More is Different: Multi-Axion Dynamics, String Bundles, and a Solution to the QCD Axion Domain Wall Problem

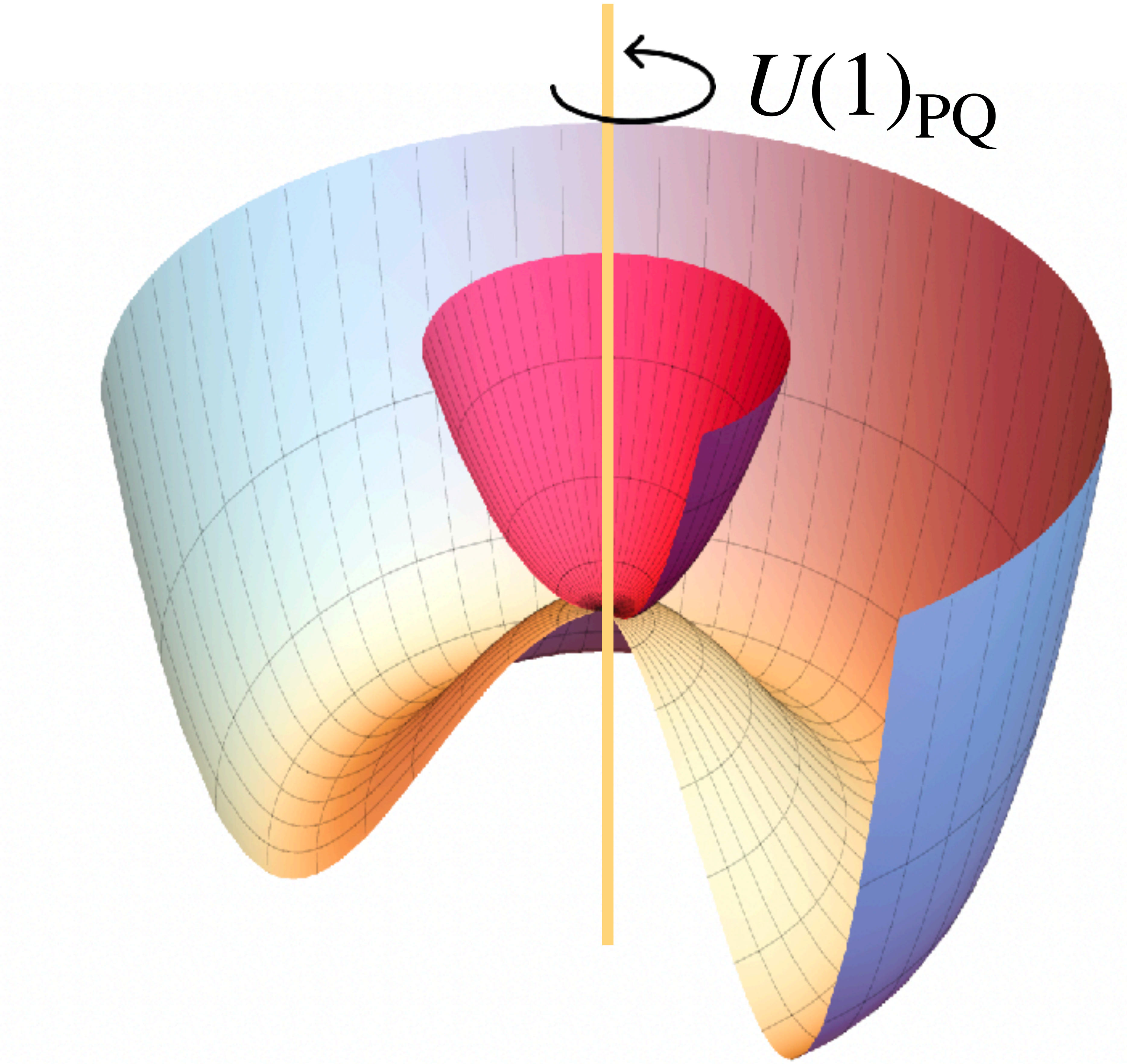
September 4, 2025@KIAS seminar

Fumi Takahashi (Tohoku University)

Based on Lee, Murai, FT and Yin 2409.09749, 2407.09478, 2507.07075

QCD axion

The QCD axion is a pseudo Nambu-Goldstone boson associated with SSB of U(1) Peccei-Quinn symmetry.



Peccei, Quinn `77, Weinberg `78, Wilczek `78

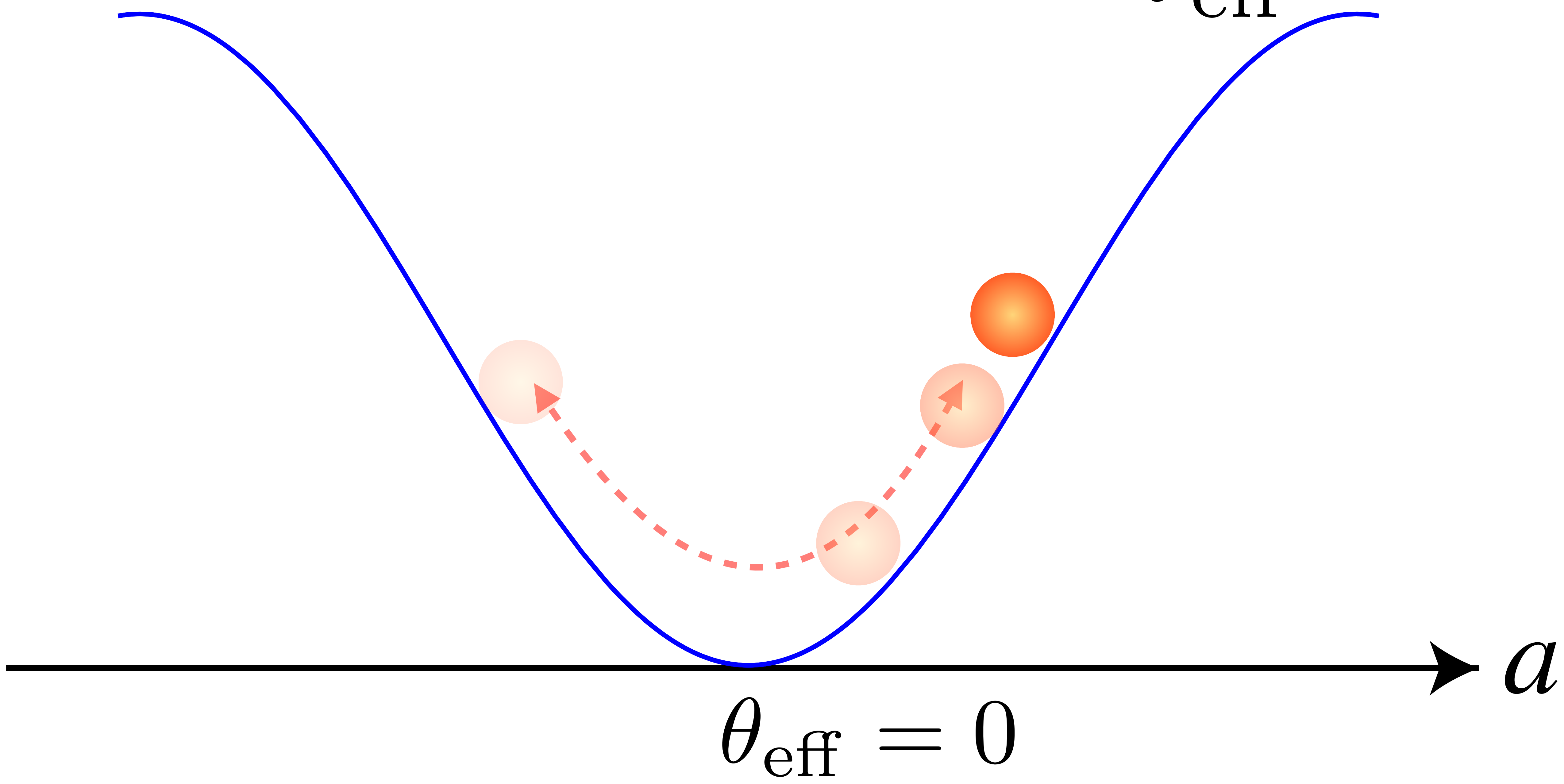
QCD axion

Dynamically solves the strong CP problem.

$$\mathcal{L}_\theta = \theta \frac{g_s^2}{32\pi^2} G^{a\mu\nu}\tilde{G}^a_{\mu\nu} \;\Rightarrow\; \mathcal{L}_\theta = \underbrace{\left(\theta + \frac{a}{f_a}\right)}_{\theta_{\rm eff}} \frac{g_s^2}{32\pi^2} G^{a\mu\nu}\tilde{G}^a_{\mu\nu}$$

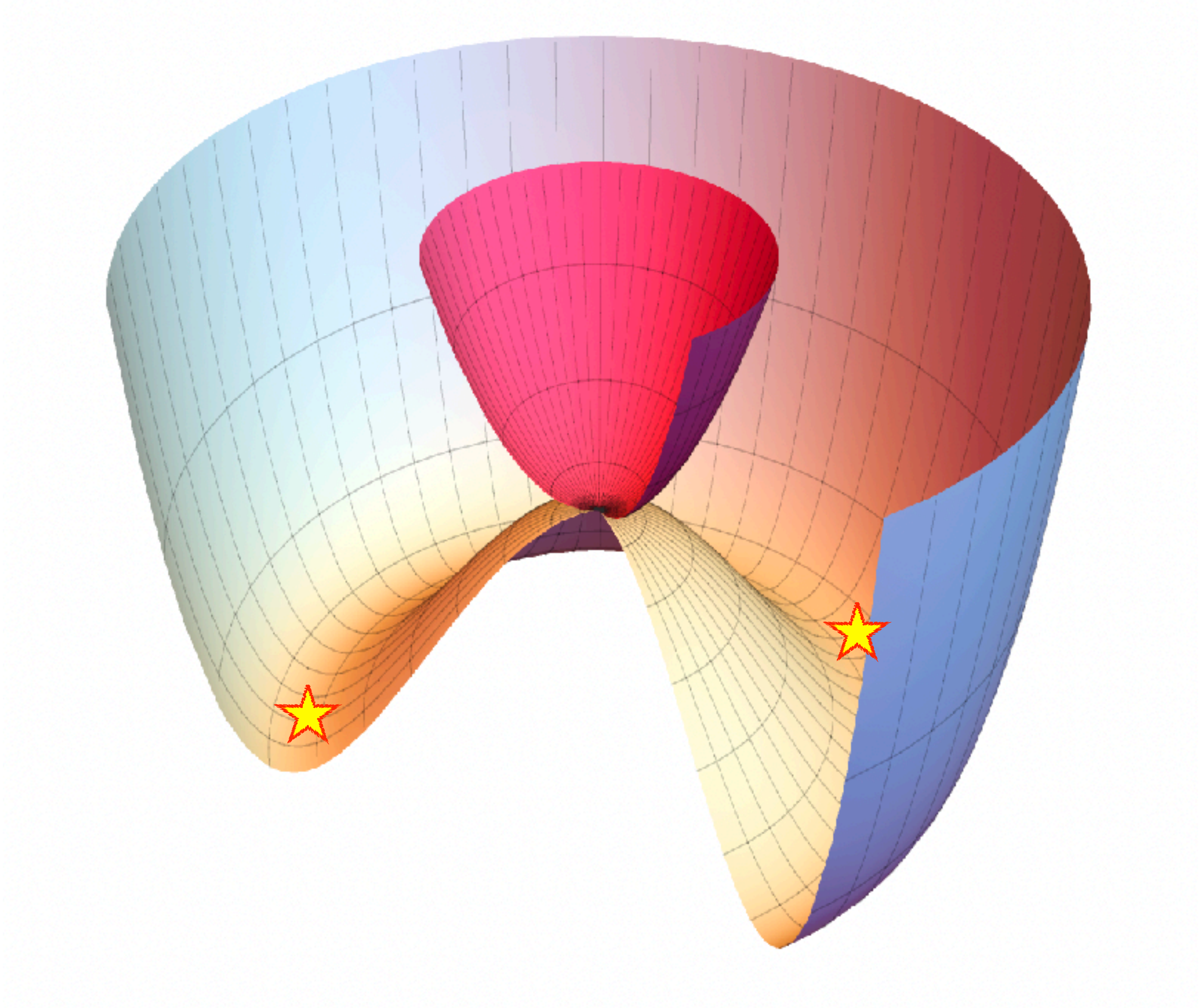
$|\bar{\theta}| \lesssim \mathcal{O}(10^{-10})$: strong CP problem

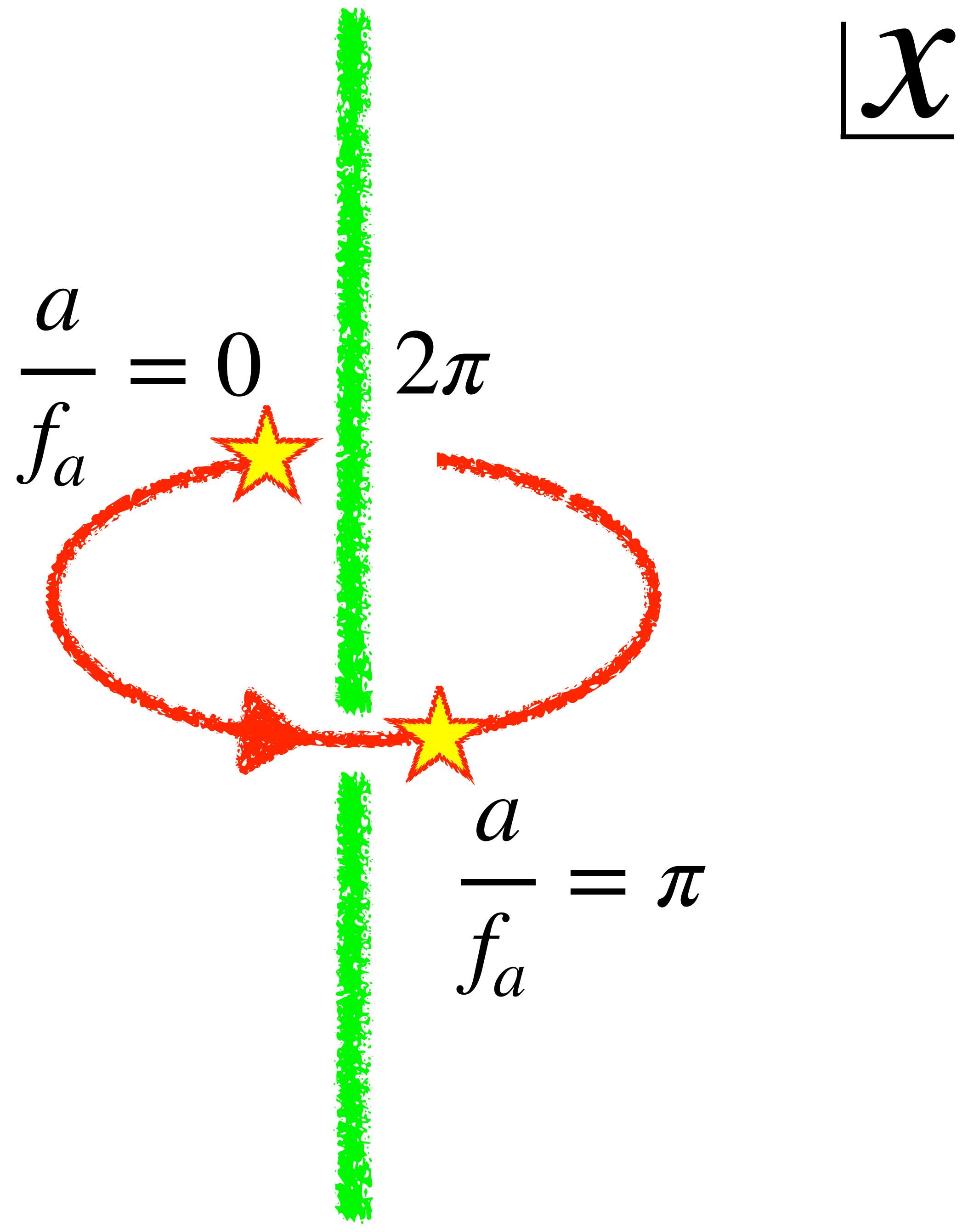
Neutron EDM,
Abel et al, 2001.11966



Axion production from strings/walls

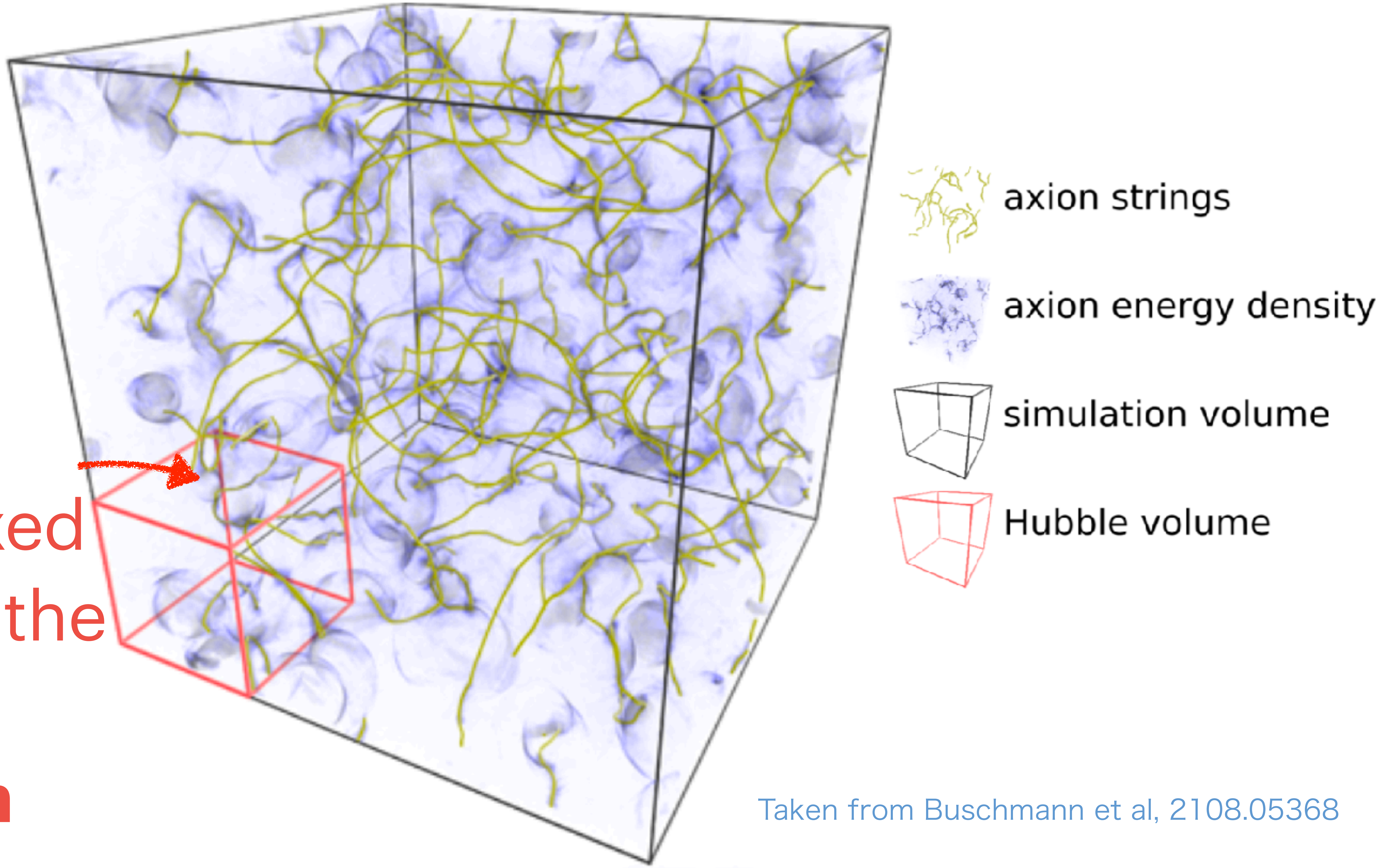
Cosmic strings and domain walls, formed in post-inflationary scenarios, produce axion dark matter.





Cosmic strings

Axion production from strings/walls



(Approximately) fixed numer of strings in the Hubble volume:
Scaling solution

Taken from Buschmann et al, 2108.05368

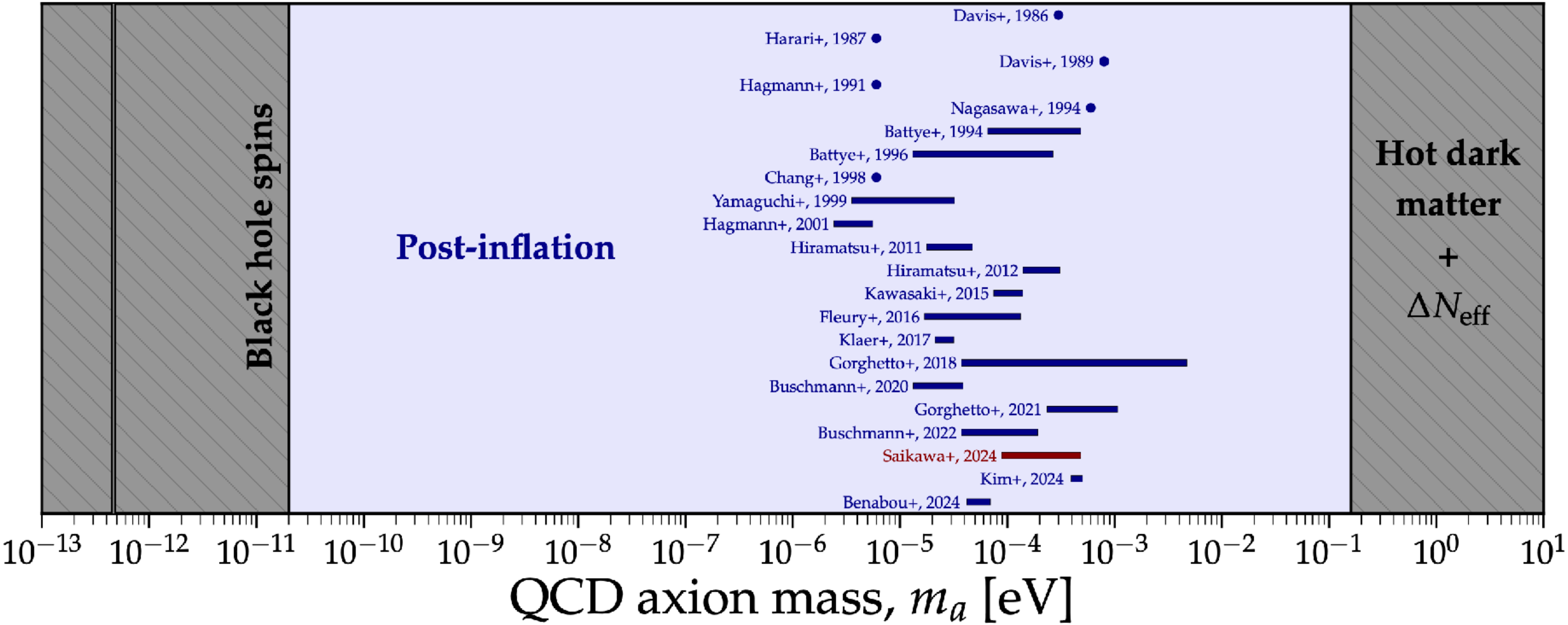
Axion production from strings/walls

$N_{\rm DW} = 1$

Hiramatsu, Kawasaki, Saikawa, Sekiguchi,, 1202.5851

Studying the evolution of strings and domain walls is crucial for predicting the mass of axion dark matter.

