Self-Resonant Dark Matter

14, October, 2021 KIAS Seminar
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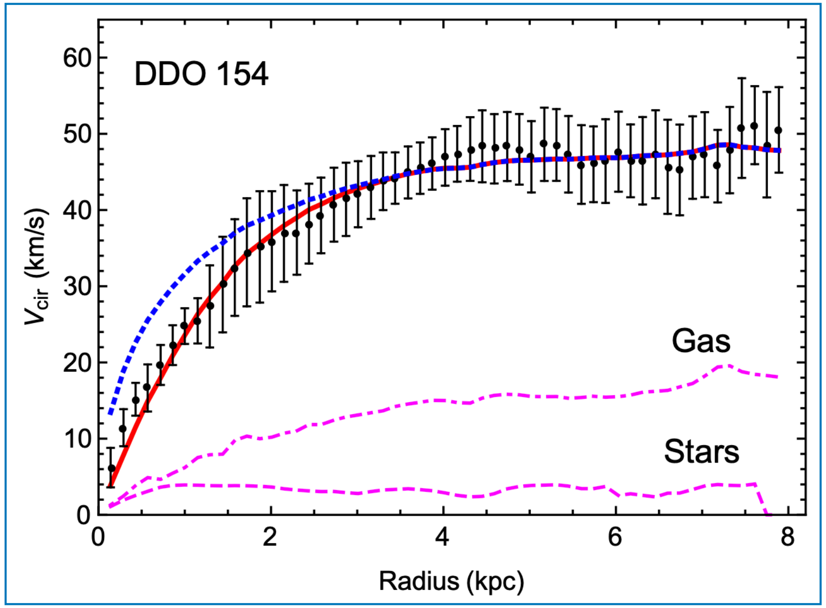
Chung-Ang University, SeongSik Kim, Bin Zhu, and Hyun Min Lee
arXiv 2108.06278

Motivations

There are some mismatches between ΛCDM, The Standard Universe Model, prediction, and observations.



Core Cusp Problem

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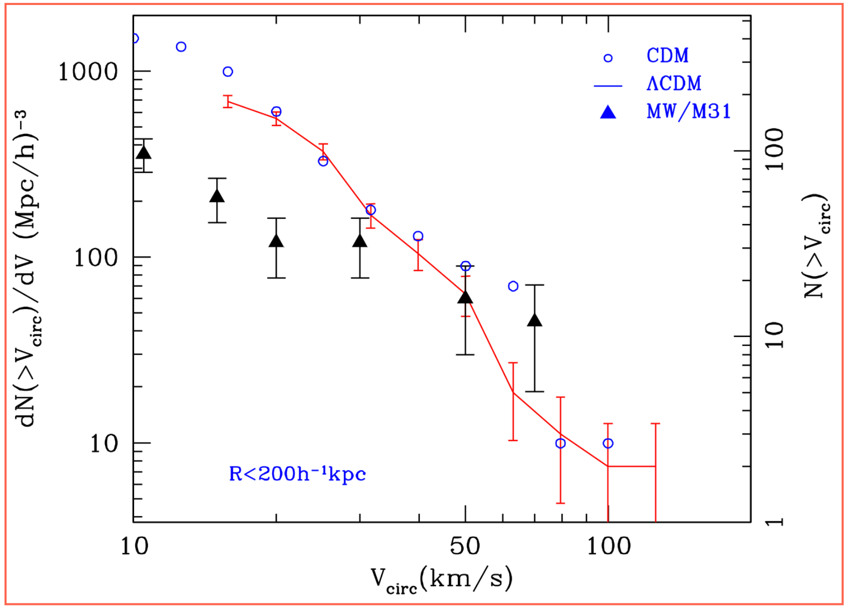
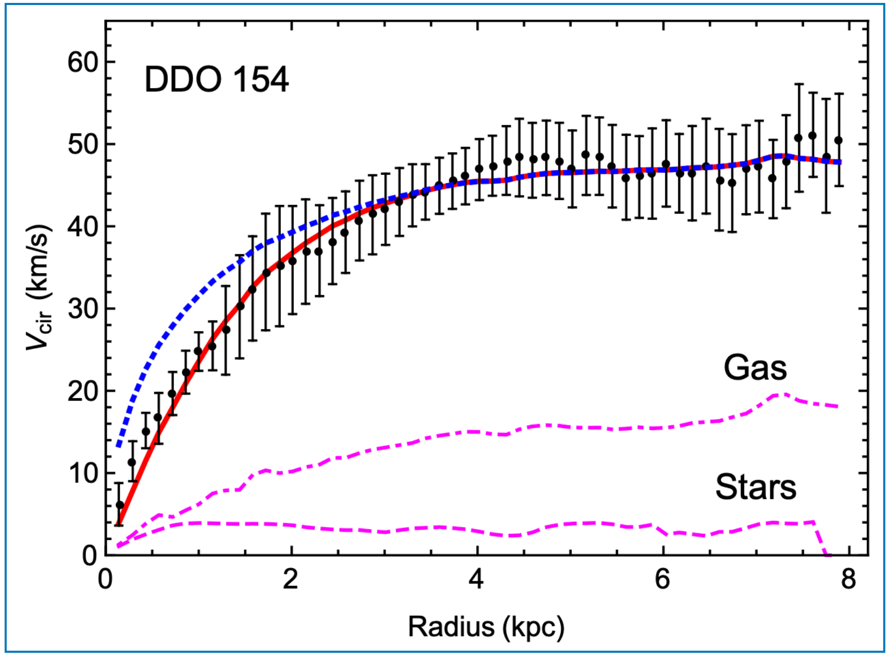


