Dark matter search with qubits

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In collaboration with

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arXiv:2212.03884 (PRL 131 (2023) 21, 211001),
arXiv:2311.10413 (PRL 133 (2024) 2, 021801),
arXiv:2407.19755

Outline

- Introduction
- DM detection with one qubit
  - arXiv:2212.03884, arXiv:2407.19755
  - We propose to use superconducting qubits as dark matter detectors
  - We may detect DM $m_{DM} \sim \omega_{qubit} \sim \mathrm{GHz} \sim 10^{-5}\ \mathrm{eV}$
- DM detection with quantum circuits
  - arXiv:2311.10413
  - We construct a quantum circuit to enhance the DM signal. With N qubits, the signal is proportional to N^2
  - I'll also talk about the noise in the circuit (ongoing work)
- Results

Introduction

Dark matter of the universe

- There are many observational evidence for the dark matter, but still its properties are unknown.
- We focus on light dark matter candidates, in particular, the dark photon dark matter and the axion, and propose a new search method **using superconducting qubits as a dark matter detector**

Quantum computation and qubits

- The fundamental piece of the quantum computation is qubit, a two-level quantum system, $|0\rangle$ and $|1\rangle$
- By the recent development of the quantum technology, many-qubit systems gradually become available
  - Currently, the system is noisy, but hopefully future development will make more cleaner quantum systems available.

Types of qubits

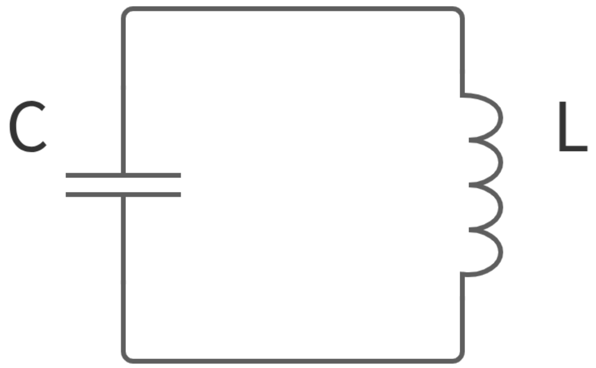
- Currently, there are several types of qubits available
  - Single photon
  - NMR
  - Ion trap
  - **Superconducting qubit (transmon qubit)** Koch et al, 07
  - …
- What is the superconducting qubit?

An example: Harmonic oscillator

- What is the *easiest* quantum system? It's a harmonic oscillator.
- Suppose to use a harmonic oscillator as a qubit:

$$|0\rangle = |0\rangle, |1\rangle = a^{\dagger}|0\rangle$$

- The simple example of a harmonic oscillator → LC circuit

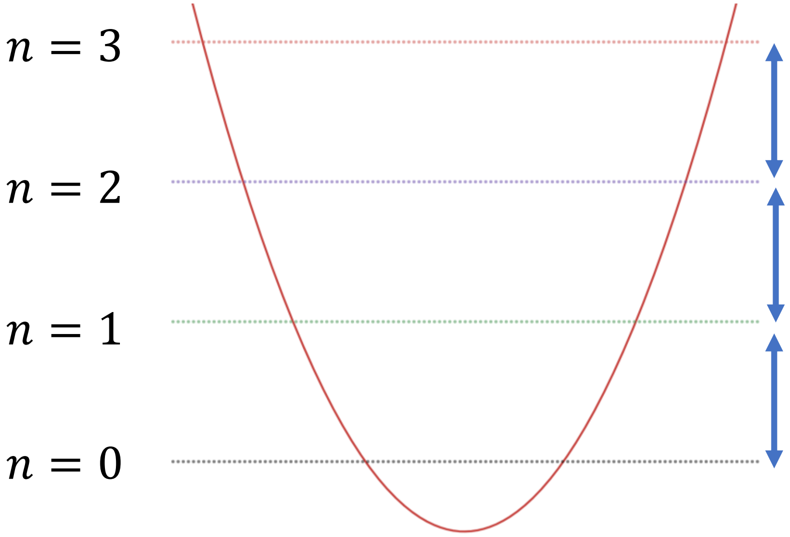


$$H = \frac{1}{2}CV^2 + \frac{1}{2}LI^2 = \frac{1}{2}CL^2\dot{I}^2 + \frac{1}{2}\,LI^2$$

We may indeed quantize the system, obtaining a quantum harmonic oscillator

Harmonic oscillators cannot be qubits

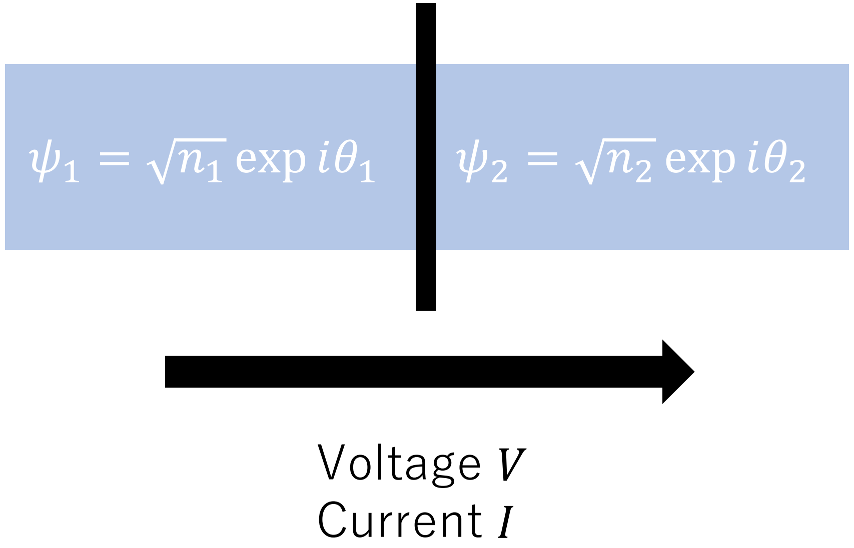
- It is NOT a two-level system!
  - Any $|n\rangle \equiv \frac{1}{\sqrt{n!}}\left(a^\dagger\right)^n|0\rangle$ is the eigenstate of the Hamiltonian
  - We cannot isolate $|g\rangle$ and $|e\rangle$



All energy differences are the same and we cannot excite only $|1\rangle$ from $|0\rangle$; then $|2\rangle$ would be excited from $|1\rangle$

Non-linearity: Josephson junction

- Josephson junction: two superconductor separated by a thin insulator



By tunneling, the Schrödinger eq is

$$i\partial_t \psi_1 = T\psi_2 - eV\psi_1$$
$$i\partial_t \psi_2 = T\psi_1 + eV\psi_2$$