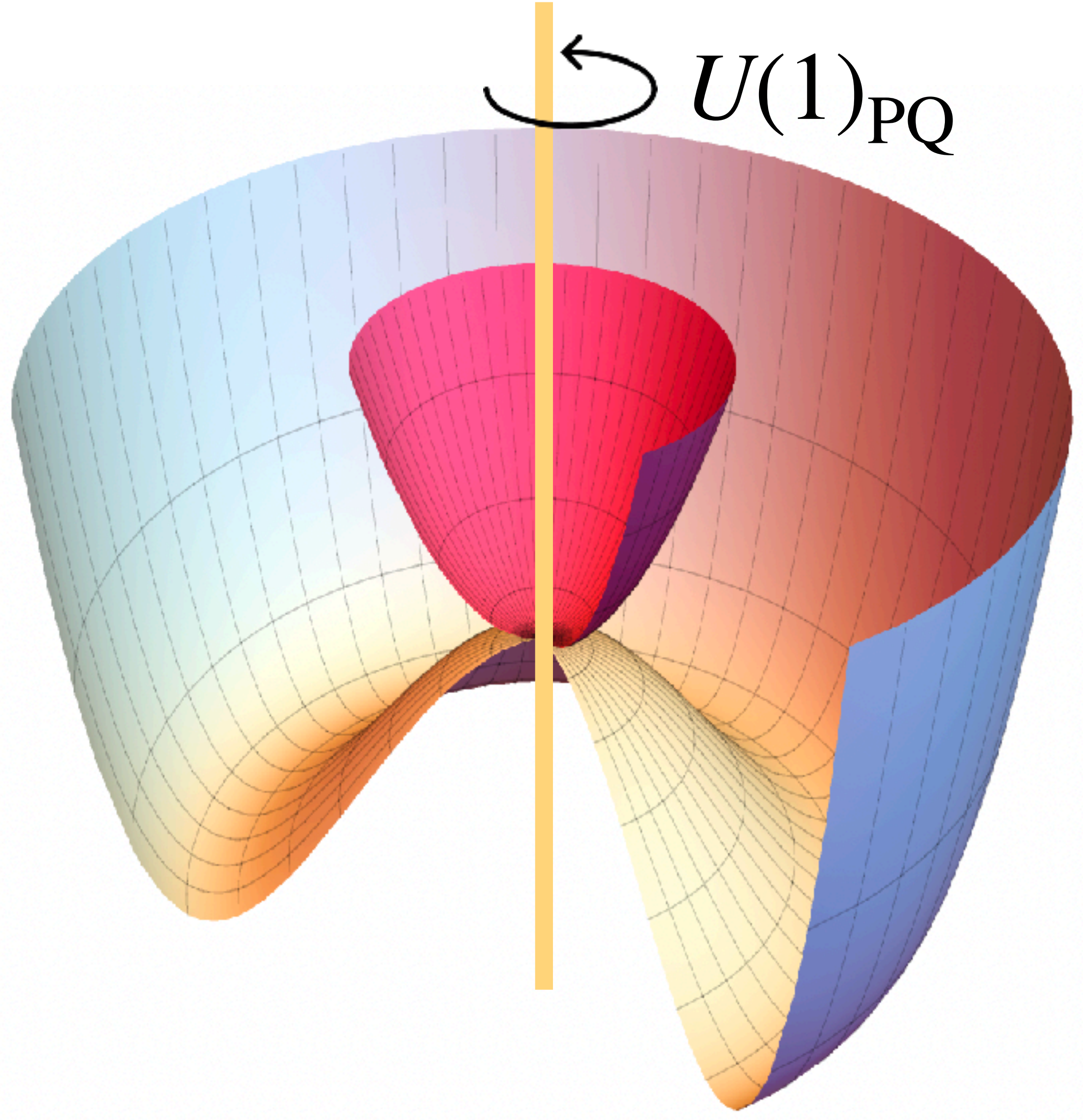
Prediction of post-inflationary scenario



Kaltschmidt, Redondo, Saikawa and Vaquero, arXiv:2502.02398

Key assumption :

Single PQ scalar model

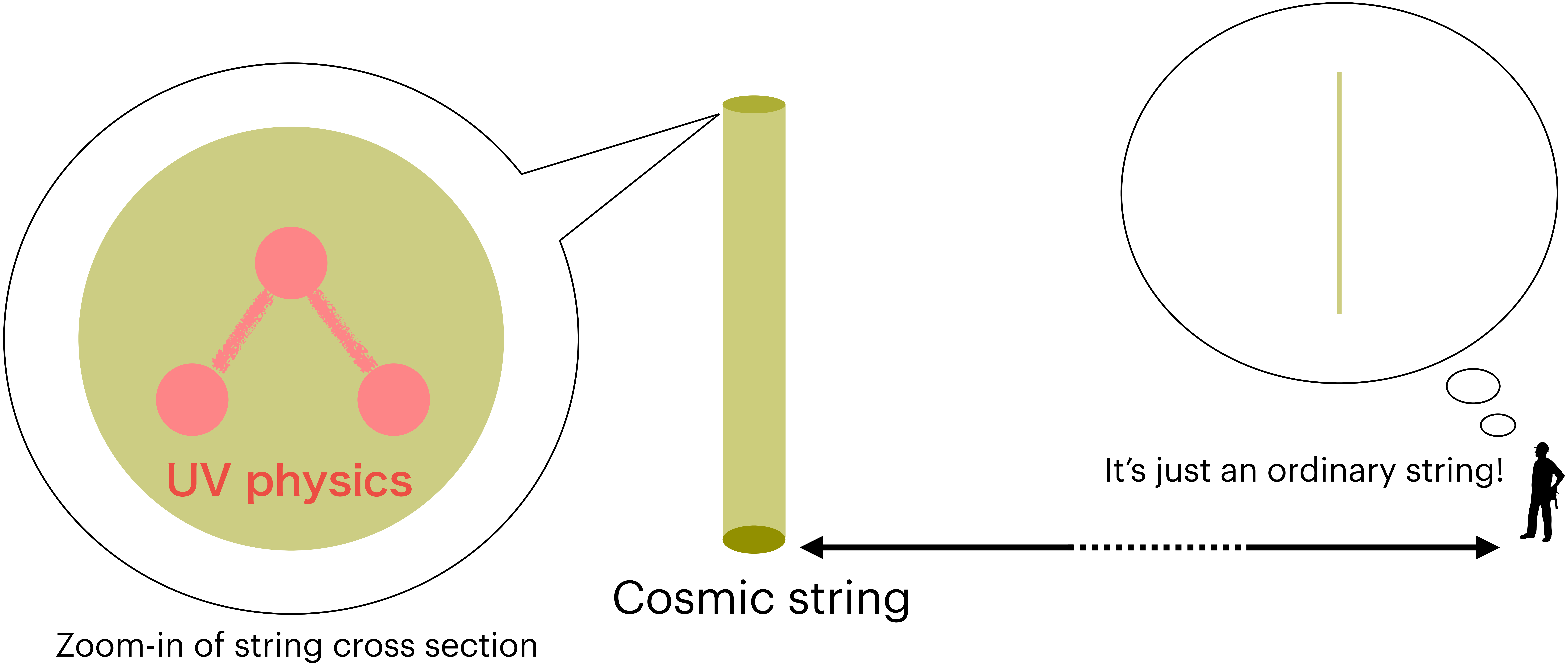


$$\Phi = \frac{f_a + s}{\sqrt{2}} e^{ia/f_a}$$

- Prevalent in high-precision lattice calculations.
- Simplifying assumption or crucial factor? 🤔

Origin and breaking of U(1) PQ are unknown!

Is UV physics always confined in the string core?



Two axion system

We introduce two PQ scalars

$$\Phi_1 = \frac{f_1}{\sqrt{2}} e^{i\frac{\phi_1}{f_1}} \quad \text{and} \quad \Phi_2 = \frac{f_2}{\sqrt{2}} e^{i\frac{\phi_2}{f_2}}$$

$\theta_1 \equiv \phi_1/f_1$ and $\theta_2 \equiv \phi_2/f_2$

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$\theta_1 \equiv \phi_1/f_1$ and $\theta_2 \equiv \phi_2/f_2$

and the potential for axions

$$V_1(\phi_1, \phi_2) = \Lambda^4 \left[1 - \cos\left(n_1 \frac{\phi_1}{f_1} + n_2 \frac{\phi_2}{f_2}\right)\right]$$

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$$V_2(\phi_1, \phi_2) = \Lambda'^4 \left[1 - \cos\left(n'_1 \frac{\phi_1}{f_1} + n'_2 \frac{\phi_2}{f_2} + \alpha\right)\right]$$

$$\Lambda \gg \Lambda'$$

$n_1, n_2, n'_1, n'_2 \in \mathbf{Z}$

with the post-inflationary initial condition.

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$$\Lambda \gg \Lambda'$$

$n_1, n_2, n_1', n_2' \in \mathbf{Z}$

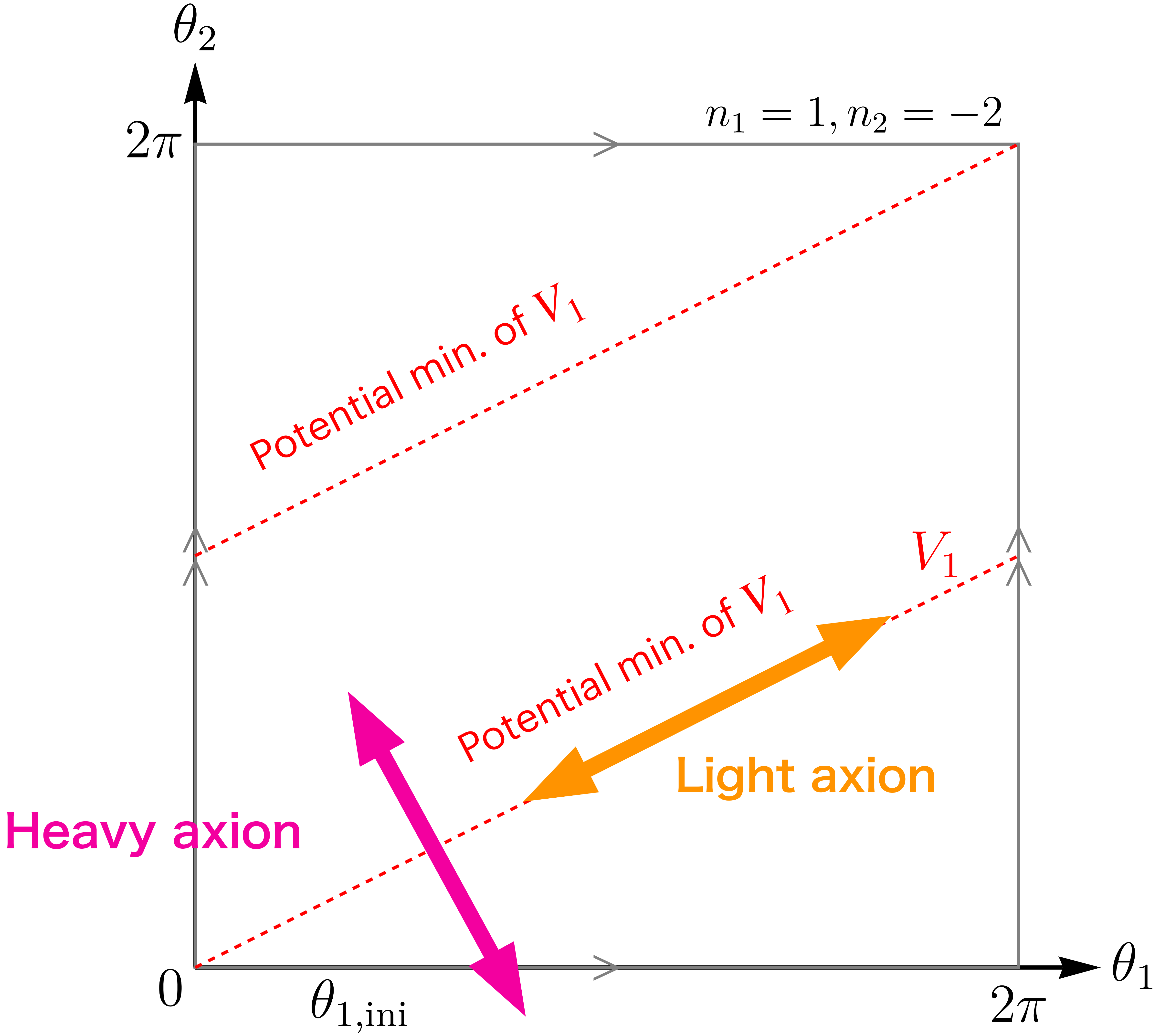
with the post-inflationary initial condition.

Two axion system

$$V_1(\phi_1, \phi_2) = \Lambda^4 \left[1 - \cos\left(n_1 \frac{\phi_1}{f_1} + n_2 \frac{\phi_2}{f_2}\right)\right]$$

One linear combination of two axions becomes heavy, leaving the orthogonal one (nearly) massless.

$$\phi_{\rm heavy} \propto n_1 \frac{\phi_1}{f_1} + n_2 \frac{\phi_2}{f_2}$$



Two types of strings and multiple DWs

Both ϕ_1, ϕ_2-strings quickly reach the scaling solution, and when V_1 becomes relevant, n_1 (n_2) domain walls appear, attached to the $\phi_1(\phi_2)$-string.

① ②

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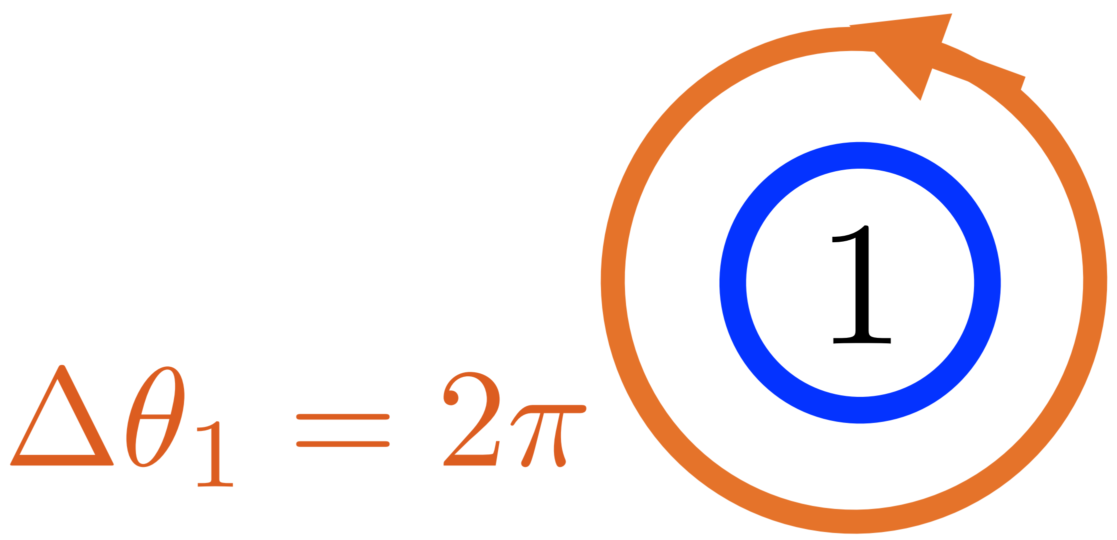

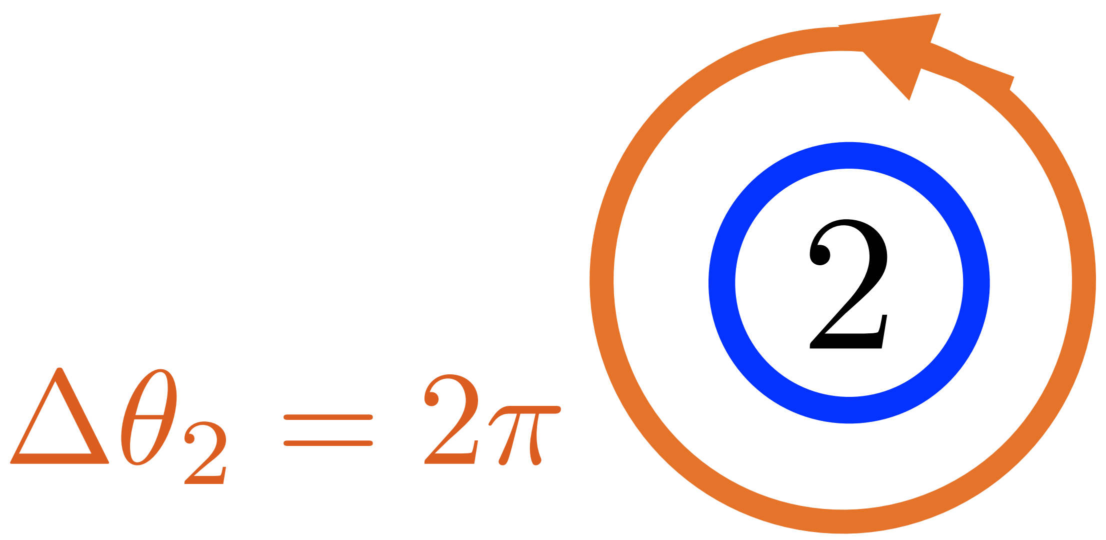

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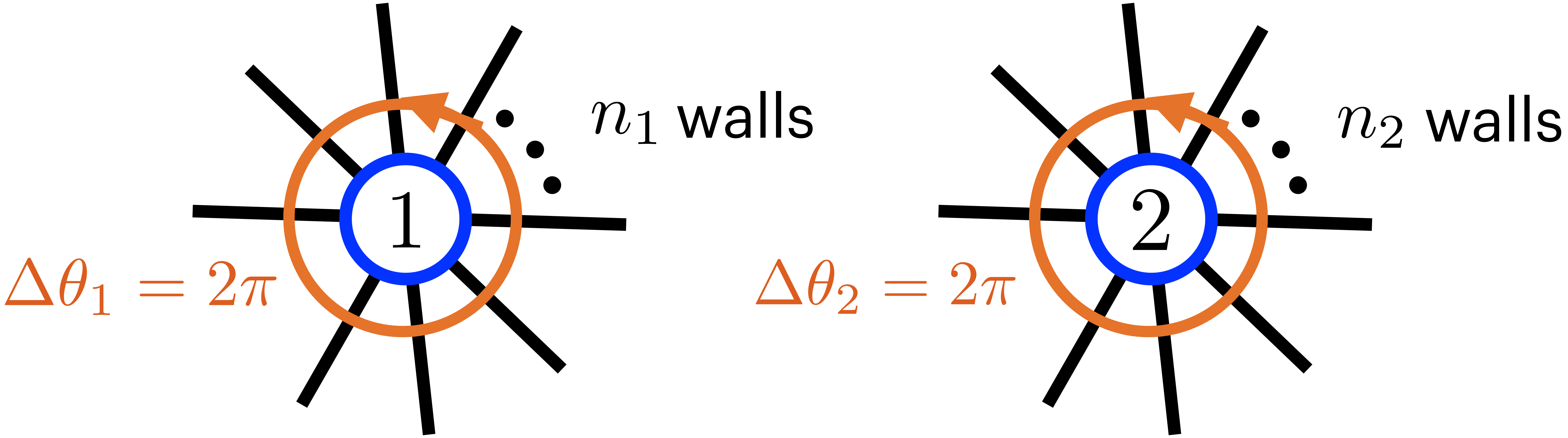

$$V_1(\phi_1, \phi_2) = \Lambda^4 \left[1 - \cos\left(n_1 \frac{\phi_1}{f_1} + n_2 \frac{\phi_2}{f_2}\right)\right] \qquad n_1, n_2 \in \mathbf{Z}$$

Case of $(n_1, n_2) = (2, 1)$

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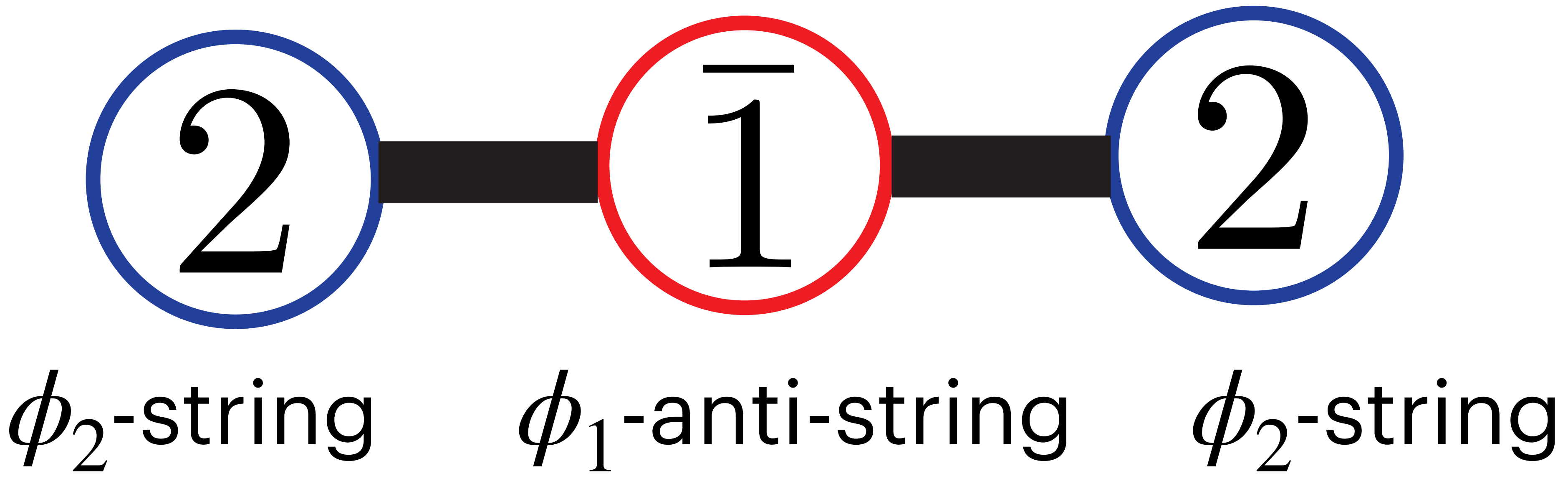

String bundle

(= ordinary cosmic string)

cf. Higaki, Jeong, Kitajima, Sekiguchi and FT, 1606.05552,
See also Eto, Hiramatsu, Saito and Sakakihara, 2309.04248

