Machine Learning for Galactic Dynamics:

Neural Stellar Density Estimation for Mapping Dark Matter in the Local Universe

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KIAS AI Workshop

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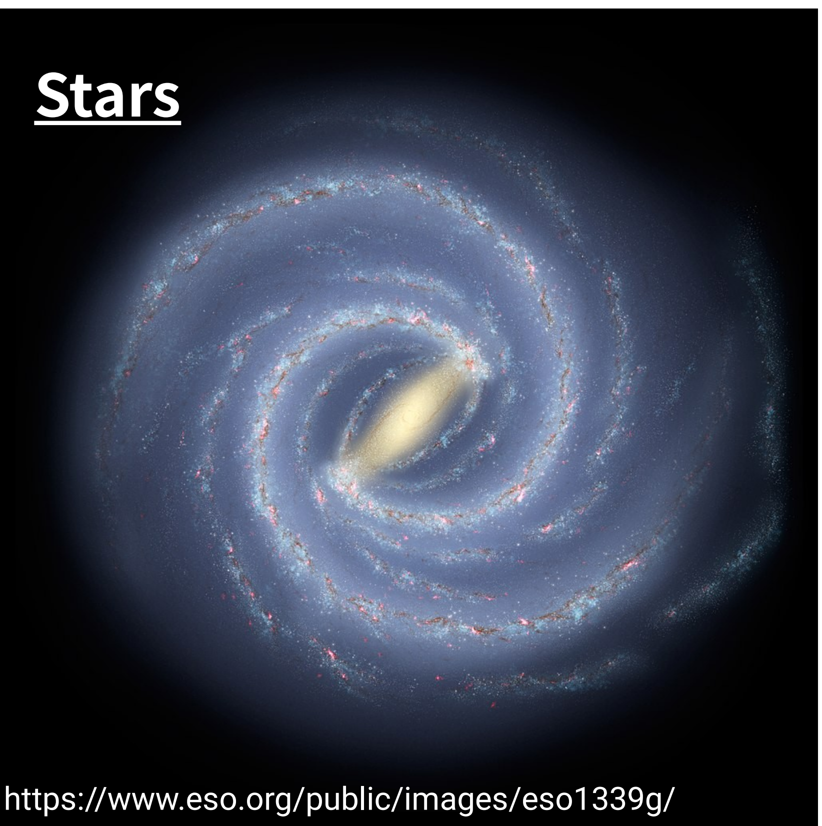
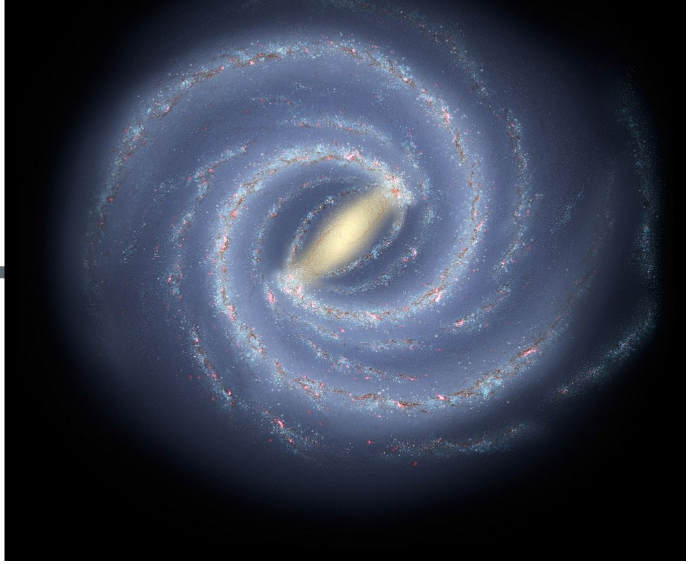
In **galactic dynamics** for studying **dark matter**,
one important and interesting task is…

Q: How to use **stellar distribution** of a galaxy to understand **its galactic dark matter density**?



In the previous talk...

Hydrodynamics and Galactic Dynamics



If we consider a galaxy as a hydrodynamic system $N \to \infty$ consisting of stars, phase-space density of a star (probability of finding a star with given position and velocity) describes the system.

$$f(\vec{x}, \vec{v})$$

Equation of motion: Boltzmann Equation

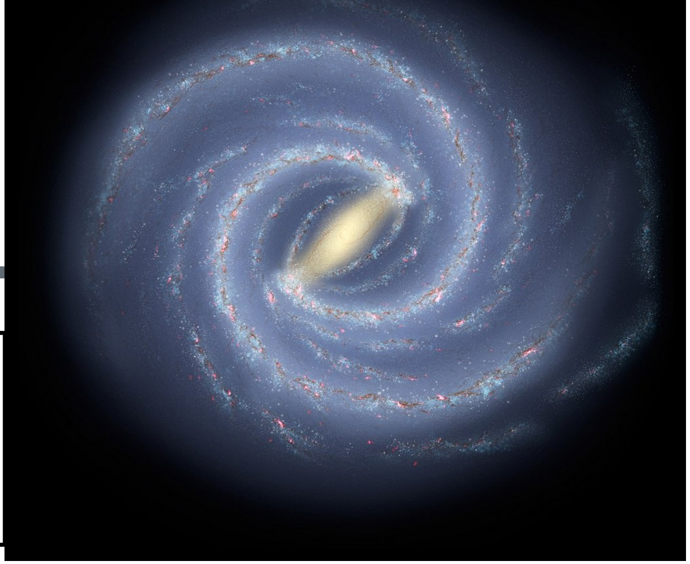
$$\left[\frac{\partial}{\partial t} + \vec{v} \cdot \frac{\partial}{\partial \vec{x}} + \vec{a} \cdot \frac{\partial}{\partial \vec{v}}\right] f(\vec{x}, \vec{v}) = 0$$

If **phase-space density** is determined...

We can estimate the **gravitational acceleration field**!

+no unnecessary assumptions are involved

Outline of Strategy



Star catalog

$\{(\vec{x}, \vec{v})\}$

Galaxy:
hydrodynamic
system

Phase space density

$f(\vec{x}, \vec{v})$

Neural Networks for Density Estimation:
Normalizing Flows

$$\vec{u}_0 \to \vec{u}_1 \to \cdots \to \vec{u}_n = (\vec{x}, \vec{v})$$

Gravitational accel.

$\vec{a}(\vec{x})$

Solving EOM (Boltzmann Equation)

$$\left[\frac{\partial}{\partial t} + \vec{v} \cdot \frac{\partial}{\partial \vec{x}} + \vec{a} \cdot \frac{\partial}{\partial \vec{v}}\right] f(\vec{x}, \vec{v}) = 0$$

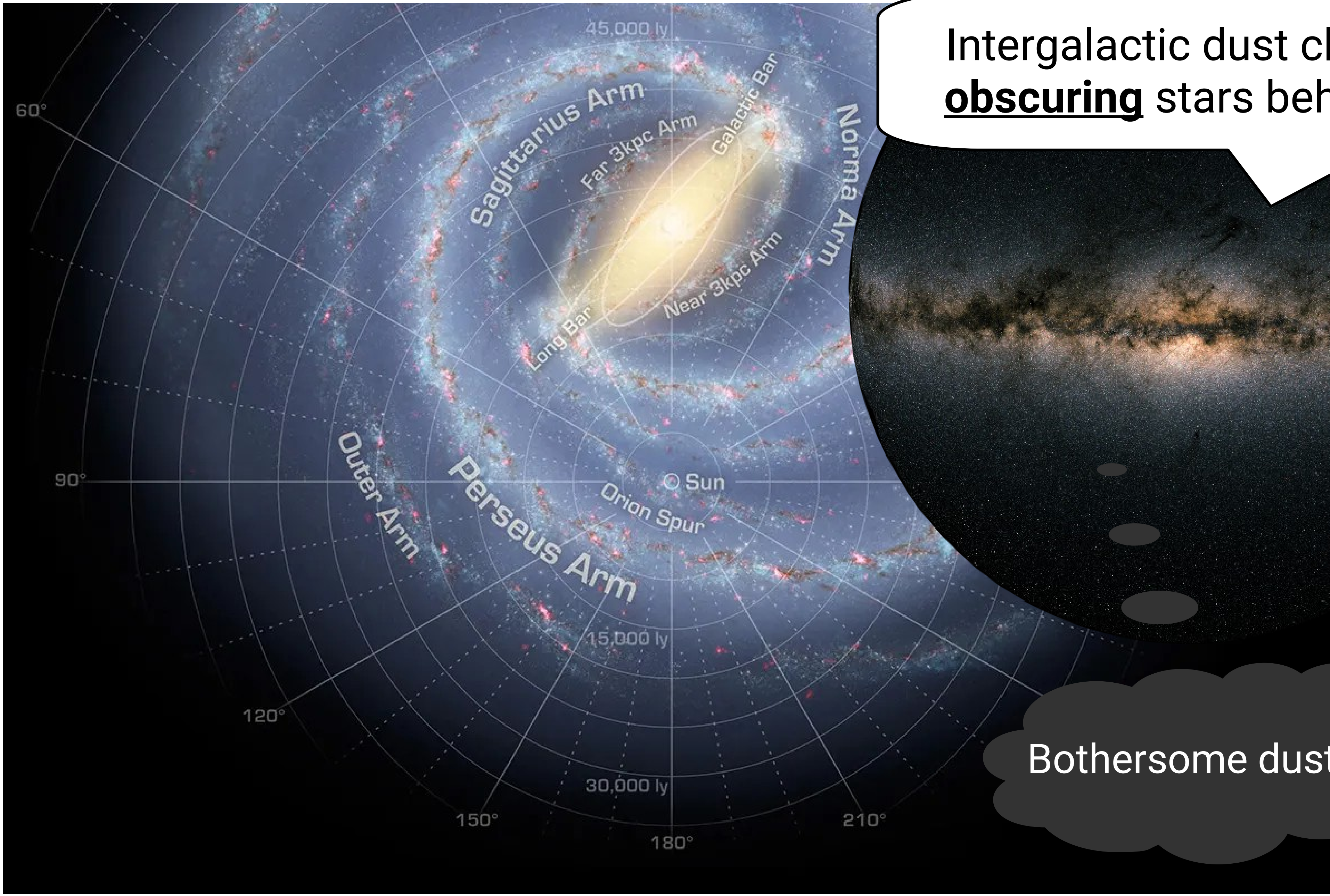
Mass density

$\rho(\vec{x})$

Solving Gauss's Equation

$$-4\pi G \rho = \nabla \cdot \vec{a}$$

arXiv: 2205.01129, 2305.13358
See also Green et. al. arXiv:2011.04673, arXiv:2205.02244

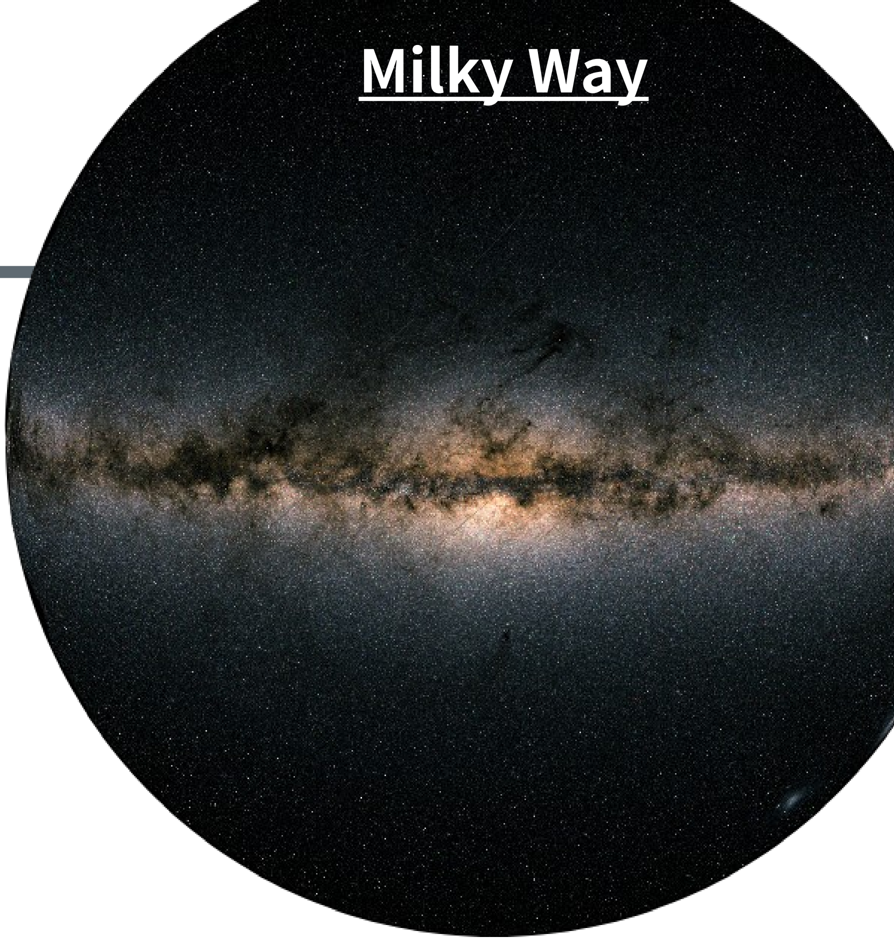


Intergalactic dust cloud **obscuring** stars behind!