Missing Satellites Problem

Reprinted From A. A. Klypin, A. V. Kravtsov, O. Valenzuela, and F. Prada, Astrophys. J. 522, 82 (1999), astro- ph/9901240.

Core-cusp Problem

Observed Galaxy Rotation Curve



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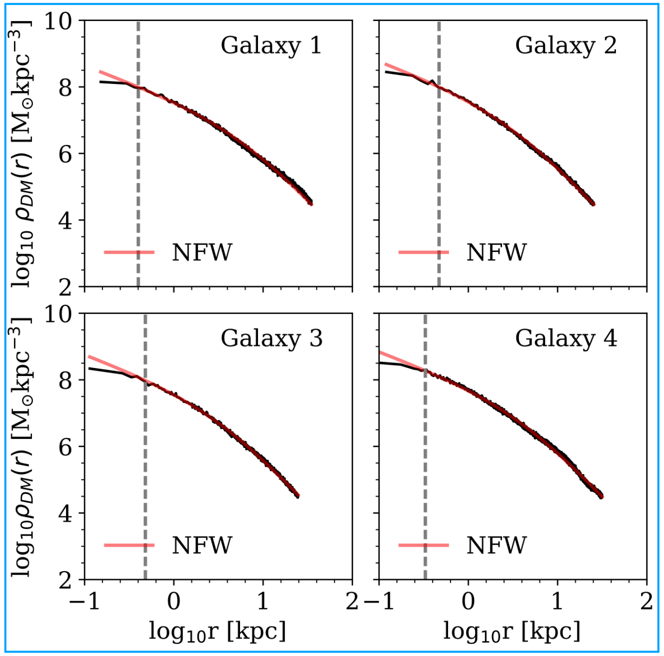
— Observation

- - - Simulation

ΛCDM Simulation



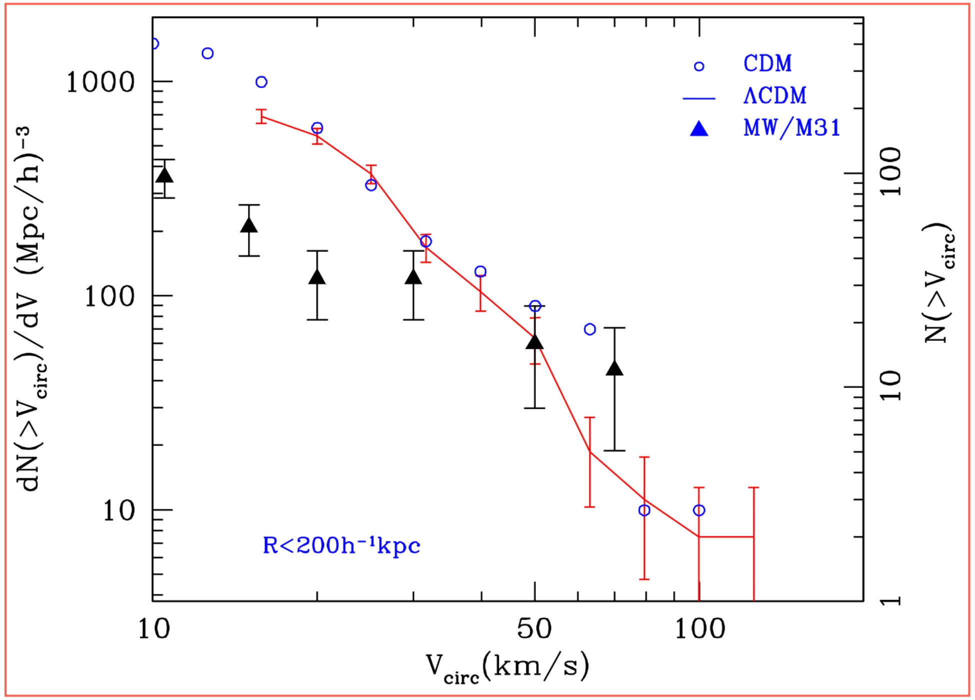
Reprinted From A. Genina at al., *Monthly Notices of the Royal Astronomical Society*, Volume 474, Issue 1, February 2018, Pages 1398–1411, arXiv 1707.06303