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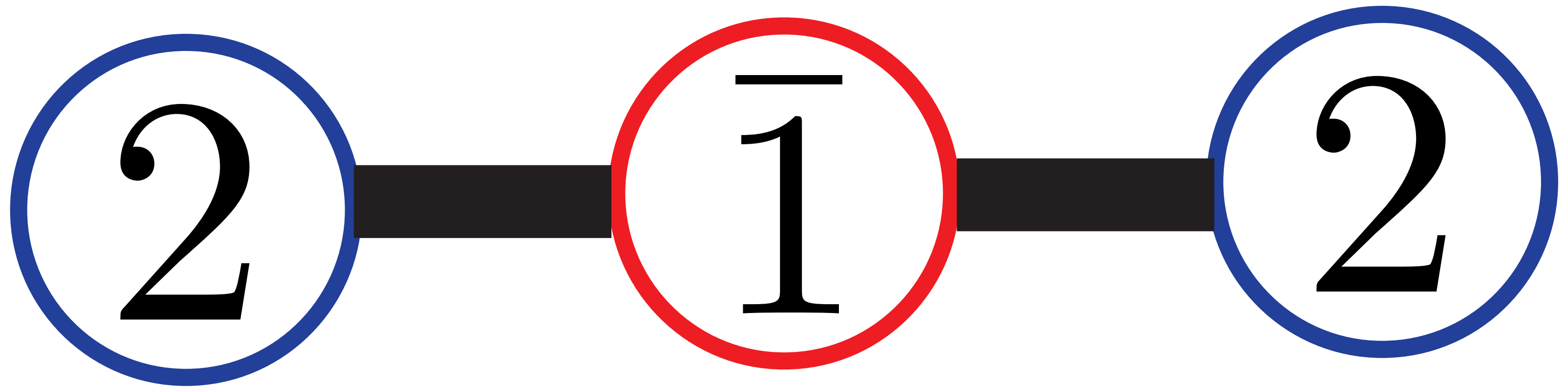

ϕ_2-string ϕ_1-anti-string ϕ_2-string

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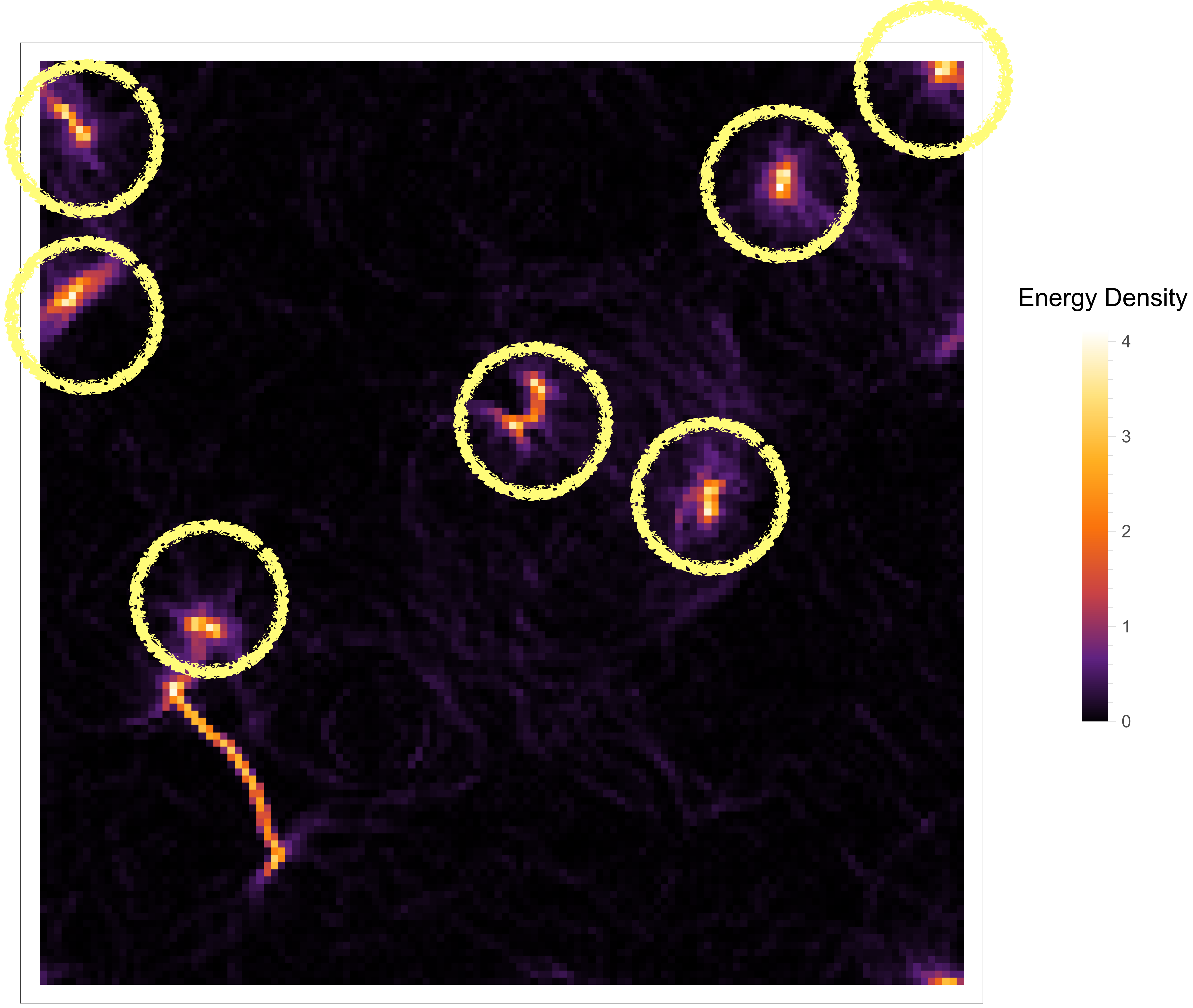

Numerical results (2D)

Lee, Murai, FT and Yin 2409.09749

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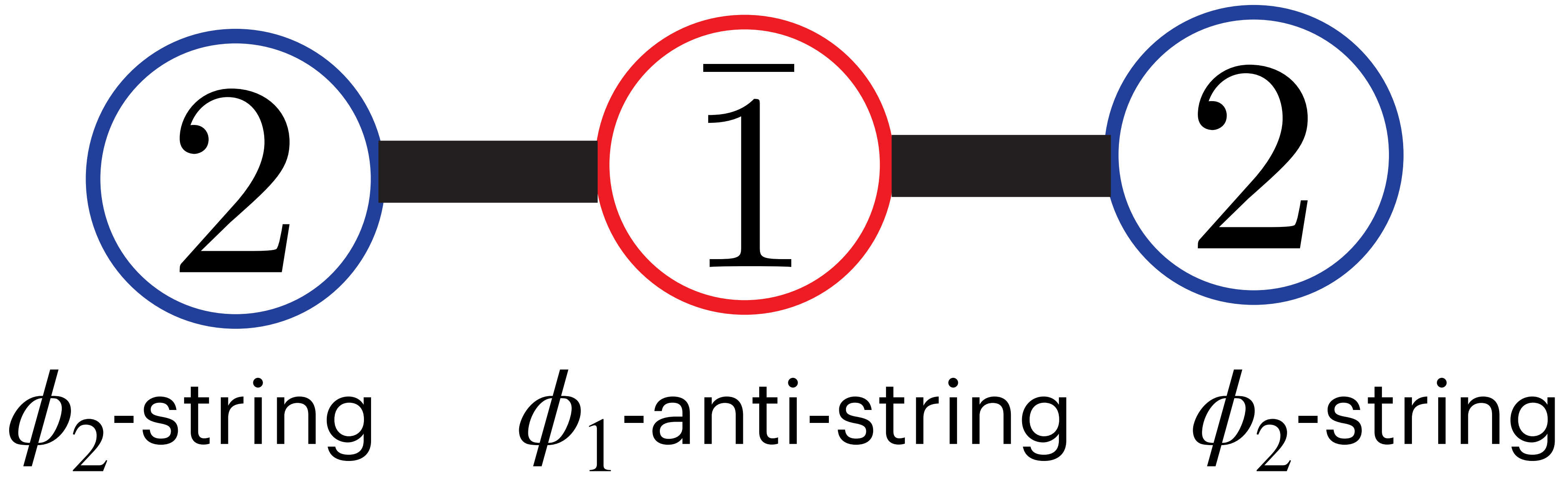

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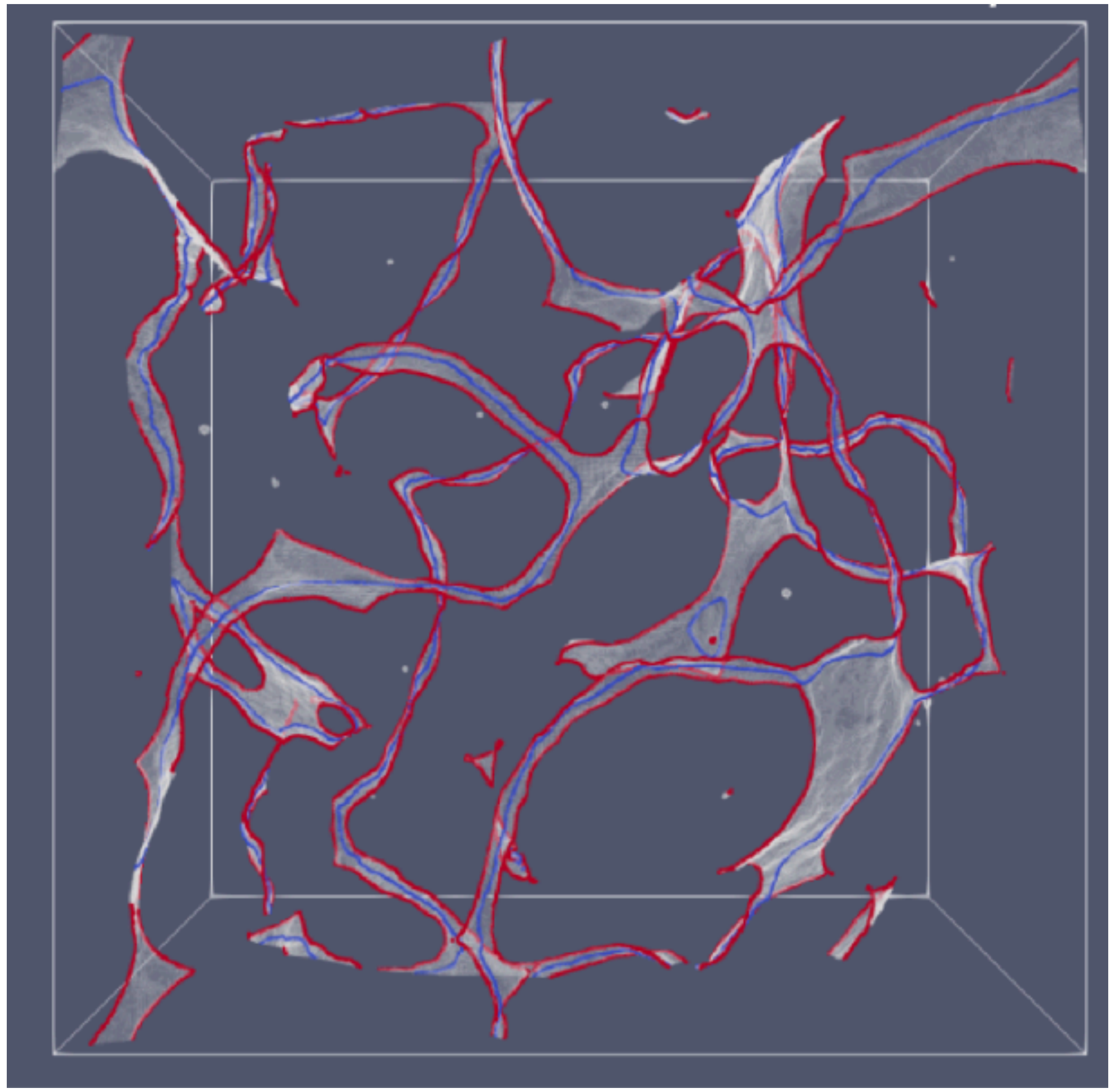

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Lee, Murai, FT and Yin 2409.09749

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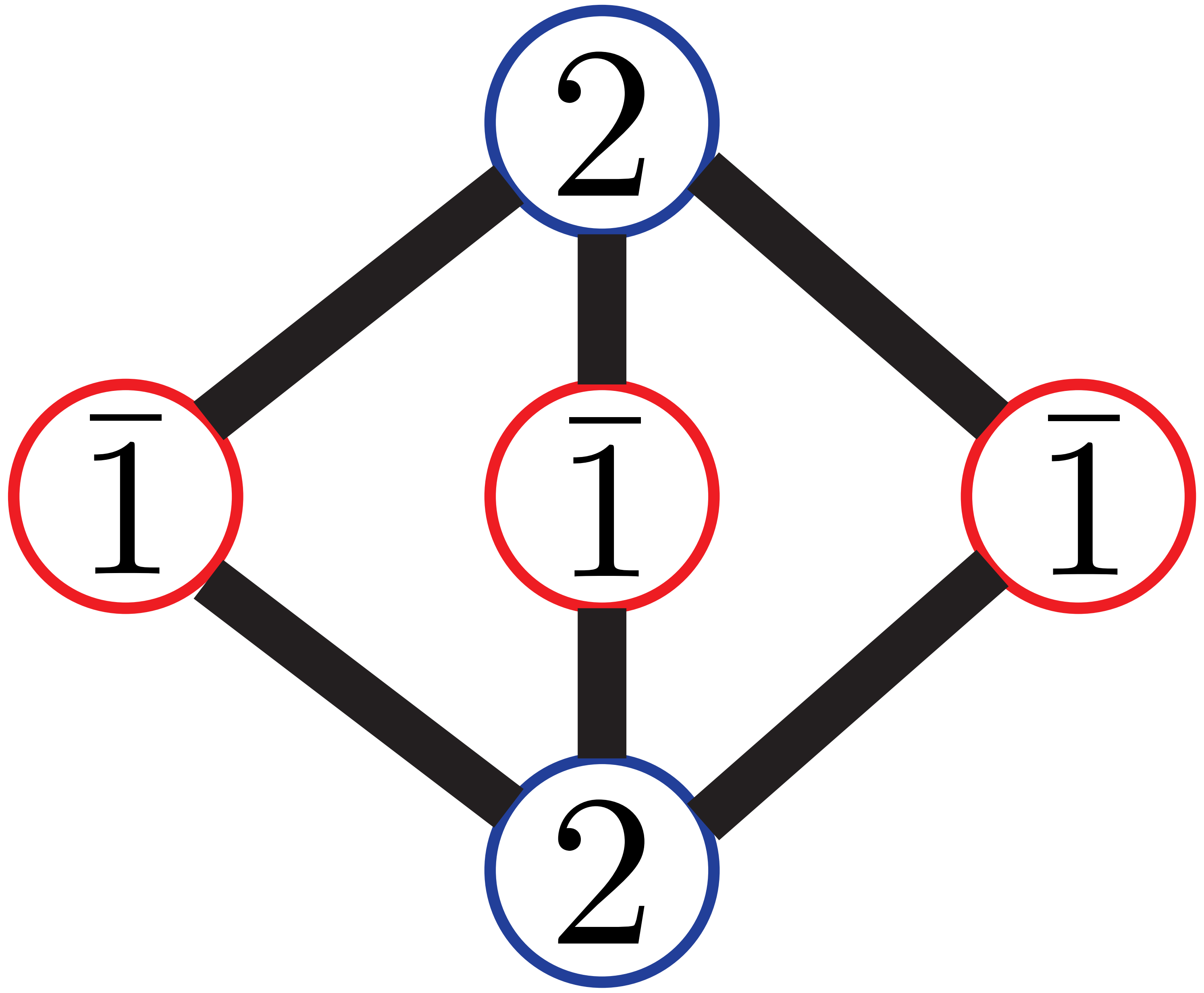

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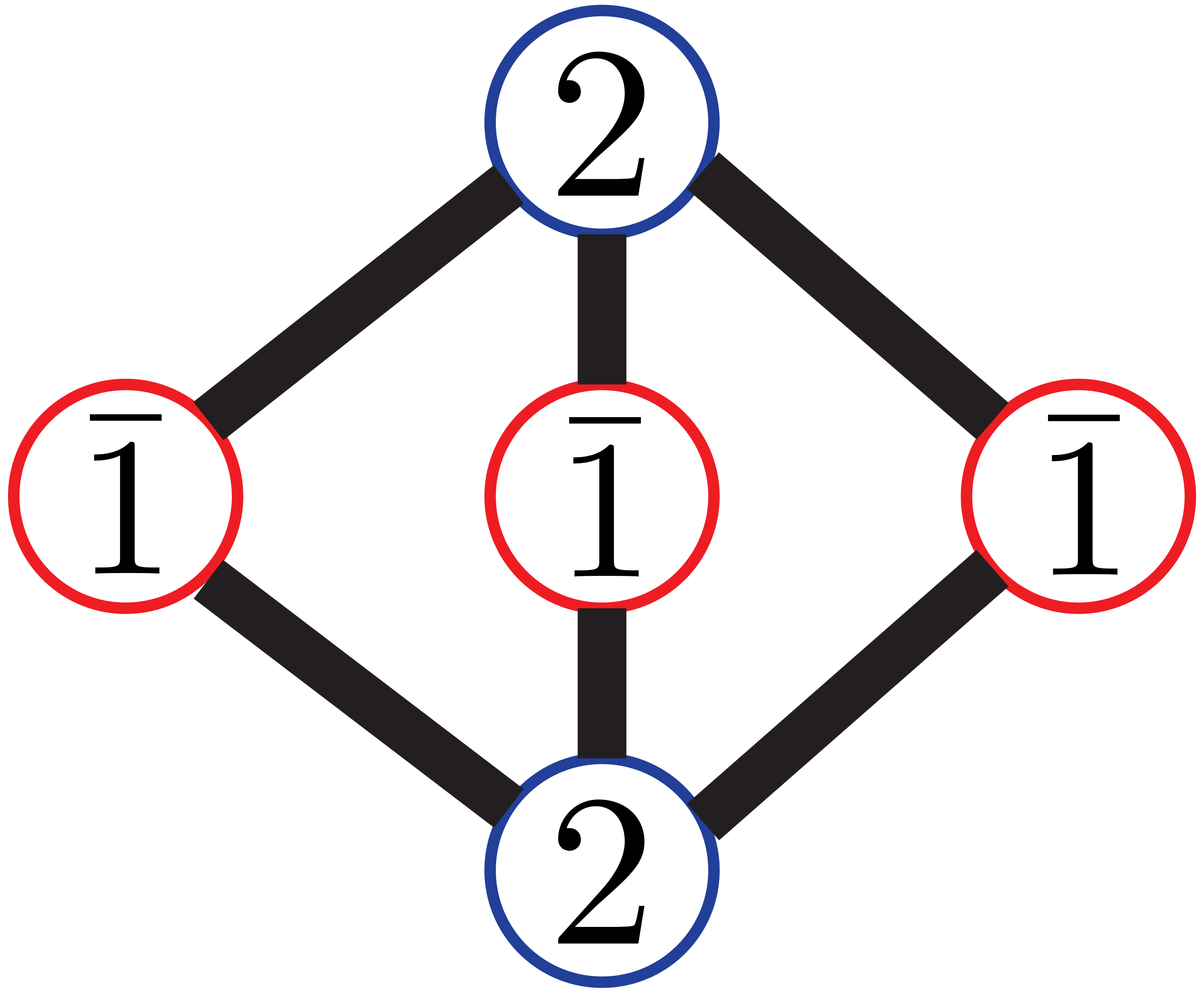

