |
| 0 | 8 | 9 | 10 | 9 | 8 |
| +1 | 7 | 8 | 9 | 8 | 7 |
| +2 | 6 | 7 | 8 | 7 | 6 |

**Legend**

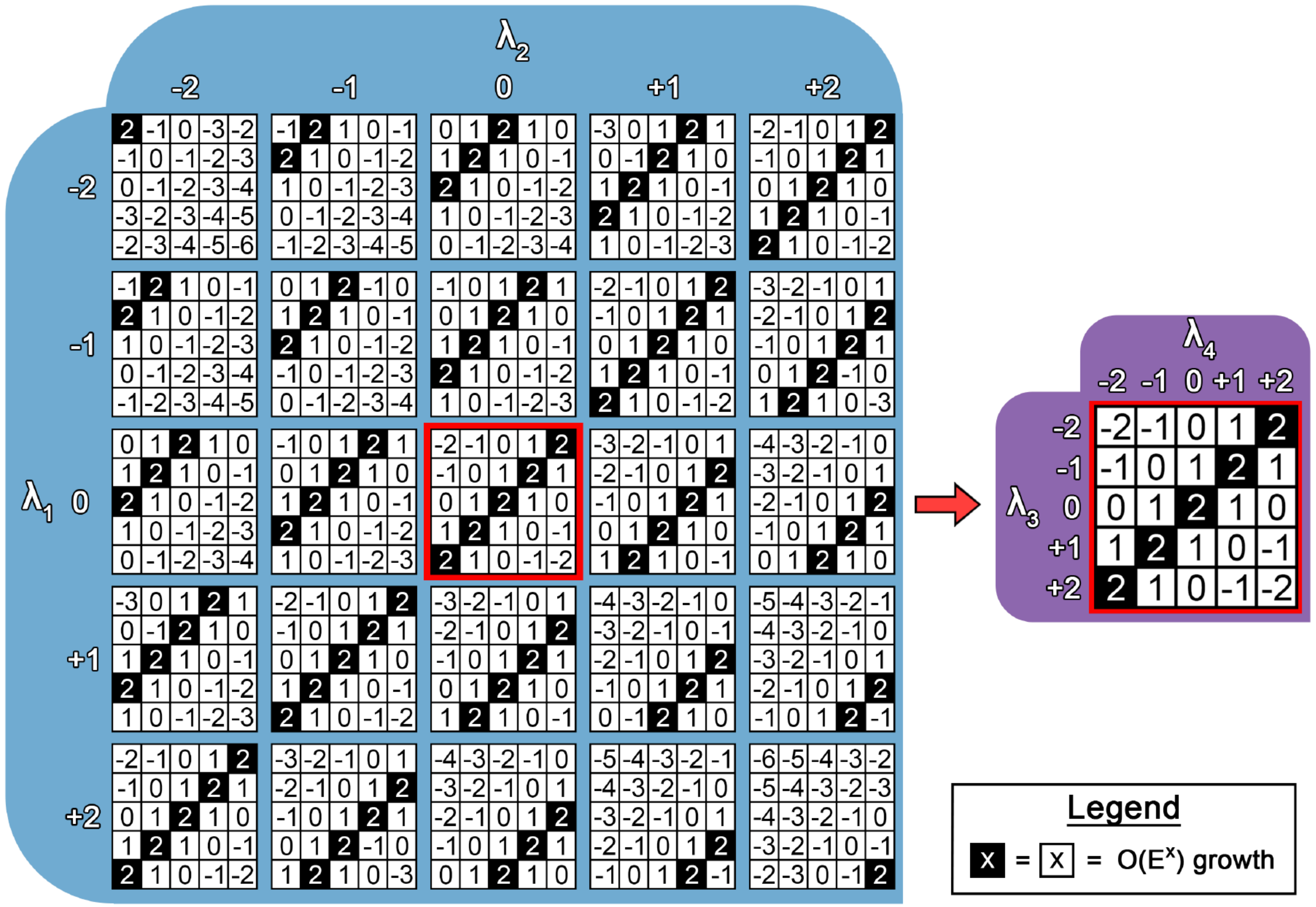
x x x = $O(E^x)$ growth

# Amplitudes and Coupling Structures on a Torus/ADS

Elastic 2-to-2 KK Mode Scattering Matrix Elements in RS1
Fastest Energy Growth per Helicity Combination: $(\lambda_1,\lambda_2) \rightarrow (\lambda_3,\lambda_4)$




Elastic 2-to-2 KK Mode Scattering Matrix Elements in RS1
Fastest Energy Growth per Helicity Combination: $(\lambda_1,\lambda_2) \rightarrow (\lambda_3,\lambda_4)$




Phys. Rev. D 107,(2023) 03505, Phys.Rev.D 101 (2020) 7, 075013 Phys.Rev.D 101 (2020) 5, 055013 Phys. Rev. D 103 (2022), 095024
Phys. Rev. D 107,(2023) 03505,

1. Can be extended to a Full Goldberger-Wise stabilized Model.
2. Matter in Bulk or brane scatterings with KK gravitons get unitarized by a similar mechanisms

# Goldstone Boson Equivalence theorems for compact extra dimensions

$$G_{MN} = e^{2A(z)} \begin{pmatrix} e^{-\kappa\varphi/\sqrt{6}}(\eta_{\mu\nu} + \kappa h_{\mu\nu}) & \frac{\kappa}{\sqrt{2}} A_\mu \\ \frac{\kappa}{\sqrt{2}} A_\mu & -\left(1 + \frac{\kappa}{\sqrt{6}}\varphi\right)^2 \end{pmatrix}$$

$$ds^2 = e^{2A(z)}(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2)$$

$$A(z) = -\ln(kz)$$

$$h_{\mu\nu}(x^\alpha, z) = \sum_{n=0}^{\infty} h_{\mu\nu}^{(n)}(x^\alpha) f^{(n)}(z)$$

$$A_\mu(x^\alpha, z) = \sum_{n=1}^{\infty} A_\mu^{(n)}(x^\alpha) g^{(n)}(z)$$

$$\varphi(x^\alpha, z) = \sum_{n=0}^{\infty} \varphi^{(n)}(x) k^{(n)}(z)$$

# Goldstone Boson Equivalence theorems for compact extra dimensions

$$G_{MN} = e^{2A(z)} \begin{pmatrix} e^{-\kappa\varphi/\sqrt{6}}(\eta_{\mu\nu} + \kappa h_{\mu\nu}) & \frac{\kappa}{\sqrt{2}} A_\mu \\ \frac{\kappa}{\sqrt{2}} A_\mu & -\left(1 + \frac{\kappa}{\sqrt{6}}\varphi\right)^2 \end{pmatrix}$$

