Axion quality from gauge flavour symmetries

Based in part on arXiv:2102.05055 (L. Darmé, EN) **and on work in progress.**

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Abridged story of the axion

Solving theoretical problems catalyses advancements in science by bringing deep new understandings along with unexpected, and often surprising, implications. Usually it also brings to light new problems of which we were previously unaware.

- QCD before 1975: U(1) problem: why the η does not behave as a 9^{th} NGB ?
- Instantons (Belavin et al. '75), Yang-Mills vacuum periodicity (Callan et al. '76; Jackiw et al. '76)
  U(1) axial anomaly + non-trivial vacuum -> no conserved axial current -> no NGB
- New problem: $\mathcal{L}_{QCD}$ -> $\mathcal{L}_{QCD}$ + $\theta \frac{\alpha_s}{8\pi} G_a^{\mu\nu} \tilde{G}^a_{\mu\nu}$ brings in QCD P,T (CP) violation. $\theta \lesssim 10^{-10}$
- PQ solution ('77): $\theta \to \theta(x)$; $V(\theta)$ s.t. $\langle\theta\rangle = 0$. It predicts a $m \approx 0$ scalar: the Axion
- Unexpectedly, the axion has also the right properties to account for the DM !
- Unsurprisingly, it raises new problems: Which is the origin of the PQ symmetry? How can it remain preserved up to the required operator dimension $d \gtrsim 10$?

- If the axion exists, these problems must be solved ! It is then conceivable that the solution could shed light on other unsolved issues of the Standard Model

Basic ingredients of the PQ solution

[Peccei, Quinn (1977), Weinberg (1978), Wilczek (1978)]

- A scalar potential invariant under a **global** U(1): $\Phi \to e^{i\xi}\,\Phi$, $\delta V(\Phi) = 0$
- U(1) **SSB**: $\Phi \to v_a\, e^{ia(x)/v_a}$. $a(x)$: $V(a) = 0 \to$ shift symmetry $a \to a + \xi v_a$
- Couplings between the scalars and some quarks $\bar{Q}_L\, \Phi\, q_R \to \bar{Q}_L\, v_a\, q_R\, e^{ia(x)/v_a}$
  U(1) is then enforced by assigning chiral PQ charges $X(Q) - X(q) = X(\Phi)$
- The symmetry must have a mixed U(1)-SU(3)$_c$ anomaly: $\Sigma_q(X_Q - X_q) \neq 0$

By redefining the quark fields in the basis of real masses $\bar{Q}_L\, v_a\, q_R$:

$$\Theta G\tilde{G} \quad \to \quad (a(x)/v_a + \Theta)\, G\tilde{G} \quad \to \quad (a(x)/v_a)\, G\tilde{G}$$

Instanton related non-perturbative QCD effects generate a potential

$V_{QCD}(a) = -(m_\pi f_\pi)^2 \cos(a/v_a)$ that drives $\langle a/v_a \rangle \to 0$ at the minimum

The PQ "origin" and "quality" problems

- $U(1)_{PQ}$ is <u>anomalous</u>. Is not a (fundamental) symmetry of the theory:

  $\int [DA_\mu D\Phi]\, D\psi D\bar{\psi}\, \exp(iS)$ is not invariant under a PQ transformation

- In benchmark axion models, Φ is a complex scalar, and a gauge singlet. Renormalizable terms $\mu^3\Phi$, $\mu^2\Phi^2$, $\mu\Phi^3$, $\lambda\Phi^4$ do not break gauge or Lorentz and are not forbidden. <u>However, they would destroy PQ invariance.</u>

- Non-pt. quantum gravity effects. Controlled solutions: $M_P^3\, e^{-S_{wh}}\, \Phi + h.c.$
  [Euclid. wormholes]. Safe suppression requires $S_{wh} > 190$ (while typical $S_{wh} \sim Log(M_P/v_a) \sim 15$)
  [Kallosh et al. '95, Alonso & Urbano '17, Alvey & Escudero '20]

- PQ breaking effective opts.: $g\, \Phi^d/\Lambda^{d-4}$ -> $g\,(v_a/\Lambda)^{d-4} < 10^{-10}\,(m_\pi f_\pi/\Lambda^2)^2$
  that is, we need to require: Eng. density eff. opt. < 10^{-10} $V_{QCD}(a)$
  E.g. $g\sim1$, $\Lambda \sim M_P$ and $v_a \sim 10^{10}$ GeV imply $d \gtrsim 10$ [with $g = g_{wh}$, $d \gtrsim 9$]
  [Barr & Seckel '92, Kamionkowski & March-Russel '92, Holman et al. '92, Ghigna et al. '92]

- The axion scale $v_a \gtrsim 10^8$ GeV contributes to the EW stability problem
  (analogously to other SM completions that involve a new large UV scale: seesaw, GUTs, etc.)

