NEW SCALAR FIELDS IN 3-3-1 MODEL WITH AXION-LIKE PARTICLE

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Motivation

$$SU(3)_{Color} \quad SU(2)_{Left} \otimes U(1)_{Y_{weak}}$$

$$Q = T_3 + \frac{Y}{2} \quad (1)$$

STANDARD MODEL

1. Extra dimesions
2. Extend the spectrum of particles
3. Extend the gauge symmetry

Strong CP problem is not verified by experiments

- massless neutrino
- gravity is absent
- unlimited fermion generation
- mass of top quark 172GeV
- there is no darkmatter candidate

SUCCESS

predict the existences of c, t, W, Z and Higgs 125GeV

BEYOND STANDARD MODEL

1. At 200GeV scale, BSM includes SM
2. Define the mixing angles of ν that fit with data from experiments
3. Explain the BAU
4. Higgs spectrum includes SMLHB
5. Exist new particles that can be dark matter candidate

Axion dark matter

- pseudoscalar proposed by Peccei - Quinn theory
- solves the Strong CP problem
- mass: $$m_a^2 \simeq \frac{f_\pi^2 m_\pi^2}{f_a^2} \frac{m_u m_d}{(m_u + m_d)^2} \quad (2)$$
- interacts with 2 gluons

3-3-1 MODEL

$$SU(3)_C \otimes SU(3)_L \otimes U(1)_X$$

$$Q = T_3 + \beta T_8 + X \quad (3)$$

$$\beta = -\frac{1}{\sqrt{3}}$$

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Phys.Lett.B **771**, 199 – 205, (2017)

3-3-1 model with axion (A331)

Axion-like particle (ALP)

XENON1T (2019)

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3-3-1 model with axion-like particle (ALP331)

ALP331 model

The particle content

Leptons in triplets

$$\psi^a_L = \begin{pmatrix} \nu^a_L \\ l^a_L \\ \nu^{ca}_L \end{pmatrix} \sim (\mathbf{1}, \mathbf{3}, -\mathbf{1/3}), \quad a = 1, 2, 3 \qquad l^a_R \sim (1, 1, -1), \quad N_{aR} \sim (1, 1, 0), \tag{4}$$

Majorana RH neutrino

2 quark generations in anti-triplets,

$$Q_{\alpha L} = \begin{pmatrix} d_{\alpha L} \\ -u_{\alpha L} \\ D_{\alpha L} \end{pmatrix} \sim (\mathbf{3}, \bar{\mathbf{3}}, \mathbf{0}), \qquad D_{\alpha R} \sim (3, 1, -1/3), \quad \alpha = 1, 2$$

one quark generation in triplet

$$Q_{3L} = \begin{pmatrix} t \\ b \\ T \end{pmatrix}_L \sim (\mathbf{3}, \mathbf{3}, \mathbf{1/3}), \qquad T_R \sim (3, 1, 2/3), \tag{5}$$

one singlet $\phi \sim (\mathbf{1}, \mathbf{1}, \mathbf{0})$, $\langle \phi \rangle = v_\phi / \sqrt{2}$,

3 Higgs triplets

η, χ - same quantum number, diffirent VEV

$$\rho = \begin{pmatrix} \rho^+_1 \\ \rho^0_2 \\ \rho^+_3 \end{pmatrix} \sim (\mathbf{1}, \mathbf{3}, \mathbf{2/3}), \quad \eta = \begin{pmatrix} \eta^0_1 \\ \eta^-_2 \\ \eta^0_3 \end{pmatrix} \sim (\mathbf{1}, \mathbf{3}, -\mathbf{1/3}), \quad \chi = \begin{pmatrix} \chi^0_1 \\ \chi^-_2 \\ \chi^0_3 \end{pmatrix} \sim (\mathbf{1}, \mathbf{3}, -\mathbf{1/3}),$$

$$\langle \rho \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & v_\rho & 0 \end{pmatrix}^T, \quad \langle \eta \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_\eta & 0 & 0 \end{pmatrix}^T, \quad \langle \chi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & v_\chi \end{pmatrix}^T, \tag{6}$$

ALP331 model

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$$SU(3)_C \otimes SU(3)_L \otimes U(1)_X$$

$V_{Higgs} \supset$ 18 non-hermite terms

$\mathcal{L}^{\nu 331}_{Yuk} \supset$ 12 independent multiplets

$$SU(3)_C \otimes SU(3)_L \otimes U(1)_X \otimes Z_{11}$$

$\mathcal{L}^{A331}_{Yuk} \supset$ 13 independent multiplets

$V_{Higgs} \supset$ 3 non-hermite terms

$$SU(3)_C \otimes SU(3)_L \otimes U(1)_X \otimes Z_{11} \otimes Z_2$$

$$\boldsymbol{(\phi, \chi, d_{aR}, U_R) \rightarrow -(\phi, \chi, d_{aR}, U_R)} \quad (7)$$