— Observation

— Simulation

Missing Satellites Problem

Cumulative number of satellites in all host halos in the ΛCDM simulation (Right Vertical Axis)



Observation

Simulation

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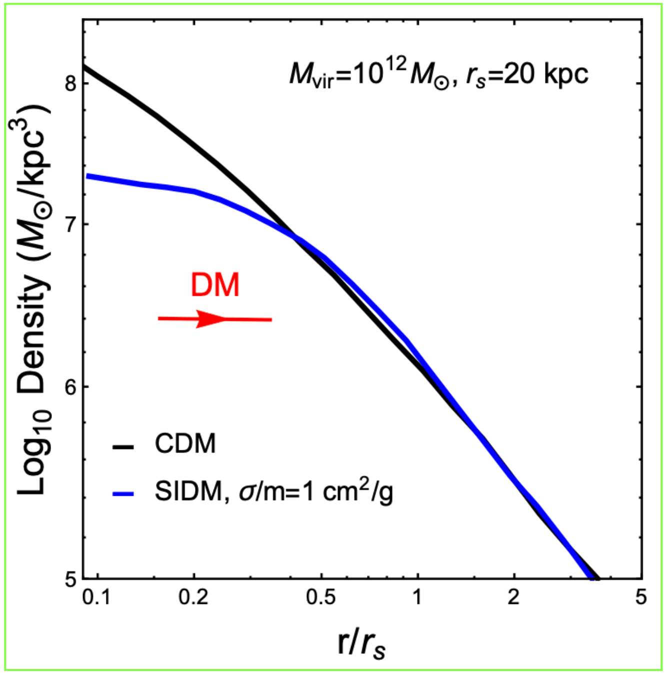
Motivation - SIDM

Self-Interacting Dark Matter (SIDM) model could resolve these mismatches when the DM scattering cross-section is proper.



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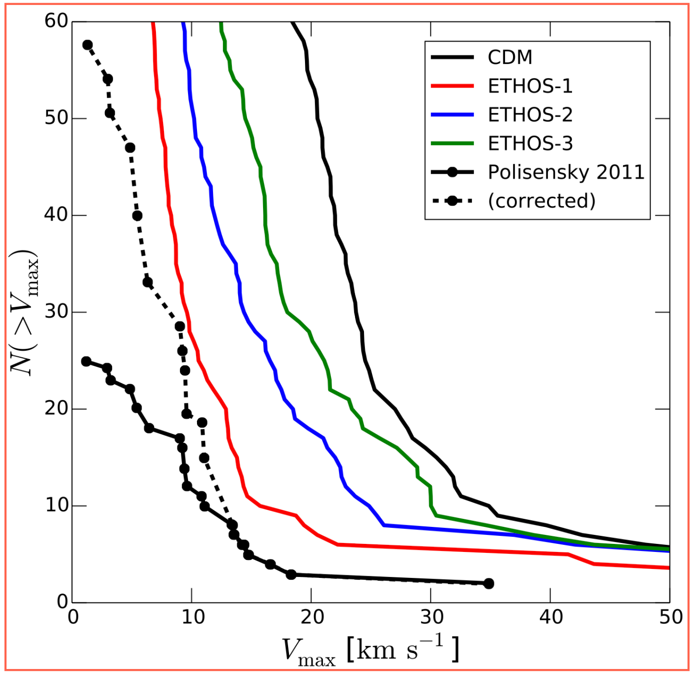

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SIDM and DM Cross-section

SIDM validity requires an accurate DM scattering cross-section.

Required DM Self-Scattering Cross-section is $\sigma/m \sim \mathcal{O}(1)\text{cm}^2/\text{g}$

Sommerfeld Effect (Enhancement)

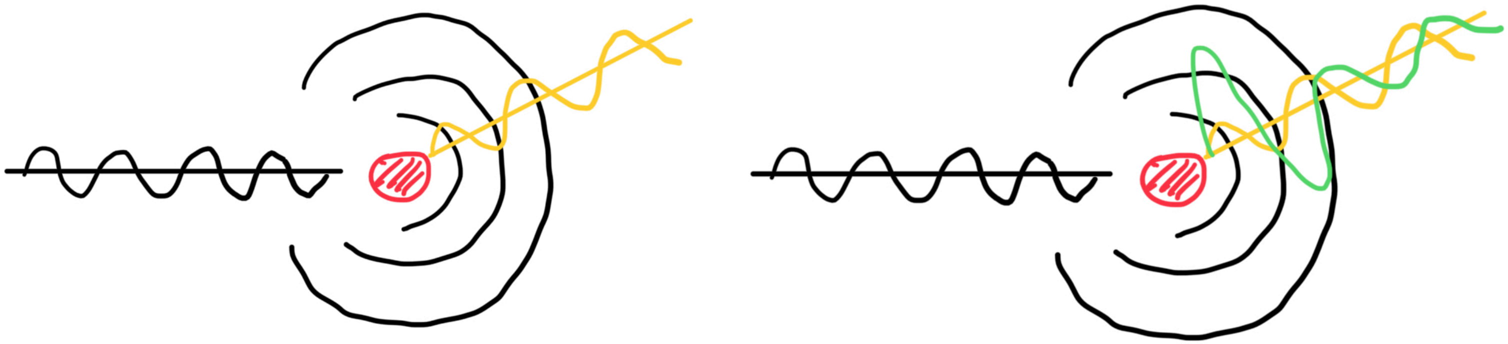
Stands out for special cases, enhance cross-section sizably,