A sample of proposed solutions

$U(1)_{PQ}$ should arise automatically as a consequence of first principles.
SSB requires VEVs ⇛ Lorentz singlets. Rely on <u>local gauge symmetries</u>

- Discrete gauge symm. $\mathbb{Z}_n$: $\Phi \to e^{i\,2\pi/n}\,\Phi$; 1st ~~PQ~~ opt. $\Lambda^{4-n}\,\Phi^n$

Requires $\mathbb{Z}_{10}$ or larger [Krauss & Wilczek '89, Dias & al. '03, Carpenter & al. '09, Harigaya & al. '13]

- Local U(1) + 2 scalars with charges $q_1+q_2 \geq 10$ 1st ~~PQ~~ : $\Lambda^{4-q_1-q_2}\,(\Phi_1^\dagger)^{q_2}\,(\Phi_2)^{q_1}$

(q1 and q2 relatively prime) [Barr & Seckel '92]

- Non-Abelian $SU(n)_L \times SU(n)_R$, $a(x) \in Y_{n\times n}$. Svd: $Y = U\,\hat{Y}\,V^\dagger\,e^{ia/v_a}$

For $n > 4$ the ren. potential is very simple: $V(Y) = (T-\mu^2)^2 \pm A$
with $T = \mathrm{Tr}(Y^\dagger Y)$, $A = \mathrm{Tr}(\mathrm{mnr}[Y^\dagger Y, 2]) = \frac{1}{2}[T^2 - \mathrm{Tr}(Y^\dagger Y Y^\dagger Y)]$

Automatic rephasing symm. $Y \to e^{i\xi}\,Y$. Anomaly from KSVZ quarks $\bar{Q}_L\,Y\,Q_R$
1st ~~PQ~~ opt. $\Lambda^{4-n}\,\det Y$ dim = n. This requires again $n \geq 10$

[Fong, EN '14 [in SU(3)xSU(3)], Di Luzio, Ubaldi, EN '17]

Can we do any better? For $V(\Phi)$ it is easy

[Darmé & EN (2021)]

- Take a local $SU(m) \times SU(n)$ ($m > n$) and a scalar multiplet $Y_{\alpha i} \sim (m, \bar{n})$

  SU invariants are constructed with Kronecker δ and Levi-Civita ε

δ-invariants can be red off the characteristic polynomial of $Y^\dagger Y$:

$$P(\xi) = \det(\xi I - Y^\dagger Y) = \Sigma_k \, (-1)^k \, C_k \, \xi^{n-k} \qquad C_k = Tr(mnr[Y^\dagger Y, k])$$

They are obviously all Hermitian $\Rightarrow$ accidental U(1): $Y \to e^{i\xi} \, Y$

ε-invariants (non-Hermitian): there is none $\varepsilon_{\alpha\beta\ldots\sigma} \, Y_{\alpha i} \, Y_{\beta j} \ldots Y_{\sigma r} = 0$ symmt.

Already for $SU(3) \times SU(2)$, $V(Y)$ enjoys <u>automatically</u> an <u>exact</u> global U(1)

Note: for a $Y_{n \times n}$ square matrix $\varepsilon_{\alpha\beta\ldots\sigma} \, \varepsilon_{ij\ldots r} \, Y_{\alpha i} \, Y_{\beta j} \ldots Y_{\sigma r} \propto \det Y \neq 0$

Such automatic U(1) symmetries are peculiar of local `rectangular' symmetries

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Can symmetries of this type be promoted to PQ symmetries ?