$$ds^2 = e^{2A(z)}(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2)$$

$$A(z) = -\ln(kz)$$

$$h_{\mu\nu}(x^\alpha, z) = \sum_{n=0}^{\infty} h_{\mu\nu}^{(n)}(x^\alpha) f^{(n)}(z)$$ → Physical KK gravitons, 0 mode + tower of massive states

$$A_\mu(x^\alpha, z) = \sum_{n=1}^{\infty} A_\mu^{(n)}(x^\alpha) g^{(n)}(z)$$ → Un-physical KK graviphotons/goldstones

$$\varphi(x^\alpha, z) = \sum_{n=0}^{\infty} \varphi^{(n)}(x) k^{(n)}(z)$$ → Only 0 mode/radion is physical, higher modes higgsed/goldstones

# Goldstone Boson Equivalence theorems for compact extra dimensions

$$G_{MN} = e^{2A(z)} \begin{pmatrix} e^{-\kappa\varphi/\sqrt{6}}(\eta_{\mu\nu} + \kappa h_{\mu\nu}) & \frac{\kappa}{\sqrt{2}} A_\mu \\ \frac{\kappa}{\sqrt{2}} A_\mu & -\left(1 + \frac{\kappa}{\sqrt{6}}\varphi\right)^2 \end{pmatrix}$$

$$ds^2 = e^{2A(z)}(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2)$$

$$A(z) = -\ln(kz)$$

$$h_{\mu\nu}(x^\alpha, z) = \sum_{n=0}^{\infty} h_{\mu\nu}^{(n)}(x^\alpha) f^{(n)}(z)$$

→ Physical KK gravitons, 0 mode + tower of massive states

$$A_\mu(x^\alpha, z) = \sum_{n=1}^{\infty} A_\mu^{(n)}(x^\alpha) g^{(n)}(z)$$

→ Un-physical KK graviphotons/goldstones

$$\varphi(x^\alpha, z) = \sum_{n=0}^{\infty} \varphi^{(n)}(x) k^{(n)}(z)$$

→ Only 0 mode/radion is physical, higher modes higgsed/goldstones

$$\mathcal{L} = \mathcal{L}_{EH} + \mathcal{L}_{\mathrm{CC}} + \Delta\mathcal{L} + \mathcal{L}_{\mathrm{GF}} + \mathcal{L}_{\mathrm{m}}$$

$$\mathcal{L}_2 = \frac{1}{2} h_{\mu\nu}^n \mathcal{D}_h^{\mu\nu\rho\sigma} h_{\rho\sigma}^n + \frac{1}{2} A_\mu^n \mathcal{D}_A^{\mu\nu} A_\nu^n + \frac{1}{2}\varphi D_\varphi \varphi$$

Propagators

$$\begin{aligned} \mathcal{D}_h^{\mu\nu\rho\sigma} &= \frac{1}{2}\left(\eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho} - \eta^{\mu\nu}\eta^{\rho\sigma}\right)(-\Box - m_n^2) \\ \mathcal{D}_A^{\mu\nu} &= -\eta^{\mu\nu}(-\Box - m_n^2), \\ \mathcal{D}_\varphi &= -\Box - m_n^2. \end{aligned}$$

$$\begin{aligned} \mathcal{L}_{\mathrm{GF}} &= F^\mu F_\mu - F_5 F_5, \\ F_\mu^n &= -\left(\partial^\nu h_{\mu\nu}^n - \frac{1}{2}\partial_\mu h^n + \frac{1}{\sqrt{2}} m_n A_\mu^n\right), \\ F_5^n &= -\left(\frac{1}{2} m_n h^n - \frac{1}{\sqrt{2}}\partial^\mu A_\mu^n + \sqrt{\frac{3}{2}} m_n \varphi^n\right) \end{aligned}$$

→ Gauge Fixing Lagrangian

# Goldstone Boson Equivalence theorems for compact extra dimensions

# Goldstone Boson Equivalence theorems for compact extra dimensions

Ward Identities $\boxed{\langle \mathbf{T} F_\mu(x)\Phi\rangle = \langle \mathbf{T} F_5(x)\Phi\rangle = 0}$

Any External on-shell physical fields after LSZ amputation

$$\boxed{\begin{aligned}&\langle \mathbf{T}\left(\partial^\nu(h^n_{\mu\nu}-\frac{1}{2}\eta_{\mu\nu}h^n)+\frac{1}{\sqrt{2}}m_n A^n_\mu\right)\Phi\rangle = 0,\\ &\langle \mathbf{T}\left(\frac{1}{2}m_n h^n - \frac{1}{\sqrt{2}}\partial^\mu A^n_\mu + \sqrt{\frac{3}{2}}m_n\varphi^n\right)\Phi\rangle = 0\end{aligned}}$$

# Goldstone Boson Equivalence theorems for compact extra dimensions

Ward Identities $\boxed{\langle \mathbf{T} F_\mu(x) \Phi \rangle = \langle \mathbf{T} F_5(x) \Phi \rangle = 0}$

Any External on-shell physical fields after LSZ amputation

$$\boxed{\begin{aligned}&\langle \mathbf{T} \left( \partial^\nu (h^n_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} h^n) + \frac{1}{\sqrt{2}} m_n A^n_\mu \right) \Phi \rangle = 0,\\ &\langle \mathbf{T} \left( \frac{1}{2} m_n h^n - \frac{1}{\sqrt{2}} \partial^\mu A^n_\mu + \sqrt{\frac{3}{2}} m_n \varphi^n \right) \Phi \rangle = 0\end{aligned}}$$