The solution is

$$I = \dot{n}_2 = -\dot{n}_1 = I_c \sin\phi$$
$$V = -\frac{1}{2e}\dot{\phi}$$

with $\phi = \theta_2 - \theta_1$

The energy is

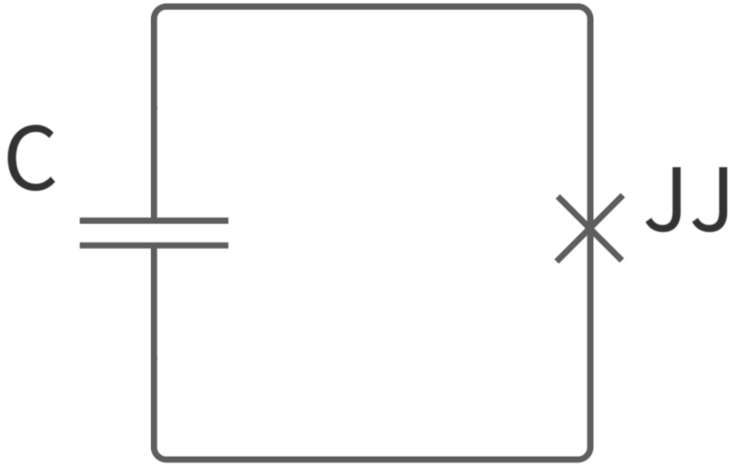
$$E = \int dt\, I\, V = \int_\phi d\phi\, I \propto -I_c \cos\phi$$

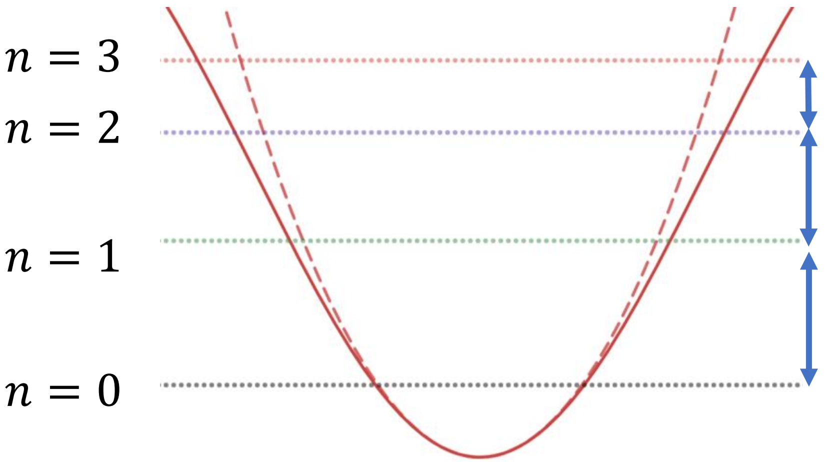
Non-linear!

Superconducting qubit

- We introduce a “non-linearity”
  - Replacing L with a Josephson junction

$$H = \frac{1}{2}CV^2 - J_0 \cos\theta$$
$$= \frac{1}{8e^2}C\dot{\theta}^2 + J_0\left[\frac{1}{2}\theta^2 - \mathcal{O}(\theta^4)\right] + \text{const.}$$

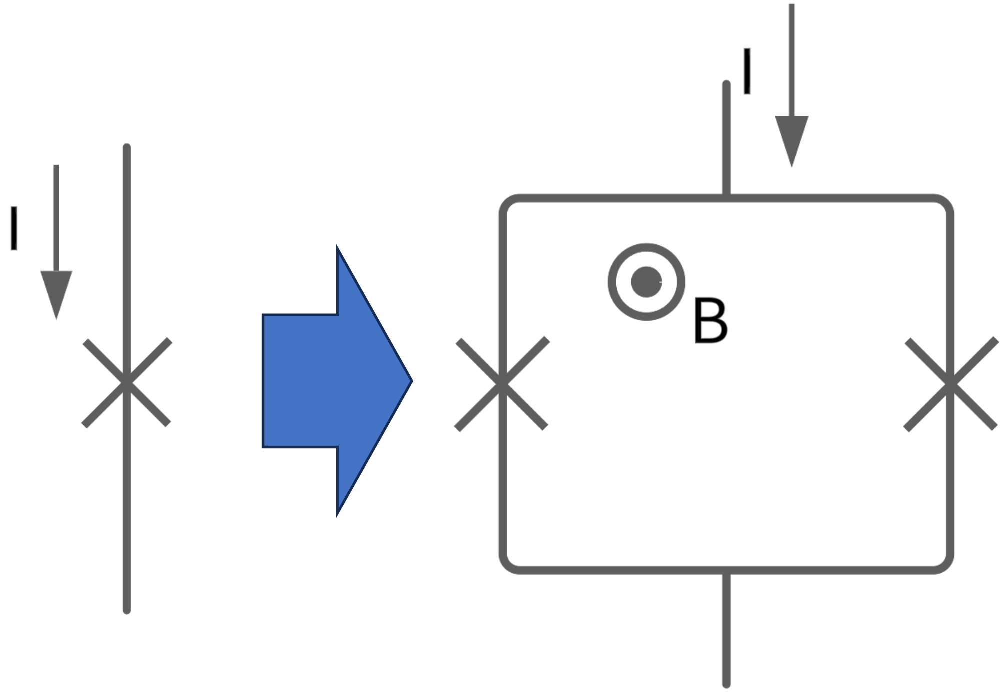




All energy differences are different;
we can use $|1\rangle$ and $|0\rangle$ as a two-level system
(For the transmon limit, $J_0 \gg \frac{e^2}{C}$, $\langle\theta^2\rangle \ll 1$ and we may regard the system as a HO)
Typically, $\omega \sim \text{GHz}$.

Frequency tuning

- We may tune the energy gap by ~one order by using SQUID
  - The tunability is one of the big advantage as the DM detector



The current of Josephson junction:

$$I = I_c \sin\phi$$

The current of SQUID (two same JJ):

