Using Gravitational Waves to Constrain the Reheating Temperature

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Based on JCAP 08 (2023) 024 e-Print: 2303.05813 [Biswas, Kar, BH Lee, HC Lee, WW Lee, Scopel, Velasco-Sevilla, Yu] and work to appear soon/ Collaboration with Kunio Kaneta
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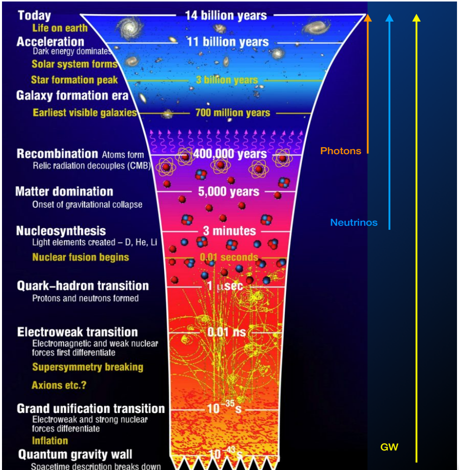
Outline

Motivation

- As we all know, Gravitational Waves offer us a unique opportunity to test theory well beyond photons and neutrinos can do.
- In particular, inflation, reheating and theories of modified gravity



- The basic inflationary paradigm is well seasoned [of course surprises can happen!] but taking it for granted, we can now test its implications through GW
- Reheating temperature can theoretically be from ~ 4 MeV up to $\sim 10^{14}$ GeV but well motivated scenarios could be predictive and the related GW could be tested by current or future experiments

Study Frames

[Both studies were motivated by two ingredients: constraints on DM and constraints on T_{RH} from GW]

1. Modified Gravities: Specific Gauss-Bonnet Model
2. Specific Inflation and Reheating Scenarios: α-attractor scenario

Modified Gravities: Specific Gauss-Bonnet Model
2303.05813 VL, et al..

$$S = \int_{\mathcal{M}} \sqrt{-g}\, d^4x \left[\frac{R}{2\kappa} - \frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi - V(\phi) + f(\phi)R^2_{\rm GB} + \mathcal{L}_m^{\rm rad} \right],$$

where $\kappa \equiv 8\pi G = 1/M_{PL}^2$ and $R^2_{\rm GB} := R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$ is the Gauss-Bonnet (GB) term.
The coupling between the scalar field, ϕ and the GB term is driven by a function of the scalar field $f(\phi)$, which is arbitrary. In this work,

$$f(\phi) = \alpha e^{\gamma\phi},$$

which is the so called dilatonic-Einstein coupling (often appearing in String Theory but can be studied independently).

The modified Friedmann equations can be written as:

$$
\begin{aligned}
H^2 &= \frac{\kappa}{3}\left(\rho_{\{\phi+\mathrm{GB}\}} + \rho_{\mathrm{rad}}\right) \equiv \frac{\kappa}{3}\rho_{\mathrm{tot}}\,, \\
\dot{H} &= -\frac{\kappa}{2}\left[\left(\rho_{\{\phi+\mathrm{GB}\}} + p_{\{\phi+\mathrm{GB}\}}\right) + \left(\rho_{\mathrm{rad}} + p_{\mathrm{rad}}\right)\right] \equiv -\frac{\kappa}{2}\left(\rho_{\mathrm{tot}} + p_{\mathrm{tot}}\right), \\
\ddot{\phi} &+ 3H\dot{\phi} + V' - f' R^2_{\mathrm{GB}} = 0\,,
\end{aligned}
$$

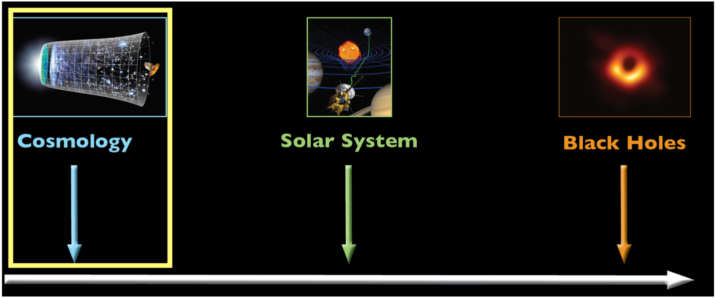
where ρ_{tot} and p_{tot} can be interpreted as the total energy density and the pressure of the Universe.