should be considered

+ Sommerfeld Effect may effect current relic Dark Matter density, with Higher Annihilation rate.

Sommerfeld Enhancement

Cross-section should be corrected
when scattering mediated by Long Range Attractive Interaction.



Since Long Range Interaction deform
2-body wavefunction significantly especially scattering origin.

$$\sigma = |A|^2 \sigma_0$$

σ_0 : cross-section calculated with ordinary approach

σ : Realistic Cross-section

A : Enhanced Amplitude ratio by Long Range Interaction

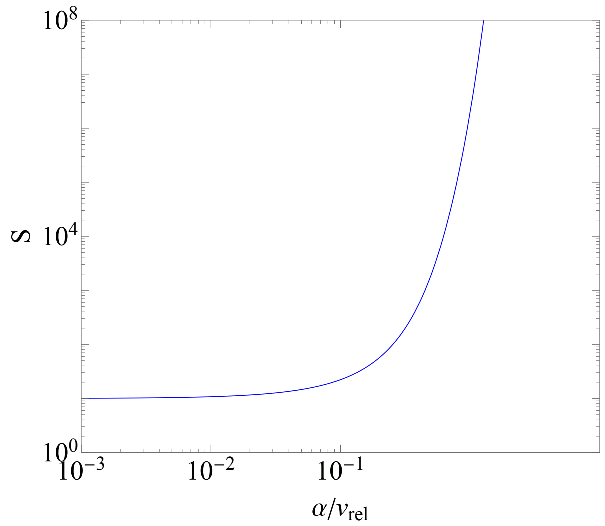
Sommerfeld Enhancement

The Correction Factor became significant for Lower Velocities

Example : Coulomb Interaction between proton and electron

$$|A|^2 = \frac{|\psi(0)|^2}{|\psi(r \to \infty)|^2} \equiv S = \frac{2\pi}{1 - e^{-2\pi\alpha/v}} \frac{\alpha}{v}, \text{ where } \alpha = \frac{e^2}{4\pi}$$

Sommerfeld Enhancement
become Significant when $\alpha/v \geq \mathcal{O}(1)$

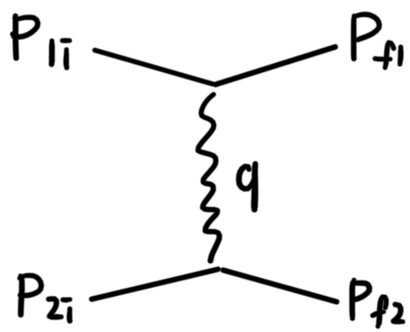


Sommerfeld Enhancement : Cross-section became larger as velocity lower

Sommerfeld Enhancement

What is implication of Sommerfeld-favored condition in QFT rule?

Coulomb scattering example



1. 3-point interaction model
2. Ingoing particle and outgoing particle are same species
3. Very light mediator (long range interaction)
4. Non-Relativistic Scattering (small relative velocity to make $\alpha/v \geq \mathcal{O}(1)$)

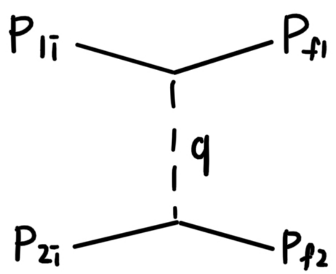
Sommerfeld Enhancement

1. 3-point interaction model
2. Ingoing particle and outgoing particle are same species
3. Non-Relativistic Scattering (small relative velocity to make $\alpha/v \geq \mathcal{O}(1)$)
4. Very light mediator (long range interaction)

Sommerfeld Enhancement Condition makes

4 point function $i\tilde{\Gamma} = -g^2 \dfrac{\text{Numerator}}{q^2 - m_{med}^2} \geq \mathcal{O}(1)$.

