MONASH University

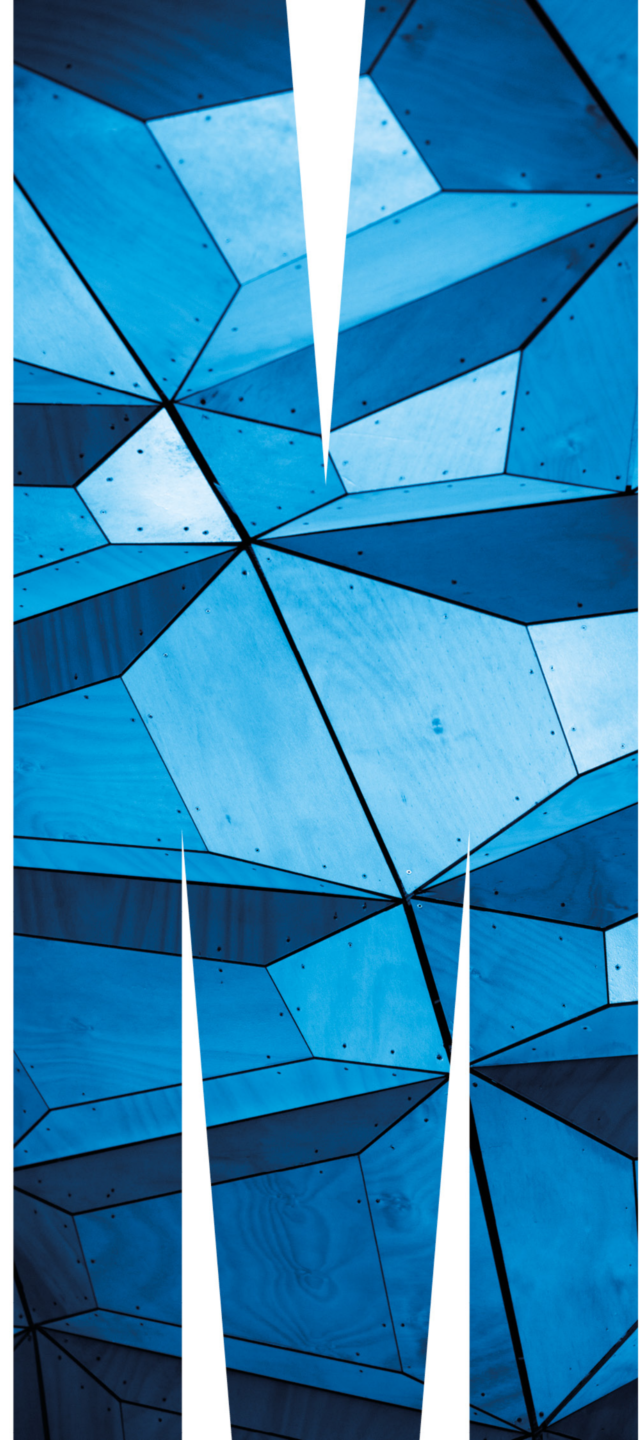
New physics in $B \to K^{(*)}\nu\bar{\nu}$

The 3rd International Joint Workshop on the Standard Model and Beyond
Jeju Island, Korea

German Valencia

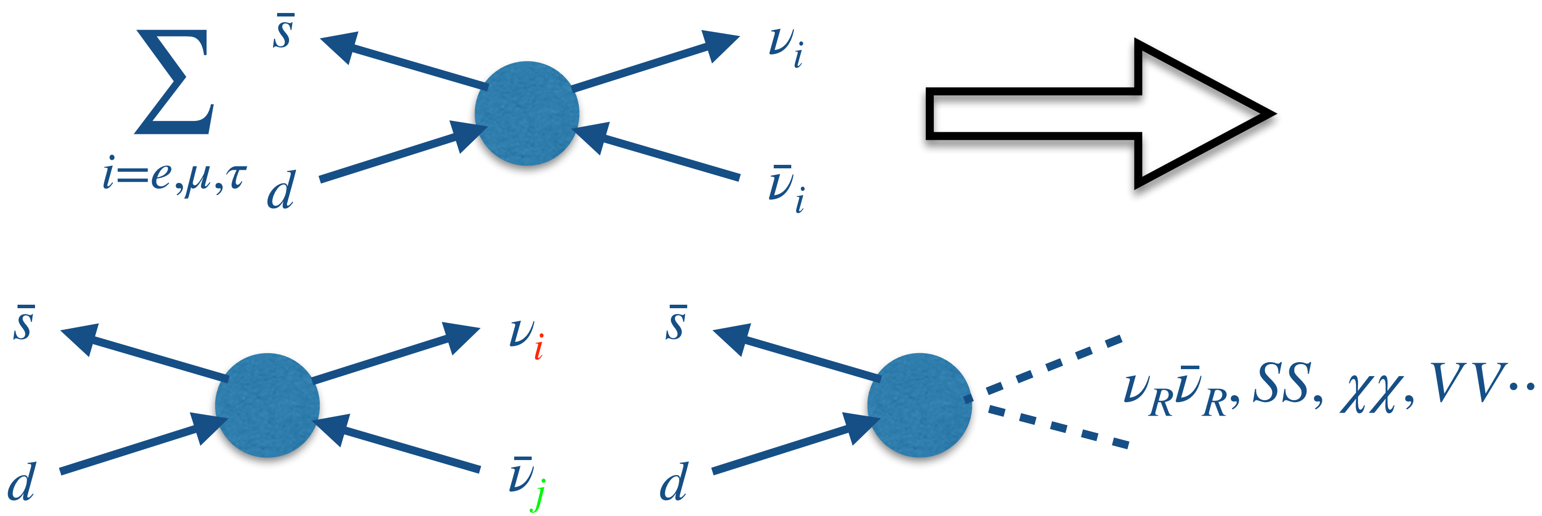
based on work with Xiao-Gang He and Xiao-Dong Ma arXiv:2309.12741, JHEP 03 (2023) 037 and Phys.Lett.B 821 (2021) 136607

GROUP OF EIGHT AUSTRALIA



the neutrinos are not seen

- The final state BSM not necessarily a $\nu_i \bar{\nu}_i$ pair:



  - different flavour neutrinos (lepton flavour violation)
  - new invisible particle pairs can be sterile neutrinos or something else
  - consider only pair produced new invisible particles
    - models with a symmetry that requires pair production
    - single invisible particle studied extensively by others and observed decay is consistent with 3-body spectrum

Quantifying the "excess"

- To constrain neutrino couplings we use

$$R_K^{\nu\nu} = \frac{\mathscr{B}(B^+ \to K^+\nu\bar{\nu})}{\mathscr{B}(B^+ \to K^+\nu\bar{\nu})_{\mathtt{SM}}} = 5.4 \pm 1.6 \qquad \text{using new result}$$

$$= 3.2 \pm 0.9 \qquad \text{or using average}$$

$$R_{K^*}^{\nu\nu} = \frac{\mathscr{B}(B \to K^*\nu\bar{\nu})}{\mathscr{B}(B \to K^*\nu\bar{\nu})_{\mathtt{SM}}} \leq 2.7 \qquad \text{Belle combined}$$

$$\leq 1.9 \qquad \text{best for neutral mode}$$

- As constraints on new invisible particles, we use instead

$$\mathscr{B}(B^+ \to K^+ + invisible)_{\mathtt{NP}} \equiv \mathscr{B}(B^+ \to K^+\nu\bar{\nu})_{\mathtt{exp}} - \mathscr{B}(B^+ \to K^+\nu\bar{\nu})_{\mathtt{SM}} = (2.0 \pm 0.7) \times 10^{-5}$$

  - combined with limits obtained from 90% c.l upper limits
  - $\mathscr{B}(B^0 \to K^0 + invisible)_{\mathtt{NP}} \leq 2.3 \times 10^{-5}$
  - $\mathscr{B}(B^+ \to K^{*+} + invisible)_{\mathtt{NP}} \leq 3.1 \times 10^{-5}$
  - $\mathscr{B}(B^0 \to K^{*0} + invisible)_{\mathtt{NP}} \leq 1.0 \times 10^{-5}$

models with additional ν final states

- We start from an effective interaction at the B scale

$$\mathcal{H}_{eff} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^{\star} \frac{e^2}{16\pi^2} \sum_{ij} \Big(C_L^{ij} \mathcal{O}_L^{ij} + C_R^{ij} \mathcal{O}_R^{ij} + C_L^{'\,ij} \mathcal{O}_L^{'\,ij} + C_R^{'\,ij} \mathcal{O}_R^{'\,ij}$$

$$+ C_9^{ij} \mathcal{O}_9^{ij} + C_{10}^{ij} \mathcal{O}_{10}^{ij} + C_{9'}^{ij} \mathcal{O}_{9'}^{ij} + C_{10'}^{ij} \mathcal{O}_{10'}^{ij} \Big) + \text{ h.c.}$$

$$\mathcal{O}_L^{ij} = (\bar{s}_L \gamma_\mu b_L)(\bar{\nu}_i \gamma^\mu (1-\gamma_5) \nu_j),\ \mathcal{O}_R^{ij} = (\bar{s}_R \gamma_\mu b_R)(\bar{\nu}_i \gamma^\mu (1-\gamma_5) \nu_j),$$

$$\mathcal{O}_L^{'\,ij} = (\bar{s}_L \gamma_\mu b_L)(\bar{\nu}_i \gamma^\mu (1+\gamma_5) \nu_j),\ \mathcal{O}_R^{'\,ij} = (\bar{s}_R \gamma_\mu b_R)(\bar{\nu}_i \gamma^\mu (1+\gamma_5) \nu_j)$$

$$\mathcal{O}_{9^{(\prime)}}^{ij} = (\bar{s}_{L(R)}, \gamma_\mu b_{L(R)})(\bar{\ell}_i \gamma^\mu \ell_j),\ \mathcal{O}_{10^{(\prime)}}^{ij} = (\bar{s}_{L(R)}, \gamma_\mu b_{L(R)})(\bar{\ell}_i \gamma^\mu \gamma_5 \ell_j)$$

- At the weak scale we have in mind leptoquarks and/or a non-universal Z' coupling the SM to the light RH neutrino
- Both cases are also constrained by $b \to s\ell^+\ell^-$ processes

low energy constants from leptoquarks

- scalar or vector leptoquarks coupling to SM fermions

$$\mathscr{L}_S = \lambda_{LS_0}\bar{q}^c_L i\tau_2 \ell_L {S_0}^\dagger + \lambda_{L\tilde{S}_{1/2}}\bar{d}_R \ell_L \tilde{S}^\dagger_{1/2} + \lambda_{LS_1}\bar{q}^c_L i\tau_2 \vec{\tau}\cdot\vec{S}^\dagger_1 \ell_L + \text{ h. c.}$$

$$\mathscr{L}_V = \lambda_{LV_{1/2}}\bar{d}^c_R \gamma_\mu \ell_L V^{\dagger\mu}_{1/2} + \lambda_{LV_1}\bar{q}_L \gamma_\mu \vec{\tau}\cdot\overrightarrow{V}^{\dagger\mu}_1 \ell_L + \text{ h. c.}$$