Gauss-Bonnet theories have been used in the past for

- Inflation [Jinsu Kim talk]
- Quintessence
- Recently for WIMPS [2303.05813]

- In [2303.05813], we have considered a model for which $V(\phi) = 0$ because we wanted to study conditions not coming from inflation but from BBN
- We put constraints on the model parameters using the fact that in a modified cosmological scenario the WIMP annihilation cross section at freeze-out $\langle \sigma v \rangle_f$ required to predict the correct relic abundance is modified compared to the standard value $3{\times}10^{-26}$ cm$^3 s^{-1}$

- By choosing the boundary condition at BBN and evolving back in time we are choosing to be agnostic about the initial conditions at higher temperatures and minimizing the effects of the modified gravities after BBN



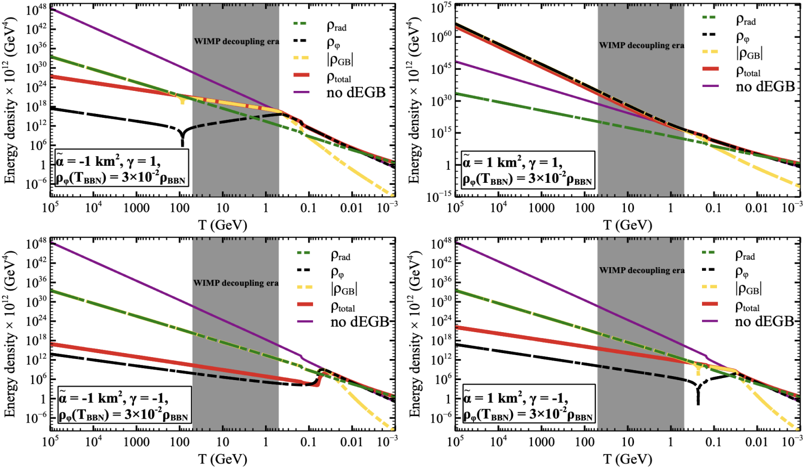
- Equations for H^2 and $\dot{H}$ can be re-arranged into a set of three coupled differential equations for the quantities ϕ, $\dot{\phi}$ and H.
- We fix the boundary condition at BBN ($T = T_{\rm BBN} = 1$ MeV) $\phi(T_{\rm BBN}) \equiv \phi_{\rm BBN}$, $\dot{\phi}(T_{\rm BBN}) \equiv \dot{\phi}_{\rm BBN}$ and $H(T_{\rm BBN}) \equiv H_{\rm BBN}$.
- We can in fact parameterize the physical observables in terms of only the following parameters

$$\phi'_{\rm BBN} = \phi_{\rm BBN} + \phi_0, \;\; \alpha' = \alpha\, e^{-\gamma\phi_0}, \;\; \gamma.$$

or equivantly

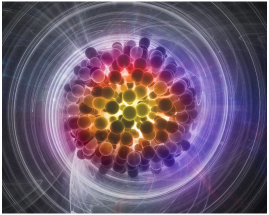
$$\rho_{\rm BBN}, \quad \alpha' = \alpha\, e^{-\gamma\phi_0}, \;\; \gamma.$$

What we found interesting is that considering different evolutions of the universe, via the GB f function leads not only to open up the WIMP parameter space but also offers different evolutions of the total density of the universe



GW Production from SM Plasma

- Physical processes ranging from microscopic particle collisions to macroscopic hydrodynamic fluctuations induce gravitational waves in any plasma in thermal equilibrium [1504.02569, J. Ghiglieri and M. Laine]. Applications studied in 2011.04731, A. Ringwald, J. Schütte-Engel and C.Tamarit / F. Muia, F. Quevedo, A. Schachner, G. Villa 2303.01548, JCAP



- For the largest wavelengths the emission rate is proportional to the shear viscosity, $\eta(T,\hat{k})$, of the plasma. In the Standard Model at $T > 160$ GeV, the shear viscosity is dominated by the most weakly interacting particles, right-handed leptons, and is relatively large.
- The evolution of the density of the GW is simply given by

$$(\partial_t + 4H)\rho(t)_{\mathrm{GW}} = 4\frac{T^4}{\overline{M}_{\mathrm{P}}^2}\int \frac{d^3k}{(2\pi)^3}\eta(T,k),$$

- All the information of the plasma is encoded in $\eta(T,k)$.