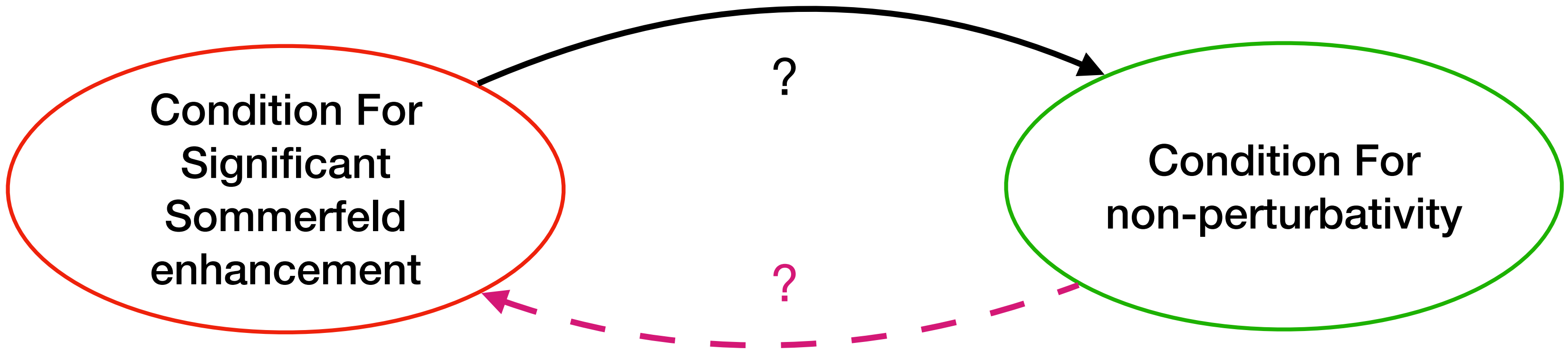
Broken perturbativity!



$$\text{denominator} = (E_o - E_i)^2 - (\vec{p}_o - \vec{p}_i)^2 - m_{med}^2$$

$$\simeq \left(\cancel{m_o} + \frac{\vec{p}_o^{\,2}}{2m_o} - \cancel{m_i} - \frac{\vec{p}_i^{\,2}}{2m_i}\right)^2 - (\vec{p}_o - \vec{p}_i)^2 - \cancel{m_{med}^2}$$

Sommerfeld Enhancement



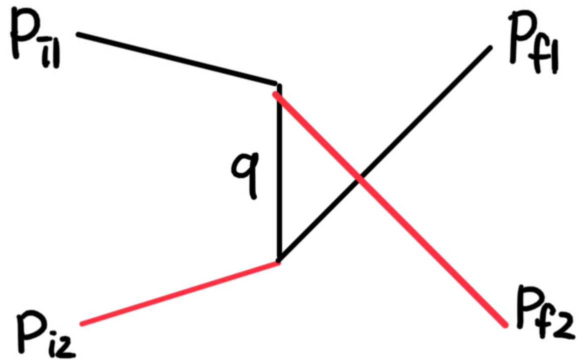
Is non-perturbativity makes significant Sommerfeld Enhancement?

Is it possible to make large 4 point function without light mediator?

Answer is, Yes for both.

New Condition for Non-Perturbativity

Is it possible 4-point function diverge without light mediator? Yes!



$$i\tilde{\Gamma} = -g^2 \frac{\text{Numerator}}{q^2 - m_{med}^2}$$

$$\text{denominator} = q^2 - m_{med}^2 = (E_{f2} - E_{i1})^2 - (\vec{p}_{f2} - \vec{p}_{i1})^2 - m_1^2$$

$$\simeq \left(m_2 + \frac{\vec{p}_{f2}^{\,2}}{2m_2} - m_1 - \frac{\vec{p}_{i1}}{2m_1} \right)^2 - (\vec{p}_{f2} - \vec{p}_{i1})^2 - m_1^2$$

$$\simeq m_2^2 - 2m_1 m_2 - \left(\sqrt{\frac{m_1}{m_2}} \vec{p}_{f2} - \sqrt{\frac{m_2}{m_1}} \vec{p}_{i1} \right)^2$$

$\mathcal{O}(v_{rel}^2)$

In Non-Relativistic limit, $2m_1 \simeq m_2$ causes non-perturbativity

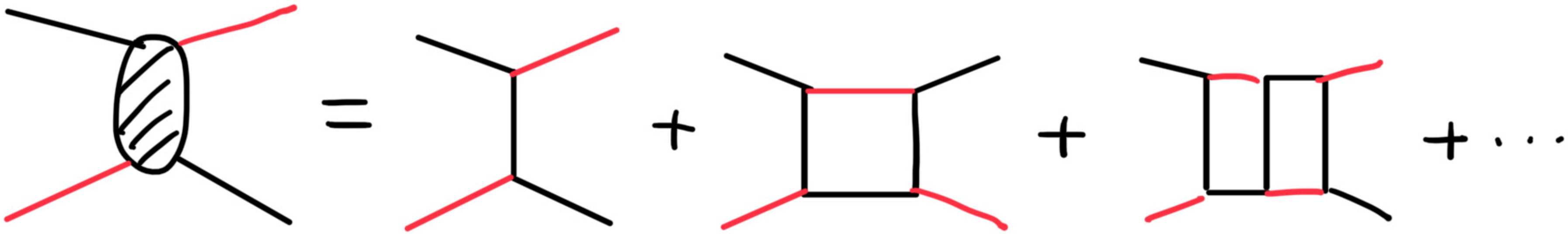
New Condition for Non-Perturbativity

1. Special mass relation between incoming/outgoing particle.
   $2m_1 \simeq m_2$
2. 3-point interaction model
3. Non-Relativistic Scattering (very small 3-momentum)
4. Massive particle itself became propagator

The particle itself became a mediator and present resonance.
That is why we call it 'Self-Resonant'!