# Goldstone Boson Equivalence theorems for compact extra dimensions

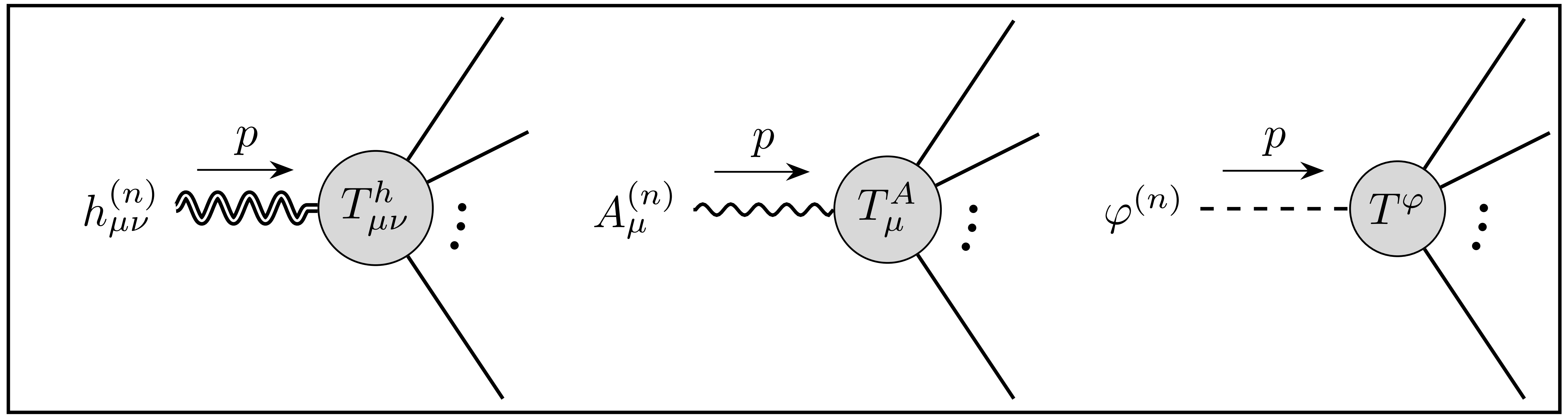
Ward Identities $\langle \mathbf{T} F_\mu(x)\Phi\rangle = \langle \mathbf{T} F_5(x)\Phi\rangle = 0$

Any External on-shell physical fields after LSZ amputation

$$\langle \mathbf{T}\left(\partial^\nu (h^n_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h^n) + \frac{1}{\sqrt{2}} m_n A^n_\mu\right)\Phi\rangle = 0,$$

$$\langle \mathbf{T}\left(\frac{1}{2} m_n h^n - \frac{1}{\sqrt{2}}\partial^\mu A^n_\mu + \sqrt{\frac{3}{2}} m_n \varphi^n\right)\Phi\rangle = 0$$

Consider an amplitude of the form




$$\mathcal{M}^h = \epsilon^{\mu\nu}(p) T^h_{\mu\nu}, \qquad \mathcal{M}^A = \epsilon^\mu(p) T^A_\mu, \qquad \mathcal{M}^\varphi = T^\varphi$$

$$\frac{i}{2} p^\nu \left(T^h_{\mu\nu} + T^h_{\nu\mu}\right) - \frac{1}{\sqrt{2}} m_n T^A_\mu = 0$$

$$-\frac{1}{2} m_n T^{h\mu}_\mu + \frac{i}{\sqrt{2}} p^\mu T^A_\mu + \sqrt{\frac{3}{2}} m_n T^\varphi = 0$$

# Goldstone Boson Equivalence theorems for compact extra dimensions

Consider the longitudinal polarizations of the KK graviton

$$\boxed{\epsilon_0^{\mu\nu} = \frac{1}{\sqrt{6}}\left(\epsilon_+^\mu\epsilon_-^\nu + \epsilon_-^\mu\epsilon_+^\nu + 2\epsilon_0^\mu\epsilon_0^\nu\right)}$$

The bad high energy growth part comes from the last term $\epsilon_0^{\mu\nu} \sim \mathcal{O}(E^2/m^2)$

Using the polarization sum $\sum_{\lambda=\pm,0} \epsilon_\lambda^\mu \epsilon_\lambda^{\nu *} = -\eta_{\mu\nu} + \frac{p^\mu p^\nu}{m^2}$

Rewrite the longitudinal polarizations as $\epsilon_0^{\mu\nu} = \frac{1}{\sqrt{6}}\left(\eta^{\mu\nu} - \frac{p^\mu p^\nu}{m^2} + 3\epsilon_0^\mu\epsilon_0^\nu\right)$

And separate out the bad high energy behaviour $\epsilon_0^\mu \equiv \frac{p^\mu}{m} + \tilde{\epsilon}_0^\mu$ $\quad \tilde{\epsilon}_0^\mu \equiv -\frac{m}{E+|\mathbf{p}|}(1, -\mathbf{p}/|\mathbf{p}|) \sim \mathcal{O}(m/E)$

Bad High Energy growth

This is fine

$$\epsilon_0^{\mu\nu} = \tilde{\epsilon}_0^{\mu\nu} + \frac{1}{\sqrt{6}}\left(\eta^{\mu\nu} + 2\frac{p^\mu p^\nu}{m^2} + 3\frac{p^\mu\tilde{\epsilon}_0^\nu + p^\nu\tilde{\epsilon}_0^\mu}{m}\right)$$

$$\mathcal{O}\left(\frac{m^2}{E^2}\right)$$

**arXiv:2311.xxxxx**

# Goldstone Boson Equivalence theorems for compact extra dimensions

Use Ward Identities to write the scattering amplitude as

$$T^h_{\mu\nu}\epsilon^{\mu\nu}_0 = T^\varphi - i\sqrt{3}\,T^A_\mu\tilde{\epsilon}^\mu_0 + T^h_{\mu\nu}\tilde{\epsilon}^{\mu\nu}_0$$

$\tilde{\epsilon}^\mu_0 \sim \mathcal{O}(m/E)$ $\quad$ $\tilde{\epsilon}^{\mu\nu}_0 \sim \mathcal{O}(m^2/E^2)$

