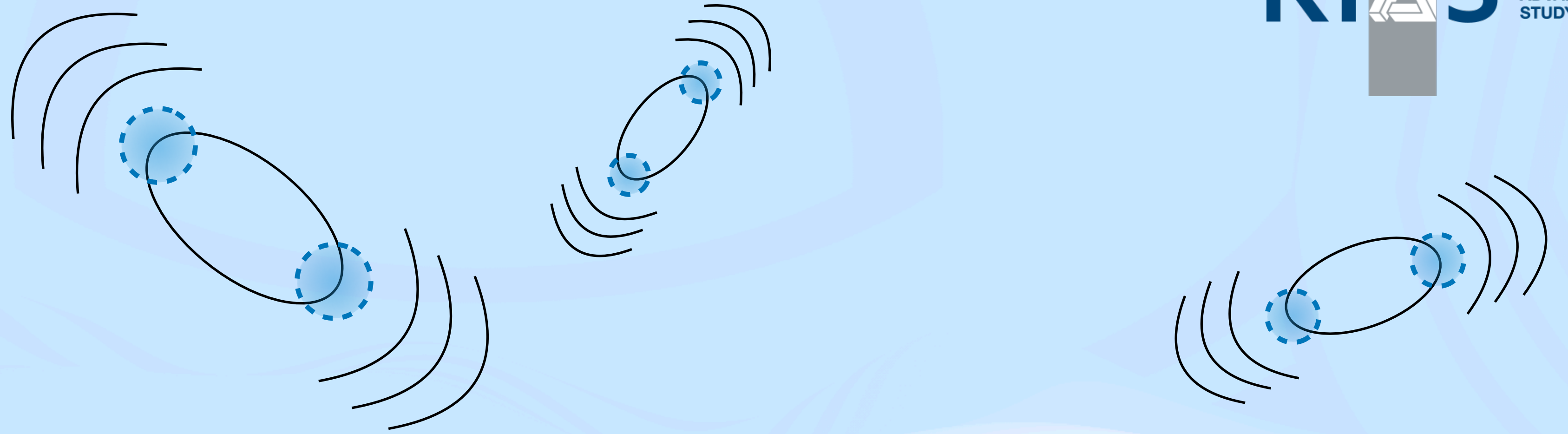




Gravitational Waves from the Early Universe: A Primordial Black Hole Perspective

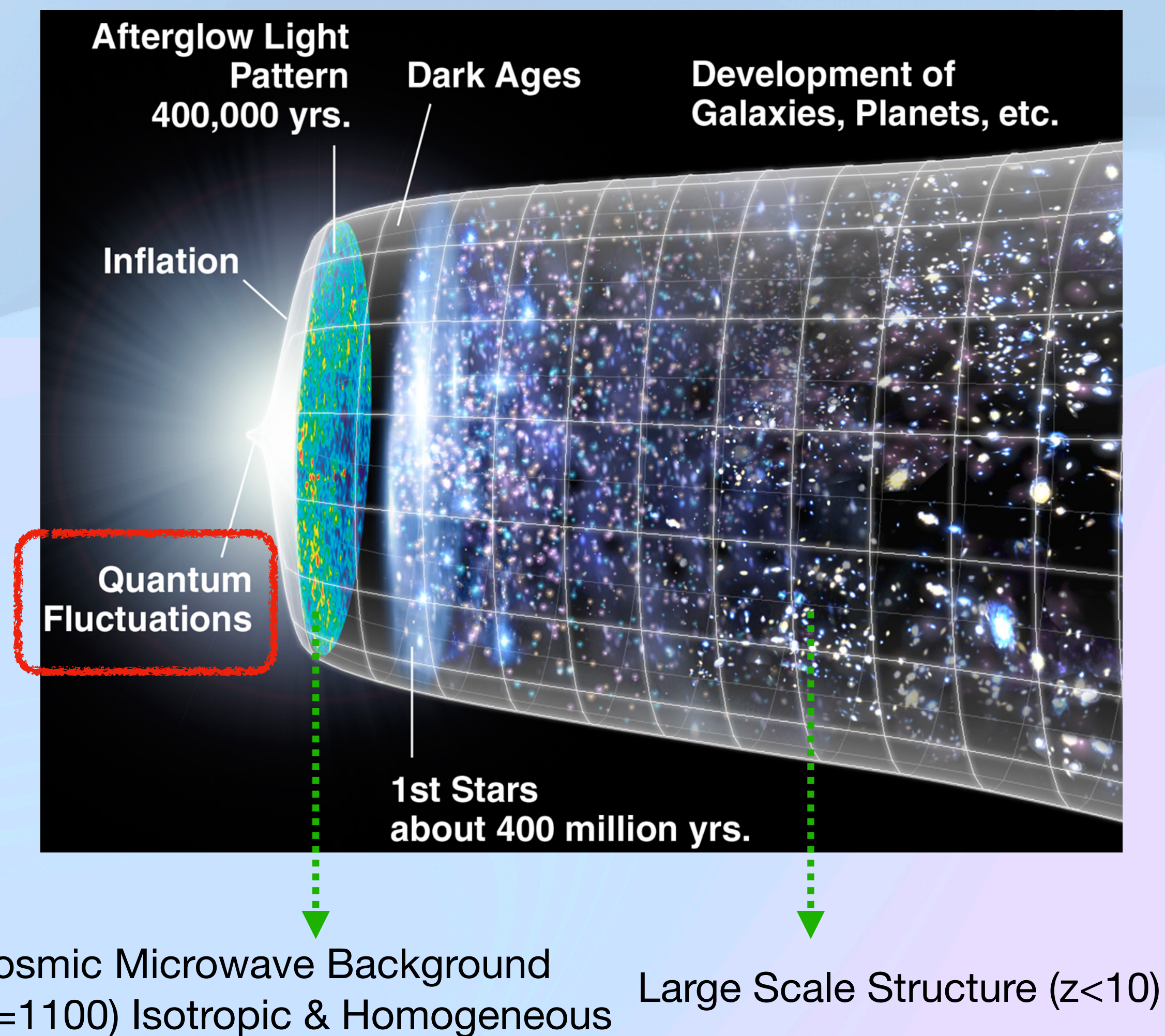
Xinpeng Wang (Kavli IPMU, The University of Tokyo)

In collaborations with:

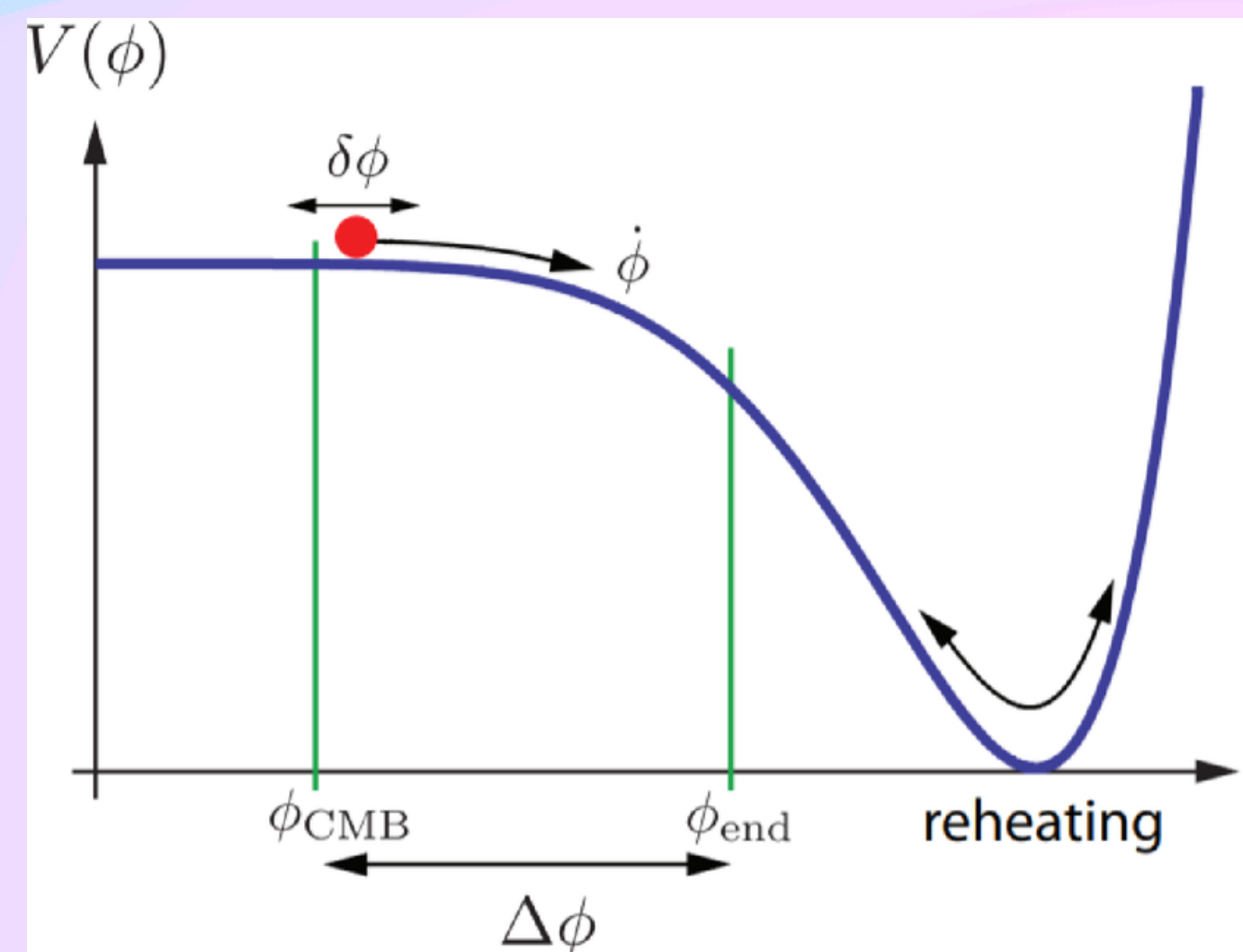
Qianhang Ding (IBS), Jinsu Kim (Tongji), Kazunori Kohri (KEK/NAOJ), Alexander Kusenko (UCLA), Yifan Lu (UCLA), Zachary S.C. Picker (UCLA), Misao Sasaki (IPMU&APCTP), Masahide Yamaguchi (IBS), Tsutomu Yanagida (IPMU), Ying-li Zhang (Tongji).

O. A Brief Introduction to Inflation and Primordial Black Holes

Inflation



Inflation (A primordial phase before Big Bang)
 A phase of semi-exponential accelerating expansion of space solving the monopole, horizon, and flatness problems.



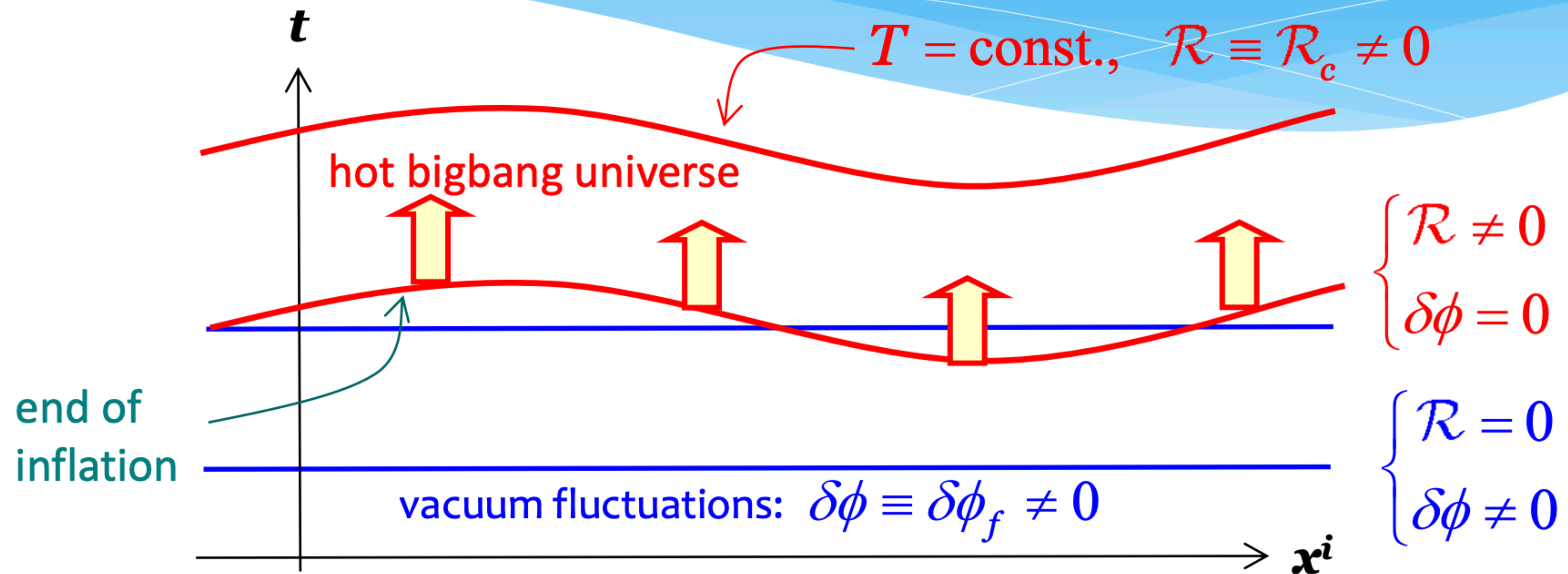
Growth of scale factor

$$\frac{a_{\text{end}}}{a_{\text{CMB}}} = e^{50 \sim 60}$$

Single field slow-roll inflation (Linde, 1983,)

From field perturbation $\delta\phi$ to curvature perturbation

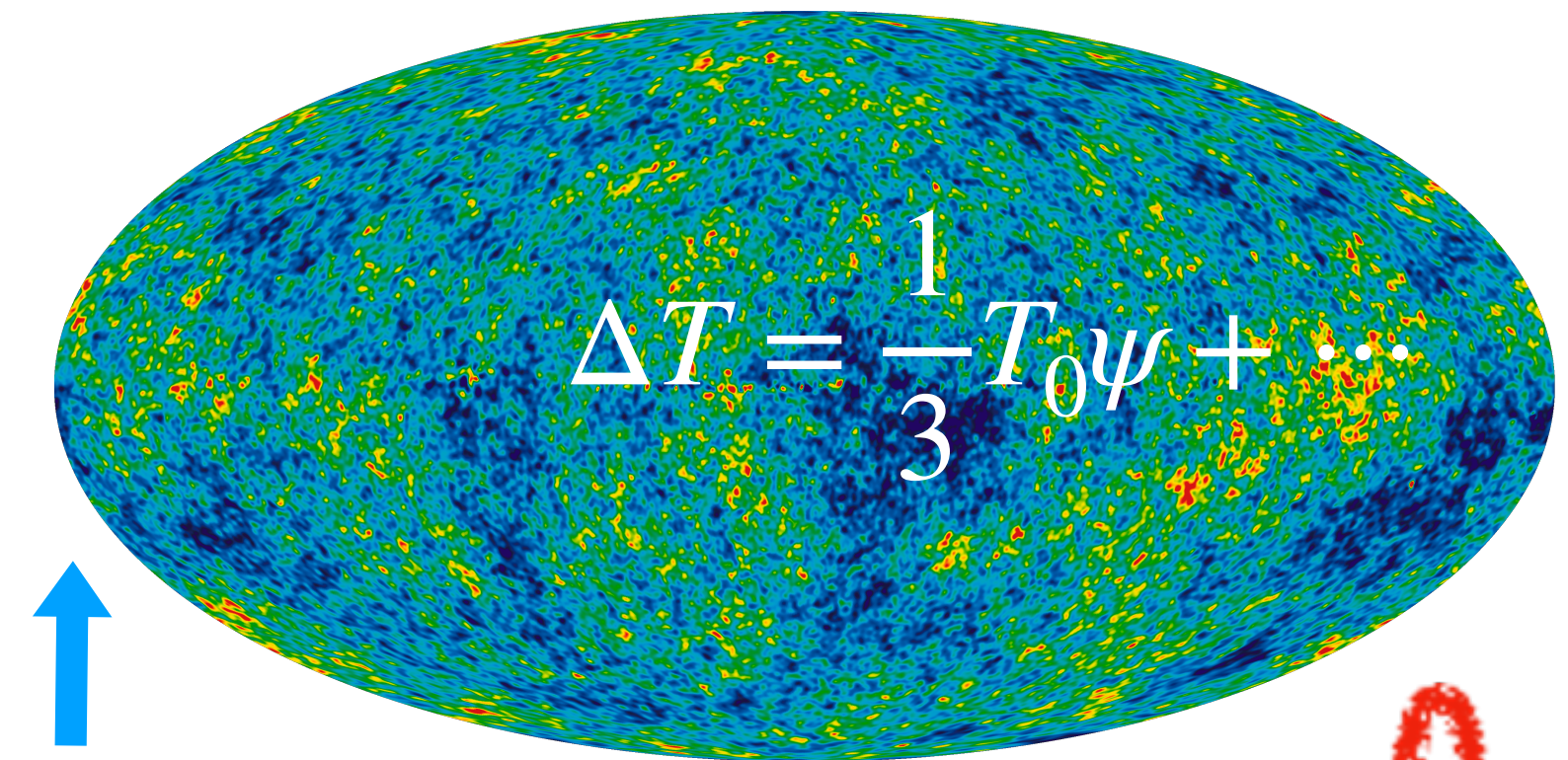
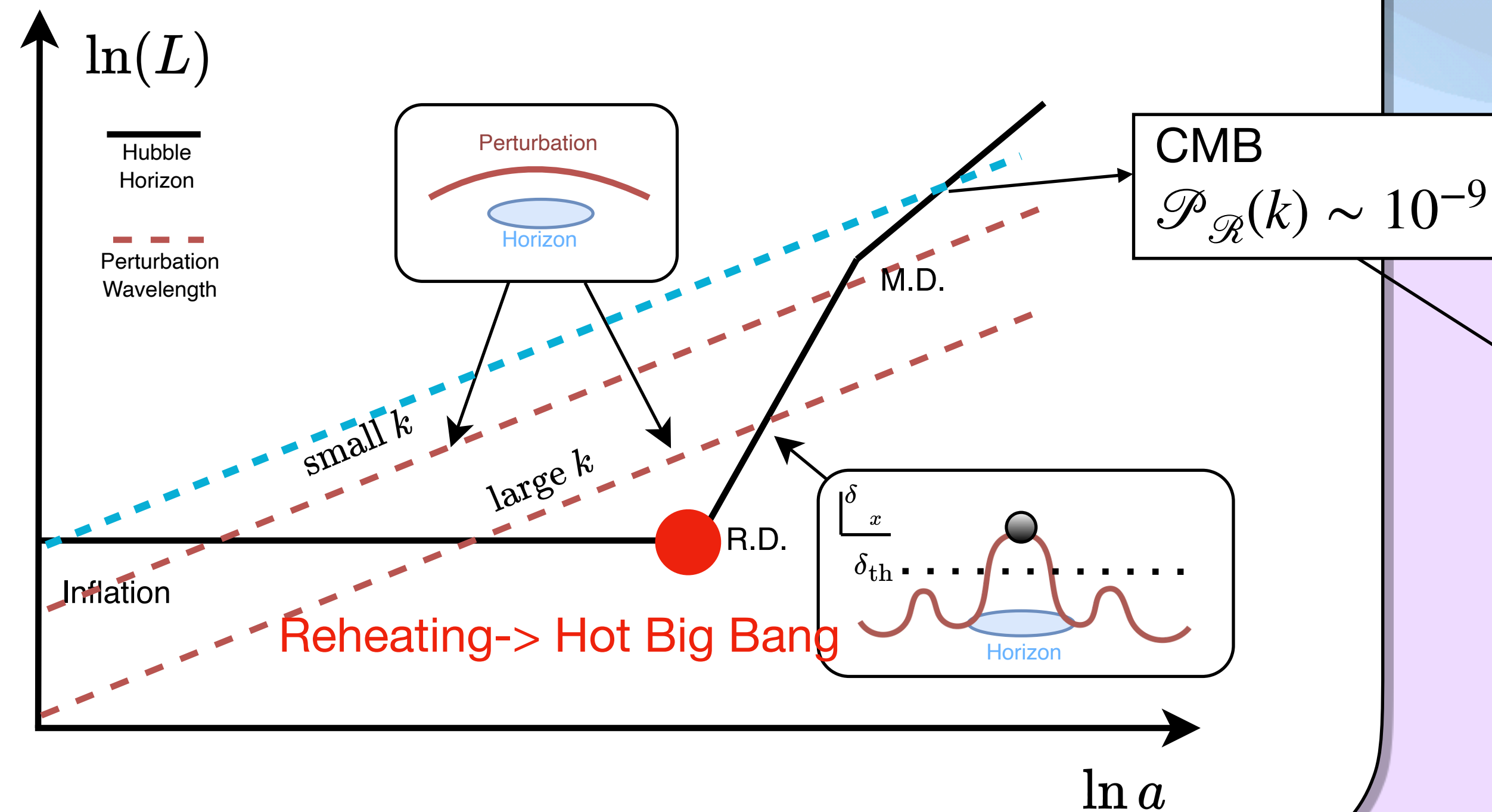
- Inflation ends/damped osc starts on “comoving” ($\phi = \text{const.}$) 3-surface.



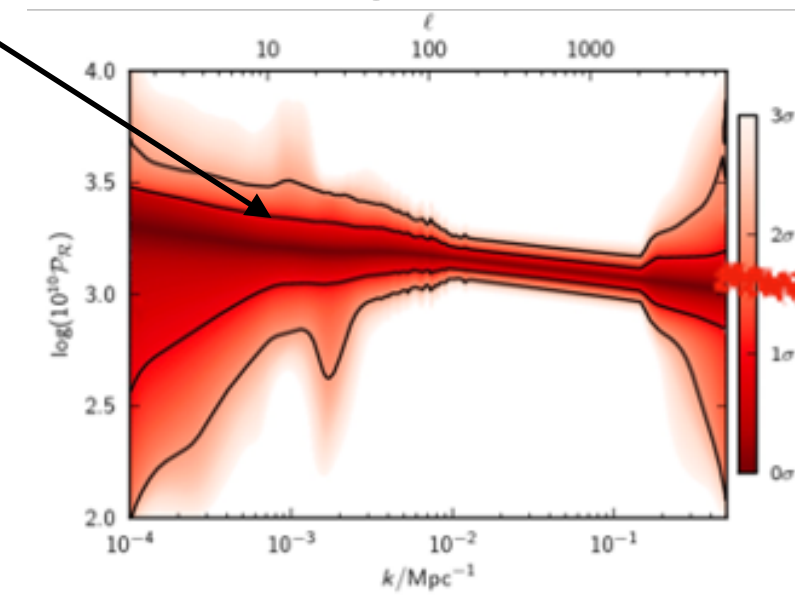
- On $\phi = \text{const.}$ surface, curvature perturbation appears $\mathcal{R} \equiv \mathcal{R}_c = -\frac{H}{\dot{\phi}} \delta\phi_f$

Primordial Black Hole seeded from Inflation

Hubble Radius VS scale factor



**Observed $\mathcal{P}_{\mathcal{R}}(k)$
@ Large scale**

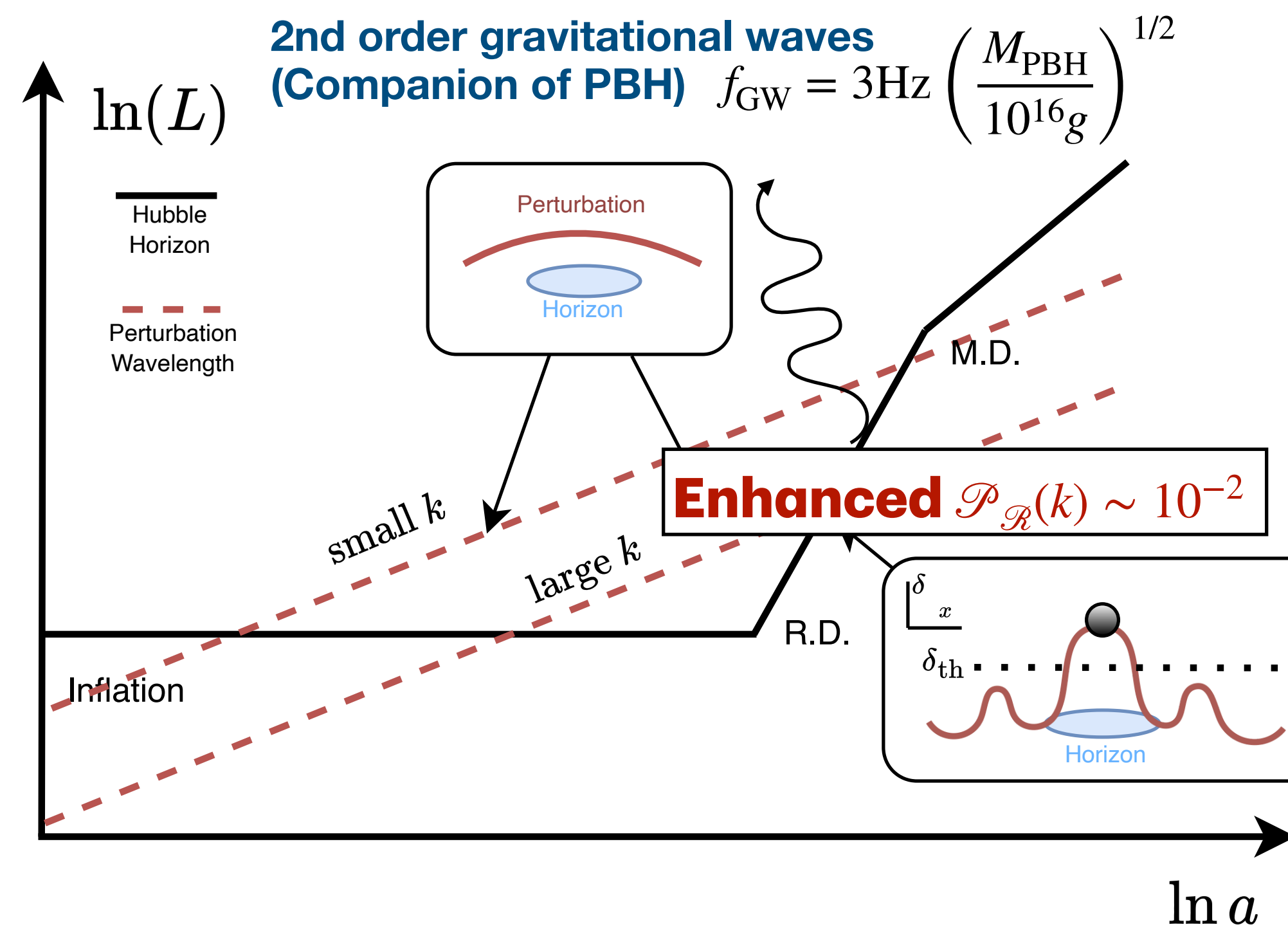


**Enhanced $\mathcal{P}_{\mathcal{R}}(k)$
@ Small scale**

The Planck Collaboration, 2018 (1807.06209)

Primordial Black Hole seeded from Inflation

Hubble Radius VS scale factor

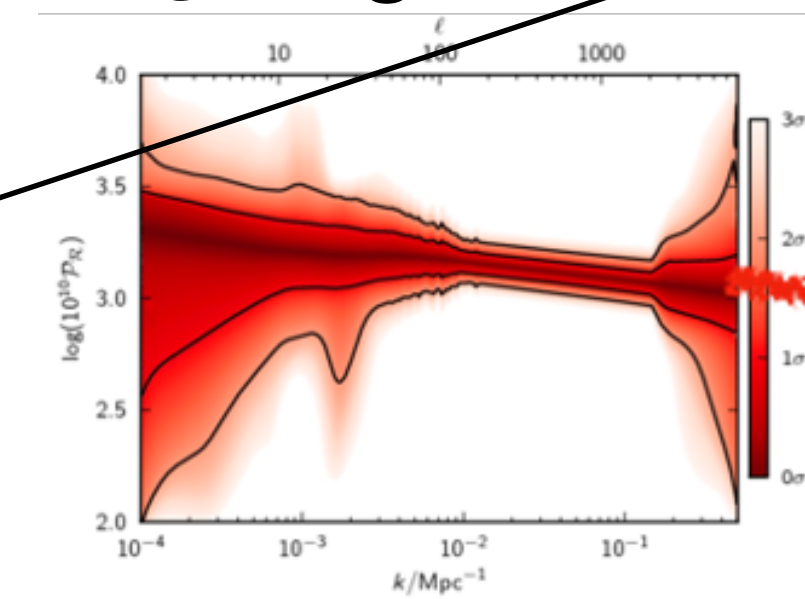


$$\delta \propto k^2 R^2 \mathcal{R} \gtrsim \delta_{\text{th}}$$

Hubble size region with $\delta \sim \mathcal{O}(1)$ collapses to form BH

Our Goal!!

**Observed $\mathcal{P}_{\mathcal{R}}(k)$
@ Large scale**

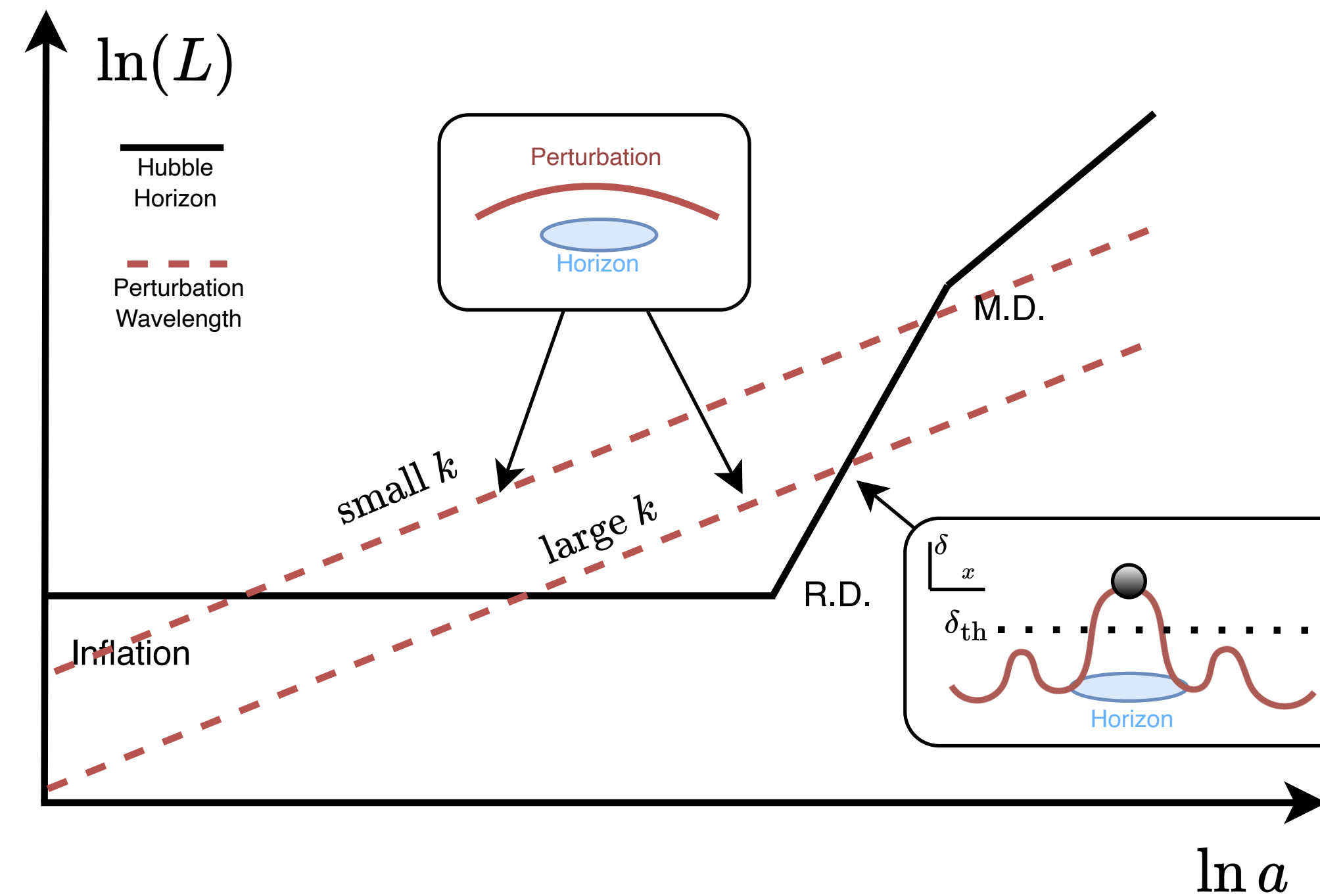


**Enhanced $\mathcal{P}_{\mathcal{R}}(k)$
@ Small scale**

The Planck Collaboration, 2018 (1807.06209)

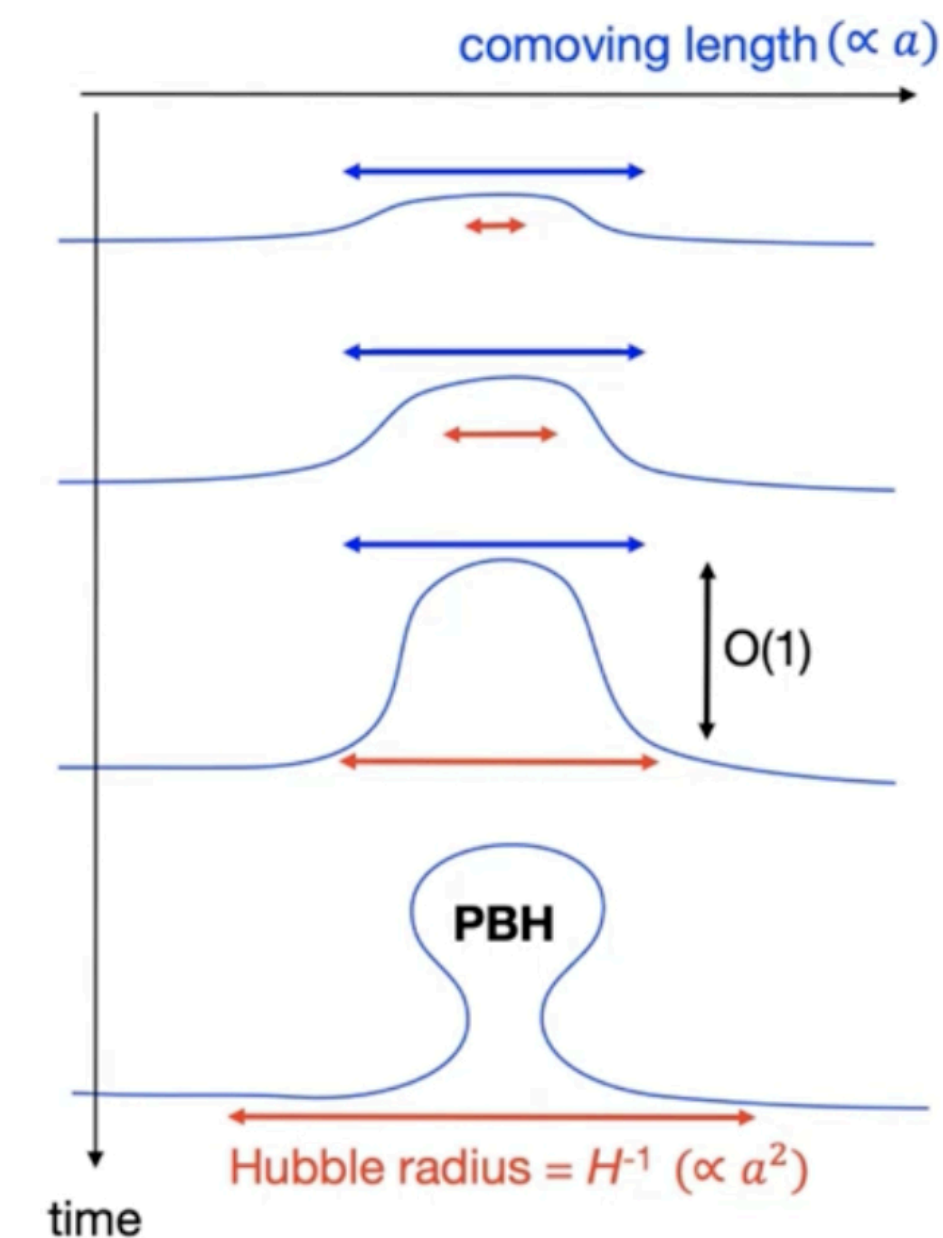
Primordial Black Hole seeded from Inflation

Hubble Radius VS scale factor



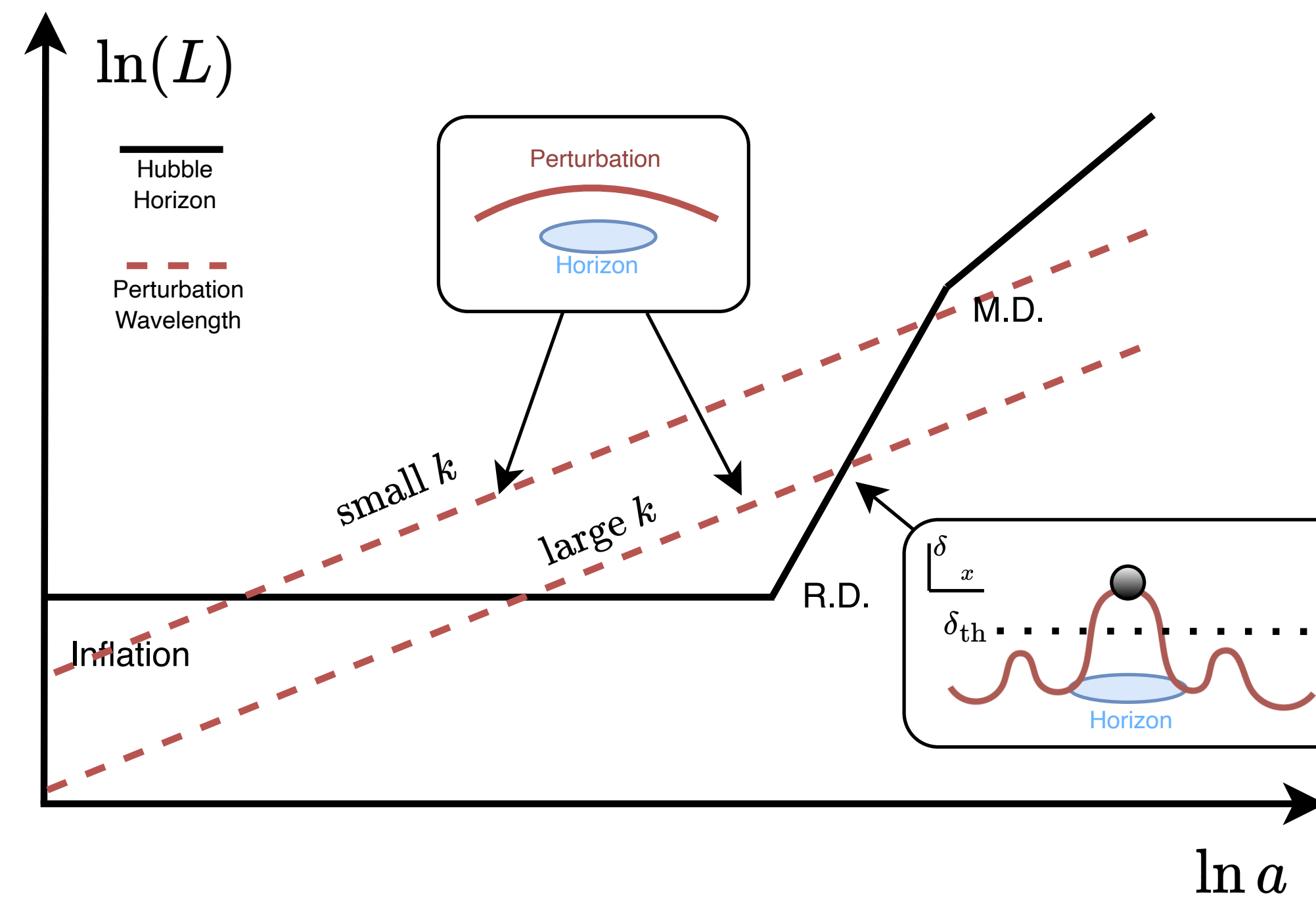
$$\delta \propto k^2 R^2 \mathcal{R} \gtrsim \delta_{th}$$