Bothersome dusts!

In the previous talk, we have discussed how to estimate **dark matter density** in a **dusty** environment of **the Milky Way**.

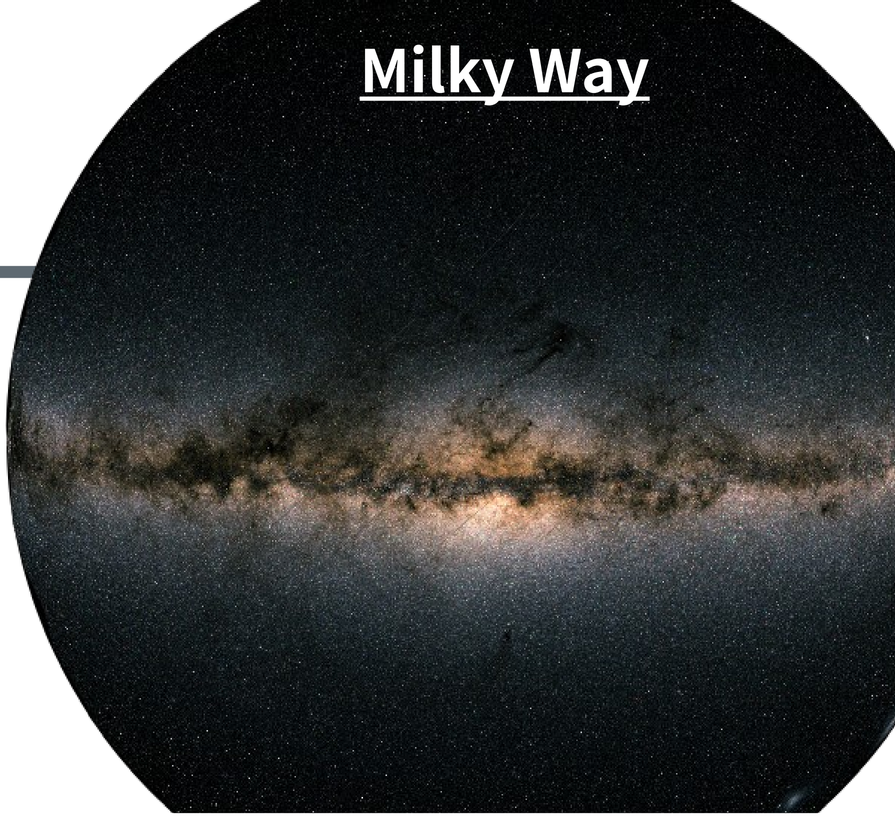
Q: Are there any **dust-free** galaxies to make this analysis simple?

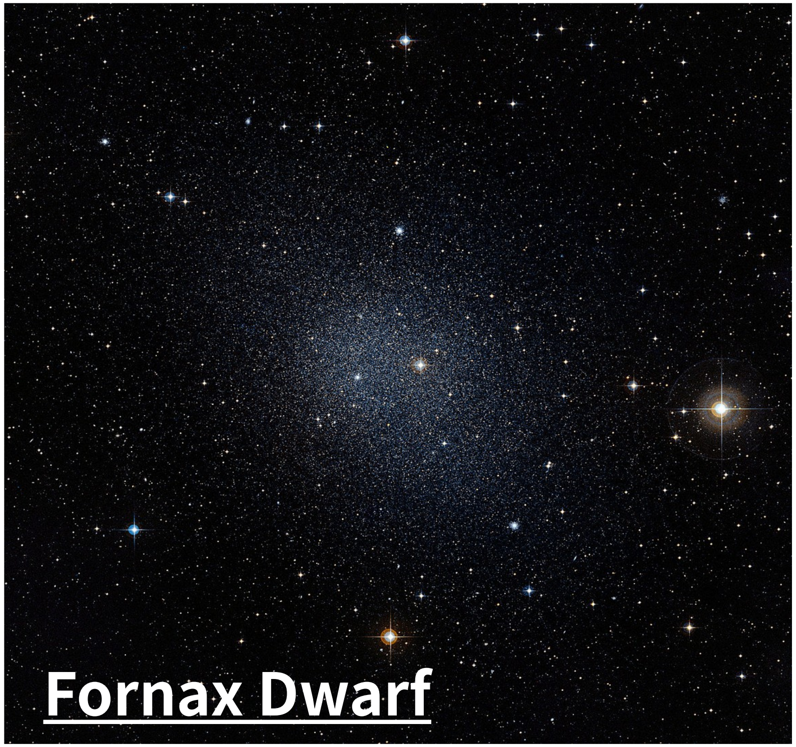


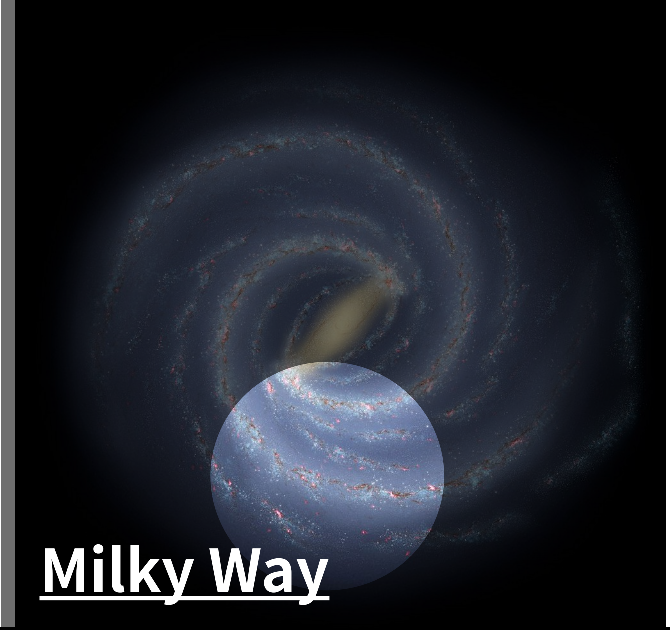
Q: Are there any **dust-free** galaxies to make this analysis simple?

Yes, there are some **dust-free satellite galaixes** of the Milky Way!

Where are they?

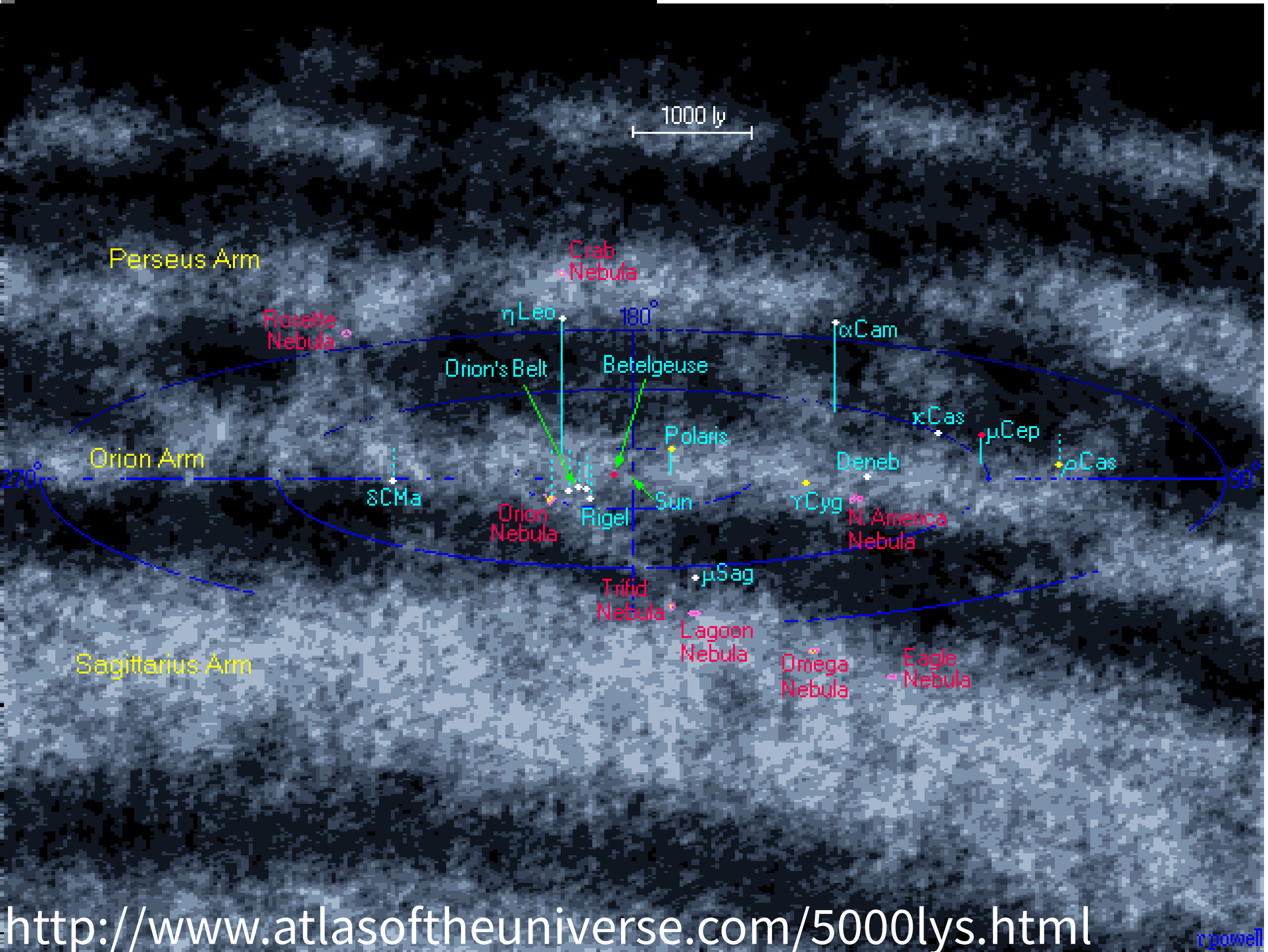






So far, we have been focused on the analysis on **our corner** of the Milky Way.

If you go further away…

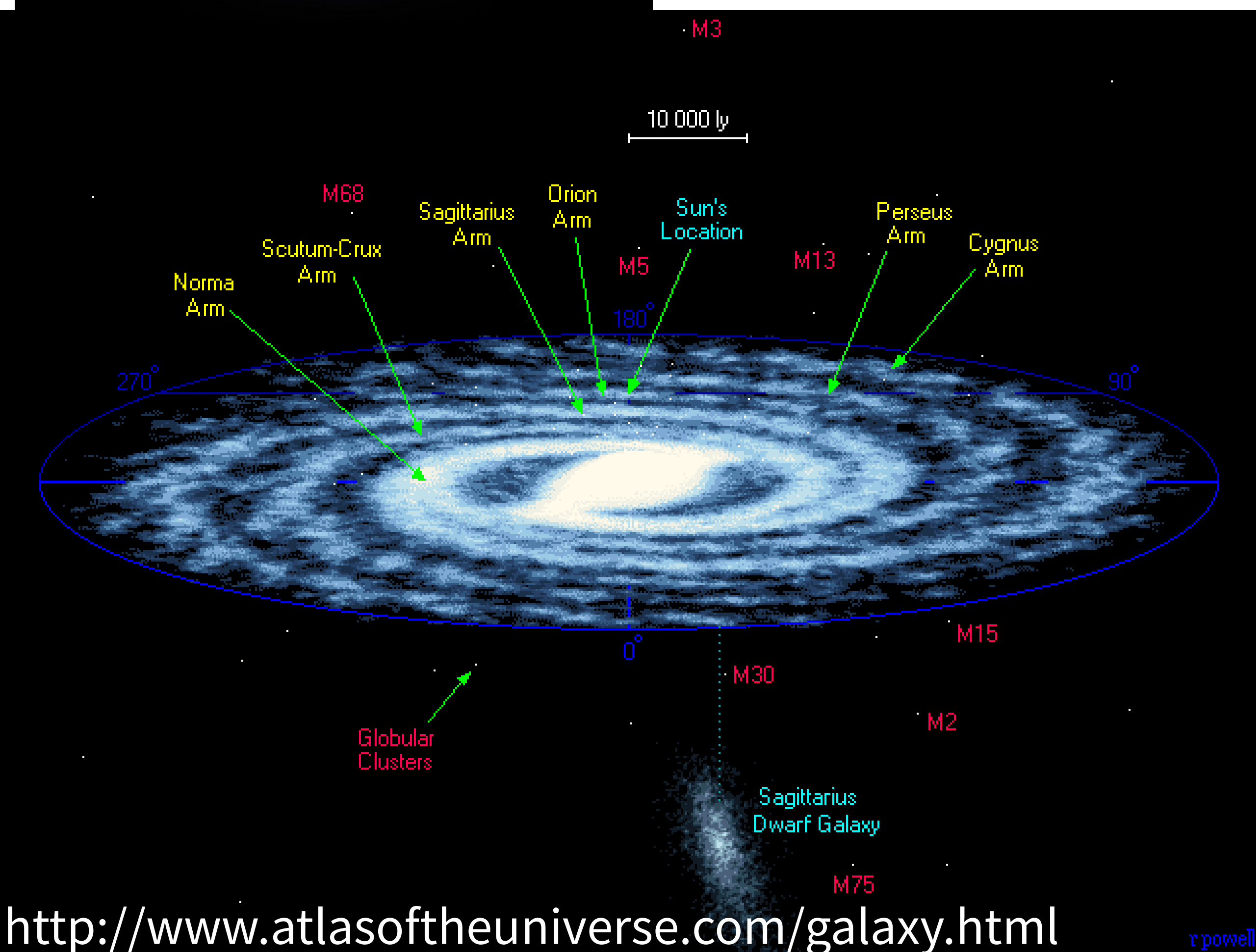


http://www.atlasoftheuniverse.com/5000lys.html

So far, we have been focused on the analysis on **our corner** of the Milky Way.