- result in

$$S^\dagger_0 : (\bar{3},1,1/3) \implies C^{ij}_L$$

$$\tilde{S}^\dagger_{1/2} : (3,2,1/6) \implies C^{ij}_{9'} = -C^{ij}_{10'} = 2C^{ij}_R$$

$$\vec{S}^\dagger_1 : (\bar{3},3,1/3) \implies C^{ij}_9 = -C^{ij}_{10} = 2C^{ij}_L$$

$$V^\dagger_{1/2} : (\bar{3},2,5/6) \implies C^{ij}_{9'} = -C^{ij}_{10'} = 2C^{ij}_R$$

$$\overrightarrow{V}^\dagger_1 : (3,3,2/3) \implies C^{ij}_9 = -C^{ij}_{10} = \frac{1}{2}C^{ij}_L$$

$$C^{ij}_L = \frac{\pi}{\sqrt{2}\alpha G_F V_{td} V^*_{ts}}\left(\frac{\lambda^{bj}_{LS_0}\lambda^{*si}_{LS_0}}{2{m_{S_0}}^2} + \frac{\lambda^{bj}_{LS_1}\lambda^{*si}_{LS_1}}{2{m_{S_1}}^2} - 2\frac{\lambda^{sj}_{LV_1}\lambda^{*bi}_{LV_1}}{{m_{V_1}}^2}\right),$$

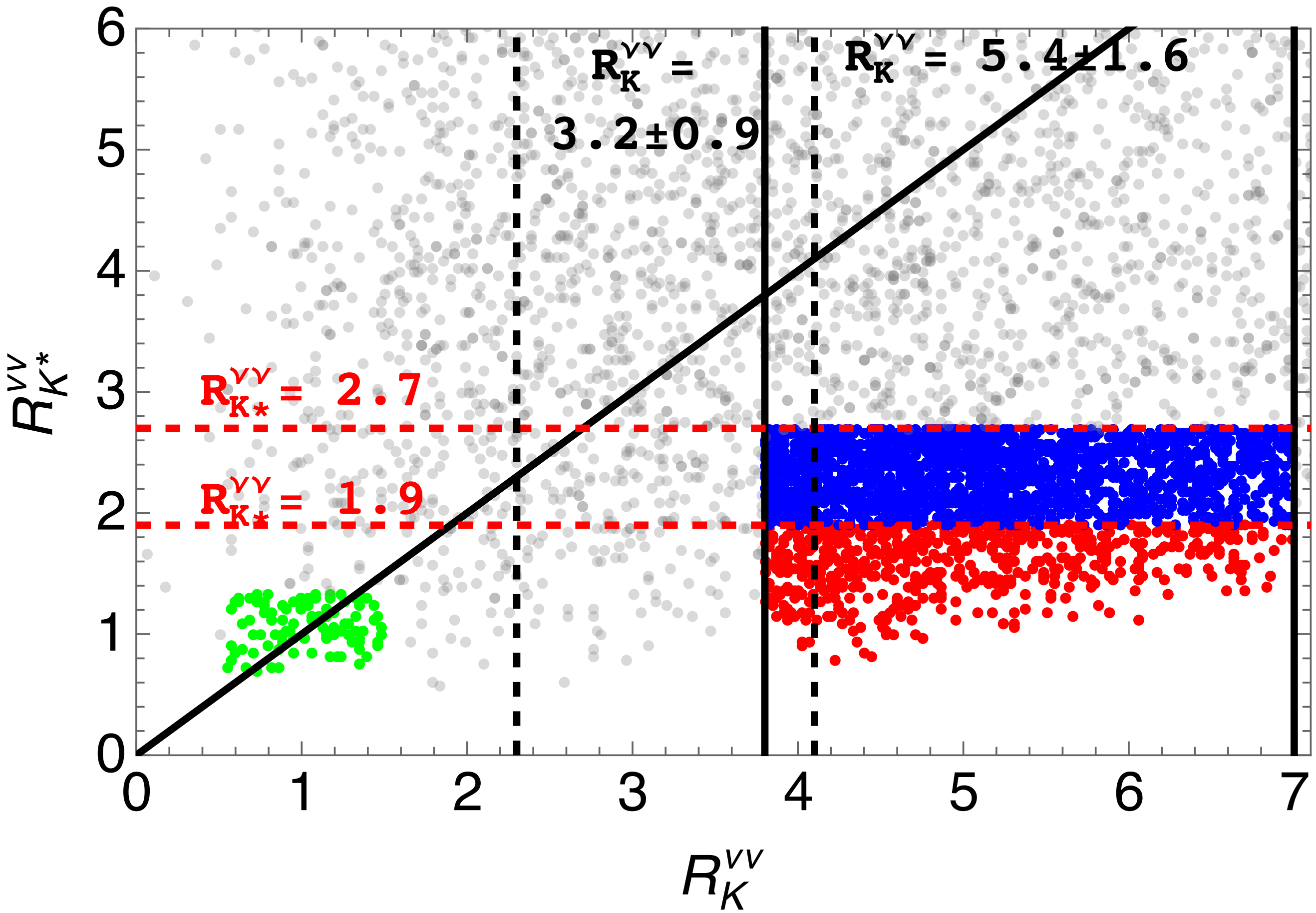
$$C^{ij}_R = C^{ij}_{9'} = -C^{ij}_{10'} = \frac{\pi}{\sqrt{2}\alpha G_F V_{td} V^*_{ts}}\left(-\frac{\lambda^{sj}_{L\tilde{S}_{1/2}}\lambda^{*bi}_{L\tilde{S}_{1/2}}}{2m^2_{\tilde{S}_{1/2}}} + \frac{\lambda^{bj}_{LV_{1/2}}\lambda^{*si}_{LV_{1/2}}}{m^2_{V_{1/2}}}\right),$$

$$C^{ij}_9 = -C^{ij}_{10} = \frac{\pi}{\sqrt{2}\alpha G_F V_{td} V^*_{ts}}\left(\frac{\lambda^{bj}_{LS_1}\lambda^{*si}_{LS_1}}{{m_{S_1}}^2} - \frac{\lambda^{sj}_{LV_1}\lambda^{*bi}_{LV_1}}{{m_{V_1}}^2}\right).$$

- S_0 also modifies $R^{(*)}_D$ via the induced operator $\bar{c}b\bar{\tau}\nu^i$

Scanning over $C_L^{ij} - C_R^{ij}$ shows solutions in general

- One LQ at a time
- S_0, S_1, V_1 generate only C_L terms: $R_K^{\nu\nu} = R_{K^*}^{\nu\nu}$
- $S_{1/2}, V_{1/2}$ with only off-diagonal terms also: $R_K^{\nu\nu} = R_{K^*}^{\nu\nu}$

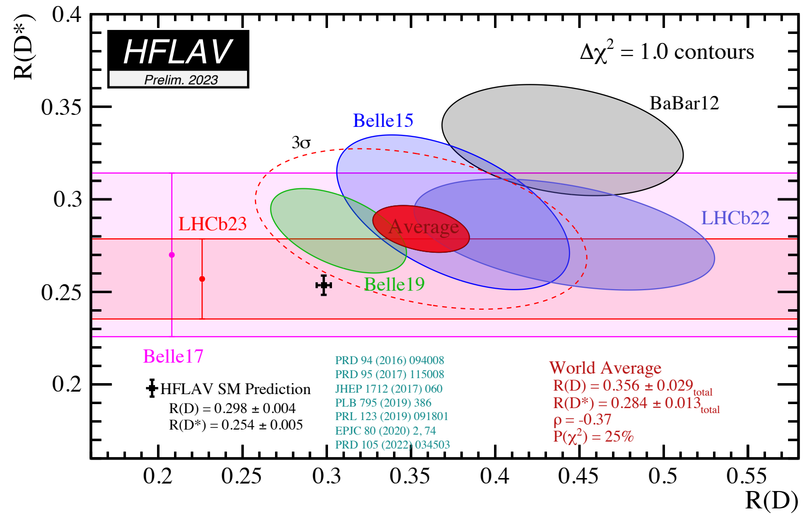


correlations with the B anomalies with C_L

- for the case of S_0 there is a correlation with $r_{D^{(*)}}$

$$r_{D^{(*)}} = \left(\frac{\alpha}{2\pi}\right)^2 |\left(C_L^{3,1}|^2 + |C_L^{3,2}|^2\right) + |1 - \frac{\alpha}{2\pi} C_L^{3,3}|^2$$

$$r_{D^{(*)}} = \frac{R_{D^{(*)}}}{R_{D^{(*)}\,SM}} \qquad \begin{aligned} r_D &= 1.19 \pm 0.10 \\ r_{D^*} &= 1.12 \pm 0.06 \end{aligned}$$



- Effect on $R_K^{\nu\nu} = R_{K^*}^{\nu\nu}$ constrains $r_{D^{(*)}}$:
- S_0 with $R_K^{\nu\nu} = R_{K^*}^{\nu\nu} \sim 3.5$ results in $r_{D^{(*)}} \lesssim 1.06$
- explaining $r_{D^{(*)}}$ with S_0 would lead to $R_K^{\nu\nu} = R_{K^*}^{\nu\nu} \gtrsim 14$

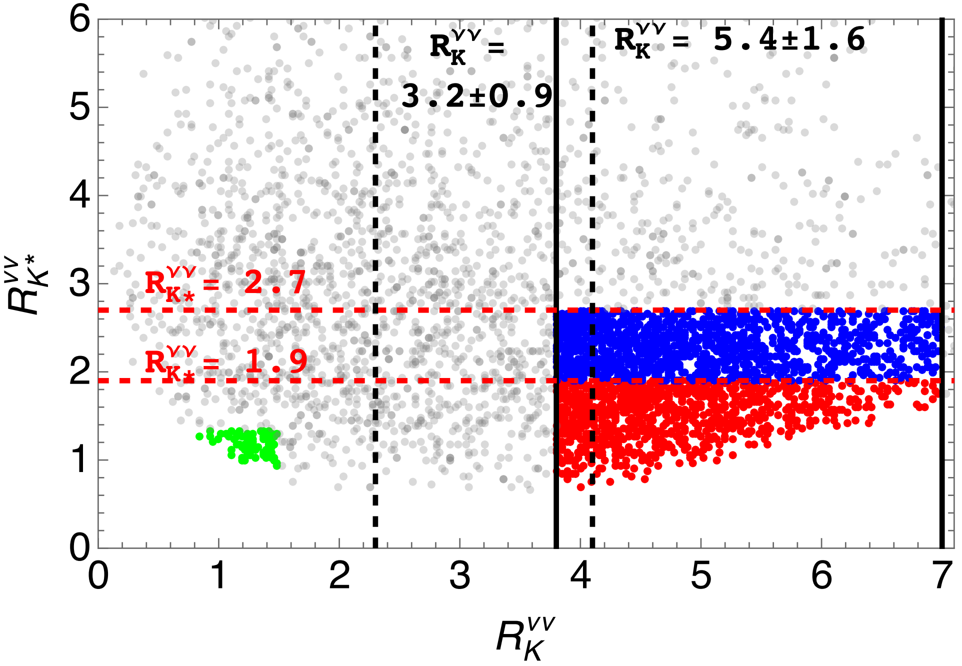
- global fits to $b \to s\ell^+\ell^-$ suggest values $C_9^{\mu\mu}, C_9^{ee} \sim -1$ and C_{10} smaller
- for $S_1,\ V_1$ this means minimal effect on $R_K^{\nu\nu} = R_{K^*}^{\nu\nu}$:

$$S_1 \Longrightarrow \ C_9^{\mu\mu} = -C_{10}^{\mu\mu} = 2C_L^{\mu\mu} \Longrightarrow \ R_K^{\nu} = R_{K^*}^{\nu} \lesssim 1.1$$

$$V_1 \Longrightarrow \ C_9^{\mu\mu} = -C_{10}^{\mu\mu} = \frac{1}{2}C_L^{\mu\mu} \Longrightarrow \ R_{K^{(*)}}^{\nu} \lesssim 1.5$$