Hubble size region with $\delta \sim \mathcal{O}(1)$ collapses to form BH

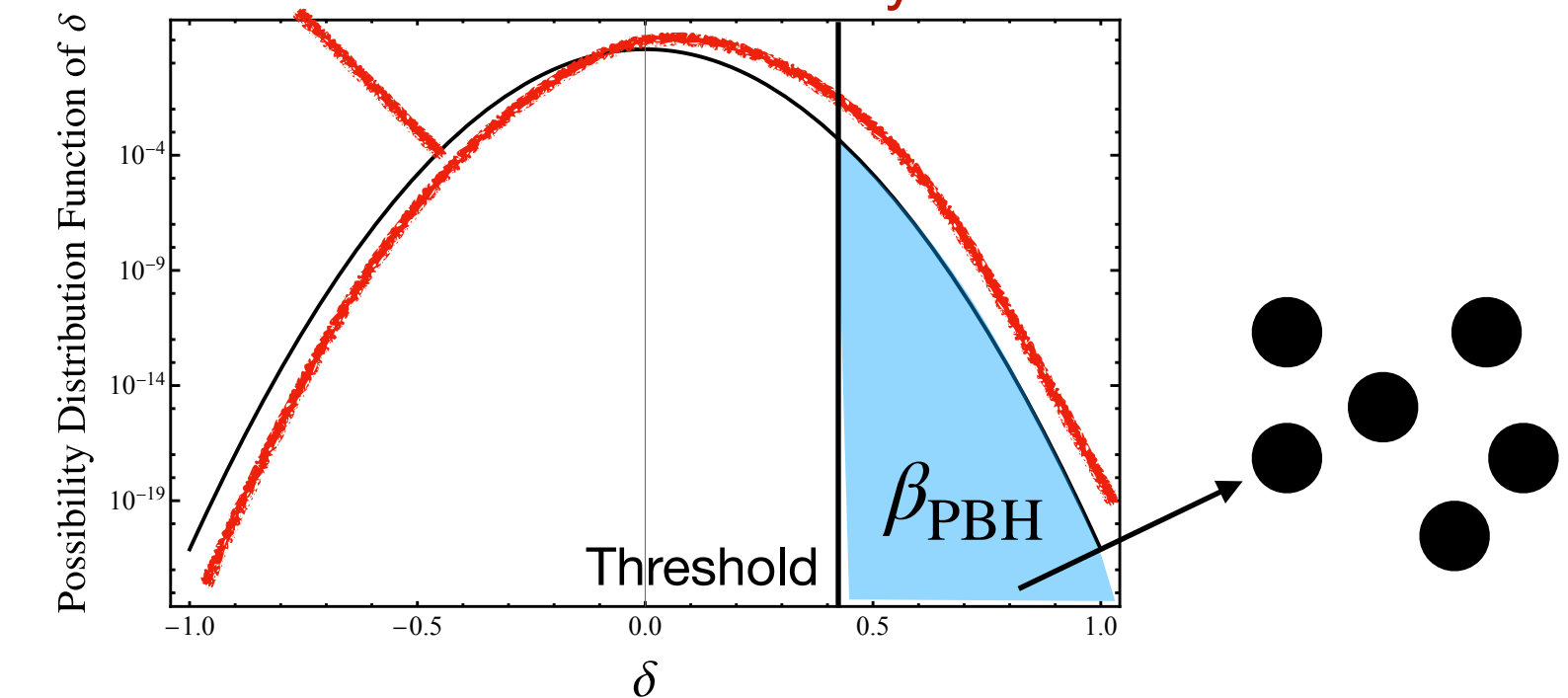


Primordial Black Hole seeded from Inflation

Hubble Radius VS scale factor



Primordial Non-Gaussianity can make differences

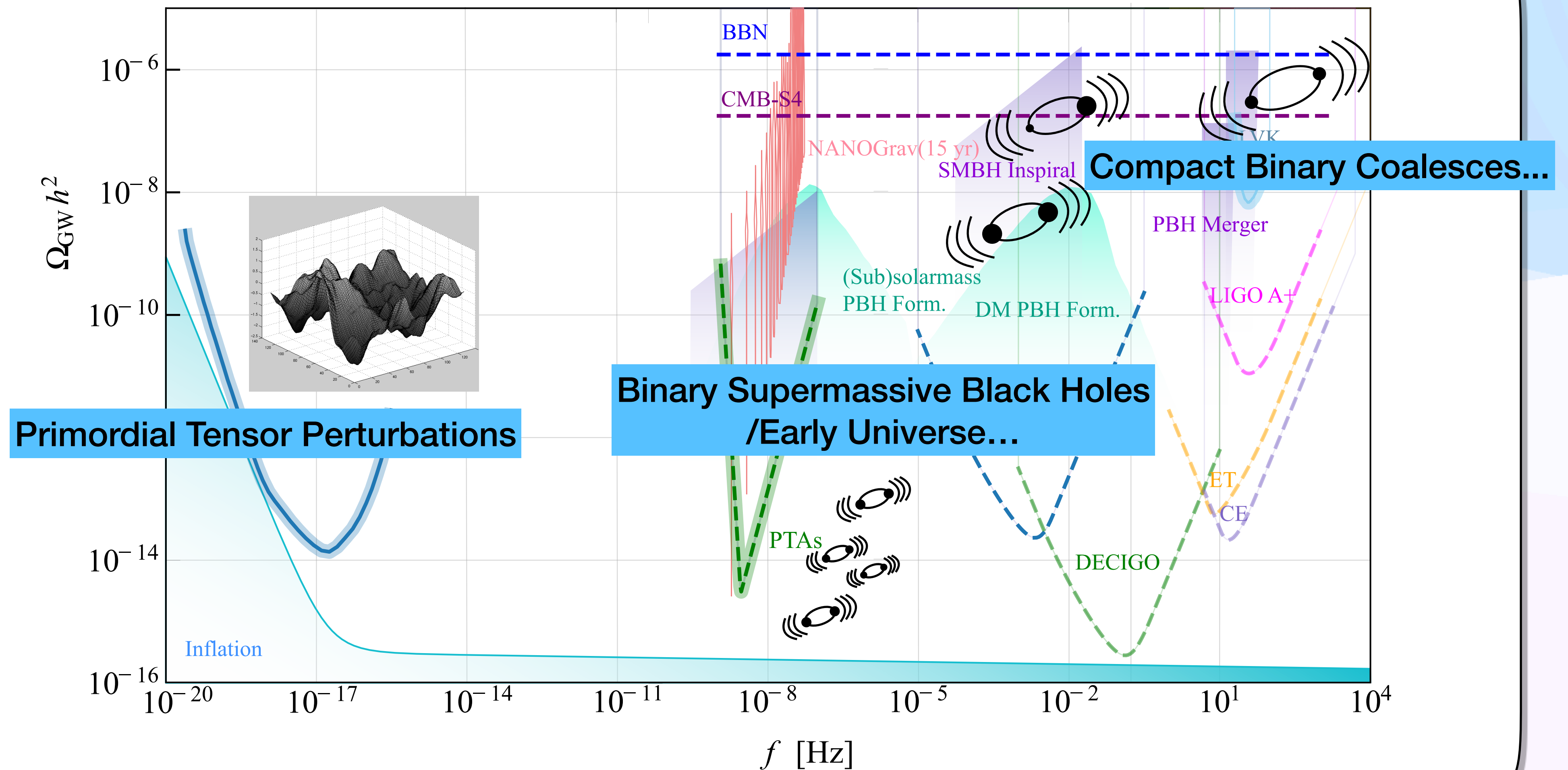


Using Press-Schechter method to estimate the PBH abundance β at formation (Gaussian case):

$$\beta(M_{\text{PBH}}) \approx \frac{1}{\sqrt{2\pi\sigma_\delta^2}} \int_{\delta_{\text{th}}}^{\infty} \exp\left(-\frac{\delta^2}{2\sigma_\delta^2}\right) d\delta$$

σ_δ^2 — — — variance of δ (typical size of fluctuations)

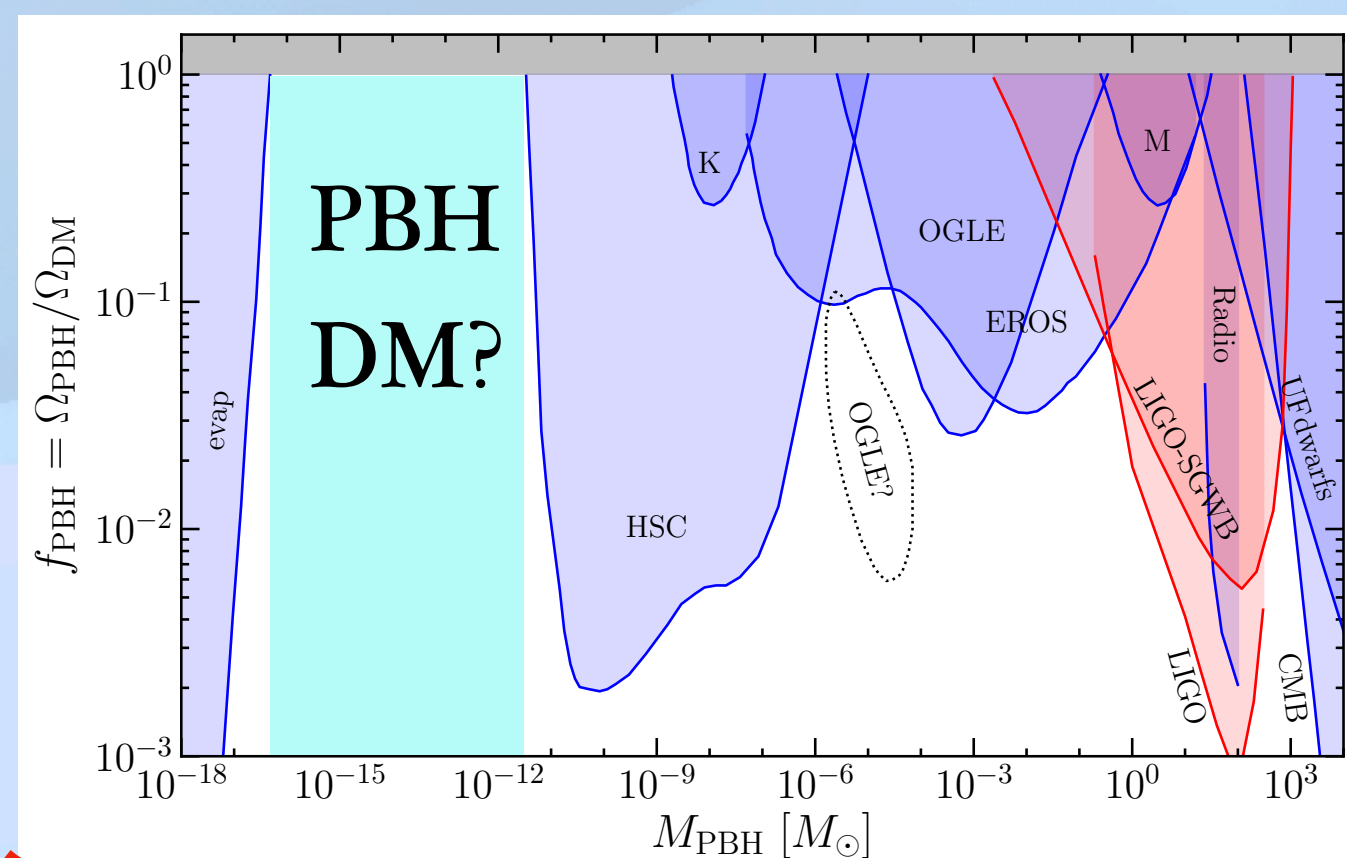
Gravitational Waves : A Great Probe to the Dark



Primordial Black Holes and Gravitational Waves

Companion: Deci hertz GW
Signals in LISA/LVK band (future)

Dark Matter PBH : $10^{17} \sim 10^{21} \text{g}$



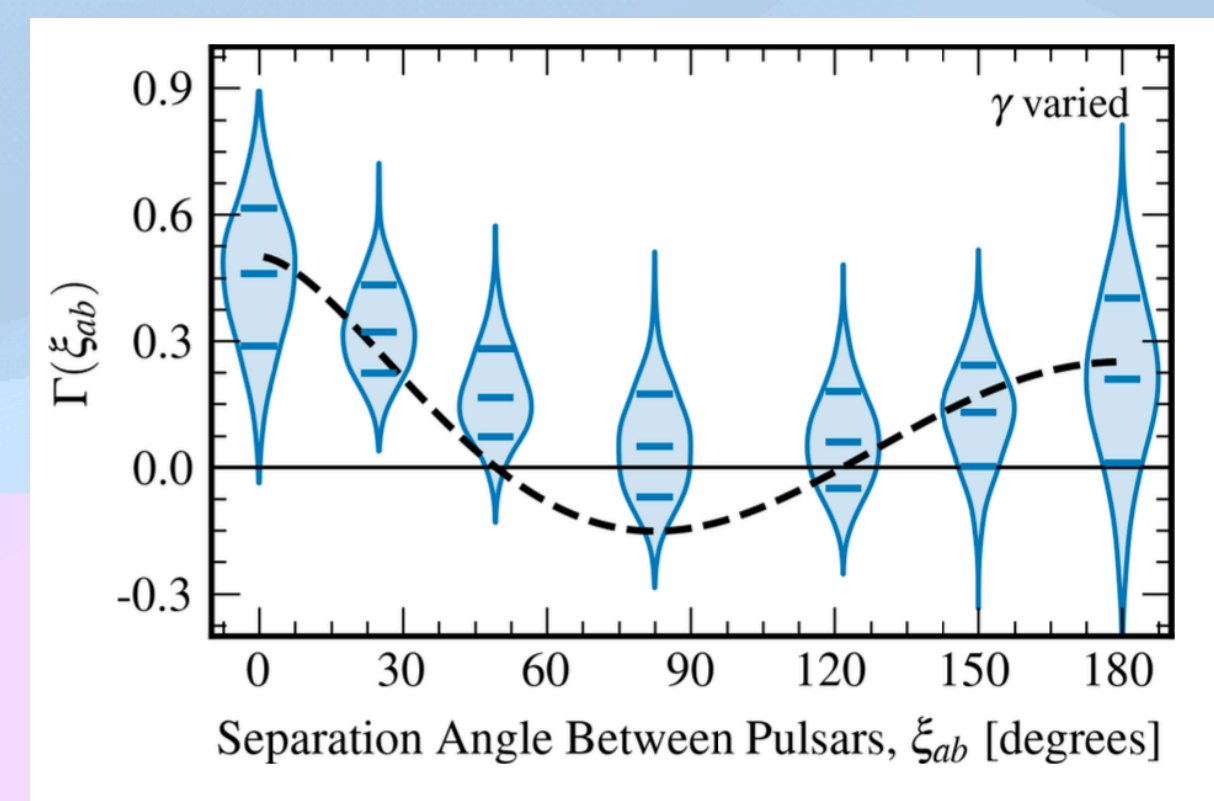
Seeds of Supermassive Black Holes $> 10^2 M_\odot$



Companion:
Nanohertz GW
Signals detected by
PTAs

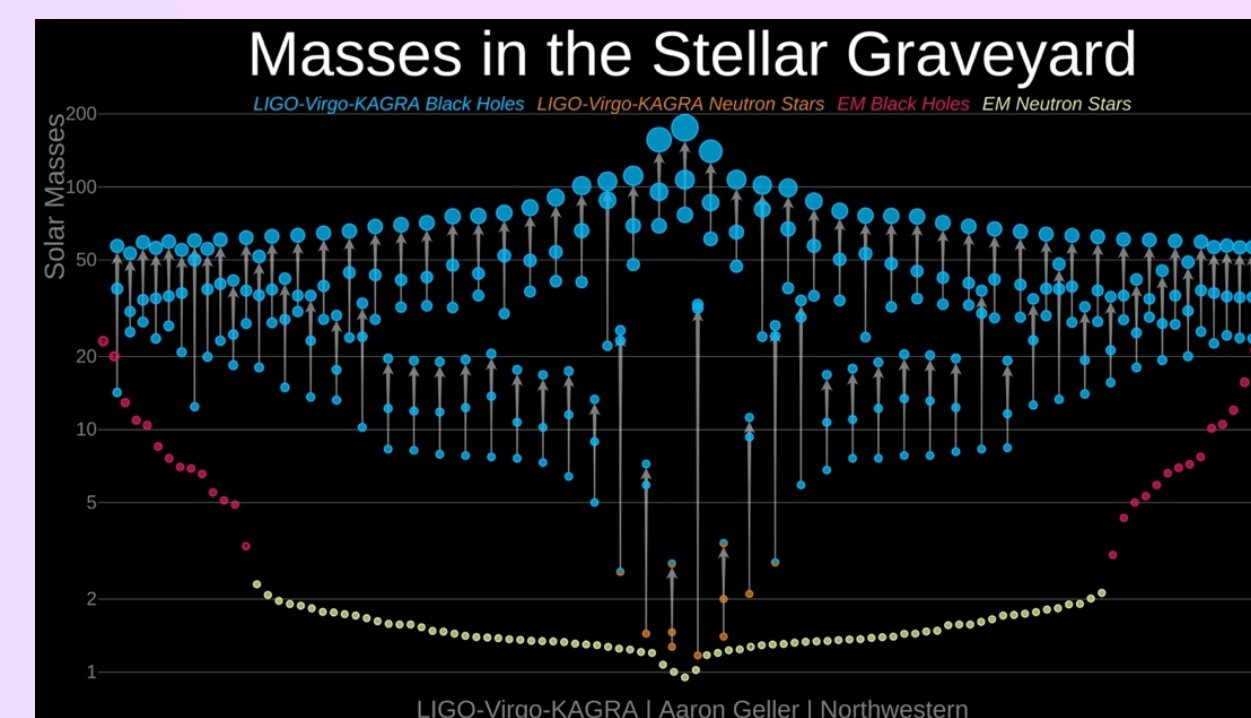
PBH

Stellar mass PBH : $\sim 10^{33} \text{g}$



Nanohertz
GW Signals
detected by
Pulsar
Timing
Arrays
(PTAs)

LIGO BH : $10 \sim 100 M_\odot$



10 hertz GW
Signals
detected by
LIGO
collaboration
(LVK)

I. Primordial Black Holes and Induced Gravitational waves from Inflation

Based on 2404.02492, 2412.16463, 2504.12035, 2505.09337, 2506.06797 + upcoming works

Amplifying the Superhorizon Perturbations

Single field slow roll inflation: $\mathcal{R}'' + 2(z'/z)\mathcal{R}' + k^2\mathcal{R} = 0$

Several e-folds after horizon crossing, $k/(aH) \ll 1$,

the e.o.m of comoving curvature $\mathcal{R}$ at leading order of spatial gradient expansion:

$$\mathcal{R}_0'' + 2(z'/z)\mathcal{R}_0' = 0$$

We observe adiabatic constant mode and decaying mode.

$$\mathcal{R}_0(\eta_f) = \mathcal{R}(\eta_k) + \mathcal{R}'(\eta_k) \int_{\eta_k}^{\eta_f} \frac{z^2(\eta_k)}{z^2(\eta')} d\eta'$$

$\propto (a/a_k)^{-3}$ Negligible outside horizon

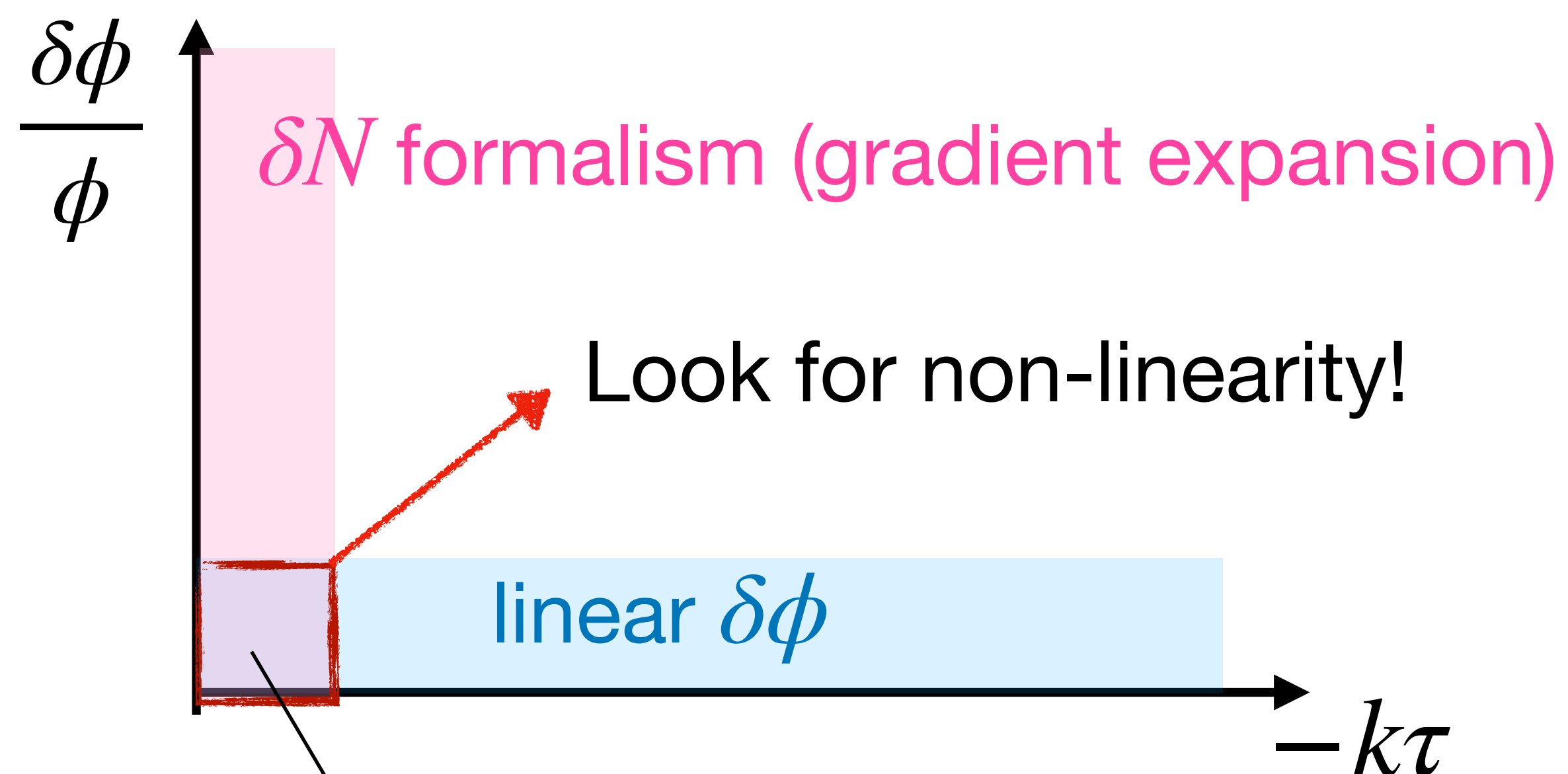
“The co-moving curvature is **frozen** after horizon crossing” ?



Not really!

Amplifying the Superhorizon Perturbations

Single field slow roll inflation:



The solution in small k limit:
adiabatic constant mode and
decaying mode.

$$\mathcal{R}_0(\eta_f) = \mathcal{R}(\eta_k) + \mathcal{R}'(\eta_k) \int_{\eta_k}^{\eta_f} \frac{z^2(\eta_k)}{z^2(\eta')} d\eta'$$

$\propto (a/a_k)^{-3}$
Negligible outside horizon

Amplifying the Superhorizon Perturbations

The super horizon behavior———Decay modes and k^2 correction

S. Leach, M. Sasaki, D. Wands and A. Liddle, 2001 (astro-ph/0101406)

Given $\mathcal{R}_k, \mathcal{R}'_k$ at (shortly after) horizon crossing $\eta = \eta_k$

$\frac{\delta\phi}{\phi}$
 δN formalism

$$\mathcal{R}_0(\eta_f) = \mathcal{R}(\eta_k) + \mathcal{R}'(\eta_k) \int_{\eta_k}^{\eta_f} \frac{z^2(\eta_k)}{z^2(\eta')} d\eta'$$

Homogenous Decay mode

$$k^2 \mathcal{R}_1(\eta_f) = k^2 \mathcal{R}(\eta_k) \int_{\eta_f}^{\eta_k} \int_{\eta_f}^{\eta'} \frac{z^2(\eta'')}{z^2(\eta')} d\eta'' d\eta'$$

1st order k^2 correction

linear $\delta\phi$

$-k\tau$

Superhorizon behavior can be important for single field inflation.

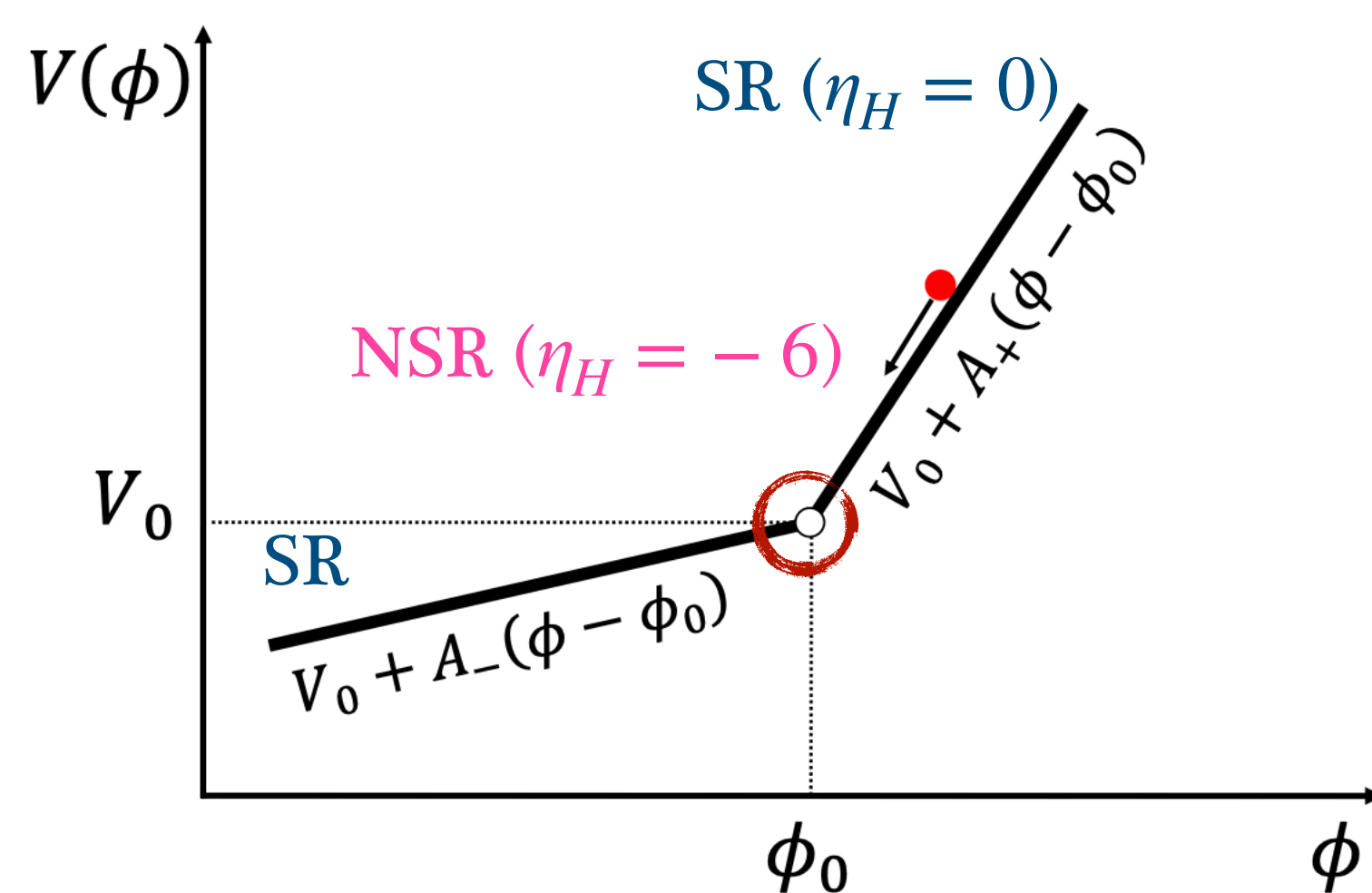
Amplifying the Superhorizon Perturbations (Single Field)

The super horizon behavior———Decay modes and k^2 correction

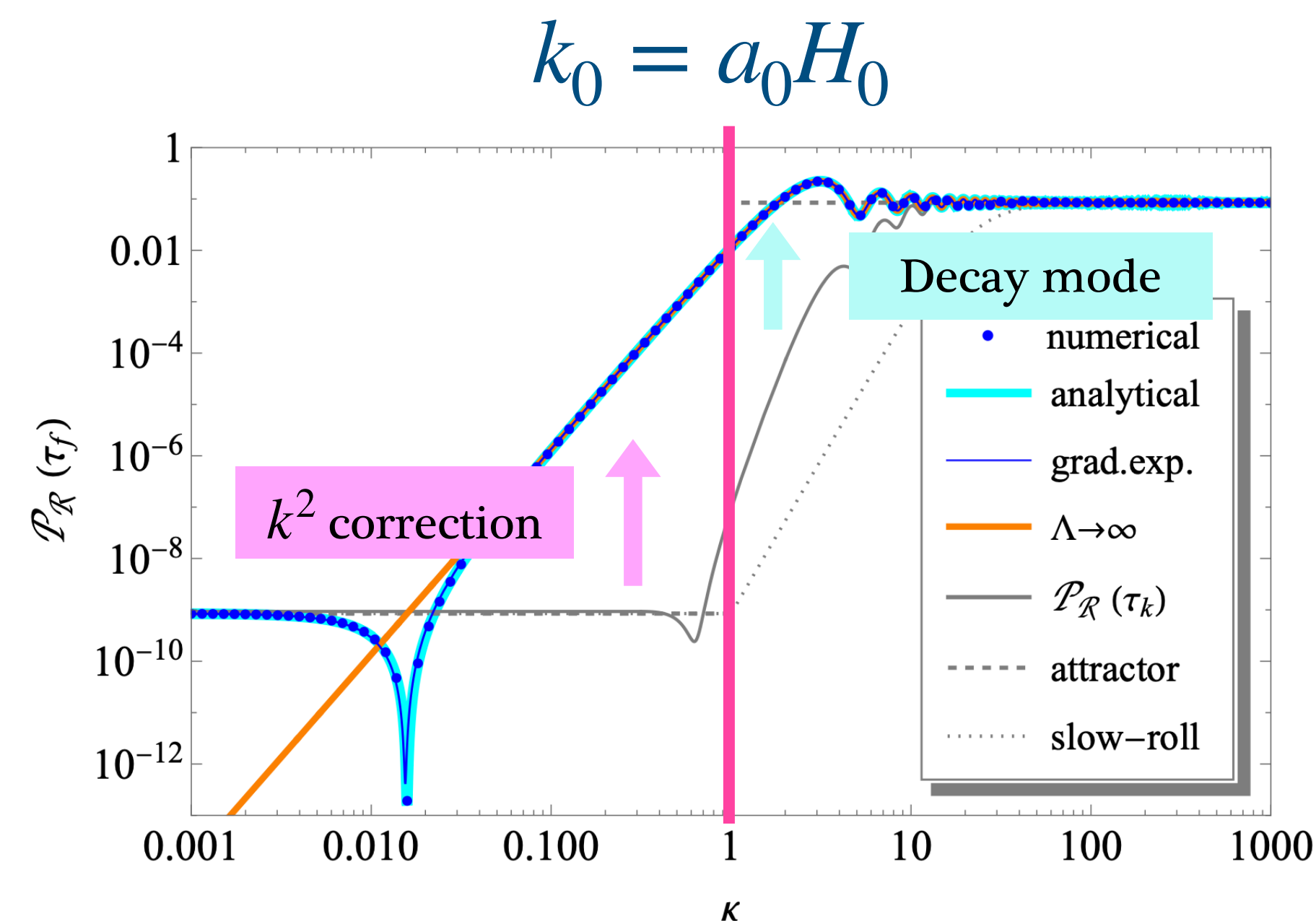
S. Leach, M. Sasaki, D. Wands and A. Liddle, 2001 (astro-ph/0101406), Shi Pi, Jianing Wang, 2022 (2209.14183);

C. T. Byrnes, P. S. Cole, and S. P. Patil, 1811.11158 **The “Steepest” growth: $\mathcal{P}_{\mathcal{R}} \propto k^4$ for single field inflation**

Starobinsky's linear piece-wise model



$\eta \ll \epsilon$ during SR phases is assumed



Specific features will appear in primordial power spectrum and full PDF, see [2412.16463](#)

Amplifying the Superhorizon Perturbations (Multi Field)

Contributions of the iso-curvature fields

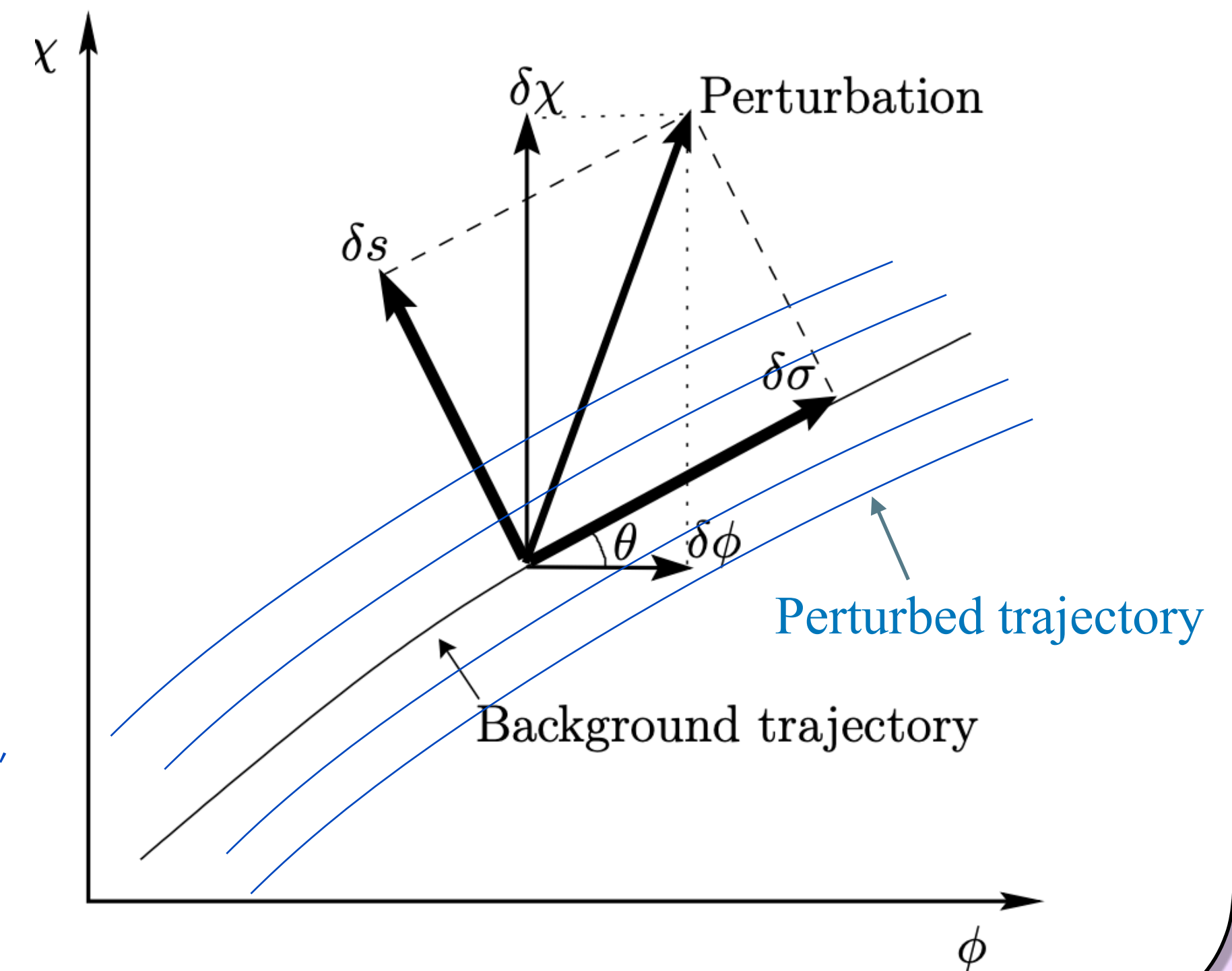
C. Gordon, D. Wands, B. Bassett and R. Maartens. 2000, (astro-ph/0009131)

$$\mathcal{S} = -3H \left(\frac{\delta p}{\dot{p}} - \frac{\delta \rho}{\dot{\rho}} \right)$$

$$\dot{\mathcal{R}} \approx -3H \frac{\dot{p}}{\dot{\rho}} \mathcal{S} \approx \sqrt{\frac{2}{\epsilon}} \dot{\theta} \delta s + \mathcal{O} \left(\frac{k^2}{a^2 H^2} \right)$$

$$\propto \frac{\dot{\chi} \delta \phi - \dot{\phi} \delta \chi}{\epsilon_H} \text{ "Field perturbations orthogonal to the background"}$$

Isocurvature sources the time evolution of curvature perturbation



Amplifying the Superhorizon Perturbations (Multi Field)

Contributions of the iso-curvature fields

R^2 inflation + non-minimally coupled χ

XW, YL. Zhang, M. Sasaki, 2024 (2404.02492)