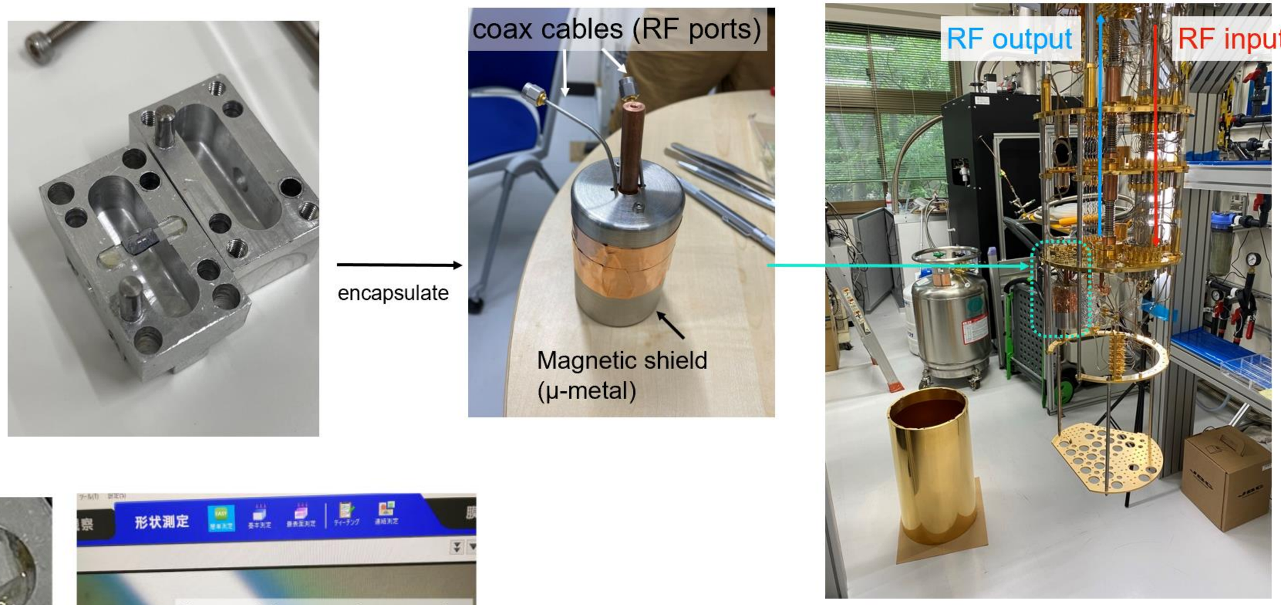
$$I = I_c \sin\phi_1 + I_c \sin\phi_2$$

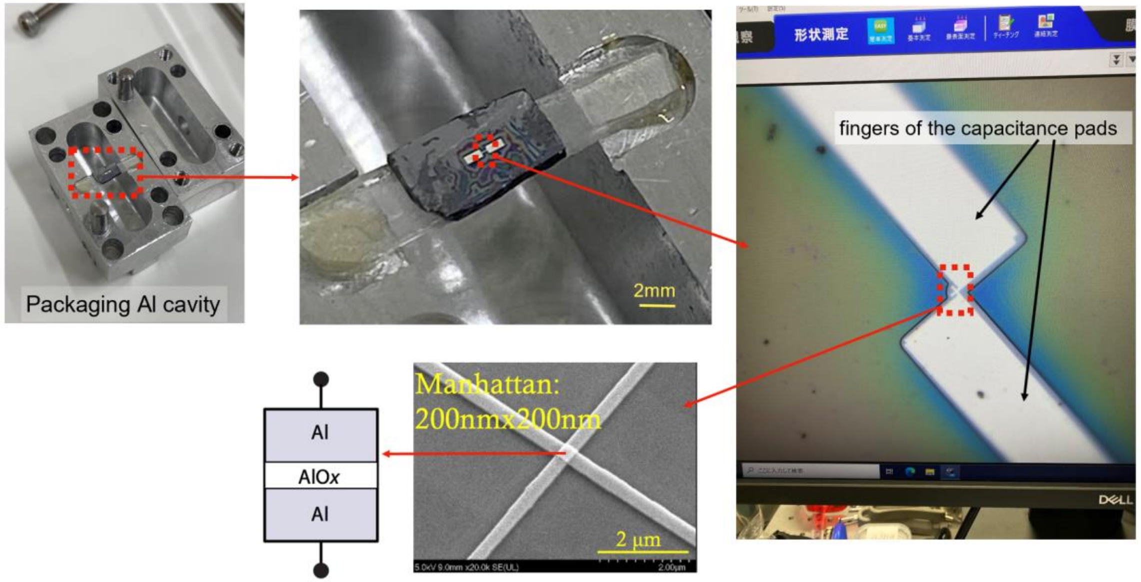
Quantization condition of superconductor:

$$2\pi n = \phi_2 - \phi_1 + 2e\, B \cdot S$$

$$\Rightarrow I = (2I_c \cos\phi_e) \sin\left(\phi_1 + \frac{\phi_e}{2}\right)$$

Garally: real qubits





Credit: S. Chen, T. Inada and T. Nitta

Dark matter detection with one qubit

Dark photon dark matter

- First DM target: dark photon dark matter with a kinetic mixing with the SM photon

$$\mathcal{L} = -\frac{1}{4}X_{\mu\nu}X^{\mu\nu} + \frac{1}{2}m_X^2 X^2 - \frac{\epsilon}{2}X_{\mu\nu}F^{\mu\nu}$$

- After solving the kinetic mixing, X_μ couples with the SM current

$$\Delta\mathcal{L} = e\left(A_\mu + \epsilon X_\mu\right)J_{\rm SM}^\mu$$

- The DM background looks like "X electric field"

$$\langle\vec{X}\rangle \simeq \bar{X}\vec{n}(t)\cos m_X t\,, \rho_{DM} \simeq \frac{1}{2}m_X^2\bar{X}^2, E_X \sim \dot{X}$$

Axion dark matter

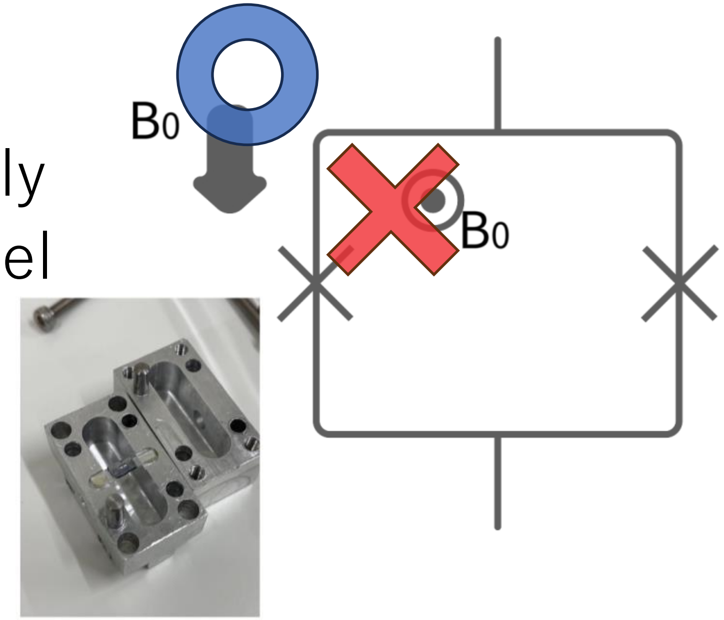
- Another target: axion (-like particle) dark matter

$$\mathcal{L} = \frac{1}{2}(\partial a)^2 - \frac{1}{2}m_a^2 a^2 + g_{a\gamma\gamma} aE \cdot B$$

- With B_0 imposed as a bg, a sources an effective electric field

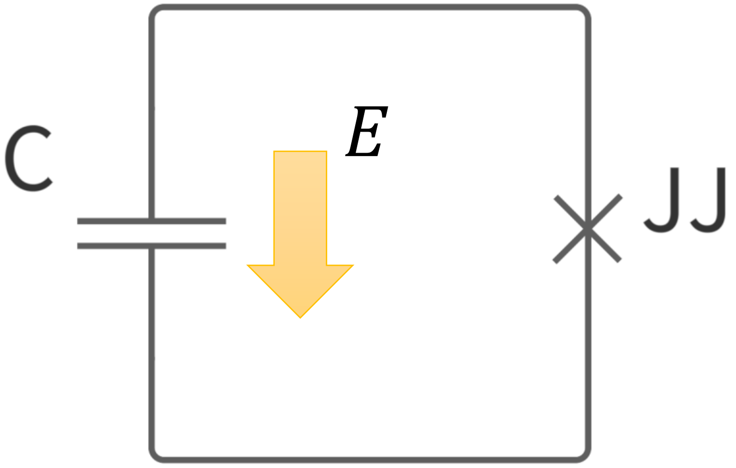
$$E = g_{a\gamma\gamma}\, aB_0$$

- Strong mag. field on SC?
  - It is reported that the magnetic field nearly 1T can be imposed if it is completely parallel (Krause *et al*, 2022)
  - We take $B_0 = 5\ \mathrm{T}$ later.
    - We of course agree that it's challenging



Interaction b/w qubits and electric fields

- How does an electric field excite the transmon?



V: canonical variable (σ_X)

E: induced from DM

$$\Delta H = CdV \cdot E$$
$$\simeq CdV \cdot \epsilon m_X \bar{X}(\vec{n}_X \cdot \vec{e}) \sin(m_{DM} t - \alpha)$$
$$\equiv 2\eta\sigma_X \sin(m_{DM} t - \alpha)$$

for DP. The last line is the same for the axion.