If you go further away, you see whole Milky Way, but it is difficult to get **all the kinematic information** of stars visible here.

No local dark matter density estimate on the **opposite corner**!



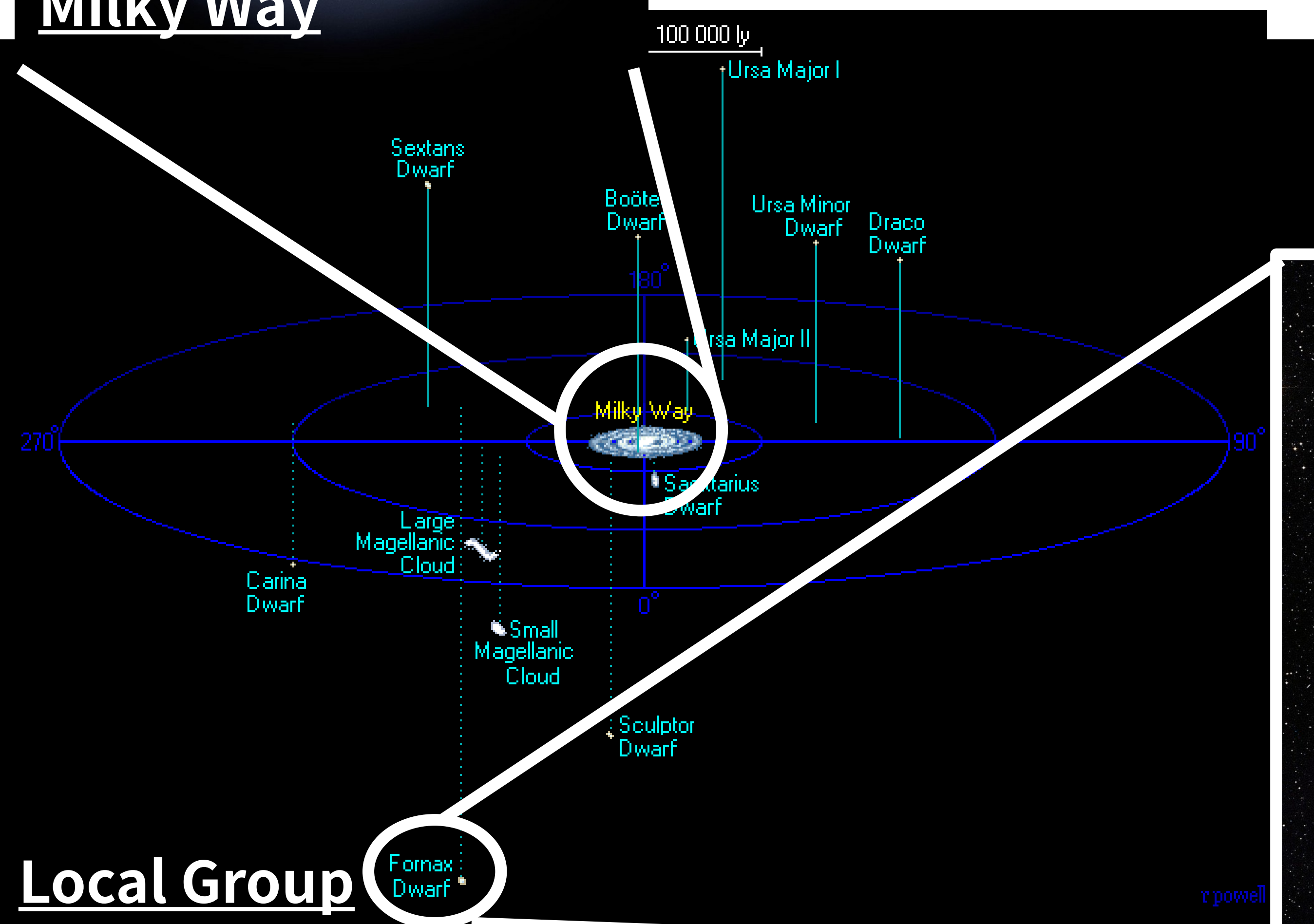
http://www.atlasoftheuniverse.com/galaxy.html

So far, we have been focused on the analysis on **our corner** of the Milky Way.

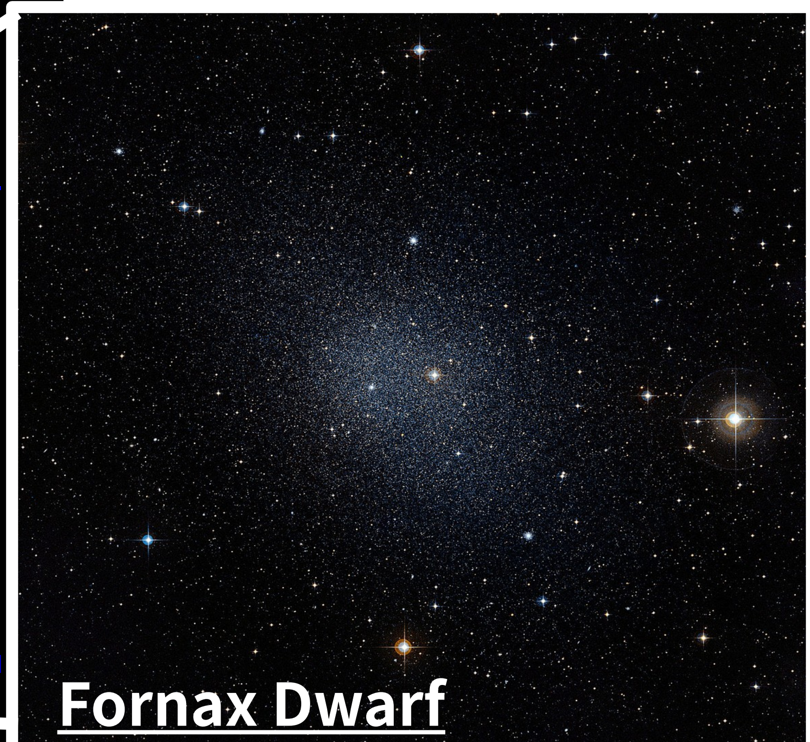
If you go further further away,

You see other satellite galaxies!