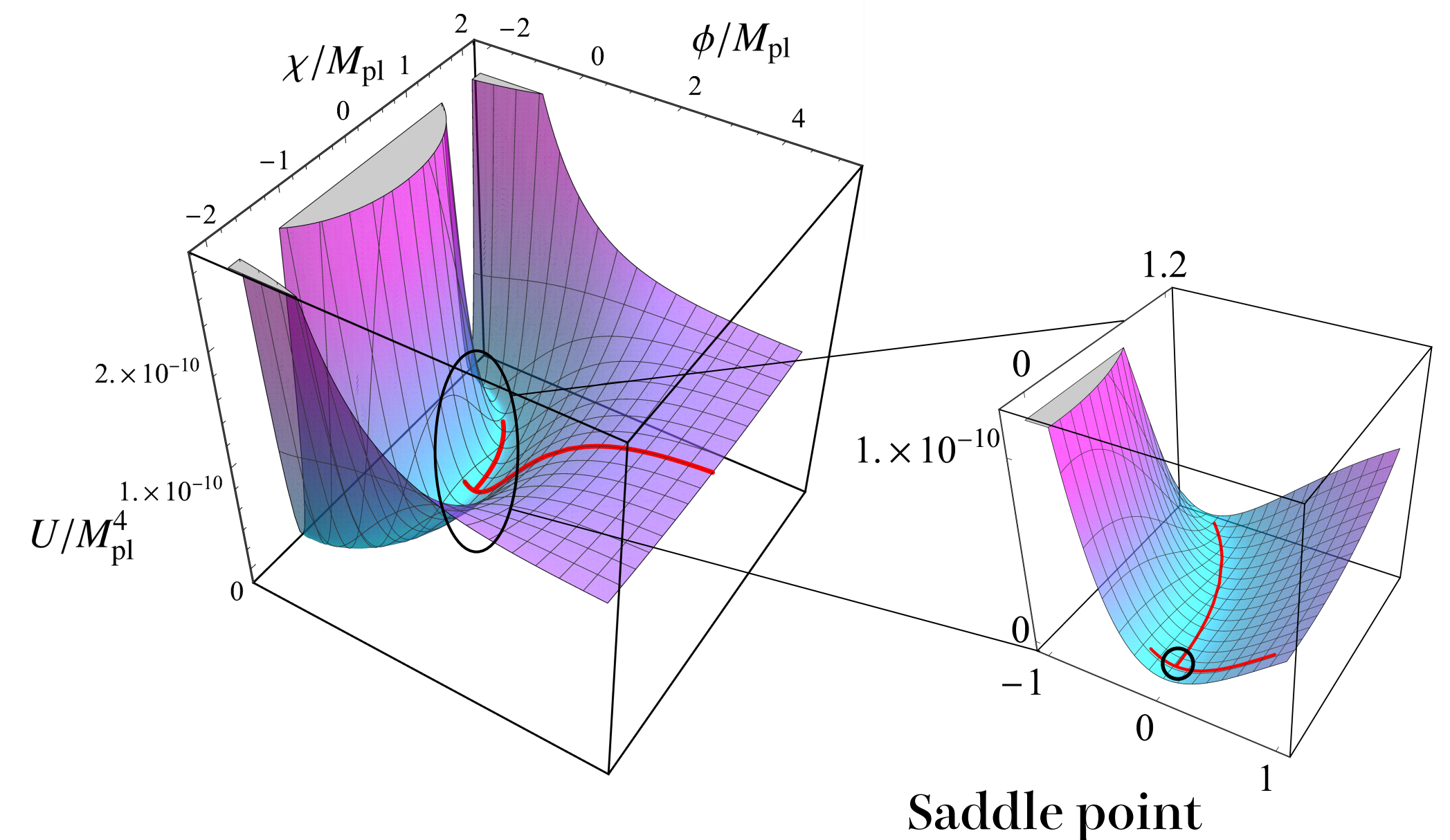
$$S_J = \int d^4x \sqrt{-g} \left[\frac{M_{\text{pl}}^2}{2} f(R, \chi) - \frac{1}{2} g^{\mu\nu} \partial_\mu \chi \partial_\nu \chi - V(\chi) \right],$$

$$f(R, \chi) = R + \frac{R^2}{6M^2} - \frac{\xi R}{M_{\text{pl}}^2} (\chi - \chi_0)^2, \text{ To break } Z_2 \text{ symmetry}$$

Avoid: 1) DW overproduction 2) eternal inflation

$$V(\chi) = V_0 - \frac{1}{2} m^2 \chi^2 + \frac{1}{4} \lambda \chi^4. \text{ To end inflation}$$

The potential is bounded from below



Amplifying the Superhorizon Perturbations (Multi Field)

R^2 inflation + non-minimally coupled χ

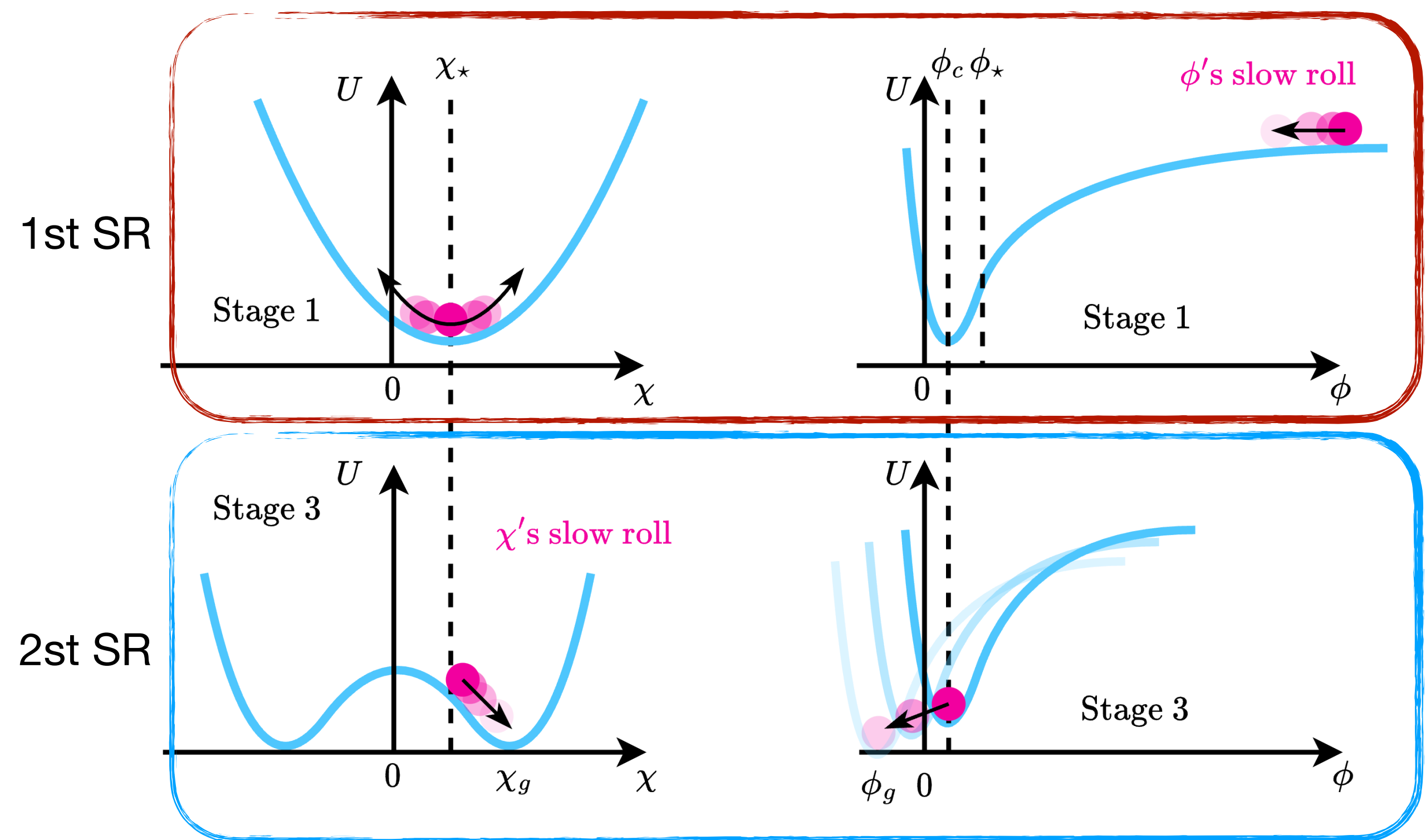
XW, YL. Zhang, M. Sasaki, 2024 (2404.02492)

In Einstein frame

$$\phi \equiv M_{\text{pl}} \sqrt{\frac{3}{2}} \ln F$$

$$U(\phi, \chi) \equiv \frac{3}{4} M^2 M_{\text{pl}}^2 W^2(\phi, \chi) + \frac{V(\chi)}{F(\phi)^2}$$

$$W(\phi, \chi) \equiv 1 - \frac{1}{F(\phi)} \left[1 - \xi \left(\frac{\chi - \chi_0}{M_{\text{pl}}} \right)^2 \right]$$

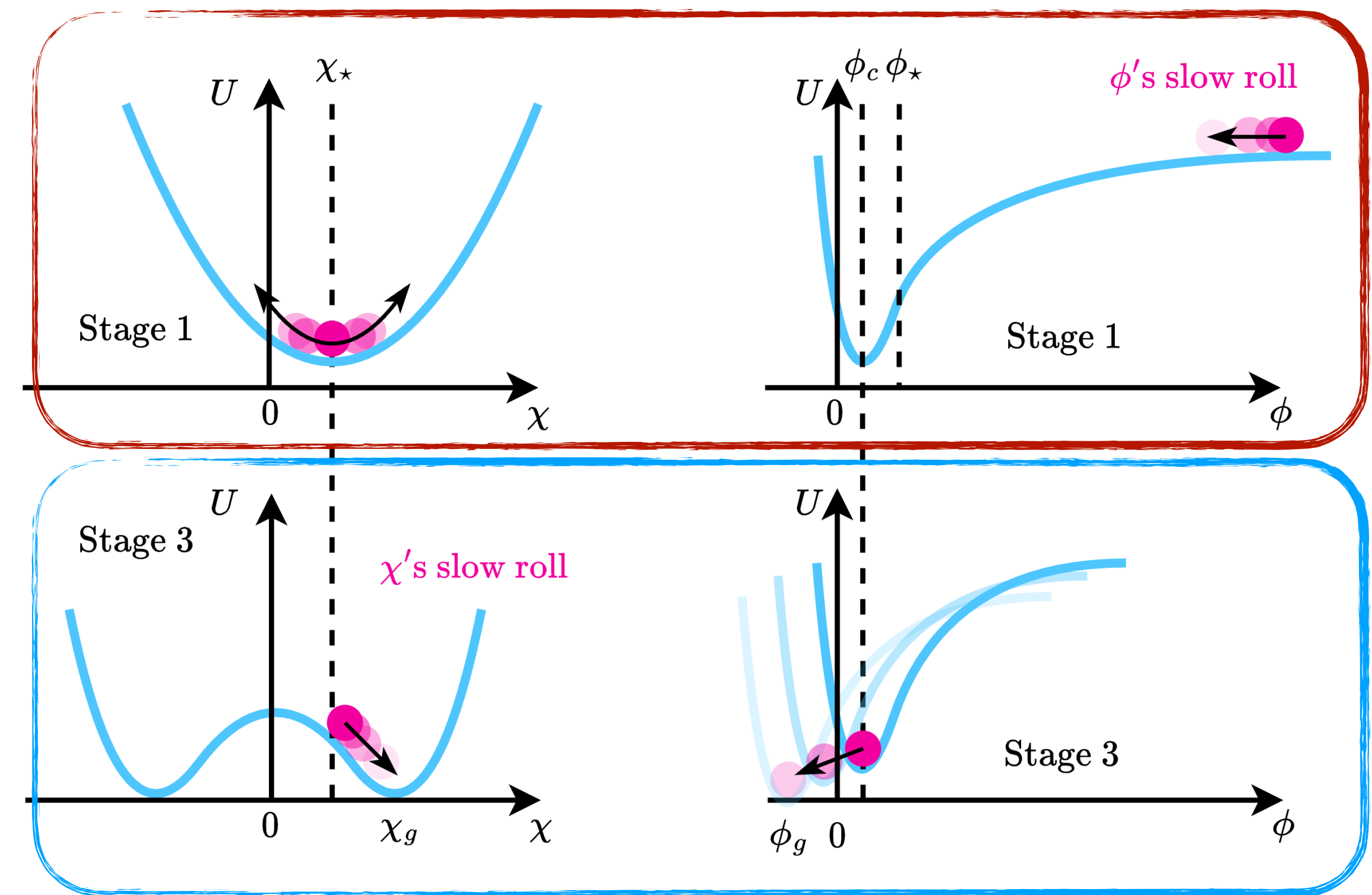
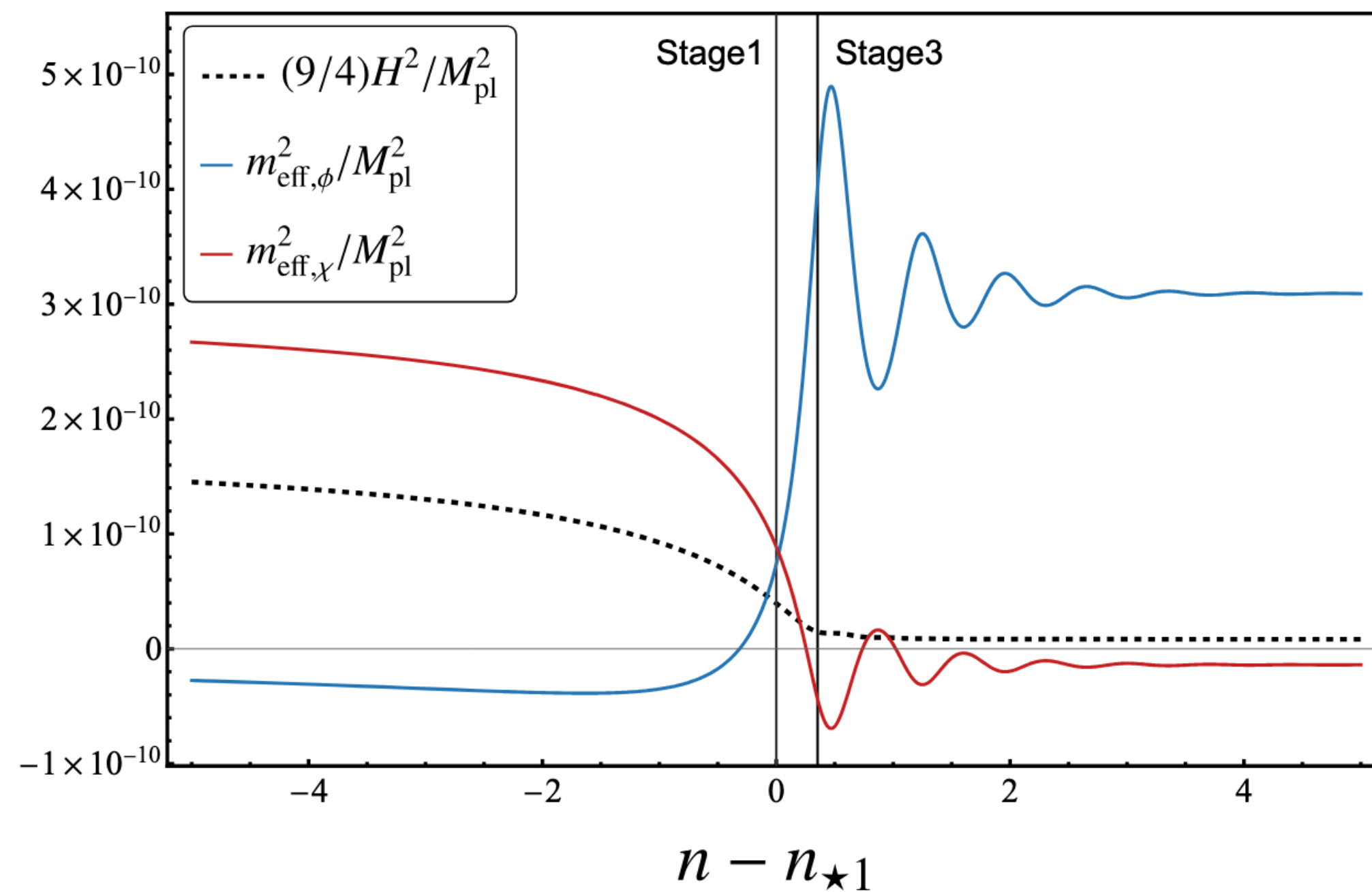


Amplifying the Superhorizon Perturbations (Multi Field)

R^2 inflation + non-minimally coupled χ

XW, YL. Zhang, M. Sasaki, 2024 (2404.02492)

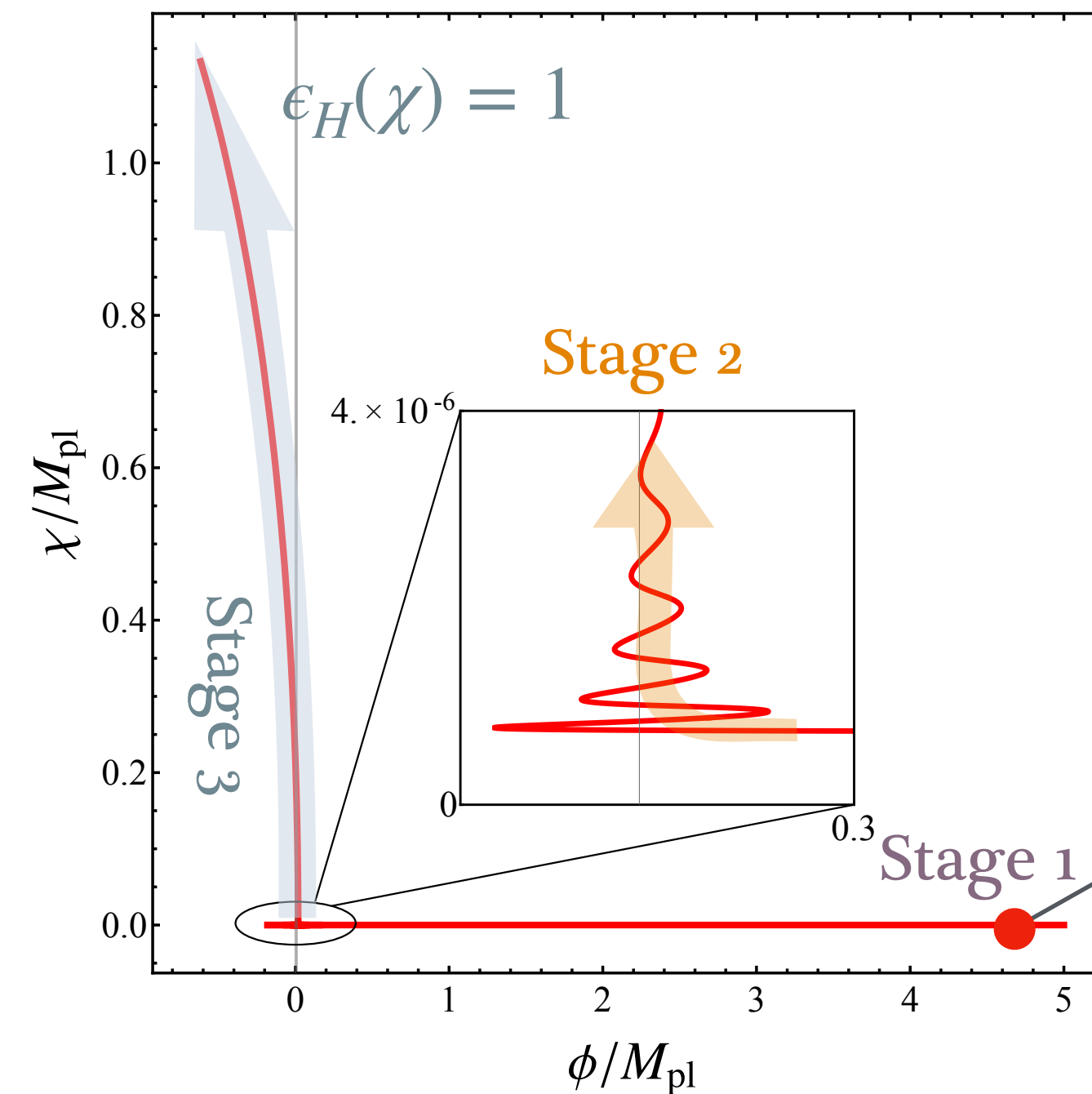
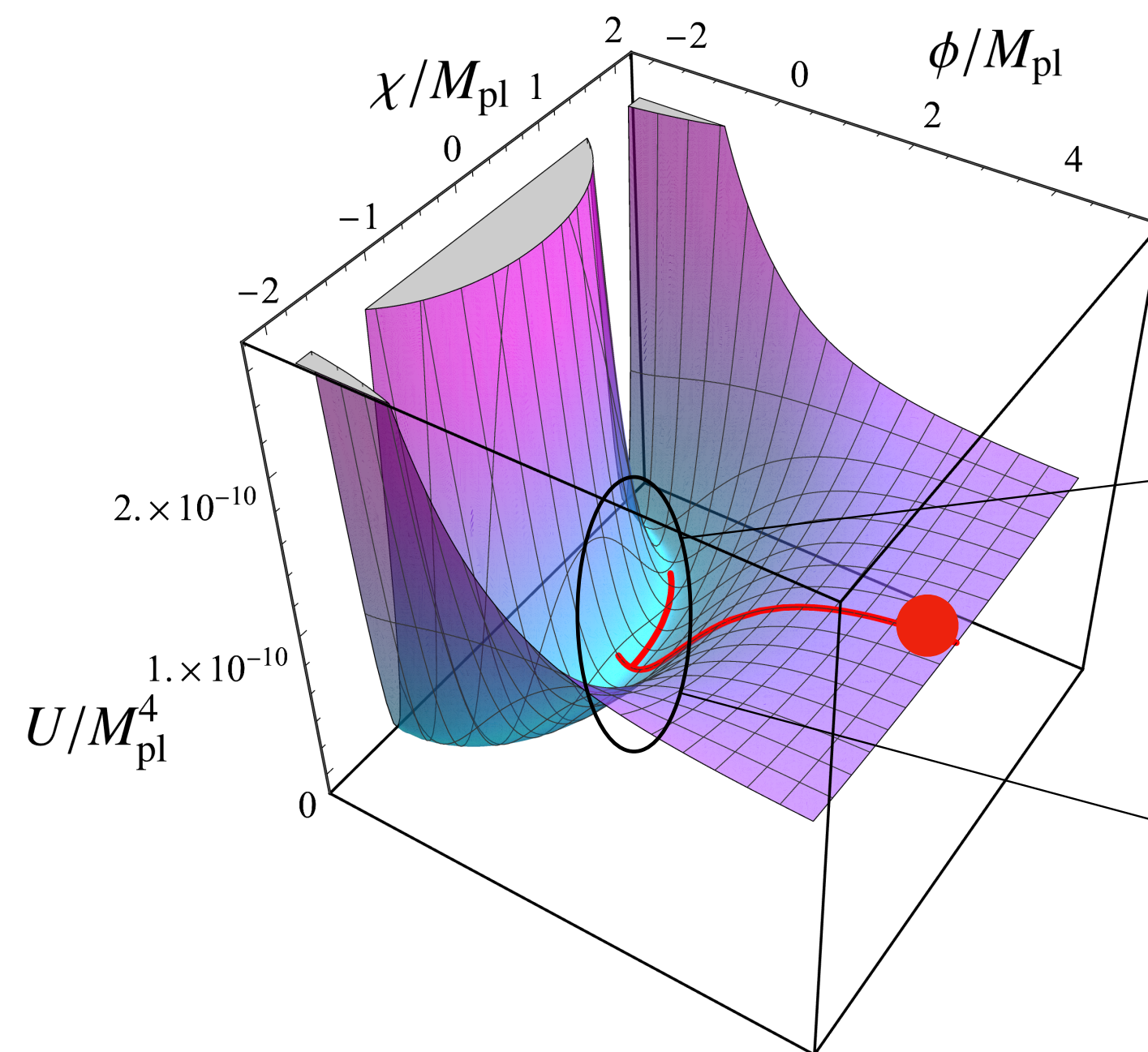
The effective mass of two fields



Amplifying the Superhorizon Perturbations (Multi Field)

R^2 inflation + non-minimally coupled χ

XW, YL. Zhang, M. Sasaki, 2024 (2404.02492)



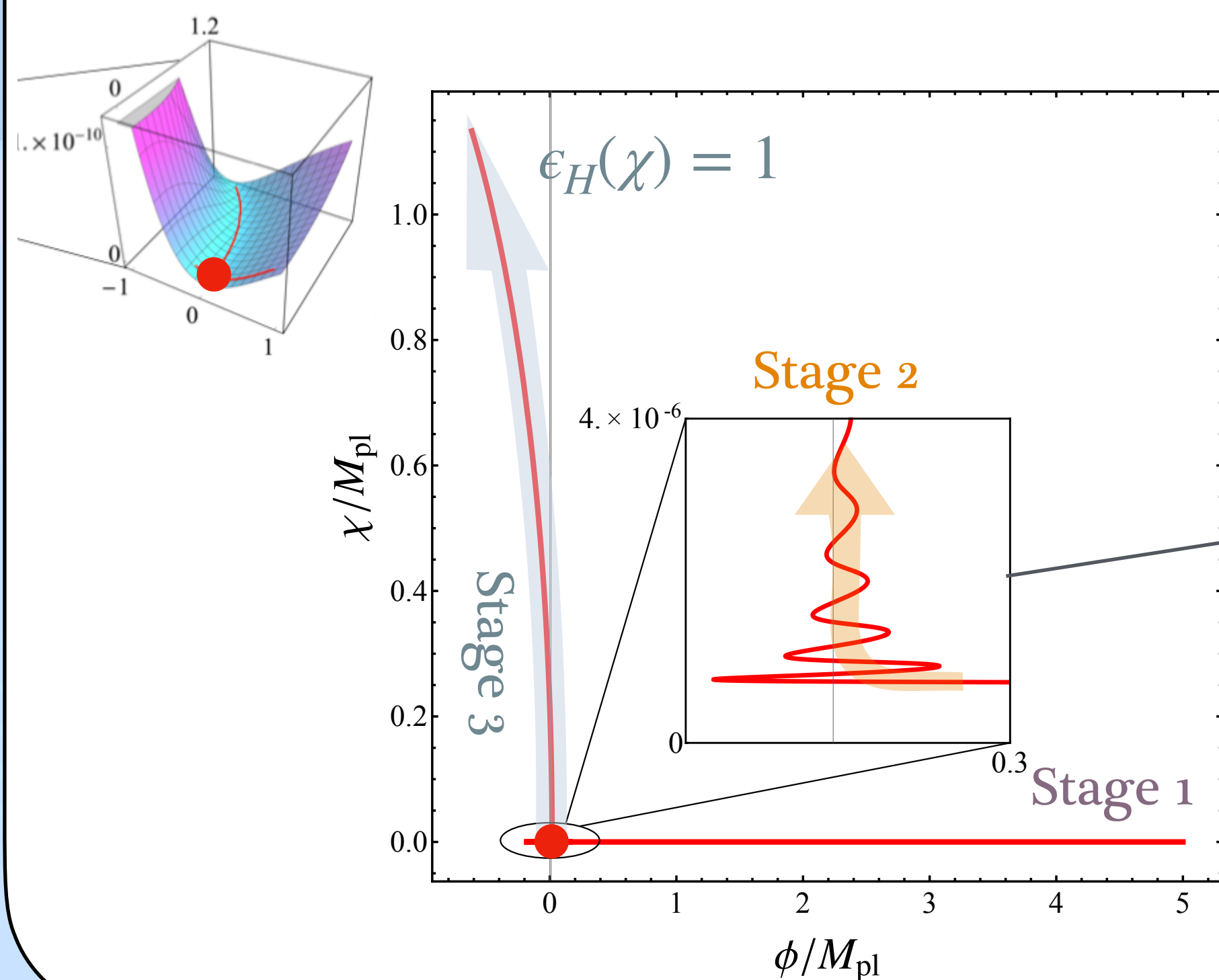
ϕ dominated inflation

Inflaton rolls along ϕ direction
(R^2 **inflation**),
 χ behaves like a damped* oscillator around χ_0 .

Amplifying the Superhorizon Perturbations (Multi Field)

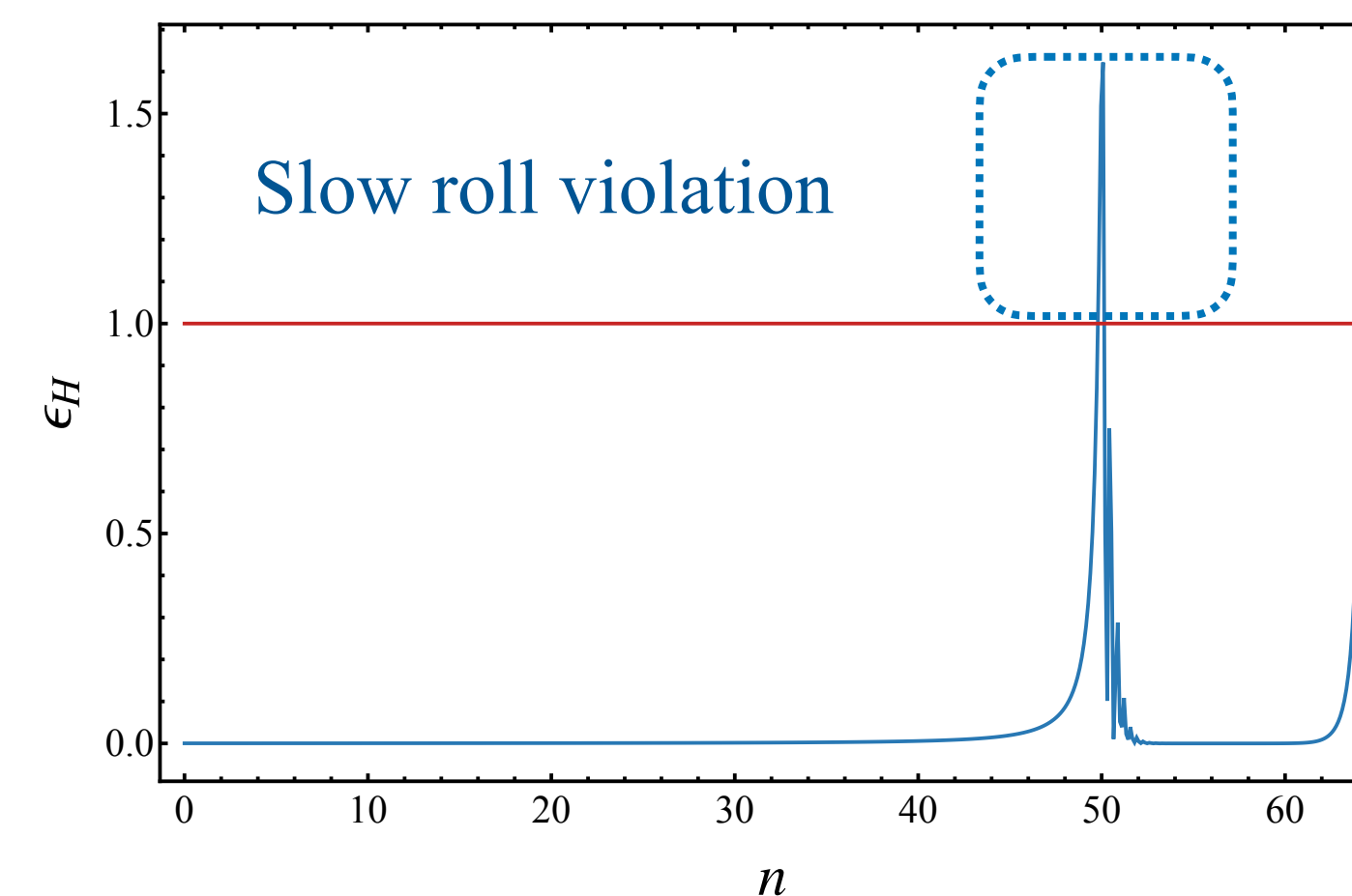
R^2 inflation + non-minimally coupled χ

XW, YL. Zhang, M. Sasaki, 2024 (2404.02492)



Baby R^2 reheating

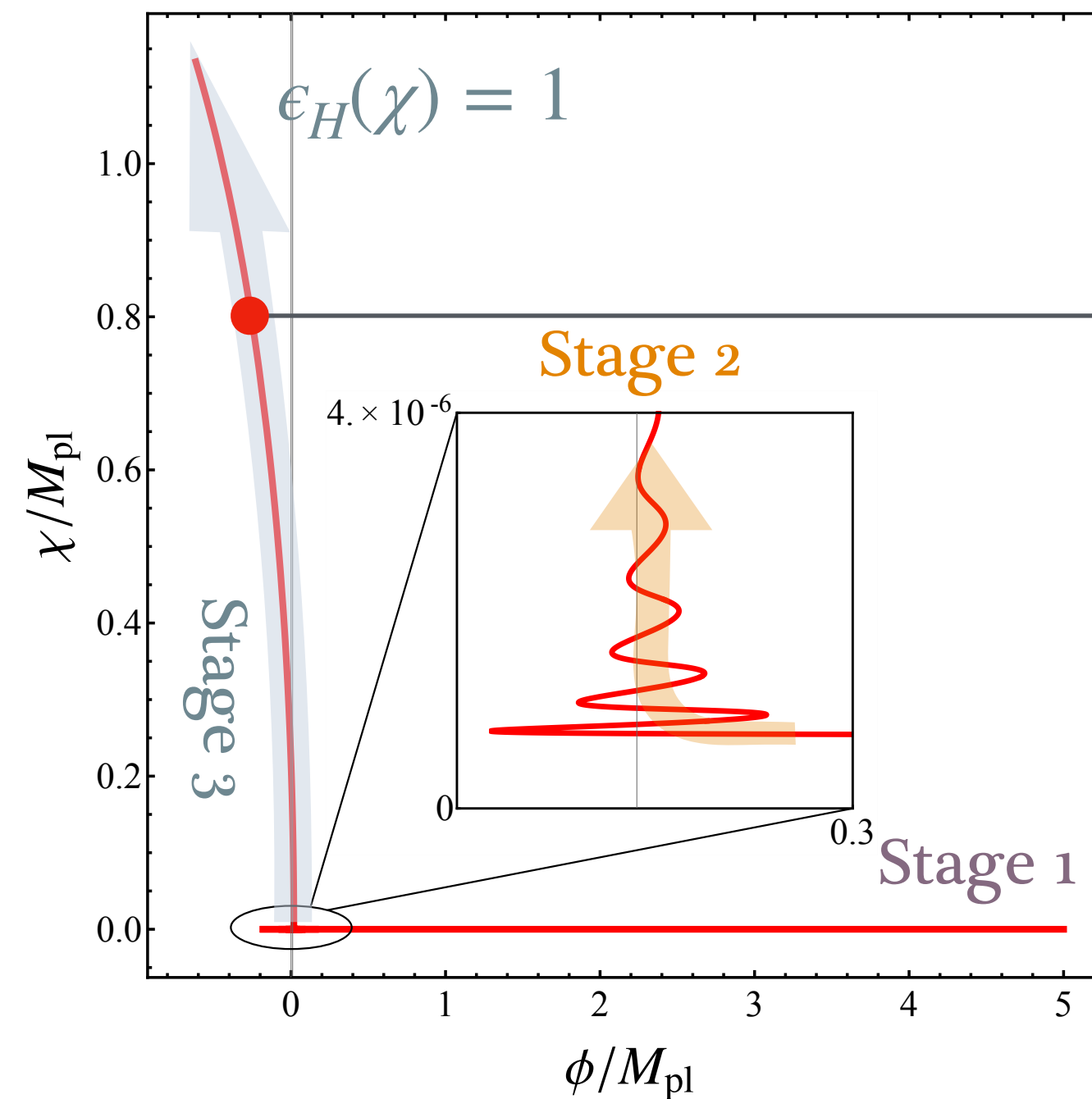
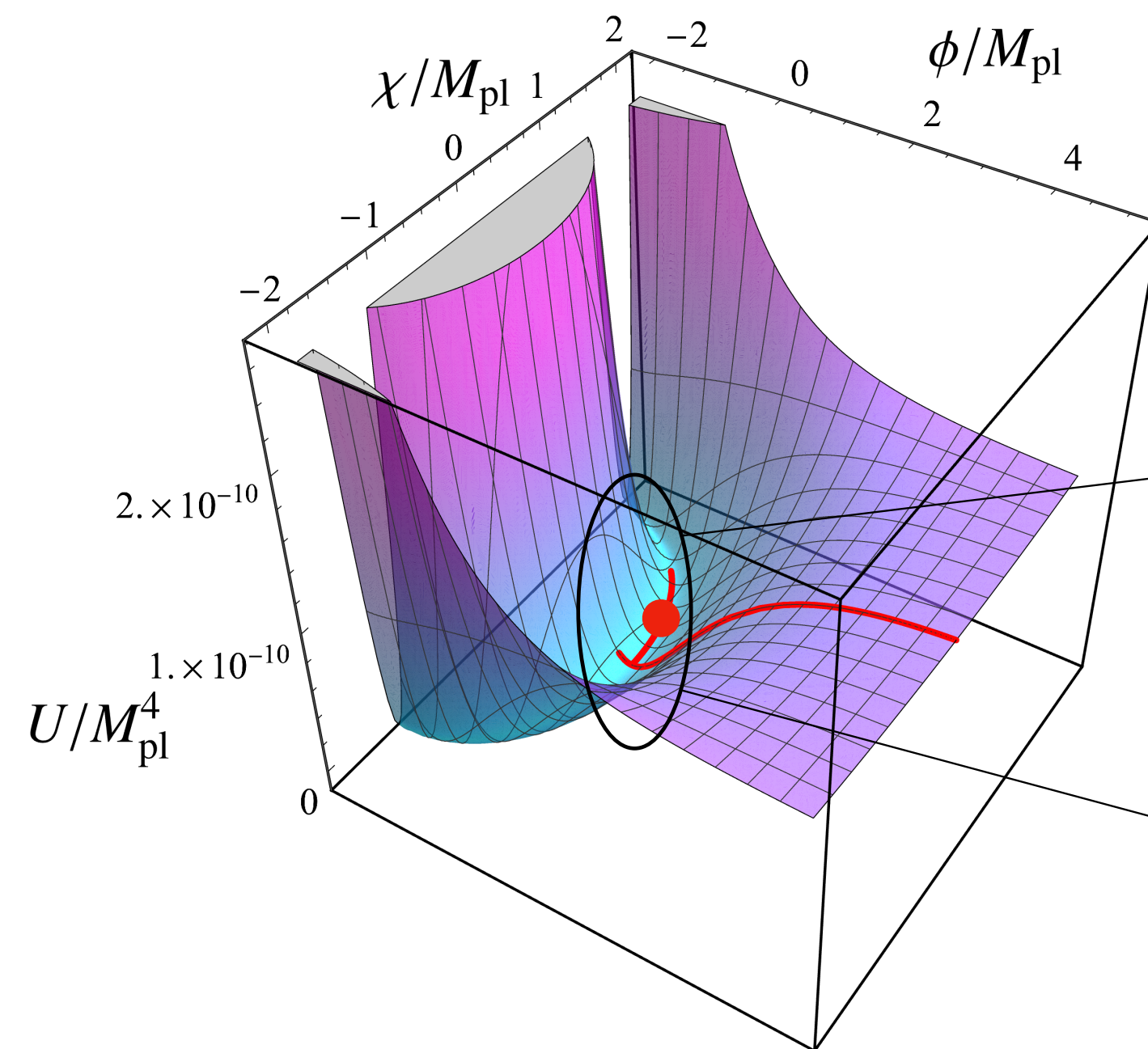
The slow roll is shortly violated ($\epsilon_H > 1$) by inflaton's oscillation along ϕ direction. χ stays in the potential valley.