(n_X, α): random (direction, phase)

Hamiltonian of qubits

In reality, the DM phase is unknown;
$\Delta H \sim 2\eta\sigma_X \sin(m_X t + \alpha)$
Then,
$H_I = \eta(\sigma_X \cos\alpha + \sigma_Y \sin\alpha)$

- The Hamiltonian of the qubit is now

$$H = H_0 + \Delta H, \qquad H_0 = -\frac{1}{2}\omega\sigma_z, \qquad \Delta H = 2\eta\sigma_X \sin m_{DM} t$$

- To solve this, we move to the interaction picture;

$$i\frac{\partial}{\partial t}\psi_I = H_I\psi_I$$
$$H_I = e^{iH_0 t}\Delta H e^{-iH_0 t}$$

- To simplify things, we adopt the rotating-wave approx.

$$H_I = \eta\sigma_X \cos(m_{DM} - \omega)t + \text{(higher freq. modes)} \simeq \eta\sigma_X$$

  - The timescale we consider is assumed to be much longer than ω^{-1}
  - We may detect DM $m_{DM} \sim \omega$

Evolution of qubits

- Everything is now simple:

$$\psi_I(t) = \exp(-iH_I t)\,\psi_I(0)$$
$$= \begin{pmatrix} \cos\eta t & -i\sin\eta t \\ -i\sin\eta t & \cos\eta t \end{pmatrix}\begin{pmatrix}\psi_0 \\ \psi_1\end{pmatrix}$$

- Evolution of qubits are clear; qubits oscillate $|0\rangle$ and $|1\rangle$.
- If we initially prepare $|0\rangle$, we may observe $|1\rangle$ if DM exists

$$|\psi(t)\rangle \simeq |0\rangle - i\,\eta t|1\rangle \Rightarrow p(0 \to 1) = |\langle 1|\psi(t)\rangle|^2 \simeq \eta^2 t^2$$

as long as t is smaller than the "coherent time" of the system

$\tau = \min(\tau_{DM}, \tau_{qubit})$

Time for DM blob to pass. $\sim 1/mv^2$

Time for the qubit to maintain the coherence
(something like Q-value/frequency)
$\sim 100\ \mu s$ by current technology

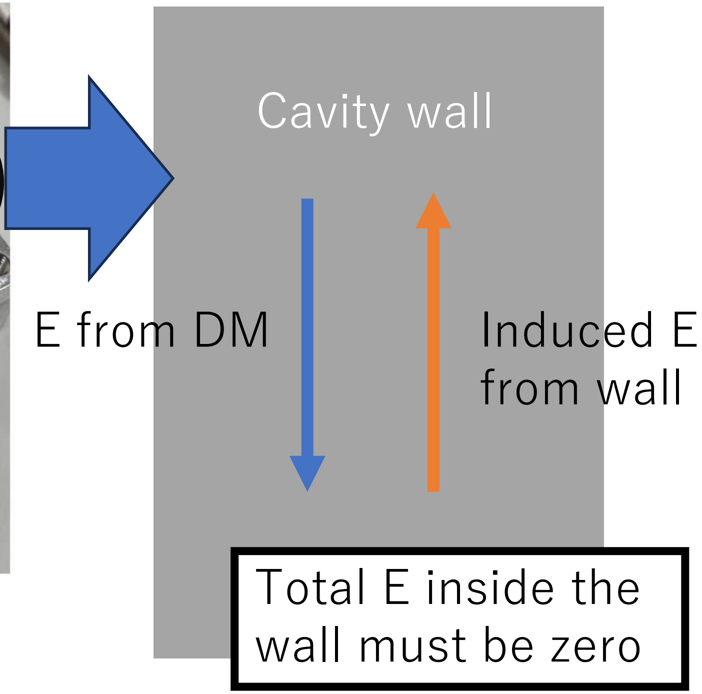
Frequency range to see is $\sim 2\pi/\tau$

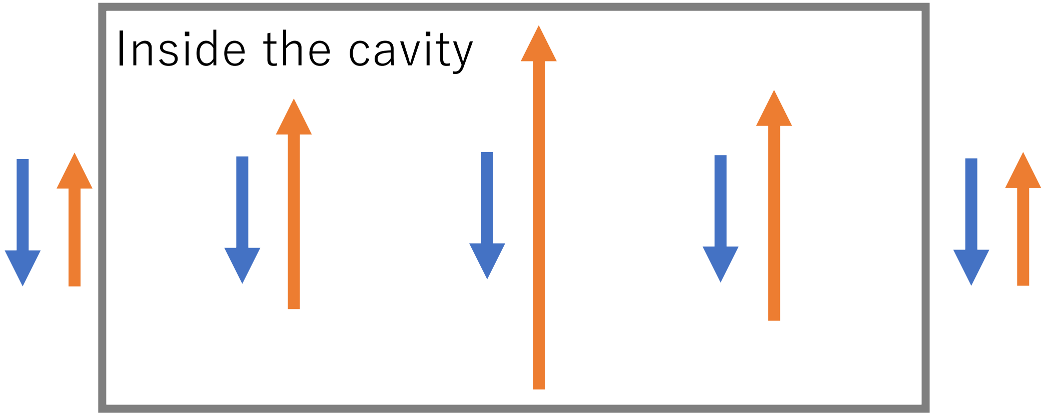
Measurement procedure

- Measurement procedure is following:
  1. Prepare all n_q qubits in the ground state $|0\rangle$
  2. Expose all qubits to the DM for the time $\min(\tau_{DM}, \tau_{qubit})$
  3. Measure all qubits to see if there are any $|1\rangle$
  4. Repeat 1-3 for some fixed time
  5. Change the frequency of the qubits by $\sim 2\pi/\tau$ by changing the magnetic flux of the SQUID
  6. Repeat 1-5 to scan some range
- The advantage of this system, compared with, say, cavity experiments, is the ease of the freq tuning

Cavity effect to enhance the signal

- We can additionally use the cavity effect to enhance the signal:





If the induced electric field is near the resonance frequency of the cavity, it is “stored” inside the cavity and the effective electric field is enhanced: $E \to \kappa E, \kappa > 1$

DM detection with quantum circuits

Quantum Enhancement

- The probability for one qubit is $p \sim \eta^2 \tau^2$. The quantum nature of *single* qubit is essential for this τ^2 dependence.
- How about n_q? In the previous procedure, we assume to use n_q qubits *independently*.
- For such "independent" qubits, as we increase the number of qubits, n_q, the probability to see $|1\rangle$ in any of the qubits increases by n_q
- Is it possible to increase the probability by using quantum nature of the system for n_q?

Summing up amplitudes

$|0\rangle \to |0\rangle + \eta\tau\,|1\rangle \Rightarrow p = \eta^2\tau^2$

$|0\rangle \to |0\rangle + \eta\tau\,|1\rangle \Rightarrow p = \eta^2\tau^2$

$\vdots$

$|0\rangle \to |0\rangle + \eta\tau\,|1\rangle \Rightarrow p = \eta^2\tau^2$

$$p_{tot} = 1 - (1-p)^{n_q} \sim n_q p$$

- Is it possible to let n_q appear in the state amplitude?
- Namely, we need to find such $|\psi\rangle$ that
$$|\psi\rangle \to |\psi\rangle + \mathcal{O}(n_q)|\psi_\perp\rangle$$
- Then, $p' = \langle\psi_\perp|\psi(t)\rangle = \mathcal{O}(n_q^2)$.