Milky Way



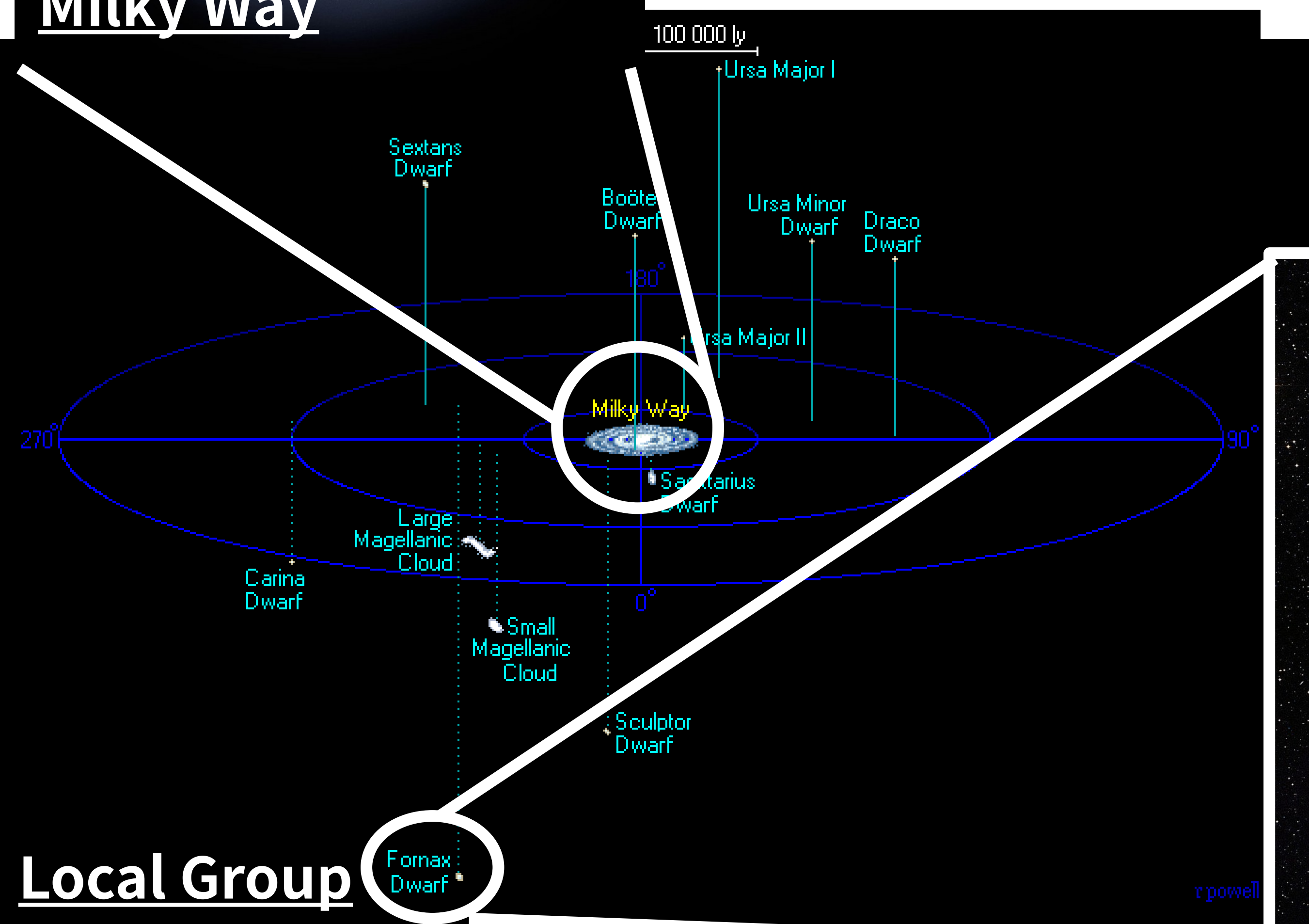
Local Group



Fornax Dwarf

http://www.atlasoftheuniverse.com/sattelit.html

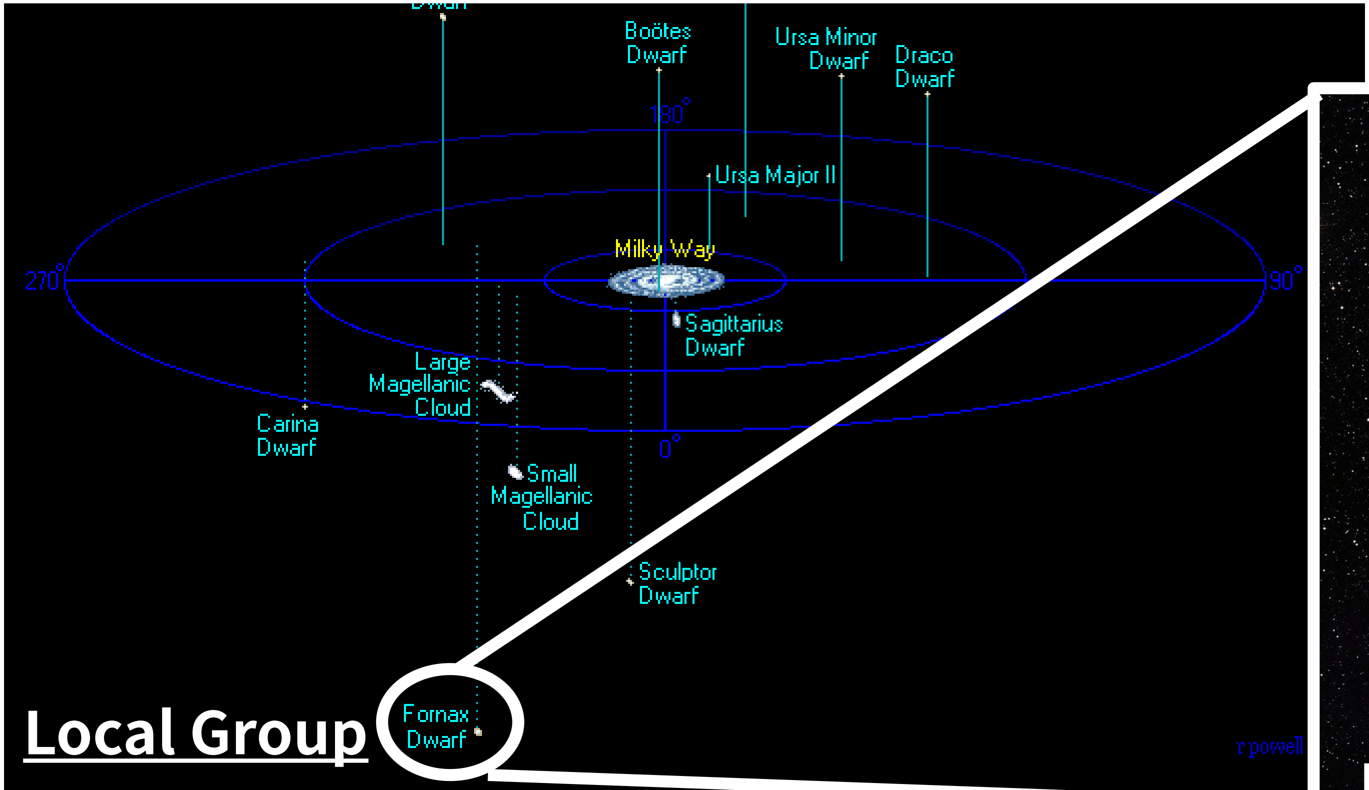
In this talk, we will focus on a type of satellite galaxy called **<u>dwarf speheroidal galaxy</u>**.

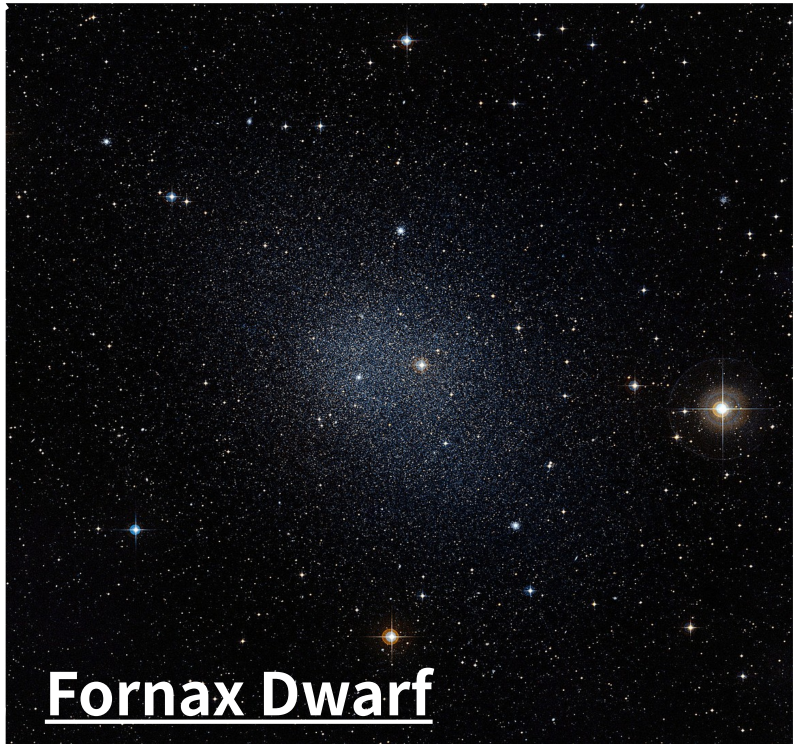


http://www.atlasoftheuniverse.com/sattelit.html

Dwarf Spheroidal Galaxy?

- A round and faint satellite galaxy, orbiting the Milky Way.
- Almost no gas and dust obscuring stars. Whole galaxy is clearly visible.



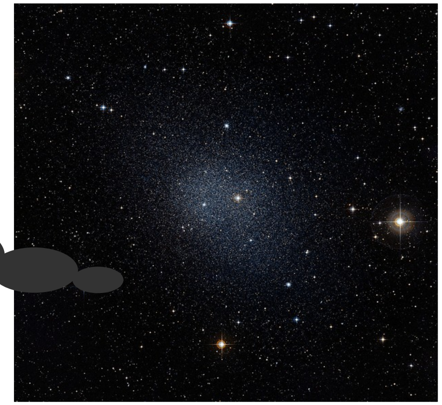


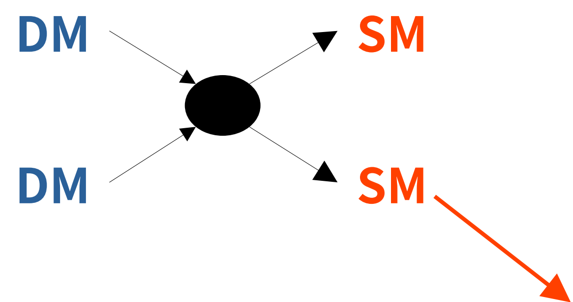
http://www.atlasoftheuniverse.com/sattelit.html

Dwarf spheroidal galaxy is a dark matter laboratory!

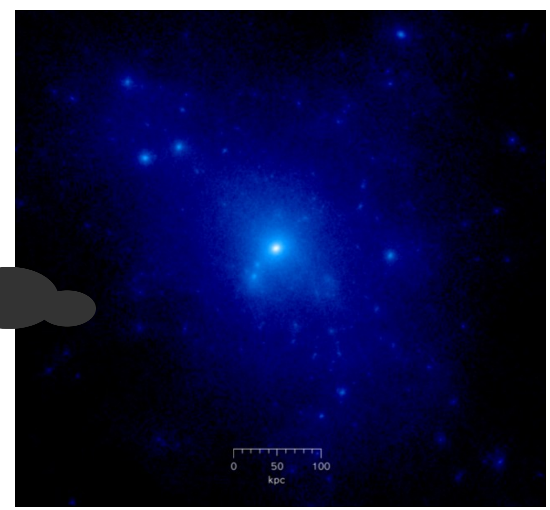
Clean signal source as dsph exhibits less baryon activity.

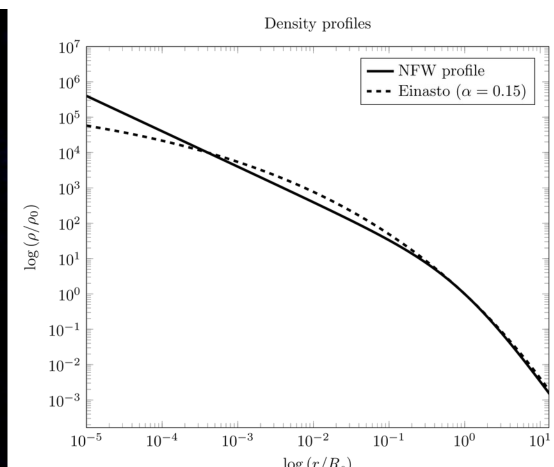
Indirect Detection experiments





Understanding the dark matter halo shape → insights on DM interactions?



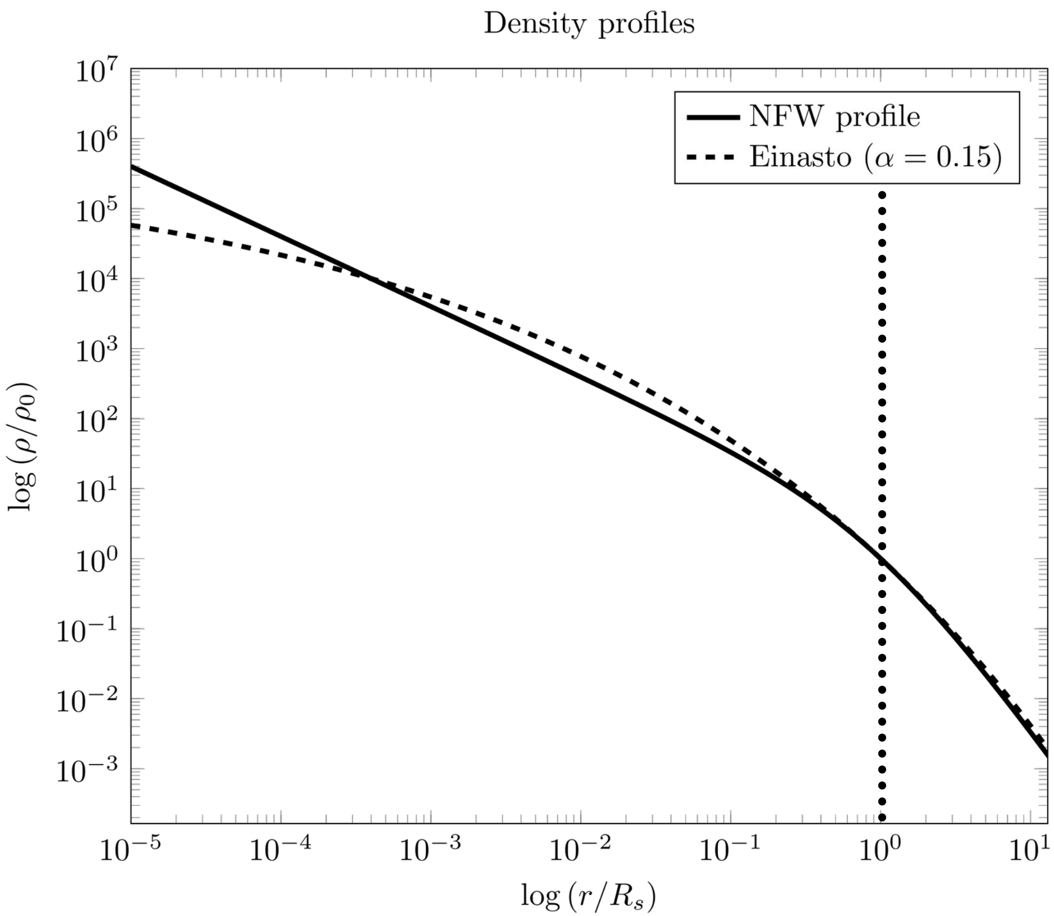


Navarro–Frenk–White (NFW) profile

A commonly used
dark matter halo model
empirically identified
in N-body simulations

$$\rho(r) = \frac{\rho_0}{\frac{r}{R_s}\left(1 + \frac{r}{R_s}\right)^2}$$

If dark matter exhibits
non-trivial interactions,
the **halo shape may vary**.



Self-interacting dark matter, wave dark matter ....

https://en.wikipedia.org/wiki/Navarro%E2%80%93Frenk%E2%80%93White_profile

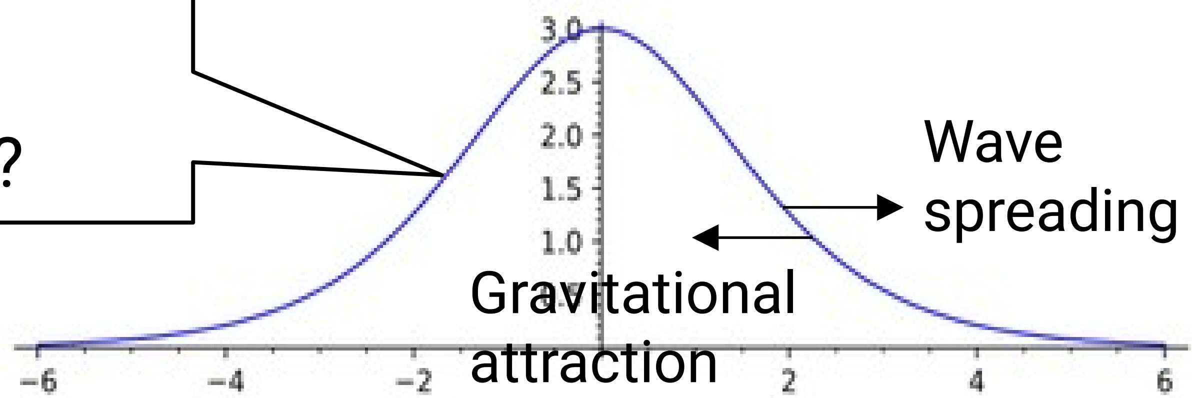
Example: Wave Dark Matter

If DM mass is so light (e.g. very light axions) so that

inter-particle spacing << de Broglie wavelength

DM exhibits **wave-like** behavior.