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$$\begin{aligned} V_{Higgs} = & \ \mu_\phi^2 \phi^*\phi + \mu_\chi^2 \chi^\dagger\chi + \mu_\rho^2 \rho^\dagger\rho + \mu_\eta^2 \eta^\dagger\eta \\ & + \lambda_1(\chi^\dagger\chi)^2 + \lambda_2(\eta^\dagger\eta)^2 + \lambda_3(\rho^\dagger\rho)^2 + \lambda_4(\chi^\dagger\chi)(\eta^\dagger\eta) \\ & + \lambda_5(\chi^\dagger\chi)(\rho^\dagger\rho) + \lambda_6(\eta^\dagger\eta)(\rho^\dagger\rho) + \lambda_7(\chi^\dagger\eta)(\eta^\dagger\chi) \\ & + \lambda_8(\chi^\dagger\rho)(\rho^\dagger\chi) + \lambda_9(\eta^\dagger\rho)(\rho^\dagger\eta) + \lambda_{10}(\phi^*\phi)^2 \\ & + \lambda_{11}(\phi^*\phi)(\chi^\dagger\chi) + \lambda_{12}(\phi^*\phi)(\rho^\dagger\rho) + \lambda_{13}(\phi^*\phi)(\eta^\dagger\eta) \\ & + \boldsymbol{(\lambda_\phi \epsilon^{ijk} \eta_i \rho_j \chi_k \phi + H.c)} \end{aligned} \quad (8)$$

$$\boldsymbol{(\eta, \rho, u_R, d_{aR}, e_{aR}, N_R) \rightarrow -(\eta, \rho, u_R, d_{aR}, e_{aR}, N_R)} \quad (9)$$

$$\begin{aligned} \mathcal{L}_{Yuk} = & \ y_1 \bar{Q}_{3L} U_{3R} \chi + \sum_{\alpha,\beta=1}^{2} (y_2)_{\alpha\beta} \bar{Q}_{\alpha L} D_{\beta R} \chi^* + \sum_{a=1}^{3} (y_3)_{3a} \bar{Q}_{3L} u_{aR} \eta + \sum_{\alpha=1}^{2}\sum_{a=1}^{3} (y_4)_{\alpha a} \bar{Q}_{\alpha L} d_{aR} \eta^* \\ & + \sum_{a=1}^{3} (y_5)_{3a} \bar{Q}_{3L} d_{aR} \rho + \sum_{\alpha=1}^{2}\sum_{a=1}^{3} (y_6)_{\alpha a} \bar{Q}_{\alpha L} u_{aR} \rho^* + \sum_{a=1}^{3}\sum_{b=1}^{3} g_{ab} \bar{\psi}_{aL} e_{bR} \rho + \sum_{a=1}^{3}\sum_{b=1}^{3} (y_\nu^D)_{ab} \bar{\psi}_{aL} \eta N_{bR} \\ & + \boldsymbol{y_{ab}^{(\rho)} \epsilon^{ijk} (\bar{\psi}_{iaL})(\psi_{bL})^c \rho^*} + \sum_{a=1}^{3}\sum_{b=1}^{3} (y_N)_{ab} \phi \bar{N}^C_{aR} N_{bR} + \text{H.c.} \end{aligned} \quad (10)$$

The minimal conditions of Higgs potential in ALP331

Let us expand these scalar fields around their VEVs.$(v_\phi \gg v_\chi \gg v_\rho, v_\eta)$.

$$\rho_2^0 = \frac{1}{\sqrt{2}}(v_\rho + R_\rho + iI_\rho)\,,\ \eta_1^0 = \frac{1}{\sqrt{2}}(v_\eta + R_\eta^1 + iI_\eta^1)\,,$$
$$\chi_3^0 = \frac{1}{\sqrt{2}}(v_\chi + R_\chi^3 + iI_\chi^3)\,, \phi = \frac{1}{\sqrt{2}}(v_\phi + R_\phi + iI_\phi)\,. \quad (14)$$

The constraints at the tree level are

$$\mu_\rho^2 + \lambda_3 v_\rho^2 + \frac{\lambda_5}{2}v_\chi^2 + \frac{\lambda_6}{2}v_\eta^2 + \frac{\lambda_{12}}{2}v_\phi^2 + \frac{A}{2v_\rho^2} = 0\,, \quad (15)$$

$$\mu_\eta^2 + \lambda_2 v_\eta^2 + \frac{\lambda_4}{2}v_\chi^2 + \frac{\lambda_6}{2}v_\rho^2 + \frac{\lambda_{13}}{2}v_\phi^2 + \frac{A}{2v_\eta^2} = 0\,, \quad (16)$$

$$\mu_\chi^2 + \lambda_1 v_\chi^2 + \frac{\lambda_4}{2}v_\eta^2 + \frac{\lambda_5}{2}v_\rho^2 + \frac{\lambda_{11}}{2}v_\phi^2 + \frac{A}{2v_\chi^2} = 0\,, \quad (17)$$

$$\mu_\phi^2 + \lambda_{10}\, v_\phi^2 + \frac{\lambda_{11}}{2}v_\chi^2 + \frac{\lambda_{12}}{2}v_\rho^2 + \frac{\lambda_{13}}{2}v_\eta^2 + \frac{A}{2v_\phi^2} = 0\,. \quad (18)$$

where $A \equiv \lambda_\phi v_\phi v_\chi v_\eta v_\rho$ (19)