Amplifying the Superhorizon Perturbations (Multi Field)

R^2 inflation + non-minimally coupled χ

XW, YL. Zhang, M. Sasaki, 2024 (2404.02492)



χ dominated inflation

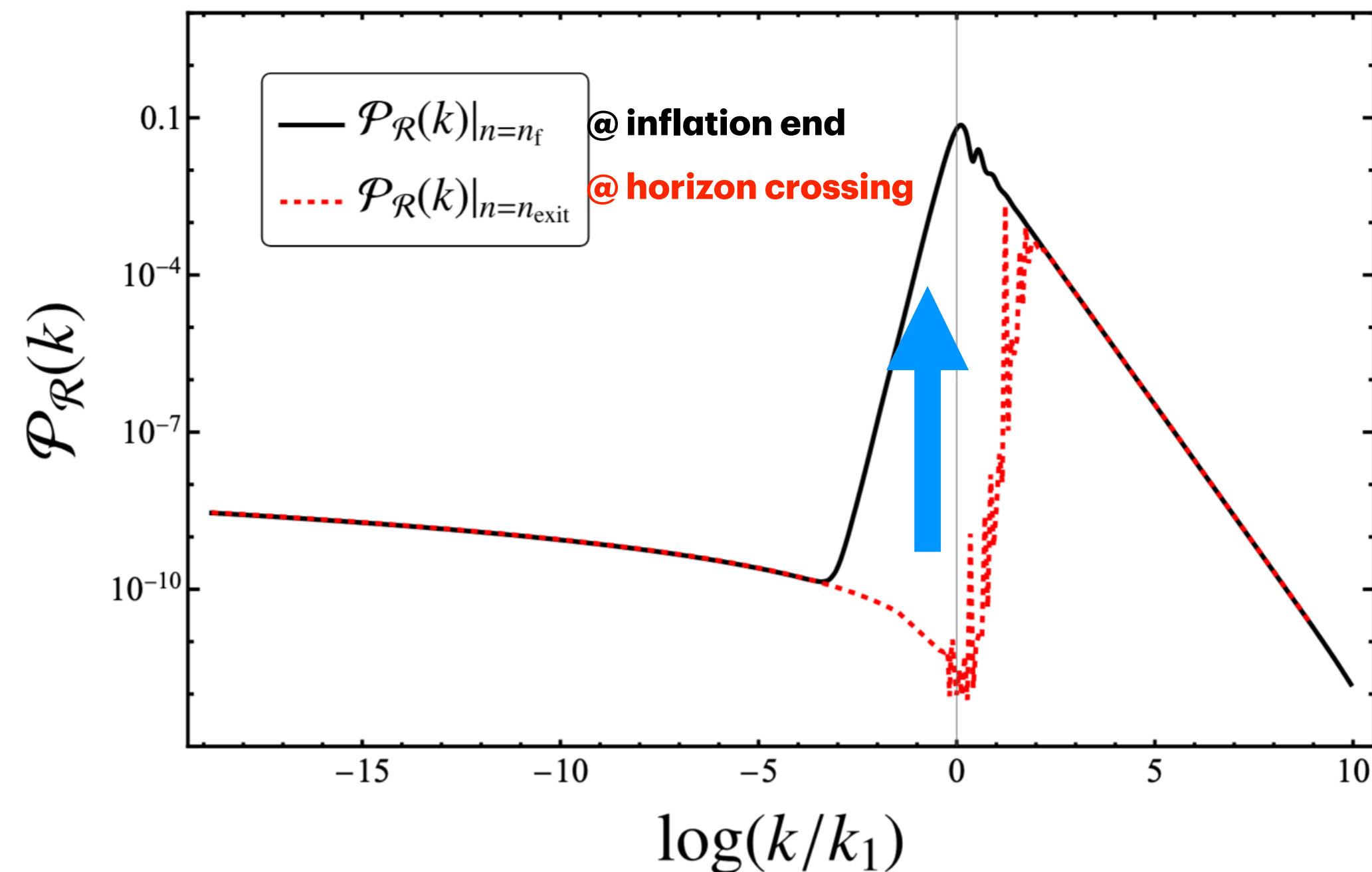
Inflation rolls along χ direction (**Higgs inflation**), ϕ behaves like a under-damped oscillator around it's potential valley. The inflation ends when $\epsilon_H = 1$.

Amplifying the Superhorizon Perturbations (Multi Field)

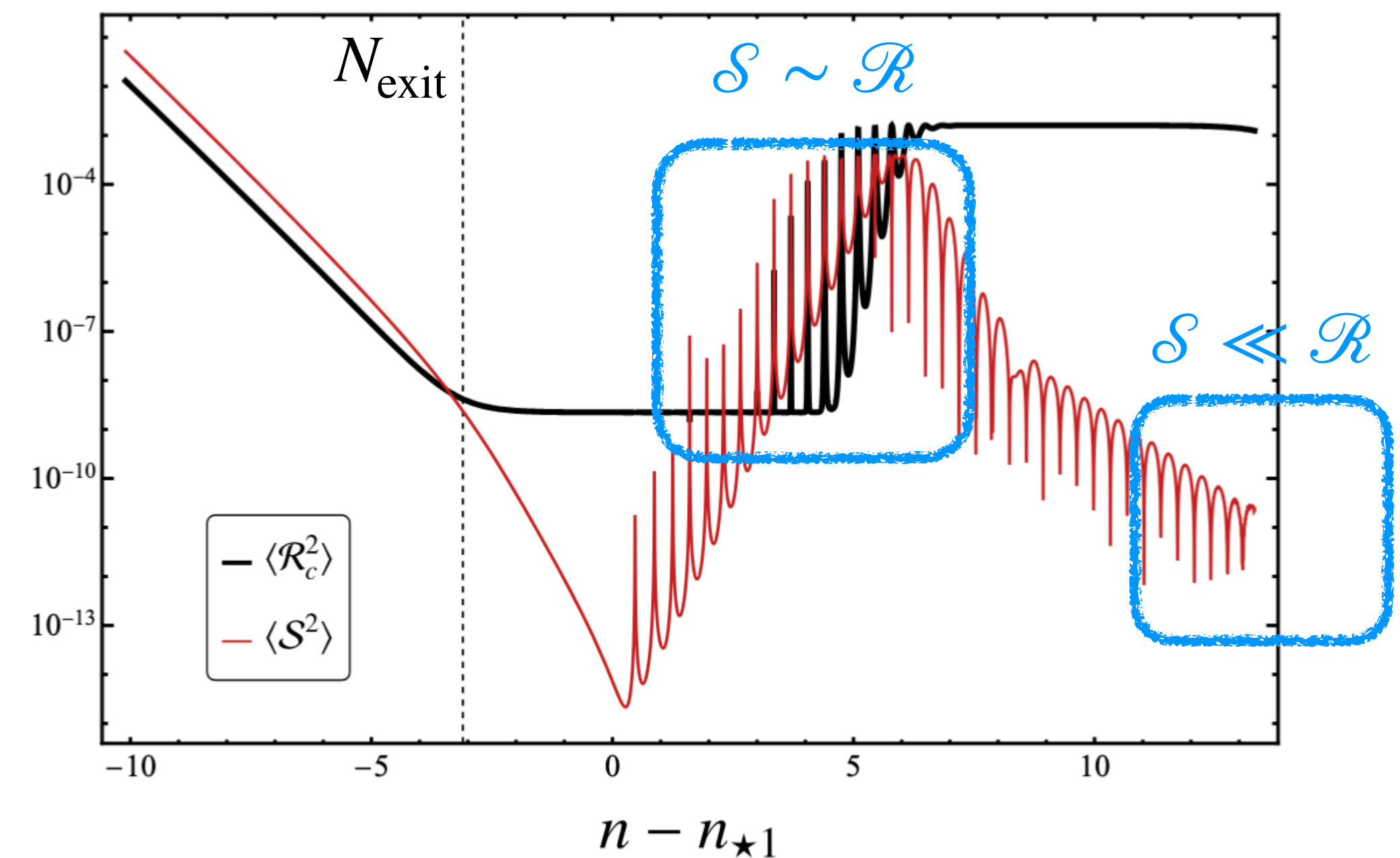
R^2 inflation + non-minimally coupled χ

XW, YL. Zhang, M. Sasaki, 2024 (2404.02492)

Comparison between the power spectrum
@ inflation end and horizon crossing



Evolution of $\mathcal{R}_c$ and $\mathcal{S}$



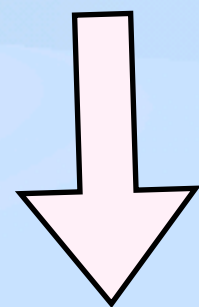
A non-linear approach: δN formalism

Kodama & Sasaki, 1984 Sasaki & Stewart, 1995 (astro-ph/9507001)

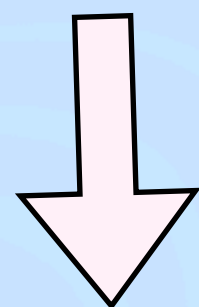
δN formalism is used to calculate the full possibility distribution function of $\mathcal{R}$,

* $\mathcal{R} = \delta N$ is based on separate universe approach, leading order of gradient expansion

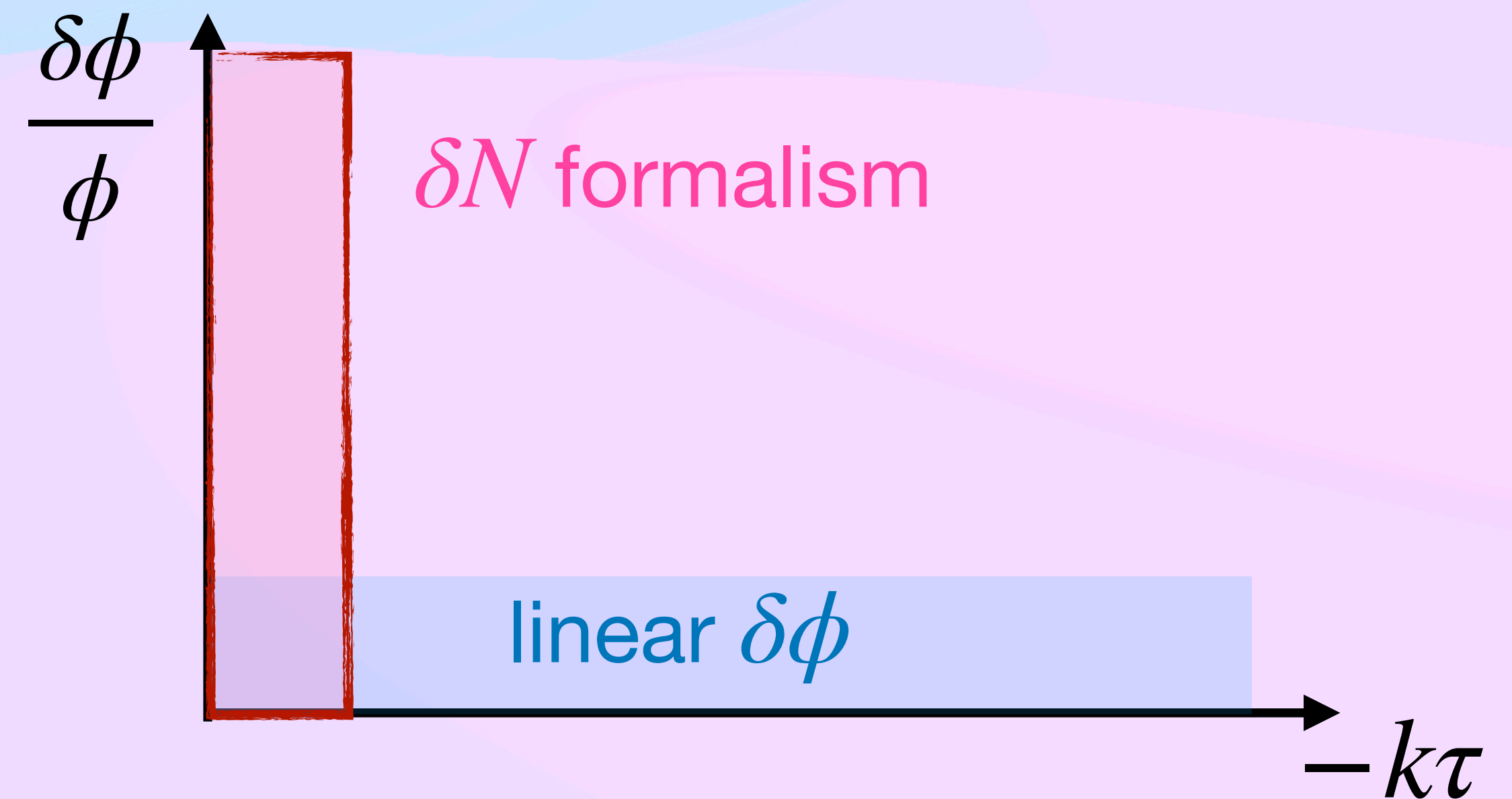
Background solution $\phi = \phi(N)$



$$\mathcal{R} \equiv \delta N = \tilde{N}_{\text{tot}} - N_{\text{tot}}$$



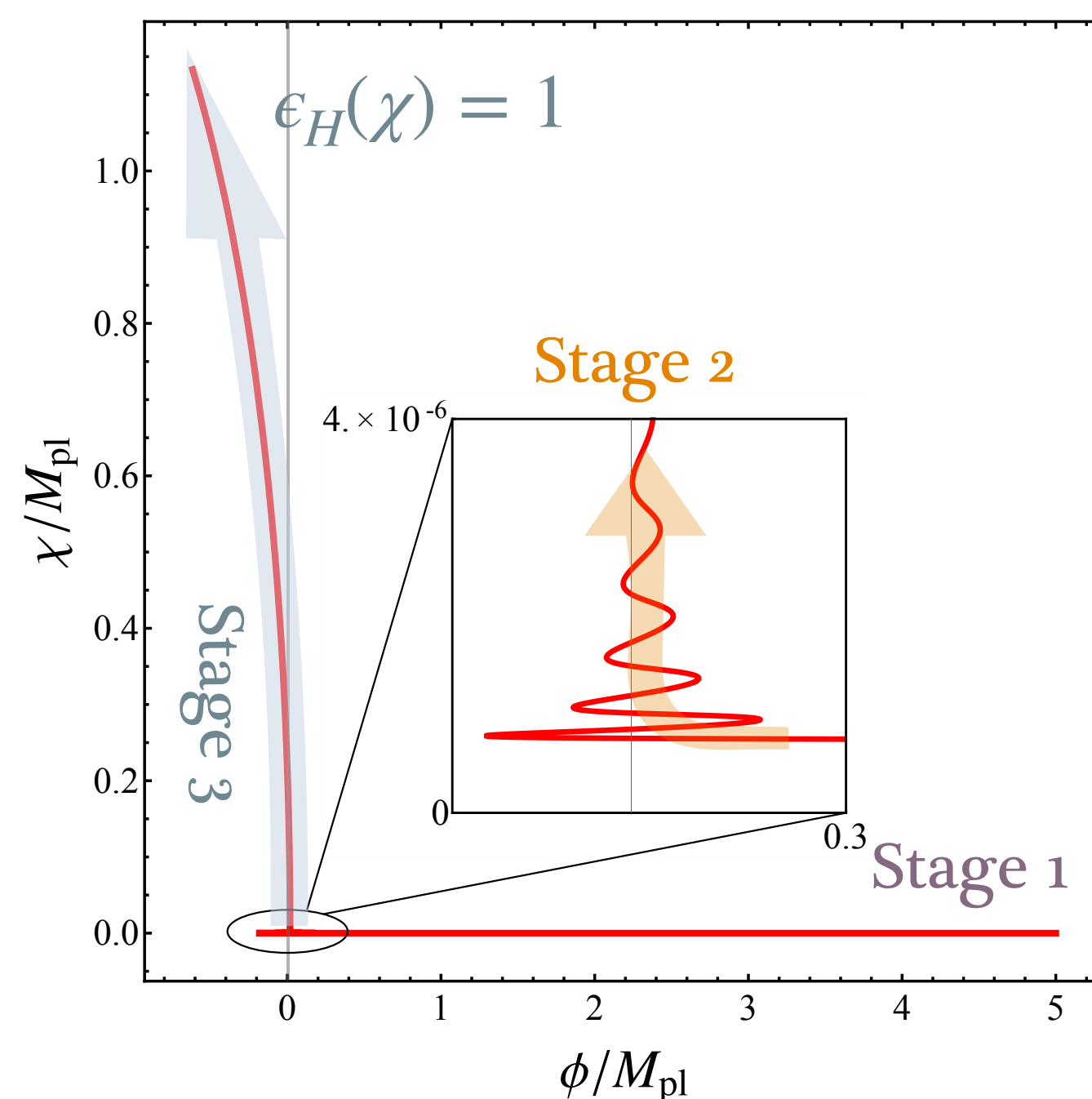
$$P[\mathcal{R}] \approx P[\mathcal{R}^{-1}(\delta\phi)]$$



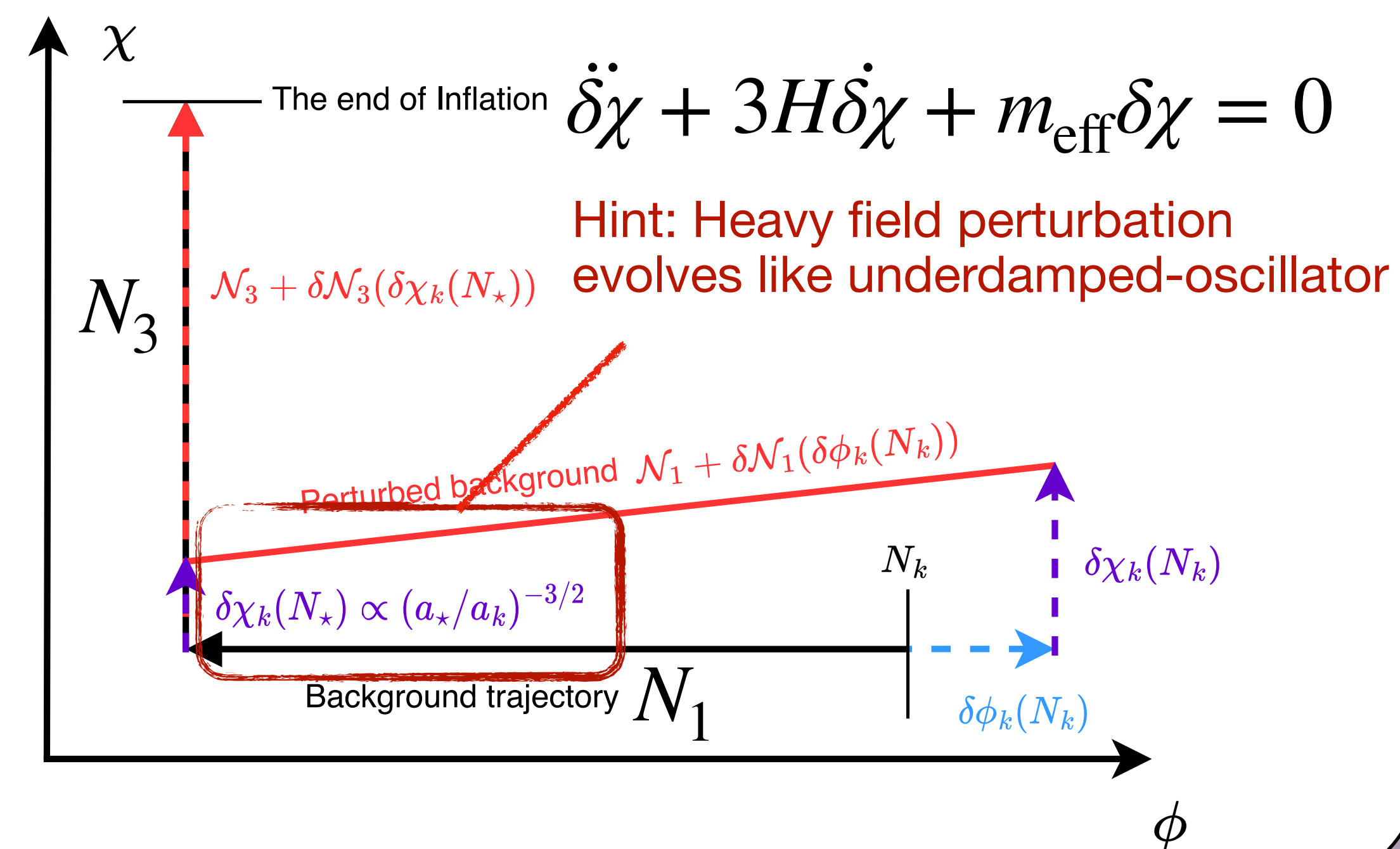
Amplifying the Superhorizon Perturbations (Multi Field)

R^2 inflation + non-minimally coupled χ

XW, YL. Zhang, M. Sasaki, 2024 (2404.02492)



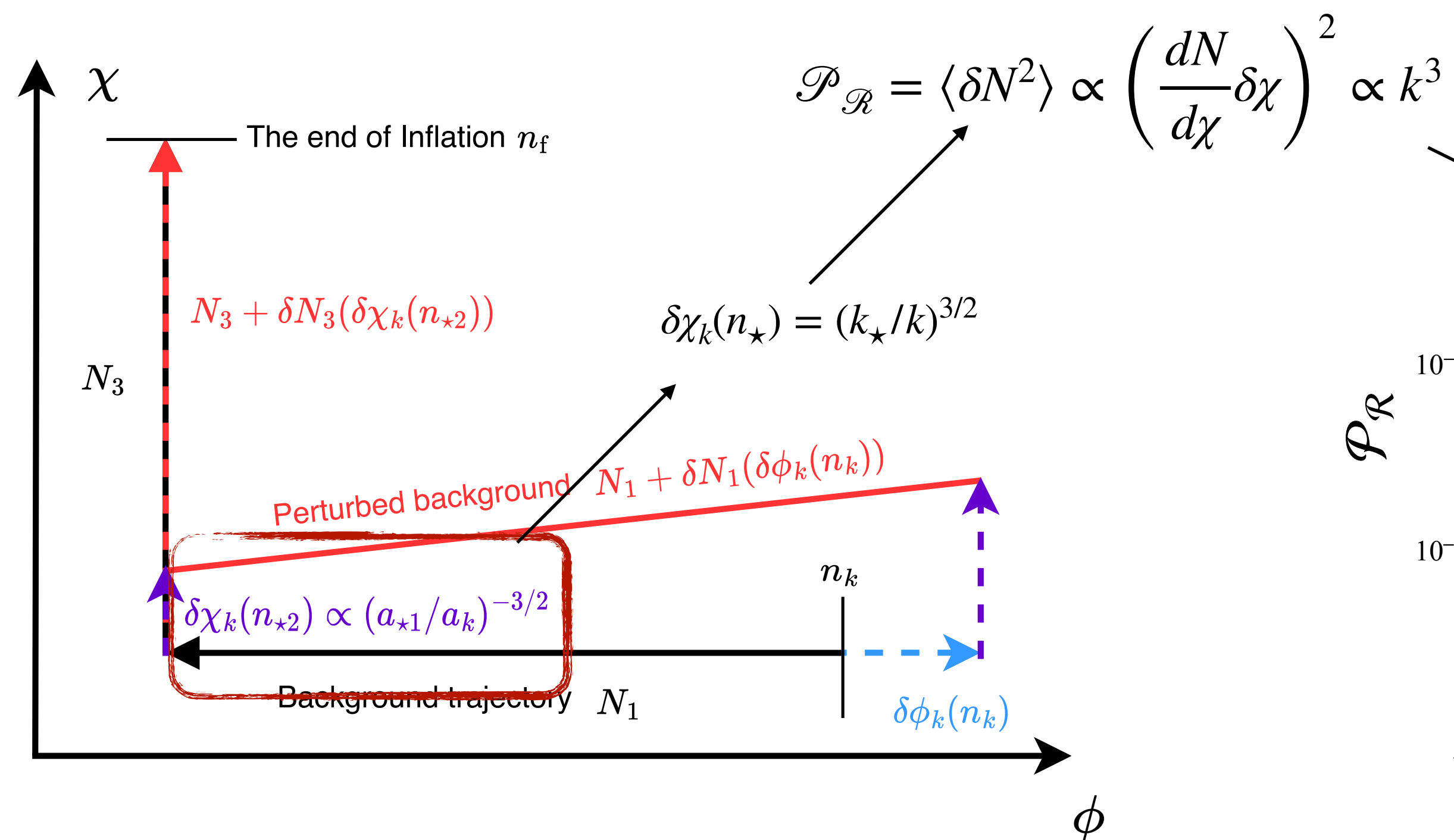
Simplified
traj.



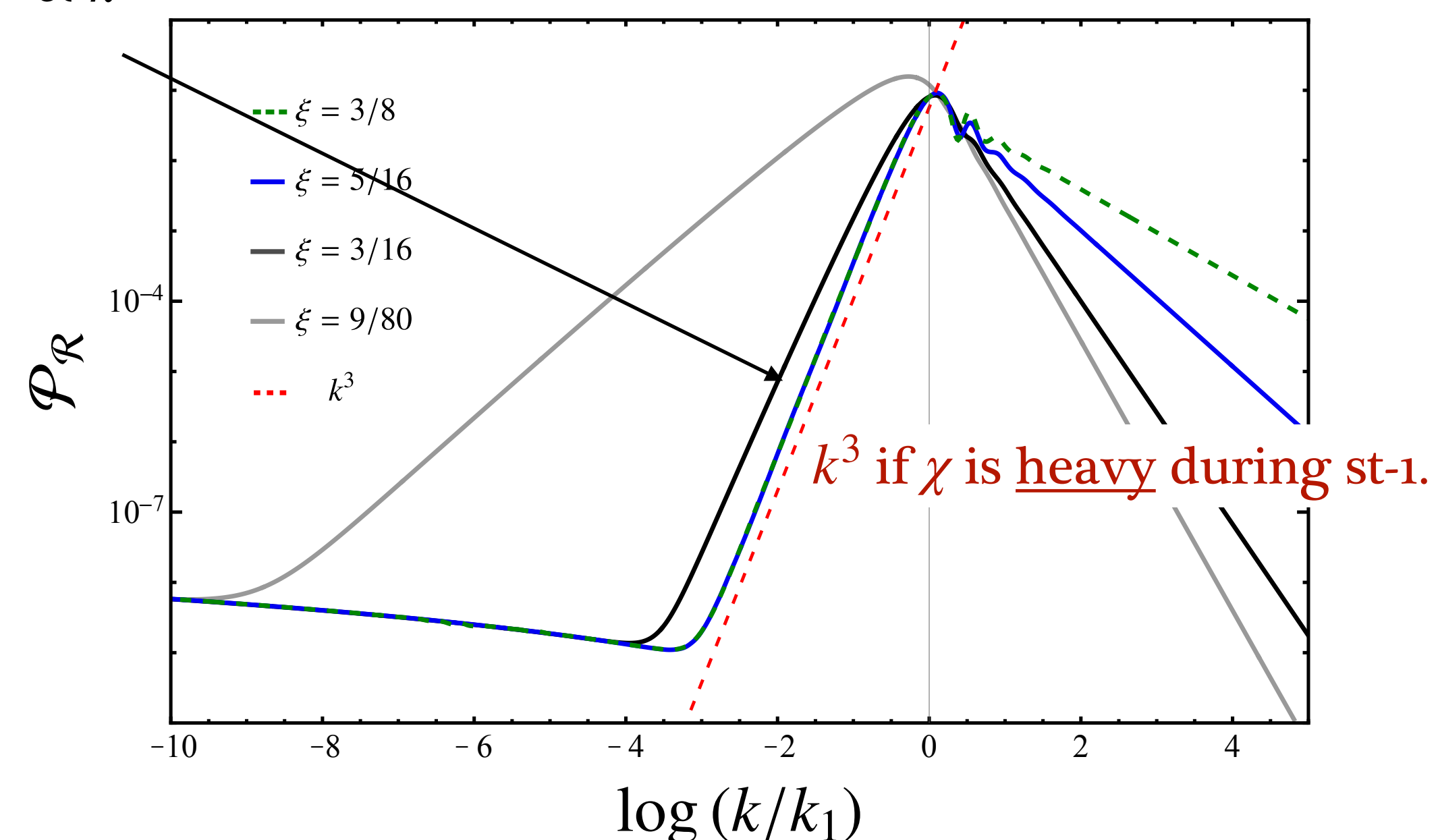
Amplifying the Superhorizon Perturbations (Multi Field)

R^2 inflation + non-minimally coupled χ

XW, YL. Zhang, M. Sasaki, 2024 (2404.02492)

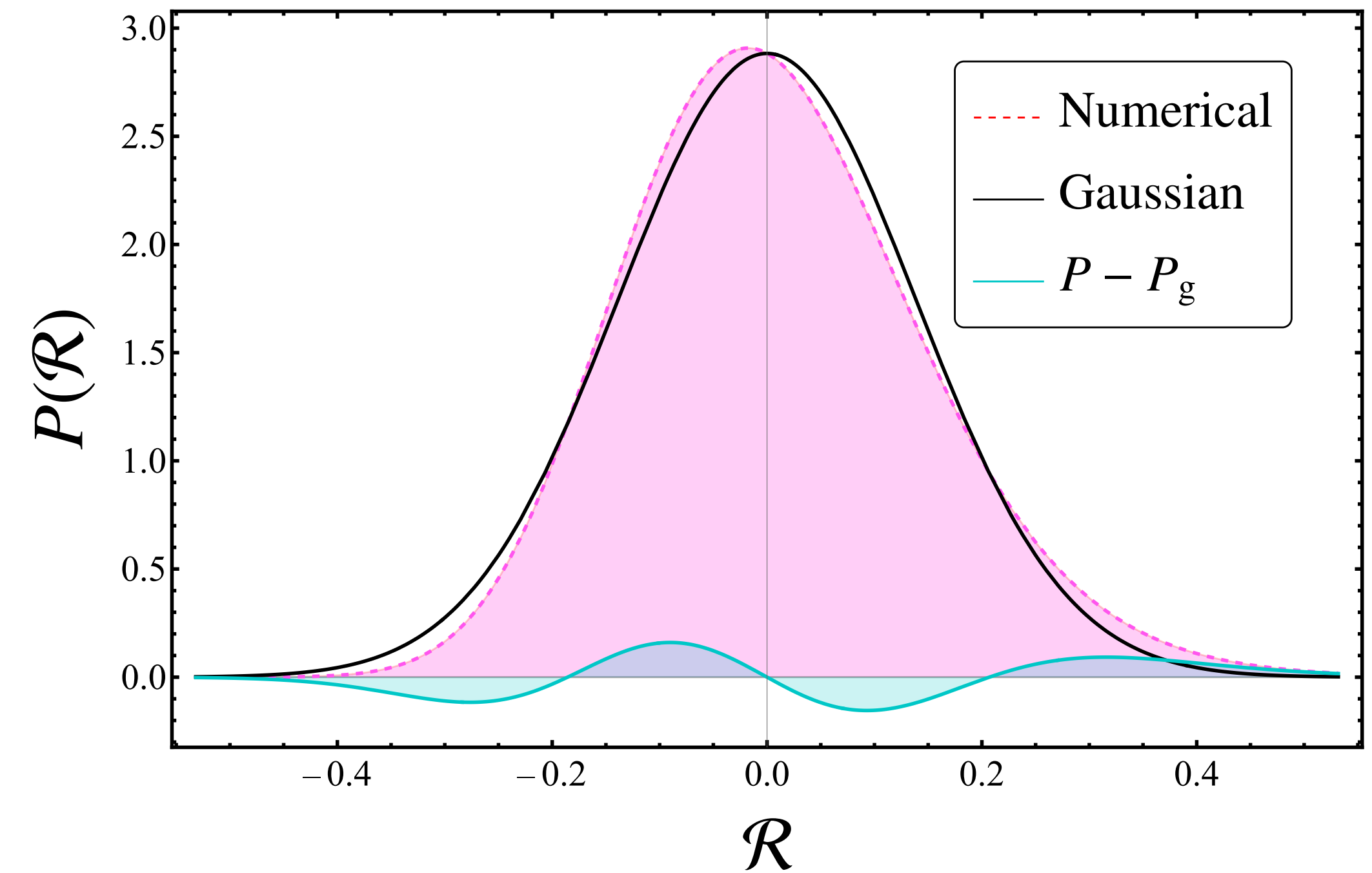
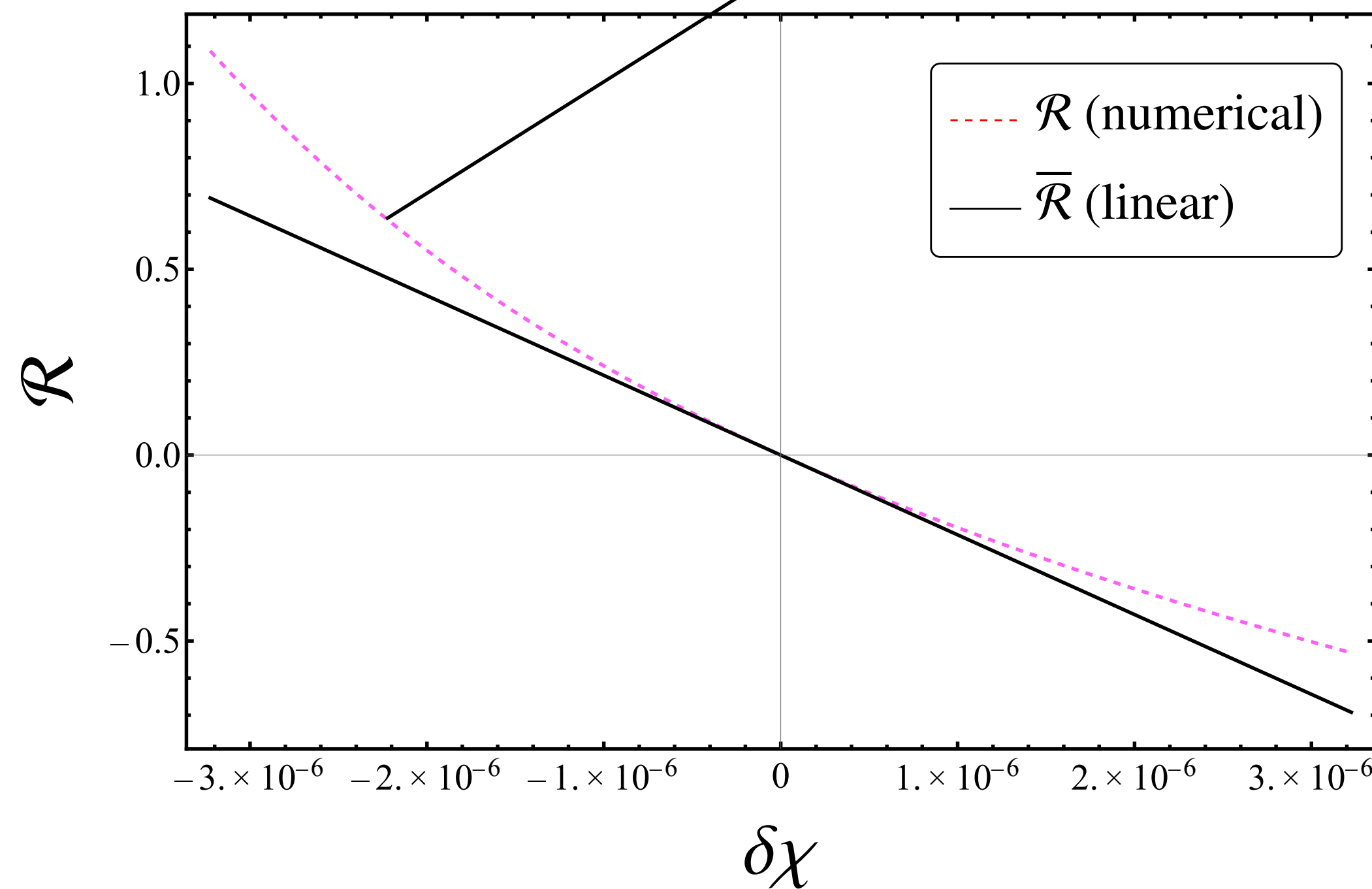
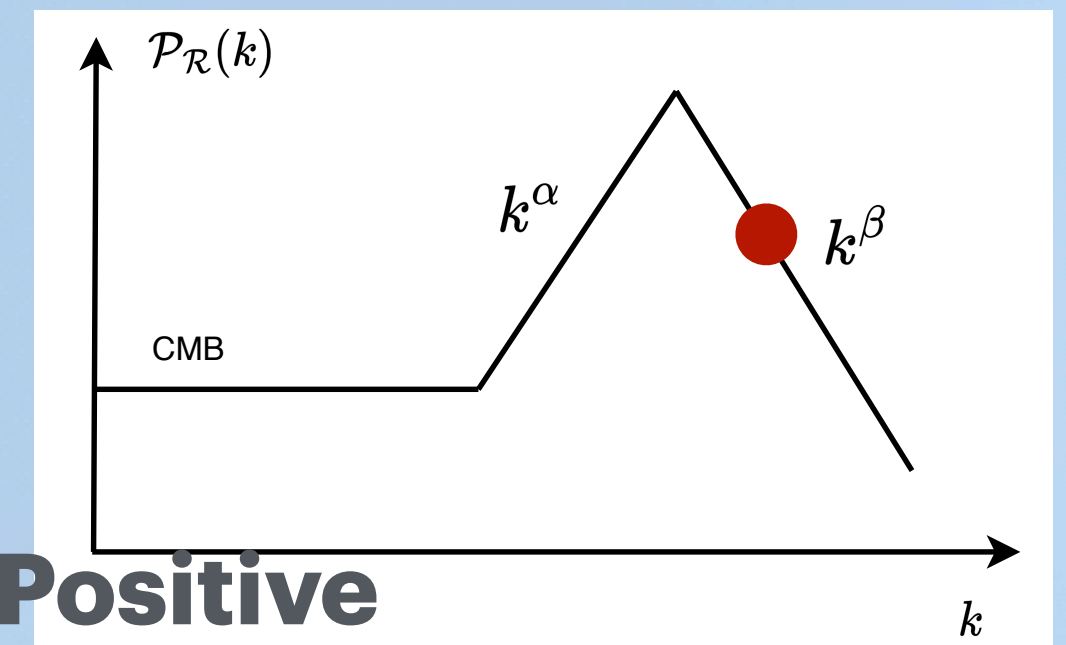


The growth rate depends on the effective mass of early isocurvature field (χ) that dominates the latter inflation.