No Bad High Energy growth

$$\boxed{T^h_{\mu\nu}\epsilon^{\mu\nu}_0 = T^\varphi + \mathcal{O}(s^0)}$$

**KK Goldstone Theorem :**

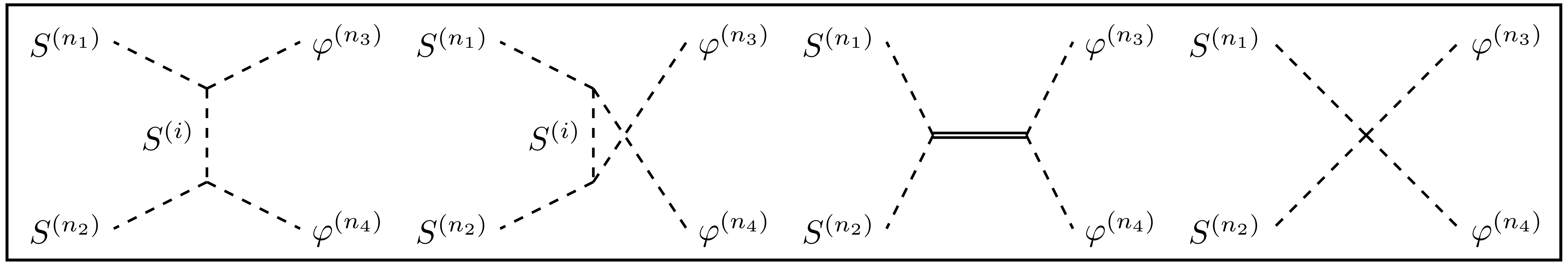
**Scattering Amplitude of longitudinally polarized KK graviton=Scalar Goldstones in the High Energy Limit**

$$\boxed{T^h_{\mu\nu}\epsilon^{\mu\nu}_{\pm1} = -iT^A_\mu\epsilon^\mu_\pm + \mathcal{O}(s^0)}$$

Similarly for Helicity 1 one states

# Goldstone Boson Equivalence theorems for compact extra dimensions

Example 1: Scattering of two bulk scalars with two KK gravitons




$$\mathcal{L}_m = \sqrt{G}\left(\frac{1}{2}G^{MN}\partial_M S\partial_N S - \frac{1}{2}M_S^2 S^2\right)$$

$$\boxed{S^{(n_1)}S^{(n_2)} \to h_L^{(n_3)}h_L^{(n_4)}}$$

Expectation from Goldstone Equivalence Theorem

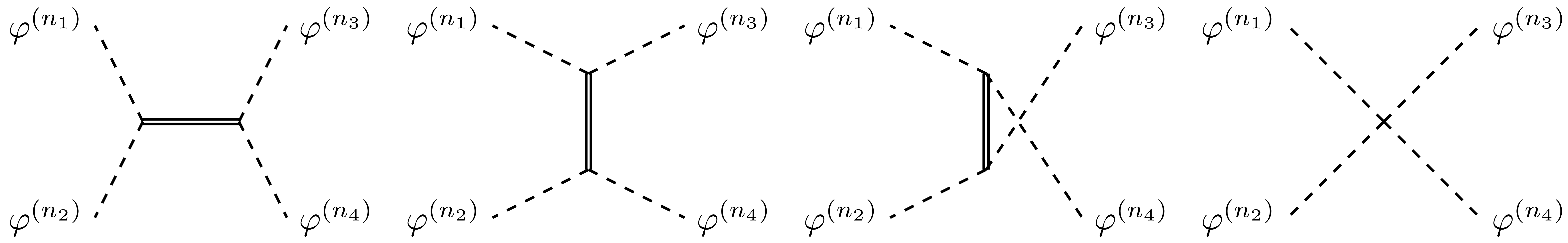
$$\boxed{\mathcal{M}\left[S^{(n_1)}S^{(n_2)} \to h_L^{(n_3)}h_L^{(n_4)}\right] = \mathcal{M}\left[S^{(n_1)}S^{(n_2)} \to \varphi^{(n_3)}\varphi^{(n_4)}\right] + \mathcal{O}(s^0)}$$

$$\mathcal{M}\left[S^{(n_1)}S^{(n_2)} \to \varphi^{(n_3)}\varphi^{(n_4)}\right] = \frac{\kappa^2 s}{32}(1-\cos 2\theta)\left\langle k^{(n)}k^{(n)}f_S^{(n)}f_S^{(n)}\right\rangle + \mathcal{O}(s^0)$$

Agrees with Unitary gauge calculation

# Goldstone Boson Equivalence theorems for compact extra dimensions

Example 2: Scattering of two gravitons with two KK gravitons




Expectation from Goldstone Equivalence Theorem

$$\mathcal{M}\left[h_L^{(n_1)} h_L^{(n_2)} \to h_L^{(n_3)} h_L^{(n_4)}\right] = \mathcal{M}\left[\varphi^{(n_1)}\varphi^{(n_2)} \to \varphi^{(n_3)}\varphi^{(n_4)}\right] + \mathcal{O}(s^0)$$

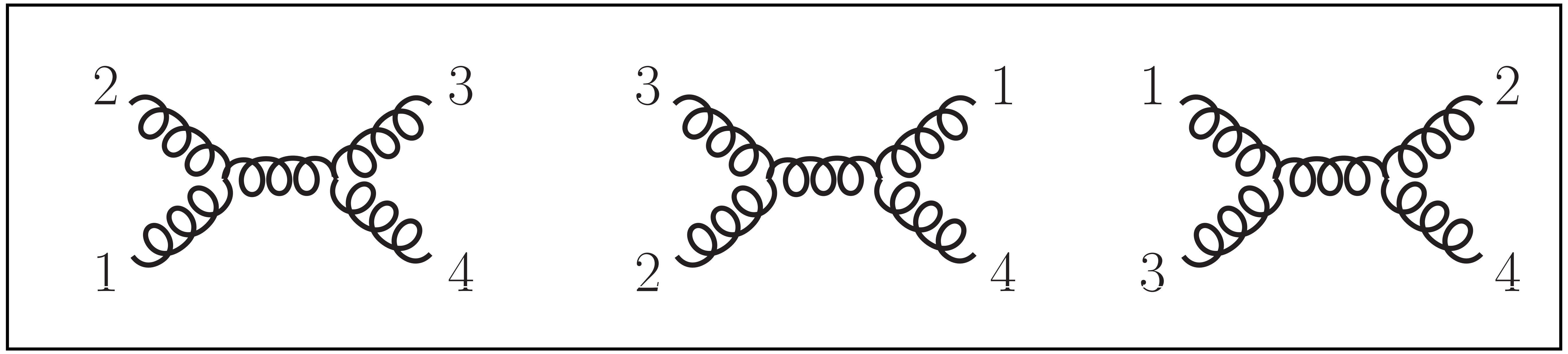
Agrees with Unitary gauge calculation

$$\mathcal{M}\left[\varphi^{(n_1)}\varphi^{(n_2)} \to \varphi^{(n_3)}\varphi^{(n_4)}\right] = \frac{\kappa^2 s}{64}\frac{(\cos 2\theta + 7)^2}{\sin^2\theta}\langle k^{(n)}k^{(n)}k^{(n)}k^{(n)}\rangle + \mathcal{O}(s^0)$$