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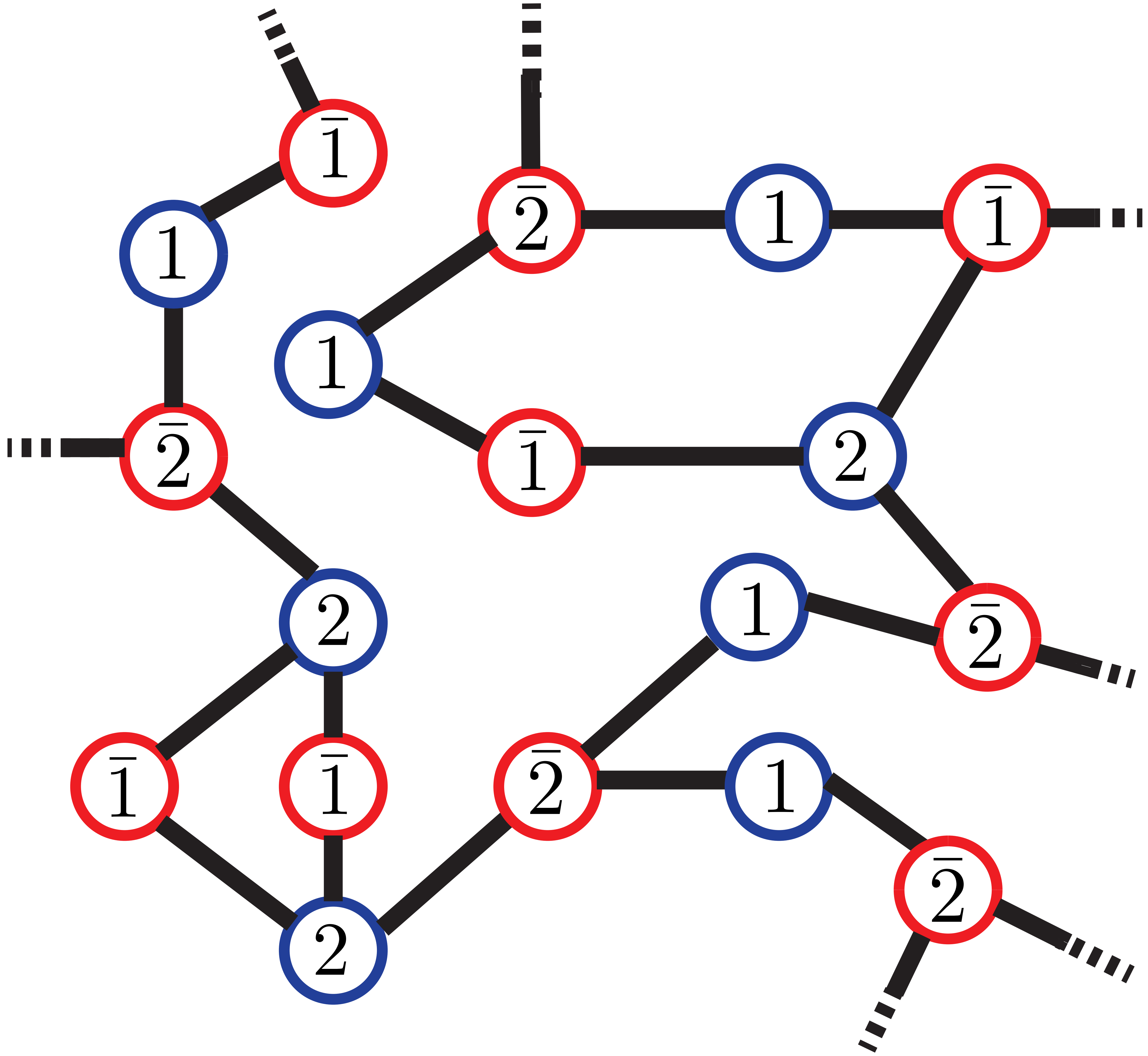
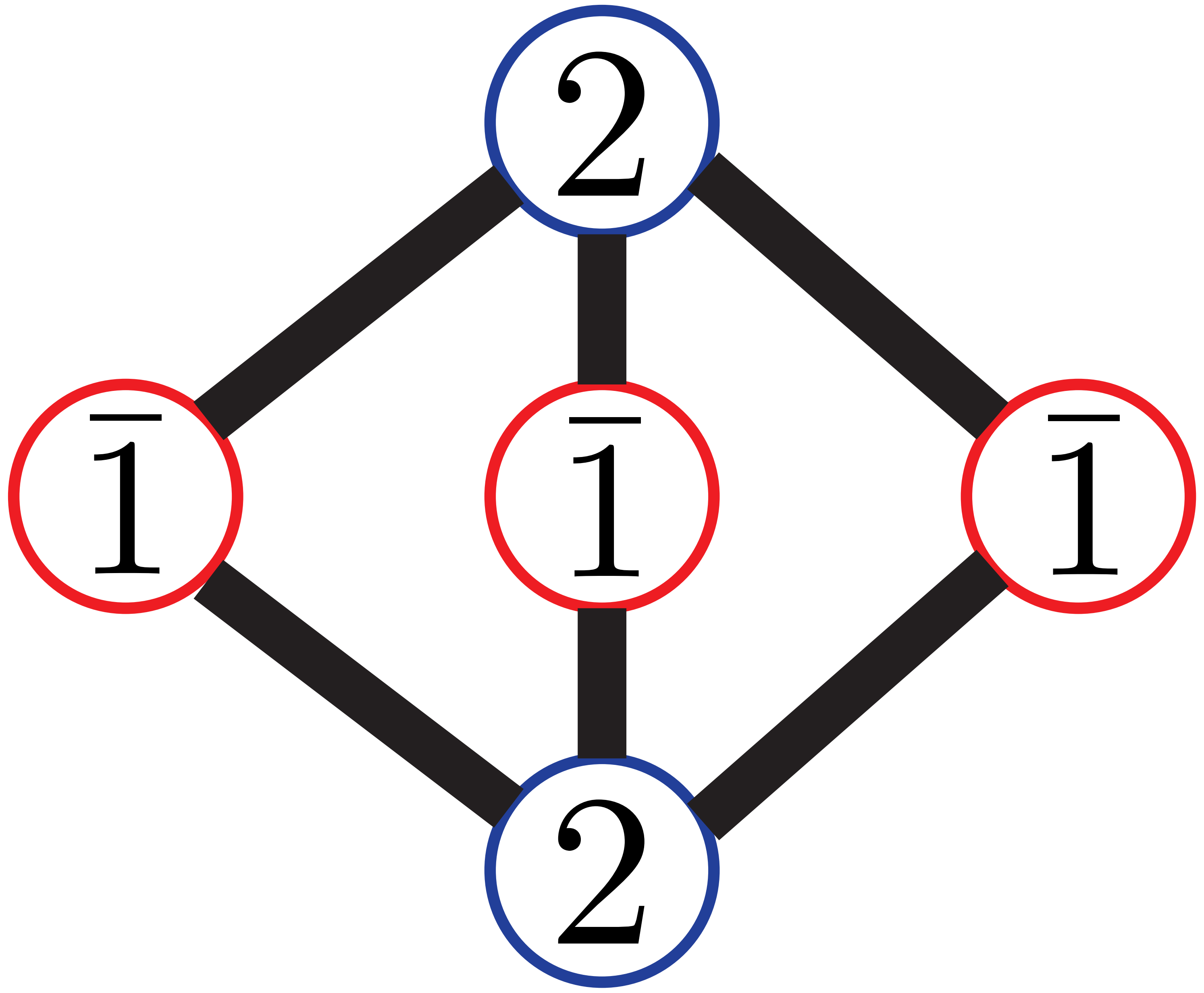


String-wall network

Case of $(n_1, n_2) = (2, 3)$

String-wall network forms in stead of string bundles.



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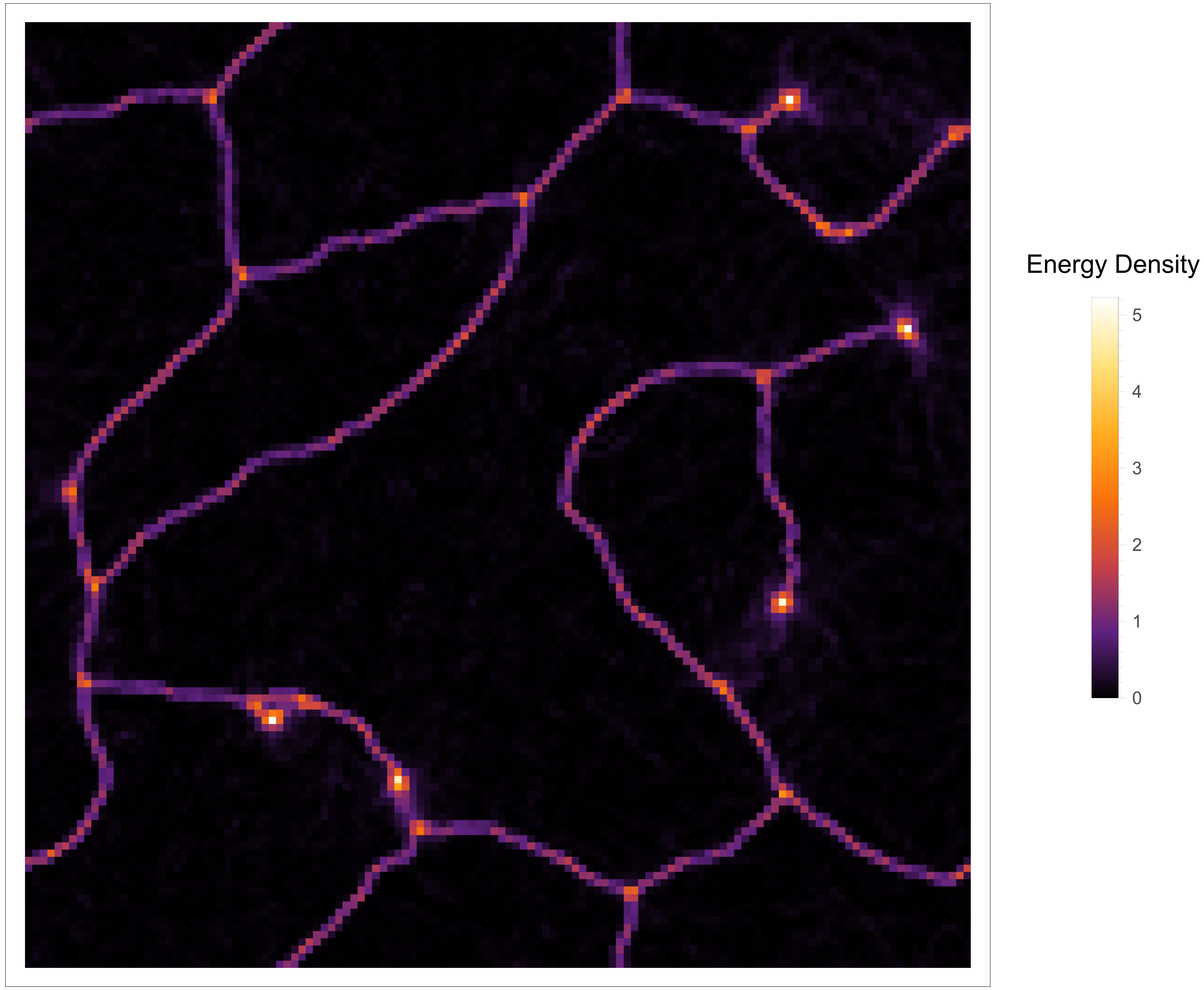

Numerical results (2D)

Lee, Murai, FT and Yin 2409.09749

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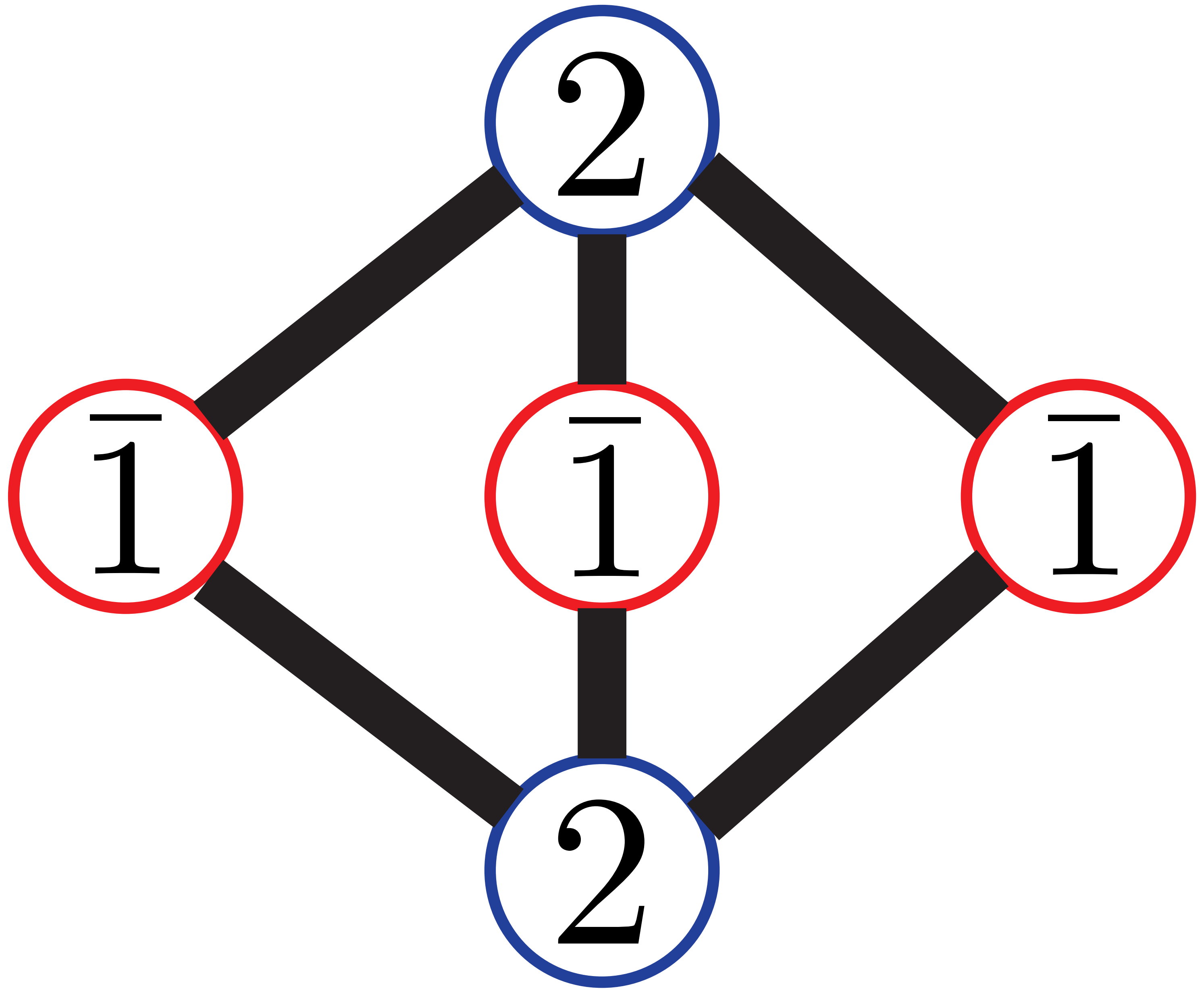

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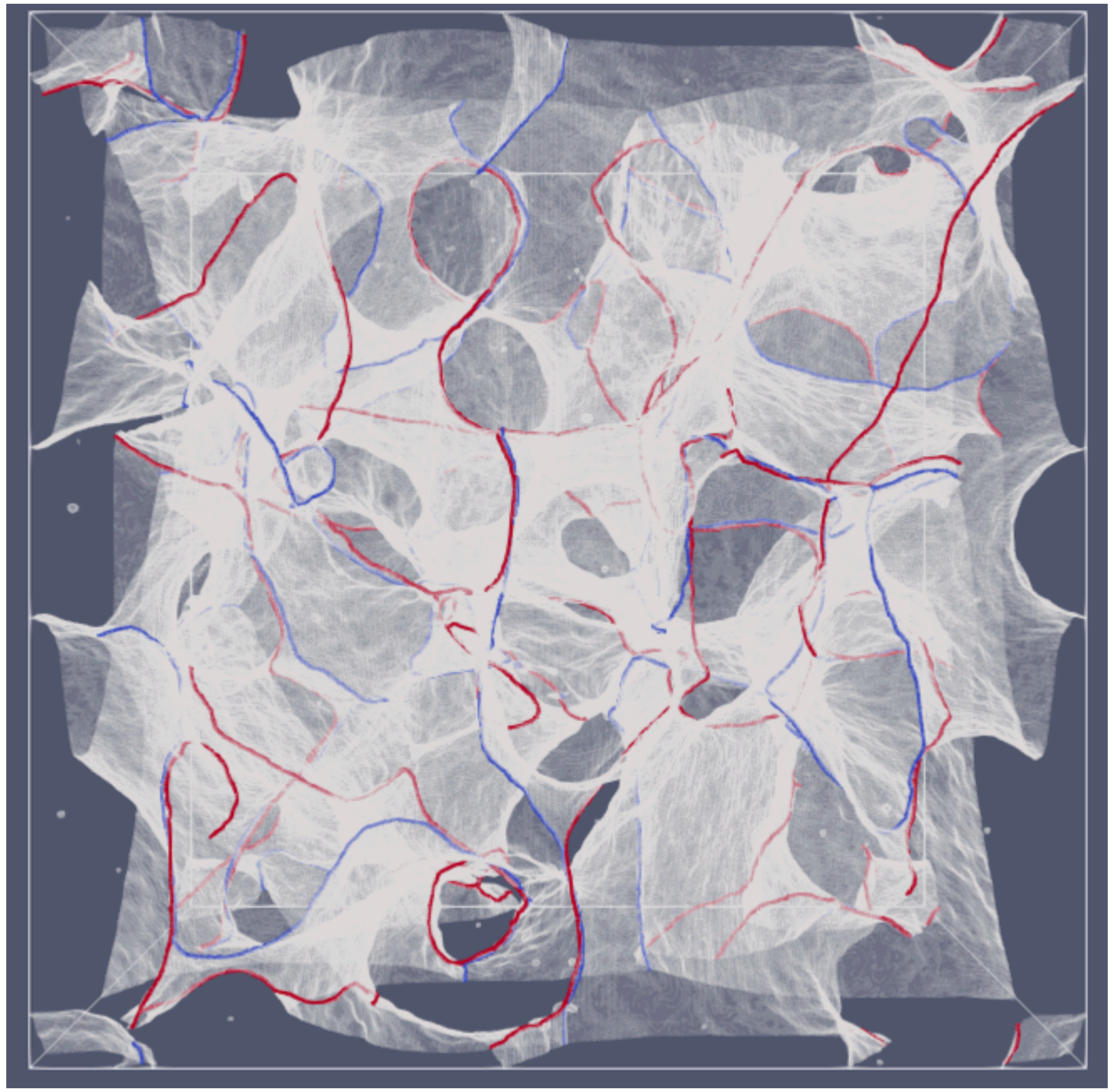
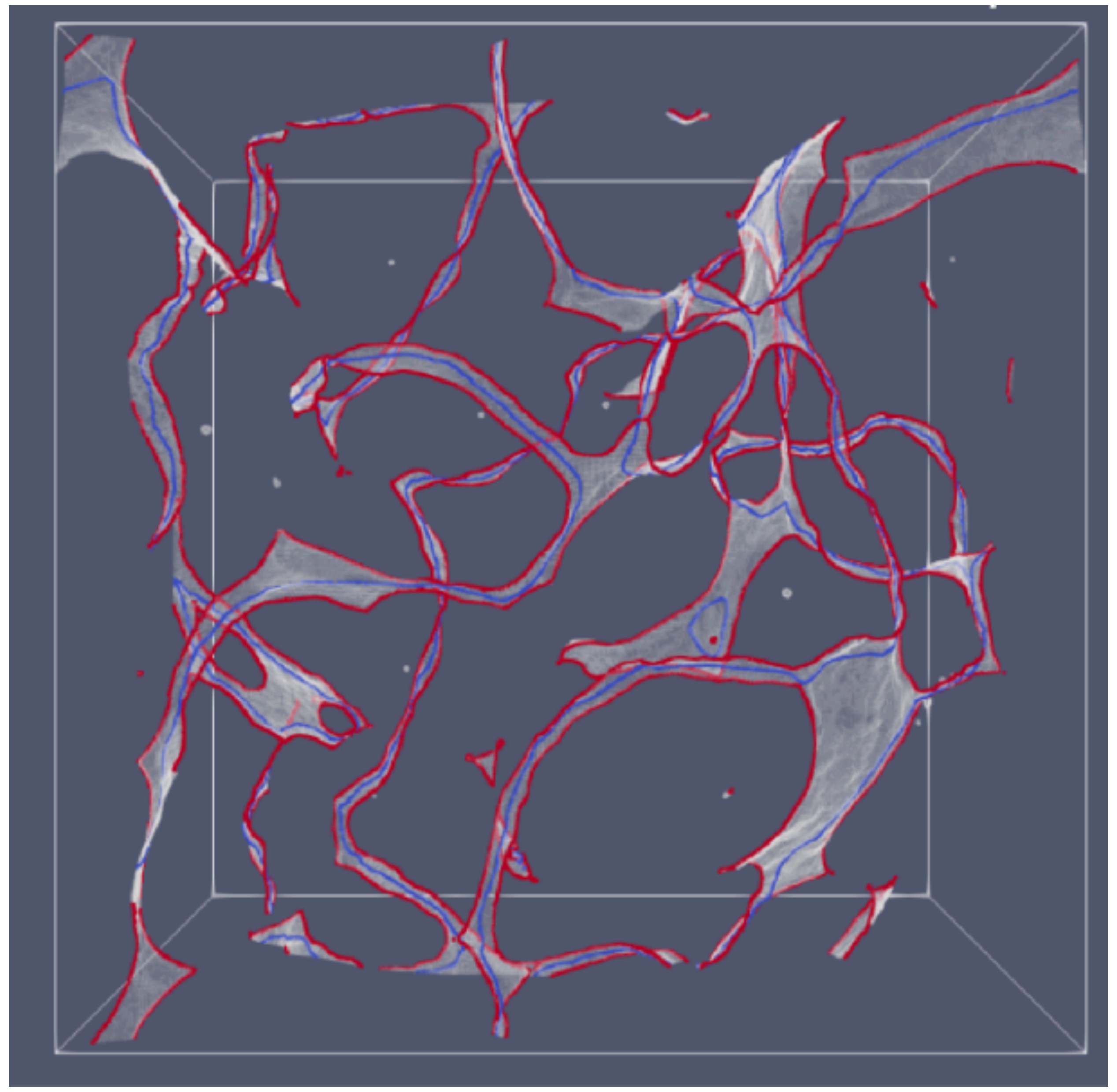


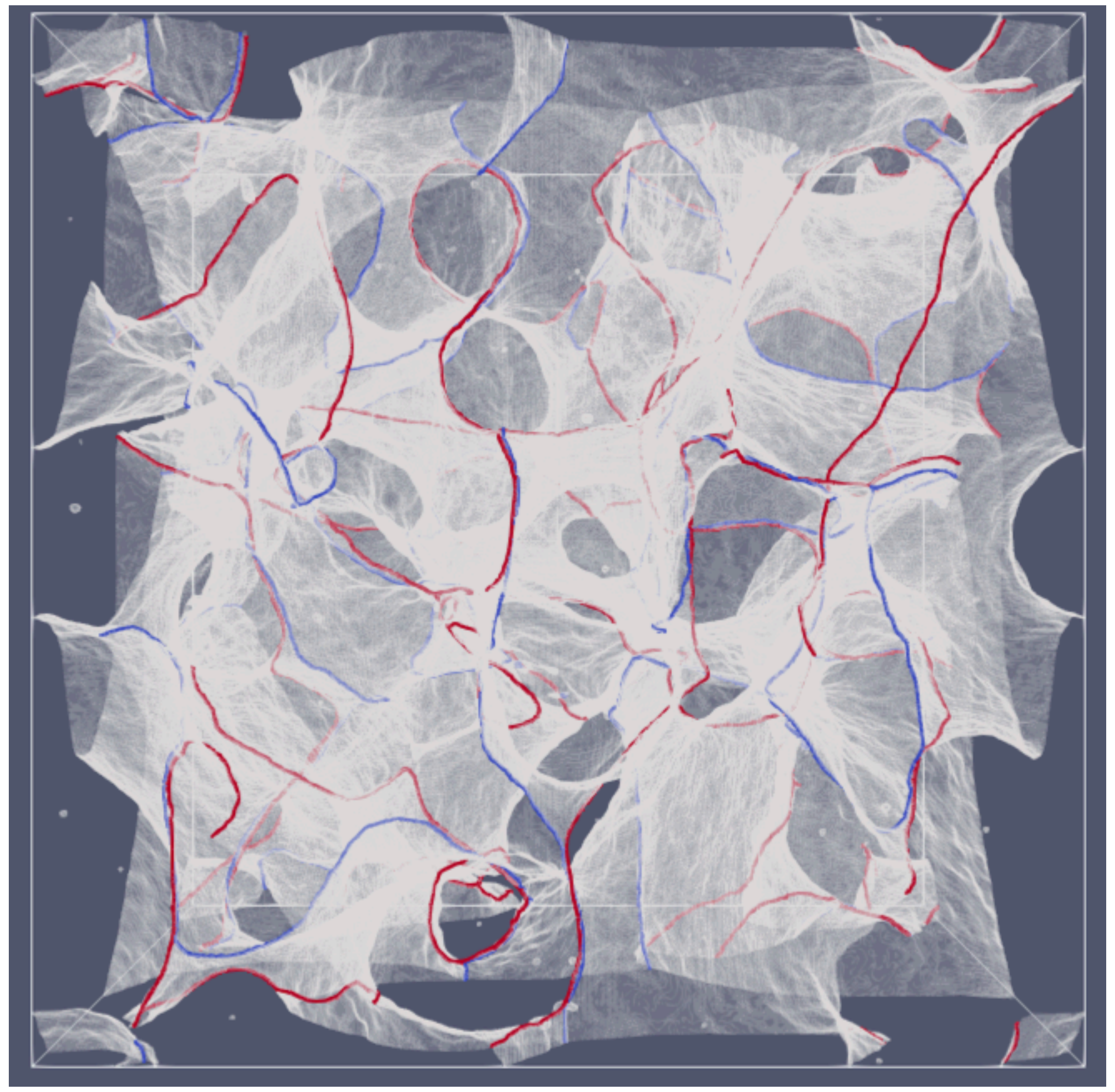
Numerical results (3D)

Coutesy of Junseok Lee

In the post-inflationary scenario, a string-wall network forms instead of ordinary cosmic strings if $n_1, n_2 \geq 2$.