Summing up phases

- Let's focus on each qubit individually:

$$H_I = \eta\sigma_X \Rightarrow H_I\,|\pm\rangle = \pm\eta|\pm\rangle$$
$$\Rightarrow \sum H_I|\pm\rangle^{\otimes n_q} = \pm n_q\eta|\pm\rangle^{\otimes n_q}$$

- Thus, the relative phase of $|\pm\rangle^{\otimes n_q}$ is $\mathcal{O}(n_q)$
- To measure the relative phase, a superposed state is needed

$$e^{i(\sum H_I)t}\left(|+\rangle^{\otimes n_q} + |-\rangle^{\otimes n_q}\right) = e^{in_q\eta t}|+\rangle^{\otimes n_q} + e^{-in_q\eta t}|-\rangle^{\otimes n_q}$$
$$\simeq \left(|+\rangle^{\otimes n_q} + |-\rangle^{\otimes n_q}\right)$$
$$+in_q\eta t\left(|+\rangle^{\otimes n_q} - |-\rangle^{\otimes n_q}\right)$$

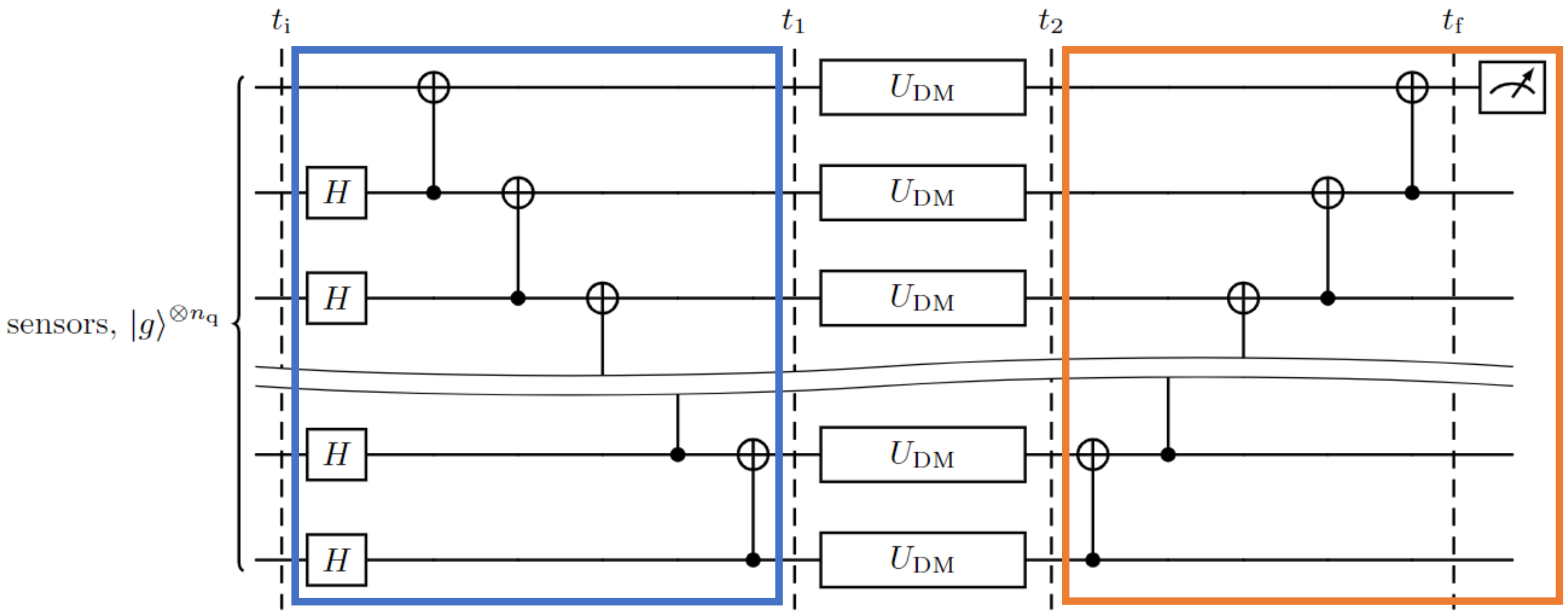
$p = n_q^2\eta^2t^2$!

- $|+\rangle^{\otimes n_q} \pm |-\rangle^{\otimes n_q}$ is called the GHZ state

Greenberger, Horne, Zeilinger, 1989,
Giovannetti *et al*, 2004

Quantum Circuit

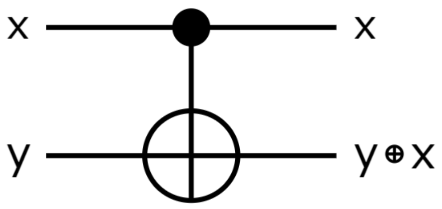
- We need to prepare $|+\rangle^{\otimes n_q} + |-\rangle^{\otimes n_q}$ (GHZ state) and measure it by $|+\rangle^{\otimes n_q} - |-\rangle^{\otimes n_q}$. This can be done by



HF Chen, Inada, Moroi, Nitta, Sichanugrist arXiv:2311.10413

Take a closed look at the circuit

- To check it, notice the equivalence on CNOT:



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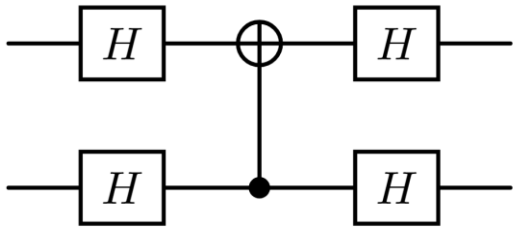


input		output	
x	y	x	y+x
$\|0\rangle$	$\|0\rangle$	$\|0\rangle$	$\|0\rangle$
$\|0\rangle$	$\|1\rangle$	$\|0\rangle$	$\|1\rangle$
$\|1\rangle$	$\|0\rangle$	$\|1\rangle$	$\|1\rangle$
$\|1\rangle$	$\|1\rangle$	$\|1\rangle$	$\|0\rangle$

From Wikipedia

Here,

$H|0\rangle = |+\rangle \sim |0\rangle + |1\rangle$

$H|1\rangle = |-\rangle \sim |0\rangle - |1\rangle$

and $H^2 = 1$

Let's check it:

$|0\rangle|0\rangle \to |+\rangle|+\rangle \to |+\rangle|+\rangle \to |0\rangle|0\rangle$

$|0\rangle|1\rangle \to |+\rangle|-\rangle \to |+\rangle|-\rangle \to |0\rangle|1\rangle$