$S_{1/2}, V_{1/2}$ with both diagonal and off-diagonal terms

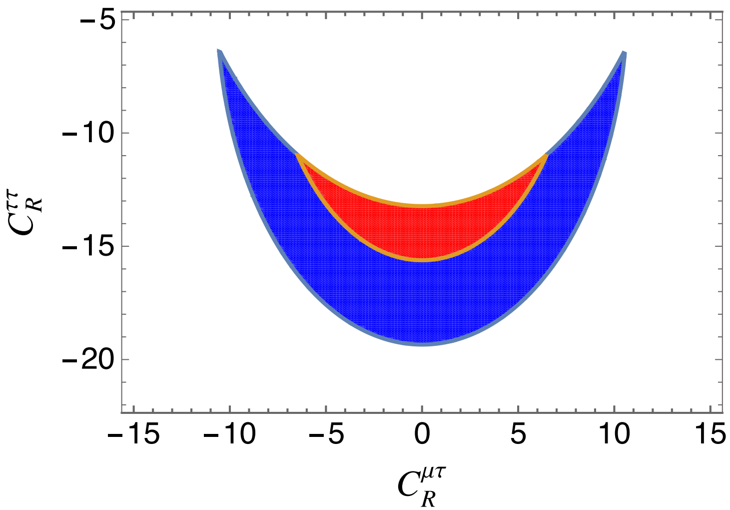
- These two LQs can reproduce the solution region

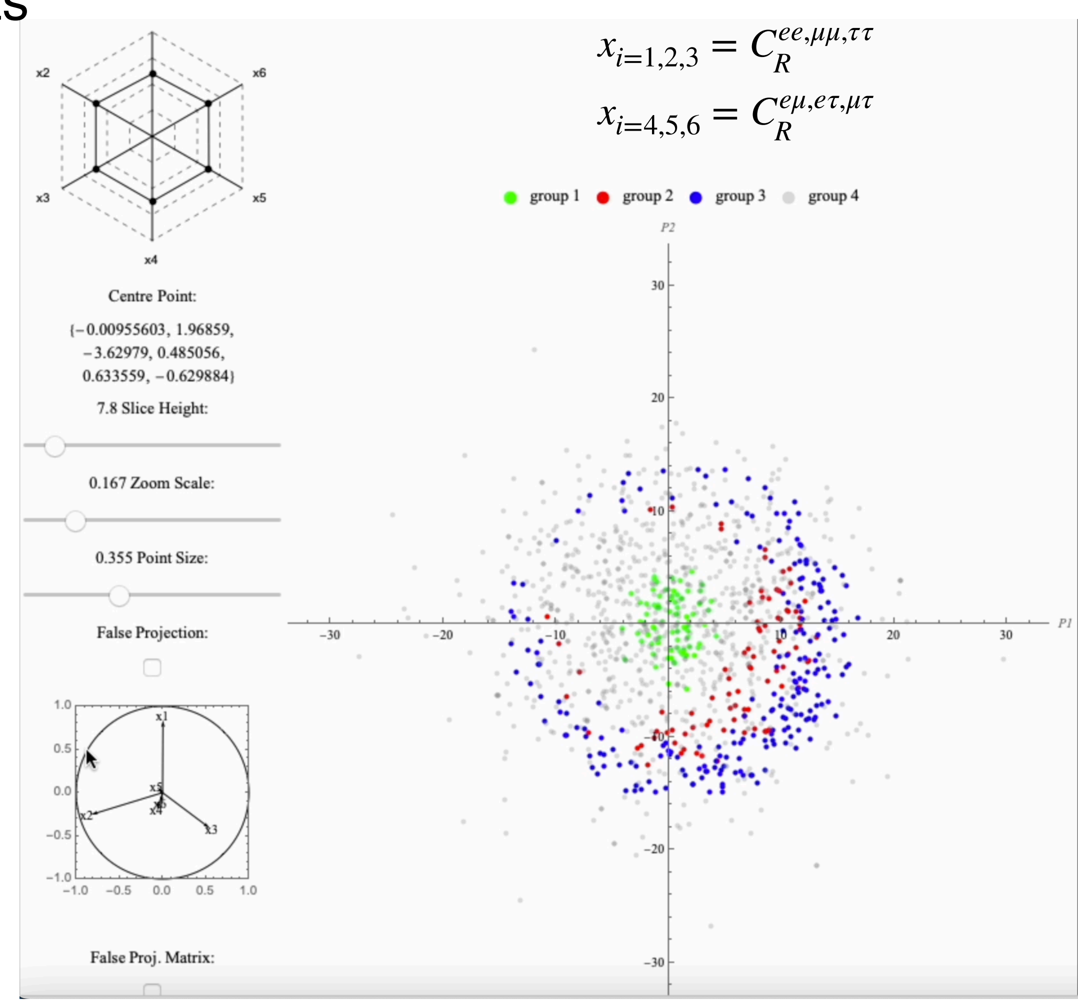


- Find the parameters and see if the models are viable
- recall that $C_R^{ij} = \frac{1}{2} C_{9'}^{ij} = -\frac{1}{2} C_{10'}^{ij}$ affecting $b \to s\ell^+\ell^-$

- at least one of the diagonal terms is large (around 10)
- cannot be $C_R^{ee,\mu\mu}$ from global fits to $b \to s\ell^+\ell^-$
- the only possibility is then to have a large $C_R^{\tau\tau}$