# A Short Summary of Double Copy: BCJ double copy

# A Short Summary of Double Copy: BCJ double copy

Consider Gluon Scattering Amplitudes




Consider Gluon Scattering Amplitudes

$$i\mathcal{A}_4^{\text{tree}} = g^2\left(\frac{n_s c_s}{s} + \frac{n_t c_t}{t} + \frac{n_u c_u}{u}\right)$$

$$c_s = -2 f^{a_1 a_2 b} f^{b a_3 a_4} \qquad c_t n_t = c_s n_s\Big|_{1\to2\to3\to1}$$

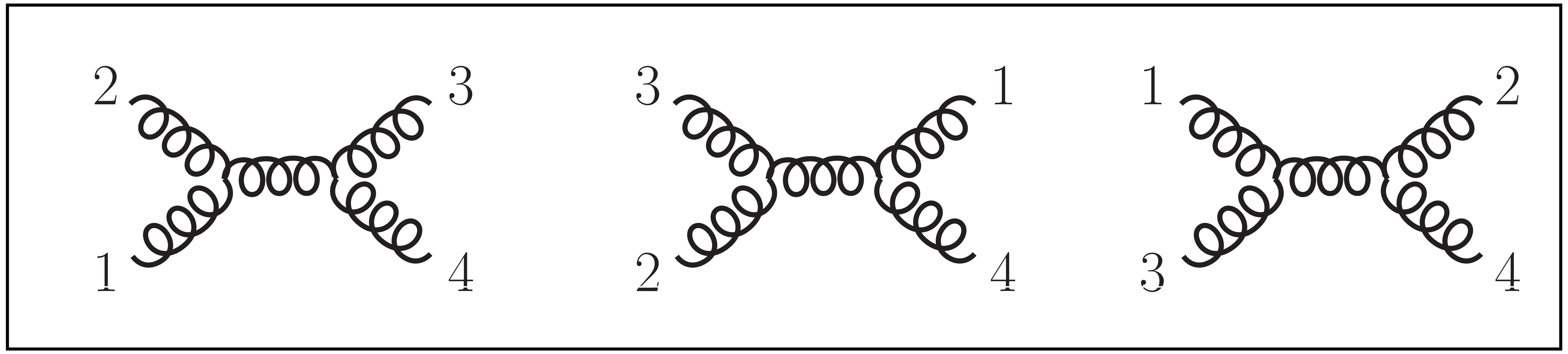
$$c_u n_u = c_s n_s\Big|_{1\to3\to2\to1}$$

$$\boxed{\begin{aligned} n_s = -\frac{1}{2}\Big\{&\Big[(\varepsilon_1\cdot\varepsilon_2)p_1^\mu + 2(\varepsilon_1\cdot p_2)\varepsilon_2^\mu - (1\leftrightarrow 2)\Big]\Big[(\varepsilon_3\cdot\varepsilon_4)p_{3\mu} + 2(\varepsilon_3\cdot p_4)\varepsilon_{4\mu} - (3\leftrightarrow 4)\Big] \\ &+ s\Big[(\varepsilon_1\cdot\varepsilon_3)(\varepsilon_2\cdot\varepsilon_4) - (\varepsilon_1\cdot\varepsilon_4)(\varepsilon_2\cdot\varepsilon_3)\Big]\Big\}, \qquad (1\end{aligned}}$$

**The Contact Interaction is factored into the above definition by suitable kinematic reshuffling**

# A Short Summary of Double Copy: BCJ double copy

Consider Gluon Scattering Amplitudes




Consider Gluon Scattering Amplitudes

$$i\mathcal{A}_4^{\text{tree}} = g^2\left(\frac{n_s c_s}{s} + \frac{n_t c_t}{t} + \frac{n_u c_u}{u}\right)$$

$$c_s = -2f^{a_1a_2b}f^{ba_3a_4} \qquad c_t n_t = c_s n_s\Big|_{1\to2\to3\to1}$$

$$c_u n_u = c_s n_s\Big|_{1\to3\to2\to1}$$

$$n_s = -\frac{1}{2}\Big\{\Big[(\varepsilon_1\cdot\varepsilon_2)p_1^\mu + 2(\varepsilon_1\cdot p_2)\varepsilon_2^\mu - (1\leftrightarrow2)\Big]\Big[(\varepsilon_3\cdot\varepsilon_4)p_{3\mu} + 2(\varepsilon_3\cdot p_4)\varepsilon_{4\mu} - (3\leftrightarrow4)\Big] + s\Big[(\varepsilon_1\cdot\varepsilon_3)(\varepsilon_2\cdot\varepsilon_4) - (\varepsilon_1\cdot\varepsilon_4)(\varepsilon_2\cdot\varepsilon_3)\Big]\Big\}, \quad (1$$

**The Contact Interaction is factored into the above definition by suitable kinematic reshuffling**

**Gauge Invariance demands that the amplitude must vanish under polarization to momentum replacement**

$$n_s\Big|_{\varepsilon_4\to p_4} = -\frac{s}{2}\Big[(\varepsilon_1\cdot\varepsilon_2)\Big((\varepsilon_3\cdot p_2) - (\varepsilon_3\cdot p_1)\Big) + \text{cyclic}(1,2,3)\Big] \equiv s\,\alpha(\varepsilon,p)$$

$$\boxed{\frac{n_s c_s}{s} + \frac{n_t c_t}{t} + \frac{n_u c_u}{u}\Big|_{\varepsilon_4\to p_4} = (c_s + c_t + c_u)\,\alpha(\varepsilon,p)}$$

Color Jacobi Identity

$$\boxed{c_s + c_t + c_u = -2(f^{a_1a_2b}f^{ba_3a_4} + f^{a_2a_3b}f^{ba_1a_4} + f^{a_3a_1b}f^{ba_2a_4}) = 0} \xrightarrow{\text{On-Shell Conditions}}$$