Nontrivial stable solution:
Soliton
→ Soliton in
dark matter halo?



Smoking gun signatures

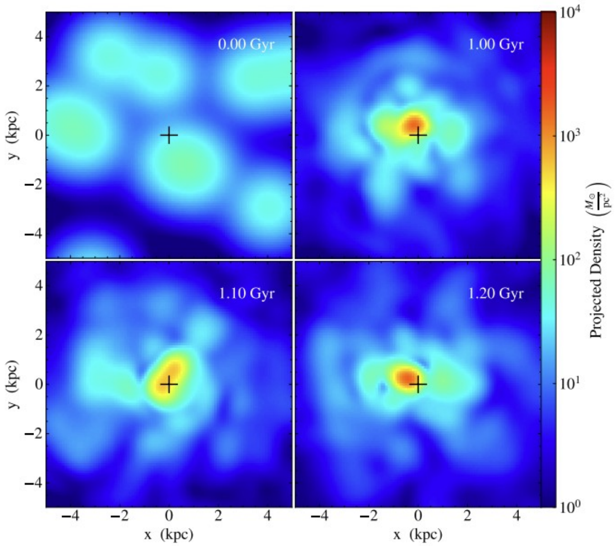
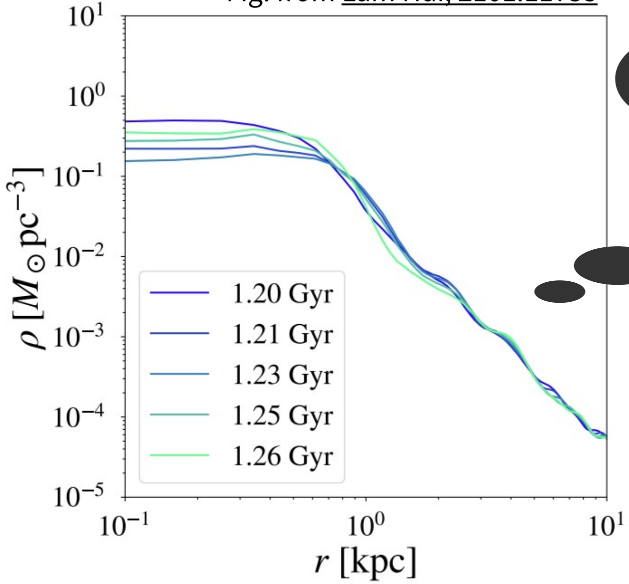


Fig. from Lam Hui, 2101.11735



Soliton Oscillations

Soliton Core

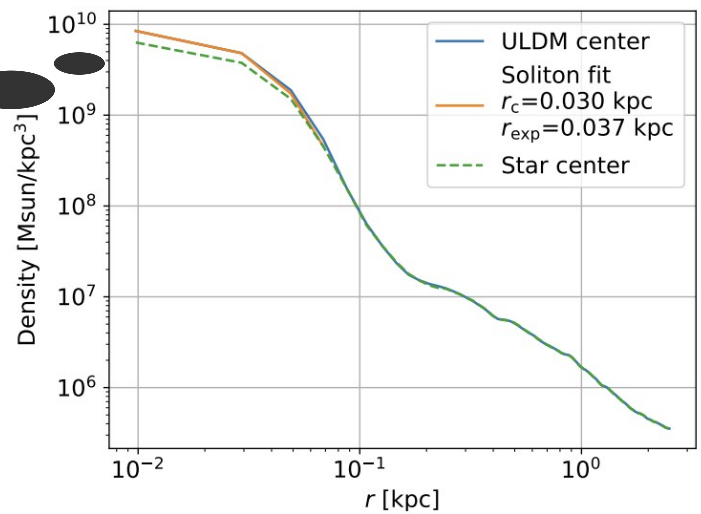
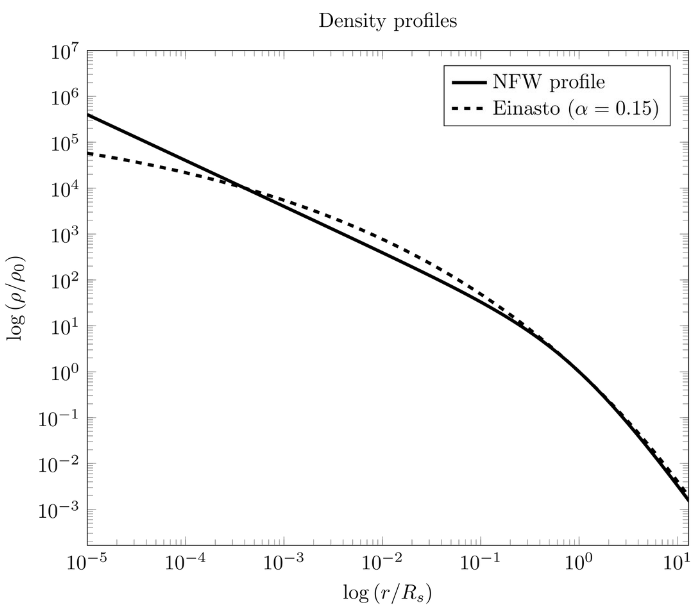
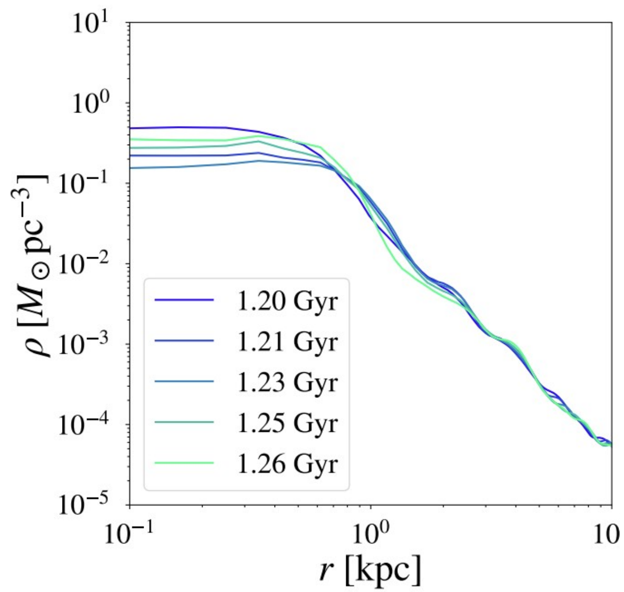
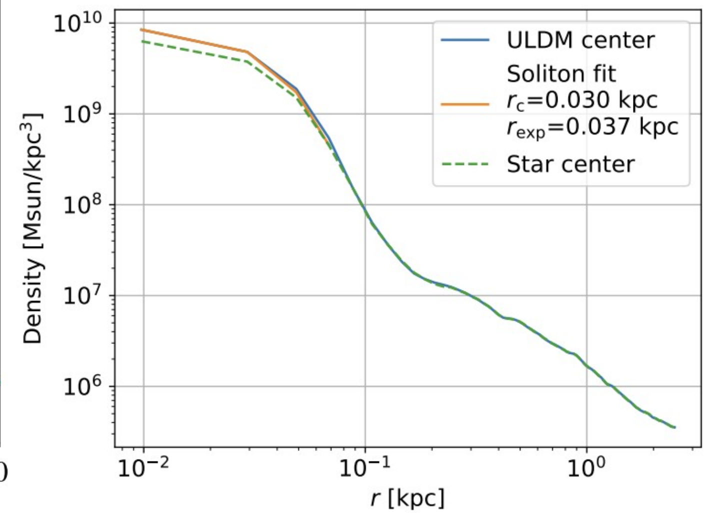


Fig. from talk by Teodori Luca, IBS Let there be light (particles) Workshop

Need for model-independent analysis





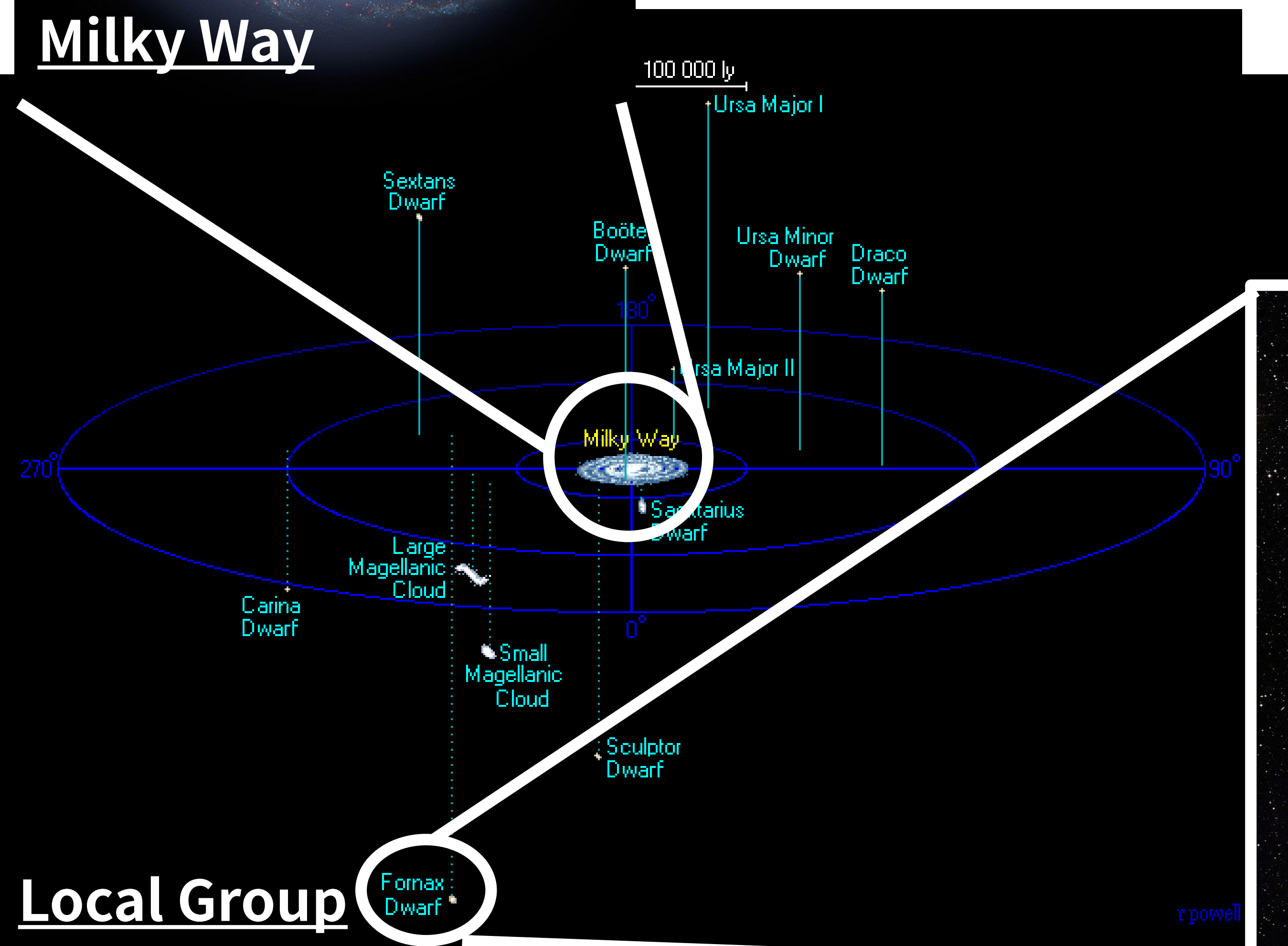


As many non-trivial DM halos are considered nowadays, we need a **free-form DM density estimation** in order to do a **model-indepdent** DM halo analysis.