Non-Gaussianity ($k \gg k_1$)

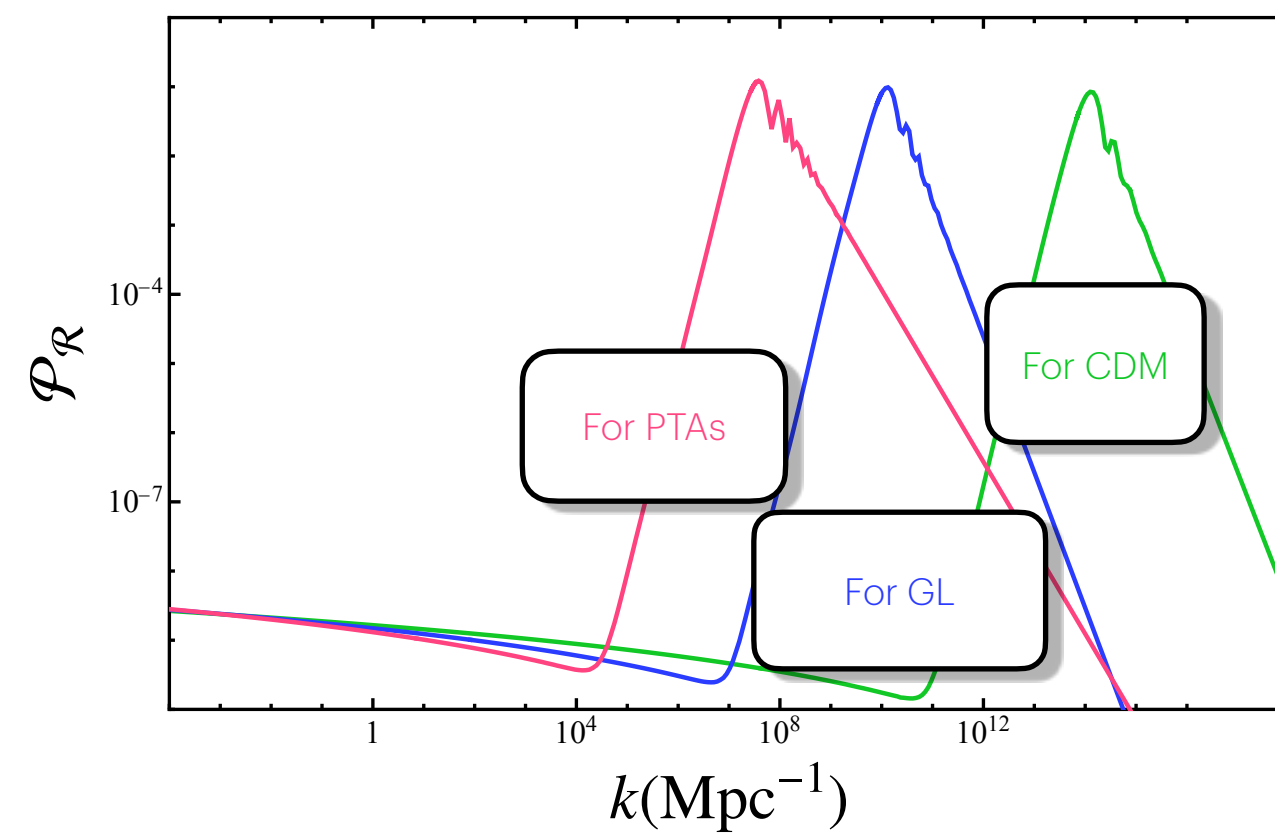
$$\mathcal{R} \approx \delta N_3 = N'_3 - N_3 \sim \frac{dN_3}{d\chi} \delta\chi + \boxed{\frac{d^2 N_3}{d\chi^2} \delta\chi^2} \Rightarrow f_{\text{NL}}^{\text{local}} \text{ Small and Positive}$$



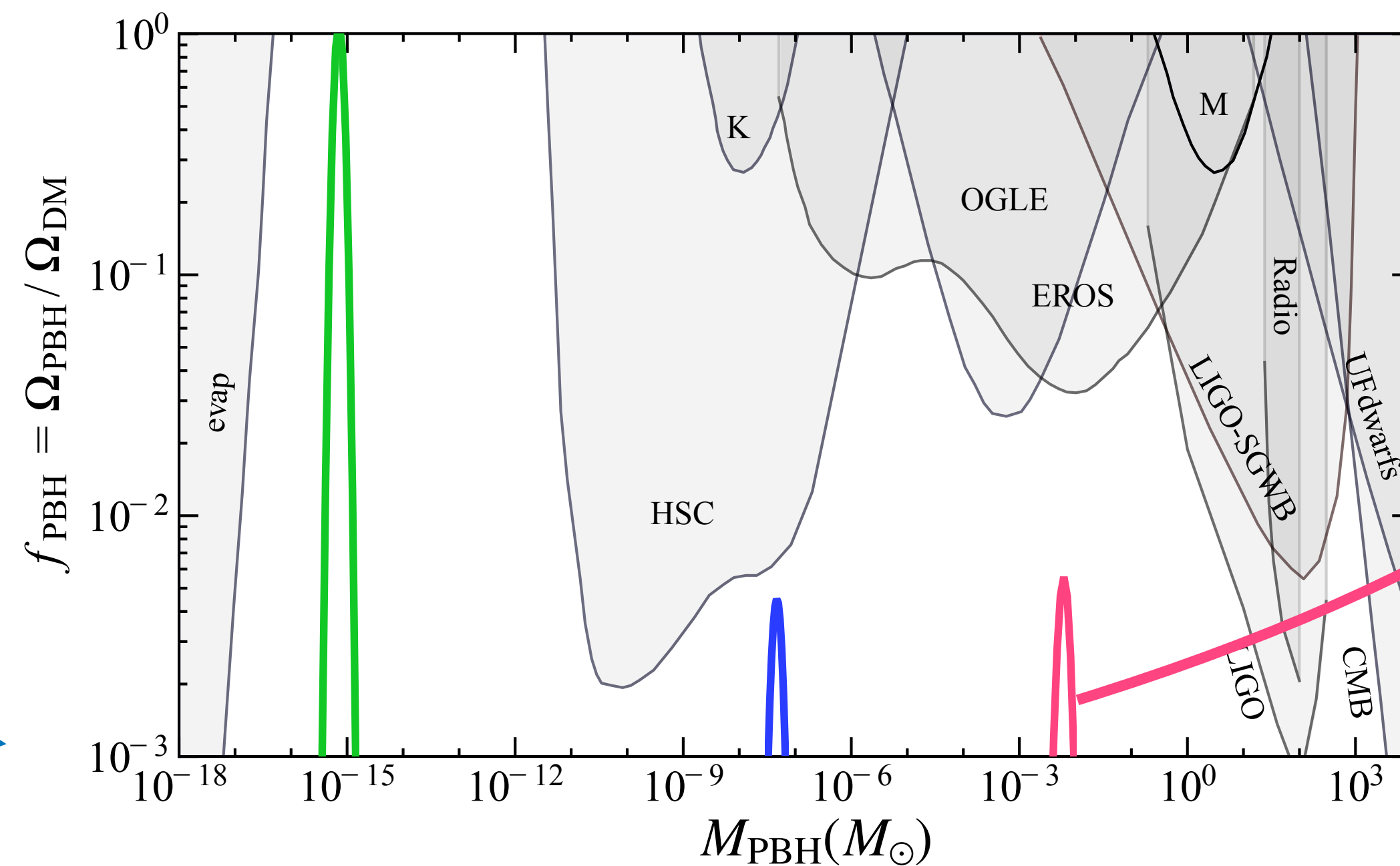
Amplifying the Superhorizon Perturbations (Multi Field)

R^2 inflation + non-minimally coupled χ

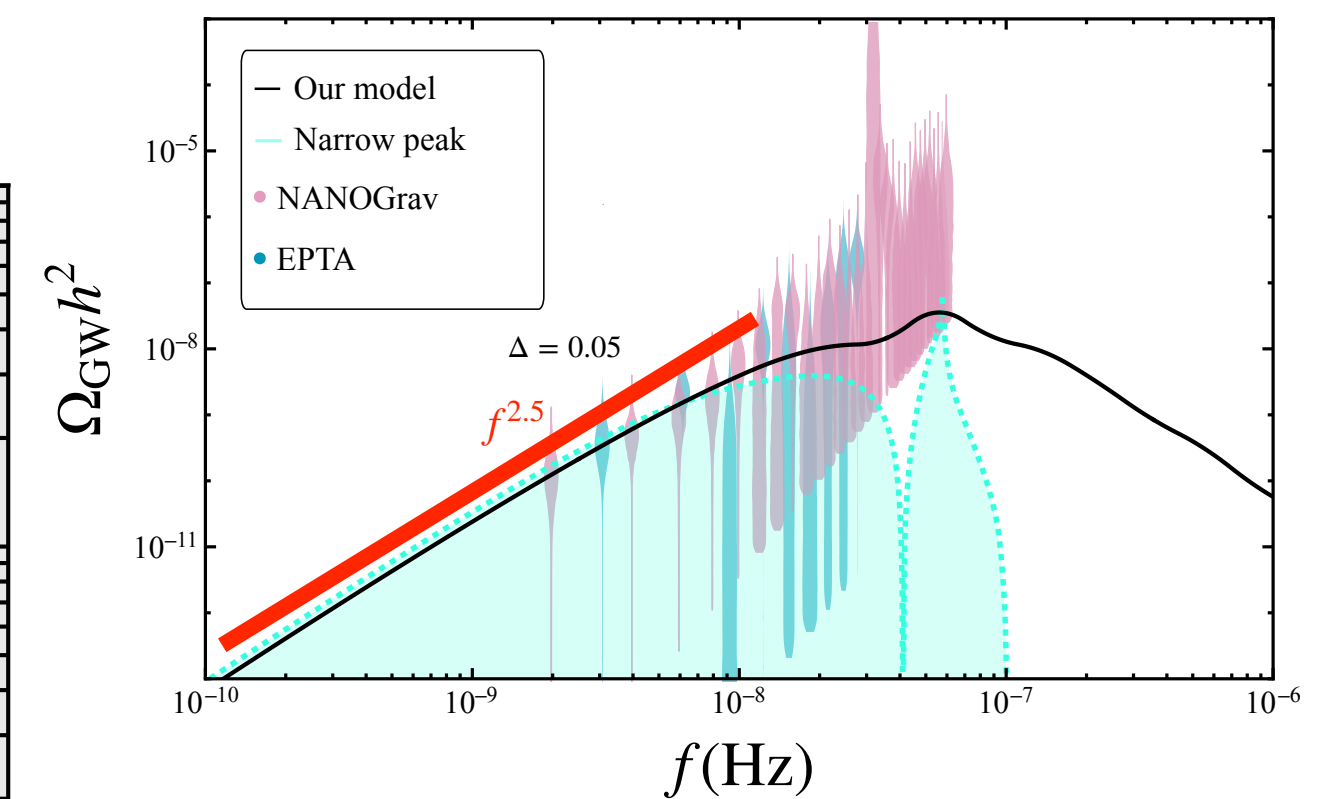
XW, YL. Zhang, M. Sasaki, 2024 (2404.02492)



Press-Schechter Formalism



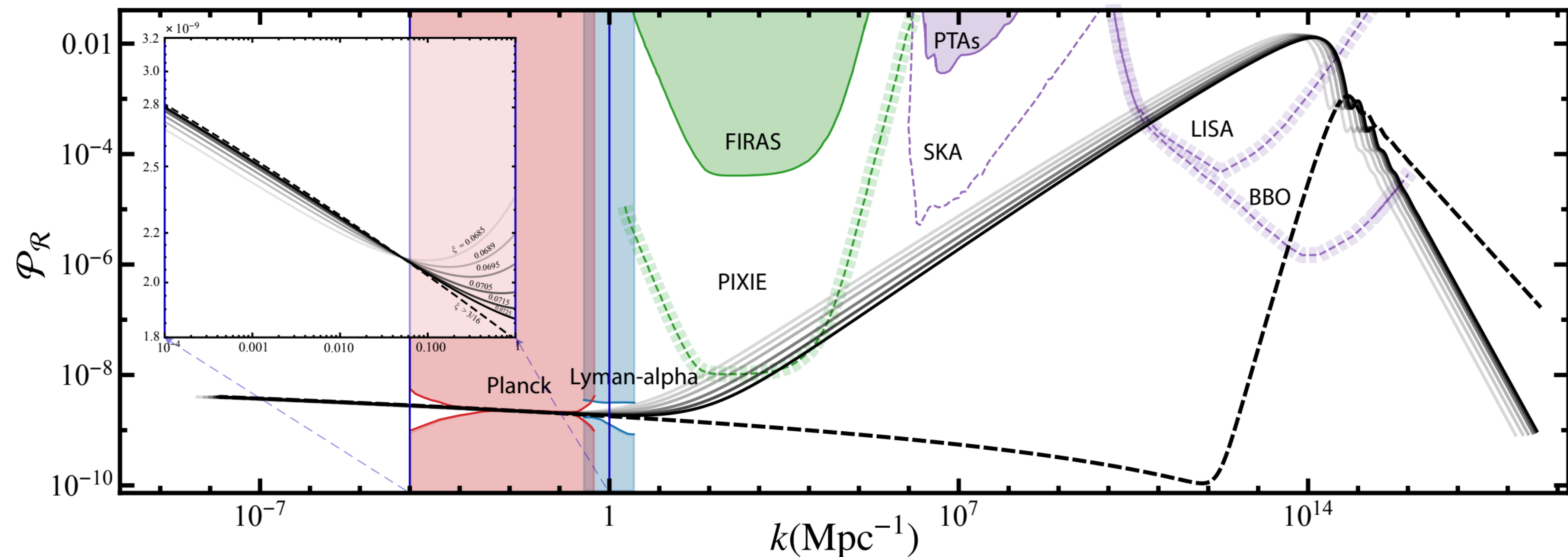
Scalar induced GW



Large scale effect: a solution to the n_s tension

R^2 inflation + non-minimally coupled χ

XW, K. Kohri, Tsutomu T. Yanagida, 2025 (2506.06797)

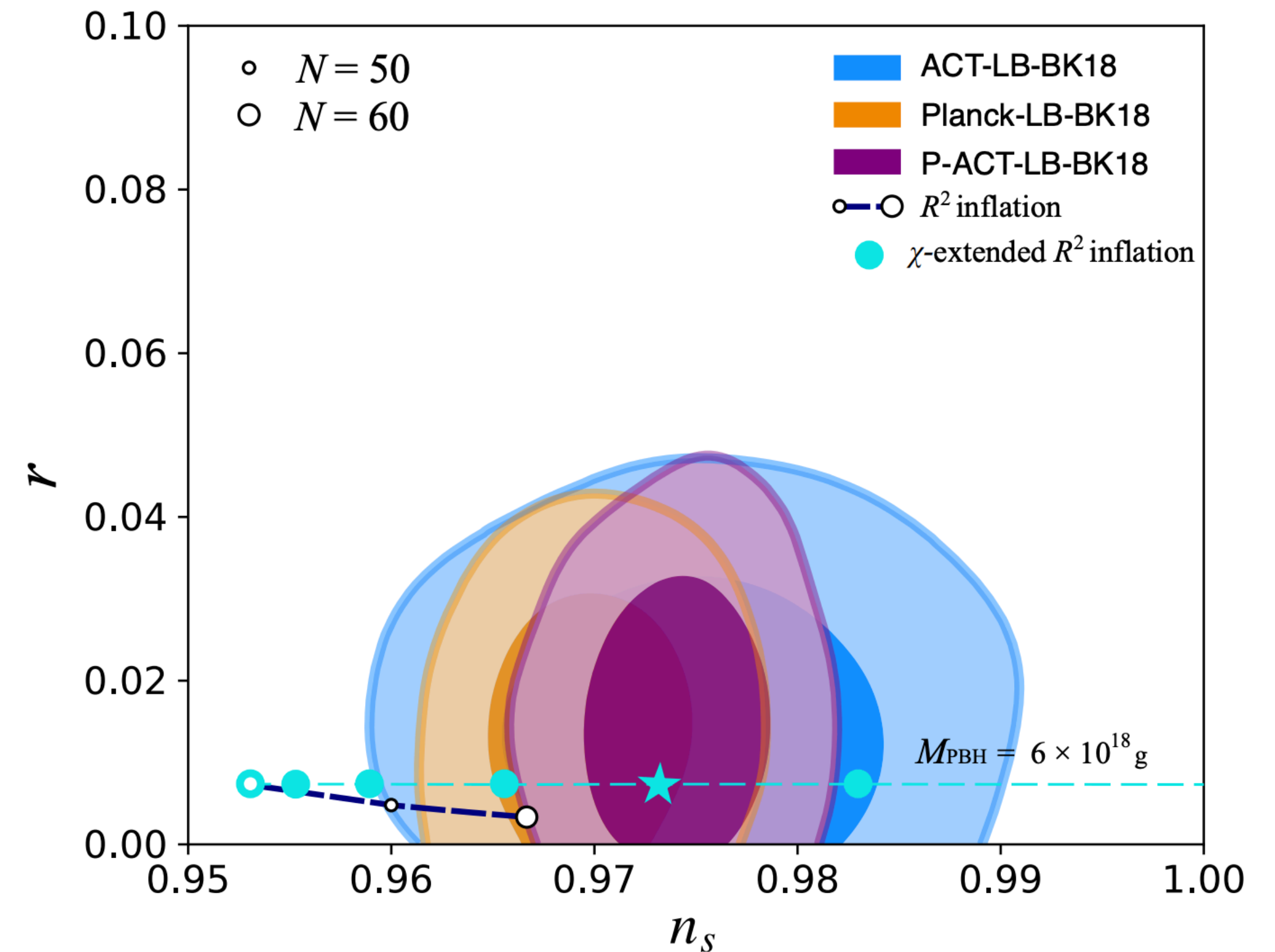
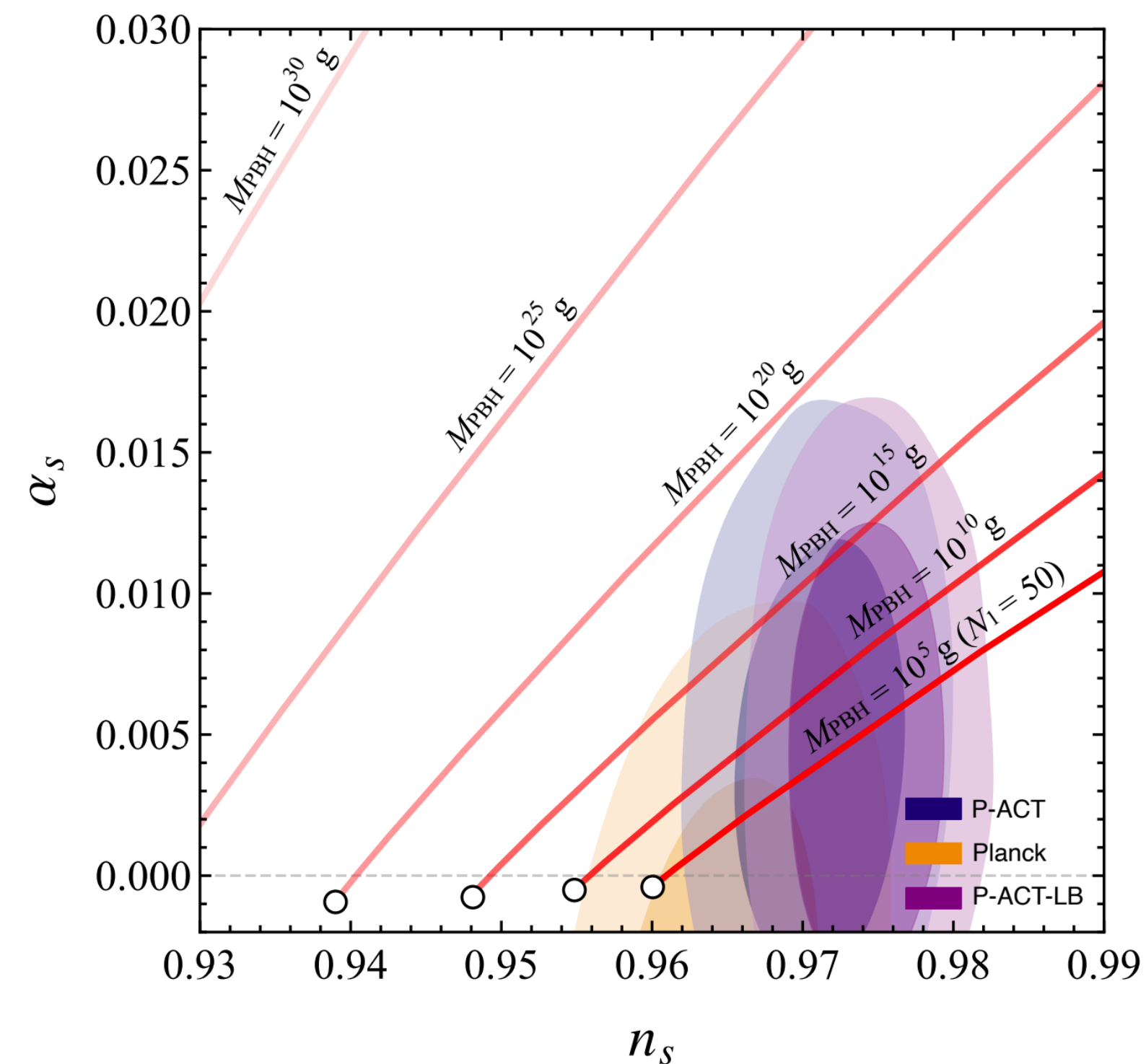


Large scale effect: a solution to the n_s tension

R^2 inflation + non-minimally coupled χ

PBHDM scenario is consistent with P-ACT result

XW, K. Kohri, Tsutomu T. Yanagida, 2025 (2506.06797)

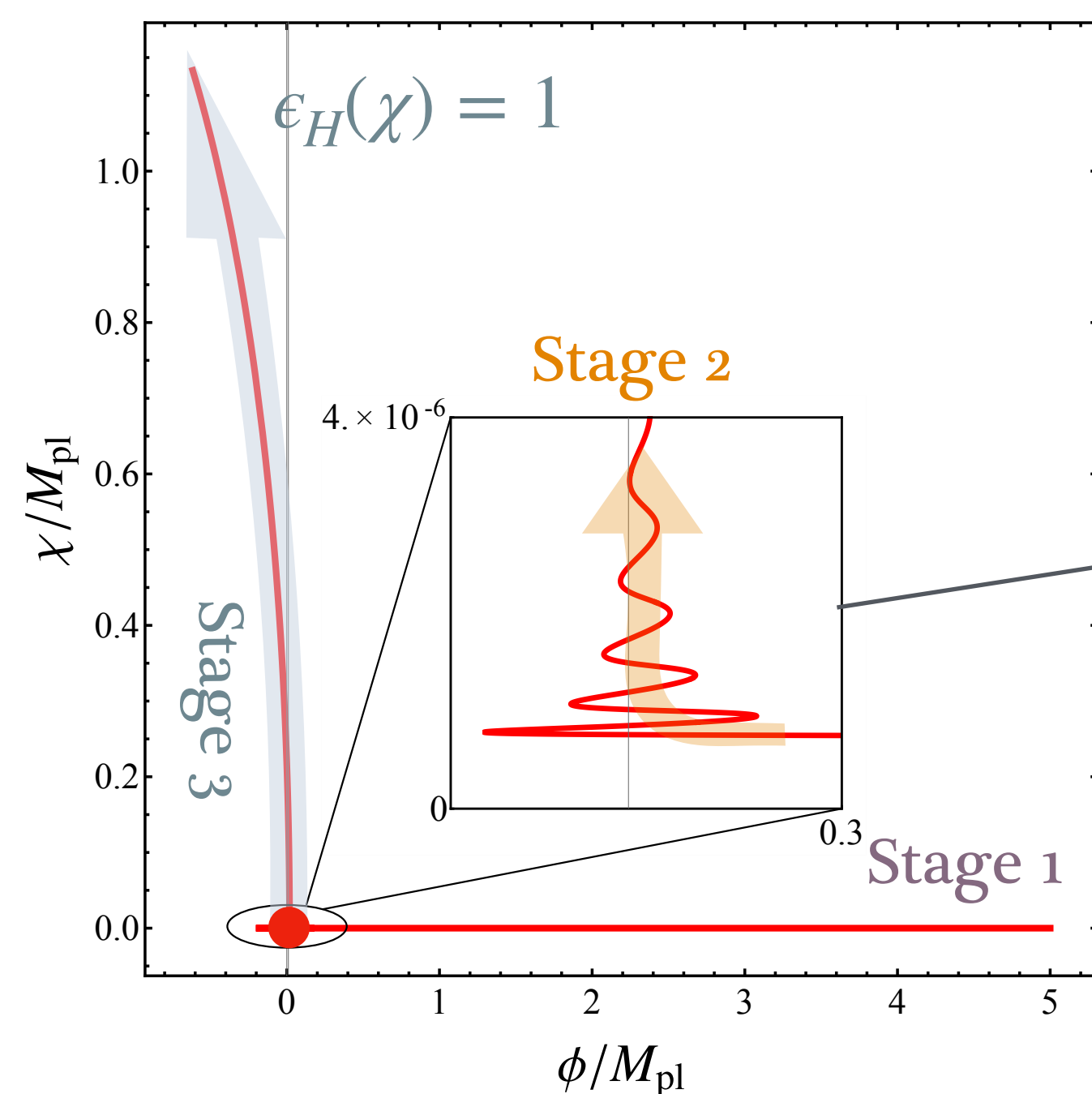


**Whether there are other interesting and unique observables
for two field models?**

Amplifying the Superhorizon Perturbations (Multi Field)

R^2 inflation + non-minimally coupled χ

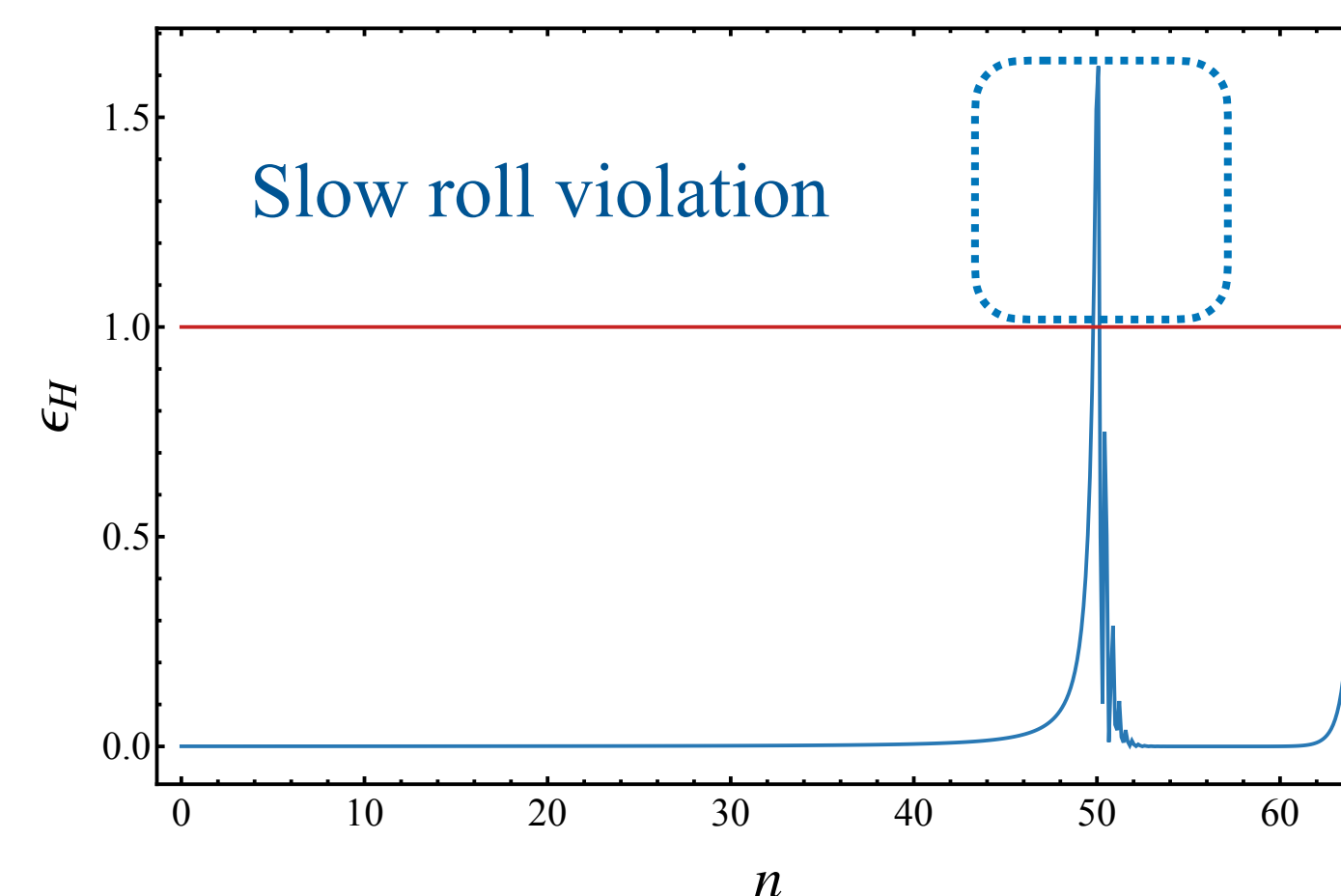
XW, YL. Zhang, M. Sasaki, 2024 (2404.02492)



Baby R^2 reheating

The slow roll is shortly violated ($\epsilon_H > 1$) by inflaton's oscillation along ϕ direction. χ stays in the potential valley.

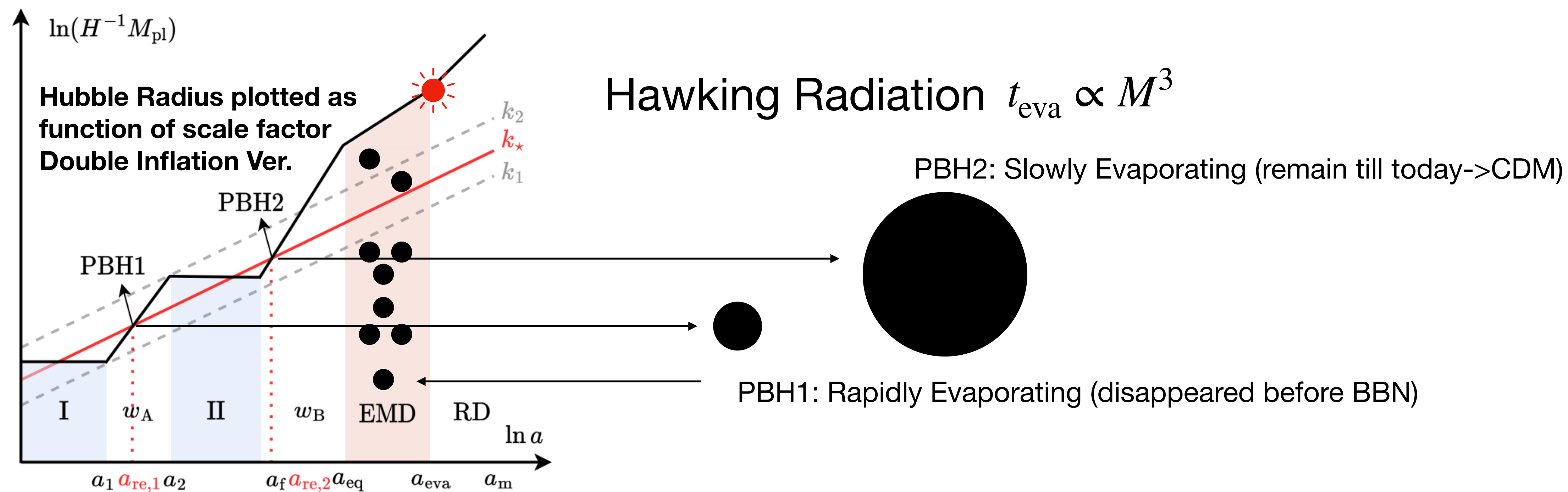
Interesting scenario happening
In hybrid-like inflationary models!



Inflation Break Stage: Dual PBH Formation

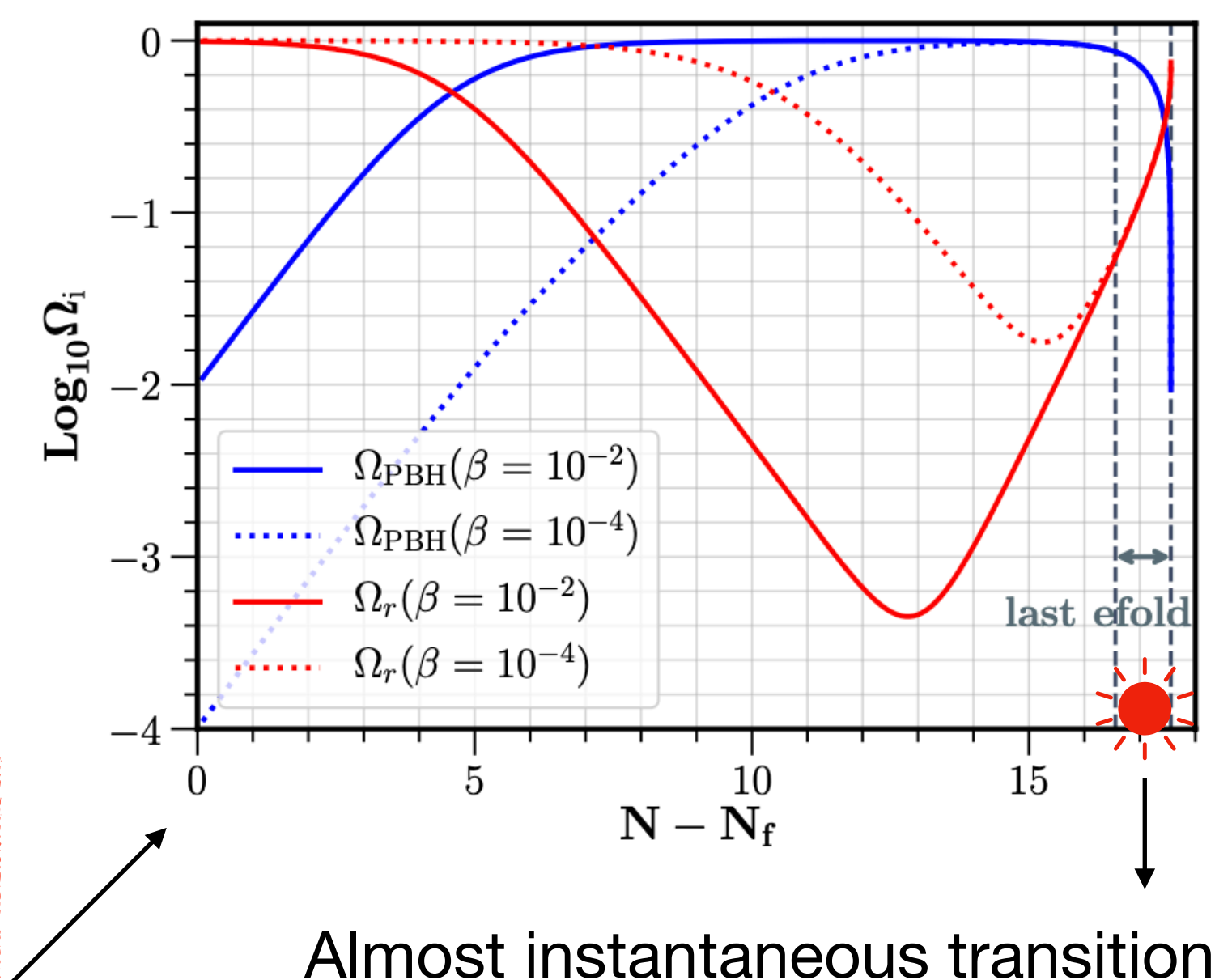
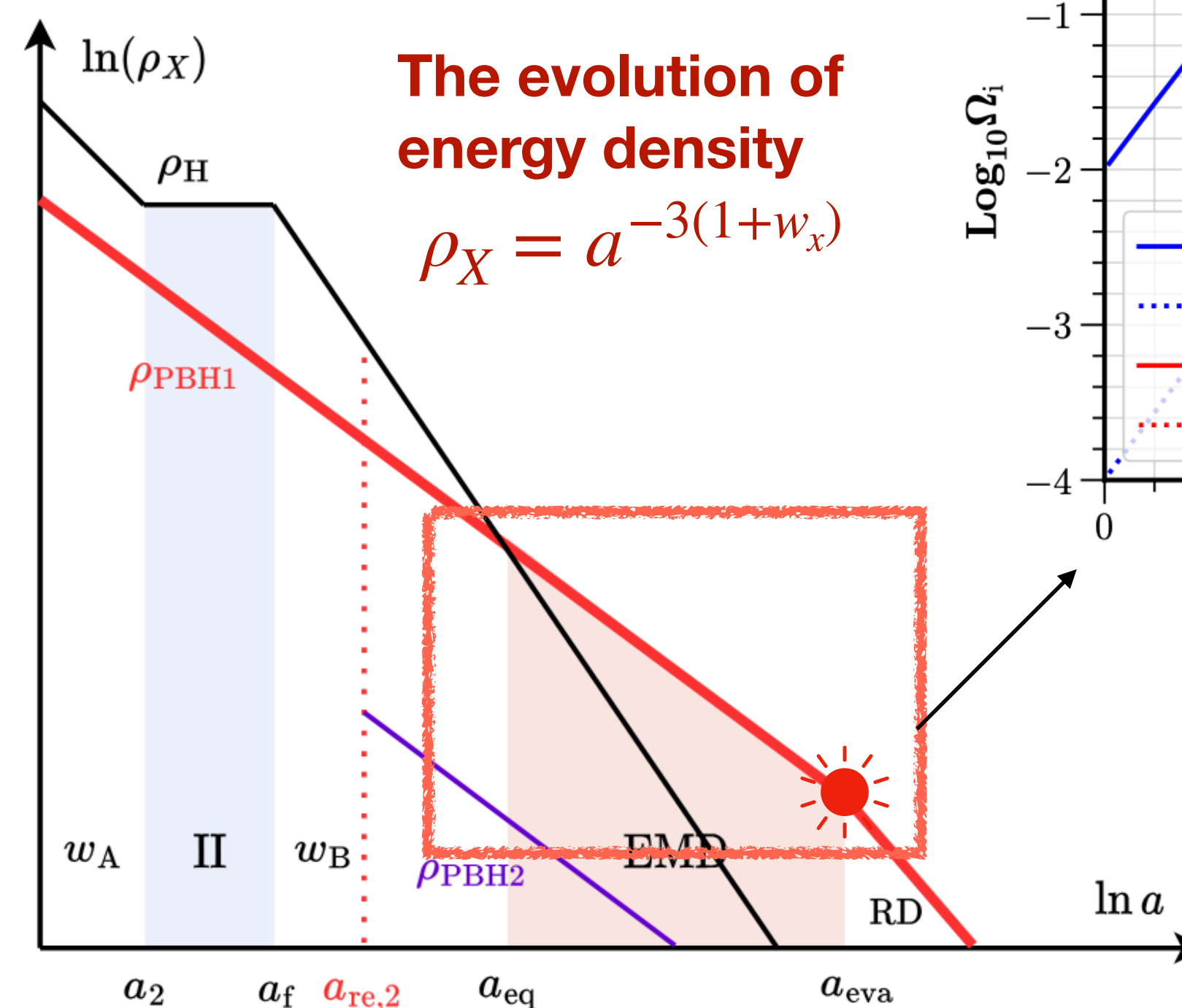
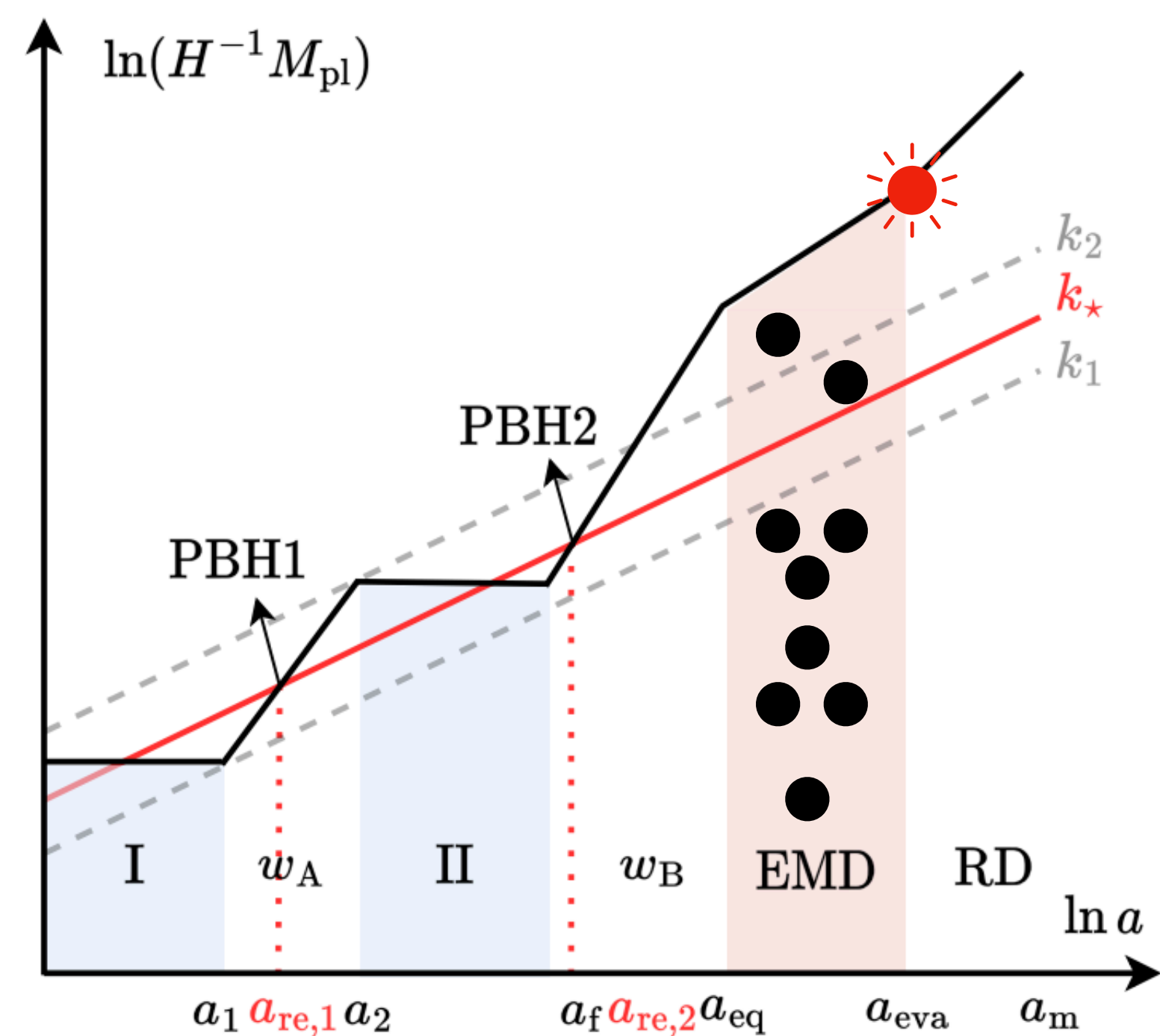
Forming two families of PBH with at nearly the same comoving scales but at two different epochs.