Near to the peak of the GW signal $\eta(T,\hat{k})$ can be computed using the HTL (Hard Thermal Logarithmic) methods Braaten, Pisarksi, Soft Amplitudes in Hot Gauge Theories: A General Analysis, Nucl. Phys B, 1990

$$\eta(T,\hat{k}) = \frac{1}{16\pi}\hat{k} f_B(\hat{k}) \sum_{i=1}^{3} d_i \hat{m}_{D_i}^2 \ln\left(4\frac{1}{\hat{m}_{D_i}}\hat{k}^2+1\right), \quad k \gtrsim 3T,$$

where the Debye masses are

$$m_{D_i}^2 = \begin{cases} d_1 \frac{11}{6} g_1^2 T^2,\, d_1 = 1, \\ d_2 \frac{11}{6} g_2^2 T^2,\, d_2 = 3, \\ d_3 2 g_3^2 T^2,\, d_3 = 8. \end{cases}$$

$\hat{k} := k/T$, $\hat{m}_{D_i} = m_{D_i}/T$.

$$\hat{k} = \frac{1}{T} 2\pi f_{\text{Today}} \frac{a_{\text{Today}}}{a}, \quad a(T) = a_0 \frac{T_0}{T} \frac{g_{*0}^{1/3}}{g_*(T)^{1/3}},$$

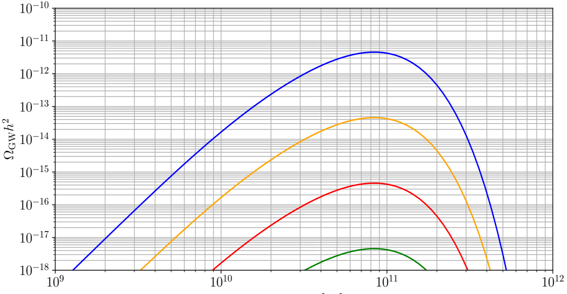
$$\hat{k} = \frac{1}{T} 2\pi f_{\text{Today}}\, a_0 \frac{T}{a_0 T_0} \left(\frac{g_*(T)}{g_{0*}}\right)^{1/3} = 2\pi \frac{1}{T_0} f_{\text{Today}} \left(\frac{g_*(T)}{g_{0*}}\right)^{1/3}.$$

- In Standard Cosmology, if the GW are emitted in the radiation era, there is a simple relation with regards the temperature [1504.02569, J. Ghiglieri and M. Laine]:

Note that if one now assumes $\eta(T, \hat{k})$, $\hat{k}$ and $g_*(T)$ are independent of the temperature [a good approximation up to the order of magnitude], we have

$$\frac{\Omega_{\rm GW}(f)h^2}{\Omega_{\gamma_0}h^2} \quad = \quad \lambda\left[\frac{a_{\rm in}T_{\rm RH}}{a_0 T_0}\right]^4 \frac{T_{\rm RH}-T_{\rm end}}{\overline{M}_{\rm P}}\frac{T_{\rm in}^2}{\sqrt{\rho}}\,\hat{k}^3\,\eta\left(T,\hat{k}\right),$$

From top to bottom $T_{\rm RH} = 10^{14}, 10^{12}, 10^{10}, 10^{8}$ GeV



- In other cosmologies, the evolution of the universe may be diverse and hence the way GW propagate

$$\rho(t_{\text{end}})_{\text{GW}} = \frac{4}{a^4(t_{\text{end}})} \int_{a_{\text{in}}}^{a_{\text{end}}} da\, \frac{a^3}{H} \int \frac{d^3k}{(2\pi)^3} \frac{T^4}{\overline{M}_{\text{P}}^2}\, \eta\left(T, \hat{k}\right),$$