The "PQ quality - flavour" connection

Any non-Abelian gauge symmetry generating a $U(1)_{PQ}$ is a flavour symmetry

$Q_L Y q_R$ ($SU(2)_L \times U(1)_Y$ vectorlike quarks) or $\frac{1}{\Lambda} Q_L Y q_R H$ (SM EW chiral quarks)

- We are led to consider models of flavour with a generic structure

$$\mathcal{L} \sim \overline{Q}\, Z\, q + \frac{1}{\Lambda}\left(\kappa_d \overline{Q}\, Y\, d\, H + \kappa_u \overline{Q}\, X\, u\, \tilde{H} + \kappa_3 \overline{Q}\, Z\, u_3\, \tilde{H}\right) + \frac{1}{\Lambda^2}\left(\kappa_q \overline{Q}\, W\, q + \dots\right) + \dots$$

with Z, X, Y scalar multiplets of some G_F. Possibly involving also combinations of scalar fields W = W[Z,X,Y]. It can contain EW vectorlike quarks (e.g. $q_R \in SU(2)_W$). SM quarks masses and mixings generated dynamically by specific ‹Z›, ‹X›, ‹Y› configurations, with hierarchical singular values [for a proof of principle of the viability, Fong & EN '13]

The guiding principle is that a PQ symmetry of the required high quality must arise automatically from G_F and the field content.

Can our U(1)'s be promoted to PQ symmetries ?

- Exercise: assume $G_F = SU(3)_L \times SU(2)_R$, take $Y_{\alpha i} \sim (3,\bar{2})$ and the quark multiplets $Q_L \sim (3,1)$; $q_R \sim (1,2)$; $t_R \sim (1,1)$ (t_R needed to avoid $SU(3)_C$ anomaly)

  $\text{Rank}(Y_{3\times 2}) = 2$, one <u>massless</u> quark. Add $Z_\alpha \sim (3,1)$: $M_q \subset \bar{Q}_L Y q_R + \bar{Q}_L Z t_R$

- Two mixed invariants $\quad I_\varepsilon = \varepsilon_{\alpha\beta\gamma}\, \varepsilon_{ij}\, Y_{\alpha i}\, Y_{\beta j}\, Z_\gamma \quad U(1)_\varepsilon: \quad 2X_Y + X_Z = 0$

  $U(1)_Y \times U(1)_Z \to U(1) \quad I_\delta = \varepsilon_{ij}\, (Z^\dagger Y)_i\, (Z^\dagger Y)_j \quad U(1)_\delta: \quad X_Y - X_Z = 0$

  Then $U(1)_Y \times U(1)_Z$ is completely broken, no residual U(1). No PQ solution ?

- Not so ! We need to consider the vacuum structure of Y and Z

$$Y = U_3\, \hat{Y}\, V_2^\dagger\, e^{i\phi_y} \quad \to \quad \langle Y \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} y_1 & 0 \\ 0 & y_2 \\ 0 & 0 \end{pmatrix} e^{i\frac{\varphi_y}{v_y}},$$

$$Z = U_3'\, \hat{Z}\, e^{i\phi_z} \quad \to \quad \langle Z \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix} e^{i\frac{\varphi_z}{v_z}}, \qquad v_{y,z}^2 = T_{y,z}$$

$$V_{\rm nH} = \mu I_\epsilon + \lambda I_\delta + \text{h.c.} \quad \longrightarrow \quad -|\mu|\langle I_\epsilon\rangle - |\lambda|\langle I_\delta\rangle$$

$$\cos[\varphi_\mu + \varphi_\epsilon(x)]\, |\mu I_\epsilon| \to -|\mu|\langle I_\epsilon\rangle$$

$$\begin{cases} \max\langle I_\epsilon\rangle & \langle Z\rangle \sim (0,0,z_3)^T, \quad \langle I_\delta\rangle = 0 \\ \max\langle I_\delta\rangle & \langle Z\rangle \sim (z_1,z_2,0)^T, \quad \langle I_\epsilon\rangle = 0 \end{cases}$$

Operators for which <O> -> 0 do not break the symmetries of the minimum, thus the vacuum can enjoy a larger symmetry than the Lagrangian. Scalar bosons associated with these symmetries remain massless [Georgi & Pais '75]

- Let us recall however that U(1) symmetry breaking operators exist that do not break the gauge symmetry, so that QCD can still produce a potential while respecting gauge invariance

- We can easily identify the NGB that remains (perturbatively) massless and enjoys the required shift symmetry.
  In the vacuum determined by I_ε, charges are related by $X_Z = -2\, X_y$

$$a(x) = \frac{v_y}{v_a}\varphi_y - 2\frac{v_z}{v_a}\varphi_z$$

$$a(x) = \frac{v_y}{v_a}\varphi_y - 2\frac{v_z}{v_a}\varphi_z, \quad v_a^2 = v_y^2 + 4v_z^2 \qquad \text{s.t.} \quad \text{for} \qquad \xi \in [0, 2\pi) \quad \begin{cases} \varphi_y & \to \quad \varphi_y + \xi\, v_y \\ \varphi_z & \to \quad \varphi_z - 2\,\xi\, v_z \\ a(x) & \to \quad a(x) + \xi\, v_a \end{cases}$$

Can this yield a viable axion model ?