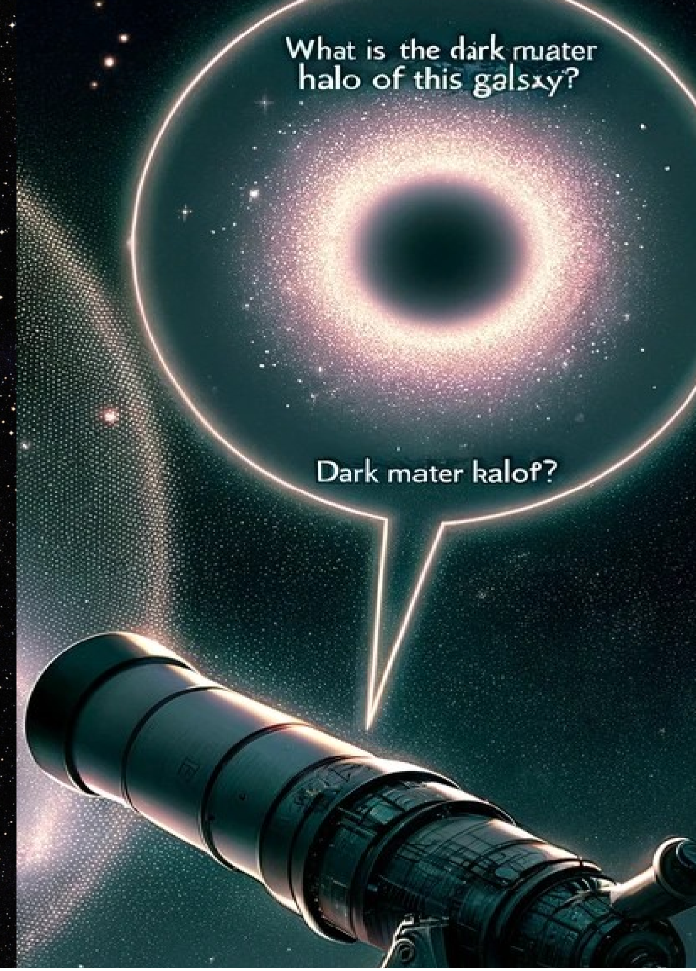
Again, unsupervised **machine learning** can help solving this type of problem!

Is the ML technique easily applicable to any of **distant dust-free galaxies, like dwarf spheroidal galaxy?**

Answer: both yes and no



http://www.atlasoftheuniverse.com/sattelit.html



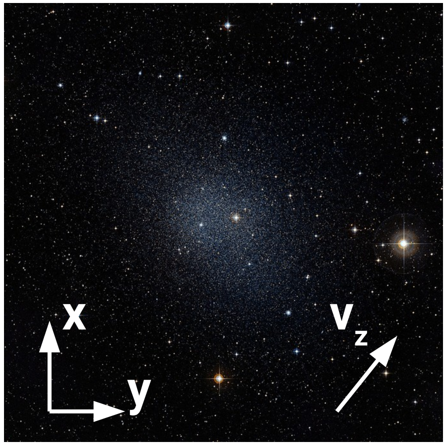
Model-Independent Spherical Jeans Analysis using Equivariant Continuous Normalizing Flows

Collaboration with
K. Hayashi (NIT, Sendai College), S. Horigome (IPMU),
S. Matsumoto (IPMU), M. M. Nojiri (KEK),

Challenges in Analyzing dSphs

- Faint galaxy
  → less number of observed stars O[100] ~ O[1000]

- Available kinematic information is **limited**!
  - Position of stars on the sky (x, y) (phot.)
  - ~~Distance to the stars (z)~~
  - ~~Proper motion of stars on the sky (v_x, v_y)~~
  - Radial velocity (v_z)   (spec.)



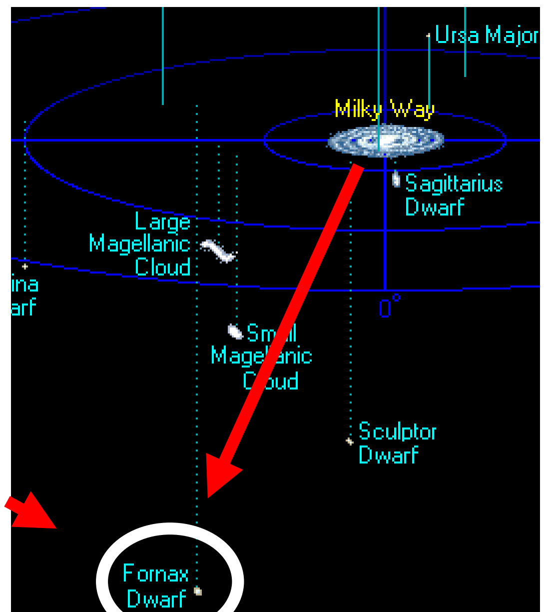
- Phase space density of stars are not accessible, and hence we cannot solve the equation of motion yet.. (Jeans equation)

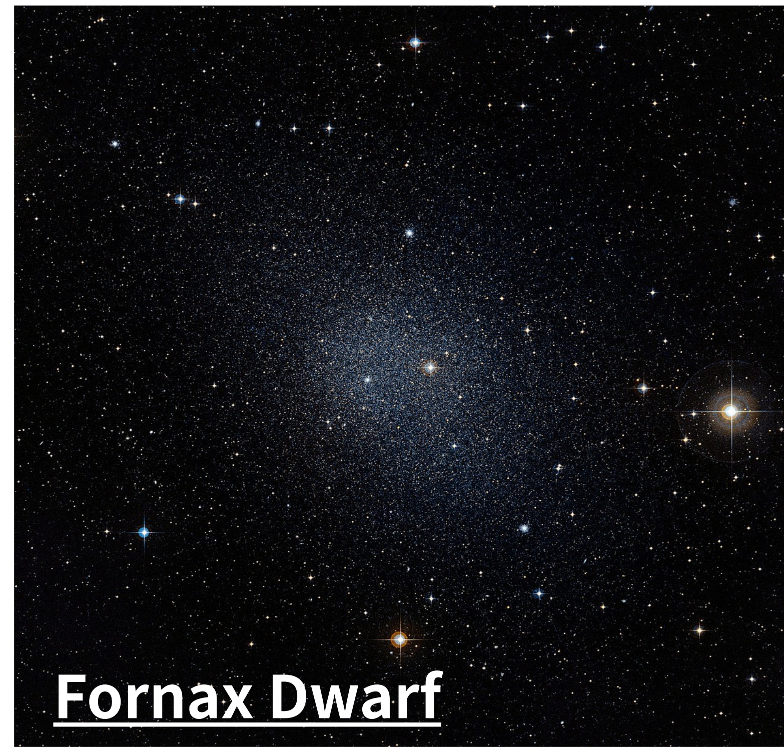
$$\frac{\partial n\langle v_j\rangle}{\partial t} + n\frac{\partial \Phi}{\partial x_j} + n\frac{\partial n\langle v_i v_j\rangle}{\partial x_i} = 0$$

Can we recover the full 6D information somehow?

Radon Transformation

- Can we recover the full 6D information somehow?
  → Yes, if we have a 3D projected snapshot of the dSph from all the direction





- This tomographic reconstruction is possible **(e.g. MRI imaging)**,
- but we only have a snapshot from only one direction…
  → Classic solution: assume **spherical symmetry**.

Spherical Jeans Equation

Introducing spherical symmetry simplifies the Jeans equation, too.

$$\frac{d}{dr} n\overline{v_r^2} + \frac{2\beta}{r} n\overline{v_r^2} = -n\frac{d\Phi}{dr}$$

List of functions needed for inferring gravitational field (Φ)

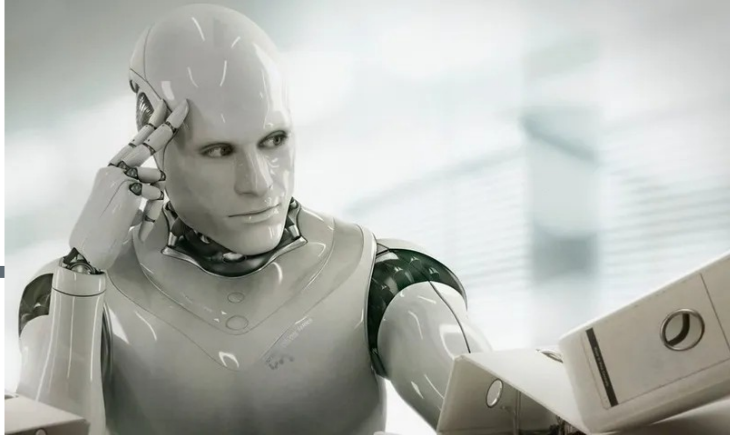
- Number density $n(r)$
- Radial velocity dispersion (variance) $\overline{v_r^2}(r)$
- Velocity anisotropy

$$\beta(r) = 1 - \frac{\overline{v_\theta^2}(r) + \overline{v_\phi^2}(r)}{2\overline{v_r^2}(r)}$$

Note: velocity anisotropy cannot be determined only using line-of-sight velocity distribution, we will provide the function (can be true or not) by hand.

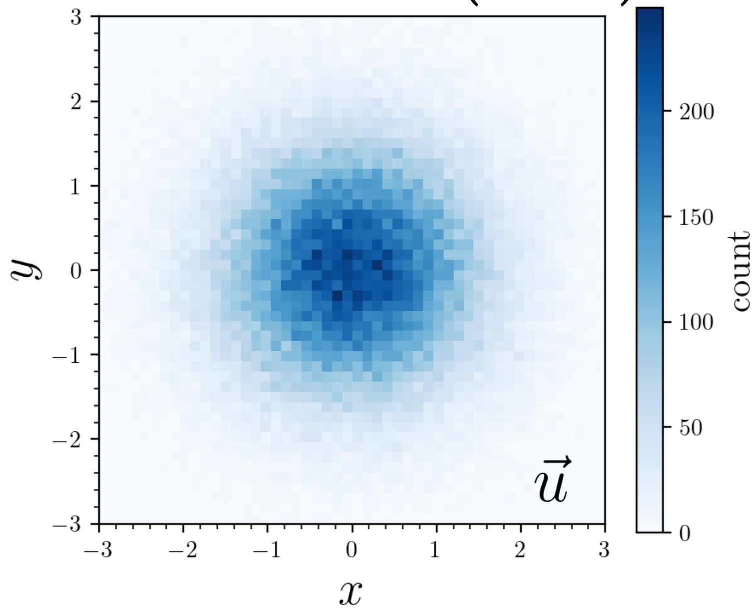
Need to estimate 2 functions from data: $n(r)$ $\overline{v_r^2}(r)$

Normalizing Flows: Neural Density Estimator



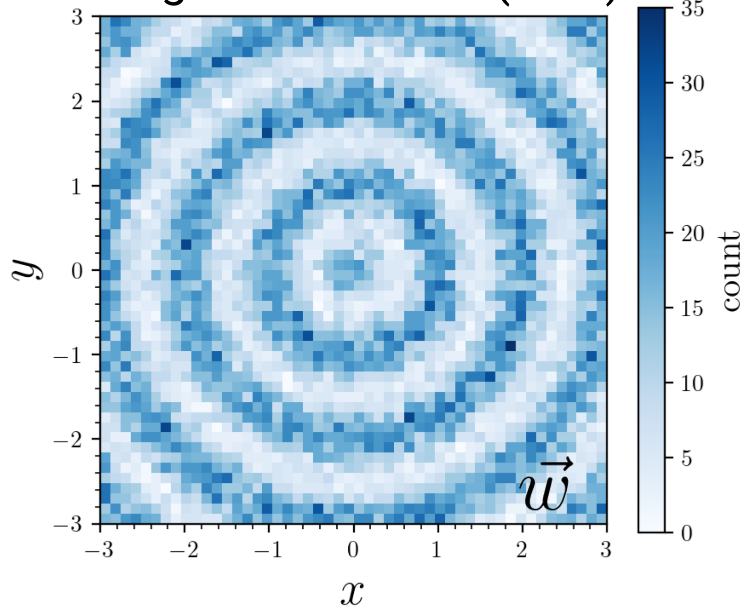
<u>Normalizing Flows</u> (NFs) is an artificial neural network that learns a transformation of random variables.

Base distribution (known)



NF

Target distribution (data)



Main idea: if we could find out such transformation, we can use the transformation formula for the density estimation:

$$p_W(\vec{w}) = p_U(\vec{u}) \cdot \left| \frac{d\vec{u}}{d\vec{w}} \right|$$

We will use this model for estimating the phase space density f(x,v) from the data.

Equivariant Continous Normalizing Flows

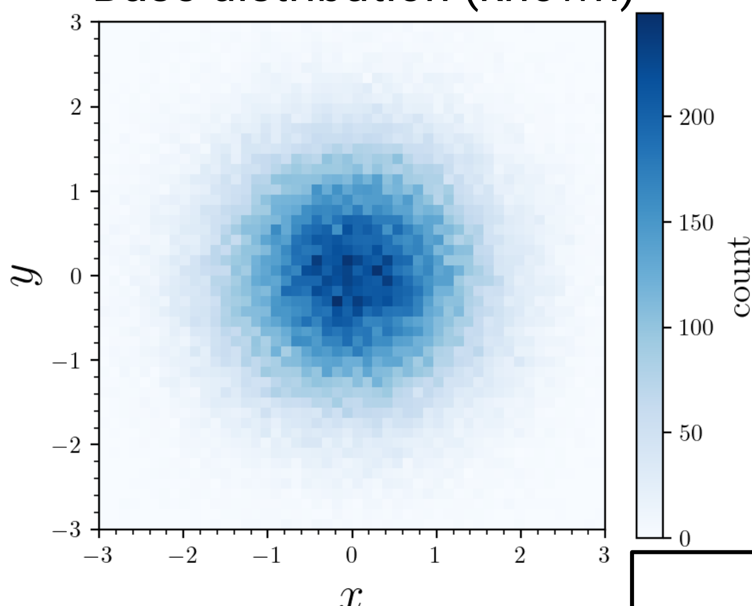
$n(r)$

How to model spherically symmetric density using normalizing flows?
→ Use Equivariant Continuous Normalizing Flows!