XW, M. Sasaki et al 2025, (2505.09337) **Let's rewrite the thermal history of the early universe!**



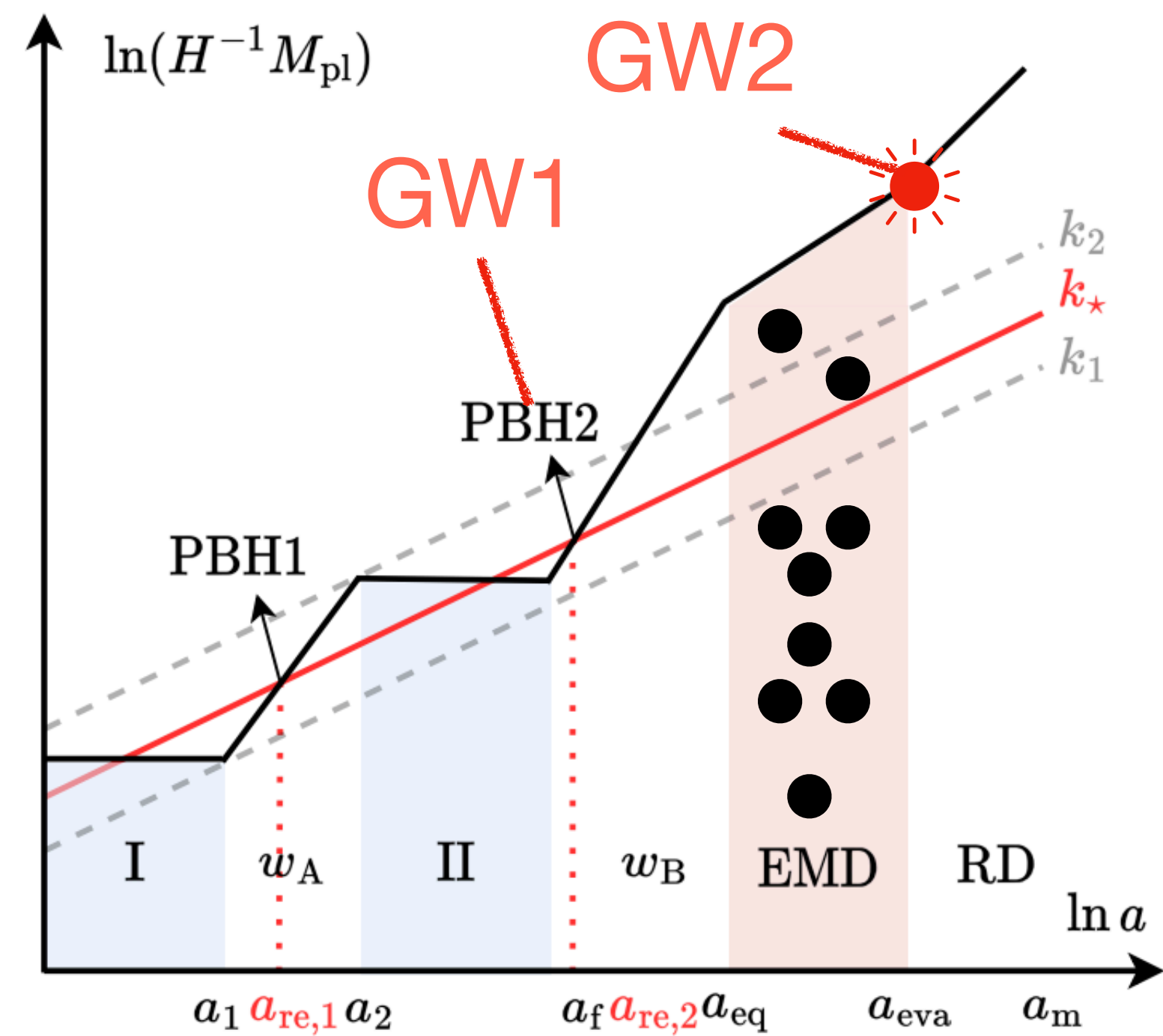
Inflation Break Stage: Dual PBH Formation

XW, M. Sasaki et al 2025, (2505.09337)

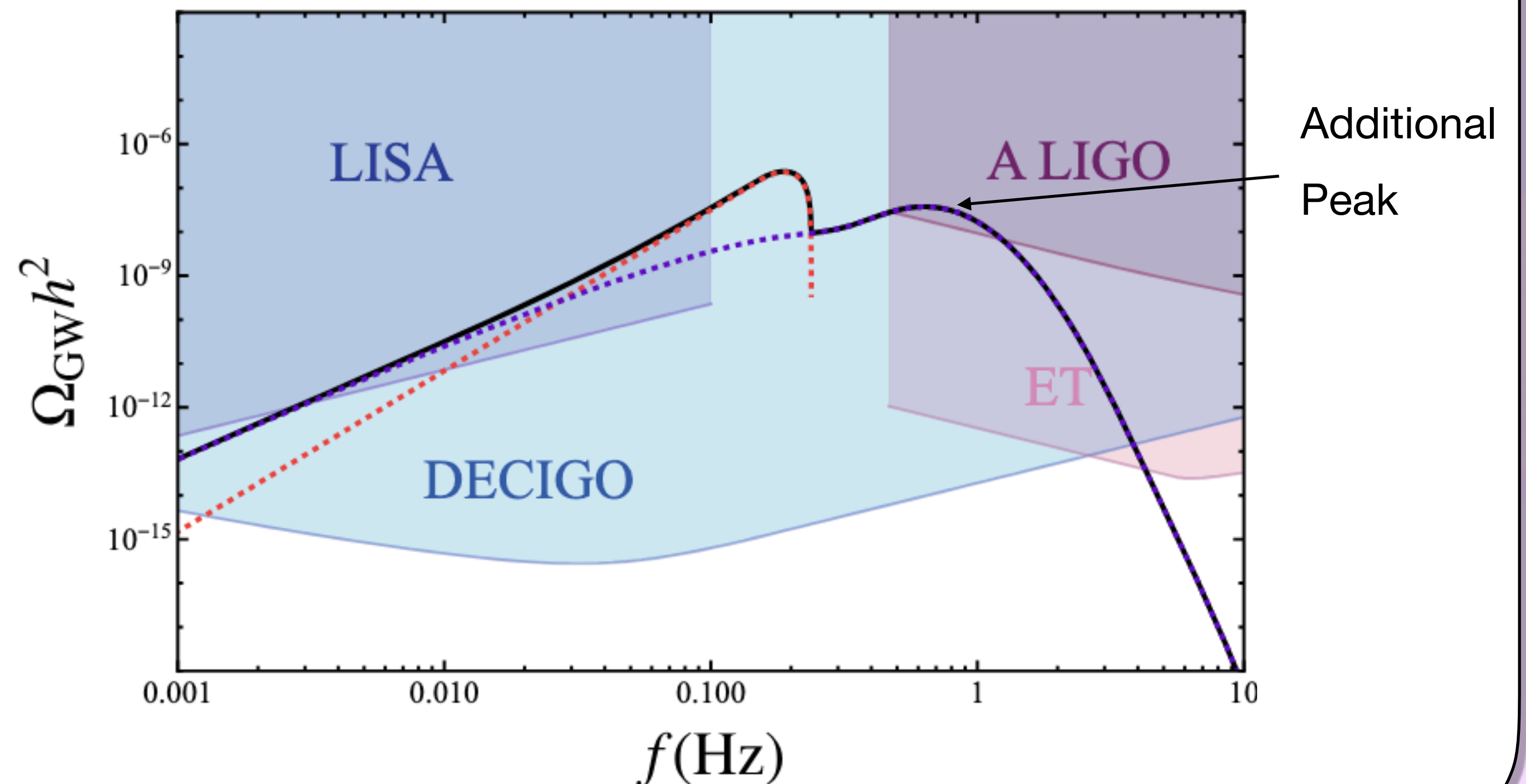


Inflation Break Stage: Dual PBH Formation

XW, M. Sasaki et al 2025, (2505.09337)



GW1: Companion of PBH2, might get enhanced during w_B
 GW2: From fast transition from EMD to RD (Guillem 2020)



II. Gravitational Waves from Yukawa Force Mergers — An Alternative to Small Scale Perturbations

Based on 2510.12984

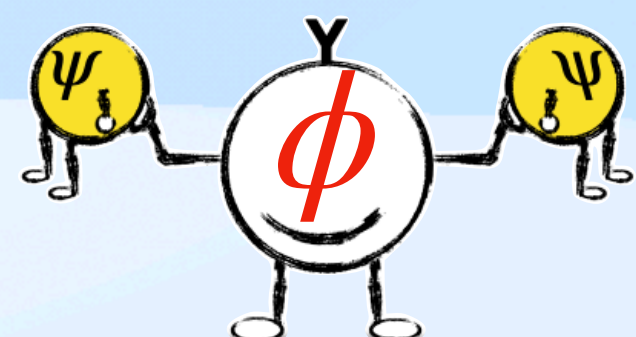
Early halo formation: Theory

Amendola, Rubio, and Wetterich, 2017
 Savastano, Amendola, Rubio, and
 Wetterich, 2019
 Flores, Kusenko 2021
 Domenech and Sasaki, 2021

The long-range Yukawa forces, if much stronger than Gravity $\beta^2 \equiv F_y/F_g \gg 1$, can efficiently form structures, halos, and black holes in the very early universe.

The Basic Lagrangian

DM&DE Interaction?



Credit. Domenech

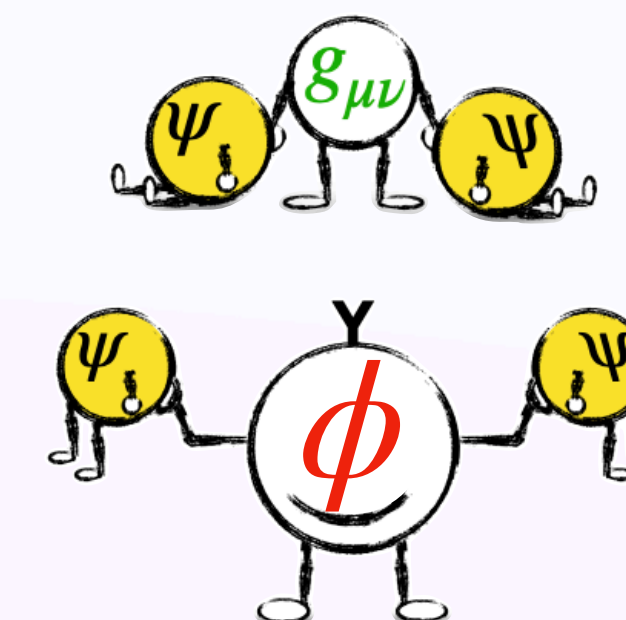
$$\mathcal{L} \supset \bar{\psi} i \Gamma^\mu D_\mu \psi - (m_\psi + y\phi) \bar{\psi} \psi - \frac{1}{2} m_\phi^2 \phi^2 - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi$$

For two fermions:

$$F_G = \frac{G m_\psi^2}{d^2}$$

$$F_y = \frac{y^2}{4\pi d^2} e^{-m_\phi d}$$

$$\frac{F_y}{F_G} \sim \frac{y^2}{G m_\psi^2} = 2\beta^2$$



For fermions in the early universe with a conserved number density: $\dot{n}_\psi + 3Hn_\psi = 0$,

The δ follows the continuity equation: $\dot{\delta}_\psi + \theta_\psi = 0$, where $\dot{\theta}_\psi + 2H\theta_\psi = -\frac{1}{a^2} \nabla^2 \phi$,

δ —number density perturbation of ψ fermions θ_ψ —divergence of the peculiar velocity $\phi = \phi_g + \phi_y$ — potential of the forces

Early halo formation: Theory

Amendola, Rubio, and Wetterich, 2017
 Savastano, Amendola, Rubio, and
 Wetterich, 2019
 Flores, Kusenko 2021
 Domenech and Sasaki, 2021

DM&DE Interaction?

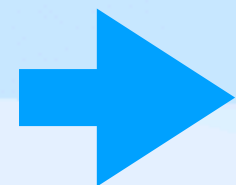


Credit. Domenech

The perturbation equation of motion for number density perturbation of ψ fermions:

$$\ddot{\delta}_k + 2H\dot{\delta}_k - \frac{3}{2}H^2\Omega_\psi \left(1 + \frac{2\beta^2}{1 + (kl_y)^{-2}} \right) \delta_k = 0$$

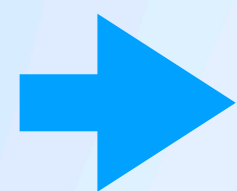
Solution
(in R.D.)



$$\delta_k \propto c_1 I_0(\sqrt{6\alpha(a/a_{\text{eq}})}) + c_2 K_0(\sqrt{6\alpha(a/a_{\text{eq}})}),$$

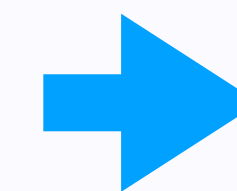
$$\alpha = f_\psi \left[1 + \frac{2\beta^2}{1 + (kl_y)^{-2}} \right]$$

When
 $6\alpha(a/a_{\text{eq}}) \ll 1$
 (Pure Gravity/
 Force Screened)

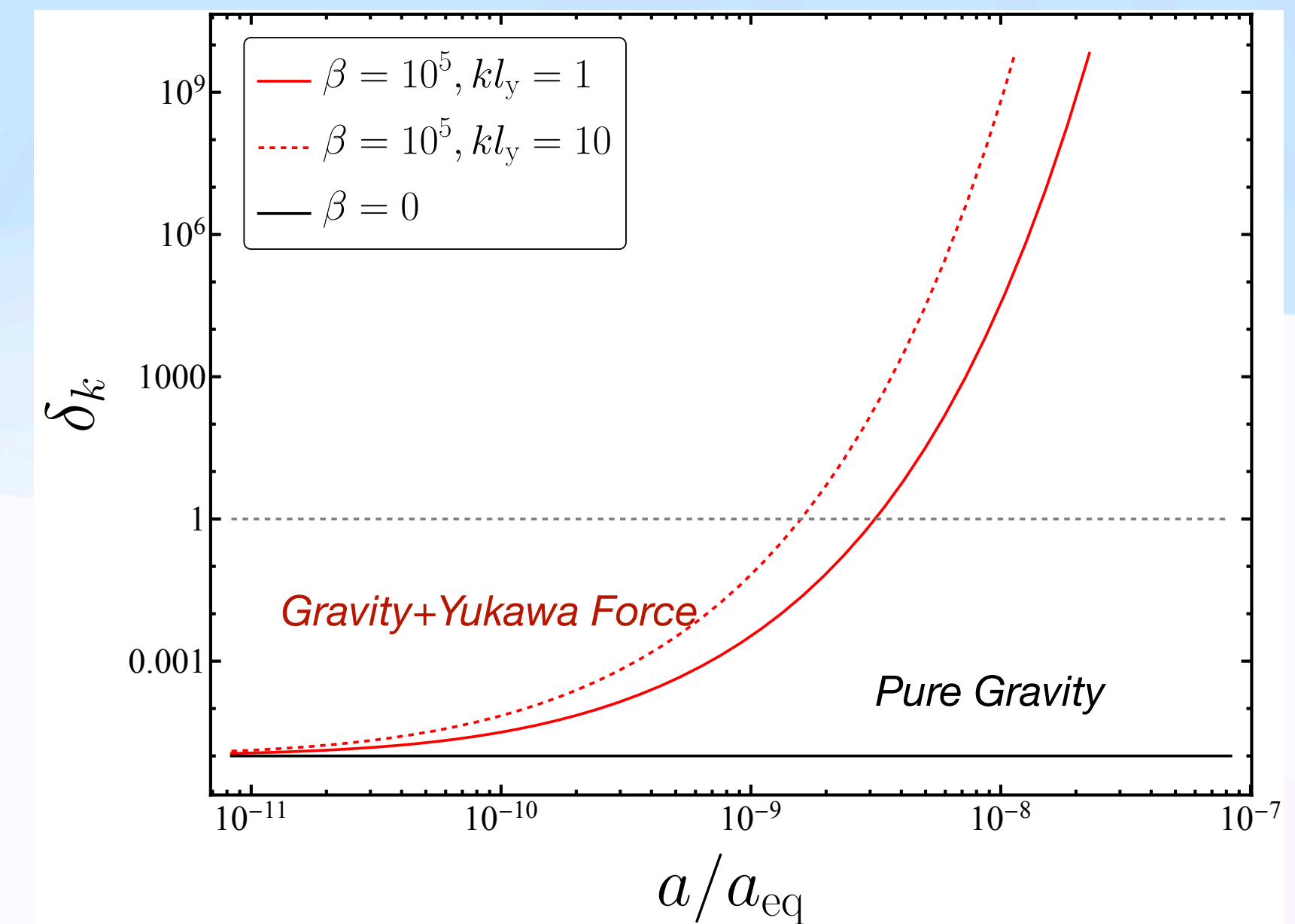


$$I_0(\sqrt{6\alpha(a/a_{\text{eq}})}) \rightarrow \text{const.}$$

When
 $6\alpha(a/a_{\text{eq}}) \gg 1$
 (Strong Force)



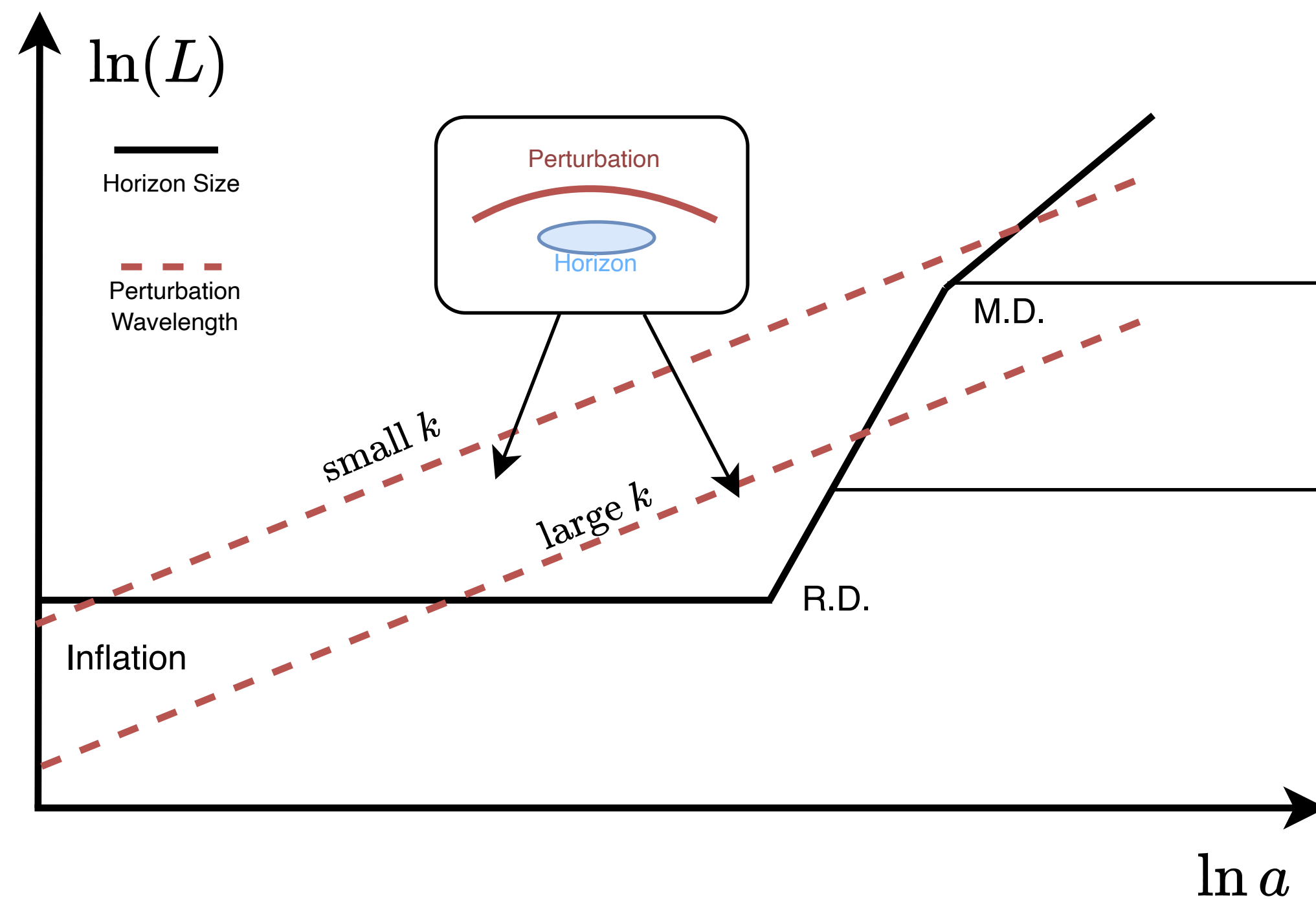
$$I_0(\sqrt{6\alpha(a/a_{\text{eq}})}) \rightarrow \frac{e^{\sqrt{6\alpha(a/a_{\text{eq}})}}}{\sqrt{2\pi\sqrt{6\alpha(a/a_{\text{eq}})}}}$$



Early halo formation: Theory

Amendola, Rubio, and Wetterich, 2017
Savastano, Amendola, Rubio, and
Wetterich, 2019
Flores, Kusenko 2021
Domenech and Sasaki, 2021
...

Gravity Only



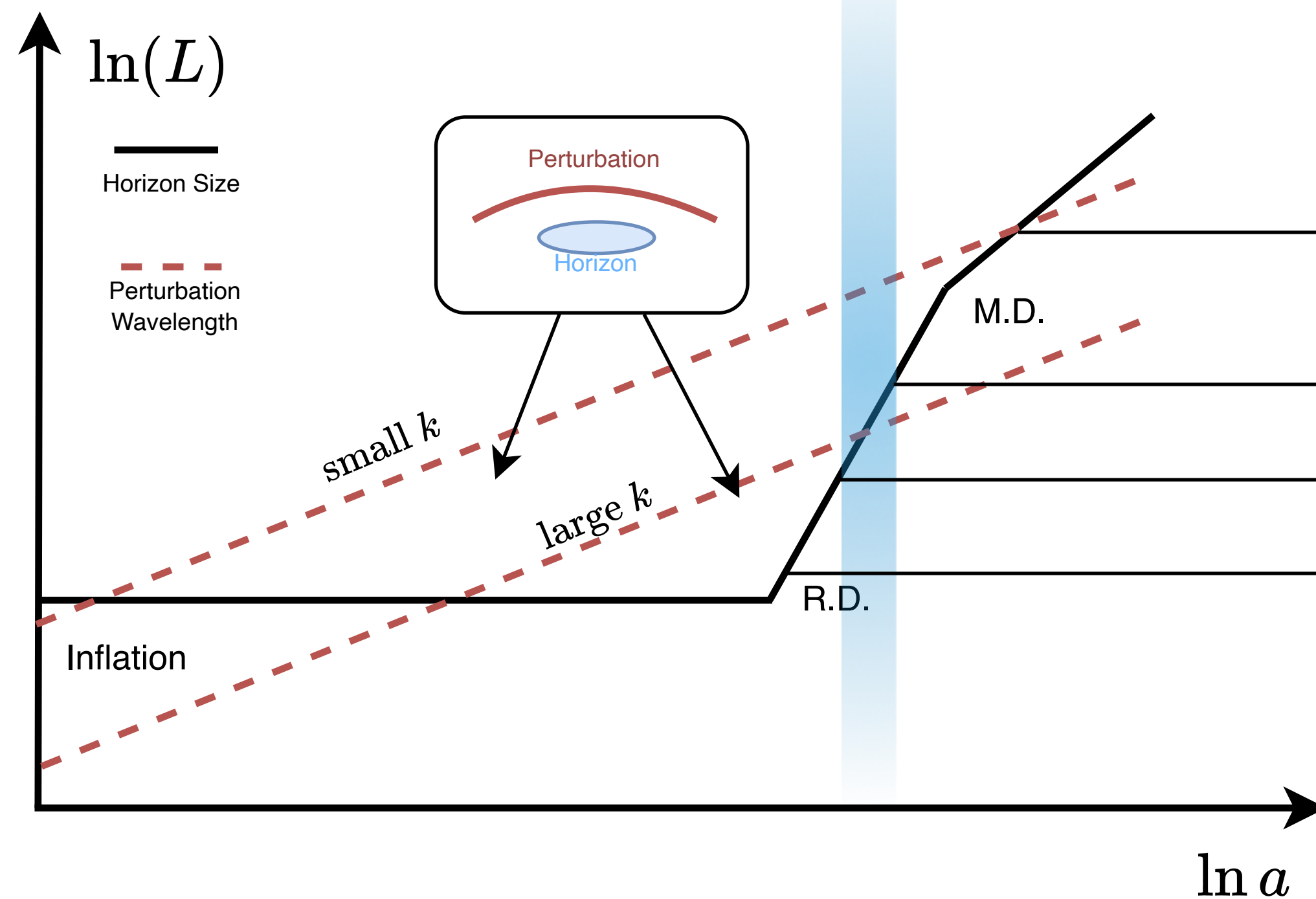
Structures Start to Form
(Large-scale, Gravitational Collapse)

Density Perturbations Barely Grow

Early halo formation: Theory

Amendola, Rubio, and Wetterich, 2017
 Savastano, Amendola, Rubio, and
 Wetterich, 2019
 Flores, Kusenko 2021
 Domenech and Sasaki, 2021
 ...

Gravity + Long-range force ($2\beta^2 \gg 1$)



Structures Start to Form
 (Large-scale, Gravitational Collapse)

Long-range Forces are Screened ($H^{-1} \gg \beta l_y$)

Structures Start to Form
 (Small-scale, Yukawa-driven Collapse)

Density Perturbations Barely Grow

Early halo formation: Theory

Amendola, Rubio, and Wetterich, 2017
 Savastano, Amendola, Rubio, and
 Wetterich, 2019
 Flores, Kusenko 2021
 Domenech and Sasaki, 2021

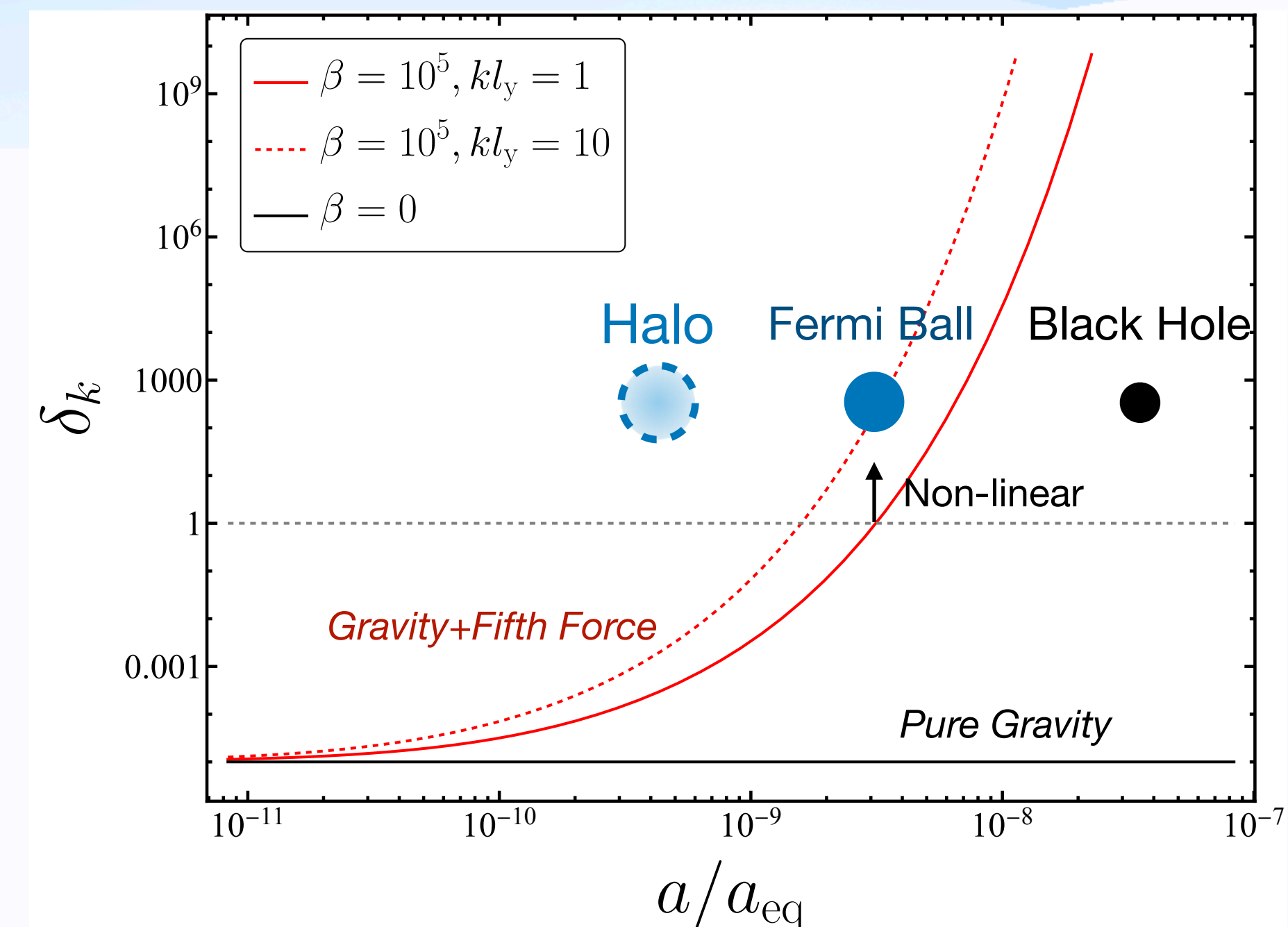
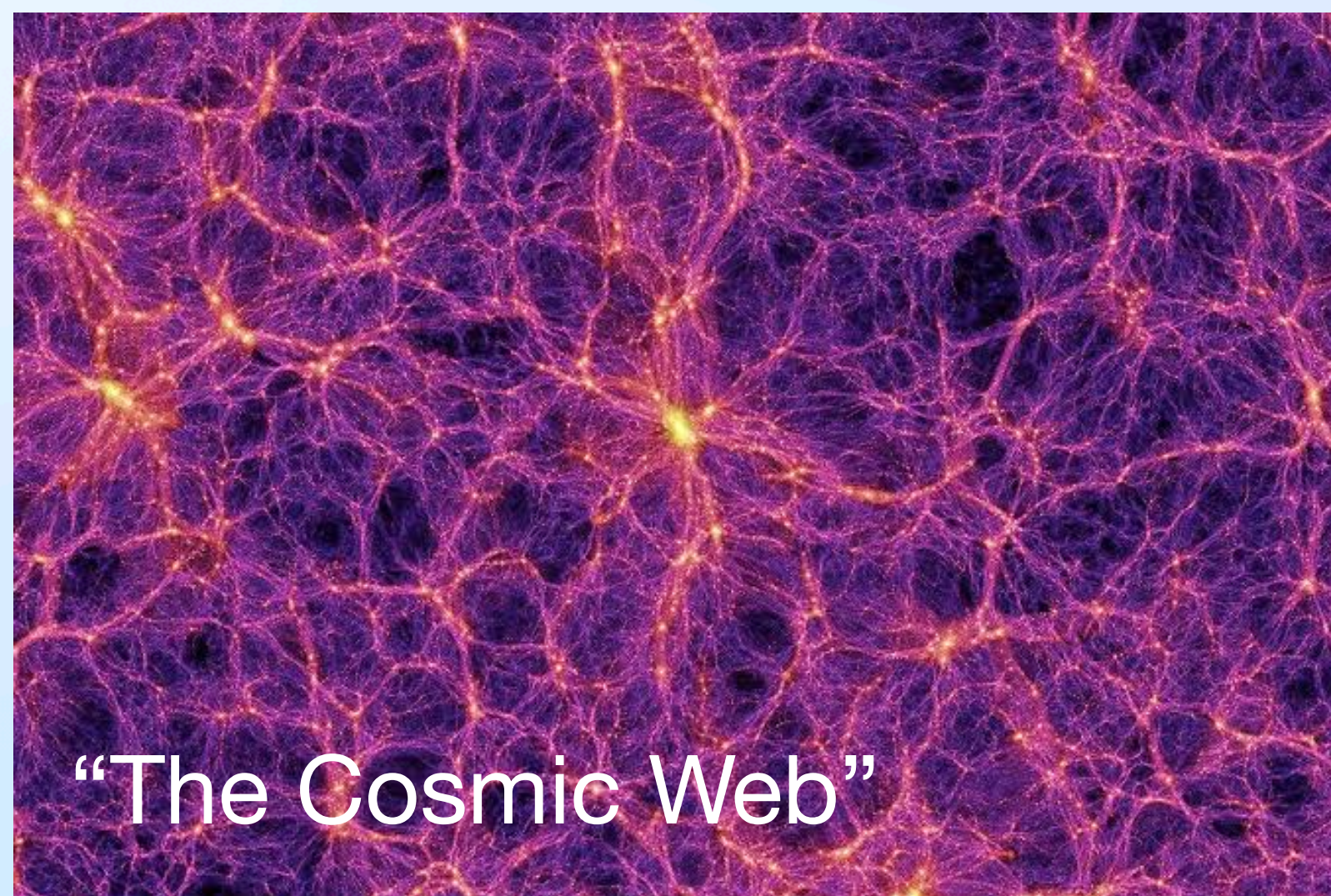
The long-range Yukawa forces, if much stronger than Gravity $\beta^2 \equiv F_y/F_g \gg 1$, can efficiently form structures, halos, and black holes in the very early universe.

DM&DE Interaction?



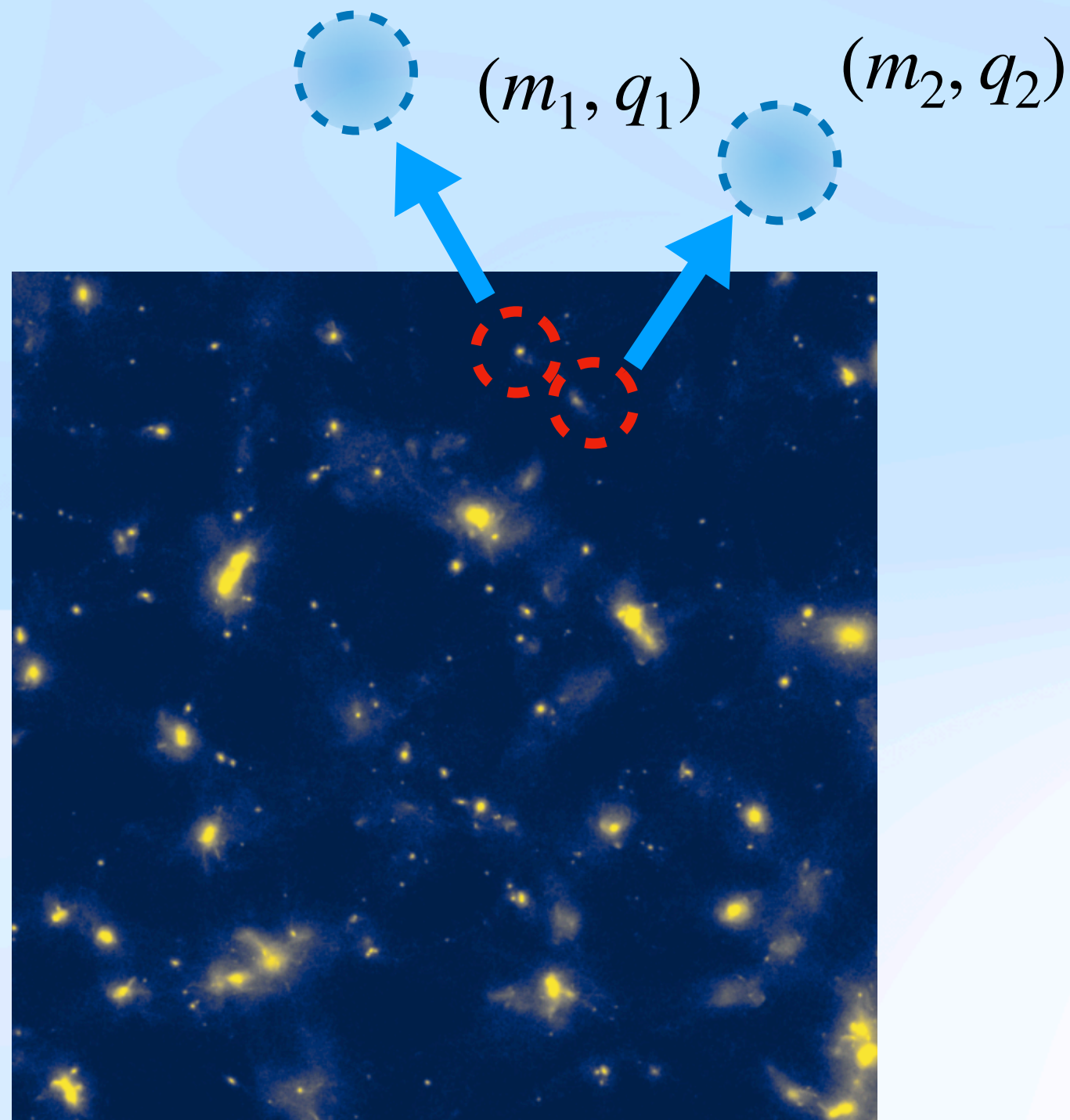
Credit. Domenech

$$\ddot{\delta}_k + 2H\dot{\delta}_k - \frac{3}{2}H^2\Omega_\psi \left(1 + \frac{2\beta^2}{1 + (kl_y)^{-2}} \right) \delta_k = 0$$



Early halo formation: Theory

Something is not yet considered..