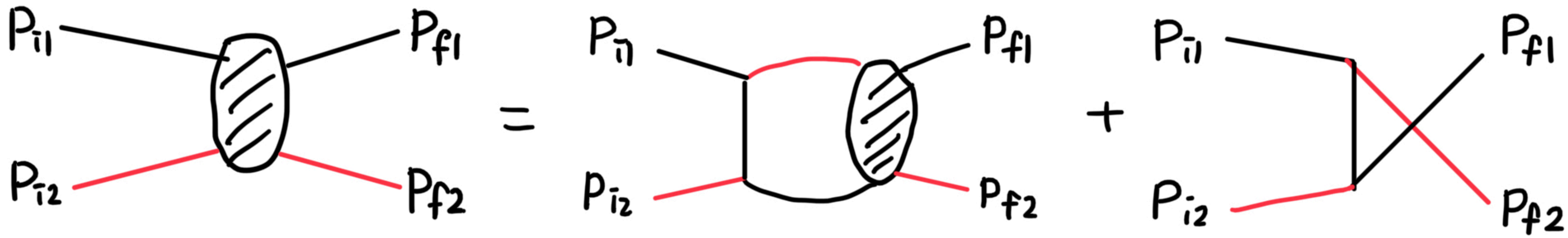
Ladder Diagram

Is non-perturbativity makes significant Sommerfeld Enhancement?



$$i\tilde{\Gamma} \qquad \mathcal{O}\left(\frac{g^2}{(m_2 - 2m_1)}\right) < \mathcal{O}\left(\frac{g^4}{(m_2 - 2m_1)^2}\right) < \mathcal{O}\left(\frac{g^6}{(m_2 - 2m_1)^3}\right)$$

for $2m_1 \simeq m_2$

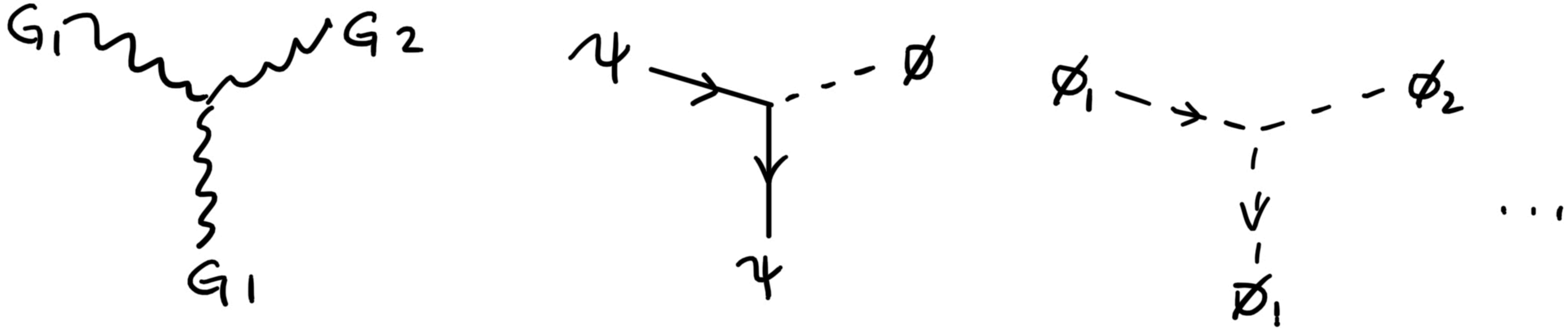


Model?

We need a model. Which model is applicable?