  - $H^2 = \frac{1}{3\overline{M_{\text{P}}}^2} \rho_{\text{Tot.}}$.
  - $H^2 = \frac{\kappa}{3}\left(\rho_{\{\phi+\text{GB}\}} + \rho_{\text{rad}}\right)$, in GB cosmologies and therefore can change drastically the way we can see GW today.
  - For an arbitrary evolution

$$\frac{\Omega_{\text{GW}}(f)h^2}{\Omega_{\gamma_0}h^2} = \frac{\lambda}{\overline{M}_{\text{P}}} \int_{T_{\text{end}}}^{T_{\text{in}}} dT \left(\frac{g_{*0}}{g*(T)}\right)^{4/3} T^2\, \hat{k}^3 \frac{\eta(T, \hat{k})}{\sqrt{\rho_{\text{Tot.}}}}\, F(T),$$

$$F(T) \approx 1,$$

assuming there is not a huge change in the degrees of freedom.

- The basic behaviour is controlled by how big $\rho_{\text{Tot.}}$ increases or decreases with respect to the radiation density, as it can be seen by looking at the temperature dependence on

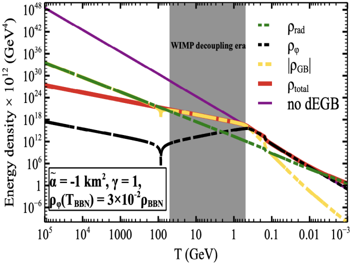
$$\frac{\Omega_{\text{GW}}(f)h^2}{\Omega_{\gamma_0}h^2} \approx \Omega_\gamma \frac{\lambda}{\overline{M}_{\text{P}}} \int_{T_{\text{end}}}^{T_{\text{in}}} dT \left(\frac{g_{*0}}{g*(T)}\right)^{4/3} T^2\, \hat{k}^3 \frac{\eta(T,\hat{k})}{\sqrt{\rho_{\text{Tot.}}}}\,,$$

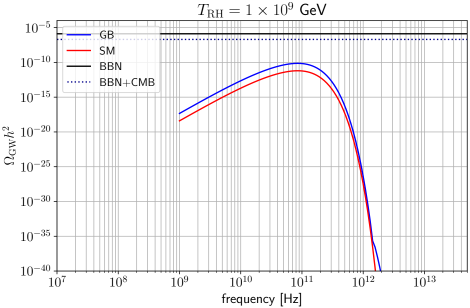
and remembering $\rho_{\text{rad.}} \propto T^4$.

- The peak frequency has only a minor dependence on the temperature and therefore it does not change much

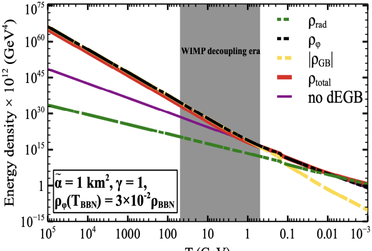
$$\hat{k} = \frac{1}{T} 2\pi f_{\text{Today}} \frac{a_{\text{Today}}}{a}, \quad a(T) = a_0 \frac{T_0}{T} \frac{g_{*0}^{1/3}}{g_*(T)^{1/3}}.$$

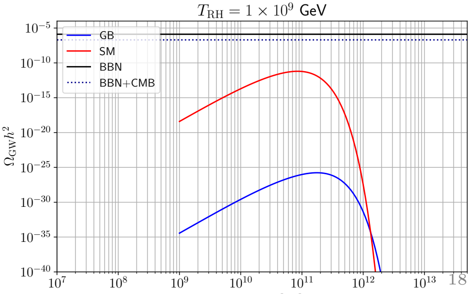
- For $\tilde{\alpha} = -1$, $\gamma = 1$:



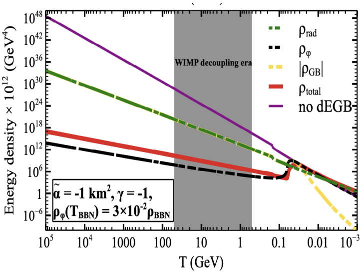


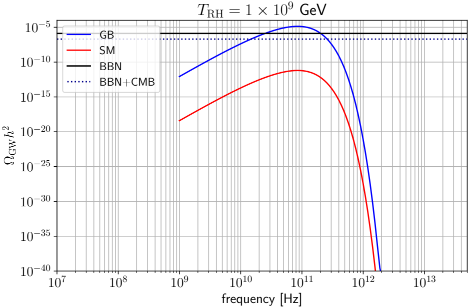
- For $\tilde{\alpha} = 1$, $\gamma = 1$:



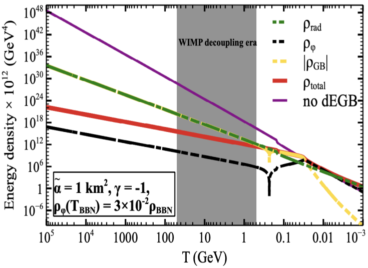


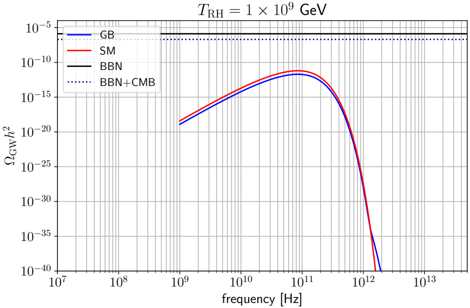
- For $\tilde{\alpha} = -1$, $\gamma = -1$:

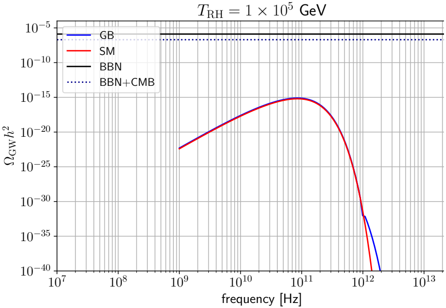


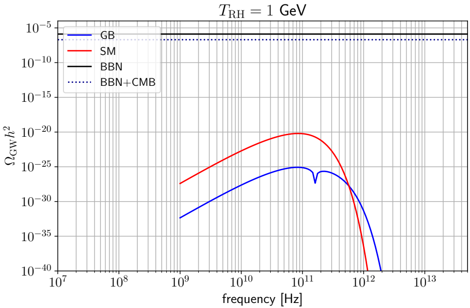


- For $\tilde{\alpha} = 1$, $\gamma = -1$:



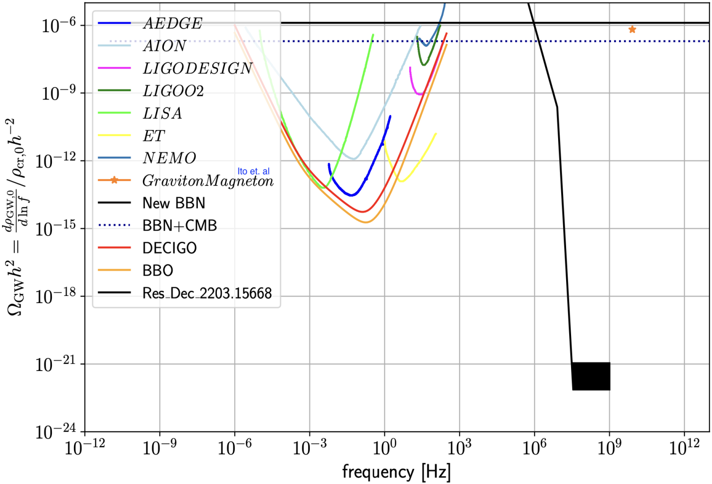






- Hence for some cases, we will be able to set a limit on the reheating temperature
- In these cases, the limit on reheating temperature can be drastically reduced in comparison to the SM
- In other cases, it could be increased and therefore change the panorama of some particle physics processes we know

- We all know however that current and future experimental efforts are far below the peak frequency for these kind of signals



Specific Assumptions of Inflation and Reheating
α attractor Inflationary potentials