$SU(3)_C \otimes SU(3)_L \otimes U(1)_X \otimes Z_{11} \otimes Z_2$

$\downarrow v_\phi \sim 10^{11}$ GeV

$SU(3)_C \otimes SU(3)_L \otimes U(1)_X \otimes Z_2$

$\downarrow v_\chi \sim 10^5$ GeV

$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes Z_2$

$\downarrow v_\rho, v_\eta \sim EW - scale$

$SU(3)_C \otimes U(1)_Q$

There are four charged scalar fields: $\eta_2^-, \rho_1^-, \rho_3^-$ and χ_2^-.

i) In the basis (η_2^-, ρ_1^-),

$$M_{c_1} = -\frac{(A-\lambda_9 v_\rho^2 v_\eta^2)}{2}\begin{pmatrix} \frac{1}{v_\eta^2} & \frac{1}{v_\eta v_\rho} \\ \frac{1}{v_\eta v_\rho} & \frac{1}{v_\rho^2} \end{pmatrix}. \quad (20)$$

We get the massless $G_1^\pm$ states ($G_1^\pm$ is Goldstone boson of the $W^\pm$) and massive $H_1^\pm$ with mass equal to

$$m^2_{H_1^\pm} = -\frac{(A-\lambda_9 v_\rho^2 v_\eta^2)}{2}\cdot\frac{(v_\rho^2+v_\eta^2)}{v_\rho^2 v_\eta^2} \quad (21)$$

The mixing angle given by $\tan\alpha = \frac{v_\eta}{v_\rho}$ (22)

ii) In the basis (χ_2^-, ρ_3^-), mass matrix has the form:

$$M_{c_2} = -\frac{(A-\lambda_8 v_\rho^2 v_\chi^2)}{2}\begin{pmatrix} \frac{1}{v_\chi^2} & \frac{1}{v_\chi v_\rho} \\ \frac{1}{v_\chi v_\rho} & \frac{1}{v_\rho^2} \end{pmatrix}. \quad (23)$$

This matrix gives the massless $G_2^\pm$ (correspond to the Goldstone boson associated with the $Y^\pm$ bilepton gauge boson) and the massive ones $H_2^\pm$ with mass are

$$m^2_{H_2^\pm} = -\frac{(A-\lambda_8 v_\rho^2 v_\chi^2)}{2}\cdot\frac{(v_\rho^2+v_\chi^2)}{v_\rho^2 v_\chi^2} \quad (24)$$

The mixing angle given by $\tan\theta_1 = \frac{v_\rho}{v_\chi}$ (25)

THE COMPLEX NEUTRAL SCALAR SECTOR OF ALP331

There are two neutral scalars: one χ_1^0 with masses

$$m^2_{\chi_1^0} = (\lambda_7 v_\eta^2 v_\chi^2 - A)\frac{(v_\eta^2 + v_\chi^2)}{v_\eta^2 v_\chi^2} \,. (26)$$

and one massless η_3^0 which is identified with Goldstone boson eaten by massive X^0. Hence

$$\eta_3^0 \equiv G_{X^0} \,. (27)$$

From the masses of χ_1^0 , it follows

$$\lambda_7 v_\eta^2 v_\chi^2 > A \,. (28)$$

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The neutral scalar CP-odd sector

$(I_\phi, I_\chi^3, I_\rho, I_\eta^1)$

$$M_I^2 = -\frac{A}{2}\begin{pmatrix} \frac{1}{v_\phi^2} & \frac{1}{v_\phi v_\chi} & \frac{1}{v_\eta v_\rho} & \frac{1}{v_\phi v_\eta} \\ & \frac{1}{v_\chi^2} & \frac{1}{v_\chi v_\rho} & \frac{1}{v_\chi v_\eta} \\ & & \frac{1}{v_\rho^2} & \frac{1}{v_\eta v_\rho} \\ & & & \frac{1}{v_\eta^2} \end{pmatrix} \quad (29)$$

$$A \equiv \lambda_\phi v_\phi v_\chi v_\eta\ v_\rho$$

- the diagonal matrix is not unitary
- 2 Goldstone boson
- a pseudoscalar (mass is unidentified)
- an axion (combination of 2 components)

$$a = \frac{1}{\sqrt{1+\frac{v_{\chi'}^2}{v_\phi^2}}}\left(I_\phi + \frac{v_{\chi'}}{v_\phi} I_{\chi'}\right) \quad (34)$$