$$C_R^{ij} = \frac{1}{2}C_{9'}^{ij} = -\frac{1}{2}C_{10'}^{ij}$$

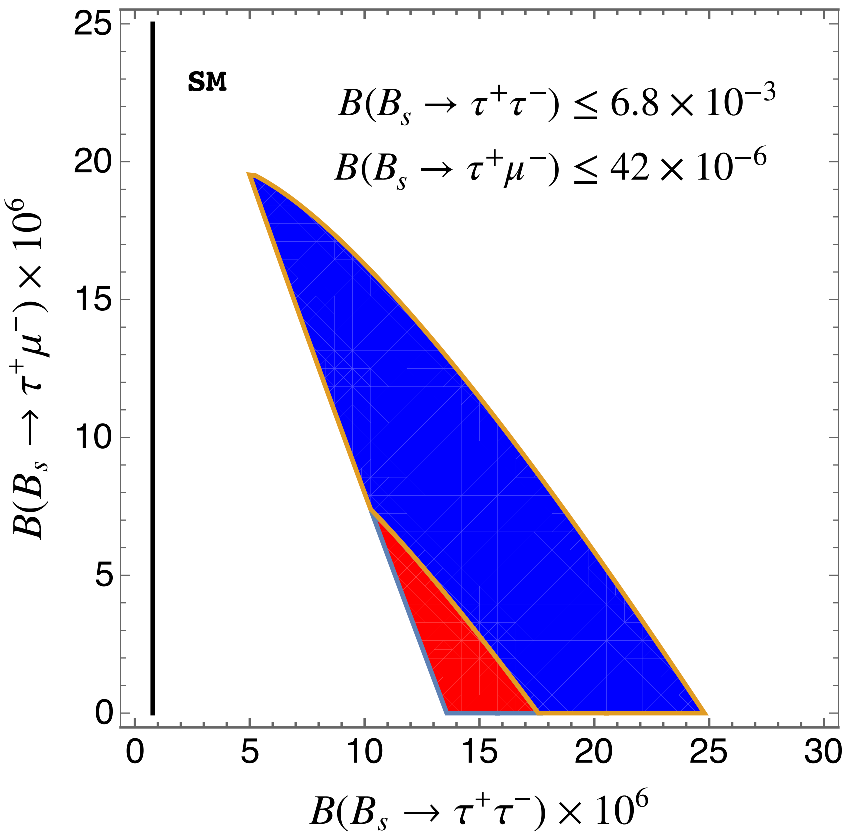


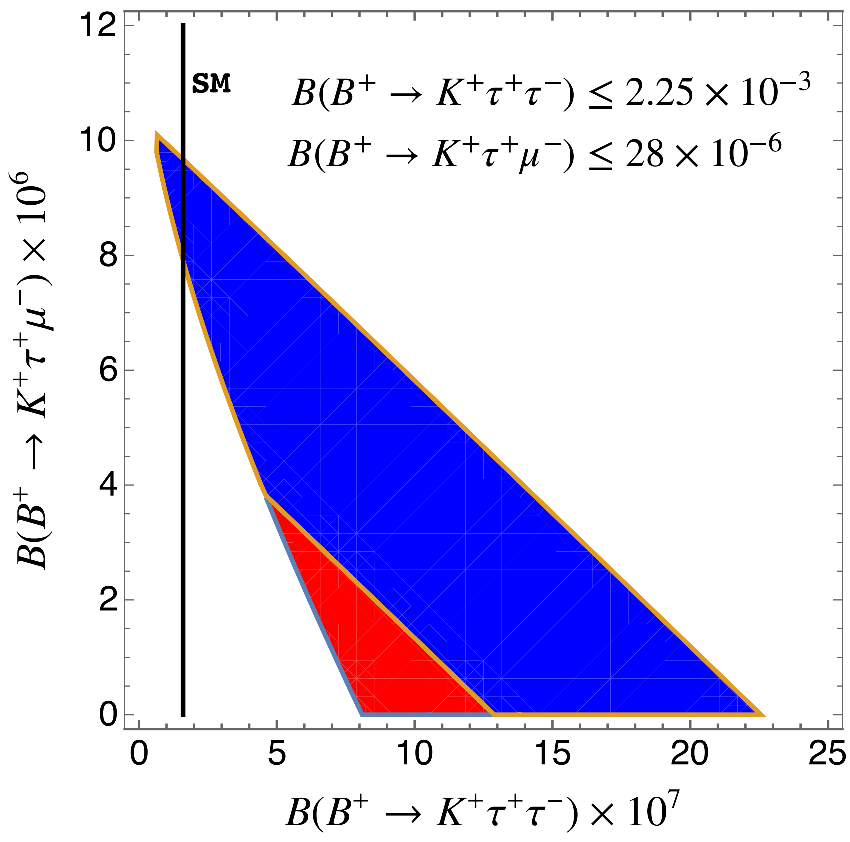


Recall:
$C_{L\ SM}^{ii} \sim -6.4$

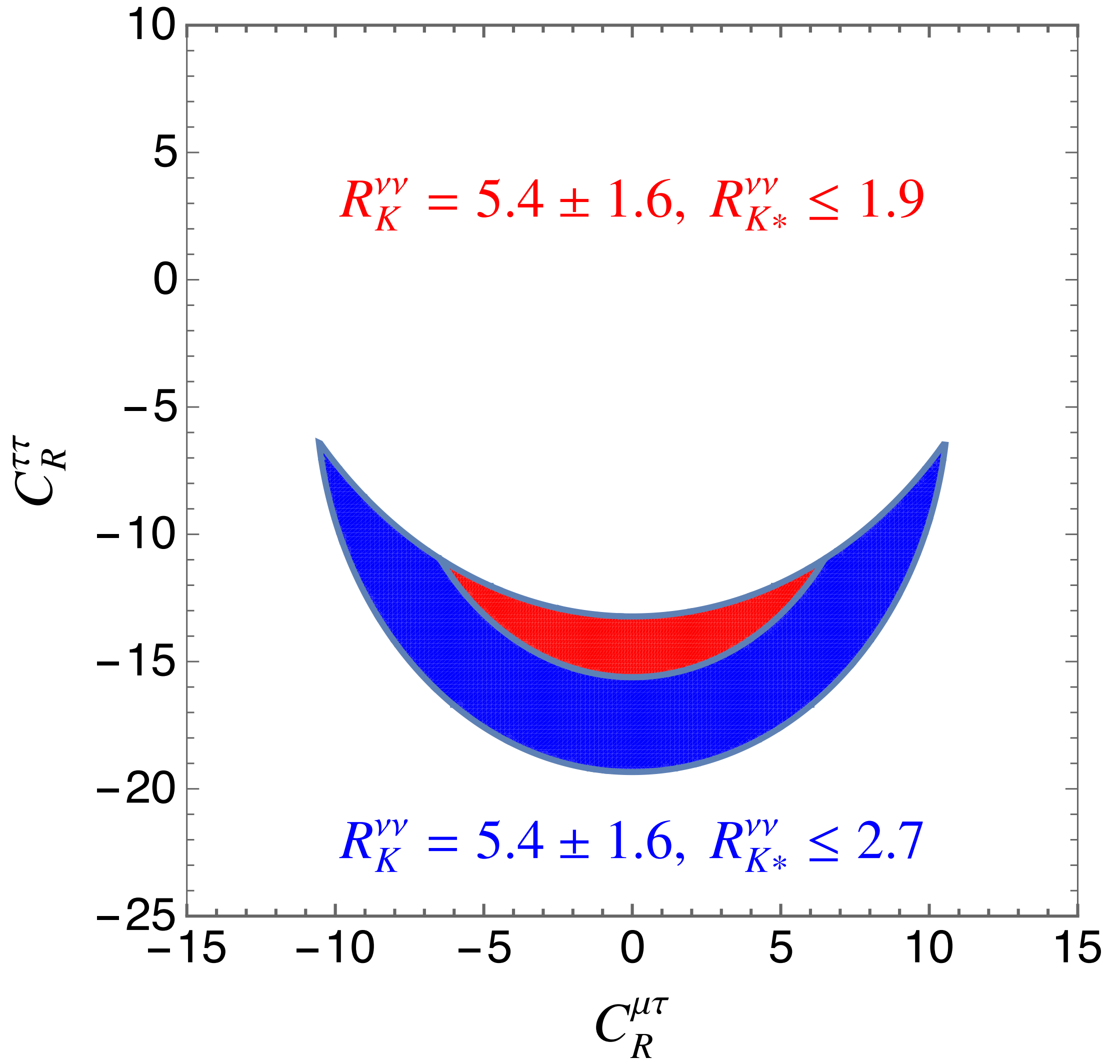
U. Laa, A. Aumann, D. Cook, GV,
JCGS 32:3, 1229 arXiv:2210.05228

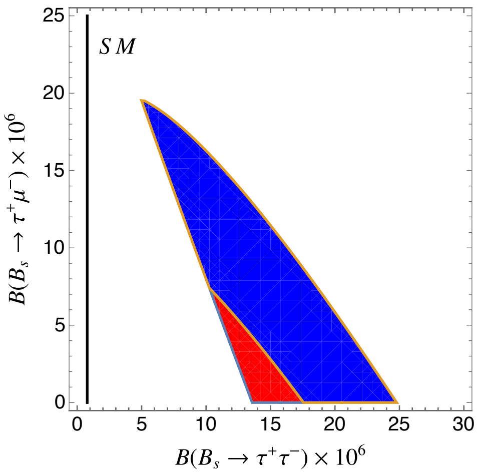
predictions for this solution



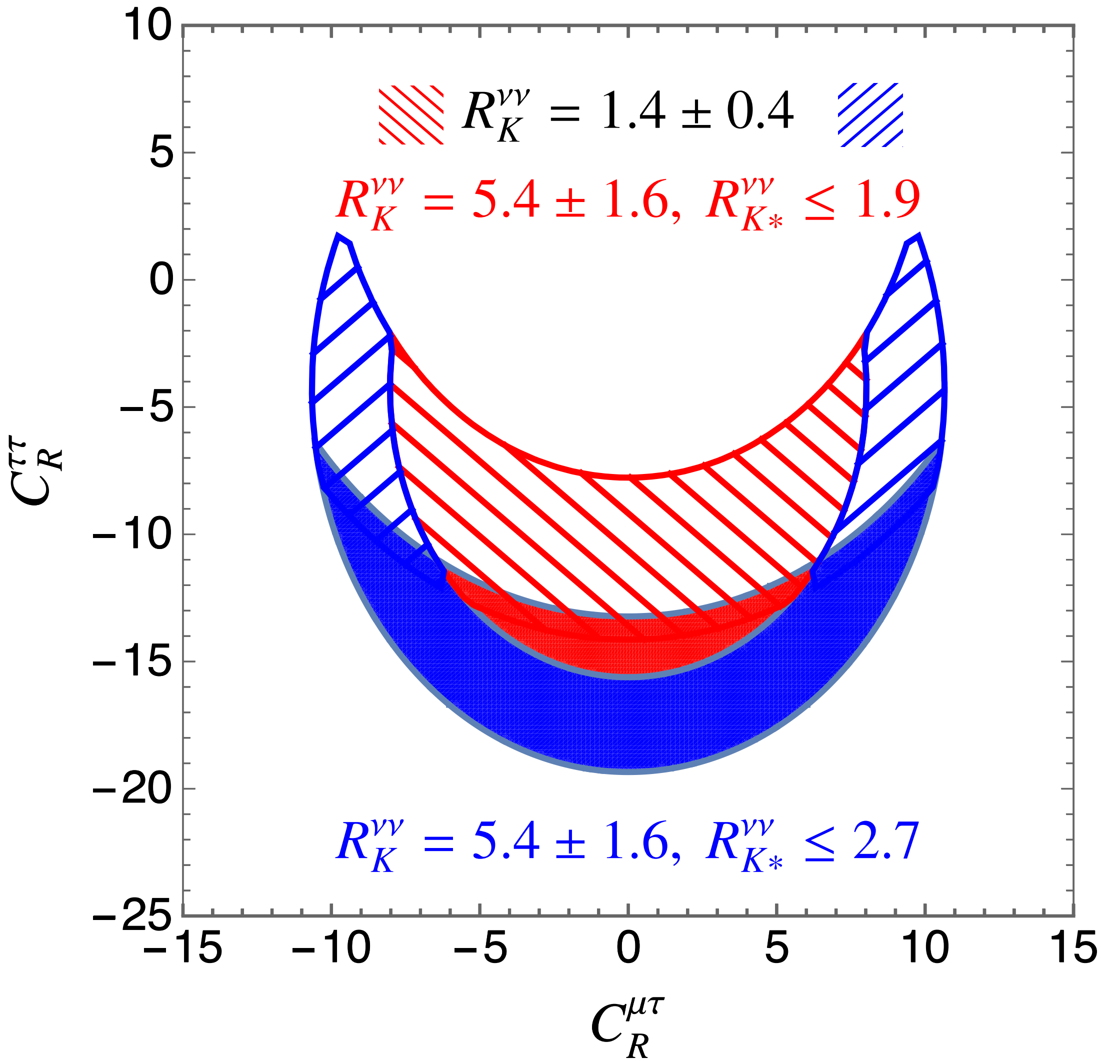


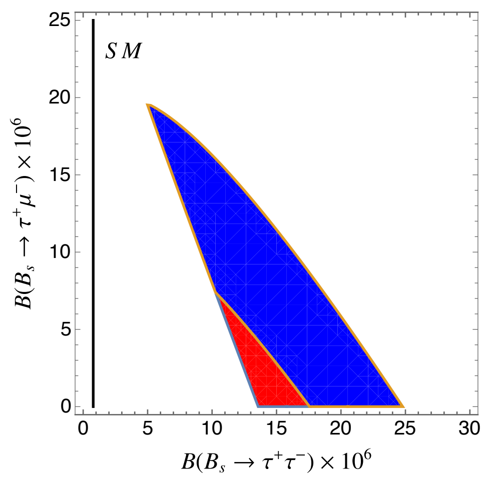
comparing the average to the new result

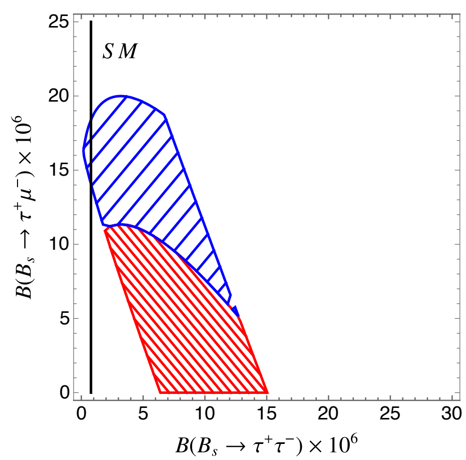




comparing the average to the new result







CLFV and $B \to K^{(*)}\nu\bar{\nu}$

- How competitive is $B \to K^{(*)}\nu\bar{\nu}$ with existing limits on CLFV?