- Standard Cosmological evolution of the universe, inflation, Einstein gravity

$$\sqrt{-g}\,\mathcal{L}_{int.} = -\frac{1}{M_{\mathrm{P}}} h_{\mu\nu}\left(T_S^{\mu\nu} M + T_\phi^{\mu\nu} + T_X^{\mu\nu}\right),$$

$$g_{\mu\nu} \approx \eta_{\mu\nu} + \frac{2}{M_{\mathrm{P}}} h_{\mu\nu}$$

- Inflation potentials of the form

$$V(\phi) = \lambda\, M_{\mathrm{P}}^4 \left|\sqrt{6}\tanh\left(\frac{\phi}{\sqrt{6}M_{\mathrm{P}}}\right)\right|^k$$

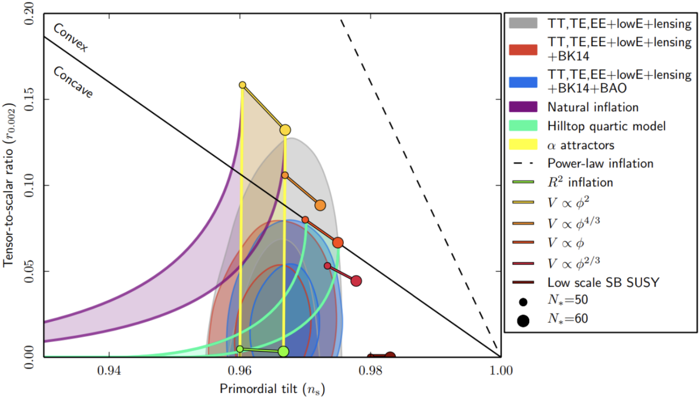
$k = 2$ is the Starobinsky inflation.

- SM only. SM couple to the inflaton through the basic interaction

$$\mathcal{L}_{\phi,\mathcal{SM}} = y\phi\bar{f}f + \eta\phi^2 H^2,$$

where f is a fermion of the SM, H the SM Higgs and y and η are their respective couplings to the inflaton, ϕ.

- We consider this kind of inflation models because are the most successful



Planck Collaboration.

Primordial GW from tensor metric perturbation during reheating while there is gravitational production of matter and radiation

- We follow 2112.15214 [Clery, Mambrini, Olive and Sarunas] and reproduce for production of matter and radiation during reheating after inflation (only gravitational interactions)
- Involved in the scattering of the inflaton or particles in the newly created radiation bath.
- In particular, the gravitational production of dark matter (scalar or fermionic) from the thermal bath as well as from scattering of the inflaton condensate.
- This process produces tensor metric perturbations in the metric, where the GWs equation of motion follows Einstein equations linearized to first order in h_{ij}

$$\partial_t^2 h_{ij} + 3H\partial_t h_{ij} - \frac{\nabla^2}{a^2} h_{ij} = 16\pi G \Pi_{ij}^{TT}$$

where Π_{ij}^{TT} transverse traceless part of the anisotropic stress tensor.

The observed relic density can be obtained from

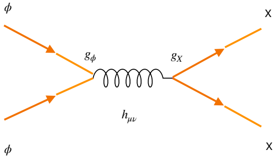
+ (a) [Gravitational portal between the thermal bath and the DM]

$$\Omega^{Ther.}h^2 \approx 1.6 \times 10^8 \frac{g_0}{g_{RH}} \sqrt{\frac{3}{\alpha_{RH}}} m_X [\mathrm{GeV}] \frac{T_{RH}^3}{M_{\mathrm{P}}^3} \frac{F[x^m, x^r, x^s]}{[1 - x^{(7-k)/3k}]^2},$$

$$g_0/g_{RH} = \frac{43}{11}\frac{4}{427},$$

m, r, s powers in k and $x = \rho_{RH}/\rho_{end}$,

$\rho_{end} \rightarrow$ at the end of inflation.