$n_1 = 1$ or $n_2 = 1$



$n_1, n_2 \geq 2.$

These heavy axion DWs also produce a large amount of GWs.

Higaki, Jeong, Kitajima, Sekiguchi and FT, 1606.05552

Mixed initial conditions

We may impose a mixed "pre-post" initial condition, i.e.,

pre-inflationary initial condition for ϕ_1

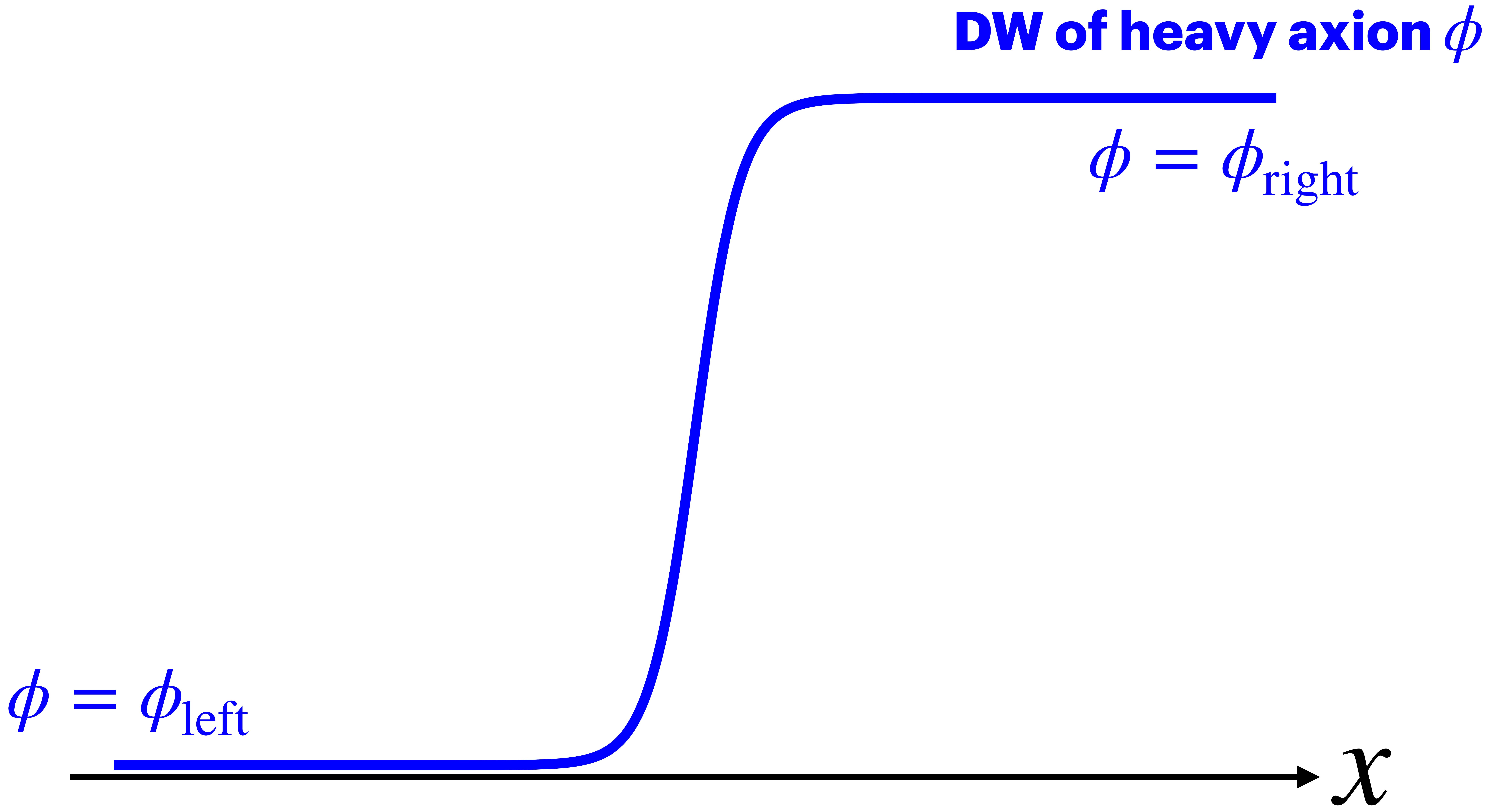
post-inflationary initial condition for ϕ_2

Then, no strings bundles are formed, and string-wall network of ϕ_2 remains if $n_2 \geq 2$ (even if $n_1 = 1$).

Mixed initial conditions makes string-wall network formation more likely.

”Induced DW” formation due to V_2

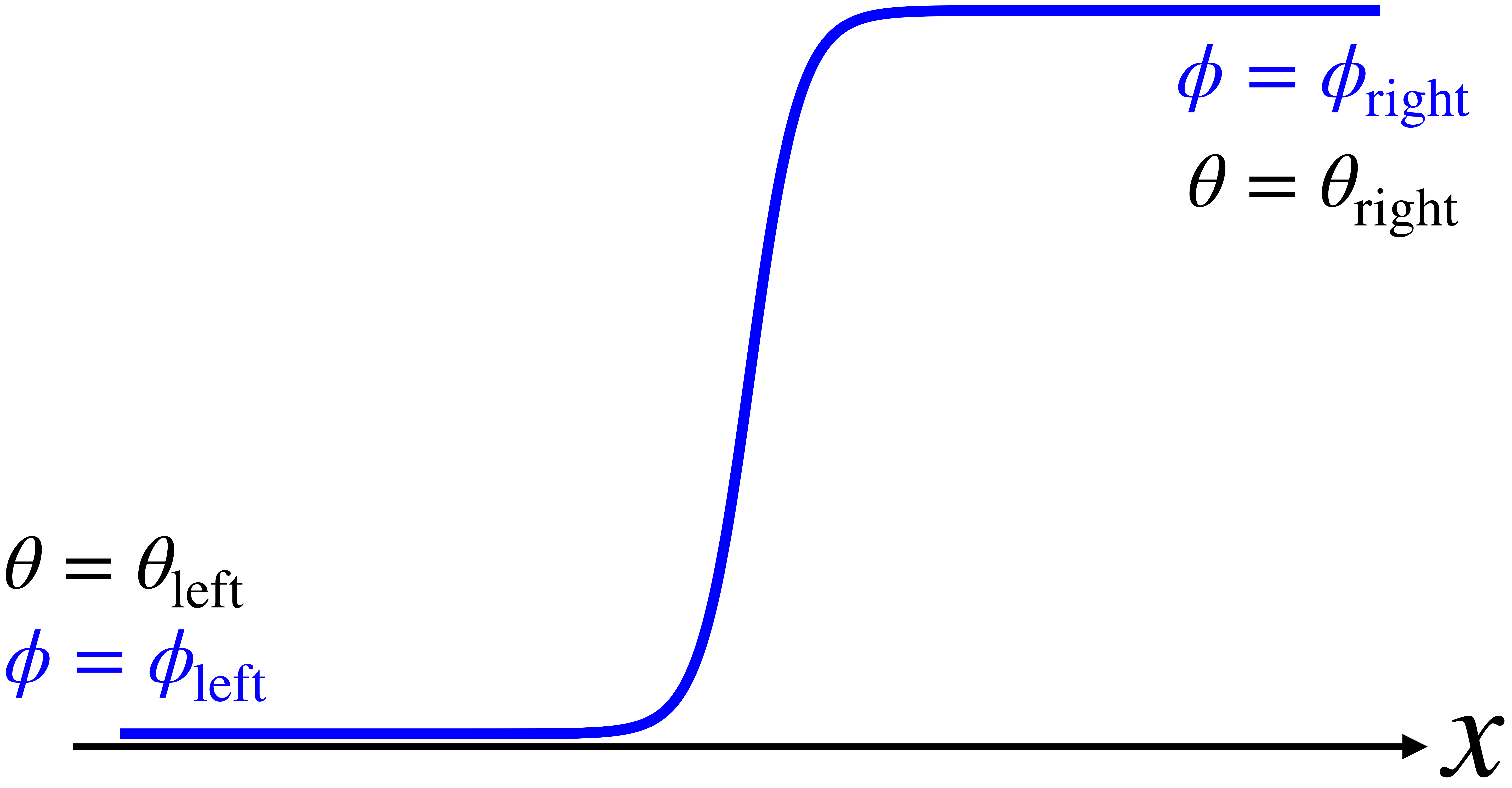
Before proceeding, let me first explain what an induced domain wall is.



DW of heavy axion ϕ
$\phi = \phi_{\rm right}$
$\phi = \phi_{\rm left}$
x

Couple ϕ to gluons: $\phi G\tilde{G}$

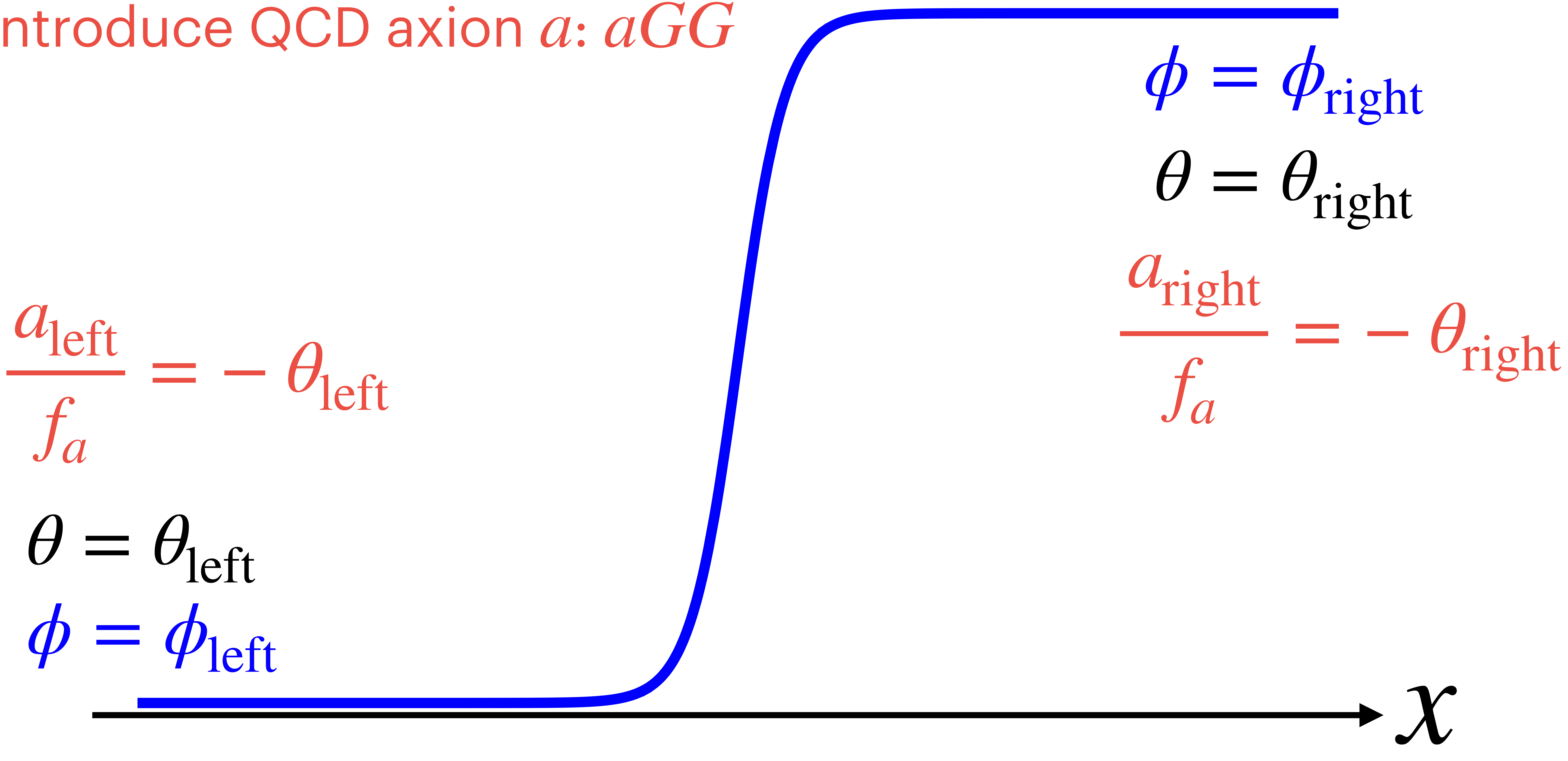
DW of heavy axion ϕ



Couple ϕ to gluons: $\phi G\tilde{G}$

DW of heavy axion ϕ

Introduce QCD axion a: $aG\tilde{G}$

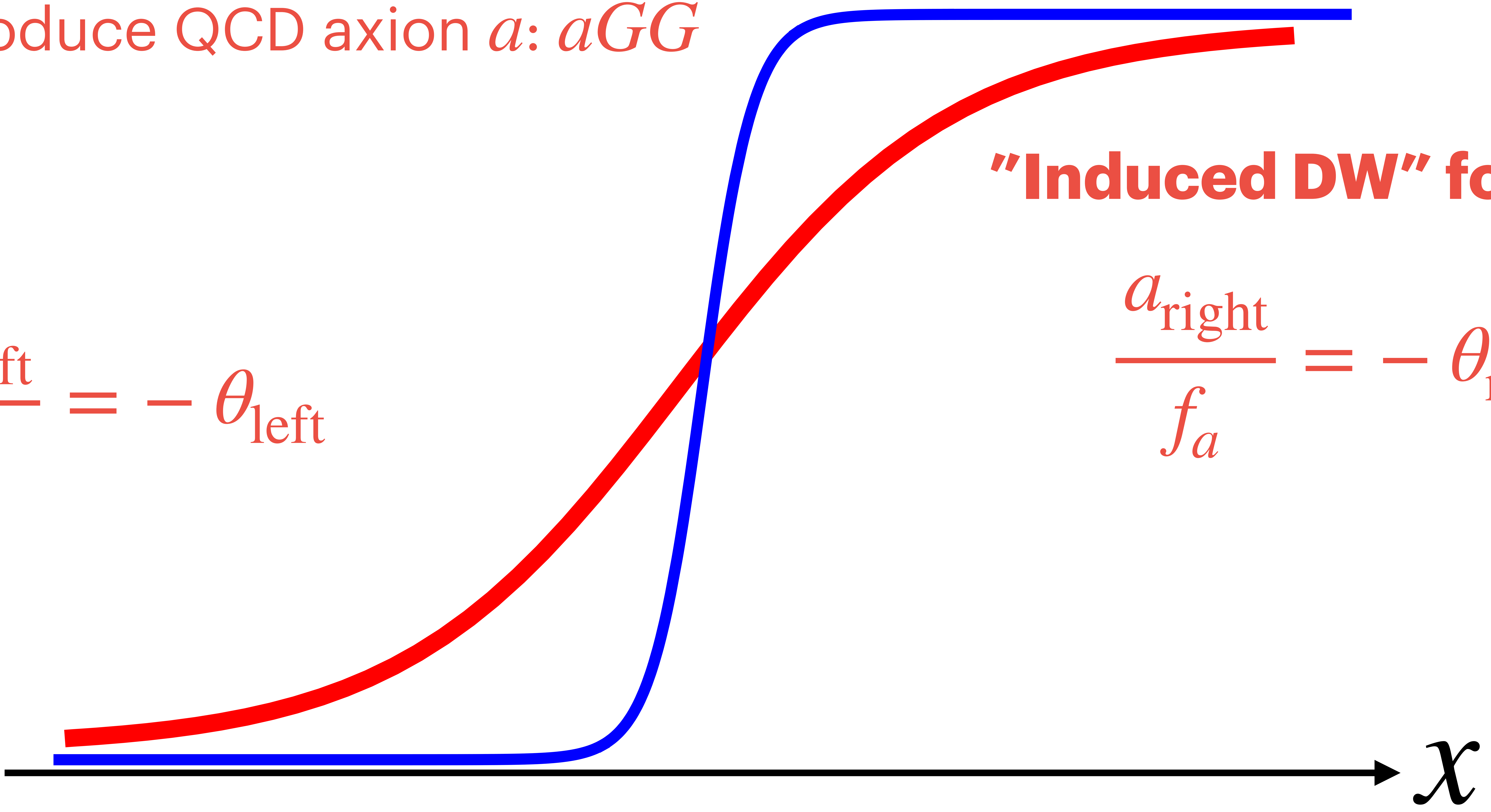


Couple ϕ to gluons: $\phi G\tilde{G}$

DW of heavy axion ϕ

Introduce QCD axion a: $aG\tilde{G}$

"Induced DW" for a



$$\frac{a_{\rm left}}{f_a} = -\theta_{\rm left}$$

$$\frac{a_{\rm right}}{f_a} = -\theta_{\rm right}$$

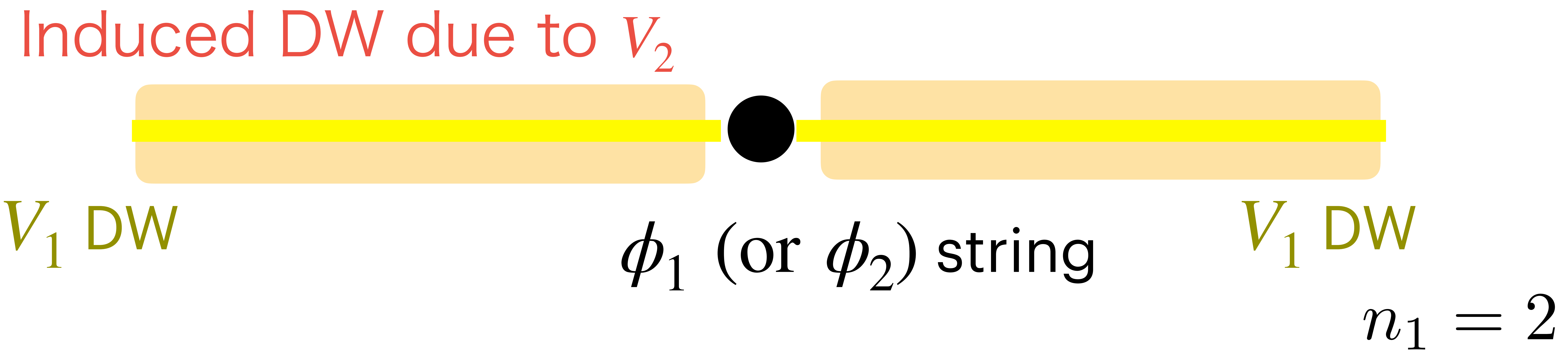
"Induced DW" formation due to V_2

Now we consider the DW formation due to $V_2 \ll V_1$.

$$V_1(\phi_1, \phi_2) = \Lambda^4 \left[1 - \cos\left(n_1 \frac{\phi_1}{f_1} + n_2 \frac{\phi_2}{f_2}\right)\right]$$

$$V_2(\phi_1, \phi_2) = \Lambda'^4 \left[1 - \cos\left(n'_1 \frac{\phi_1}{f_1} + n'_2 \frac{\phi_2}{f_2}\right)\right] \qquad \Lambda' \ll \Lambda$$

Both V_1 and V_2 can be minimized in any domains, and so there is no potential bias at the minimum.



Induced DW due to V_2
V_1 DW
ϕ_1 (or ϕ_2) string
V_1 DW
$n_1 = 2$

The string-wall network persists even after V_2 is included, since induced DWs appear and the domains remain degenerate in energy.

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Thus, even a minimal extension with two PQ scalars leads to the cosmological DW problem if n_1, $n_2 \geq 2$.

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Thus, even a minimal extension with two PQ scalars leads to the cosmological DW problem if $n_1, n_2 \geq 2$.

The simplest solution to the DW problem is to introduce a potential bias along the heavy axion, leaving the PQ mechanism intact. (I'll present an alternative approach shortly.)

Induced domain walls of QCD axion

Lee, Murai, FT and Yin 2407.09478

We consider induced domain walls of the QCD axion a, arising from its mixing with a heavy axion ϕ.