The 3-3-1 model with axion -
the cold dark matter candidate

$$M_{odd}^2 = -\frac{A}{2}\begin{pmatrix} \frac{1}{v_\phi^2} & \frac{1}{v_\phi v_\chi} & \frac{1}{v_\phi v_\rho} & \frac{1}{v_\phi v_\eta} \\ & \frac{1}{v_\chi^2} & \frac{1}{v_\chi v_\rho} & \frac{1}{v_\chi v_\eta} \\ & & \frac{1}{v_\rho^2} & \frac{1}{v_\eta v_\rho} \\ & & & \frac{1}{v_\eta^2} \end{pmatrix} \quad (30)$$

- the matrix which is used to diagonalize the mass matrix is unitary:

$$\begin{pmatrix} a \\ G_{Z'} \\ G_Z \\ A_5 \end{pmatrix} = \begin{pmatrix} c_{\theta_\phi} & -s_{\theta_3}s_{\theta_\phi} & -s_\alpha c_{\theta_3} s_{\theta_\phi} & -c_\alpha c_{\theta_3} s_{\theta_\phi} \\ 0 & c_{\theta_3} & -s_\alpha s_{\theta_3} & -c_\alpha s_{\theta_3} \\ 0 & 0 & c_\alpha & -s_\alpha \\ s_{\theta_\phi} & s_{\theta_3} c_{\theta_\phi} & s_\alpha c_{\theta_3} c_{\theta_\phi} & c_\alpha c_{\theta_3} c_{\theta_\phi} \end{pmatrix} \begin{pmatrix} I_\phi \\ I_\chi^3 \\ I_\rho \\ I_\eta^1 \end{pmatrix} \quad (31)$$

- mass of the pseudoscalar

$$m_{A_5}^2 = -\frac{A}{2}\left(\frac{1}{v_\phi^2} + \frac{1}{v_\chi^2} + \frac{1}{v_\rho^2} + \frac{1}{v_\eta^2}\right) \simeq -\frac{\lambda_\phi v_\phi v_\chi}{\sin 2\alpha} \quad (32)$$

$$\lambda_\phi < 0 \quad \lambda_\phi = -\frac{m_{A_5}^2 \sin 2\alpha}{v_\phi v_\chi} \quad (33)$$

- axion-like particle (combination of 4 components)

$$a = I_\phi c_{\theta_\phi} - I_\chi^3 s_{\theta_\phi} s_{\theta_3} - I_\rho c_{\theta_3} s_\alpha s_{\theta_\phi} - I_\eta^1 c_\alpha c_{\theta_3} s_{\theta_\phi} \quad (35)$$

The 3-3-1 model with axion-like particle (ALP331)

THE NEUTRAL SCALAR CP-EVEN SECTOR OF ALP331

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Neutral scalar CP-even sector $(R_\eta, R_\rho, R_\chi, R_\phi)$

$$M_R^2 = \begin{pmatrix} 2\lambda_2 v_\eta^2 - \frac{A}{2v_\eta^2} & \lambda_6 v_\eta v_\rho + \frac{\lambda_\phi\ v_\chi v_\phi}{2} & \lambda_4 v_\eta v_\chi + \frac{\lambda_\phi\ v_\rho v_\phi}{2} & \frac{\lambda_\phi v_\rho v_\chi}{2} + \lambda_{13} v_\eta v_\phi \\ \lambda_6 v_\eta v_\rho + \frac{\lambda_\phi\ v_\chi v_\phi}{2} & 2\lambda_3 v_\rho^2 - \frac{A}{2v_\rho^2} & \frac{\lambda_\phi v_\eta v_\phi}{2} + \lambda_5 v_\rho v_\chi & \frac{\lambda_\phi\ v_\eta v_\chi}{2} + \lambda_{12} v_\rho v_\phi \\ \lambda_4 v_\eta v_\chi + \frac{\lambda_\phi\ v_\rho v_\phi}{2} & \frac{\lambda_\phi v_\eta v_\phi}{2} + \lambda_5 v_\rho v_\chi & 2\lambda_1 v_\chi^2 - \frac{A}{2v_\chi^2} & \frac{\lambda_\phi\ v_\eta v_\rho}{2} + \lambda_{11} v_\chi v_\phi \\ \frac{\lambda_\phi\ v_\rho v_\chi}{2} + \lambda_{13} v_\eta v_\phi & \frac{\lambda_\phi v_\eta v_\chi}{2} + \lambda_{12} v_\rho v_\phi & \frac{\lambda_\phi\ v_\eta v_\rho}{2} + \lambda_{11} v_\chi v_\phi & 2\lambda_{10} v_\phi^2 - \frac{A}{2v_\phi^2} \end{pmatrix} \quad (36)$$