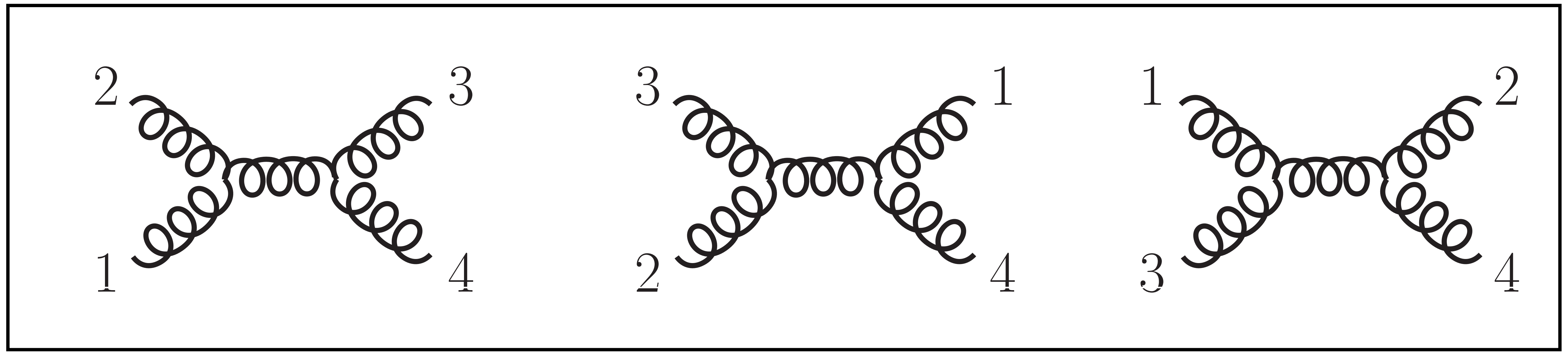
Kinematic Jacobi Identity

$$\boxed{n_s + n_t + n_u = 0}$$

# Amplitudes and Coupling Structures on a Stabilized Torus/ADS

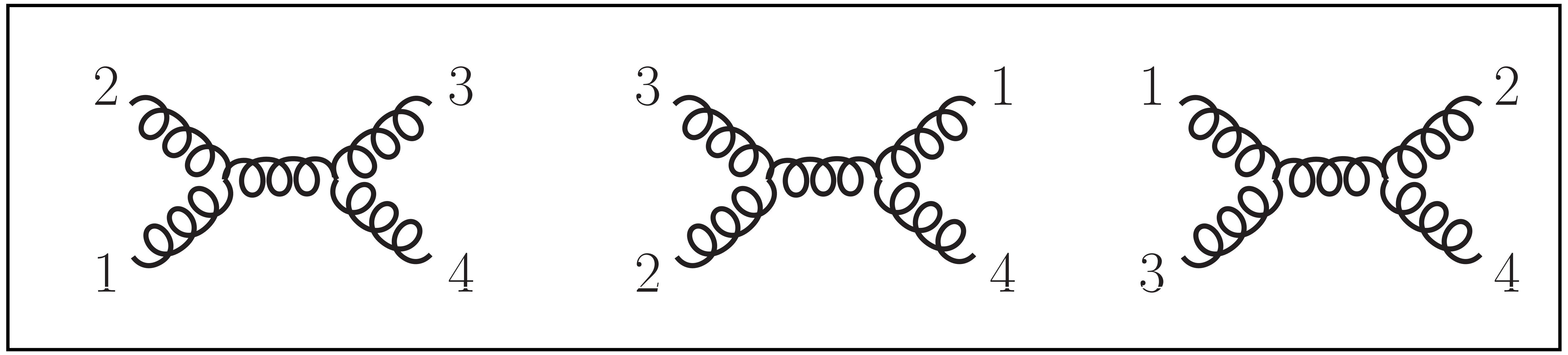
# Amplitudes and Coupling Structures on a Stabilized Torus/ADS

Color and Kinematic Factors are mutually Interchangable



# Amplitudes and Coupling Structures on a Stabilized Torus/ADS

Color and Kinematic Factors are mutually Interchangable

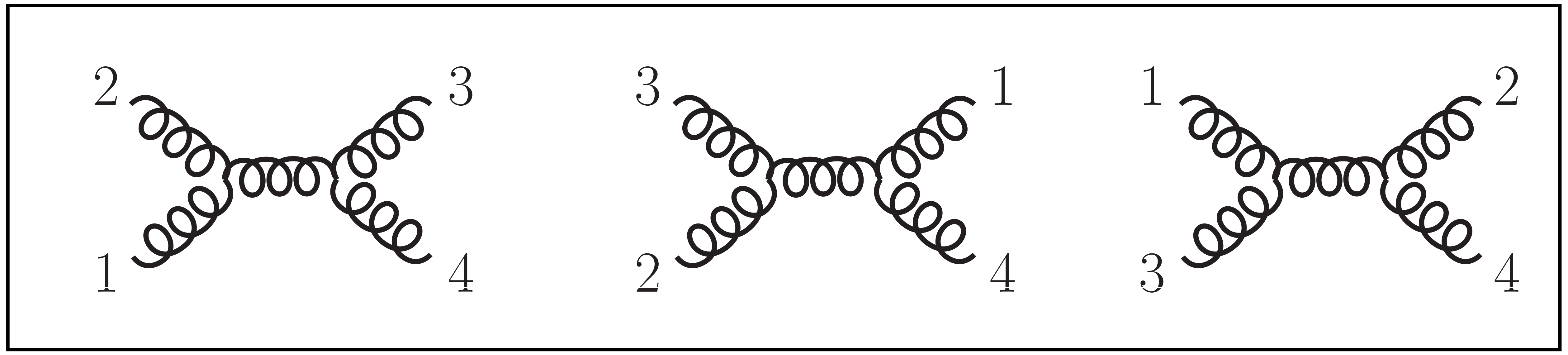



$$i\mathcal{A}_4^{\text{tree}} = g^2\left(\frac{n_s c_s}{s} + \frac{n_t c_t}{t} + \frac{n_u c_u}{u}\right)$$

Massless Gravity amplitude

# Amplitudes and Coupling Structures on a Stabilized Torus/ADS

Color and Kinematic Factors are mutually Interchangable




$$iA_4^{\text{tree}} = g^2\left(\frac{n_s c_s}{s} + \frac{n_t c_t}{t} + \frac{n_u c_u}{u}\right)$$

Massless Gravity amplitude

$$iA_4^{\text{tree}}\Big|_{\substack{c_i \to \tilde{n}_i \\ g \to \kappa/2}} \equiv i\mathcal{M}_4^{\text{tree}} = \left(\frac{\kappa}{2}\right)^2\left(\frac{n_s^2}{s} + \frac{n_t^2}{t} + \frac{n_u^2}{u}\right)$$