For two fermionic minihalos:

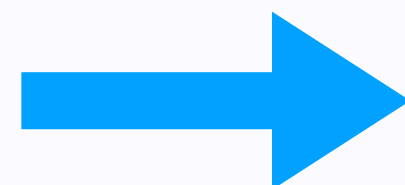
$$F_G = \frac{Gq_1q_2m_\psi^2}{d^2}$$

$$F_y = \frac{y^2q_1q_2}{4\pi d^2}e^{-m_\phi d}$$

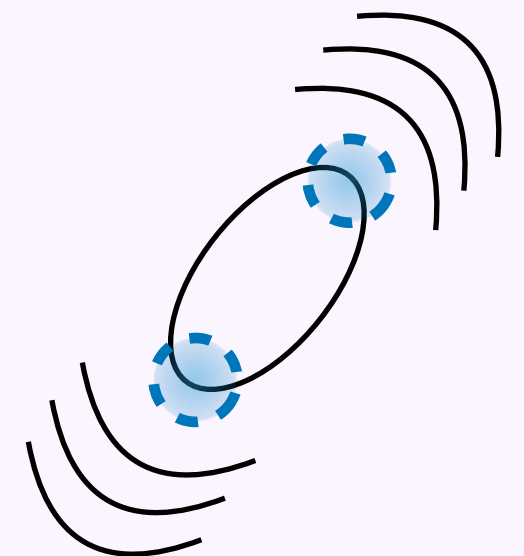
$$\frac{F_y}{F_G} \sim \frac{y^2}{Gm_\psi^2} = 2\beta^2$$

Attractive Potential V.S. Hubble Flow

$$\mu H^2 d^2 - \frac{Gm_1m_2}{d} - \frac{y^2q_1q_2}{d} < 0$$



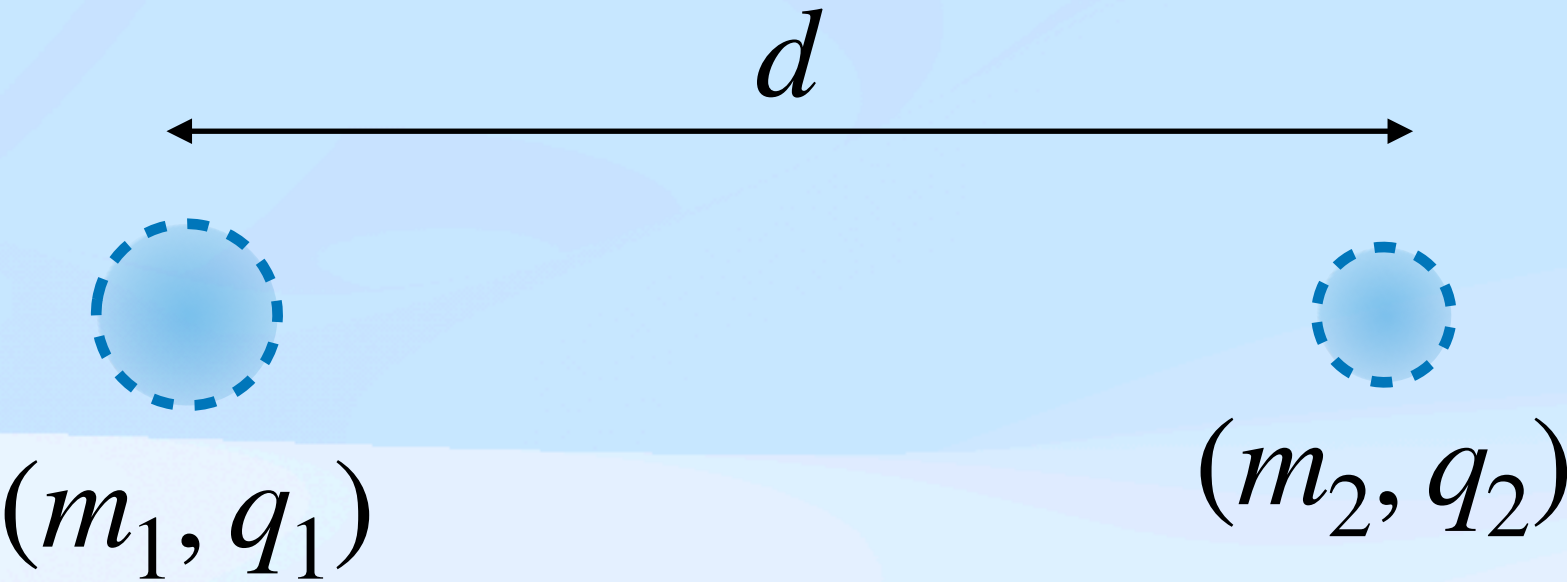
Isolated Bound System
+ Tidal Force



Radiations from halo binaries:

GW and SW from the Halo Inspirals

The simplest case: $dm_\phi \ll 1$ $\frac{F_y}{F_G} = 2\beta^2$



$$P_{\text{GW}} \propto G |\ddot{Q}_g^{ab}|^2.$$

$$P_{\text{SW}} \propto y^2 |\ddot{Q}_q^{ab}|^2.$$

Quadrupole **Scalar Waves** and **Gravitational Waves** are radiated at the same time.

(Dipole momentum vanishes)

Gravity V.S. Yukawa Forces

	Gravitationally-Bound	Yukawa-Bound
Strain h_z	$\frac{4}{d_L c^4} (\pi f_z)^{2/3} (G\mathcal{M}_z)^{5/3} \quad (\star)$	$(\star) \times (2\beta^2)^{2/3}$
Frequency $f_z = f/(1+z)$	$\frac{1}{\pi(1+z)} \sqrt{\frac{G(m_1+m_2)}{a^3}} \quad (\circ)$	$(\circ) \times \sqrt{2\beta^2}$
Orbital Evolution df_z/dt_z	$\frac{96}{5} \pi^{8/3} \left(\frac{G\mathcal{M}_z}{c^3}\right)^{5/3} f_z^{11/3} \quad (\Delta)$	$(\Delta) \times (2\beta^2)^{5/3}$

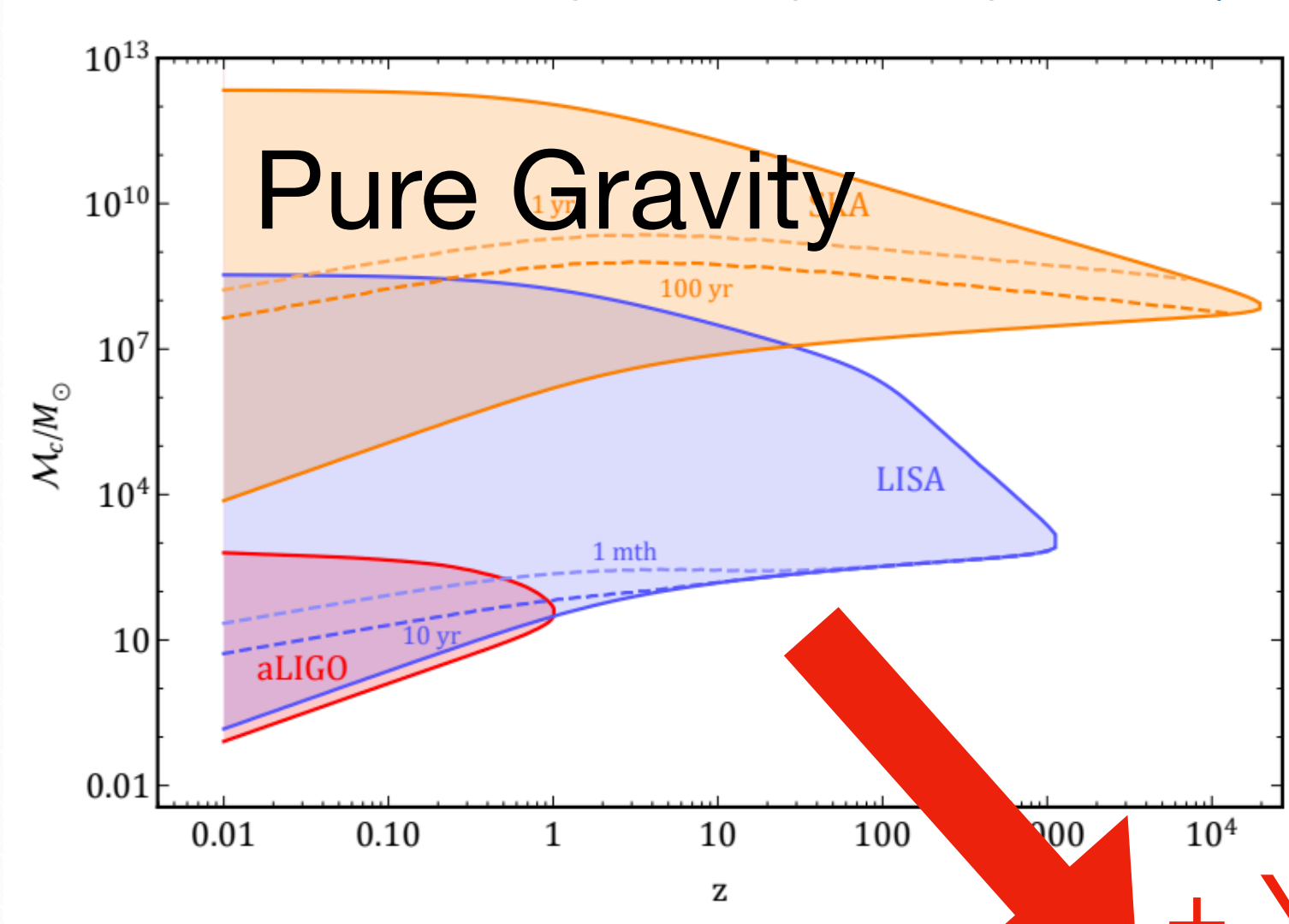
Detectability

Gravity V.S. Yukawa Forces

	Gravitationally-Bound	Yukawa-Bound
Strain h_z	$\frac{4}{d_L c^4} (\pi f_z)^{2/3} (G\mathcal{M}_z)^{5/3} \quad (\star)$	$(\star) \times (2\beta^2)^{2/3}$
Frequency $f_z = f/(1+z)$	$\frac{1}{\pi(1+z)} \sqrt{\frac{G(m_1+m_2)}{a^3}} \quad (\circ)$	$(\circ) \times \sqrt{2\beta^2}$
Orbital Evolution df_z/dt_z	$\frac{96}{5} \pi^{8/3} \left(\frac{G\mathcal{M}_z}{c^3}\right)^{5/3} f_z^{11/3} \quad (\Delta)$	$(\Delta) \times (2\beta^2)^{5/3}$

$d_L \propto (1+z)$
 $f_z = f/(1+z)$
 $\mathcal{M}_z = \mathcal{M}(1+z)$

Detectable CBC by LIGO/LISA/SKA (Credit. Qianhang)



High-redshift events are not detectable if BBH

- 1) Cut-off Frequency $\propto 1/\mathcal{M}_z$
- 2) Strain related to $\mathcal{M}$

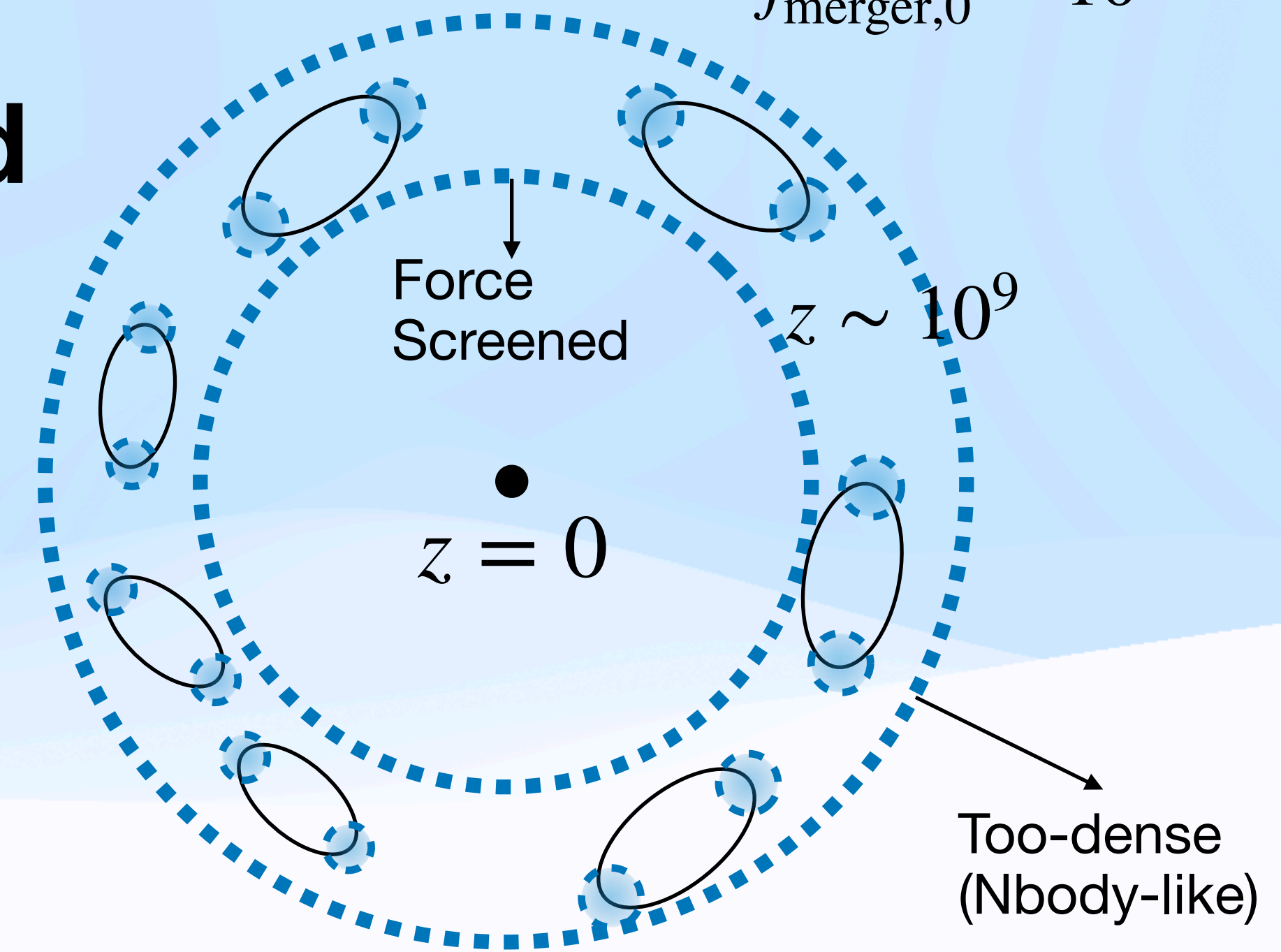
+ Yukawa Force

Radiations from halo binaries:

Individual event & stochastic background

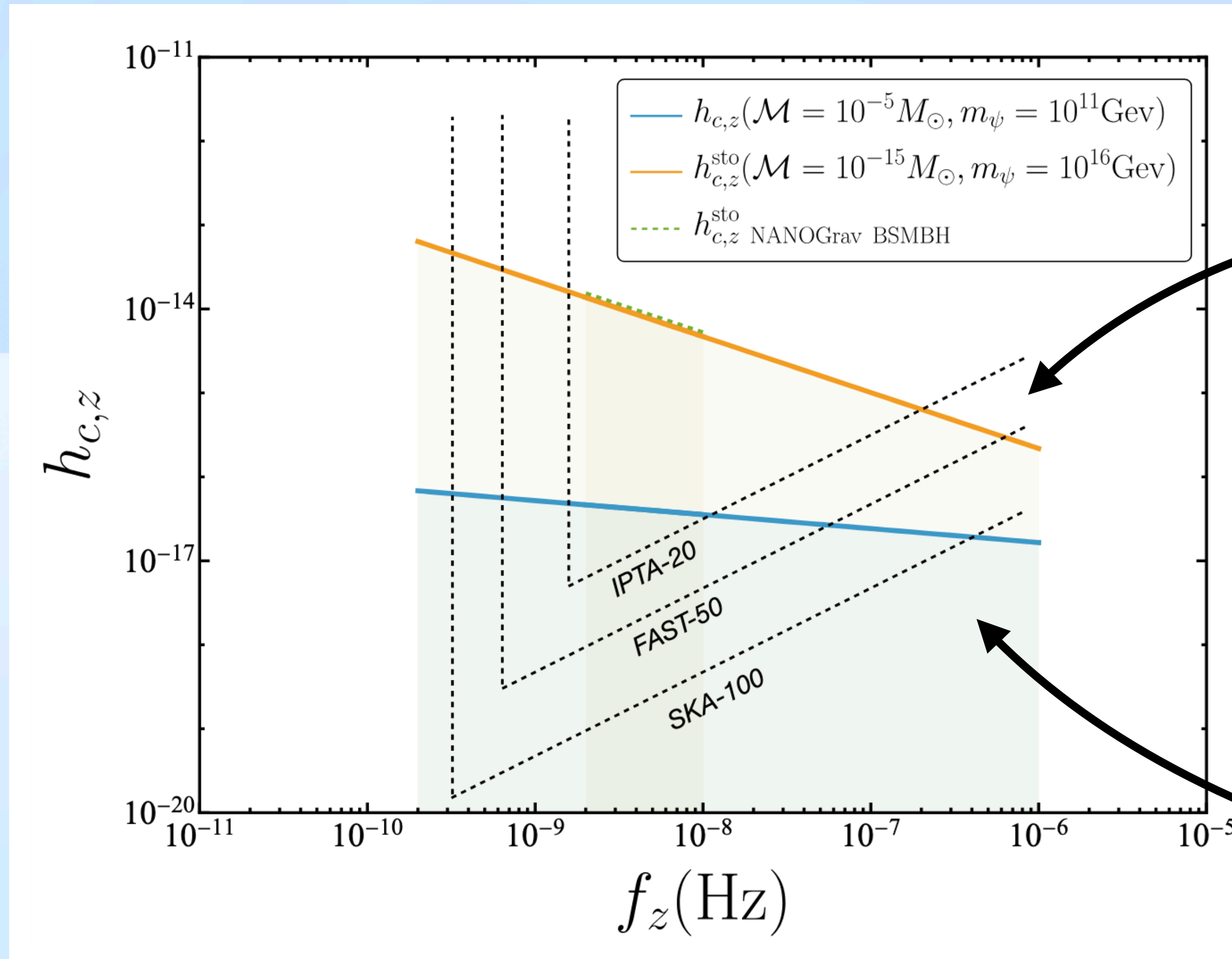
The fraction of merger relics in dark matter

$$f_{\text{merger},0} = 10^{-6}$$



Stochastic GWB @ $z = 10^9 \pm \delta z$

Single Event @ $z = 10^9$

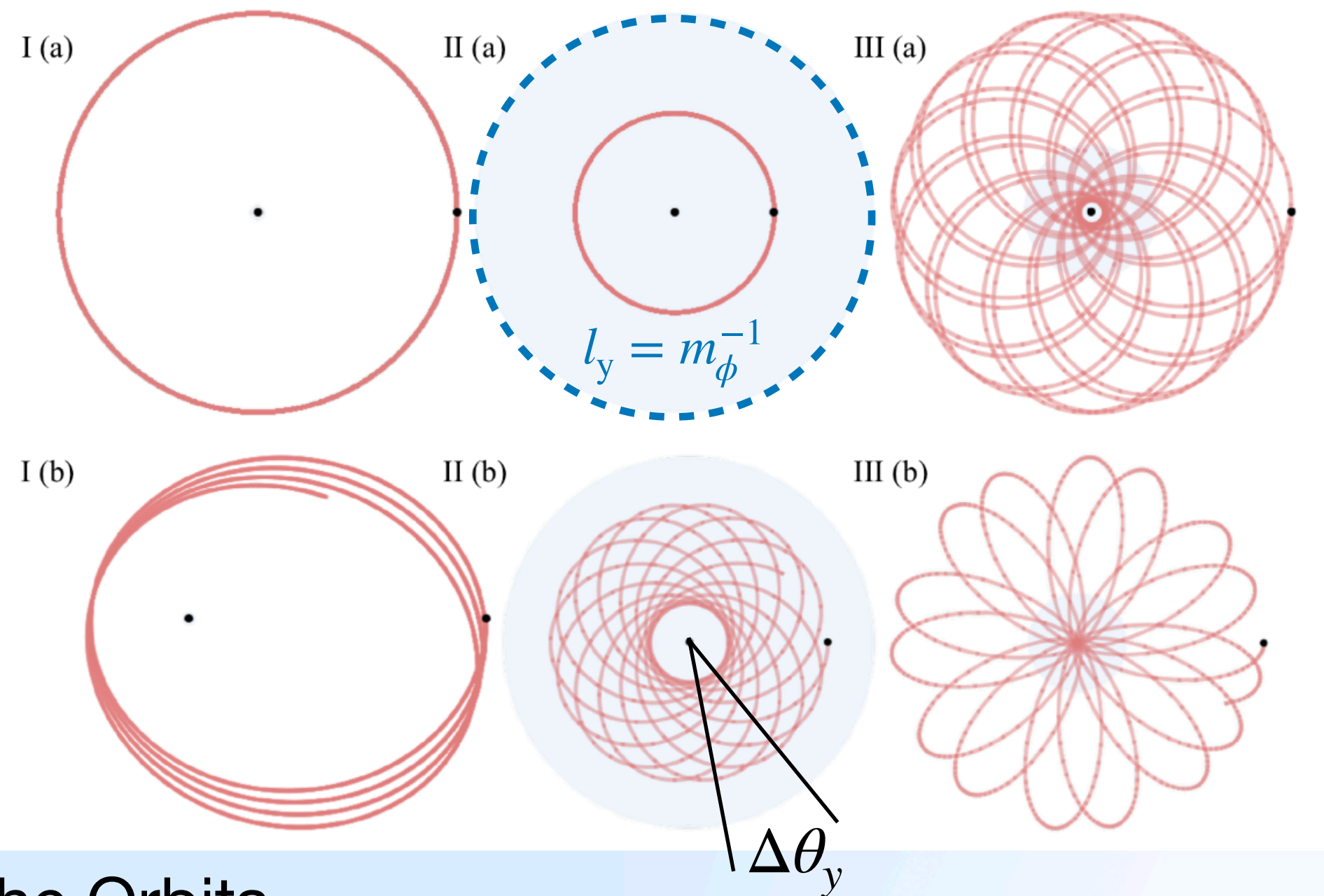


Radiations from halo binaries:

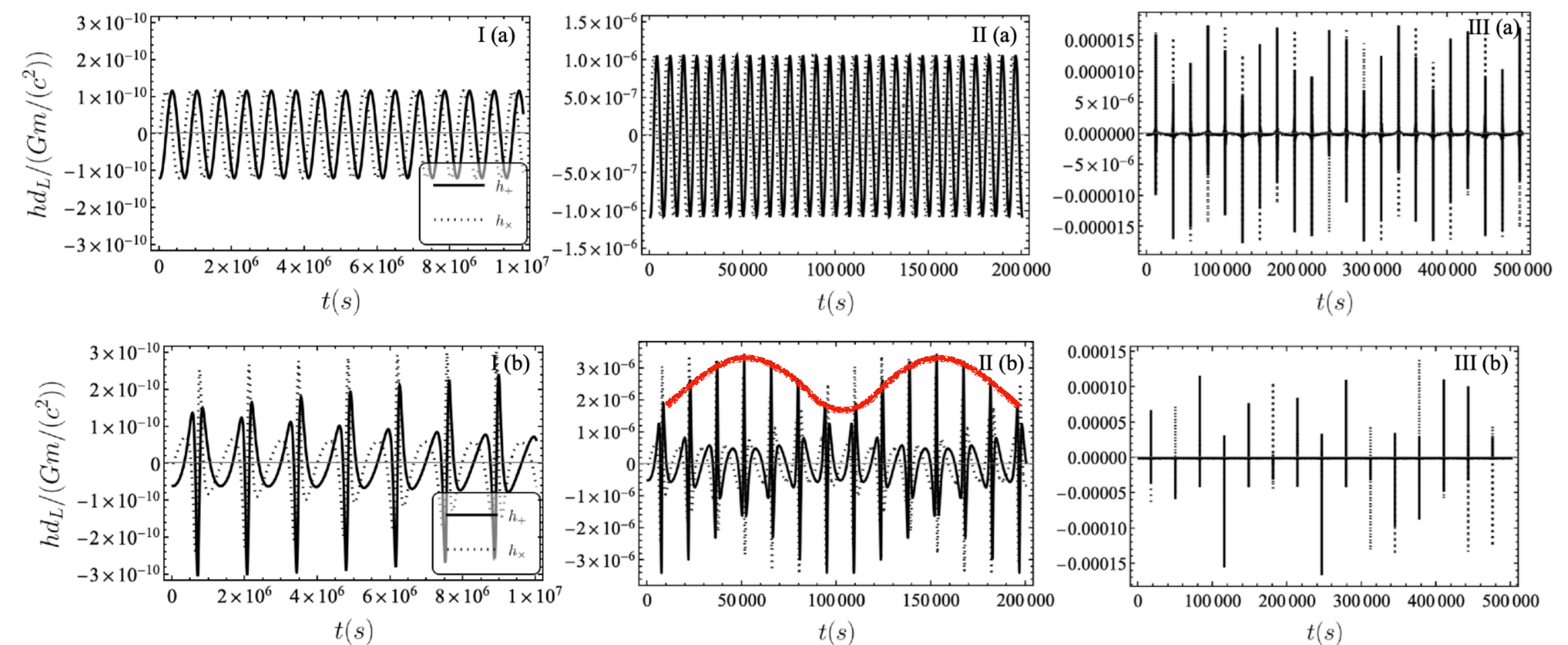
A Smoking Gun: The Precession of the Orbit

If we consider the potential with Screening:

$$V = -\frac{1}{r} \left(Gm_1m_2 + \frac{y^2 q_1 q_2}{4\pi} e^{-m_\phi r} \right).$$



The Orbits



The GW Strain

The Precession Angle per orbit $\Delta\theta_{\text{GR}} \sim \frac{GM}{r(1-\epsilon^2)c^2}$ (1PN) $\Delta\theta_y \sim 2\pi \left(\sqrt{\frac{1+r/l_y}{1+r/l_y-(r/l_y)^2}} - 1 \right)$ (0PN)

Cooling of the halo

Flores, Kusenko 2021

Flores, Lu, Kusenko 2023

After virialization...

Without dissipation, halos will remain virialized until the constituent particles decay...

sparse 

I. Radiation from non-relativistic incoherent motions

II. Radiation from $\psi - \psi$ bremsstrahlung

(Bound state formation)

III. Radiation from the surface

dense 

If Asymmetric: $\eta_\psi = (n_\psi - n_{\bar{\psi}})/s \neq 0$

Fermi Ball Formation

Black Hole Formation

Halo Annihilation

The scalar charge of the halo

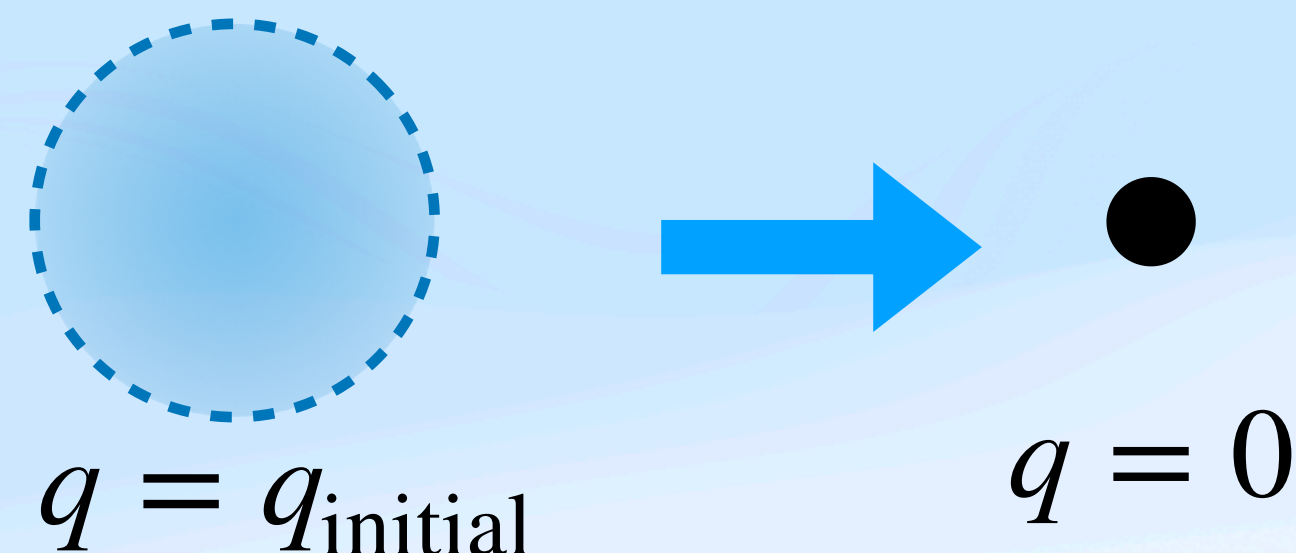
If the degeneracy pressure cannot support anymore...

The ‘Chandrasekhar Mass’
for strong force:

$$m_h > m_{\text{critical}} \sim \frac{M_{\text{pl}}^2}{m_\psi}$$

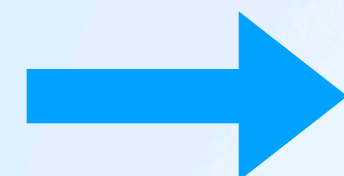
A black hole will form eventually!

“No hair theorem”



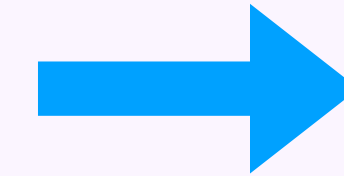
If the degeneracy pressure can still support the fermi ball...

Scalar
condensation



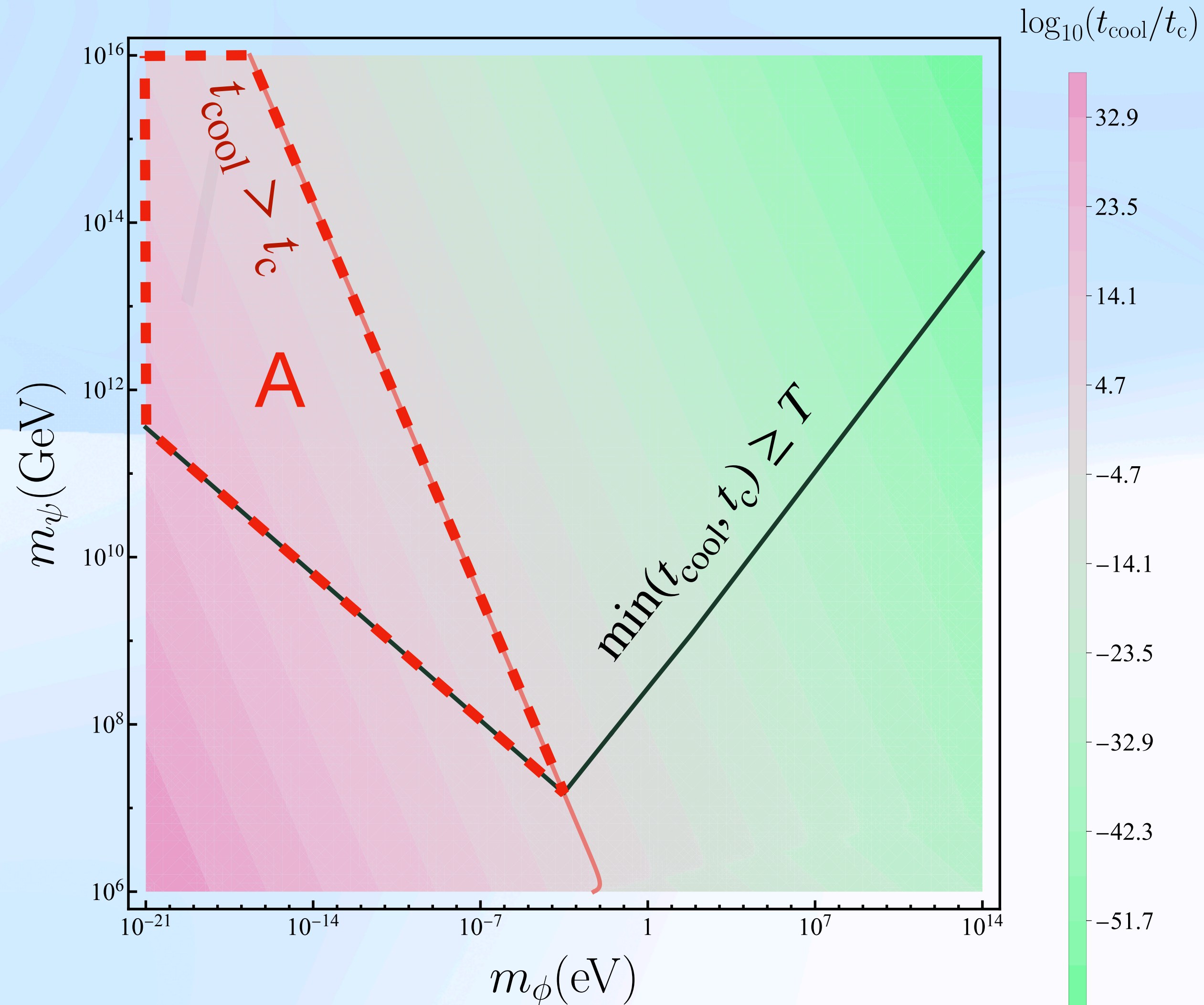
$$m_{\phi, \text{eff}}^2 = \underbrace{V_{, \phi \phi}}_{\lambda \phi^4} - y \underbrace{\frac{\partial \langle \bar{\psi} \psi \rangle}{\partial \phi}}_{m_{\psi, \text{eff}}^2(\phi)}$$

Diagram illustrating the effective mass squared of the scalar field, $m_{\phi, \text{eff}}^2$, which is the sum of the scalar potential term $V_{, \phi \phi}$ (labeled $\lambda \phi^4$) and the fermion mass term $m_{\psi, \text{eff}}^2(\phi)$ (labeled $y \frac{\partial \langle \bar{\psi} \psi \rangle}{\partial \phi}$). A red arrow points upwards from the fermion mass term.

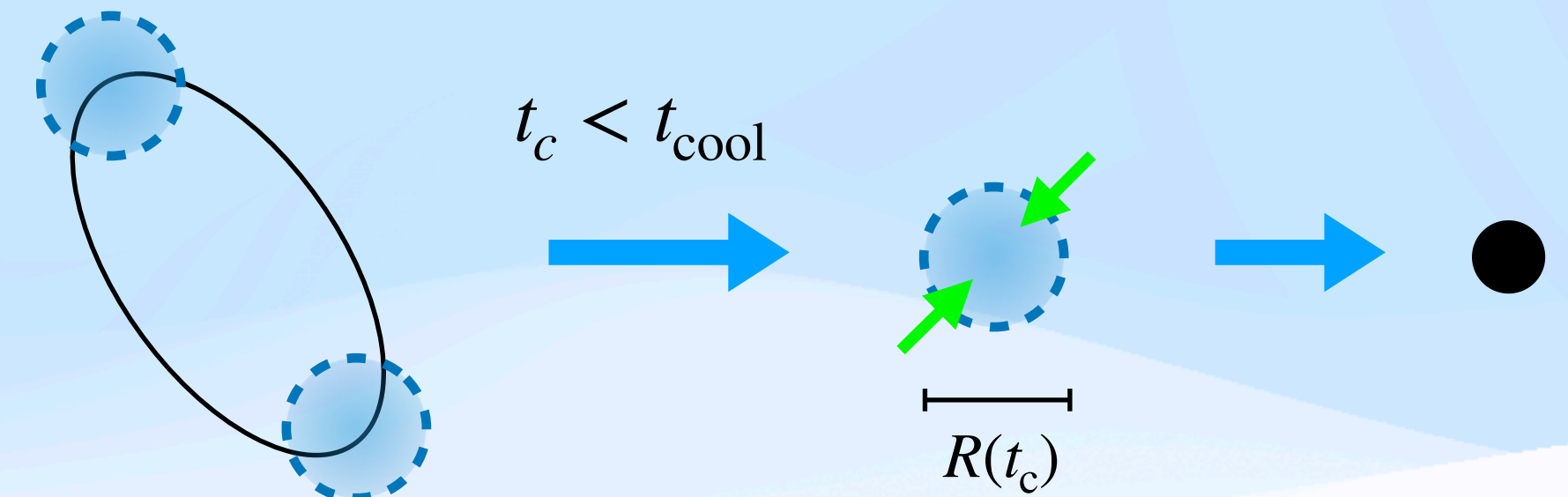


Force/
Charge
Screened

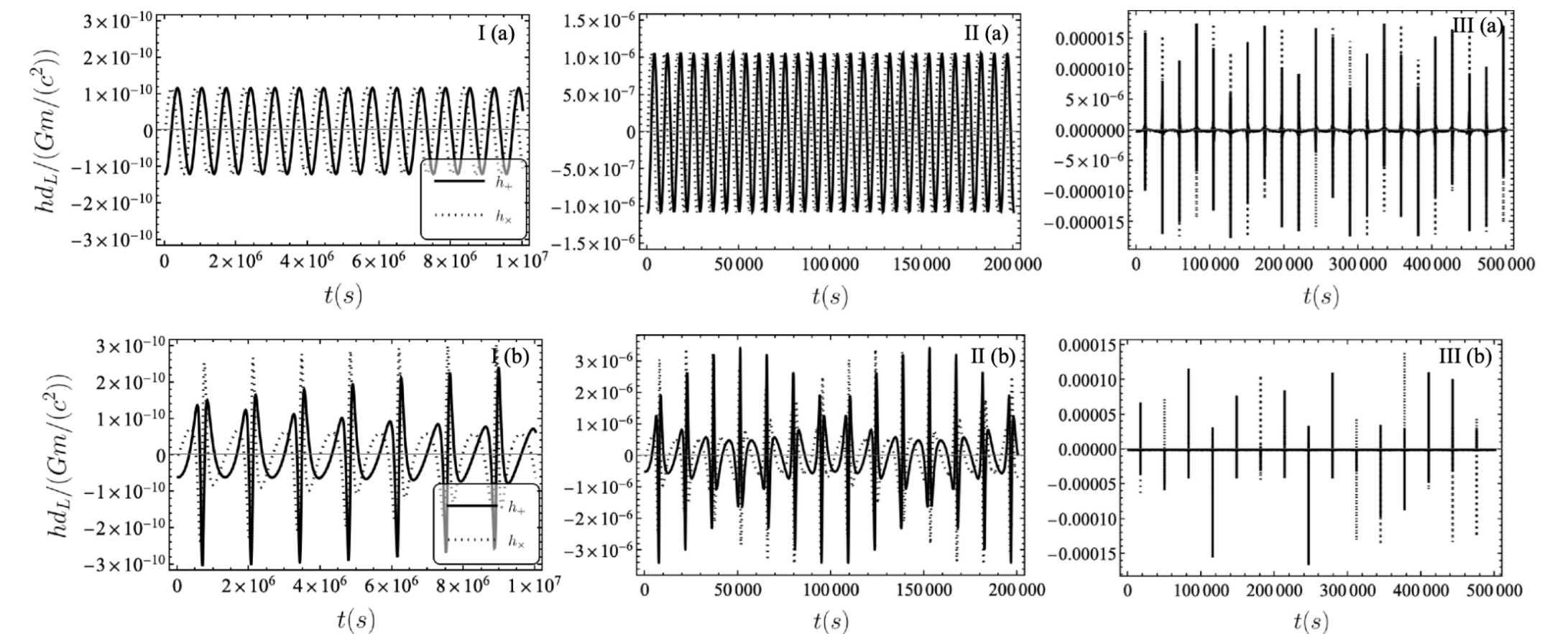
Comparing the time scale



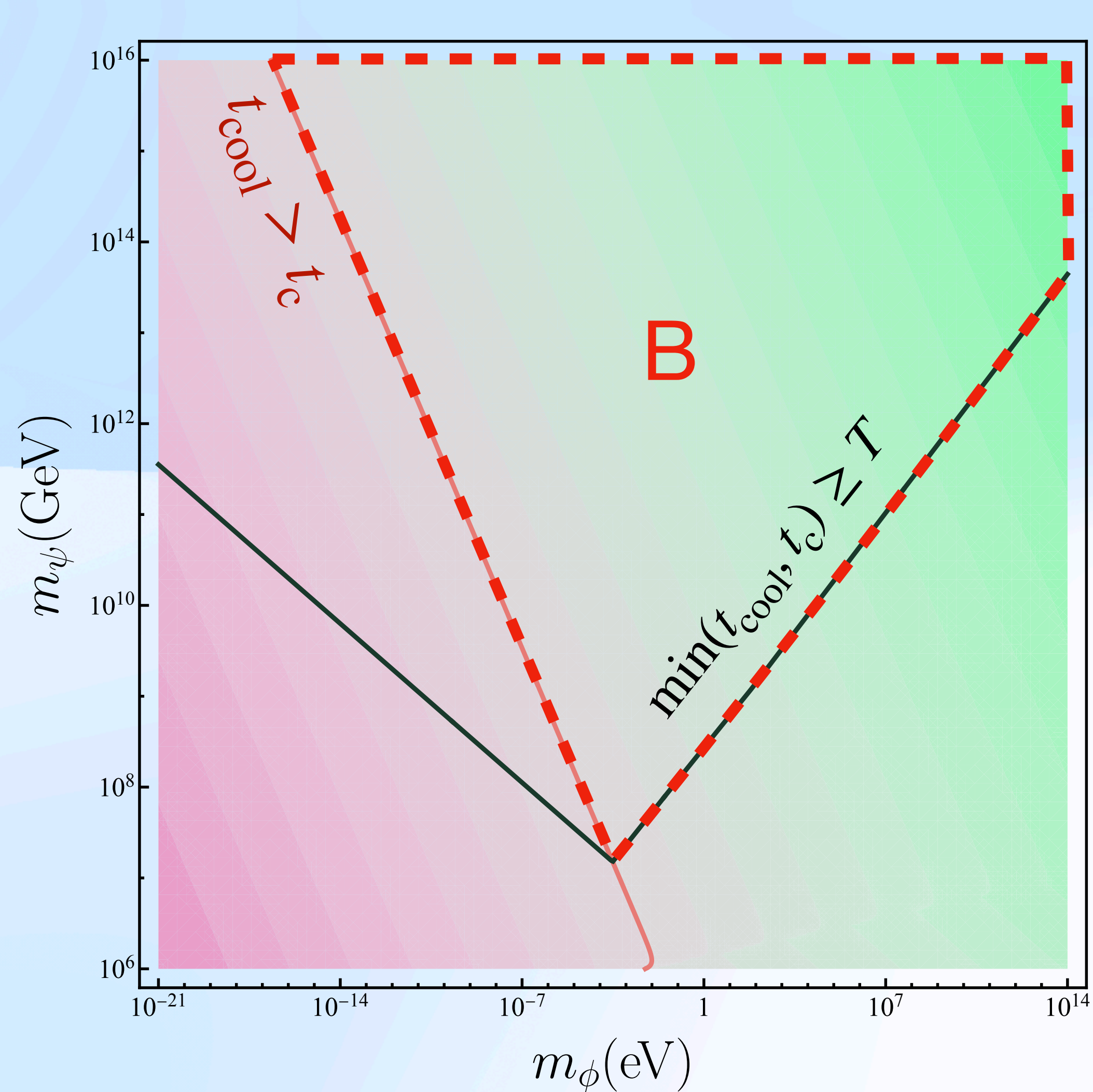
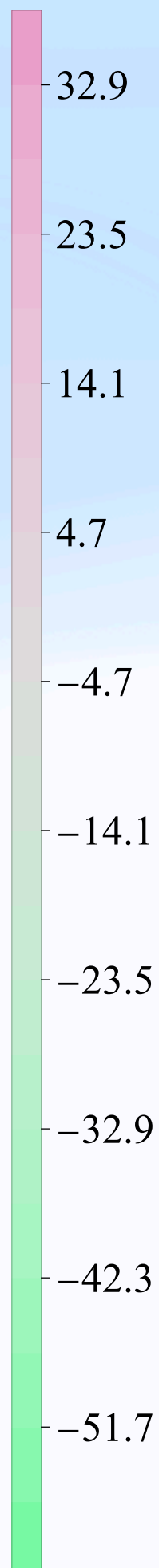
Complete Merger