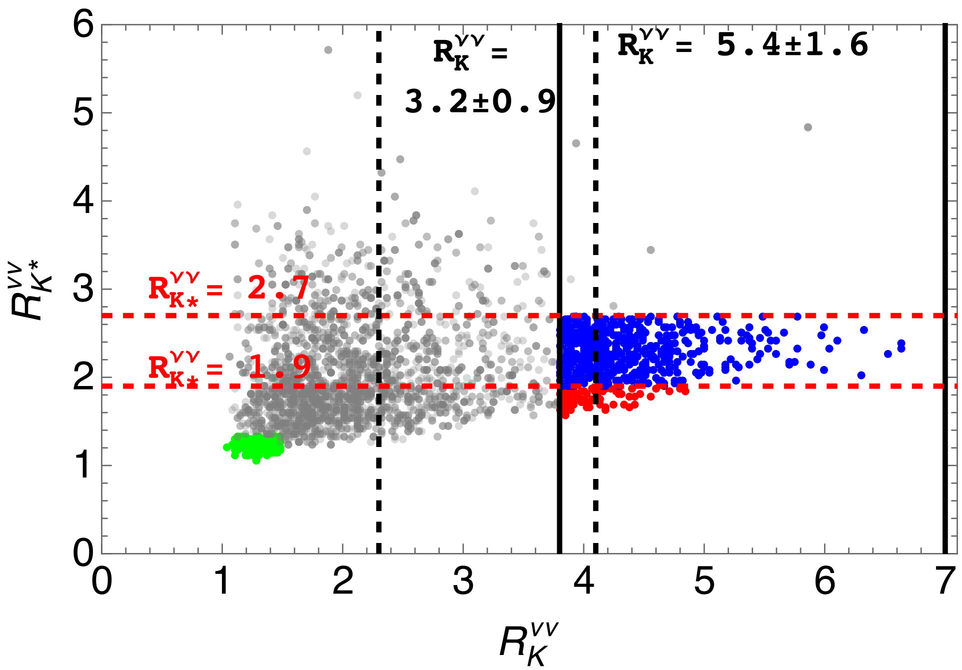
Mode	**90% c.l**
$\mathcal{B}(B_s \to e^\pm \mu^\mp)$	5.4×10^{-9}
$\mathcal{B}(B_s \to \mu^\pm \tau^\mp)$	4.2×10^{-5}
$\mathcal{B}(B^+ \to K^+ e^- \mu^+)$	6.4×10^{-9}
$\mathcal{B}(B^+ \to K^+ e^- \tau^+)$	1.5×10^{-5}
$\mathcal{B}(B^+ \to K^+ \mu^- \tau^+)$	2.8×10^{-5}
$\mathcal{B}(B^+ \to K^{*+} e^- \mu^+)$	9.9×10^{-7}
$\mathcal{B}(B^0 \to K^{*0} e^- \mu^+)$	1.2×10^{-7}

CLFV and $B \to K^{(*)}\nu\bar{\nu}$

LQ	**upper bound on** $C^{ij}_{L,R}$			$R^{\nu}_{K} = R^{\nu}_{K^*}$		
				μe	$e\tau$	$\mu\tau$
$\tilde{S}_{1/2}$	$\lvert C^{\mu e}_{R}\rvert \lesssim 0.4$	$\lvert C^{e\tau}_{R}\rvert \lesssim 26$	$\lvert C^{\mu\tau}_{R}\rvert \lesssim 35$	1.001	6.4	11
S_1	$\lvert C^{\mu e}_{L}\rvert \lesssim 0.2$	$\lvert C^{e\tau}_{L}\rvert \lesssim 13$	$\lvert C^{\mu\tau}_{L}\rvert \lesssim 18$	1.0003	2.4	3.5
$V_{1/2}$	$\lvert C^{\mu e}_{R}\rvert \lesssim 0.4$	$\lvert C^{e\tau}_{R}\rvert \lesssim 26$	$\lvert C^{\mu\tau}_{R}\rvert \lesssim 35$	1.001	6.4	11
$V^{\dagger}_{1}$	$\lvert C^{\mu e}_{L}\rvert \lesssim 0.8$	$\lvert C^{e\tau}_{L}\rvert \lesssim 52$	$\lvert C^{\mu\tau}_{L}\rvert \lesssim 70$	1.005	23	40

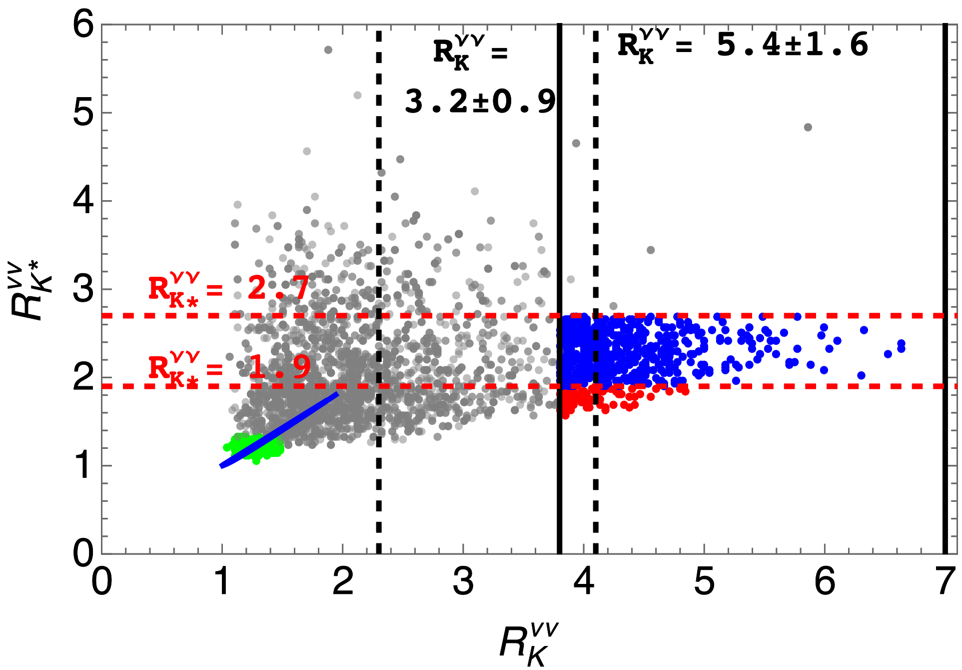
- Each LQ, allowing only off-diagonal terms, produces only C_L or only C_R resulting in $R^{\nu\nu}_{K} = R^{\nu\nu}_{K*}$
- The current upper bound from CLFV processes is less restrictive than the bound from $R^{\nu\nu}_{K}$ except for μe flavours
- This bound for the case of S_1 is comparable to that from $R^{\nu\nu}_{K}$ but less restrictive than the one from $R^{\nu\nu}_{K*}$

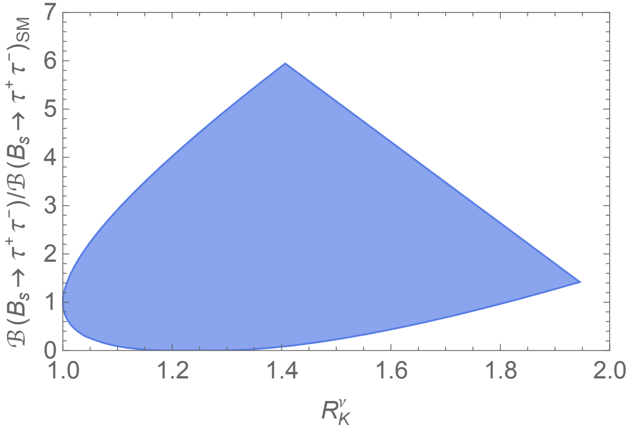
Scanning over $C_L^{\prime ij} - C_R^{\prime ij}$



- in general, this scenario can also reproduce the new result

Scanning over $C_L^{\prime ij} - C_R^{\prime ij}$





- in general, this scenario can also reproduce the new result
- in our specific model, $C_L^{\prime 33} << C_R^{\prime 33}$ (one loop vs tree) resulting in $R_{K*}^{\nu\nu} \sim R_K^{\nu\nu}$
- B_s mixing further restricts $C_L^{\prime 33}$, $C_R^{\prime 33}$ to $R_{K*}^{\nu\nu} \sim R_K^{\nu\nu} \lesssim 2$
- $B_s \to \tau^+\tau^-$ can be enhanced by up to $5 \sim 6$ (could also be suppressed)

New light invisible particles

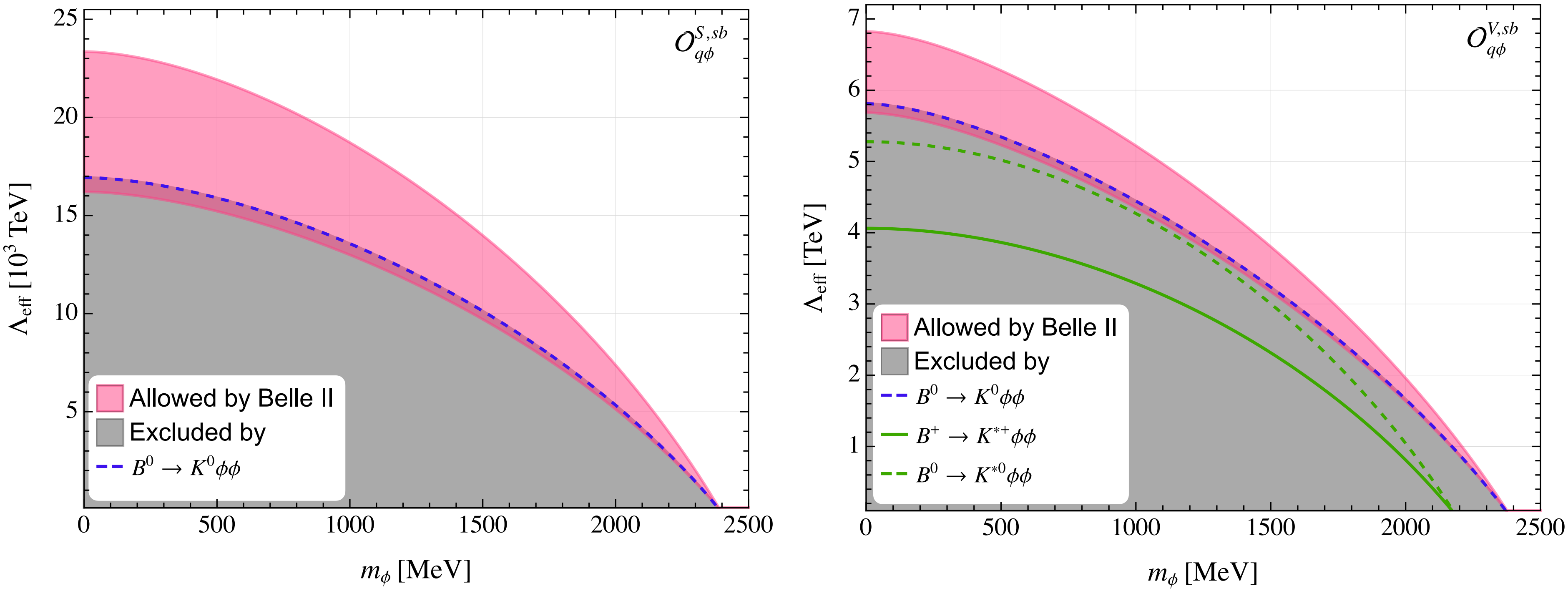
- mass window to invisible light particles: $m < m_B - m_K$
- we assume they are pair-produced (3 body decay)
- we consider spins $0, \frac{1}{2}, 1$
- Mediators are assumed at the weak scale and integrated out to produce a LEFT of the form $\mathscr{L} = \sum C_i\, O_i$
- We define a window to match the new Belle II result as

$$\mathscr{B}(B^+ \to K^+ + invisible)_{\mathtt{NP}} \equiv \mathscr{B}(B^+ \to K^+\nu\bar{\nu})_{\mathtt{exp}} - \mathscr{B}(B^+ \to K^+\nu\bar{\nu})_{\mathtt{SM}}$$
$$= (2.0 \pm 0.7) \times 10^{-5}$$