+ (b) [Gravitational portal between the inflaton and the DM]

(i) For scalar DM

$$\Omega_0\, h^2 \approx c_k \left[\frac{m_X}{2.4\times 10^{24/k-6}\ \mathrm{GeV}}\right]\left[\frac{\rho_{end}}{10^{64}\ GeV^4}\right]^{1-1/k}\left[\frac{10^{40}\ \mathrm{GeV}^4}{\rho_{RH}}\right]^{1/4-1/k},$$

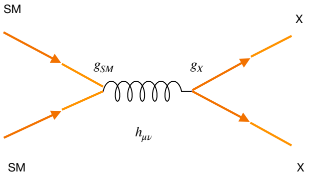
$$c_k \sim \frac{k+2}{6k-6}$$

(ii) For fermionic DM

$$\Omega_0\, h^2 \approx c_k \frac{1}{2.4^{8/k}}\left[\frac{m_X}{8.3\times 10^{6+6/k}\ \mathrm{GeV}}\right]^3\left[\frac{10^{-11}}{\lambda}\right]^{2/k}\left[\frac{10^{40}\ \mathrm{GeV}^4}{\rho_{RH}}\right]^{1/4-1/k}$$

$$\times \left[\frac{\rho_{end}}{10^{64}\ \mathrm{GeV}^4}\right]^{1/k},$$

$$c_k \sim \frac{k+2}{k(k-1)}$$



$$g_\phi = \frac{T_\phi^{\mu\nu}}{M_P}$$

$$g_X = \frac{T_X^{\mu\nu}}{M_P}$$

$$g_{SM} = \frac{T_{SM}^{\mu\nu}}{M_P}$$

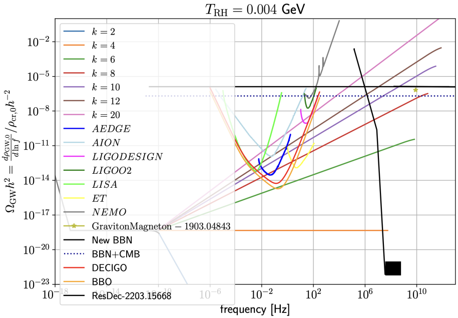
- The GW spectrum today can be parameterized as

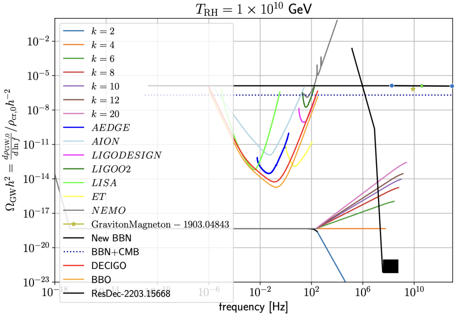
$$\Omega_{\rm GW}(f)h^2 \sim \Omega_{\gamma_0}h^2 \begin{cases} f^{2n-4}, & f < f_{\rm peak} \\ 0, & f > f_{\rm peak} \end{cases},$$

  n related to the parameter k appearing in the inflationary potential.

- $k = 2\pi f_{\rm Today}\frac{a_{\rm Today}}{a}$, $a(T_{\rm RH}) = a_0\frac{T_0}{T_{\rm RH}}\frac{g_{*0}^{1/3}}{g_*(T_{\rm RH})^{1/3}}$,

- This can put constrains on the reheating temperature





- We see then that α-attractor models for inflation can be constrained by BBN+CMB and from future experiments like LISA DECIGO and BB0 for the $(10^{-5}, 10^{2})$ Hz region and experimental concepts using Graviton-Magnon resonances and probing the GHz region [Ito, Ikeda, Miuchi, Soda. Eur. Phys. J. C (2020) 80:179]. For some cases, even using only BBN bounds.

Summary

- GW will probe the evolution of our universe in a way photons or neutrinos cannot
- In particular GW can probe the reheating temperature
- Standard General Relativity or theories of Modified gravity can be tested using basic assumptions, for example the GW coming from the Standard Model plasma
- No experiments are currently planned to probe the required region in frequency and sensitivity but this kind of studies constitute a solid science case that can provide a motivation to develop experiments probing that region
- Being optimistic, some cases can be ruled out just by using the BBN bound
- Using further assumptions, like a specific inflation and/or reheating scenario we can constrain reheating temperature in lower frequencies that can be potentially observed by future experiments.