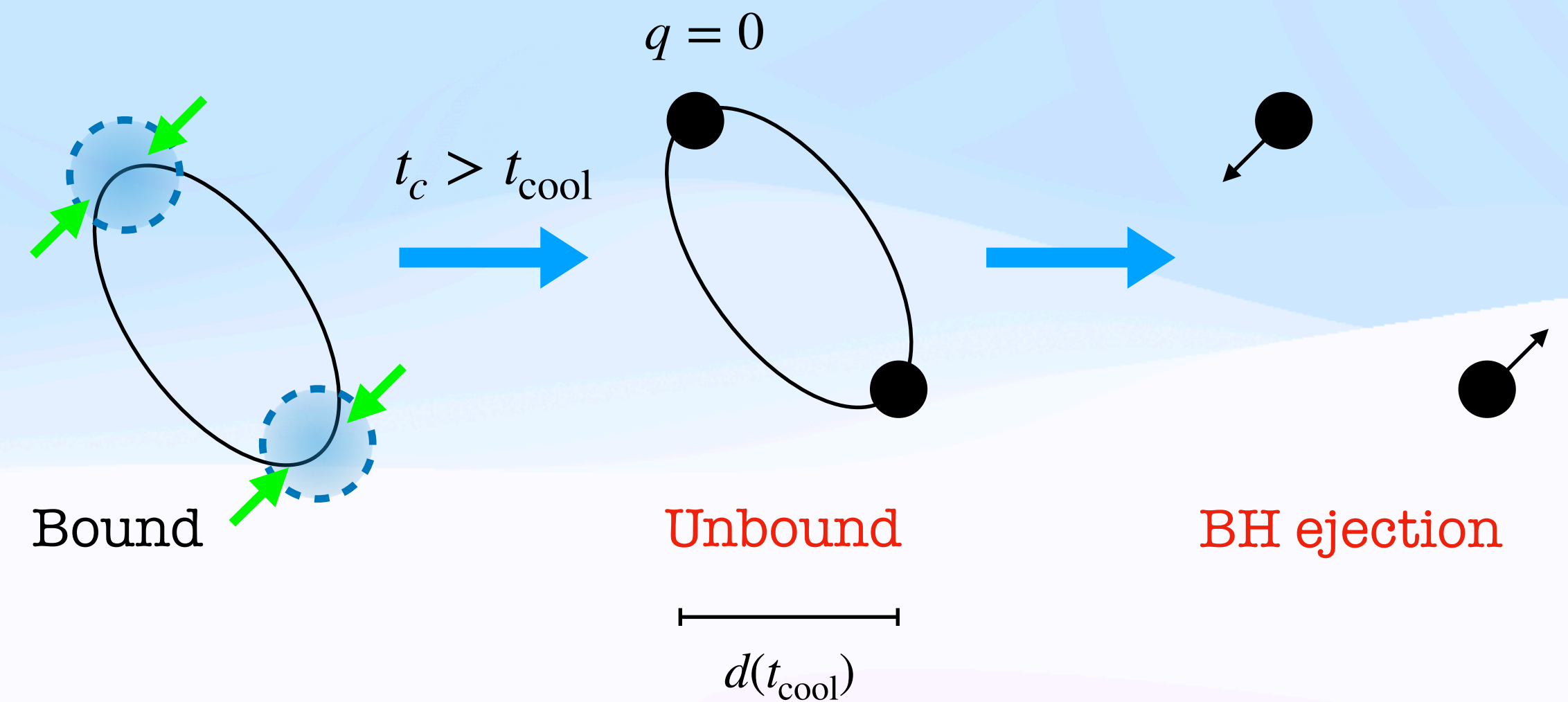
Gravitational Waves (Inspiral-Merger-Ringdown)



Comparing the time scale


 $\log_{10}(t_{\text{cool}}/t_c)$


Interrupted Merger

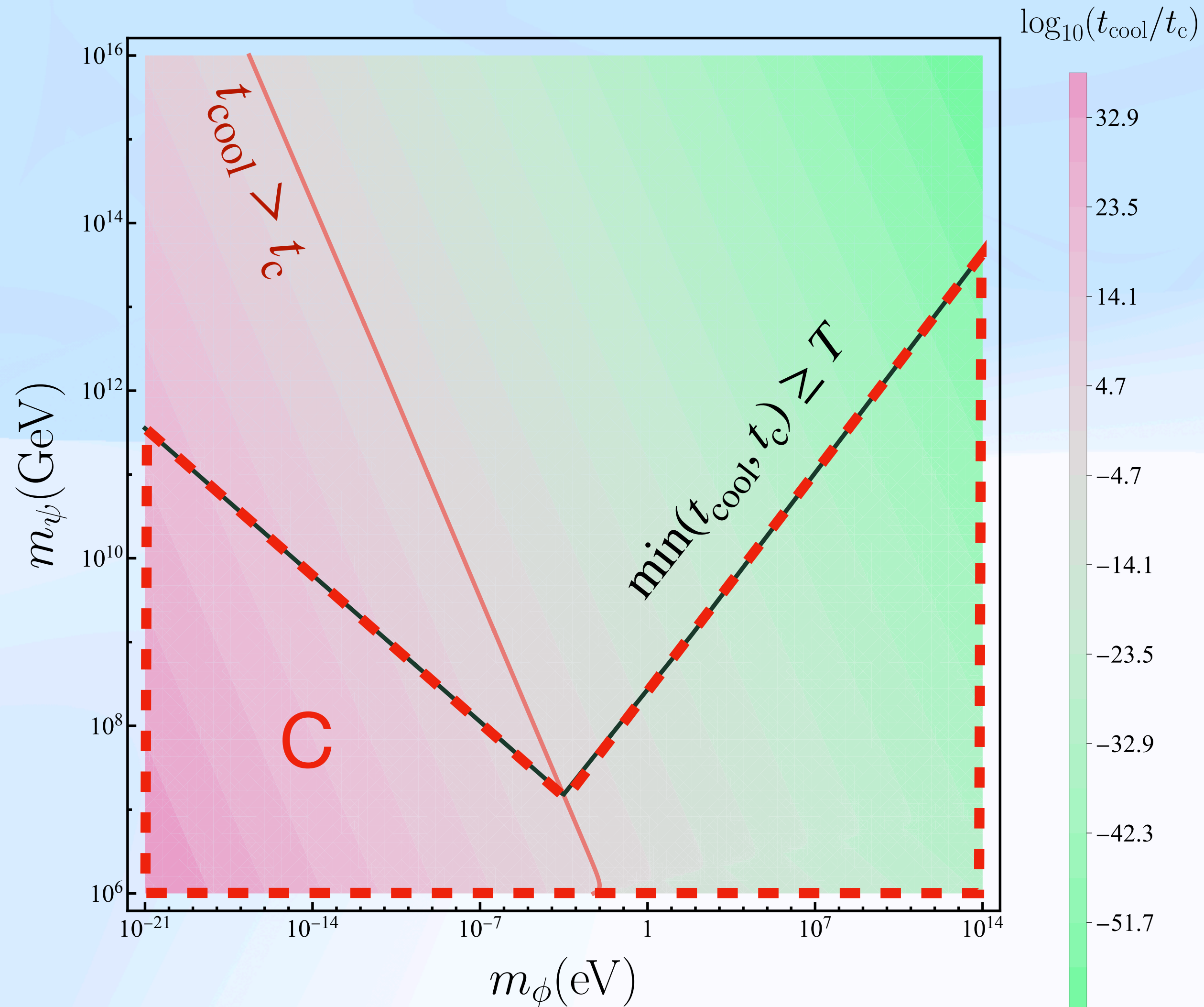


$E_y \rightarrow E_k$ Ejection @ v_{rel}

$$v_{\text{rel}} \sim \sqrt{\frac{4G(m_1 + m_2)\beta^2}{d}} \rightarrow 1 \text{ when } y^2 \sim \frac{m_\psi}{m_\phi q}$$

Free-Floating Planets in Micro-lensing?

Comparing the time scale

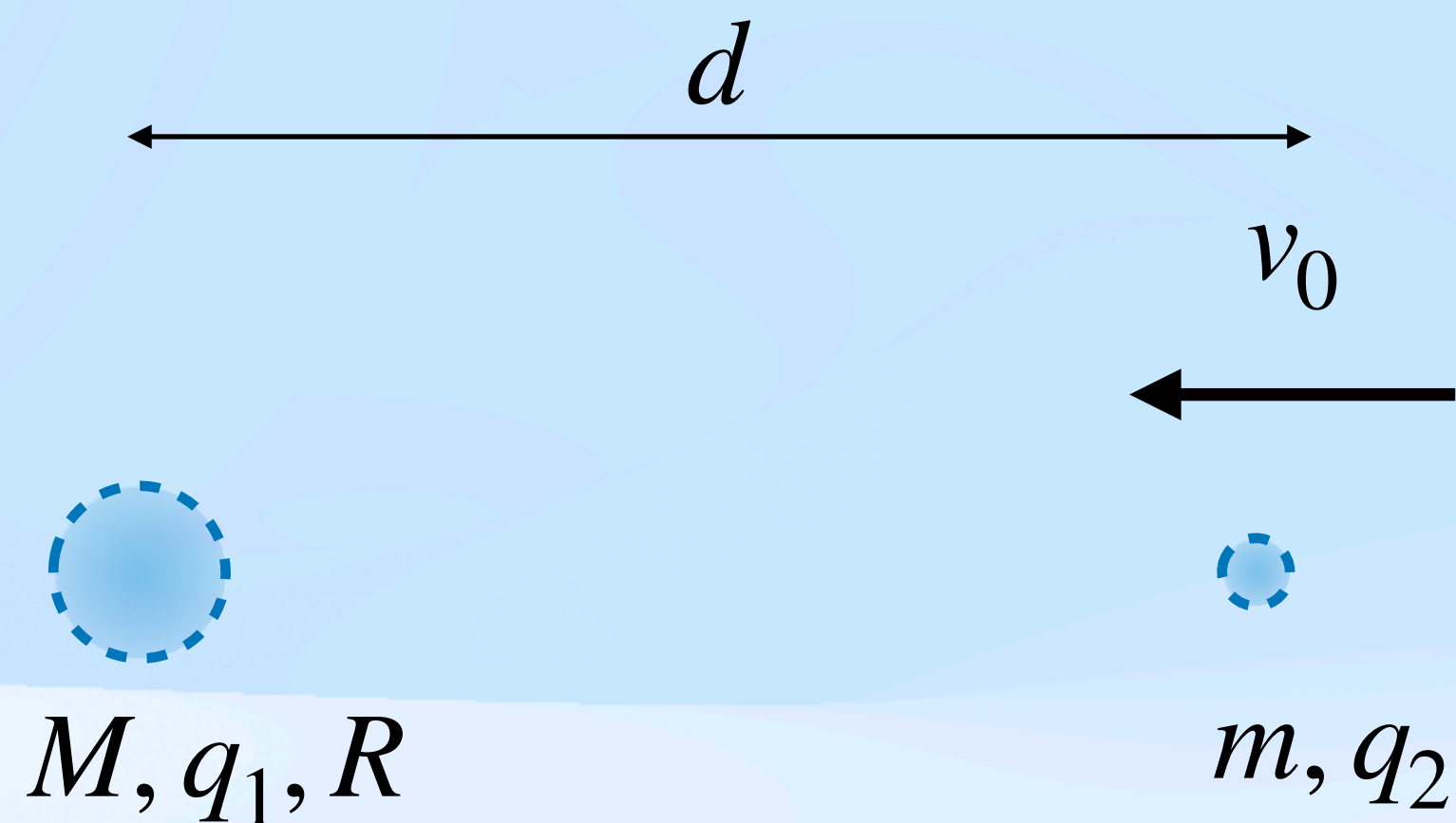


Prompt Merger

The merger concludes before completing a full orbital period.

Prompt Merger: Short-time GW Burst

When throwing a relativistic point-like charge to a halo...

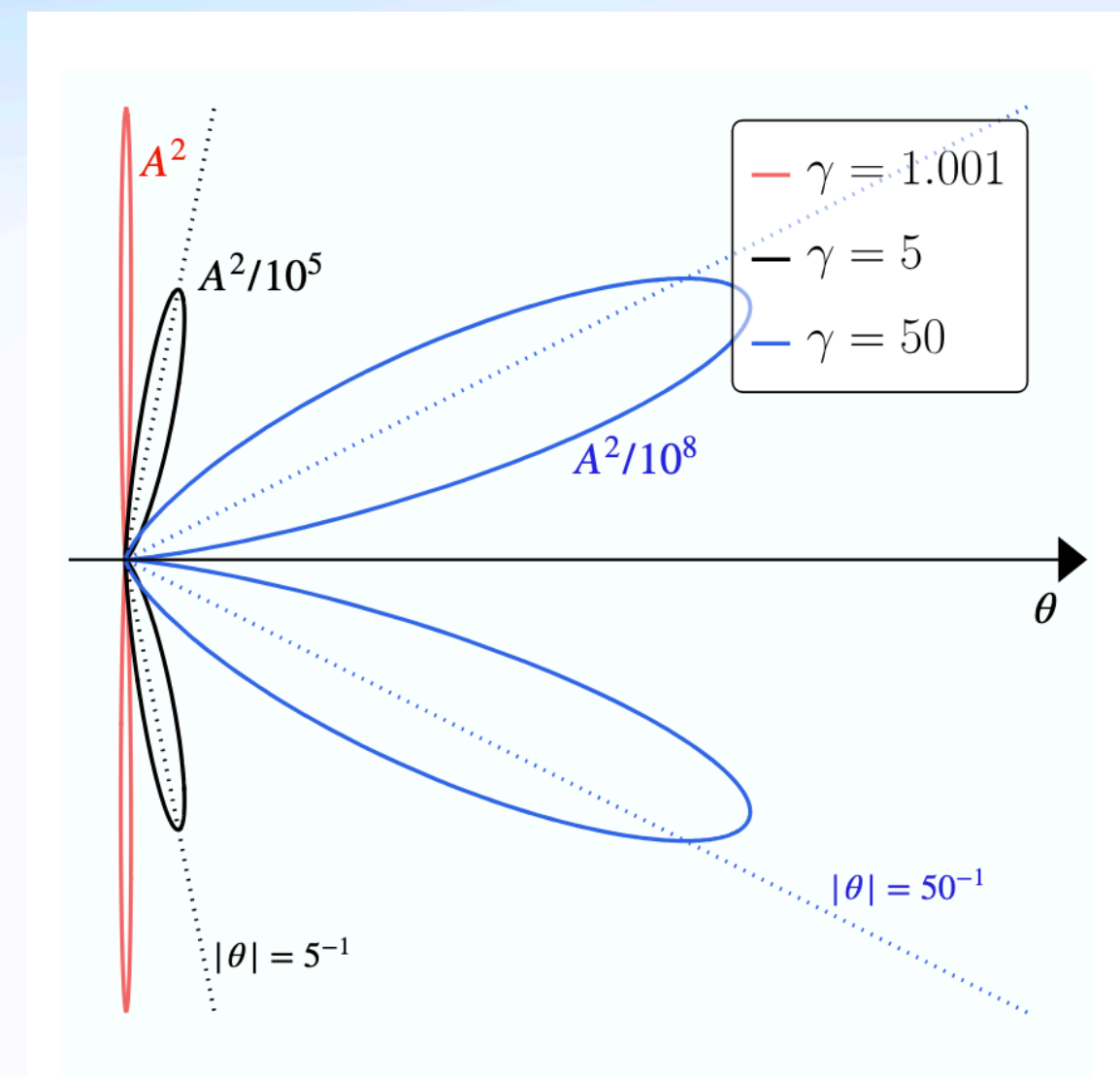


Assuming $R \gg 2M$:

$$\ddot{d} = -\frac{y^2 q_1 q_2}{4\pi\gamma^3 d^2}$$

The Gravitational Wave Strain

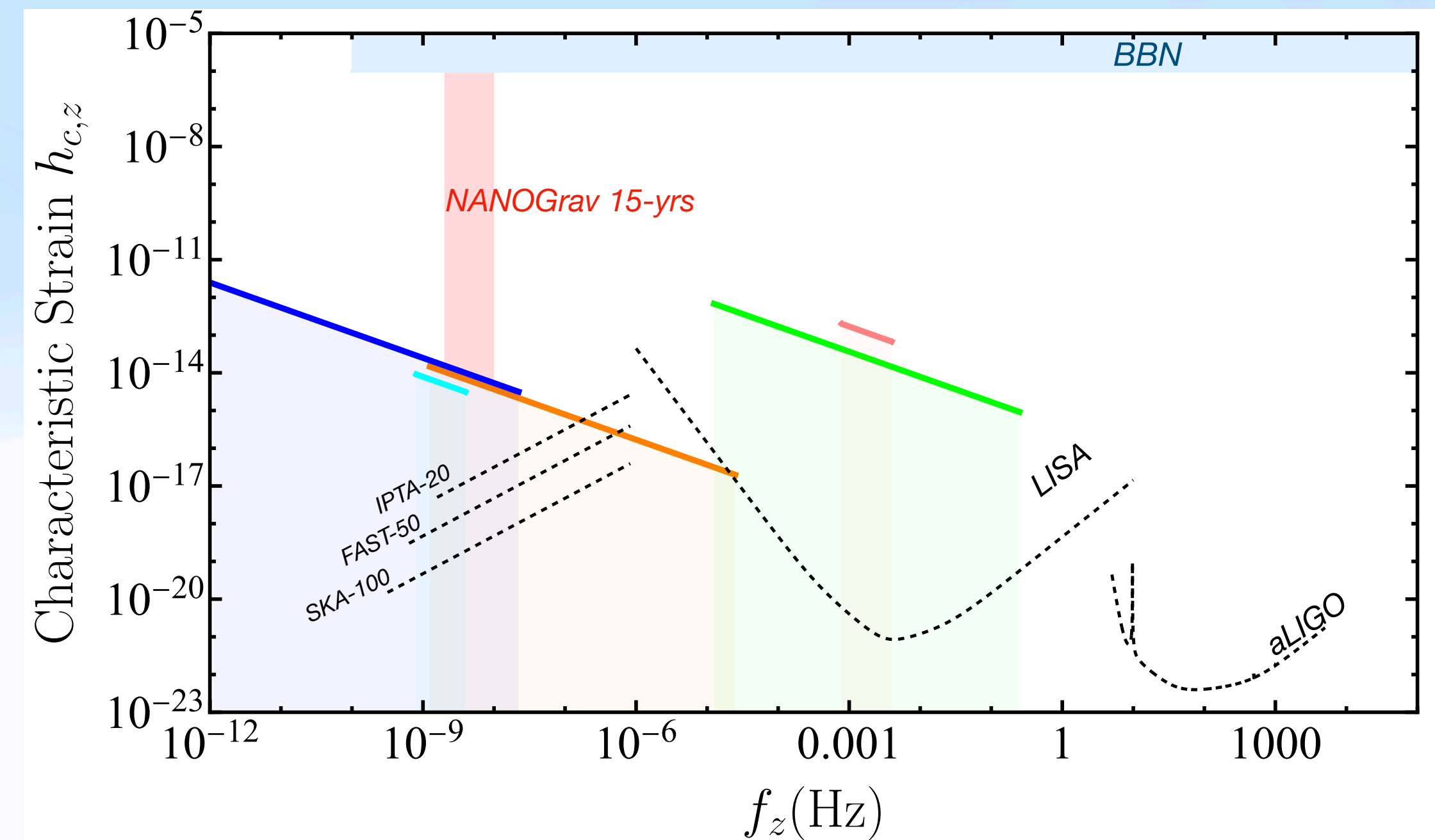
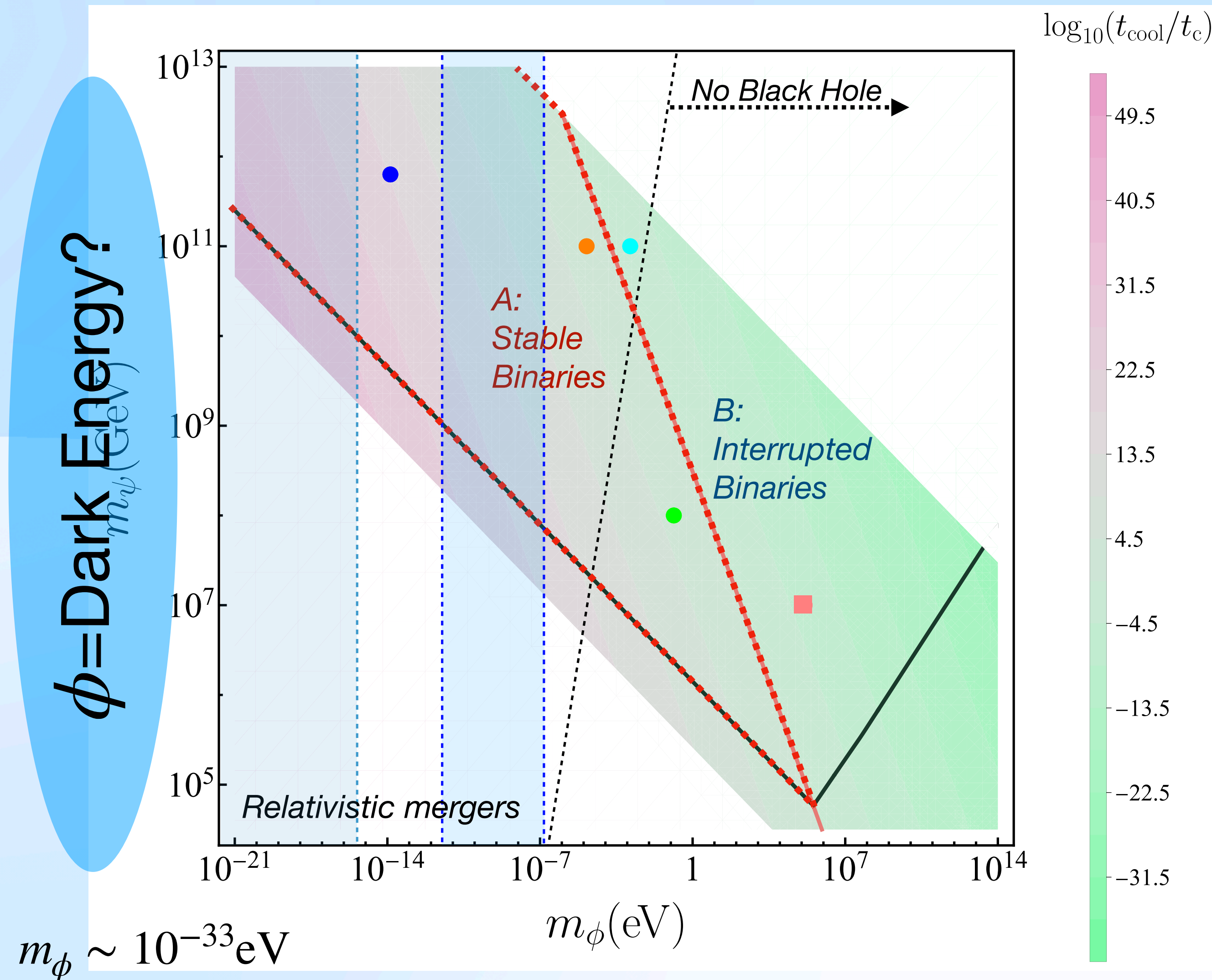
$$h_{\theta\theta} = -h_{\phi\phi} = \frac{2m}{D_L} \left[\frac{\gamma \dot{d}^2 \sin^2 \theta}{1 - \dot{d} \cos \theta} \right]$$



The energy emitted is concentrated in a opening angle γ^{-1} , just like a γ -ray burst.

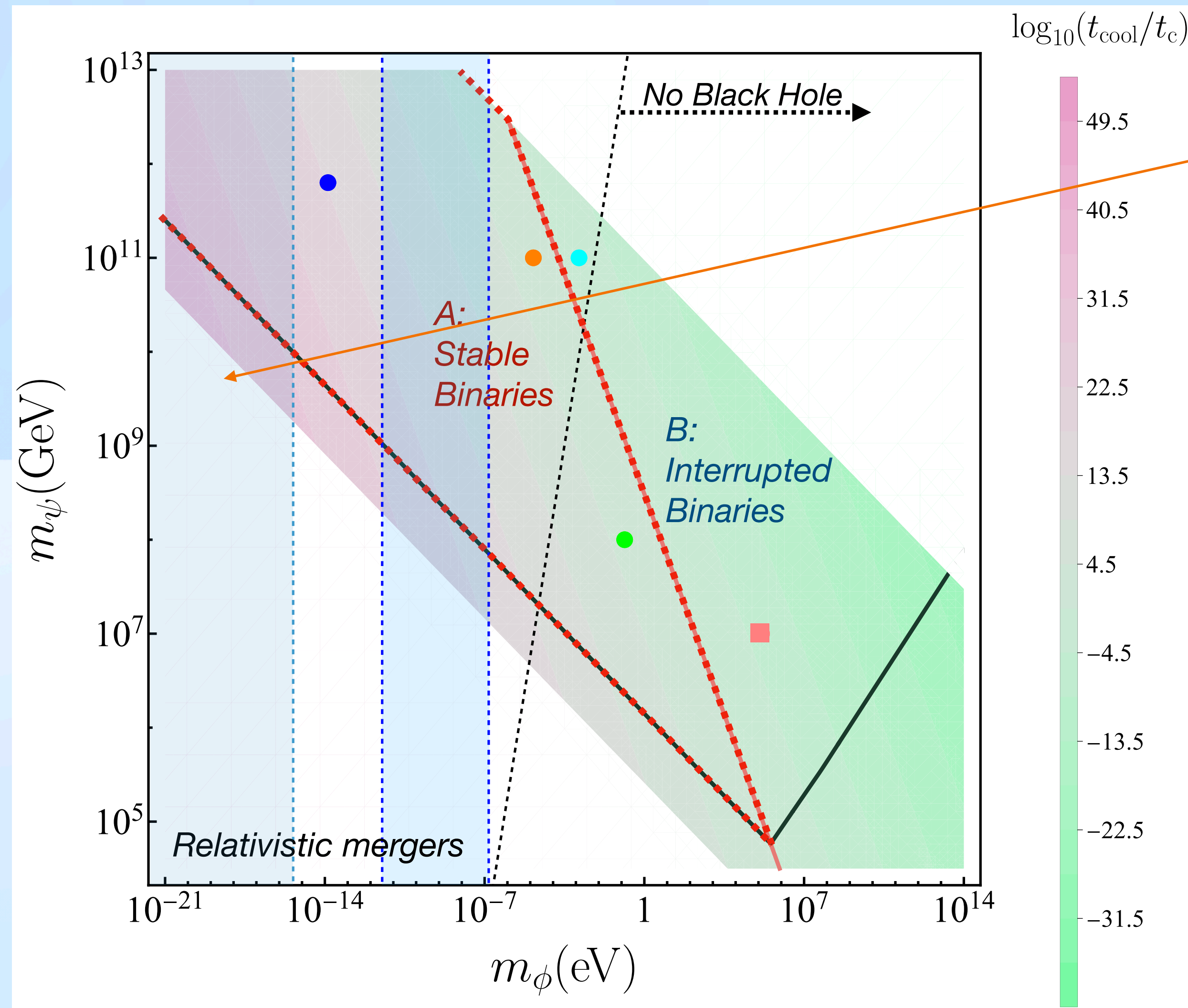
In the ultra-relativistic limit, $\Delta E_{\text{GW}} \propto \gamma_0^2$

A plot of parameter space constraints



GWB comes from the minihalo Inspirals

A plot of parameter space constraints



Observational Constraints

if FB=DM (Bullet Cluster)

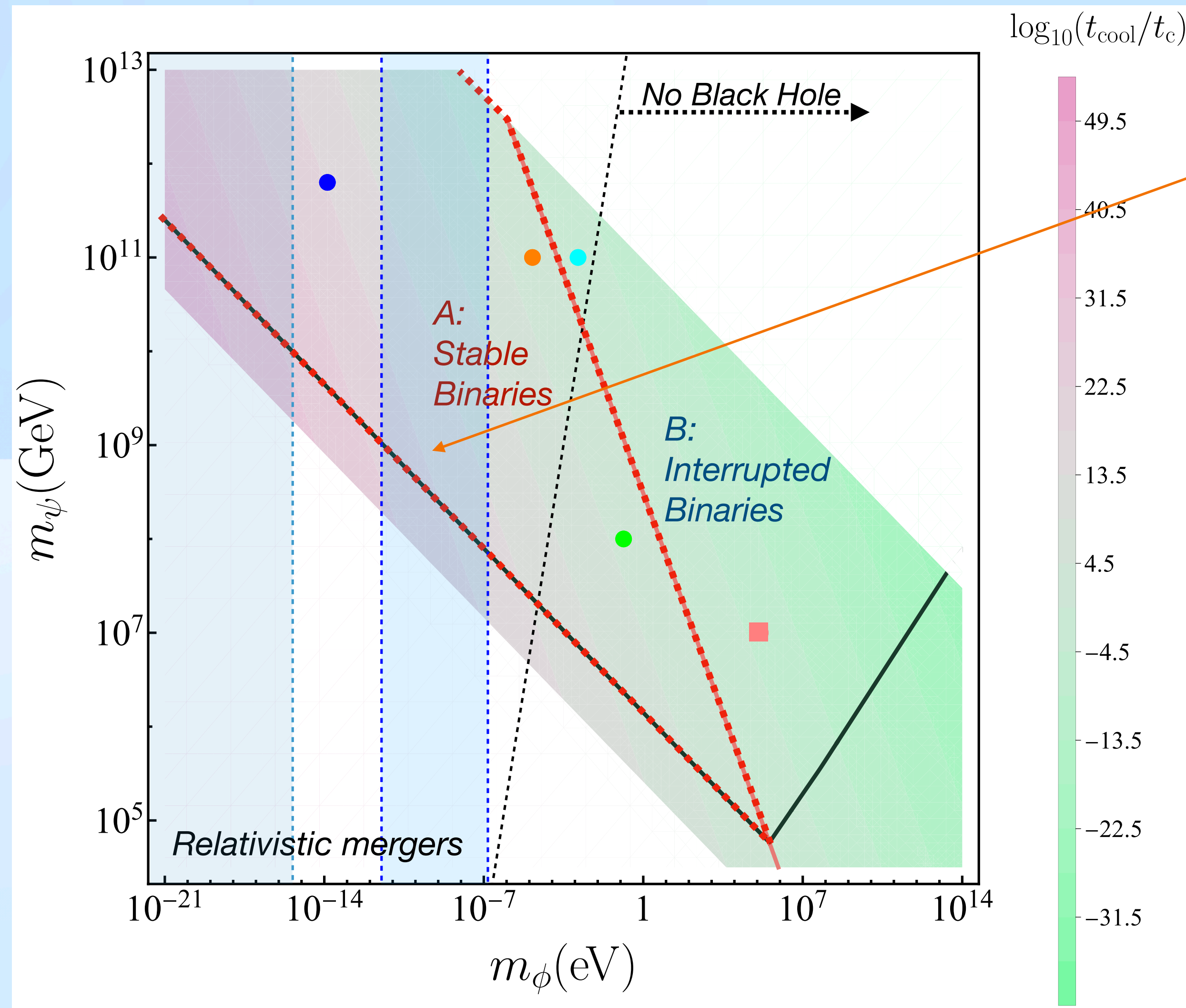
$$\sigma/m_{\text{FB}} < 1\text{cm}^2/\text{g}$$

$$\sigma \sim \pi m_\phi^{-2}$$

if FB=small fraction of DM

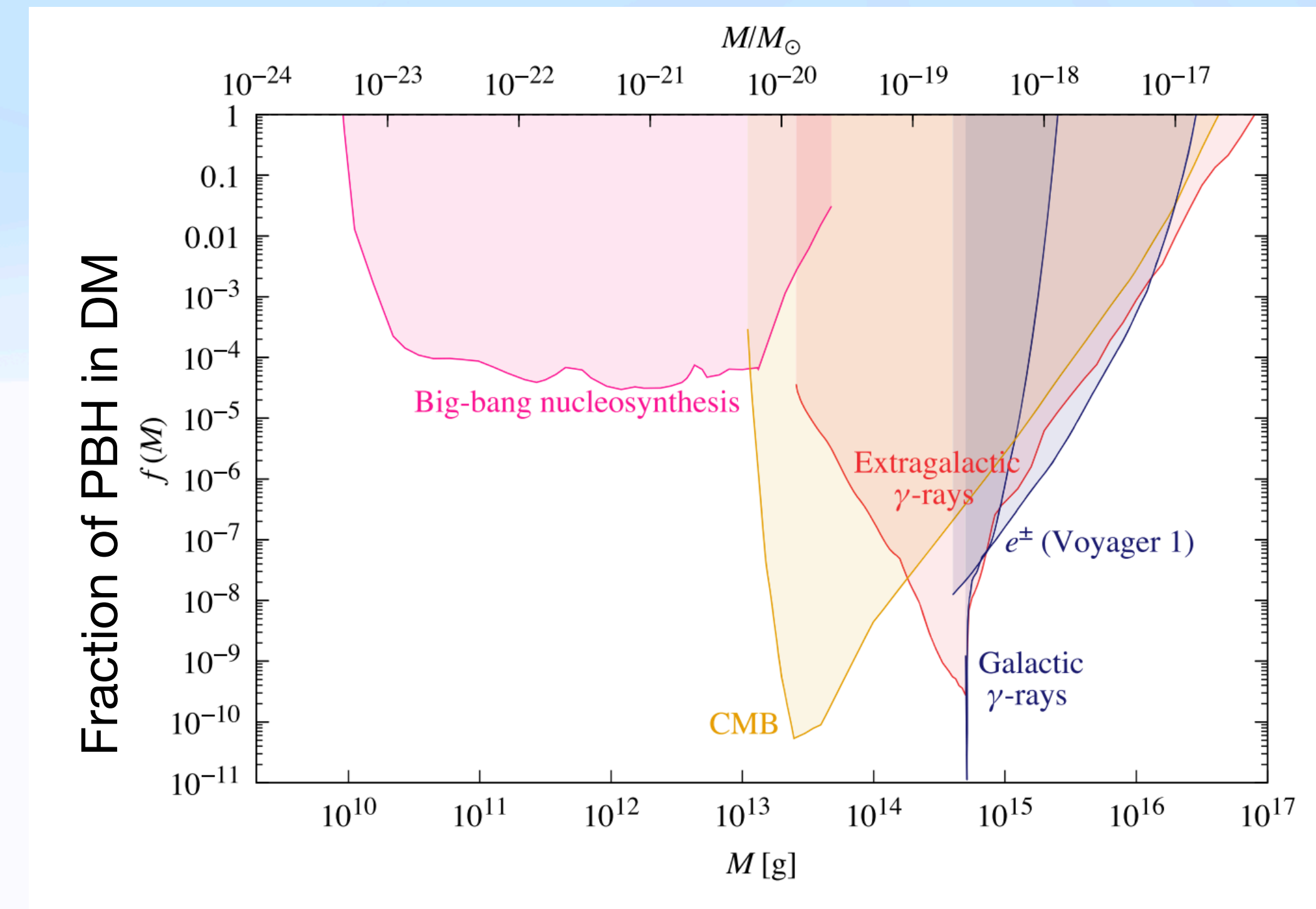
No Constraints!

A plot of parameter space constraints



Observational Constraints

if BH=DM (BBN & γ -ray)



Take Home

- I. **Primordial Black Holes (PBHs)** are promising dark matter candidates.
- II. They can form from enhanced small-scale primordial density perturbations, from attractive interactions among dark matter particles, or through other non-standard early-Universe mechanisms...
- III. PBHs also act as sources of **gravitational waves** across a wide range of frequency bands, offering distinctive signals that can be probed by future experiments.