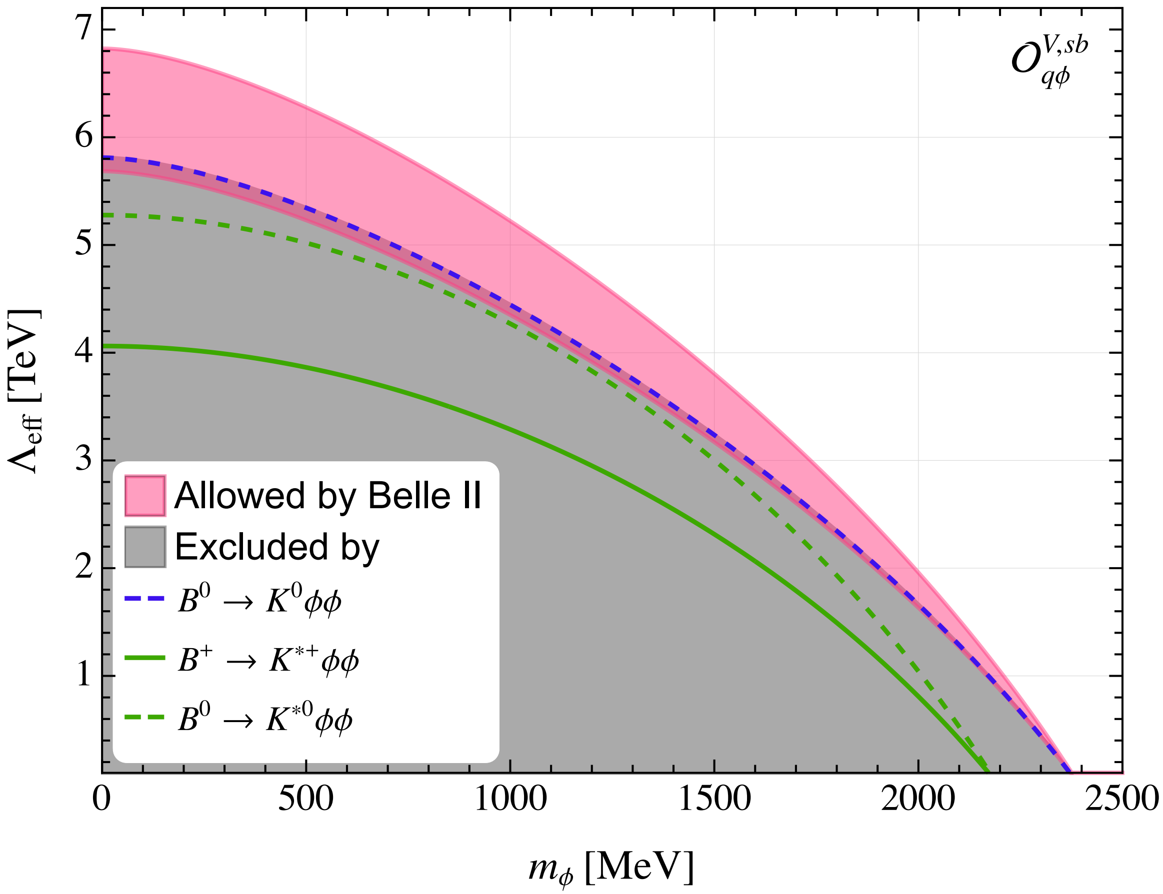
- Consider constraints from other modes
- finally we look for an enhancement in $3 \leq q^2 \leq 7\ \mathrm{GeV}^2$

scalars up to dim 6 that contribute to $B^+ \to K^+ + invisible$

- $\mathcal{O}_{q\phi}^{S,sb} = (\bar{s}b)(\phi^\dagger\phi)$, $\quad \mathcal{O}_{q\phi}^{V,sb} = (\bar{s}\gamma^\mu b)(\phi^\dagger i\overleftrightarrow{\partial}_\mu\phi)$
- use $C_{q\phi}^{S,sb} \equiv \Lambda_{\text{eff}}^{-1}, \quad C_{q\phi}^{V,sb} \equiv \Lambda_{\text{eff}}^{-2}$
- they both arise at dim 6 in SMEFT (blue vanishes for real scalar fields)



plots explained



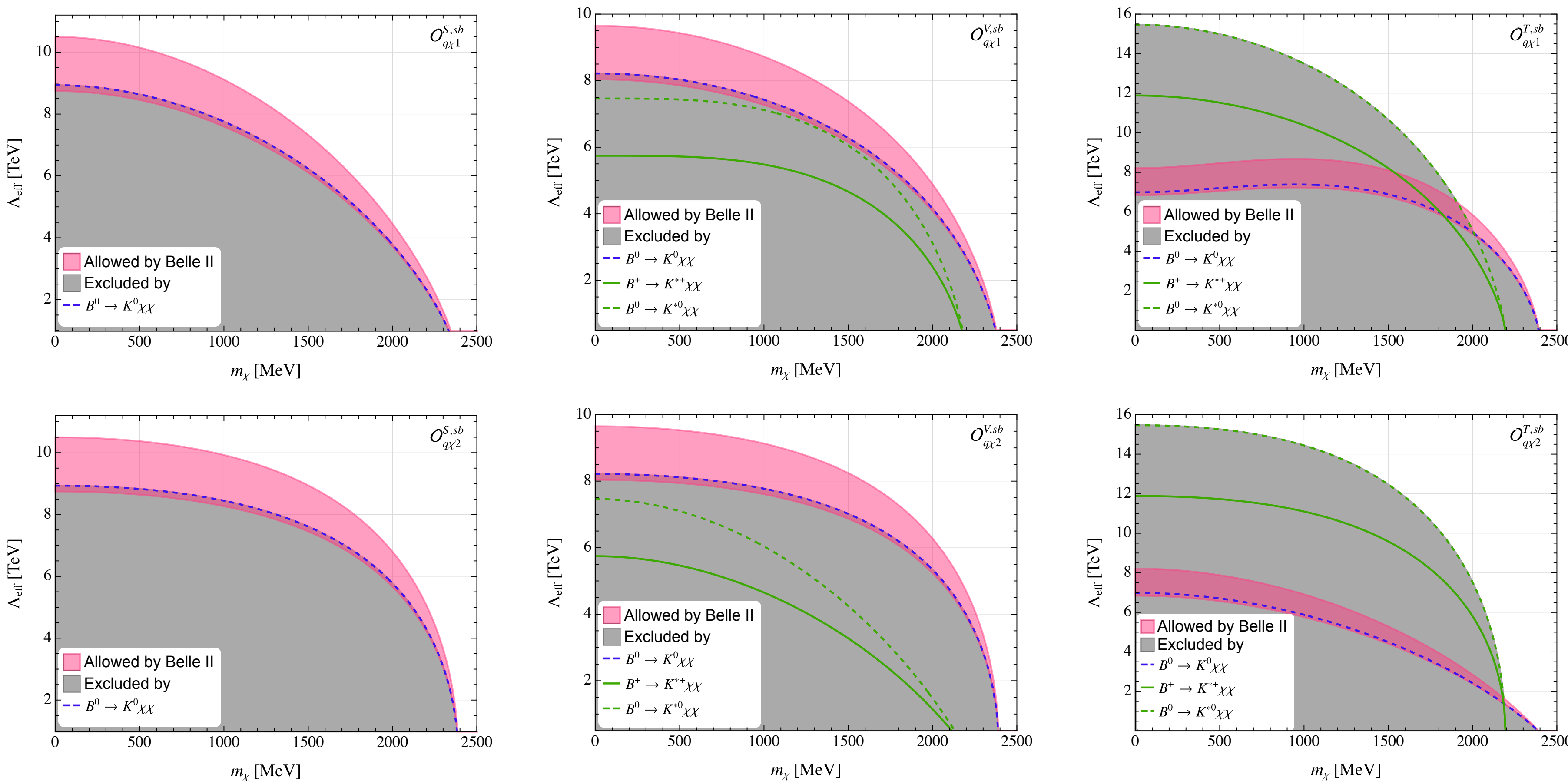
- pink shaded region reproduces the new Belle II result:
  $\mathscr{B}(B^+ \to K^+ + \text{ invisible}) = (2.4 \pm 0.7) \times 10^{-5}$
- region below the solid green line gives a rate for $B^+ \to K^{*+} + \text{ invisible}$ that is too large
- similarly for the dashed blue and green lines for the modes $B^0 \to K^0 + \text{ invisible}$ and $B^0 \to K^{*0} + \text{ invisible}$ respectively
- the region of interest is then the pink region that is above all the other lines.

fermions: six operators at dim 6

$$\mathcal{O}^{S,sb}_{q\chi 1} = (\bar{s}b)(\bar{\chi}\chi), \qquad \mathcal{O}^{S,sb}_{q\chi 2} = (\bar{s}b)(\bar{\chi}i\gamma_5\chi),$$

$$\mathcal{O}^{V,sb}_{q\chi 1} = (\bar{s}\gamma^{\mu}b)(\bar{\chi}\gamma_{\mu}\chi), \qquad \mathcal{O}^{V,sb}_{q\chi 2} = (\bar{s}\gamma^{\mu}b)(\bar{\chi}\gamma_{\mu}\gamma_5\chi), \qquad C_i^j \equiv \Lambda_{\text{eff}}^{-2}$$

$$\mathcal{O}^{T,sb}_{q\chi 1} = (\bar{s}\sigma^{\mu\nu}b)(\bar{\chi}\sigma_{\mu\nu}\chi), \qquad \mathcal{O}^{T,sb}_{q\chi 2} = (\bar{s}\sigma^{\mu\nu}b)(\bar{\chi}\sigma_{\mu\nu}\gamma_5\chi),$$



vectors up to dim 6

- vector field formulation

$$\mathcal{O}_{qX}^{S,sb} = (\bar{s}b)(X_\mu^\dagger X^\mu),$$

$$\mathcal{O}_{qX1}^{T,sb} = \frac{i}{2}(\bar{s}\sigma^{\mu\nu}b)(X_\mu^\dagger X_\nu - X_\nu^\dagger X_\mu),$$

$$\mathcal{O}_{qX2}^{T,sb} = \frac{1}{2}(\bar{s}\sigma^{\mu\nu}\gamma_5 b)(X_\mu^\dagger X_\nu - X_\nu^\dagger X_\mu),$$

$$\mathcal{O}_{qX2}^{V,sb} = (\bar{s}\gamma_\mu b)\partial_\nu(X^{\mu\dagger}X^\nu + X^{\nu\dagger}X^\mu)$$

$$\mathcal{O}_{qX3}^{V,sb} = (\bar{s}\gamma_\mu b)(X_\rho^\dagger \overleftrightarrow{\partial}_\nu X_\sigma)\epsilon^{\mu\nu\rho\sigma},$$

$$\mathcal{O}_{qX4}^{V,sb} = (\bar{s}\gamma^\mu b)(X_\nu^\dagger i\overleftrightarrow{\partial}_\mu X^\nu),$$

$$\mathcal{O}_{qX5}^{V,sb} = (\bar{s}\gamma_\mu b)i\partial_\nu(X^{\mu\dagger}X^\nu - X^{\nu\dagger}X^\mu),$$

$$\mathcal{O}_{qX6}^{V,sb} = (\bar{s}\gamma_\mu b)i\partial_\nu(X_\rho^\dagger X_\sigma)\epsilon^{\mu\nu\rho\sigma}$$