1. Special mass relation between incoming/outgoing particle. $2m_1 \simeq m_2$
2. 3-point interaction model
3. ~~Non-Relativistic Scattering (very small 3-momentum)~~ not related to model
4. Massive particle itself became propagator

Ex)



There are Flexibility to construct model,

But Simple model could determine common Physics

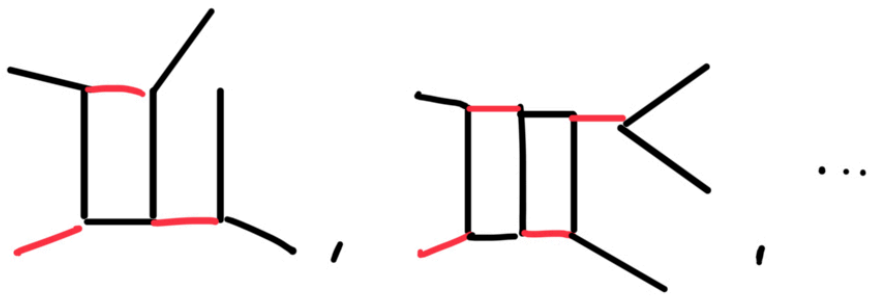
Simple Model

DM candidates : One Real Scalar & One Complex Scalar

$$\mathscr{L} = |\partial_\mu \phi_1|^2 - m_1^2 \phi_1^2 + \frac{1}{2}(\partial_\mu \phi_2)^2 - \frac{1}{2} m_2^2 \phi_2^2 - 2 g m_1 \phi_2 |\phi_1|^2$$

m_1 in coupling term makes g dimensionless.

We consider $(m_1 <)\ m_2 < 2m_1$ case.



Because $m_2 \geq 2m_1$ model has the drawback.

Decay mode became significant as the order goes higher.

Notations

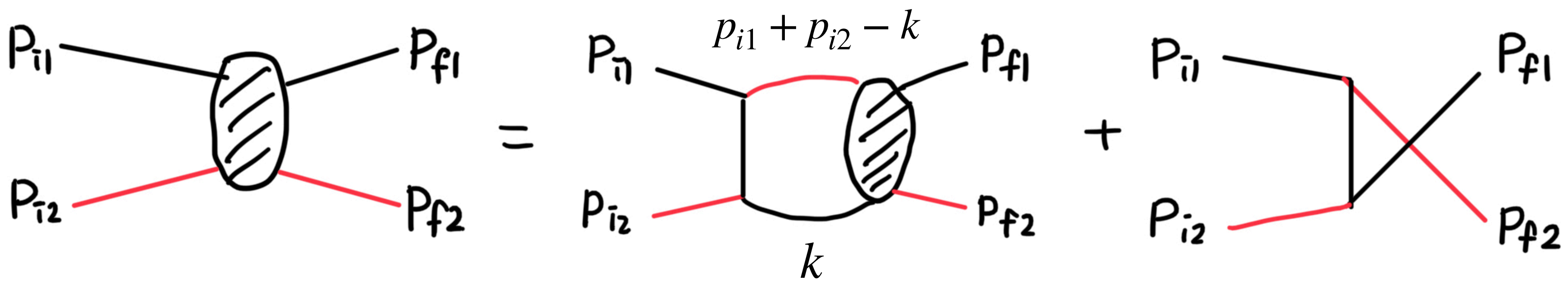
- $G_i(q)$ $(i = 1,2)$: particle i propagator with momentum transfer q. $\dfrac{i}{q^2 - m_i^2}$,

- $i\tilde{\Gamma}$: 4 point function via single mediator exchange.
  $$-\frac{4im_1^2g^2}{q^2 - m_1^2} \simeq 4im_1^2g^2\left[m_2(2m_1 - m_2) + \left(\sqrt{\frac{m_1}{m_2}}\vec{p}_{f2} - \sqrt{\frac{m_2}{m_1}}\vec{p}_{i1}\right)^2\right]^{-1}$$ for $q = p_{f2} - p_{i1}$
  Note that ϕ_1 always mediate u-channel interaction.

- $i\Gamma$: total 4 point function

Bethe-Salpeter Equation

This is What we are going to calculate. To confirm whether enhancement occurs or not.



$$i\Gamma(p_{i1}, p_{i2}; p_{f1}, p_{f2}) = -\int \frac{d^4k}{(2\pi)^4} \tilde{\Gamma}(p_{i1}, p_{i2}; k, p_{i1} + p_{i2} - k) G_1(k) G_2(p_{i1} + p_{i2} - k) \Gamma(k, p_{i1} + p_{i2} - k; p_{f1}, p_{f2})$$

$$+ i\tilde{\Gamma}(p_{i1}, p_{i2}, p_{f1}, p_{f2})$$

Seems like Integral Form of Schrodinger equation

: Approximation. First order contribution is much smaller than higher-order terms

Bethe-Salpeter Equation

$$i\Gamma(p_{i1}, p_{i2}; p_{f1}, p_{f2}) \simeq -\int \frac{d^4k}{(2\pi)^4} \tilde{\Gamma}(p_{i1}, p_{i2}; k, p_{i1} + p_{i2} - k) G_1(k) G_2(p_{i1} + p_{i2} - k) \Gamma(k, p_{i1} + p_{i2} - k; p_{f1}, p_{f2})$$

Bethe-Salpeter Wavefunction (in 4-momentum space)

$$\chi(p_{i1}, p_{i2}; p_{f1}, p_{f2}) \equiv G_1(p_{i1}) G_2(p_{i2}) \Gamma(p_{i1}, p_{i2}; p_{f1}, p_{f2}) \equiv \chi(p_{i1}, p_{i2})$$