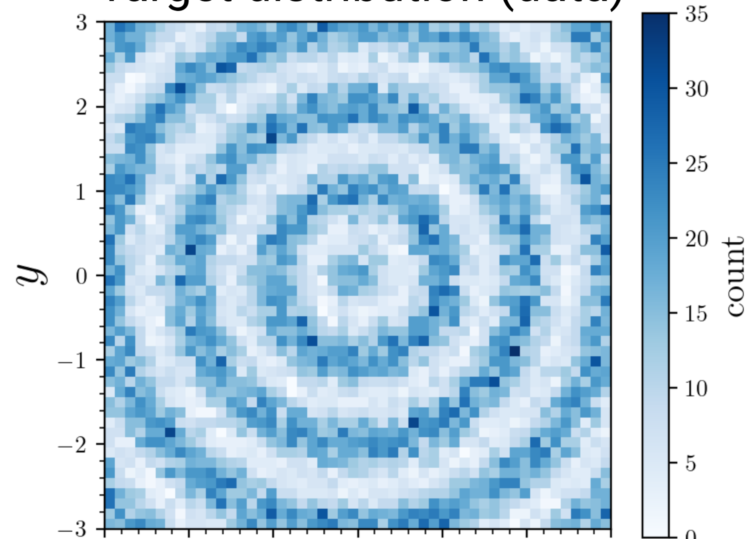
$$\frac{d\vec{x}}{dt} = \vec{F}(\vec{x}, t) \longrightarrow \frac{d\vec{x}}{dt} = \hat{r} f(\vec{x}, t)$$

- Invariant (Gaussian) base distribution
- Equivariant vector field

Base distribution (known)



Target distribution (data)

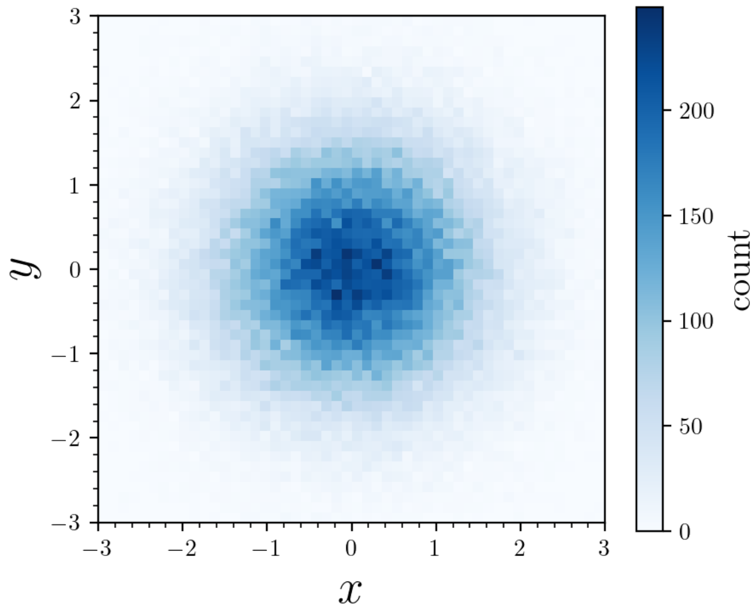


This setup is very flexible. You may add **physics constraints** to **neural networks**, too!

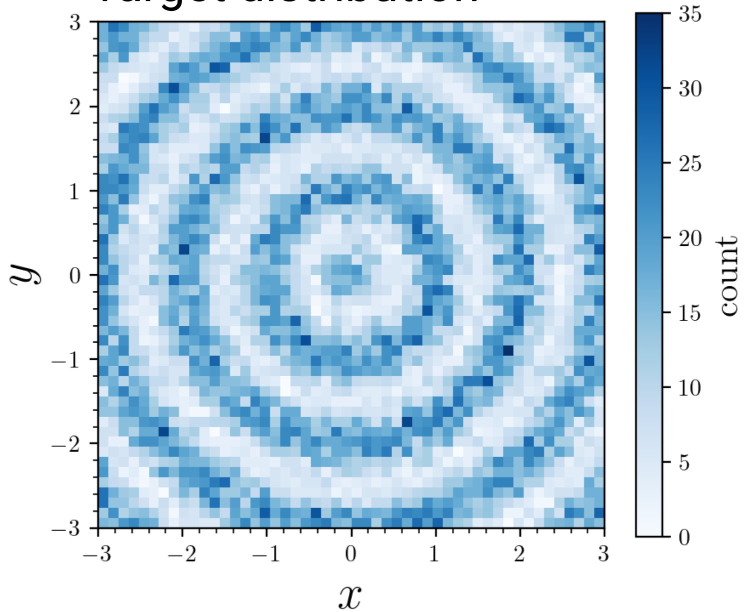
Normalizing Flows: How it works?

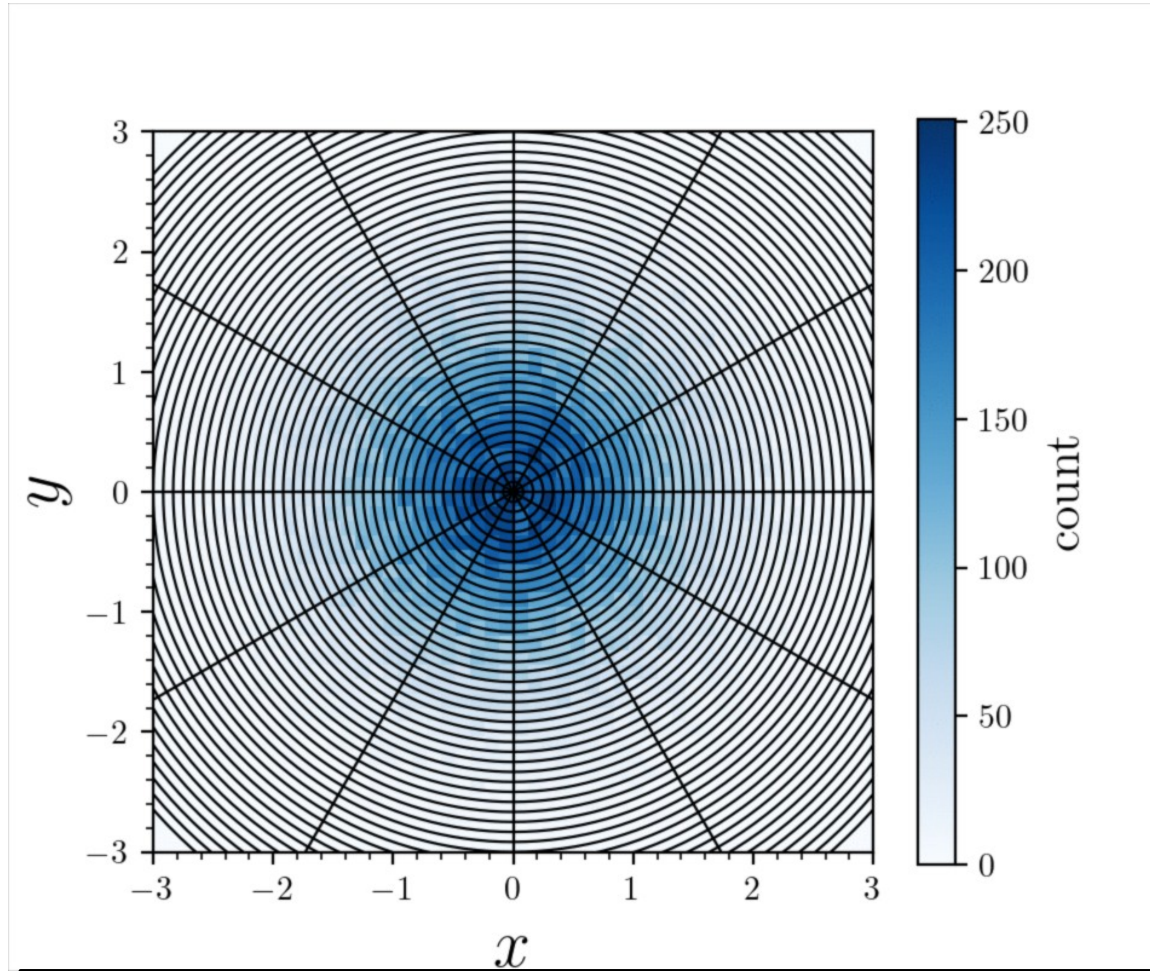
$n(r)$

Base distribution



Target distribution





Normalizing flows can fit **arbitrary** probability density, suitable for **model-independent** analysis!

* result of a continuous normalizing flow learning infinitesimal transformations

Cored Spherical Density Model

In dSph analysis, we may further constrain the density model as conventional analysis often only consider the following type of densities.

- **Cored** density (constant density at r << 0)
- **Cuspy** density

ex) plummer sphere:

$$p(r) = \left(1 + \frac{r^2}{r_0^2}\right)^{-5/2}$$

Equivariant CNF for modeling cored density profile

$$\frac{d\vec{x}}{dt} = \hat{r} f(\vec{x}, t) \longrightarrow \frac{d\vec{x}}{dt} = \hat{r} \tanh\left(\frac{|\vec{x}|}{r_0}\right) f(\vec{x}, t)$$

Transformation at the origin is suppressed, remaining as Gaussian-shape. → cored density

Cuspy Spherical Density Model

In dSph analysis, we may further constrain the density model as conventional analysis often only consider the following type of densities.

- **Cored** density (constant density at r << 1)
- **Cuspy** density

Equivariant CNF for modeling cuspy density profile

ex) NFW profile:

$$p(r) = \left(\frac{r}{r_0}\right)^{-1}\left(1+\frac{r}{r_0}\right)^{-2} \to \frac{1}{r}$$

Apply power-law transform to radial component

$$|r| \to |r|^{c+1} \qquad \text{Jacobian} \propto r^{-\frac{3c}{1+c}}$$

to **cored** spherical symmetric density model

Velocity Dispersion Estimation

The velocity dispersion can be simply estimaed using Gaussian model conditioned on position, as the MLE on variance parameter of Gaussian is a variance estimator.

$$\Sigma(r;\theta) = \begin{pmatrix} \overline{v_r^2}(r;\theta) & 0 & 0 \\ 0 & \overline{v_\theta^2}(r;\theta) & 0 \\ 0 & 0 & \overline{v_\phi^2}(r;\theta) \end{pmatrix}$$

Note that only radial velocity dispersion is modeled by a neural network, others are given by velocity anisotropy function provided.

$$\overline{v_\theta^2}(r;\theta) = \overline{v_\phi^2}(r;\theta) = \overline{v_r^2}(r;\theta) \cdot (1 - \beta(r))$$

Here is a 6D density model, but...

Now we have a full 6D phase-space density model ready for solving spherical Jeans equation.