- Either masses or the physical states of Higgs boson are undefined.
- predict $m_{R_\phi}^2 \sim \lambda_{10} v_\phi^2$ (37)

$$\begin{pmatrix} h_5 \\ h \\ H_\chi \\ \Phi \end{pmatrix} = \begin{pmatrix} -c_{\alpha_2} & -s_{\alpha_2} c_{\alpha_3} & -s_{\alpha_2} s_{\alpha_3} c_{\alpha_\phi} & s_{\alpha_2} s_{\alpha_3} s_{\alpha_\phi} \\ s_{\alpha_2} & -c_{\alpha_2} c_{\alpha_3} & -c_{\alpha_2} s_{\alpha_3} c_{\alpha_\phi} & c_{\alpha_2} s_{\alpha_3} s_{\alpha_\phi} \\ 0 & s_{\alpha_3} & -c_{\alpha_3} c_{\alpha_\phi} & c_{\alpha_3} s_{\alpha_\phi} \\ 0 & 0 & s_{\alpha_\phi} & c_{\alpha_\phi} \end{pmatrix} \begin{pmatrix} R_\eta \\ R_\rho \\ R_\chi \\ R_\phi \end{pmatrix} \quad (38)$$

SMLHB

$$m_h^2 \simeq \frac{3}{2}\lambda_3 v^2 \quad (42)$$

$$m_\Phi^2 \approx 2\lambda_{10} v_\phi^2 \quad (39)$$

(inflaton)

$$m_{H_\chi}^2 \approx 2\lambda_1 v_\chi^2 + \frac{\lambda_5^2}{2\lambda_1} v_\rho^2 \quad (40)$$

$$m_{h_5}^2 \simeq \frac{\lambda_3 v^2}{2} + m_{A_5}^2 \quad (43)$$

$\lambda_2 \approx \lambda_3 \approx \lambda_6$ ← $v_\eta = v_\rho = \frac{v}{\sqrt{2}}$

$$m_{h,h_5}^2 \approx \lambda_2 v_\eta^2 + \lambda_3 v_\rho^2 - \frac{A v^2}{4 v_\eta^2 v_\rho^2} \pm \sqrt{(\lambda_2 v_\eta^2 - \lambda_3 v_\rho^2)^2 + \lambda_6^2 v_\eta^2 v_\rho^2 + \dots} \quad (41)$$

In the limit $v_\phi \gg v_\chi \gg v_\rho, v_\eta$, one has

$$\eta \simeq \begin{pmatrix} \frac{1}{\sqrt{2}}(v_\eta + h_5 + iA_5) \\ H_1^- \\ G_{X^0} \end{pmatrix}, \; \chi \simeq \begin{pmatrix} \chi_1^0 \\ G_{Y^-} \\ \frac{1}{\sqrt{2}}(v_\chi + H_\chi + iG_{Z'}) \end{pmatrix},$$

$$\rho \simeq \begin{pmatrix} G_{W^+} \\ \frac{1}{\sqrt{2}}(v_\rho + h + iG_Z) \\ H_2^+ \end{pmatrix}, \phi = \frac{1}{\sqrt{2}}(v_\phi + \Phi + ia).$$

Notice:

- A_5 is a new CP - odd scalar field.
- a is an axion-like particle, a new CP - odd scalar field
- h_5, H_χ, Φ are new CP - even scalar fields.
- χ_1^0 is a bilepton DM [C. A. de S. Pires, P. S. Rodrigues da Silva, JCAP 0712:012 (2007)]

$\mathcal{L}_{Yuk}$ | $FCNC$ → **SM quark vs SMLHB (h)**

V_{Higgs} | $g_{haa}, g_{hA_5A_5}, g_{h_5aa}, g_{h_5A_5A_5}$

$$M_u = \begin{pmatrix} (y_6)_{11}\frac{v_\rho}{v_\eta} & (y_6)_{12}\frac{v_\rho}{v_\eta} & (y_6)_{13}\frac{v_\rho}{v_\eta} \\ (y_6)_{21}\frac{v_\rho}{v_\eta} & (y_6)_{22}\frac{v_\rho}{v_\eta} & (y_6)_{23}\frac{v_\rho}{v_\eta} \\ (y_3)_{31} & (y_3)_{32} & (y_3)_{33} \end{pmatrix} \frac{v_\eta}{\sqrt{2}} = V_{uL}\widetilde{M}_u V_{uR}^\dagger, \quad (29)$$

$$\widetilde{M}_u = diag(m_u, m_c, m_t)$$

$$-\mathcal{L}_Y^{(u)} = \sum_{n=1}^{2}\sum_{a=1}^{3}(y_6)_{na}\bar{u}_{nL}\frac{v_\rho + R_\rho - iI_\rho}{\sqrt{2}}u_{aR} + \sum_{a=1}^{3}(y_3)_{3a}\bar{u}_{3L}\frac{v_\eta + R_\eta^1 + iI_\eta^1}{\sqrt{2}}u_{bR} + h.c \quad (30)$$