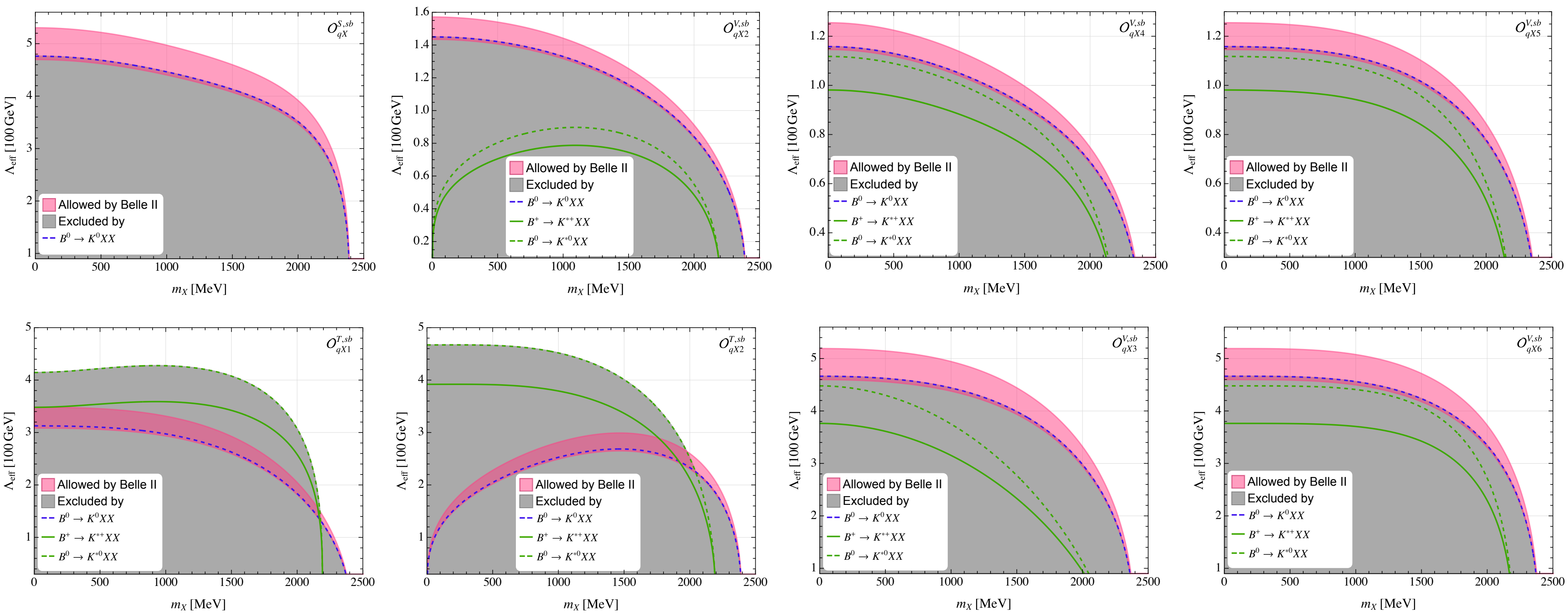
- operators in blue vanish for real fields
- these operators produce amplitudes that diverge in the massless limit
- this known problem is addressed by assuming that X is a gauge boson, and gauge invariance forbids its direct appearance
- these operators are thus assumed to inherit a coefficient that vanishes for massless X

results with vector operators

- scaling including mass factors to address divergence

$$C^S_{qX} \equiv \frac{m^2}{\Lambda^3_{\rm eff}}, \quad C^T_{qX1,2} \equiv \frac{m^2}{\Lambda^3_{\rm eff}}, \quad C^V_{qX2,4,5} \equiv \frac{m^2}{\Lambda^4_{\rm eff}}, \quad C^V_{qX3,6} \equiv \frac{m}{\Lambda^3_{\rm eff}}$$

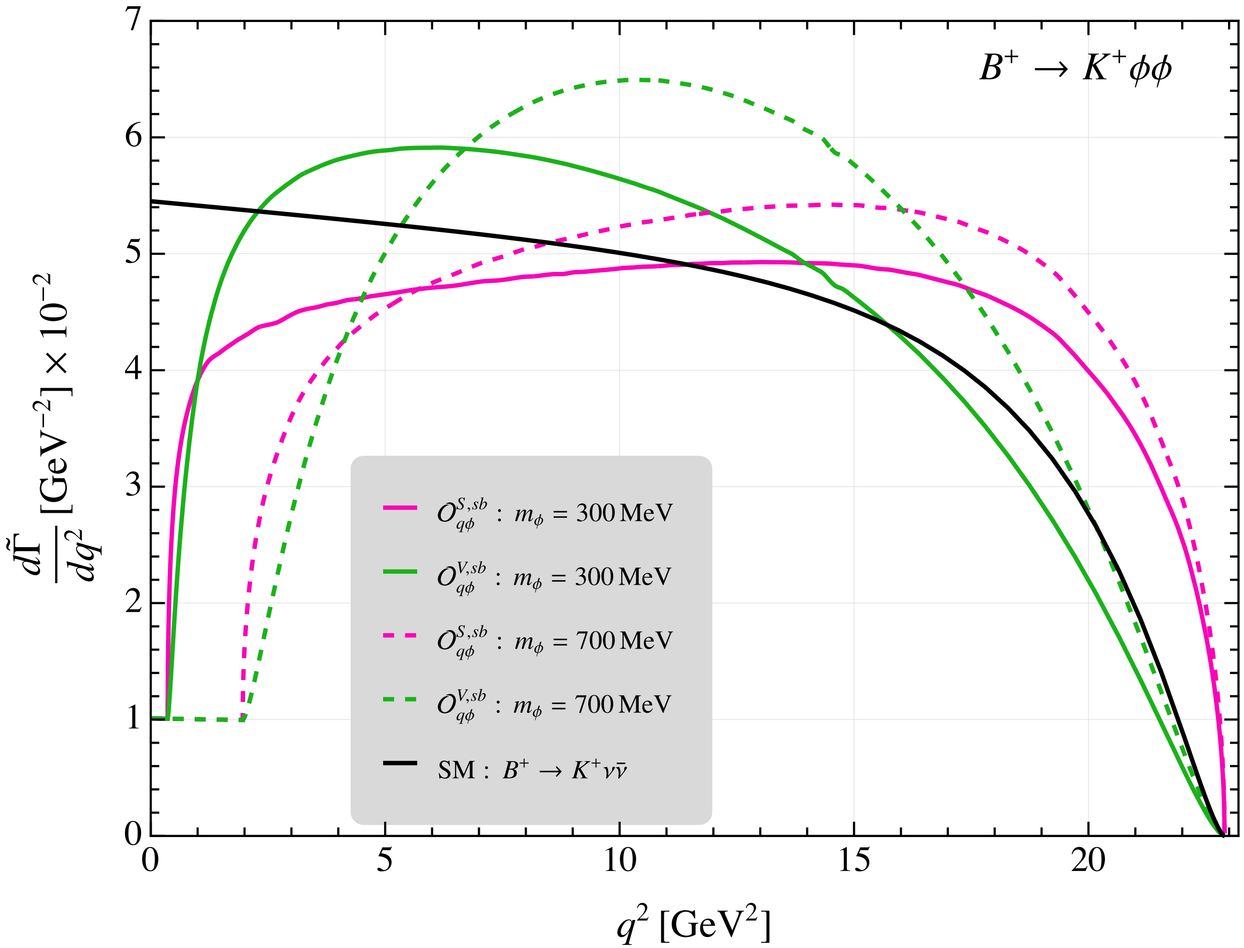
- Two of the operators, $\mathcal{O}^{T,sb}_{qX1}$, $\mathcal{O}^{T,sb}_{qX2}$, are mostly ruled out by other modes



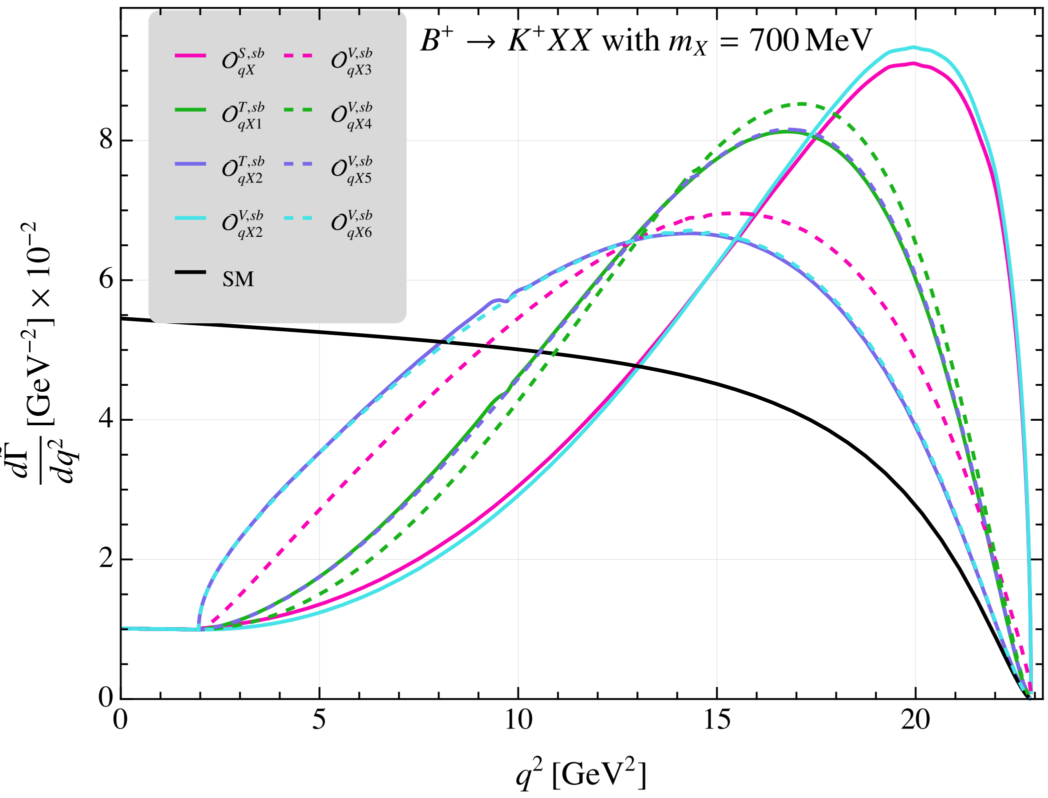
q^2 spectrum scalars

- first Belle II result suggests excess over SM shape around $3 \leq q^2 \leq 7\ \mathrm{GeV}^2$
- $\mathcal{O}_{q\phi}^{V,sb} = (\bar{s}\gamma^\mu b)(\phi^\dagger i\overleftrightarrow{\partial}_\mu \phi)$ with lower m_ϕ appears preferred

$$\frac{d\tilde{\Gamma}}{q^2} \equiv \frac{\frac{d\Gamma_{\rm SM}}{dq^2} + \frac{d\Gamma_{\rm NP}}{dq^2}}{\Gamma_{\rm SM} + \Gamma_{\rm NP}} = \frac{\frac{d\tilde{\Gamma}_{\rm SM}}{dq^2} + (R^K_{\nu\nu} - 1)\frac{d\tilde{\Gamma}_{\rm NP}}{dq^2}}{R^K_{\nu\nu}}$$