Diffeomorphism Invariance

$$\frac{n_s^2}{s} + \frac{n_t^2}{t} + \frac{n_u^2}{u}\Big|_{\varepsilon_4^{\mu\nu} \to p_4^\mu \varepsilon_4^\nu + p_4^\nu \varepsilon_4^\mu} = 2(n_s + n_t + n_u)\,\alpha(\varepsilon, p) = 0\,.$$

arXiv:2311.xxxxx

# Amplitudes and Coupling Structures on a Stabilized Torus/ADS

Calculate the 4 D KK amplitudes of a Gauge theory

arXiv:2311.xxxxx

# Amplitudes and Coupling Structures on a Stabilized Torus/ADS

Calculate the 4 D KK amplitudes of a Gauge theory

$$\boxed{\hat{A}_0^a(\hat{p}_1)\hat{A}_0^b(\hat{p}_2) \to \hat{A}_0^c(\hat{p}_3)\hat{A}_0^d(\hat{p}_4)}$$

$$\boxed{\hat{p}^M = (p^\mu, \pm p^5)}$$

On -Shell Conditions

$$p_i^\mu p_{i\mu} = E^2 \cos^2 \omega \equiv M_n^2$$

arXiv:2311.xxxxx

# Amplitudes and Coupling Structures on a Stabilized Torus/ADS

Calculate the 4 D KK amplitudes of a Gauge theory

$$\boxed{\hat{A}_0^a(\hat{p}_1)\hat{A}_0^b(\hat{p}_2) \to \hat{A}_0^c(\hat{p}_3)\hat{A}_0^d(\hat{p}_4)}$$

$$\boxed{\hat{p}^M = (p^\mu, \pm p^5)}$$

On -Shell Conditions

$$p_i^\mu p_{i\mu} = E^2 \cos^2 \omega \equiv M_n^2$$

$$\boxed{\mathcal{M}_5 \equiv \frac{1}{4} \sum_{\eta_i = \pm} \mathcal{M} \left[ \hat{A}_0^a(\hat{p}_{1,\eta_1})\hat{A}_0^b(\hat{p}_{2,\eta_2}) \to \hat{A}_0^c(\hat{p}_{3,\eta_3})\hat{A}_0^d(\hat{p}_{4,\eta_4}) \right] \delta_{\eta_1+\eta_2-\eta_3-\eta_4,0} =}$$

arXiv:2311.xxxxx

# Amplitudes and Coupling Structures on a Stabilized Torus/ADS

Calculate the 4 D KK amplitudes of a Gauge theory

On -Shell Conditions

$$\boxed{\hat{A}_0^a(\hat{p}_1)\hat{A}_0^b(\hat{p}_2) \to \hat{A}_0^c(\hat{p}_3)\hat{A}_0^d(\hat{p}_4)}$$

$$\boxed{\hat{p}^M = (p^\mu, \pm p^5)}$$

$$p_i^\mu p_{i\mu} = E^2 \cos^2\omega \equiv M_n^2$$

$$\boxed{\mathcal{M}_5 \equiv \frac{1}{4}\sum_{\eta_i=\pm} \mathcal{M}\left[\hat{A}_0^a(\hat{p}_{1,\eta_1})\hat{A}_0^b(\hat{p}_{2,\eta_2}) \to \hat{A}_0^c(\hat{p}_{3,\eta_3})\hat{A}_0^d(\hat{p}_{4,\eta_4})\right]\delta_{\eta_1+\eta_2-\eta_3-\eta_4,0} =} \; {g_5}^2\left(-\frac{3\,c\,C^{abe}C^{cde}}{2} + \frac{3\,(c+3)\,C^{ace}C^{dbe}}{2\,(c-1)} - \frac{3\,(c-3)\,C^{ade}C^{bce}}{2\,(c+1)}\right)$$

arXiv:2311.xxxxx

# Amplitudes and Coupling Structures on a Stabilized Torus/ADS

Calculate the 4 D KK amplitudes of a Gauge theory

On -Shell Conditions

$$\boxed{\hat{A}_0^a(\hat{p}_1)\hat{A}_0^b(\hat{p}_2) \to \hat{A}_0^c(\hat{p}_3)\hat{A}_0^d(\hat{p}_4)}$$

$$\boxed{\hat{p}^M = (p^\mu, \pm p^5)}$$

$$p_i^\mu p_{i\mu} = E^2 \cos^2\omega \equiv M_n^2$$

$$\boxed{\mathcal{M}_5 \equiv \frac{1}{4}\sum_{\eta_i=\pm} \mathcal{M}\left[\hat{A}_0^a(\hat{p}_{1,\eta_1})\hat{A}_0^b(\hat{p}_{2,\eta_2}) \to \hat{A}_0^c(\hat{p}_{3,\eta_3})\hat{A}_0^d(\hat{p}_{4,\eta_4})\right]\delta_{\eta_1+\eta_2-\eta_3-\eta_4,0} =} \; g_5{}^2\left(-\frac{3\,c\,C^{abe}C^{cde}}{2} + \frac{3\,(c+3)\,C^{ace}C^{dbe}}{2\,(c-1)} - \frac{3\,(c-3)\,C^{ade}C^{bce}}{2\,(c+1)}\right)$$

$$\frac{1}{g_5^2}\mathcal{M}_5 - \frac{1}{g^2}\mathcal{M}_4 = -\frac{4\,c\,(\kappa^4+\kappa^2-2)\left(C^{abe}C^{cde}+C^{ace}C^{dbe}+C^{ade}C^{bce}\right)}{\kappa^2}$$

arXiv:2311.xxxxx

# Amplitudes and Coupling Structures on a Stabilized Torus/ADS

Calculate the 4 D KK amplitudes of a Gauge theory

On -Shell Conditions

$$\boxed{\hat{A}_0^a(\hat{p}_1)\hat{A}_0^b(\hat{p}_2) \to \hat{A}_0^c(\hat{p}_3)\hat{A}_0^d(\hat{p}_4)}$$