$$\left(\Gamma_u^{h,h_5,A_5}\right)_{ij}$$

$$M_d = \begin{pmatrix} (y_4)_{11}\frac{v_\eta}{v_\rho} & (y_4)_{12}\frac{v_\eta}{v_\rho} & (y_4)_{13}\frac{v_\eta}{v_\rho} \\ (y_4)_{21}\frac{v_\eta}{v_\rho} & (y_4)_{22}\frac{v_\eta}{v_\rho} & (y_4)_{23}\frac{v_\eta}{v_\rho} \\ (y_5)_{31} & (y_5)_{32} & (y_5)_{33} \end{pmatrix} \frac{v_\rho}{\sqrt{2}} = V_{dL}\widetilde{M}_d V_{dR}^\dagger, \quad (31)$$

$$\widetilde{M}_d = diag(m_d, m_s, m_b)$$

$$-\mathcal{L}_Y^{(d)} = \sum_{n=1}^{2}\sum_{a=1}^{3}(y_4)_{na}\bar{d}_{nL}\frac{v_\eta + R_\eta^1 - iI_\eta^1}{\sqrt{2}}d_{aR} + \sum_{a=1}^{3}(y_5)_{3a}\bar{d}_{3L}\frac{v_\rho + R_\rho + iI_\rho}{\sqrt{2}}d_{bR} + h.c \quad (32)$$

$$\left(\Gamma_d^{h,h_5,A_5}\right)_{ij}$$

$$\left(\Gamma^h_{u,d}\right)_{ij} = \frac{c_{\alpha_2}}{v_\rho}\sum_{n=1}^{2}\sum_{a=1}^{3}\left(\left(V_L^{(u)}\right)^\dagger\right)_{in}\left(V_{uL}\widetilde{M}_u V_{uR}^\dagger\right)_{na}\left(V_R^{(u)}\right)_{aj} - \frac{s_{\alpha_2}}{v_\eta}\sum_{a=1}^{3}\left(\left(V_L^{(u)}\right)^\dagger\right)_{i3}\left(V_{uL}\widetilde{M}_u V_{uR}^\dagger\right)_{3a}\left(V_R^{(u)}\right)_{aj} = \frac{c_{\alpha_2}}{v_\rho}\left(\widetilde{M}_{u,d}\right)_{ij} - \frac{c_{\alpha_2}}{v_\eta}(\tan\alpha + \tan\alpha_2)\left(\Gamma_h^{\prime(u,d)}\right)_{ij} \quad (33)$$

$$\tan\alpha = -\tan\alpha_2 \quad (34)$$

SMLHB decays

$h \to \bar{b}b$ (35)

$$g_{hb\bar{b}} = \frac{c_{\alpha_2}}{s_\alpha}(\tan\alpha - \tan\alpha_2)\frac{m_b}{v} = a_{hb\bar{b}}g_{hb\bar{b}}^{SM}$$

$h \to \bar{l}l$

$$g_{hl\bar{l}} = \sum_{a=1}^{3}\frac{v c_{\alpha_2}}{v_\rho}\frac{(M_l)_{aa}}{v} = a_{hl\bar{l}}g_{hl\bar{l}}^{SM} \quad (36)$$

$$a_{hf\bar{f}} = \frac{g_{hf\bar{f}}^{ALP331}}{g_{hf\bar{f}}^{SM}} \quad (37)$$

Meson oscillation

$\delta = K, B_d, B_s$

$$\mathcal{H}_{eff}^{(\delta^0-\bar{\delta}^0)} = \frac{G_F^2 m_W^2}{16\pi^2}\sum_{i=1}^{3}C_i^{(\delta^0-\bar{\delta}^0)}(\mu)O_i^{(\delta^0-\bar{\delta}^0)}(\mu) + \frac{4\sqrt{2}G_F c_W^4 m_Z^2}{(3-4s_W^2)m_{Z'}^2}|(V_{DL}^*)_{32}(V_{DL})_{31}|^2 O_4^{(\delta^0-\bar{\delta}^0)} \quad (38)$$

$$C_i^{(\delta^0-\bar{\delta}^0)} \propto \frac{1}{m_h^2}, \frac{1}{m_{h_5}^2}, \frac{1}{m_{A_5}^2}$$

$O_i^{(\delta^0-\bar{\delta}^0)} \supset$ axion currents

$O_4^{(\delta^0-\bar{\delta}^0)} \supset$ vector currents

$i = 1, 2, 3$

t quark decays

$t \to hc(u)$ $\quad t \to h_5c(u)$

$$Br(t \to Hq) = \frac{\frac{g_{tHq}^2}{4\pi}\frac{(m_t^2 - m_H^2)^2}{2m_t m_H}}{\Gamma_t} \quad (39)$$

$H = h, h_5$, $q = c, u$

$t \to c(u)\gamma$

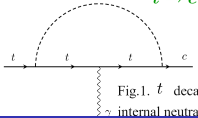


Fig.1. t decay with internal neutral Higgs

Meson oscillation

Meson mass spliting:
$\Delta m_\delta = (\Delta m_\delta)_{SM} + \Delta m_\delta^{(NP)}$, $\delta = K, B_s, B_d$ (40)