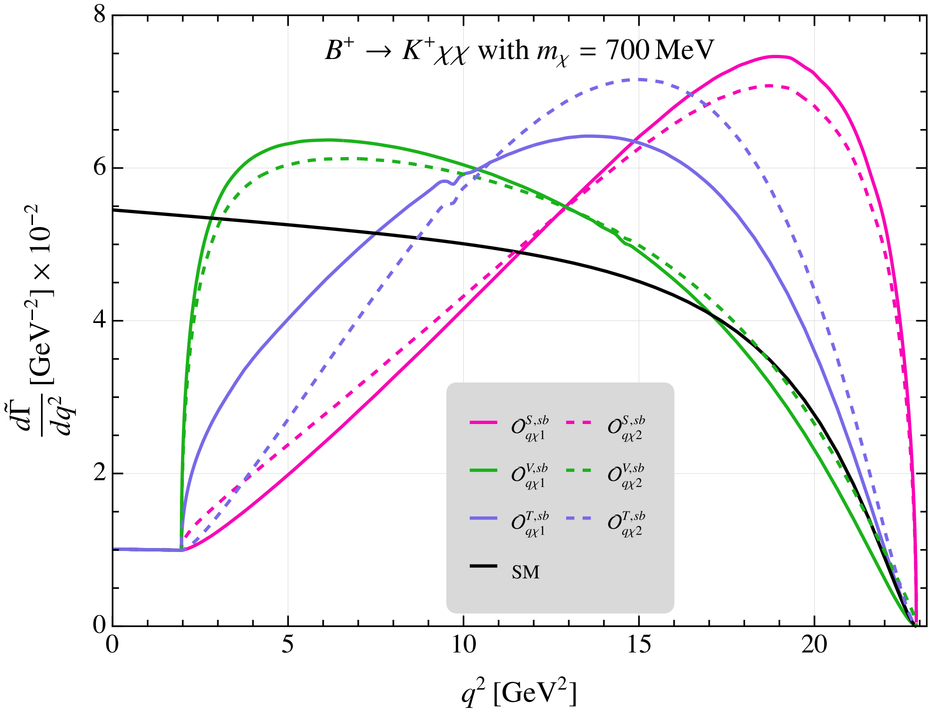


q^2 spectrum vectors



- pairs of spin one particles appear disfavoured by the shape of the spectrum
- will be useful to discriminate among models if excess confirmed

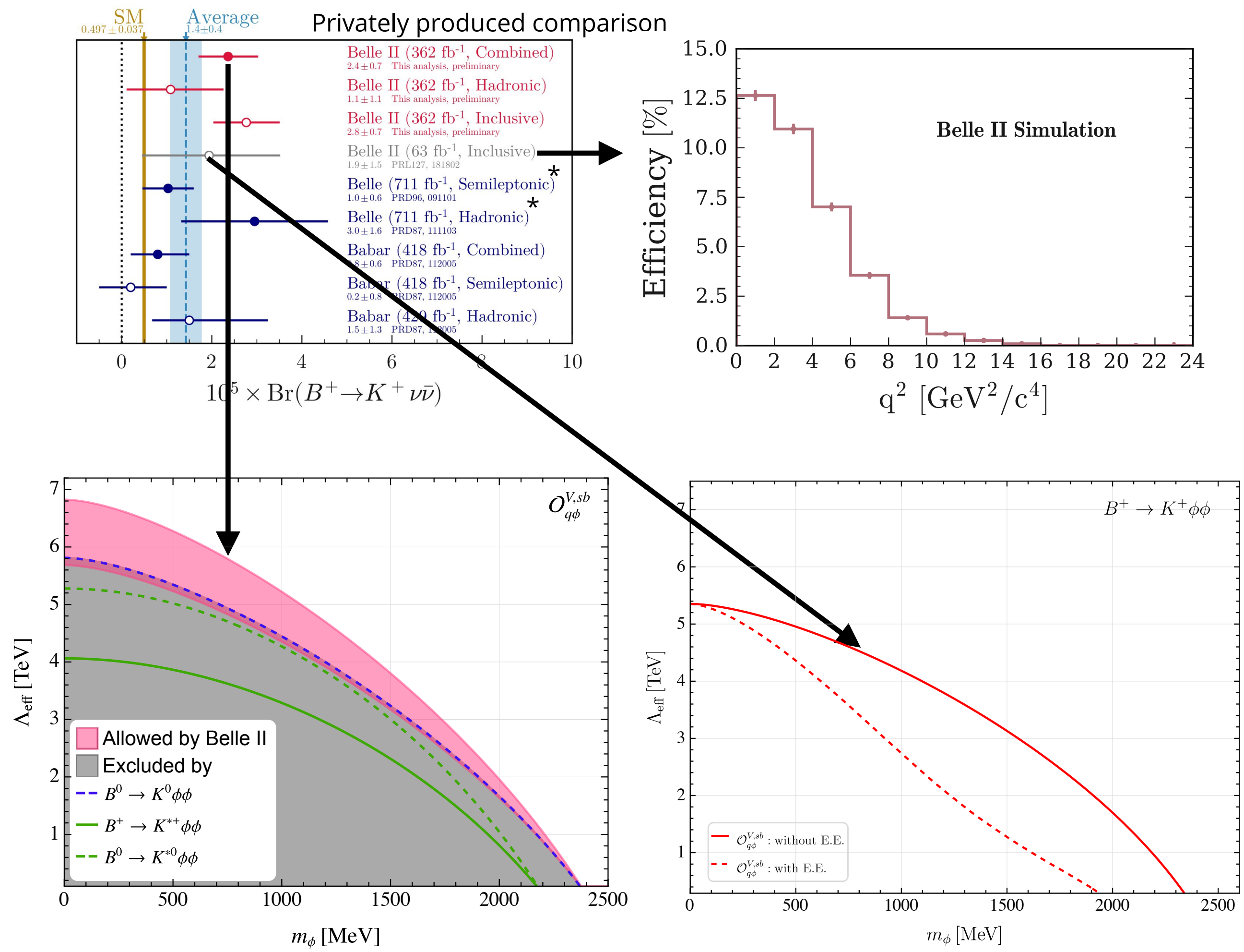
q^2 spectrum fermions



- two operators have a spectrum that peaks in the low q^2
- vector couplings of fermions would thus be preferred $\mathcal{O}^{V,sb}_{q\chi2} = (\bar{s}\gamma^\mu b)(\bar{\chi}\gamma_\mu\chi)$, $\quad (\bar{s}\gamma^\mu b)(\bar{\chi}\gamma_\mu\gamma_5\chi)$ and with masses on the low kinematic end

experimental efficiency

- affects constraints on NP because BR limits assume the SM spectrum
- not enough information in the latest results but we can use 2021 result for illustration



conclusions

- motivated by the recent Belle II result, we have looked at BSM physics that could enhance the mode $B^+ \to K^+ \nu\bar{\nu}$ over its SM value, broadly interpreting it as $B^+ \to K^+ + \quad \text{invisible}$
- at the same time we require consistency with existing 90% c.l upper bounds on the related modes $B \to K^{(*)}\nu\bar{\nu}$ and $B^0 \to K^0\nu\bar{\nu}$
- we also consider correlations with charged lepton modes

- neutrino LFV couplings with only LH neutrinos can reproduce the rates for these modes
  - when induced by a single LQ exchange, $S_{1/2}, V_{1/2}$ can reproduce the rates provided at least one LF diagonal coupling is ~ 10
  - the $b \to s\ell^+\ell^-$ global fits rule out this possibility for $e^+e^-, \mu^+\mu^-$ modes
  - This solution results in enhanced modes with taus that can be probed experimentally.

conclusions continued

- a light sterile (RH) neutrino
  - the parameter space in LEFT can produce the desired pattern in $B \to K^{(*)}\nu\bar{\nu}$ rates
  - a specific model with a non-universal Z' mediator cannot sufficiently enhance $B^+ \to K^+\nu\bar{\nu}$ due to B_s mixing constraints
- pairs of new invisible scalars, vectors or fermions
  - we constructed the lowest dimension LEFT for these three cases and selected the operators relevant for these modes
  - there are viable regions of parameter space to explain the desired pattern in $B \to K^{(*)}\nu\bar{\nu}$ rates for all three cases
  - only a few of them (one with scalars, two with fermions) achieve the enhanced $B^+ \to K^+\nu\bar{\nu}$ rate with a q^2 shape similar to the preliminary result from Belle II.

form factors

$$
\begin{aligned}
\langle P(k)|\bar{q}b|B(p)\rangle &= \frac{m_B^2-m_P^2}{m_b-m_q} f_0(q^2),\\
\langle P(k)|\bar{q}\gamma^\mu b|B(p)\rangle &= \left[(p+k)^\mu - \frac{m_B^2-m_P^2}{q^2}q^\mu\right] f_+(q^2) + \frac{m_B^2-m_P^2}{q^2}q^\mu f_0(q^2),\\
\langle P(k)|\bar{q}\sigma^{\mu\nu} b|B(p)\rangle &= \frac{2i}{m_B+m_P}(p^\mu q^\nu - p^\nu q^\mu) f_T(q^2),
\end{aligned}
$$

$$
f_0(s) = \frac{r_2}{1-s/m_{\rm fit}^2},\; f_{+(T)}^{\pi}(s) = \frac{r_1}{1-s/m_R^2} + \frac{r_2}{1-s/m_{\rm fit}^2},\; f_{+(T)}^{K}(s) = \frac{r_1}{1-s/m_R^2} + \frac{r_2}{(1-s/m_R^2)^2}.
$$

use LCSR results from P. Ball and R. Zwicky Phys. Rev. D 71, 014015 (2005)

$$
\begin{aligned}
\langle V(k)|\bar{q}\gamma_5 b|B(p)\rangle &= -i\epsilon^*_{V,\nu} q^\nu \frac{2m_V}{m_b+m_q} A_0,\\
\langle V(k)|\bar{q}\gamma^\mu b|B(p)\rangle &= \epsilon^{\mu\nu\rho\sigma}\epsilon^*_{V,\nu} p_\rho k_\sigma \frac{2}{m_B+m_V} V_0,\\
\langle V(k)|\bar{q}\gamma^\mu\gamma_5 b|B(p)\rangle &= i\epsilon^*_{V,\nu}\left[g^{\mu\nu}(m_B+m_V)A_1 - \frac{(p+k)^\mu q^\nu}{m_B+m_V}A_2 - q^\mu q^\nu \frac{2m_V}{q^2}(A_3-A_0)\right]\\
\langle V(k)|\bar{q}\sigma_{\mu\nu} b|B(p)\rangle &= i\epsilon_{\mu\nu\rho\sigma}\epsilon^*_{V,\alpha}\left\{ g^{\alpha\rho}(p+k)^\sigma T_1 - g^{\alpha\rho}q^\sigma \frac{m_B^2-m_V^2}{q^2}(T_1-T_2)\right.\\
&\quad \left. + \; 2q^\alpha p^\rho k^\sigma \left[\frac{1}{m_B^2-m_V^2}T_3 - \frac{1}{q^2}(T_1-T_2)\right]\right\},
\end{aligned}
$$

$\{A_0, A_1, A_{12}, V_0, T_1, T_2, T_{23}\}$ as $F_{1,2,3,4,5,6,7}$:

$$
F_i(s) = \frac{1}{1-s/m_{R,i}^2}\sum_k \alpha_k^i [z(s)-z(0)]^k, \quad z(s) \equiv \frac{\sqrt{s_+-s}-\sqrt{s_+-s_0}}{\sqrt{s_+-s}+\sqrt{s_+-s_0}},
$$

use LCSR results from A. Bharucha, D. M. Straub and R. Zwicky JHEP 08, 098 (2016)