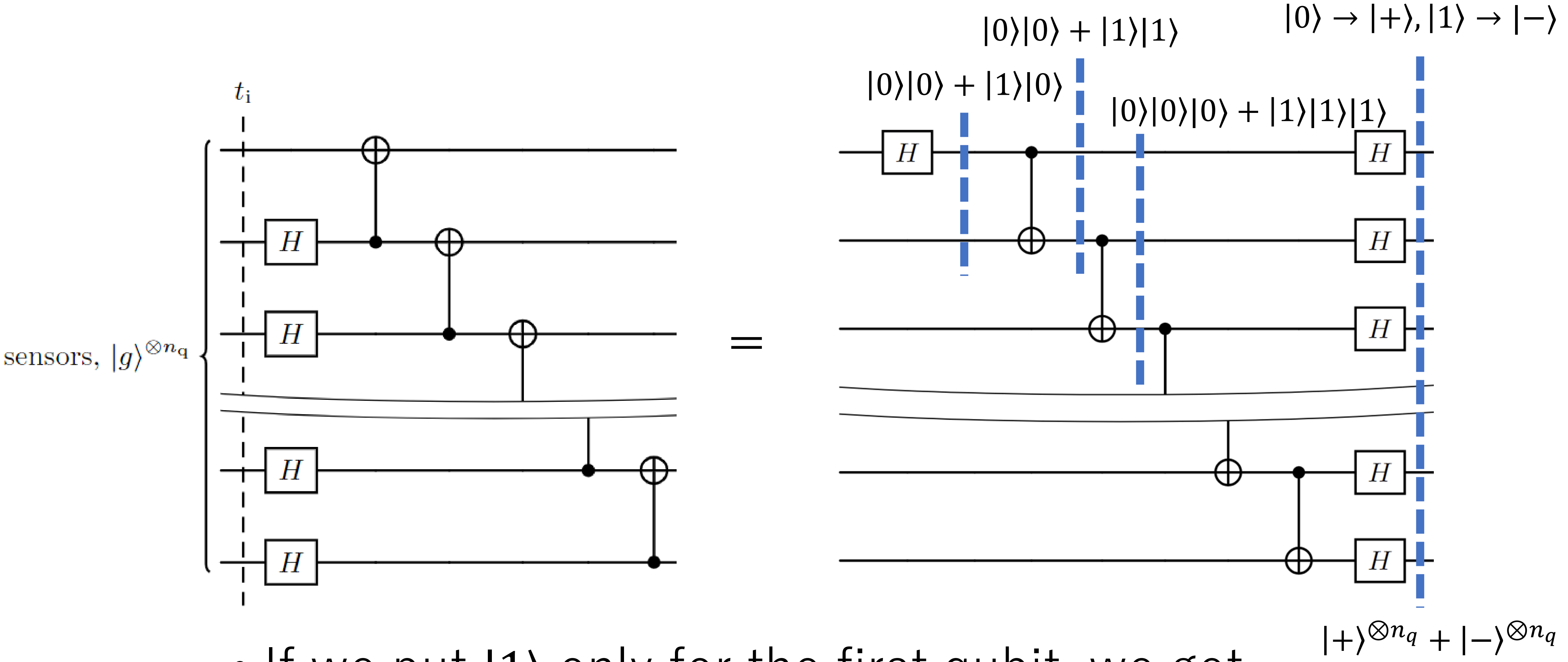
$|1\rangle|0\rangle \to |-\rangle|+\rangle \to |-\rangle|-\rangle \to |1\rangle|1\rangle$

$|1\rangle|1\rangle \to |-\rangle|-\rangle \to |-\rangle|+\rangle \to |1\rangle|0\rangle$

Here,

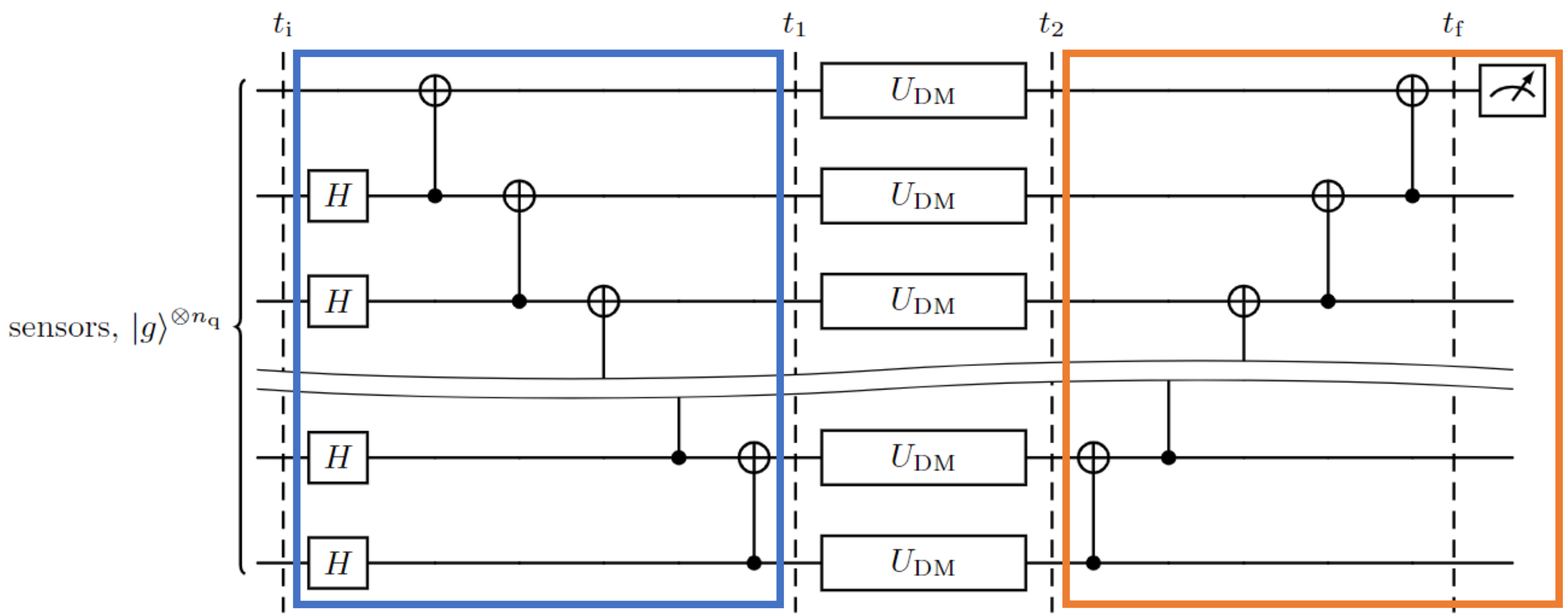
$\mathrm{CNOT}|\psi\rangle|\pm\rangle = |\psi\rangle|0\rangle \pm (X|\psi\rangle)|1\rangle$

Take a closed look at the circuit



- If we put $|1\rangle$ only for the first qubit, we get $|+\rangle^{\otimes n_q} - |-\rangle^{\otimes n_q}$ instead

Take a closed look at the circuit



- The orange part is the inverse of the blue part (but for the last Hadamard gates)
- $|+\rangle^{\otimes n_q} \pm |-\rangle^{\otimes n_q}$ are converted back to $|0\rangle$ and $|1\rangle$, respectively.