$i, j = 1, 2, 3, i \neq j$

$$\Delta m_\delta^{(NP)} = \frac{4\sqrt{2} G_F c_W^4 m_Z^2}{(3-4s_W^2) m_{Z'}^2} \left|(V_{DL}^*)_{3i}(V_{DL})_{3j}\right|^2 f_\delta^2 B_\delta \eta_\delta m_\delta + \frac{G_F^2 m_W^2}{6\pi^2} m_\delta f_\delta^2 \eta_\delta B_\delta \left[P_2^{(\delta^0-\bar{\delta}^0)} C_3^{(\delta^0-\bar{\delta}^0)} + P_1^{(\delta^0-\bar{\delta}^0)} \left(C_1^{(\delta^0-\bar{\delta}^0)} + C_2^{(\delta^0-\bar{\delta}^0)} \right) \right], \quad (41)$$

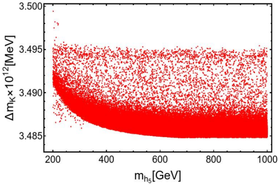


Fig.2. Corellation between mass spliting Δm_K and m_{h_5}

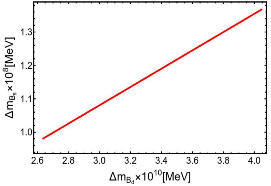


Fig.3. Corellation between mass splitings Δm_{B_d} and Δm_{B_s}

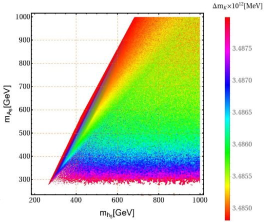


Fig.4. Allowed region of Δm_K, Δm_{B_d}, Δm_{B_s} in the plane (m_{A_5}, m_{h_5}),

- m_{h_5} increases, Δm_K decreases.

- the number of solutions consistent with the meson oscillation constraints is increased when $m_{h_5} \sim TeV$

$$\Delta m_\delta \propto \frac{1}{m_h^2}, \frac{1}{m_{h_5}^2}, \frac{1}{m_{A_5}^2} \quad (42)$$

$\Rightarrow$ The larger m_{h_5}, m_{A_5} are, the easier to find more solutions consistent with the meson oscillation constraints

$$(\Delta m_K)_{\text{exp}} = (3.484 \pm 0.006) \times 10^{-12} MeV$$
$$(\Delta m_K)_{SM} = 3.483 \times 10^{-12} MeV$$
$$(\Delta m_{B_d})_{\text{exp}} = (3.334 \pm 0.013) \times 10^{-10} MeV,$$
$$(\Delta m_{B_d})_{SM} = (3.653 \pm 0.037 \pm 0.019) \times 10^{-10} MeV,$$
$$(\Delta m_{B_s})_{\text{exp}} = (1.1683 \pm 0.0013) \times 10^{-8} MeV,$$
$$(\Delta m_{B_s})_{SM} = (1.1577 \pm 0.022 \pm 0.051) \times 10^{-8} \text{MeV}, \quad (43)$$

$$200GeV \leqslant m_{h_5}$$
$$100GeV \leqslant m_{A_5} \quad (44)$$

SMLHB decays

$$\Gamma(h \to \bar{f}f) = \int d\Gamma = \frac{g^2_{(h,f,f)}}{8\pi} m_h \left(1 - \frac{4m_f^2}{m_h^2}\right)^{\frac{3}{2}}. \quad (45)$$

$h \to \bar{b}b$

$$a_{hb\bar{b}} = \frac{c_{\alpha_2}}{s_\alpha}(\tan\alpha - \tan\alpha_2) = 2\frac{c_{\alpha_2}}{c_\alpha} \quad (46)$$

$$a^{exp}_{hb\bar{b}} = 0.91^{+0.17}_{-0.16} \quad (47)$$

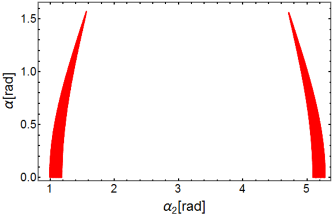


Fig.5. Corellation between mixing angles α and α_2 in $h \to \bar{b}b$ decay

$\alpha \in (0, 1.6)$ rad

$\alpha_2 \in (1 \div 1.6)$ rad

hoặc $(4.75 \div 5.4)$ rad (48)

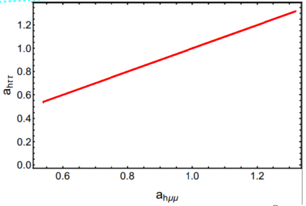


Fig.6. Corellation between parameters $a_{h\tau\tau}$ and $a_{h\mu\mu}$.

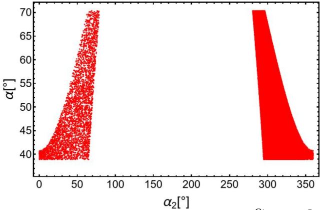


Fig.7. Corellation between mixing angles α_2 and α in $h \to \mu\mu$, $h \to \tau\tau$ decays.