Pre-inflationary condition for the QCD axion $A = \frac{f_a}{\sqrt{2}} e^{i\frac{a}{f_a}}$,

Post-inflationary condition for the heavy axion $\Phi = \frac{f_\phi}{\sqrt{2}} e^{i\frac{\phi}{f_\phi}}$

The axion potential:

Lee, Murai, FT and Yin 2407.09478

$$V(a,\phi) = \chi(T)\left[1-\cos\left(\frac{a}{f_a}+\frac{\phi}{f_\phi}\right)\right] + \Lambda^4\left[1-\cos\left(2\frac{\phi}{f_\phi}\right)\right]$$

$$\chi(T) \equiv \frac{m_a^2(T) f_a^2}{N_a^2} = \begin{cases} \chi_0 & (T < T_{\rm QCD}) \\ \chi_0 \left(\frac{T}{T_{\rm QCD}}\right)^{-n} & (T \geq T_{\rm QCD}) \end{cases},$$

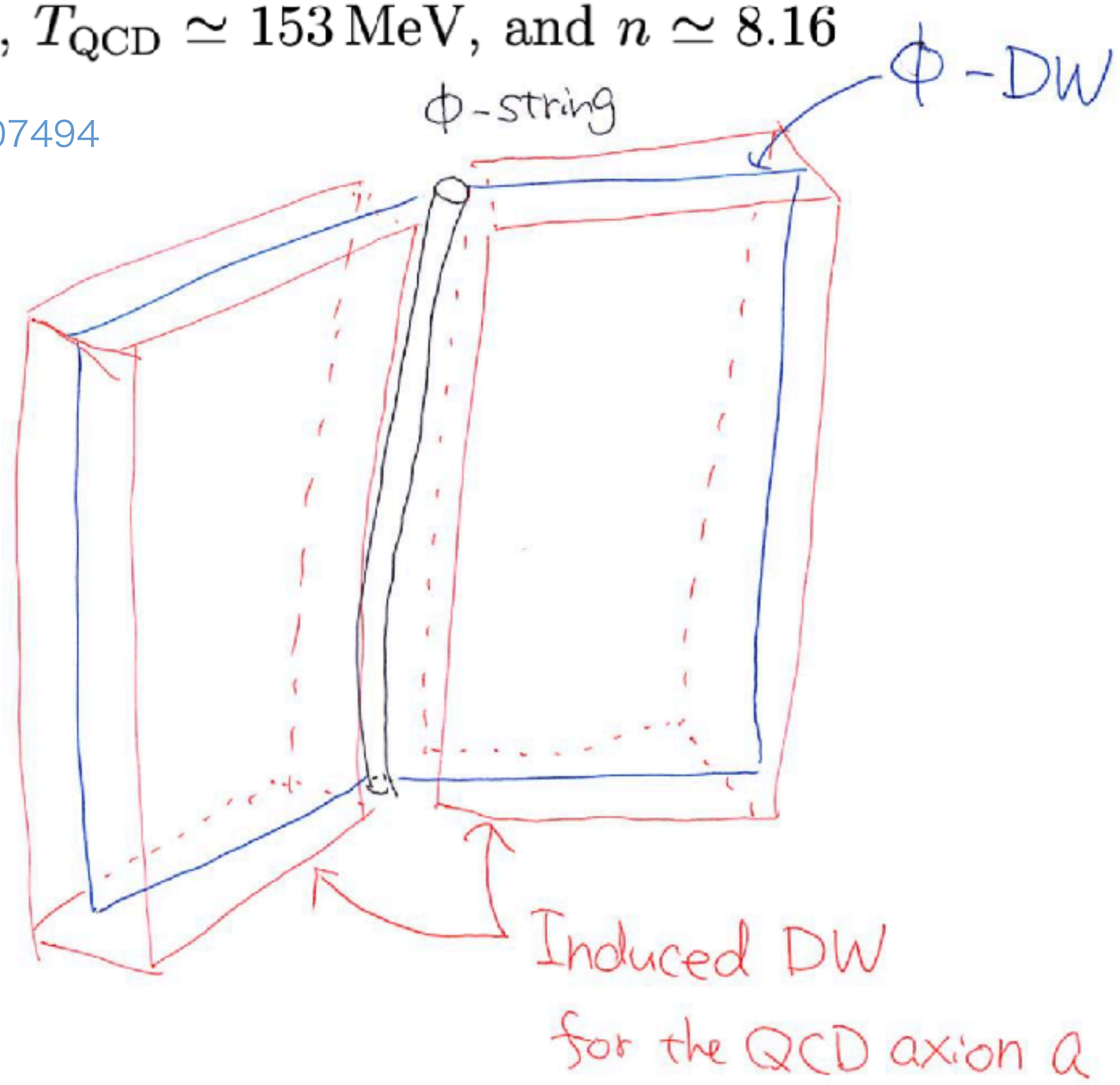
$\chi_0 \simeq (75.6\,\mathrm{MeV})^4$, $T_{\rm QCD} \simeq 153\,\mathrm{MeV}$, and $n \simeq 8.16$

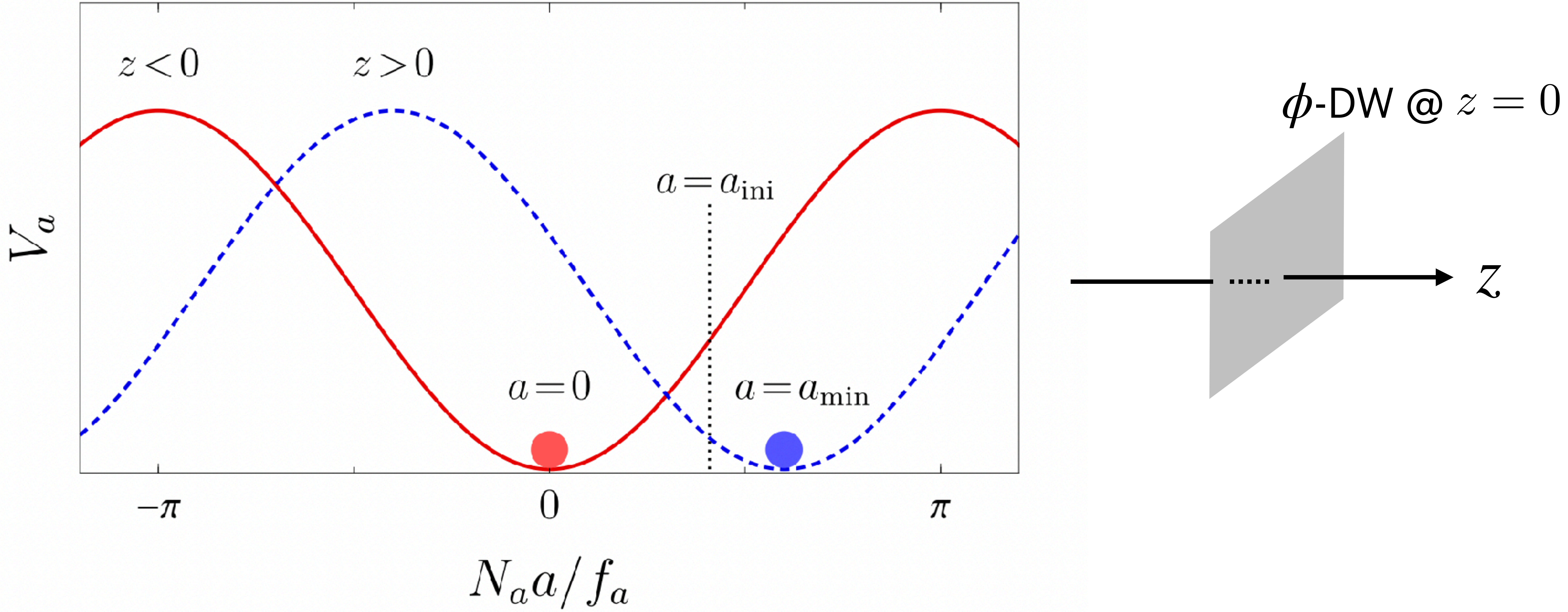
A. Borsanyi et al, 1606.07494

For simplicity we assume a hierachy in the mass and the tension:

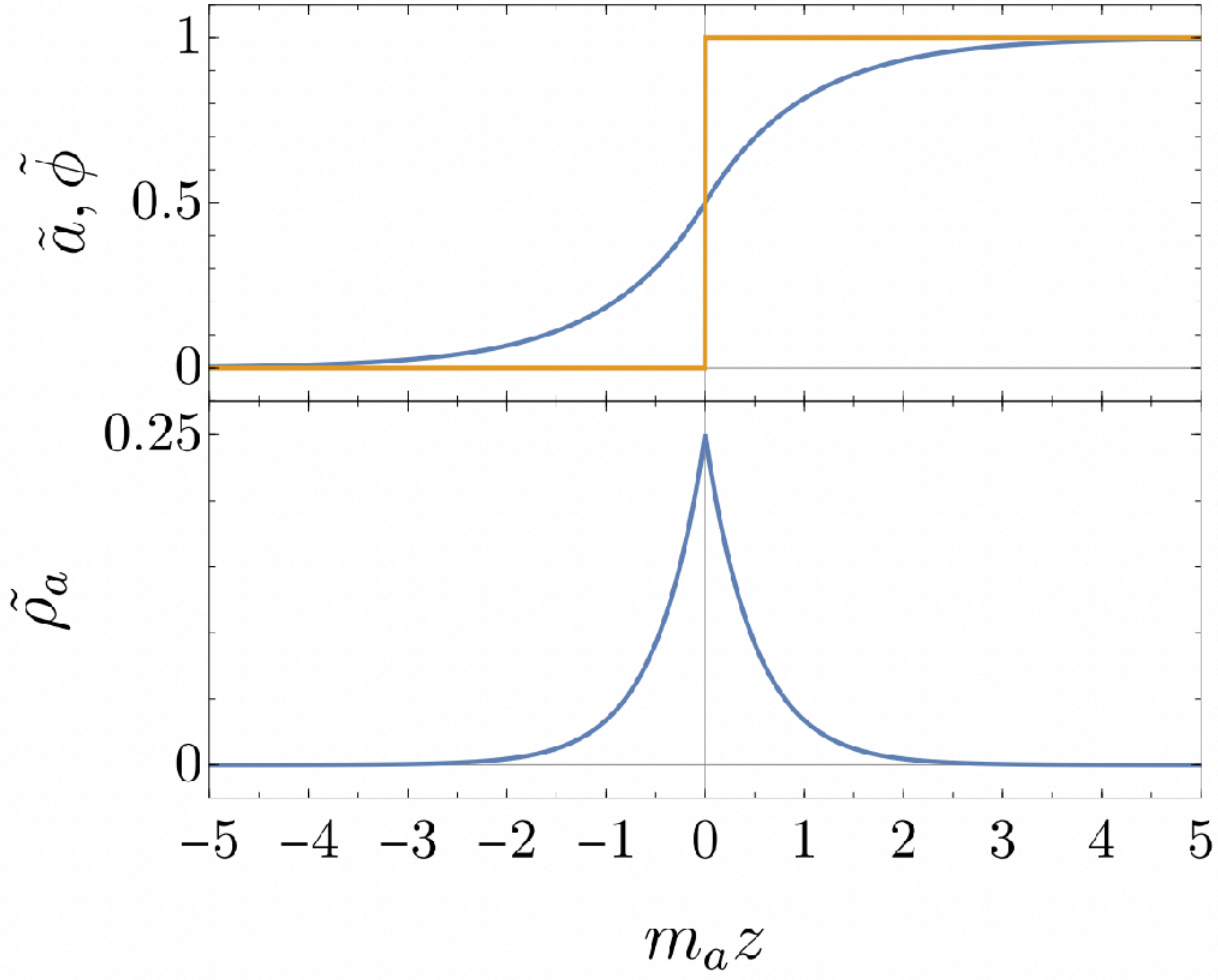
$$m_\phi \sim \frac{\Lambda^2}{f_\phi} \gg m_{a0} = m_a(T=0)$$

$$m_\phi f_\phi^2 \gg m_{a0} f_a^2$$

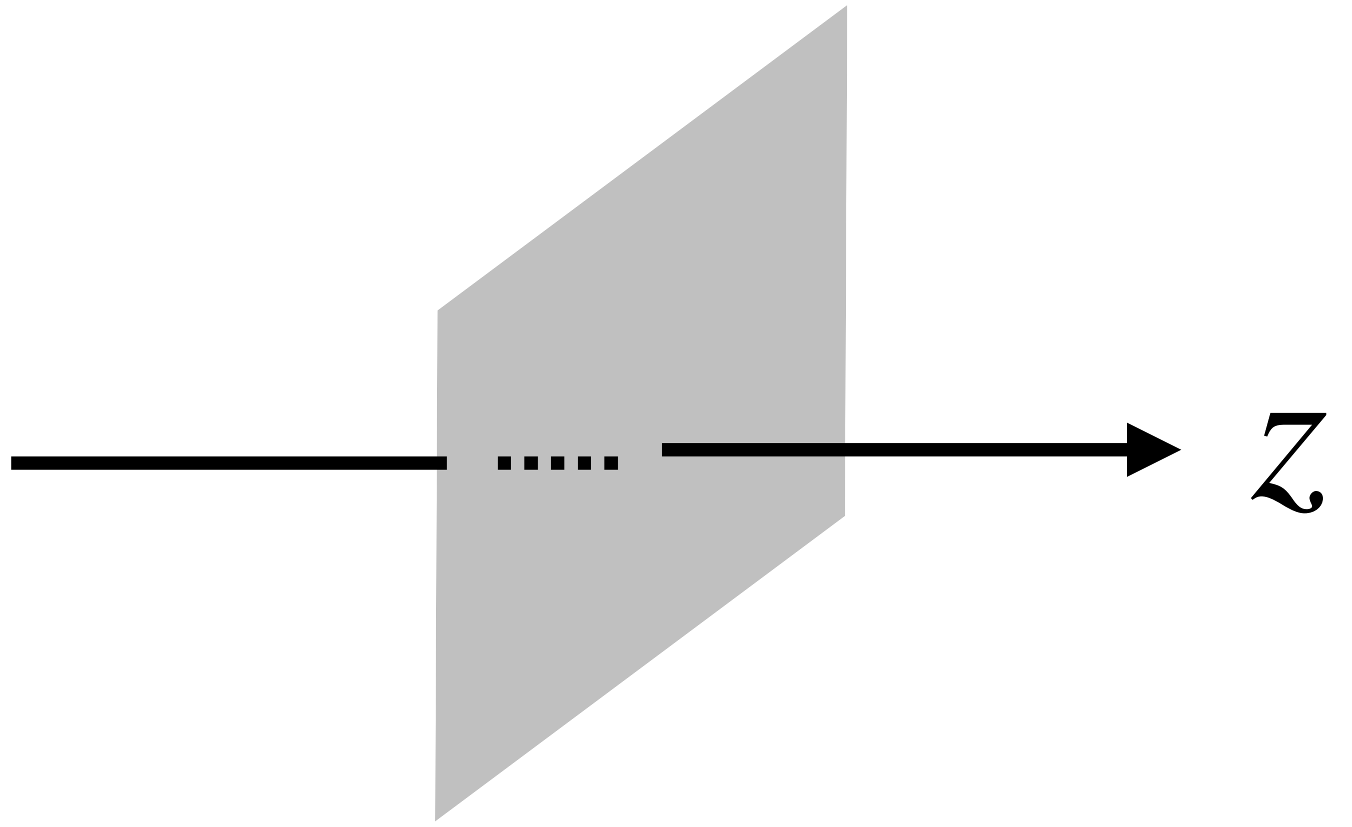




$z<0$
$z>0$
$a=a_{\rm ini}$
$a=0$
$a=a_{\rm min}$
V_a
$-\pi$
0
π
$N_a a/f_a$
ϕ-DW @ $z=0$
z



ϕ-DW @ $z = 0$

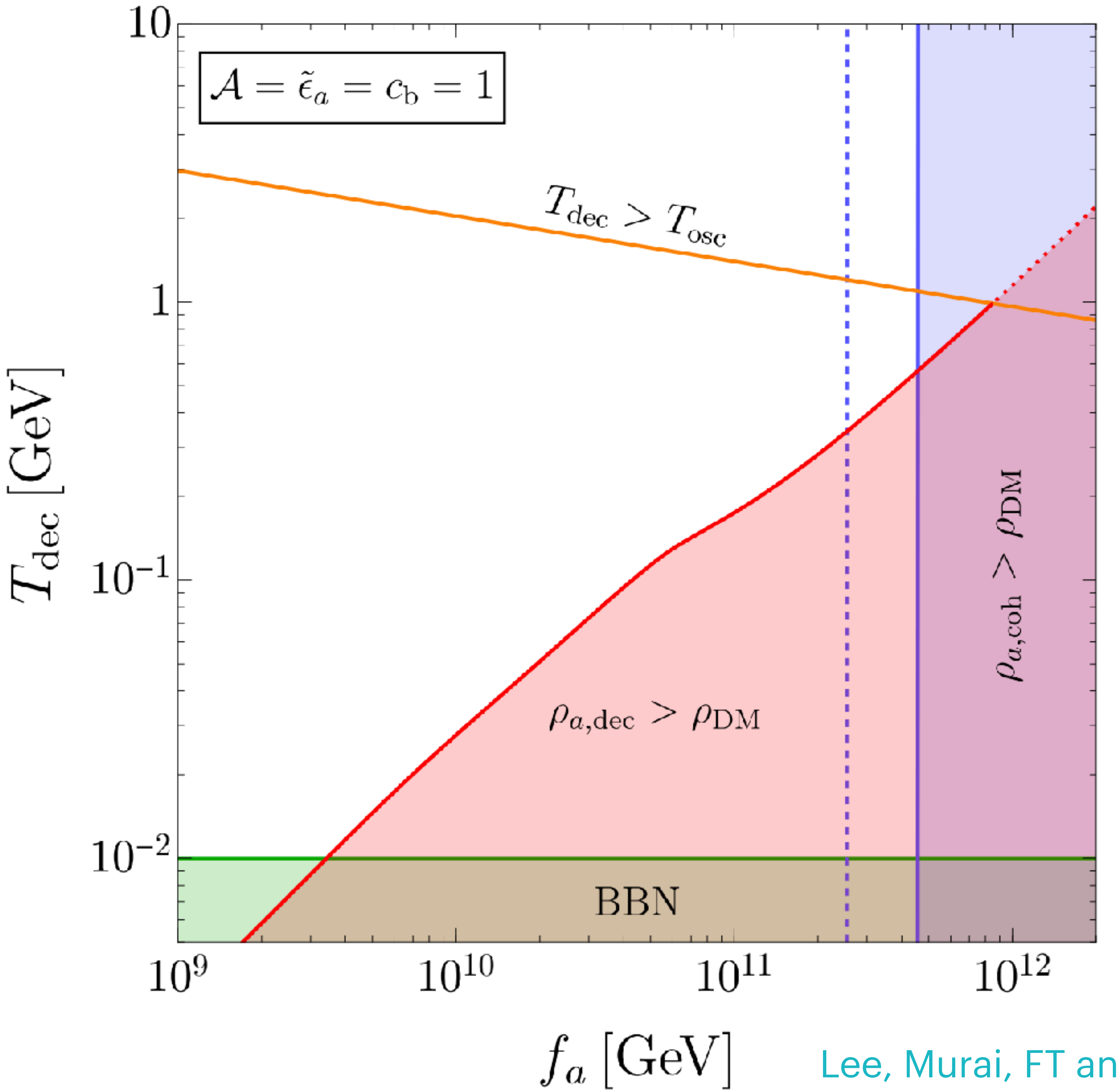


$$\sigma_a \sim m_a a_{\min}^2 < 8 m_a f_a^2$$

The tension of induced DWs is always smaller than that of ordinary DWs.

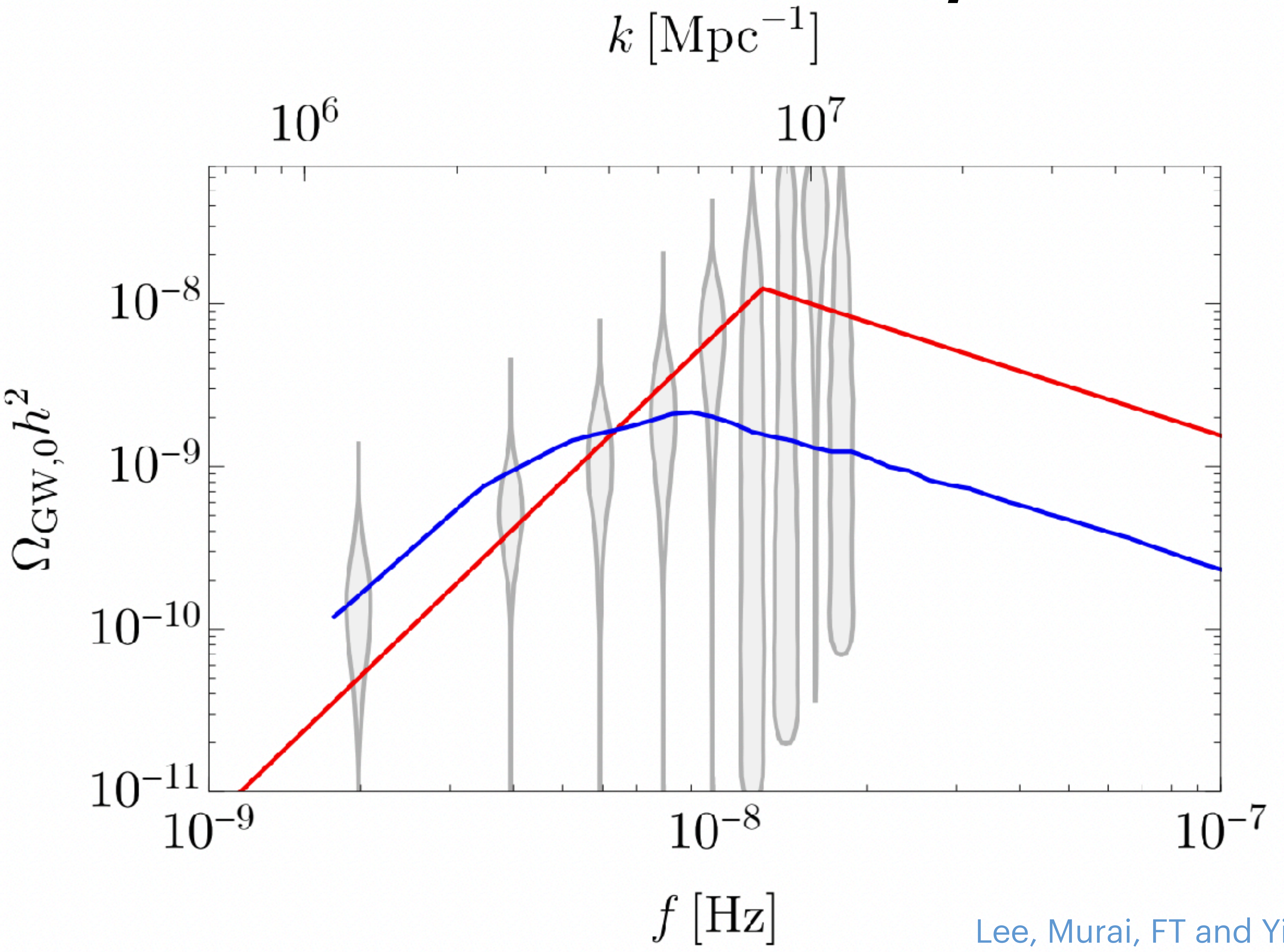
Decay temperature of the ϕ-DWs

(We introduce a small potential bias along ϕ-direction to make DWs unstable)



Lee, Murai, FT and Yin 2407.09478

GW spectrum from the ϕ-DW collapse



Red:
Analytical estimate with
$\sigma_\phi = 6 \times 10^{15}\,\mathrm{GeV}^3$
$T_{\mathrm{dec}} = 100\,\mathrm{MeV}$

Blue:
Numerical estimate with
$\sigma_\phi = 1.3 \times 10^{15}\,\mathrm{GeV}^3$
$T_{\mathrm{dec}} = 150\,\mathrm{MeV}$