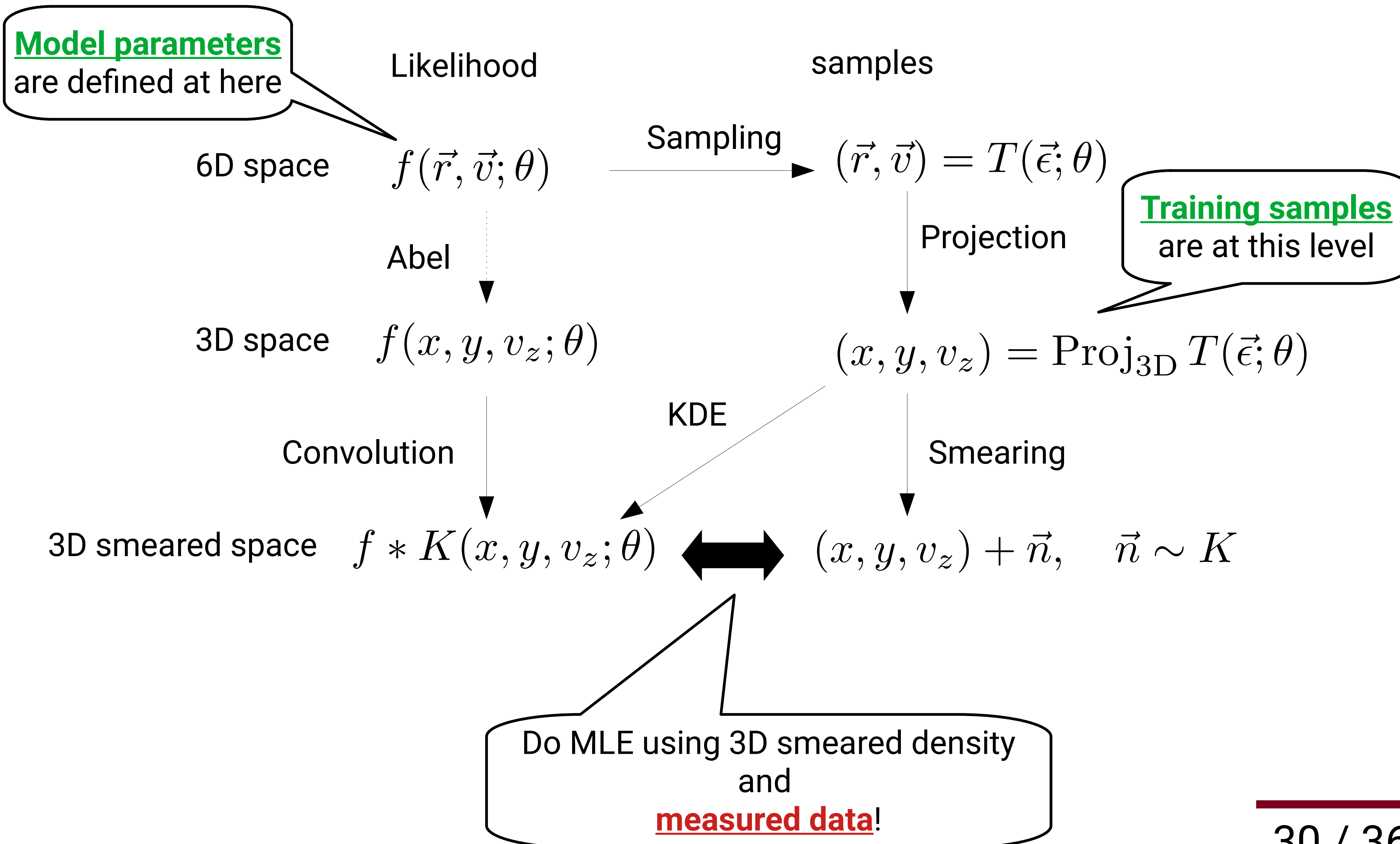
$p(\vec{r}) = n(r;\theta)$ modeled by equivariant CNF for cuspy halos

$$p(\vec{v}|\vec{r}) = \mathrm{GaussPDF}(\vec{v}; \mu = 0, \Sigma(r;\theta))$$

$$f(\vec{r}, \vec{v}) = p(\vec{r}) \times p(\vec{v}|\vec{r})$$

Wait, we only have x, y, vz.
How can we train this network by MLE?
We cannot use a conventional loss function.

How to train this model?



Loss Function for Modeling Dwarf Spheroidal Galaxy

- In order to train the normalizing flow with spherical symmetry using limited kinematic information, we minimize the following entropy:

$$\mathcal{L}(\theta) = \int d\vec{w}_\perp \; p * K_h(\vec{w}_\perp) \; \log \hat{p} * K_h(\vec{w}_\perp; \theta)$$

- Importance sampling: N_T training sample (stars) ~ p, N_K noise samples ~ K_h

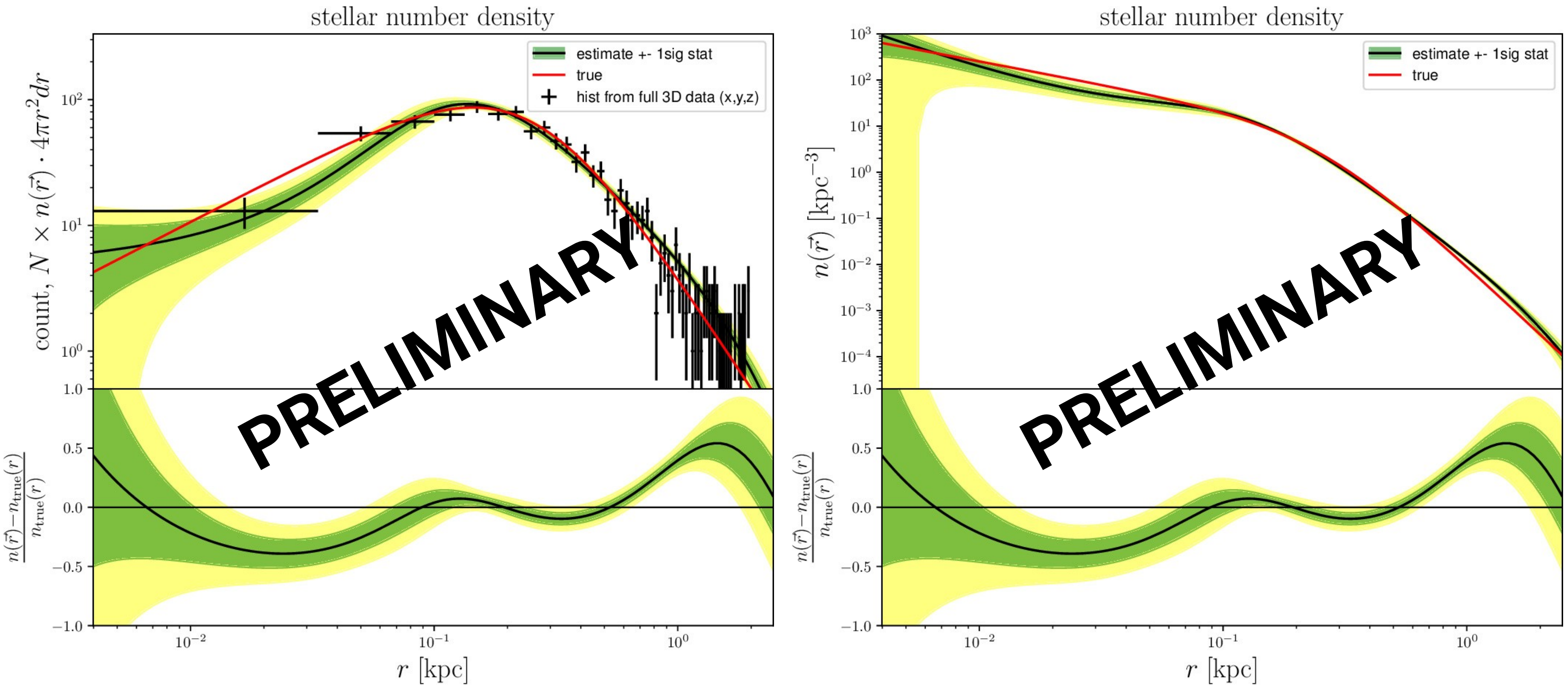
$$\mathcal{L}(\theta) = \frac{1}{N N_K} \sum_{a=1}^{N} \sum_{b=1}^{N_K} \log \hat{p} * K_h(\vec{w}_\perp^{(a)} + \vec{\epsilon}^{(b)}; \theta)$$

- KDE for the smeared likelihood model:
  N_G generated stars from the normalizing flows~ \hat{p}

$$\mathcal{L}(\theta) = \frac{1}{N N_K} \sum_{a=1}^{N} \sum_{b=1}^{N_K} \log \frac{1}{N_G} \sum_{c=1}^{N_G} K_h \left[\vec{w}_\perp^{(a)} + \vec{\epsilon}^{(b)} - \vec{T}(\vec{z}^{(c)}; \theta) \right]_\perp$$

Results: stellar number density

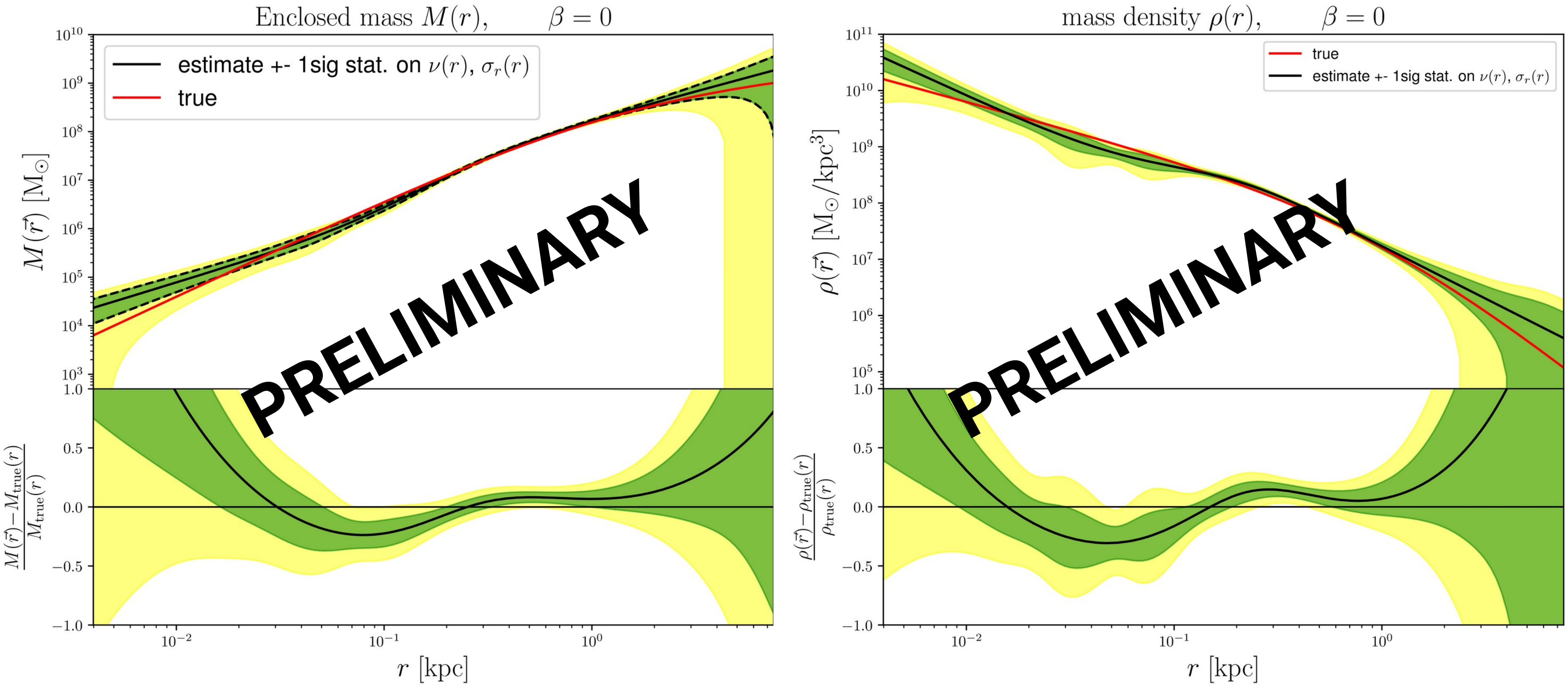
Here we present inferred stellar number density trained on 2D position information (x, y).



Dataset: simulated dwarf spherodal galaxy from Gaia Challenge Dataset
https://astrowiki.surrey.ac.uk/doku.php?id=tests:sphtri

Results: dark matter mass density

Here we present inferred mass density calculated from
stellar density and velocity dispersion trained on 3D information (x, y, vz).



Dataset: simulated dwarf spherodal galaxy from Gaia Challenge Dataset
https://astrowiki.surrey.ac.uk/doku.php?id=tests:sphtri

Conclusions

- We introduce a model-independent and unbinned spherical Jeans analysis using **normalizing flows**, a neural density estimator utilizing transformation of random variables.
- We **invented a loss function** for training normalizing flows modeling dSphs only using projected information, without performing Abel transformation.
- Using a mock spherical galaxy from Gaia Challenge dataset, we demonstrated that normalizing flows are capable of estimating **phase-space density** information for required solving Jeans equation.
- To do?:
  - Generalizing the framework to axisymmetric system.
  - Applying our analysis to real dwarf spheroidal galaxies, and estimate the effect to J-factors when the assumptions are relaxed.

AI+HEP in East Asia

24–28 Feb 2025
IBS
Asia/Seoul timezone

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- Visa Information
- Code of Conduct

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Registration is open :D
https://indico.ibs.re.kr/event/789/

This regional workshop aims to connect researchers in East Asia working in the interdisciplinary field of Artificial Intelligence and High Energy Physics (AI+HEP). The main topics covered include machine learning for particle theory, phenomenology and experiments, astrophysics and cosmology, as well as HEP tools for AI theory.

The workshop will have invited plenary talks, contributed presentations, and ample time for discussions. Both domain experts and those who are interested in exploring the field are welcome to participate, especially postdocs and graduate students. The goal is to foster a regional research community and to stimulate more collaborations.

Invited Speakers:

- Cheng-Wei Chiang (National Taiwan University (NTU))
- Ahmed Hammad (KEK)
- Ji-hoon Kim (Seoul National University)
- Congqiao Li (Peking University)
- Vinicius Mikuni (NERSC, Berkeley Lab)
- Masahiro Morinaga (ICEPP, University of Tokyo)
- Myeonghun Park (Seoultech)
- to be updated

Thank you
for listening!