$h \to \bar{l}l$

$$a_{h\bar{l}l} = \frac{v c_{\alpha_2}}{v_\rho} = \frac{c_{\alpha_2}}{c_\alpha} \quad (49)$$

$$a^{exp}_{h\mu\mu} = 0.72^{+0.50}_{-0.72} \quad (50)$$
$$a^{exp}_{h\tau\tau} = 0.93^{+0.13}_{-0.13}$$

$\alpha \in (40^o \div 75^o)$

$\alpha_2 \in (10^o \div 80^o)$

or $(275^o \div 340^o)$ (51)

$\alpha \in (0.7, 1.4)$ rad

$\alpha_2 \in (0.2 \div 1.4)$ rad

or $(4.8 \div 5.9)$ rad (52)

$\alpha \in (0.7 \div 1.4)$ rad,

$\alpha_2 \in (1 \div 1.4)$ rad

or $(4.8 \div 5.4)$ rad

(53)

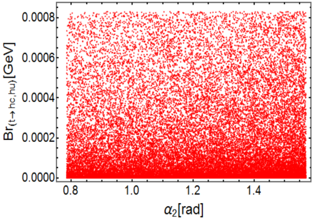
Rare t quark decays

$t \to ch\,,\ t \to uh$

$$Br(t\to hc)_{SM} \simeq 10^{-15}, \quad Br(t\to hu)_{SM} \simeq 10^{-17}. \quad (54)$$

$$Br(t\to hq)_{exp} < 0.47\% \ (@CL95\%) \quad (55)$$

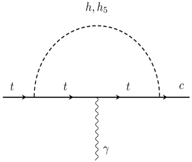
$$Br(t\to hq) = \frac{\frac{g_{thq}^2}{4\pi}\frac{(m_t^2-m_h^2)^2}{2m_t m_h}}{\Gamma_t} \quad (56) \qquad q = c, u$$



H3.7. Colleration between mixing angle α_2 and $Br_{(t\to hc,hu)}$.

$$\alpha_2 \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right) \qquad Br_{(t\to hc,hu)} < 10^{-3} \quad (57)$$

$t \to c\gamma\,,\ t \to u\gamma$



$$A_h = -\frac{s_{\alpha_2}}{s_\alpha}, \quad B_h = \frac{s_{\alpha_2}}{s_\alpha} + \frac{c_{\alpha_2}}{c_\alpha}, \quad A_{h_5} = \frac{c_{\alpha_2}}{s_\alpha}, \quad B_{h_5} = -\frac{c_{\alpha_2}}{s_\alpha} + \frac{s_{\alpha_2}}{c_\alpha}, \quad (58)$$

$$\Gamma(t\to q\gamma) = \frac{\alpha G_F m_t^3 |y_{hqt}|^2}{192\pi^4}\left|\left(f_1\frac{m_h}{m_t} + f_2\frac{m_h}{m_t}\right)A_h B_h + \left(f_1\frac{m_{h_5}}{m_t} + f_2\frac{m_{h_5}}{m_t}\right)A_{h_5}B_{h_5}\right|^2 \quad (59)$$

$$Br(t\to c\gamma)_{exp} < 2.2\times 10^{-4}, \quad Br(t\to u\gamma)_{exp} < 6.1\times 10^{-5} \quad (60)$$

$$Br(t\to c\gamma)_{SM} < 4.6\times 10^{-14}, \quad Br(t\to u\gamma)_{SM} < 3.7\times 10^{-16} \quad (61)$$

$$\Gamma_{top} = 1.42^{+0.19}_{-0.15}\ \text{GeV}, \quad m_h = 126GeV, \quad 96GeV \leqslant m_{h_5} \leqslant 200GeV, \quad 10^{-2}\ \text{GeV} \le y_{hct}, y_{hut} \le 1.2\times 10^{-2} \quad (62)$$

4 to 6 orders smaller

$$Br(t\to c\gamma)^{max}_{ALP331} < 2.2\times 10^{-8}, \quad Br(t\to c\gamma)_{ALP331} < 1.9\times 10^{-10} \quad (63)$$

CONCLUSIONS

1. We define exactly the physical states of scalar fields in CP-odd sector. The results show that just axion like particle exists in the ALP331, not QCD axion.
2. There is a pseudoscalar field A_5 gets mass with lower limit should be 250 GeV.
3. We diagonalize approximately the 4×4 mass mixing matrix in CP-even sector. We define the SMLHB - which can't be solved in ALP331 model through 2 decades.
4. There is a very heavy Higgs boson with mass $\sim 10^{11}$ GeV which can use to explain the inflaton in the Early universe.
5. We predict the existence of a new light Higgs h_5 with mass near the mass of SMLHB (96GeV or 150GeV). The mass spliting between h_5 and A_5 should be in EW scale.

THANK YOU FOR YOUR ATTENTION!