Quantum noises

- Up to this point, we haven't thought of the quantum noises.
- What is *quantum* noises?
- Quantum noises is, the effect of environments and effectively let the state jump into another state with *classical* probability

$$|\psi\rangle \rightarrow \begin{cases} |\psi\rangle & \text{with probability } p \\ E|\psi\rangle & \text{with probability } 1-p \end{cases}$$

- The final state is a classical mixture of the state and not a pure state anymore. It can be written in the density matrix.
- E need not to be unitary, although I ignore the normalization here; e.g. $E = a$, the de-excitation noise

See e.g. Nielsen & Chuang

Effect of quantum noises

- Entangled states such as the GHZ states are, very generally speaking, more fragile to the quantum noise than separated system

Huelga et al., 1997

Separated system

$$|\psi_1\rangle, |\psi_2\rangle, \cdots |\psi_{n_q}\rangle$$

$n_q \times \eta^2 t^2$ signals,
$n_q \times (1 - e^{-\gamma t})$ errors

Entangled system

$$|\psi_1\rangle|\psi_2\rangle \cdots |\psi_{n_q}\rangle + \cdots$$

$\sim n_q^2 \eta^2 t^2$ signals,
$(1 - e^{-n_q \gamma t})$ errors

Errors on any qubits are counted as the error of the system…

$\tau_{\text{entangled}} \sim \tau_{\text{separated}} / n_q$ and NO signal enhancement anymore?

Quantum error correction

- Actually, the GHZ state is strong against some error.
  - Suppose a special error, $|\pm\rangle \to |\mp\rangle$ on the first qubit only. The state is then

$$a\left(|+\rangle^{\otimes N} + |-\rangle^{\otimes N}\right) + b\left(|+\rangle^{\otimes N} - |-\rangle^{\otimes N}\right)$$

$$\to a\left(|-\rangle|+\rangle^{\otimes N-1} + |+\rangle|-\rangle^{\otimes N-1}\right) + b\left(|-\rangle|+\rangle^{\otimes N-1} - |+\rangle|-\rangle^{\otimes N-1}\right)$$

  - We can measure X_0X_1 and $X_{n_q}X_1$, which are always $+1$ for the states before the error. We can locate the error and correct it.
- However, such procedure cannot be performed for general error.

Kessler et al, 2014, Dür et al, 2014, Jeske et al, 2014

Effect of the quantum noise on our circuit

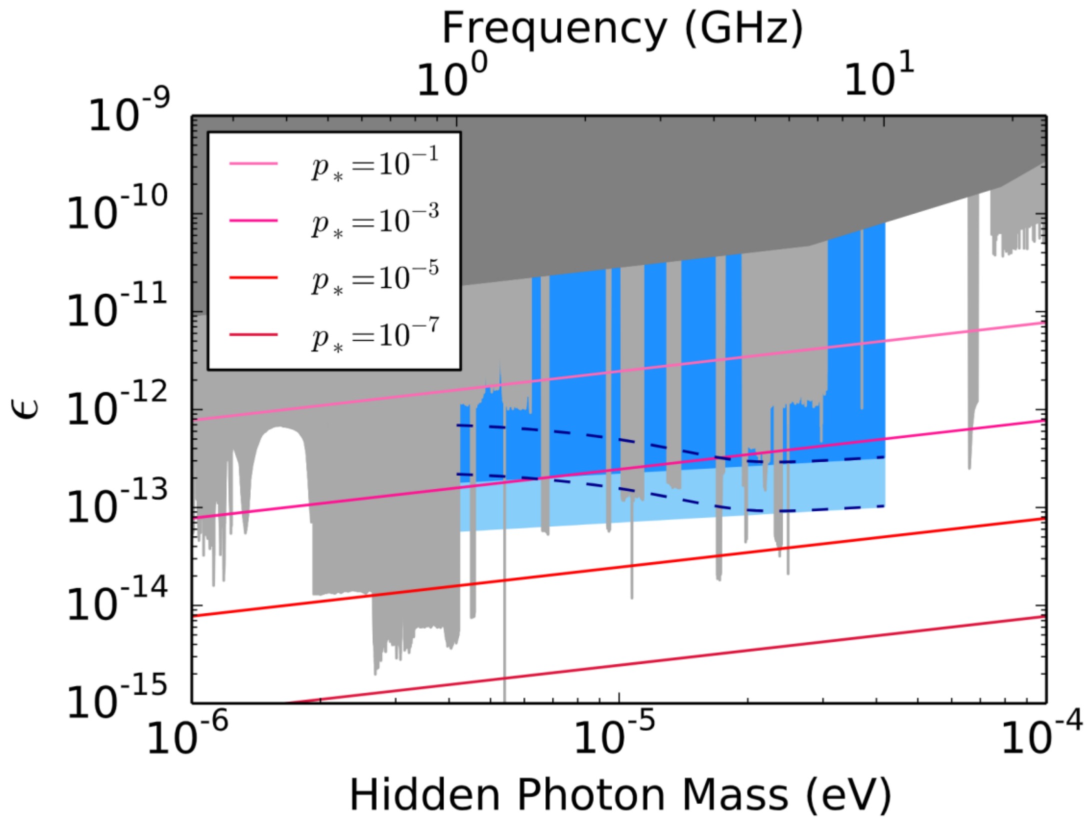
- There are possible ways to evade the nightmare;

1. The coherent time of the system is $\tau = \min(\tau_{DM}, \tau_{qubit})$. Thus, if $\tau_{qubit} \gg n_q \tau_{DM}$, the system is constrained by the DM coherent time.
2. Assume that the state prep error is negligible. Then, if we take $\tau_{GHZ} = \tau / n_q$, the probability is $\mathcal{O}(n_q^0)$, the time to perform one measurement is $\mathcal{O}(n_q^{-1})$, the frequency range we may scan by one measurement is $\mathcal{O}(n_q^1)$. In total, the signal is still $\mathcal{O}(n_q^2)$

HF, Matsuzaki, Moroi, Sichanugrist et al., in prep

Results

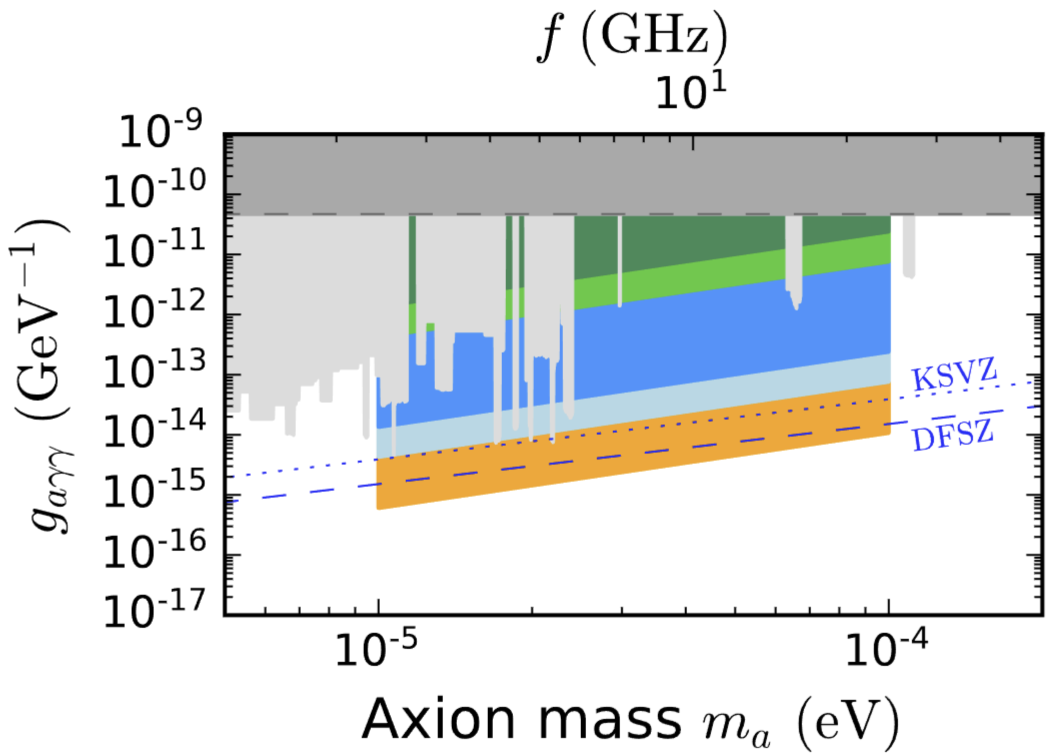
Result for 1-year measurement (DP)