Kitajima, Lee, Murai, FT and Yin 2306.17146

Lee, Murai, FT and Yin 2407.09478

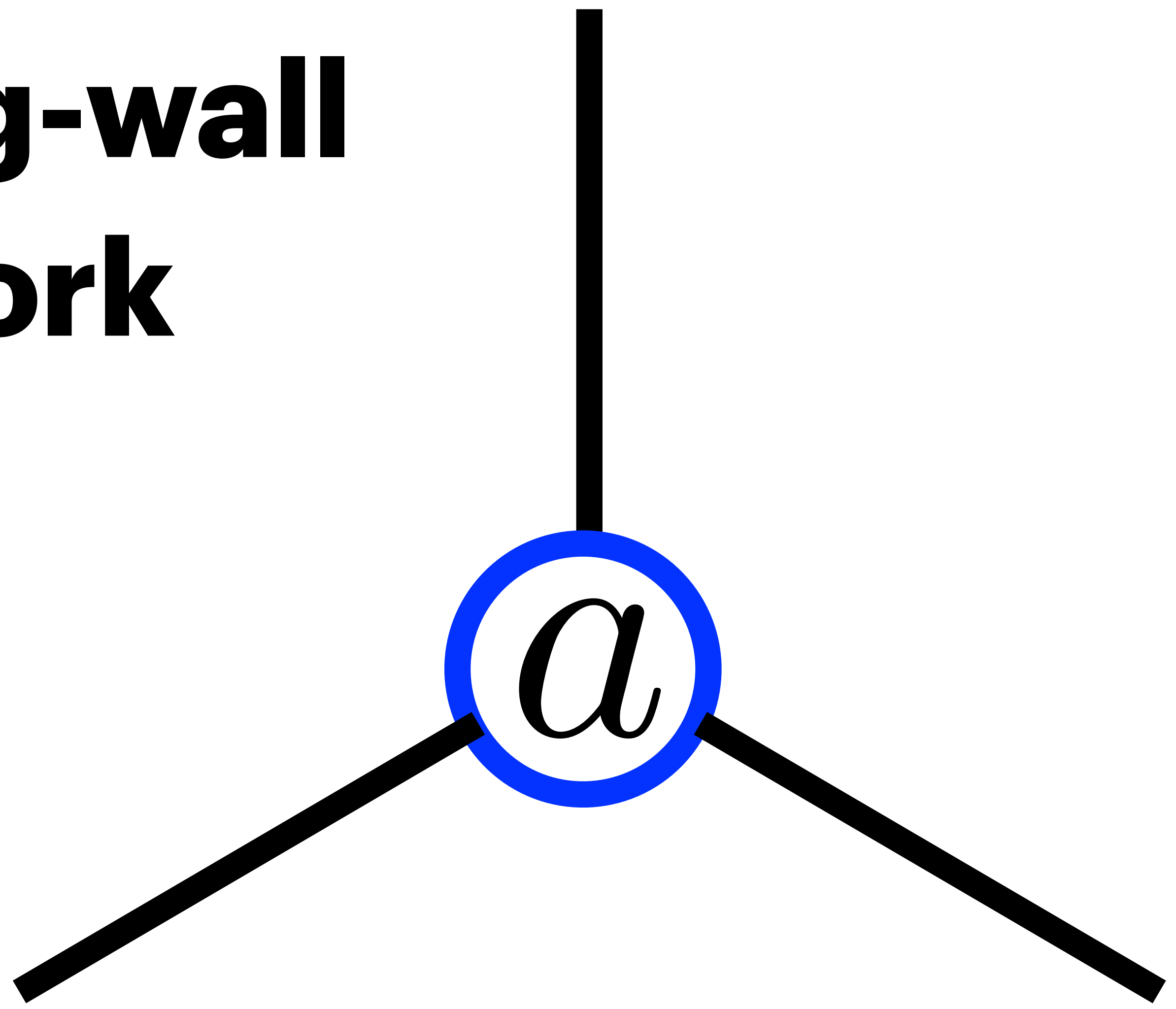
New solution to the QCD axion DW problem

FT and Yin 2012.11576, Lee, Murai, FT and Yin 2507.07075, see also Kondo, Murayama 2507.07973

$$\mathcal{L} = -N_{\rm DW}\frac{g_s^2}{32\pi^2}\frac{a}{v_a}G\tilde{G}$$

$N_{\rm DW} = 3 \text{ or } 6$ for DFSZ axion

String-wall network



New solution to the QCD axion DW problem

FT and Yin 2012.11576, Lee, Murai, FT and Yin 2507.07075,

$$\mathcal{L} = -N_{\rm DW}\frac{g_s^2}{32\pi^2}\frac{a}{v_a}G\tilde{G} \quad \Rightarrow \quad \mathcal{L} = -\frac{g_s^2}{32\pi^2}\left(N_{\rm DW}\frac{a}{v_a} + \frac{\varphi}{v_\varphi}\right)G\tilde{G}$$

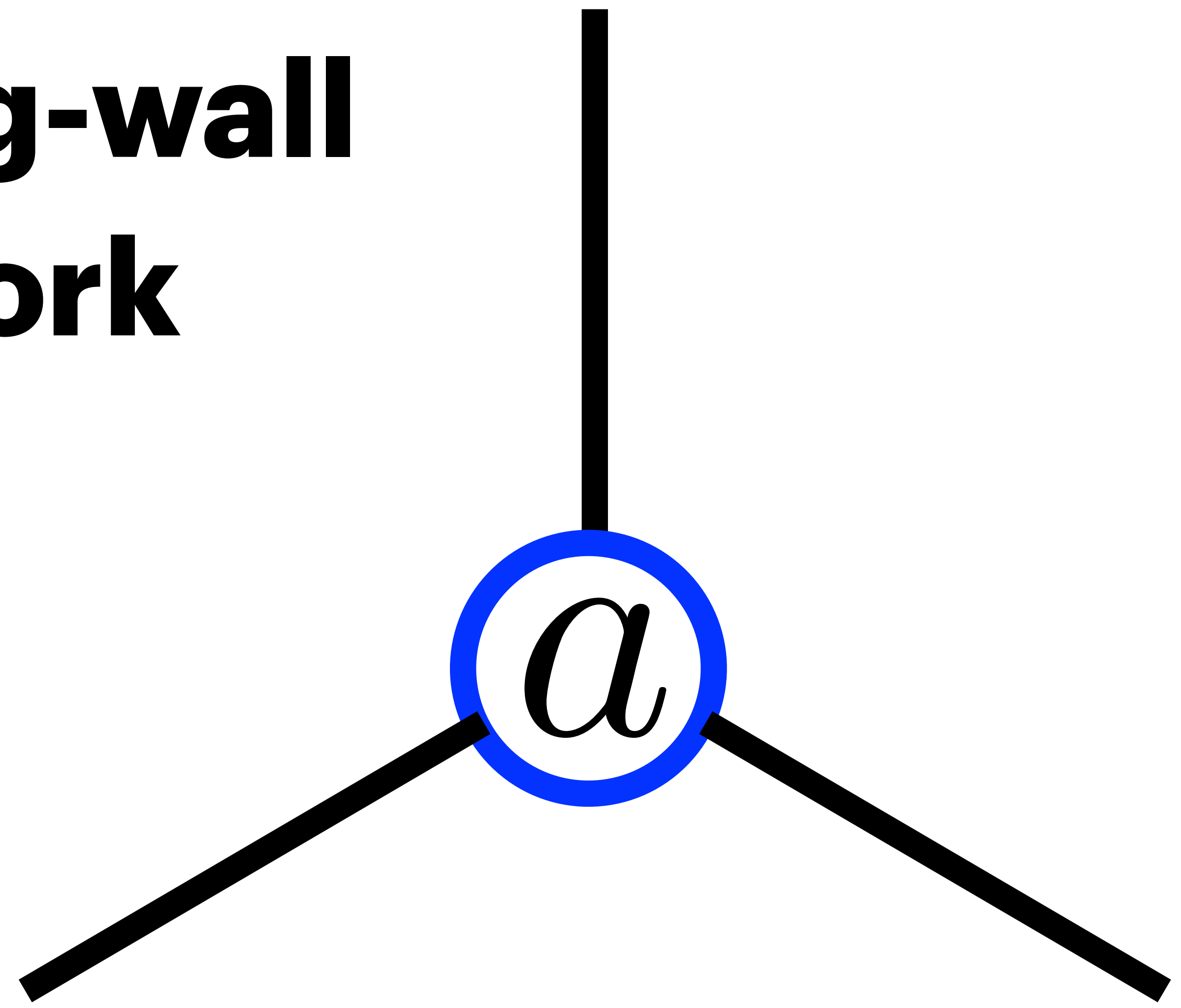
$N_{\rm DW} = 3$ or 6 for DFSZ axion

φ : Massless (or very light) axion

e.g. KSVZ axion

(different from a)

String-wall network



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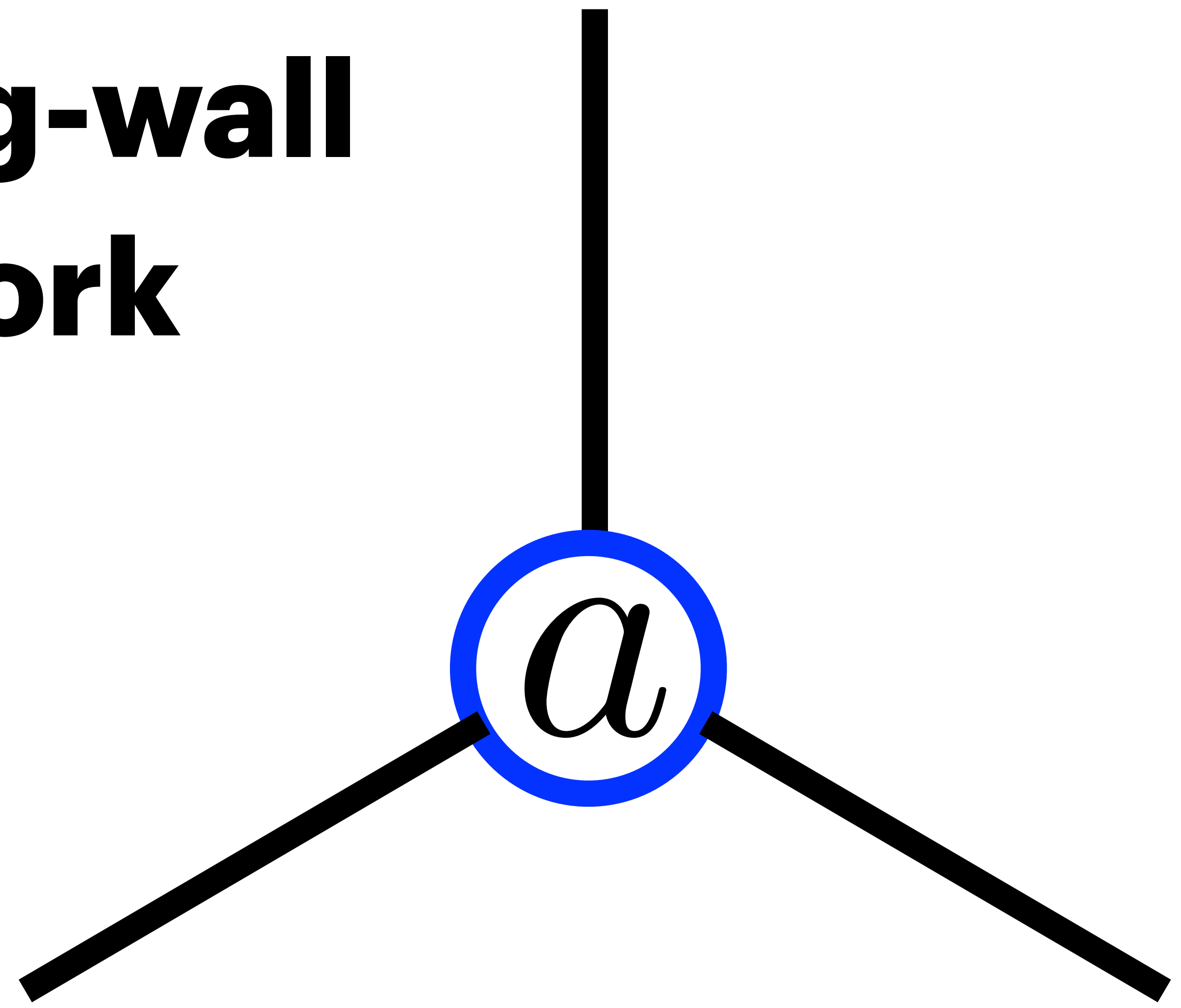
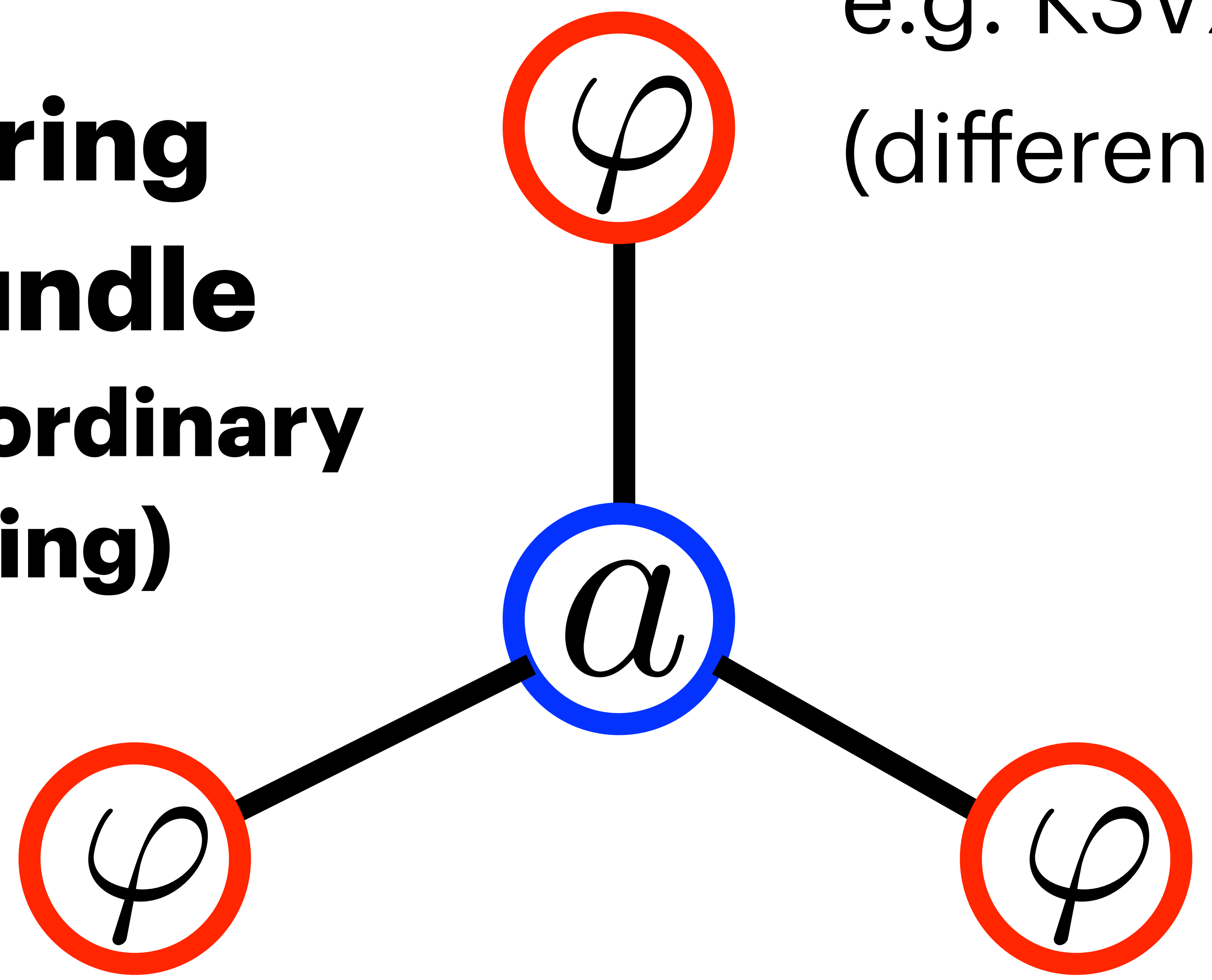


String bundle
(= ordinary string)



New solution to the QCD axion DW problem

FT and Yin 2012.11576, Lee, Murai, FT and Yin 2507.07075,

Adding another light axion can solve the DW problem by converting the string-wall network into string bundes.

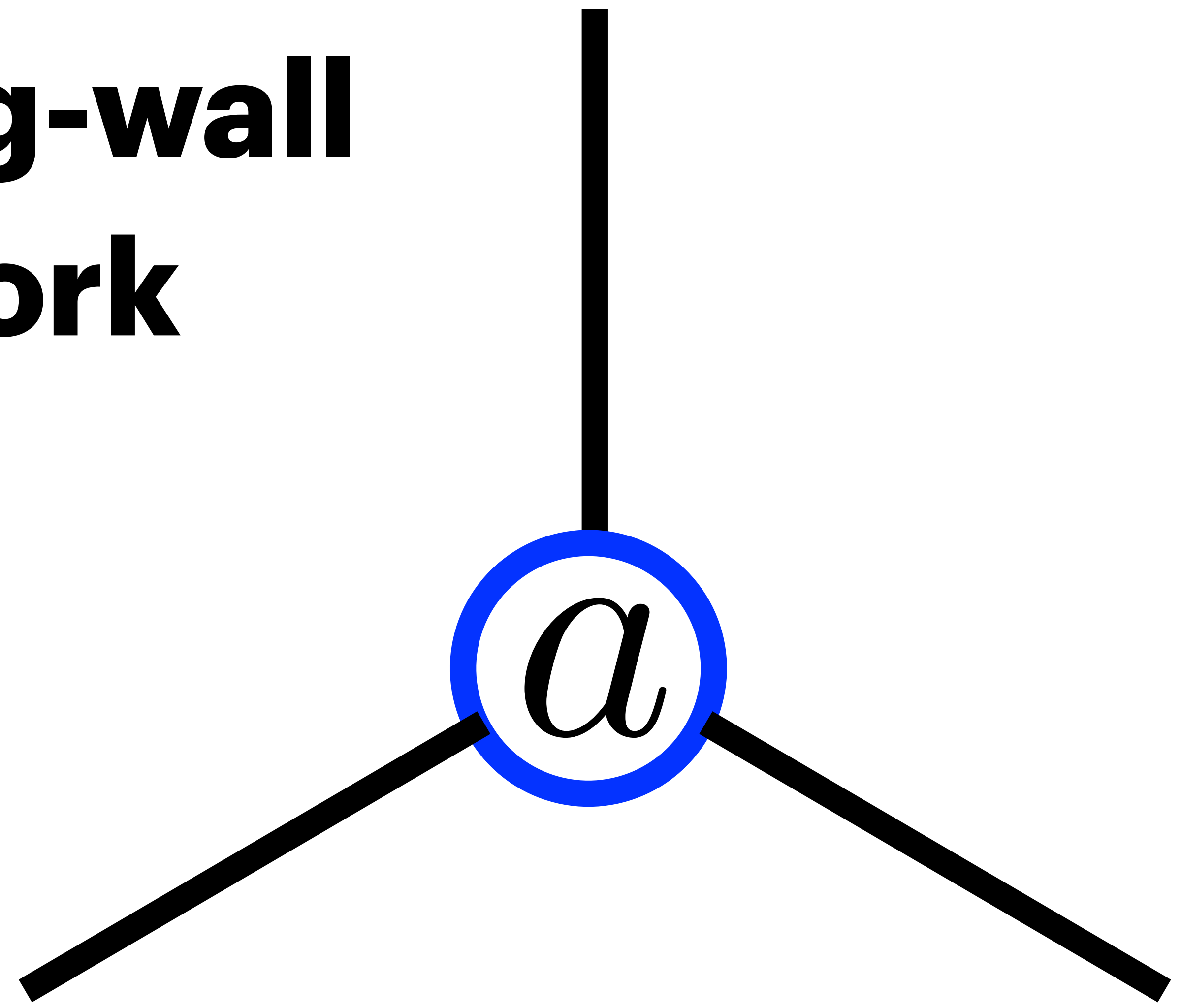
$N_{\rm DW} = 3 \text{ or } 6$ for DFSZ axion

φ : Massless (or very light) axion

e.g. KSVZ axion

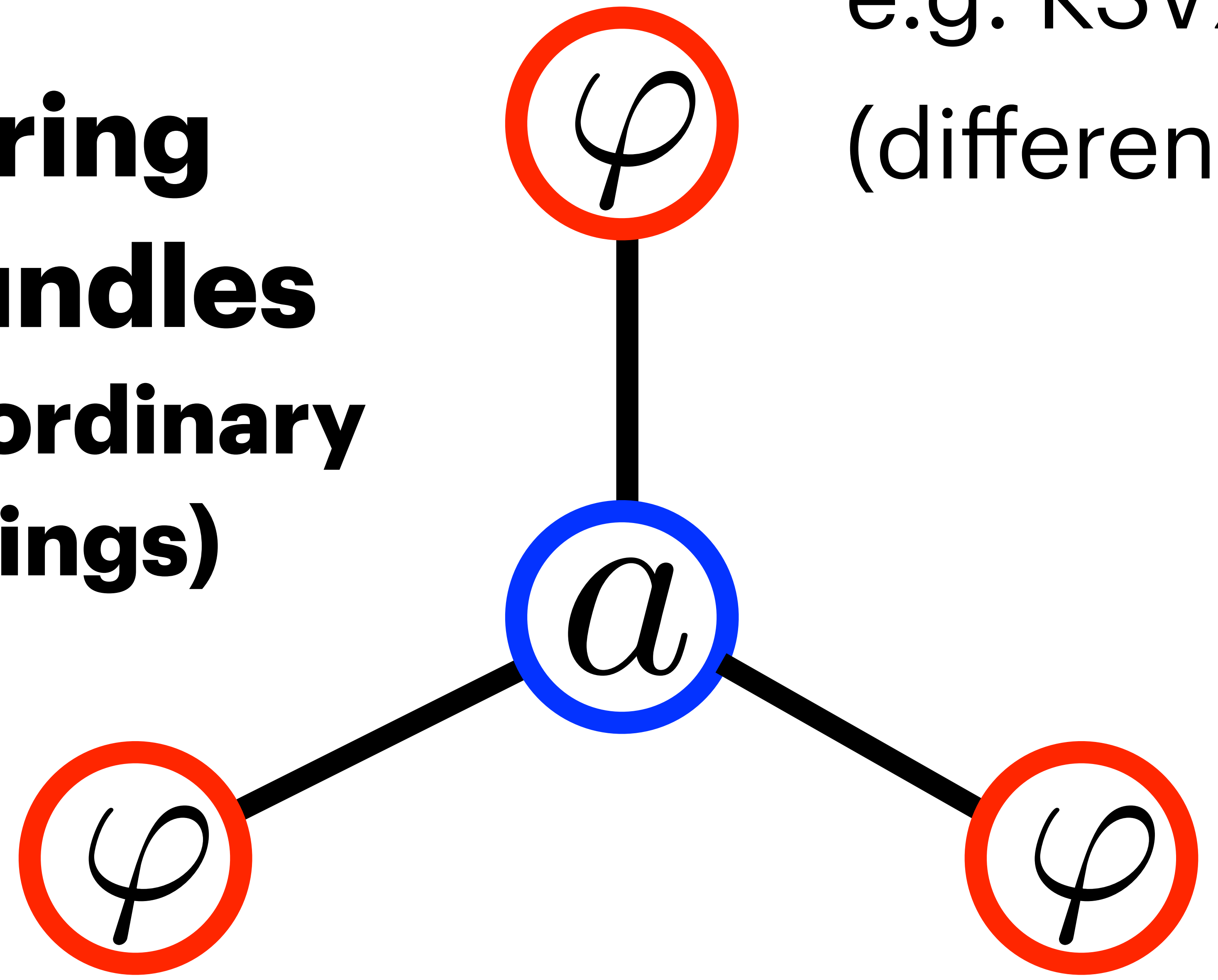
(different from a)