- Recall $\langle Y \rangle \sim (y_1, y_2, 0)^T$. To ensure a mass for t_R, we need to choose $\langle Z \rangle \sim (0,0,z_3)^T$ that is $\langle Y \rangle$ and $\langle Z \rangle$ must be "misaligned".

The vacuum is defined by $\langle I_\delta \rangle = 0$ and $\langle I_\varepsilon \rangle \neq 0$ $\quad X(I_\varepsilon) = 2X_Y + X_Z = 0$

- Let us now compute the anomaly $\quad A_{PQ} = \Sigma_{q_L} X_L - \Sigma_{q_R} X_R$

$$3\, X_Q - 2\, X_q - X_t = 2(X_Q - X_q) + (X_Q - X_t) = 2X_Y + X_Z = X(I_\varepsilon) = 0$$

Thus $\langle I_\varepsilon \rangle$ breaks $U(1)_Y \times U(1)_Z \rightarrow U(1)_\varepsilon$ which is non-anomalous !
Then $U(1)_\varepsilon$ is not a PQ symmetry, and its (exactly massless) NGB does not solve the strong CP problem.

Is this just an unlucky accident occurring with the flavour $SU(3)_L \times SU(2)_R$ gauge symmetry ?

An upper limit on the quality of the PQ symmetry

Consider a gauge symmetry $G_F = [\Pi_\ell\, SU(m_\ell)]_L \times [\Pi_r\, SU(n_r)]_R$ acting on a certain set of scalar multiplets in bi-fundamentals $Y^{\ell r} \in SU(m_\ell) \times SU(n_r)$ of the m_ℓ, n_r gauge factors, and on $N=\Sigma_\ell\, \lambda_\ell\, m_\ell = \Sigma_r\, \lambda_r\, n_r$ LH and RH quarks also in fundamentals ($\lambda_{\ell,r}$: isospin multiplicity). We can write a certain number of gauge invariant quark-scalar couplings: $\Sigma\, \eta^{\ell r} \bar{Q}_\ell\, Y^{\ell r} q_r$ ($\eta^{\ell r}$: O(1) constants; $^{\ell r}$ \`names' not indices; H/Λ when needed)

Assuming that all the quarks acquire masses ($\det M_q \neq 0$), it can be shown that:

1. for any global U(1) there exists at least one scalar operator $O(Y)$ with a non-vanishing VEV and charge equal to the U(1)-$SU(3)_c$ anomaly: $X_{O(Y)} = A_C \neq 0$

2. modulo the coupling constants $\eta^{\ell r}$ we have: $\langle X_{O(Y)} \rangle \simeq \Lambda^{4-N} \det \mathcal{Y}_q^{eff}$

1. implies that any anomalous U(1) suffers explicit breaking at least at $d = N$. This provides an upper limit on the quality of G_F-protected PQ symmetries.

2. implies that this source of breaking is removed as $\det \mathcal{Y}_q^{eff} \to 0$. Providing an unexpected connection between PQ quality and Yukawa hierarchies !

A glimpse on the generation of Yukawa hierarchies

Consider $G_F = SU(4)_L \times [SU(3)_d \times SU(2)_u]_R$ and the quark/scalar multiplets:

$$Q_L \sim (4,1,1), \quad q_R \sim (1,1,1), \quad d_R \sim (1,\bar{3},1), \quad u_R \sim (1,1,\bar{2}), \quad t_R \sim (1,1,1)$$

$$Y \sim (4,3,1), \quad X \sim (4,1,2), \quad Z \sim (4,1,1)$$

The Yukawa couplings originate from the effective Lagrangian

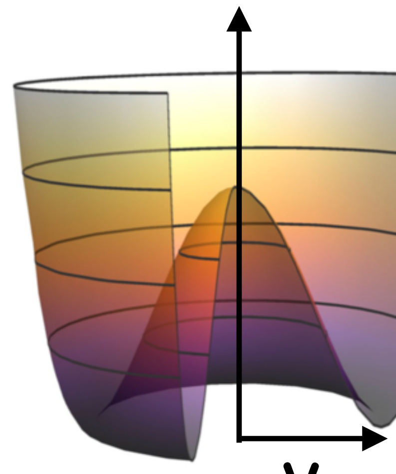
$$\mathcal{L}_\Upsilon \sim \overline{Q}\, Z\, q + \frac{1}{\Lambda}\left(\kappa_d \overline{Q}\, Y\, d\, H + \kappa_u \overline{Q}\, X\, u\, \tilde{H} + \kappa_3 \overline{Q}\, Z\, u_3\, \tilde{H}\right)$$