- We plot 5σ discovery reach
- We assume
  - Coherent time $2\pi Q/\omega$, $Q \sim 10^6$
  - Scanning with $\Delta\omega = 2\pi/\tau$
  - 0.1% readout error
  - “thermal noise” for 1 or 30 mK
- blue: 1 qubit
- lightblue: separated 100 qubits
- With the GHZ state, the sensitivity for ϵ is improved by $\sqrt{n_q}$

HF Chen, Inada, Moroi, Nitta, Sichanugrist arXiv:2212.03884

Result for 1-year measurement (axion)



- We plot 5σ discovery reach
- $B = 5$ T and the same parameters as the previous one but for the "thermal noise"
  - We suspect it is already included in τ
- (light)green: 1 (sep. 100) qubit
- (light)blue: Use of the cavity effect, $\kappa = 100$
- orange: $\kappa = 100$ + entangled 100 qubits

HF Chen, Inada, Moroi, Nitta, Sichanugrist arXiv: 2407.19755

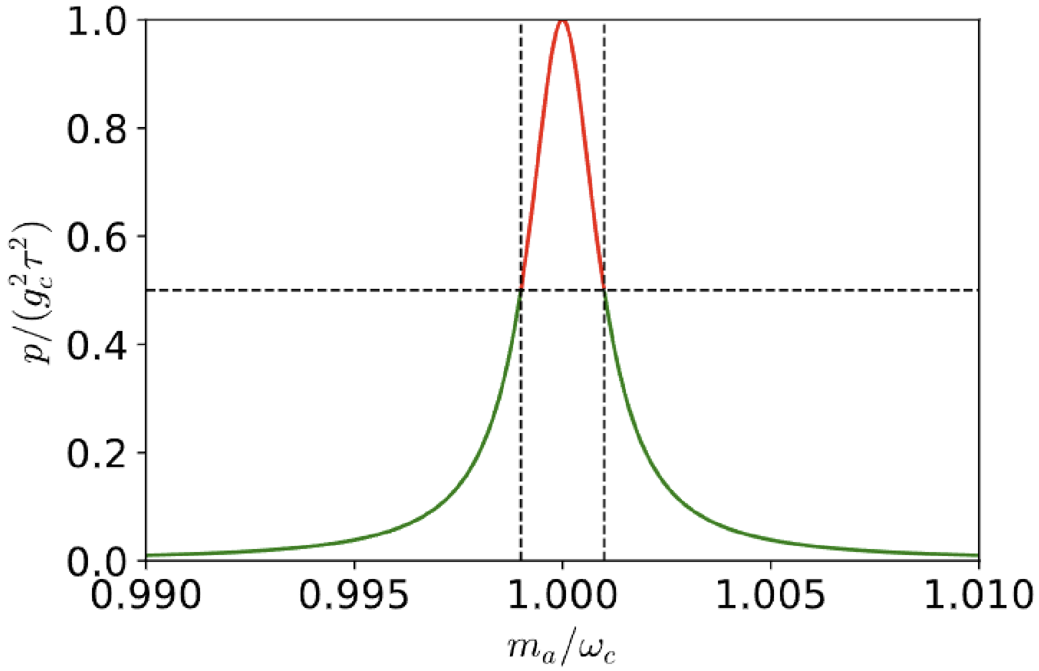
Summary

Summary

- We proposed to use transmon qubits as a dark matter detector
- It could constrain unexplored regions of the dark photon and axion dark matter parameter regions
- For the axion DM, it may reach the QCD axion bound
- The use of entangled initial states may improve the sensitivity
  - The evaluation of quantum noises are non-trivial, but we can show even with large noises the entangled states may have an advantage

Backup

Fully mixed mode and $\boldsymbol{\kappa}$



$$\kappa \sim \frac{m}{m - \omega_c} \lesssim \frac{m}{\lambda}$$

ω_c: cavity freq

λ: cavity-qubit mixing

Probability dependence

$$p_{ge}(\tau) \simeq 0.12 \times \kappa^2 \cos^2\Theta \left(\frac{\epsilon}{10^{-11}}\right)^2 \left(\frac{f}{1\ \mathrm{GHz}}\right) \times \left(\frac{\tau}{100\ \mu\mathrm{s}}\right)^2 \left(\frac{C}{0.1\ \mathrm{pF}}\right) \left(\frac{d}{100\ \mu\mathrm{m}}\right)^2 \times \left(\frac{\rho_{\mathrm{DM}}}{0.45\ \mathrm{GeV/cm^3}}\right),$$

$$p^{(1)}_{g\to e} \simeq 0.11 \times \left(\frac{g_{a\gamma\gamma}}{10^{-10}\ \mathrm{GeV}^{-1}}\right)^2 \left(\frac{m_a}{1\ \mu\mathrm{eV}}\right)^{-1} \left(\frac{B_0}{1\ \mathrm{T}}\right)^2 \left(\frac{\tau}{100\ \mu\mathrm{s}}\right)^2 \kappa^2 \times \left(\frac{C}{0.1\ \mathrm{pF}}\right) \left(\frac{d}{100\ \mu\mathrm{m}}\right)^2 \left(\frac{\rho_{\mathrm{DM}}}{0.45\ \mathrm{GeV/cm^3}}\right).$$

Syndrome measurement

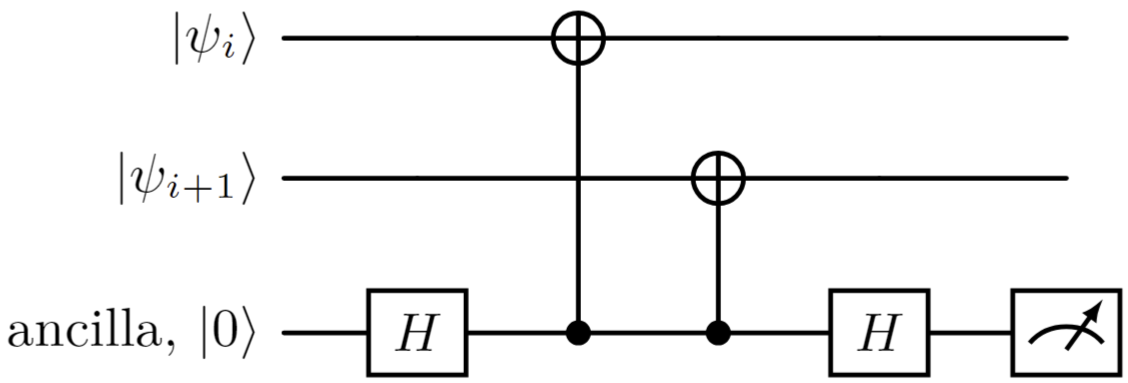


FIG. 2: Quantum circuit for the syndrome $X_i X_{i+1}$ measurement.