String-wall network



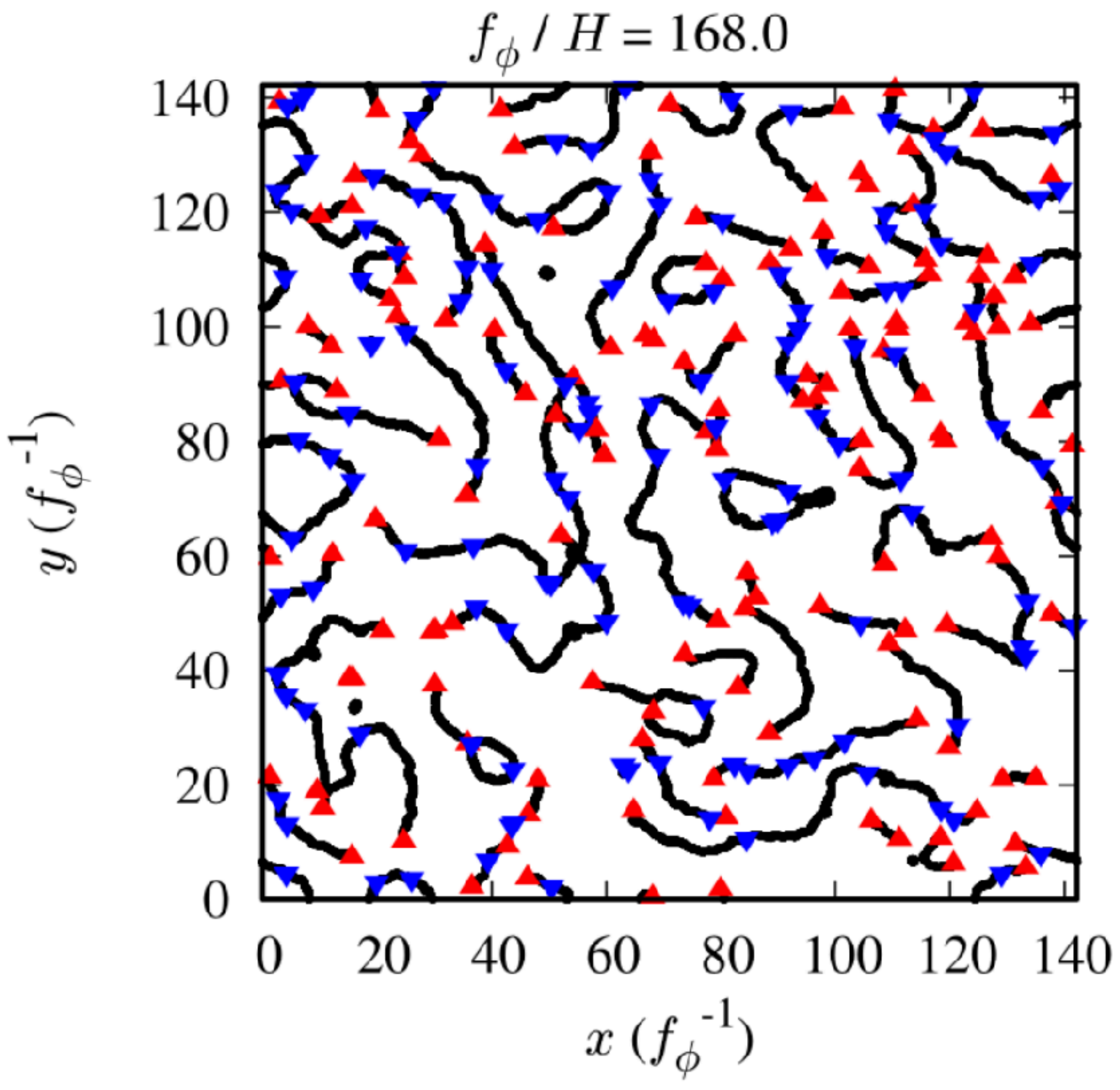
String bundles

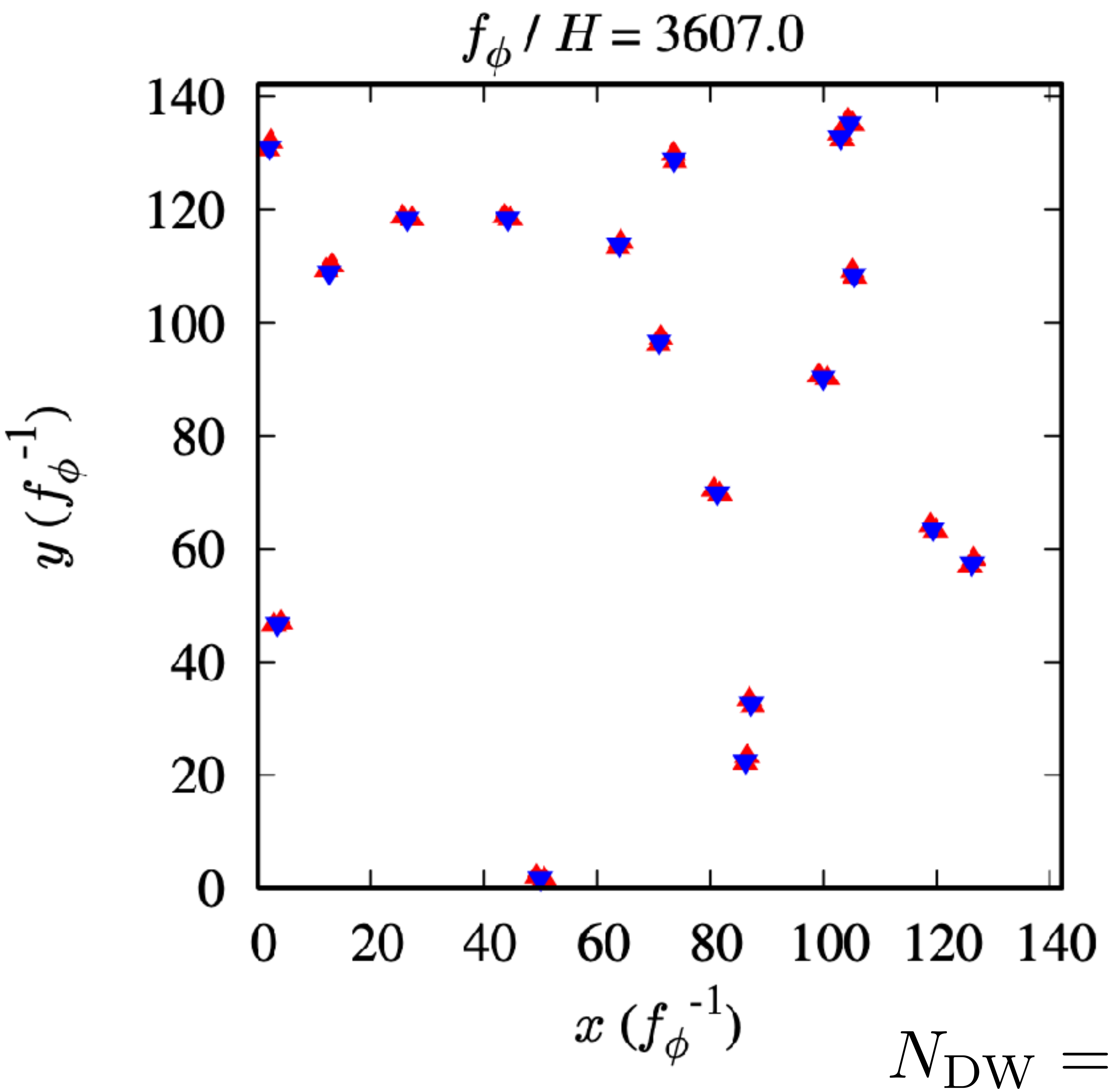
(= ordinary strings)



New solution to the QCD axion DW problem

Lee, Murai, FT and Yin 2507.07075,



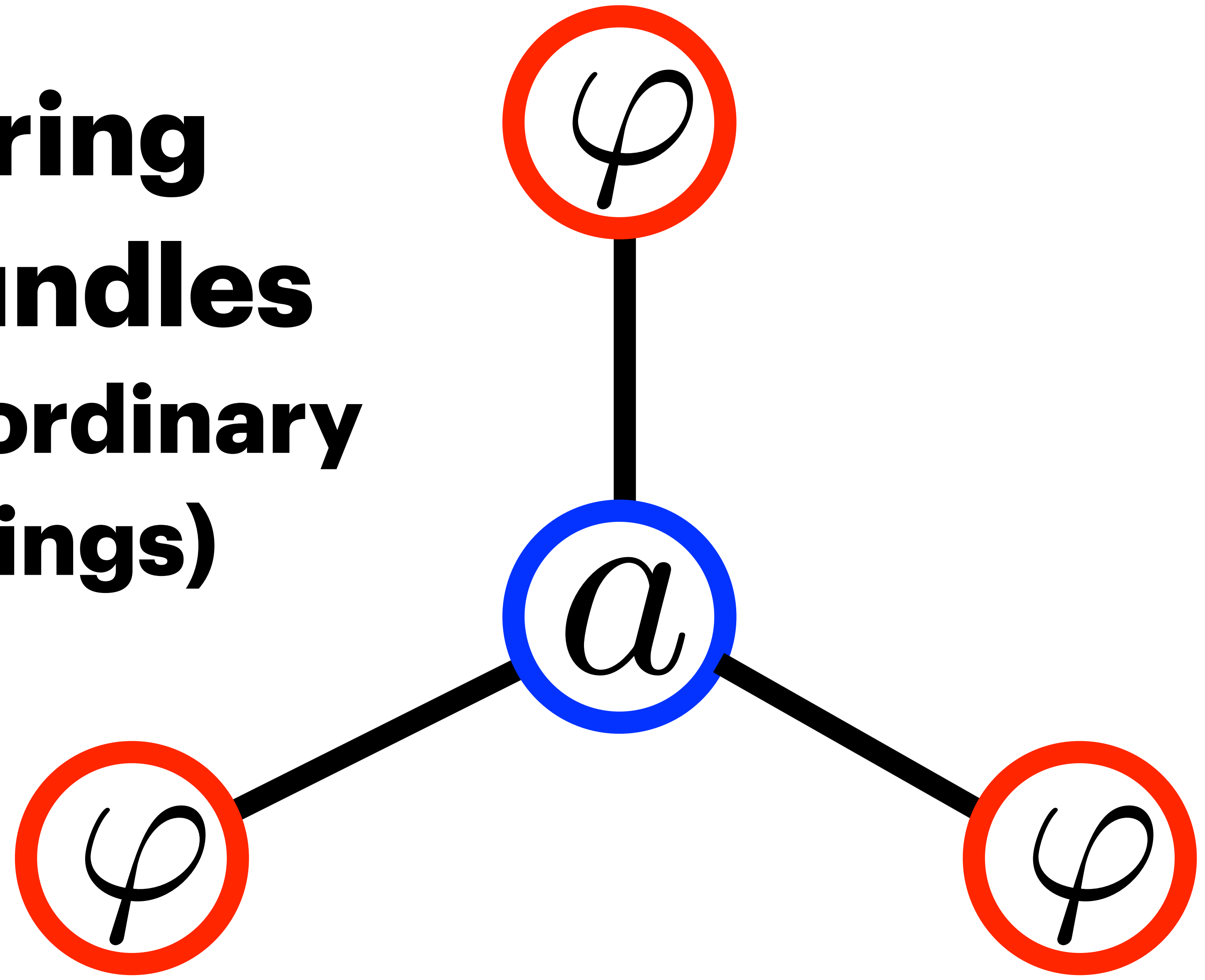


$N_{\rm DW} = 2$

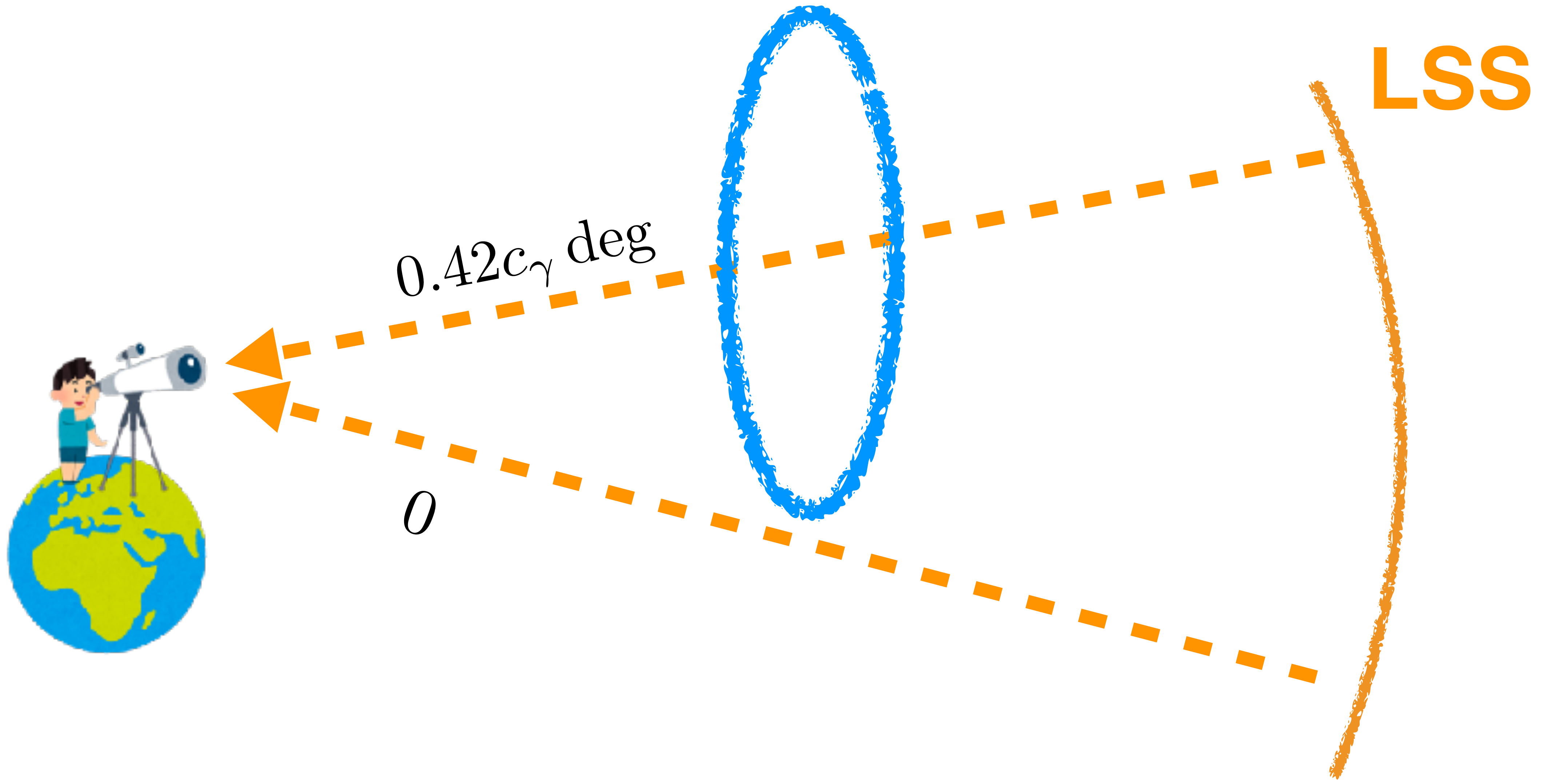
New solution to the QCD axion DW problem

Lee, Murai, FT and Yin 2507.07075, FT and Yin 2012.11576

String bundles (= ordinary strings)

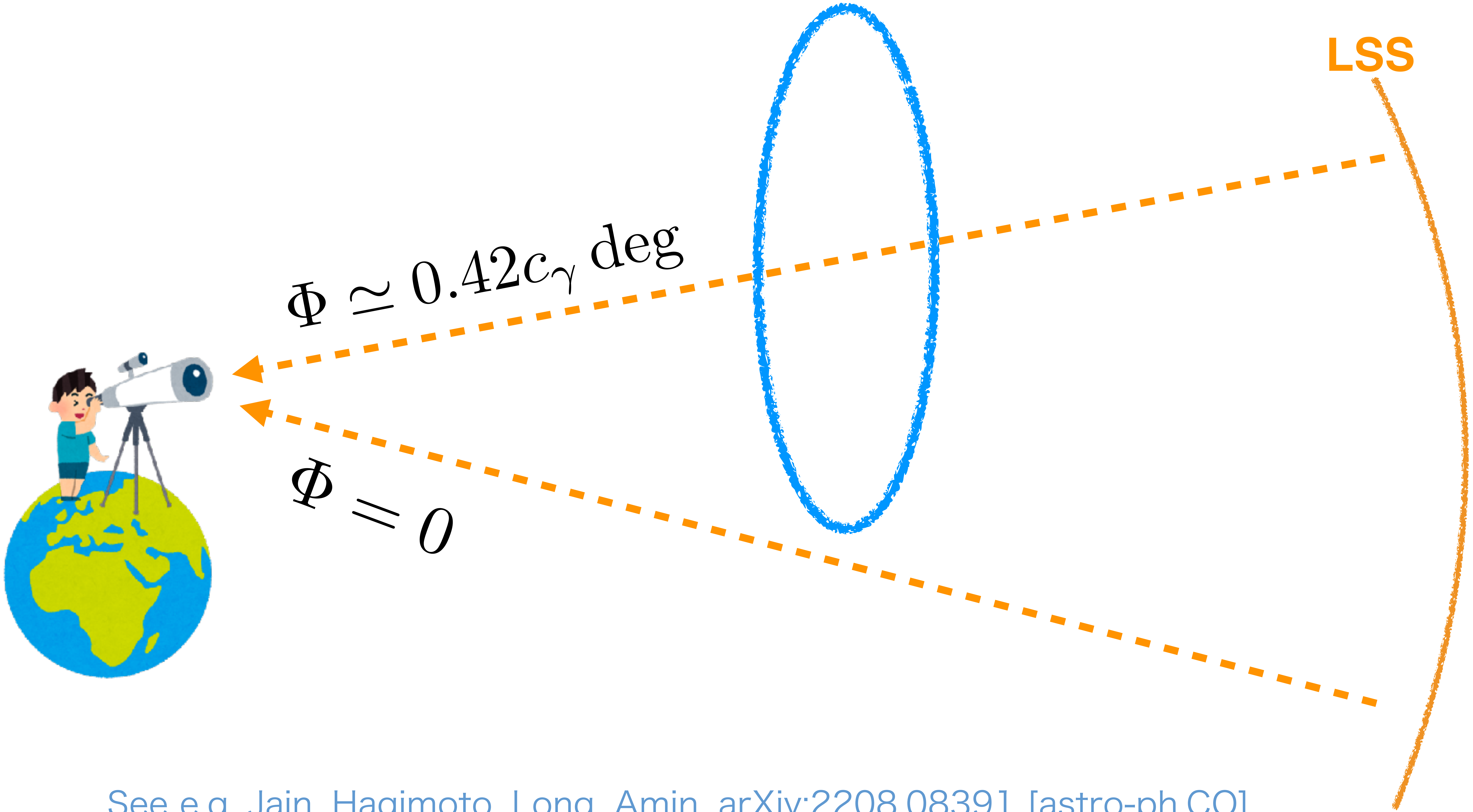


Unlike the usual QCD axion strings, the string bundles are long-lived, and could contribute to the cosmic birefringence!



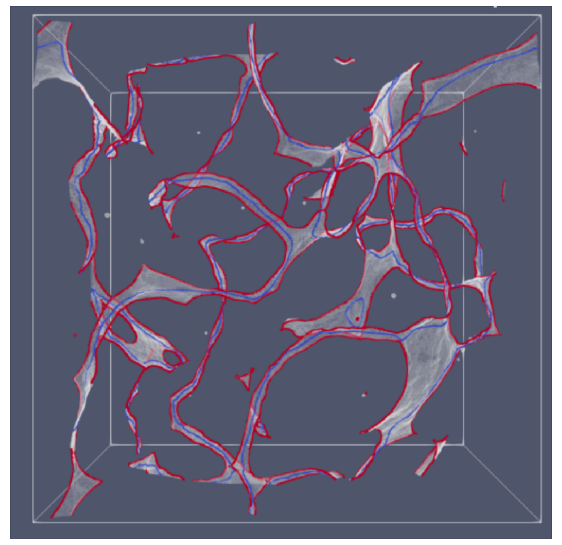
See e.g. Jain, Hagimoto, Long, Amin, 2208.08391

$$\mathcal{L}_{\phi\gamma} = -c_\gamma \frac{\alpha}{4\pi}\frac{\phi}{f_\phi}F_{\mu\nu}\tilde{F}^{\mu\nu} = -\frac{1}{4}g_{\phi\gamma\gamma}\phi F_{\mu\nu}\tilde{F}^{\mu\nu},$$

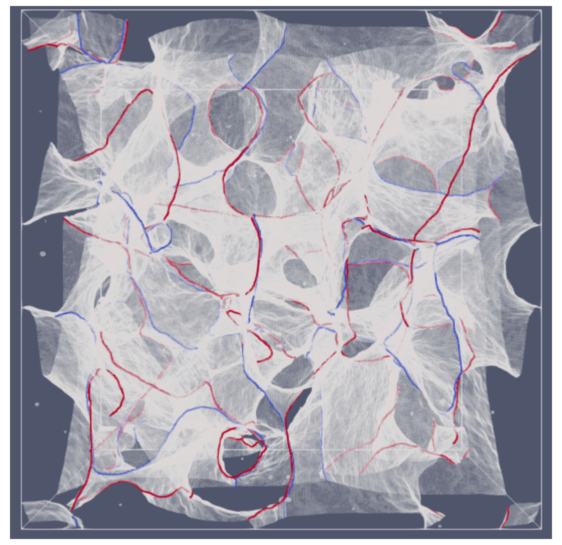


See e.g. Jain, Hagimoto, Long, Amin, arXiv:2208.08391 [astro-ph.CO].

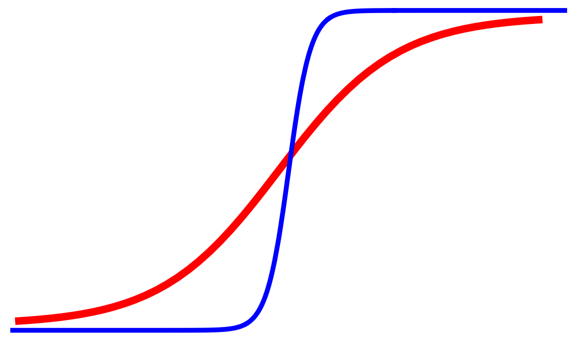
Summary



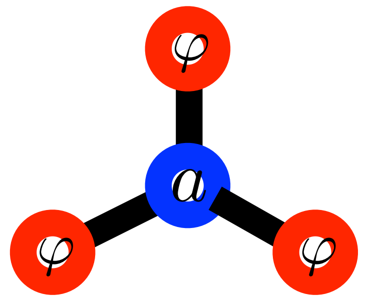
- The origin and breaking of U(1) PQ are unknown.



- A minimal extension from one to two PQ scalars often leads to stable DWs with large tension.
- Their decay can produce GWs and QCD axion DM even for small f_a.



- Adding another light axion can solve the QCD axion DW problem, leading to stable string bundles.



In multi-axion systems, cosmology changes drastically —
More is different!