$$\boxed{\hat{p}^M = (p^\mu, \pm p^5)}$$

$$p_i^\mu p_{i\mu} = E^2 \cos^2\omega \equiv M_n^2$$

$$\boxed{\mathcal{M}_5 \equiv \frac{1}{4}\sum_{\eta_i=\pm} \mathcal{M}\left[\hat{A}_0^a(\hat{p}_{1,\eta_1})\hat{A}_0^b(\hat{p}_{2,\eta_2}) \to \hat{A}_0^c(\hat{p}_{3,\eta_3})\hat{A}_0^d(\hat{p}_{4,\eta_4})\right]\delta_{\eta_1+\eta_2-\eta_3-\eta_4,0} =} \; {g_5}^2\left(-\frac{3\,c\,C^{\text{abe}}C^{\text{cde}}}{2} + \frac{3\,(c+3)\,C^{\text{ace}}C^{\text{dbe}}}{2\,(c-1)} - \frac{3\,(c-3)\,C^{\text{ade}}C^{\text{bce}}}{2\,(c+1)}\right)$$

$$\frac{1}{g_5^2}\mathcal{M}_5 - \frac{1}{g^2}\mathcal{M}_4 = -\frac{4\,c\,(\kappa^4+\kappa^2-2)\left(C^{\text{abe}}C^{\text{cde}} + C^{\text{ace}}C^{\text{dbe}} + C^{\text{ade}}C^{\text{bce}}\right)}{\kappa^2}$$

Combine two copies of different helicities and replace color factors by kinematic functions

arXiv:2311.xxxxx

# Amplitudes and Coupling Structures on a Stabilized Torus/ADS

Calculate the 4 D KK amplitudes of a Gauge theory

$$\boxed{\hat{A}^a_0(\hat{p}_1)\hat{A}^b_0(\hat{p}_2) \to \hat{A}^c_0(\hat{p}_3)\hat{A}^d_0(\hat{p}_4)}$$

On -Shell Conditions

$$\boxed{\hat{p}^M = (p^\mu, \pm p^5)}$$

$$p_i^\mu p_{i\mu} = E^2 \cos^2\omega \equiv M_n^2$$

$$\boxed{\mathcal{M}_5 \equiv \frac{1}{4}\sum_{\eta_i=\pm} \mathcal{M}\left[\hat{A}^a_0(\hat{p}_{1,\eta_1})\hat{A}^b_0(\hat{p}_{2,\eta_2}) \to \hat{A}^c_0(\hat{p}_{3,\eta_3})\hat{A}^d_0(\hat{p}_{4,\eta_4})\right]\delta_{\eta_1+\eta_2-\eta_3-\eta_4,0} =} \; g_5{}^2\left(-\frac{3\,c\,C^{abe}C^{cde}}{2} + \frac{3\,(c+3)\,C^{ace}C^{dbe}}{2\,(c-1)} - \frac{3\,(c-3)\,C^{ade}C^{bce}}{2\,(c+1)}\right)$$

$$\frac{1}{g_5^2}\mathcal{M}_5 - \frac{1}{g^2}\mathcal{M}_4 = -\frac{4\,c\,(\kappa^4+\kappa^2-2)\,(C^{abe}C^{cde} + C^{ace}C^{dbe} + C^{ade}C^{bce})}{\kappa^2}$$

Combine two copies of different helicities and replace color factors by kinematic functions

$$\boxed{\mathbf{M} = \left(\frac{\kappa}{2}\right)^2\left(\frac{n_s n'_s}{s} + \frac{n_t n'_t}{t} + \frac{n_u n'_u}{u}\right)}$$

arXiv:2311.xxxxx

# Amplitudes and Coupling Structures on a Stabilized Torus/ADS

Calculate the 4 D KK amplitudes of a Gauge theory

$$\boxed{\hat{A}_0^a(\hat{p}_1)\hat{A}_0^b(\hat{p}_2) \to \hat{A}_0^c(\hat{p}_3)\hat{A}_0^d(\hat{p}_4)}$$

On -Shell Conditions

$$\boxed{\hat{p}^M = (p^\mu, \pm p^5)}$$

$$p_i^\mu p_{i\mu} = E^2 \cos^2\omega \equiv M_n^2$$

$$\boxed{\mathcal{M}_5 \equiv \frac{1}{4}\sum_{\eta_i=\pm} \mathcal{M}\left[\hat{A}_0^a(\hat{p}_{1,\eta_1})\hat{A}_0^b(\hat{p}_{2,\eta_2}) \to \hat{A}_0^c(\hat{p}_{3,\eta_3})\hat{A}_0^d(\hat{p}_{4,\eta_4})\right]\delta_{\eta_1+\eta_2-\eta_3-\eta_4,0} =} \; {g_5}^2\left(-\frac{3\,c\,C^{\mathrm{abe}}C^{\mathrm{cde}}}{2} + \frac{3\,(c+3)\,C^{\mathrm{ace}}C^{\mathrm{dbe}}}{2\,(c-1)} - \frac{3\,(c-3)\,C^{\mathrm{ade}}C^{\mathrm{bce}}}{2\,(c+1)}\right)$$

$$\frac{1}{g_5^2}\mathcal{M}_5 - \frac{1}{g^2}\mathcal{M}_4 = -\frac{4\,c\,(\kappa^4+\kappa^2-2)\left(C^{\mathrm{abe}}C^{\mathrm{cde}} + C^{\mathrm{ace}}C^{\mathrm{dbe}} + C^{\mathrm{ade}}C^{\mathrm{bce}}\right)}{\kappa^2}$$

Combine two copies of different helicities and replace color factors by kinematic functions

$$\boxed{\mathbf{M} = \left(\frac{\kappa}{2}\right)^2\left(\frac{n_s n_s'}{s} + \frac{n_t n_t'}{t} + \frac{n_u n_u'}{u}\right)}$$

Equivalent to a 4D KK graviton amplitude on a flat torus

**arXiv:2311.xxxxx**

# Conclusions

- Compactified theories of extra dimensions -> No low energy cut-off
- Cancellations due to different diagrams reduce $O(s^5)$ growth to $O(s)$.
- No low energy cut-off for consistent models of stabilization
- Uncovered sum rules enforcing this cancellation
- Can show -> Analysis extends to matter on brane or bulk
- Consistent with literature on massive gravity.
- Possible to double-copy a compactified gauge theory to compactified gravity for flat toroidal compactification
- Pheno papers : Doing an unitarity analysis for DM models, ultralight radion as a candidate …
- Theory papers : Spinor Helicity/Goldstone Equivalence calculation.
- More connections with massive gravity community …