"Flavour relevant" renormalizable invariants and their action in the approx. in which the svd L-matrices $U_X, U_Z \to I_4$ (neglecting mixings, only hierarchies)

$$\begin{array}{llllll}
A_Y & = \mathrm{Tr}\,[\mathrm{Mnr}(Y^\dagger Y, 2)] & = \; y_1^2\, y_2^2 + y_1^2\, y_3^2 + y_2^2\, y_3^2, & (\eta_A > 0) & \hat{Y} \to (y, 0, 0) \\
D_X & = \mathrm{Det}[X^\dagger X] & = \; x_1^2\, x_2^2, & (\eta_D < 0) & \hat{X} \to (x, x) \\
\mathcal{E}_{YZ} & = \epsilon_3 \epsilon_4\, YYYZ & \to \; -2\, y_1\, y_2\, y_3\, z_4 & & \hat{Y} \to (y, \epsilon_y, \epsilon_y) \\
T_{XY} & = \mathrm{Tr}\,[XX^\dagger YY^\dagger] & \to \; x_1^2\, y_1^2 + x_2^2\, y_2^2, & (\eta_T < 0) & \hat{Y} \to (y, \epsilon_y, \epsilon'_y), \; \hat{X} \to (x, \epsilon_x) \\
T_{ZX} & = \mathrm{Tr}\,[ZZ^\dagger XX^\dagger], & T_{ZY} = \mathrm{Tr}\,[ZZ^\dagger YY^\dagger] \to 0 & &
\end{array}$$

A different approach: scale vs. compact space radius

Consider the usual Mexican hat potential for a complex Φ hosting the axion



- Scale of PQ symm. breaking: $\langle\Phi\rangle = v_a$
  (phase transition, primordial GW,…)
- Axion compact field space radius $a \in [0, 2\pi f_a)$
  (suppression of axion couplings: $a(x)/f_a$)

Here $v_a = f_a$, but conceptually they are different quantities.

When the axion is hosted in more than one scalar multiplet: $\Phi_i \sim v_i\, e^{a_i/v_i}$

$a = \Sigma_i (v_i/f_a)\, a_i$ with $f_a^2 = \Sigma_i X_i^2\, v_i^2$ enhancement by large charge values

[Clockwork mechanism: Choi & Im '16, Kaplan & Rattazzi '16, Giudice & McCullough '17 …]

Consider a gauge group $[SU(3)\times SU(2)]^{n+1}$ and $Y \sim (1_{n-1}, 2_n, 3_n)$, $\Sigma \sim (3_{n-1}, \bar{2}_n, \bar{3}_n)$

The potential $V = \Sigma_n\, \varepsilon_3\, \varepsilon_2\, Y_n\, Y_n\, \Sigma_{n+1}\, Y_{n+1}$ has automatic symm. $X_{n+1} = 2\, X_n$ ($X_\Sigma = 0$)

Then $f_a^2 = \Sigma_n X_n^2\, v_n^2 \approx (1/3)\, v^2\, 4^{n+1}$ (after taking all $v_n \approx v$)

If quarks couple to Y_1 : $\bar{Q}\, Y_1\, q$ so that X_q are small, all axion interactions are suppressed as $1/f_a$. For $n \sim 20$, $v \sim 100$ GeV, $v/M_P \sim 10^{-17}$

Summary and conclusions

- The PQ mechanism provides an elegant and convincing solution to the strong CP problem. However, we do not yet have a similarly elegant and convincing model enforcing this mechanism in a natural way

- The scalar potential that breaks spont. $U(1)_{PQ}$ can automatically be $U(1)$ invariant and protected from higher-dim ~~PQ~~ opts. if the scalars transform under some suitable local symmetry (Abelian continuous/discrete or non-Abelian)

- Non-Abelian symm. can be directly interpreted as flavour symm. (whether for KSVZ and/or SM quarks). A certain type of symm. have particularly interesting features w.r. $U(1)_{PQ}$ protection, flavour hierarchies, etc.

- So far, they suggest that: some quark masses should have a different origin than others; additional vectorlike quarks are most likely needed (viol. CKM unit.); some flavour gauge bosons with $m_F \sim v_a (m_u/m_t)$; etc.

Thanks for your attention