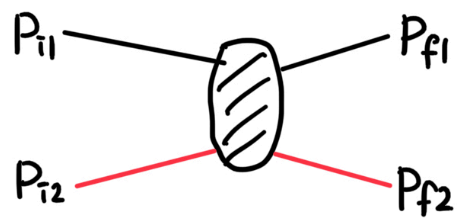
Abbreviation : $\tilde{\Gamma}(p_{i1}, p_{i2}; k, p_{i1} + p_{i2} - k) = \dfrac{4m_1^2 g^2}{(p_{i2} - k)^2 - m_1^2} \equiv \tilde{U}((p_{i2} - k)^2)$

$$i\chi(p_{i1}, p_{i2}) = -G_1(p_{i1}) G_2(p_{i2}) \int \frac{d^4k}{(2\pi)^4} \tilde{U}((p_{i2} - k)^2) \chi(q, p_{i1} + p_{i2} - k)$$

Bethe-Salpeter Equation

CM Frame Adoption

$$P = \frac{1}{2}(p_{i1} + p_{i2}) = (P_0, \overrightarrow{0}) \,, \; Q = \mu \left(\frac{p_{i2}}{m_2} - \frac{p_{i1}}{m_1} \right)$$



BS wavefunction with CM Frame

$$\chi(p_{i1}, p_{i2}) = \tilde{\chi}(P, Q)$$

Equation Became

$$\begin{aligned} i\tilde{\chi}(P,Q) &= -G_1\left(-Q + \frac{2\mu}{m_2}P\right) G_2\left(Q + \frac{2\mu}{m_1}P\right) \int \frac{d^4k}{(2\pi)^4} \tilde{U}((p_{i2}-k)^2)\tilde{\chi}\left(P, \frac{2\mu}{m_2}P - k\right) \\ &= -G_1\left(-Q + \frac{2\mu}{m_2}P\right) G_2\left(Q + \frac{2\mu}{m_1}P\right) \int \frac{d^4k'}{(2\pi)^4} \tilde{U}((p_{i2}-k)^2)\tilde{\chi}\left(P, k'\right) \end{aligned}$$

$$\mu = \frac{m_1 m_2}{m_1 + m_2} \qquad k' = \frac{2\mu}{m_2} - k$$

Bethe-Salpeter Equation

χ to 3-momentum wavefunction, $\tilde{\psi}_{BS}(\overrightarrow{Q}) \equiv \int \frac{dQ_0}{2\pi} \tilde{\chi}(Q)$

$$i\tilde{\psi}_{BS}(\overrightarrow{Q}) = \left[-\int \frac{dQ_0}{2\pi} G_1\left(-Q + \frac{2\mu}{m_2}P\right) G_2\left(Q + \frac{2\mu}{m_1}P\right)\right] \left[\int \frac{d^3\overrightarrow{k'}}{(2\pi)^3} \tilde{U}\left(\left|\sqrt{\frac{m_1}{m_2}}\overrightarrow{Q} + \sqrt{\frac{m_2}{m_1}}\overrightarrow{k'}\right|\right) \tilde{\psi}_{BS}(\overrightarrow{k'})\right]$$

$$= \frac{i}{4m_1m_2}\left(\frac{\overrightarrow{Q}^2}{2\mu} - E\right)^{-1} \left[\int \frac{d^3\overrightarrow{k'}}{(2\pi)^3} \tilde{U}\left(\left|\sqrt{\frac{m_1}{m_2}}\overrightarrow{Q} + \sqrt{\frac{m_2}{m_1}}\overrightarrow{k'}\right|\right) \tilde{\psi}_{BS}(\overrightarrow{k'})\right]$$

Major Substitutions ends here. Left is mathematics.

Result : Schrodinger-like Equation

$$-\frac{1}{2\mu}\nabla^2\psi(\vec{r}) - \frac{\alpha}{r}e^{-Mr}\psi(-\frac{m_2}{m_1}\vec{r}) = E\psi(\vec{r}) \qquad E = P_0 - \frac{m_1 + m_2}{2}, M = m_2\sqrt{2 - \frac{m_2}{m_1}}, \alpha = \frac{g^2}{4\pi}$$

Answer to First Question

$$-\frac{1}{2\mu}\nabla^2\psi(\vec{r}) - \frac{\alpha}{r}e^{-Mr}\psi(-\frac{m_2}{m_1}\vec{r}) = E\psi(\vec{r})$$

$$E = P_0 - \frac{m_1 + m_2}{2}, M = m_2\sqrt{2 - \frac{m_2}{m_1}}, \alpha = \frac{g^2}{4\pi}$$

- In $2m_1 \simeq m_2$ limit, the potential seems attractive, Coulombic, and long-range.
- The Equation is based and satisfied within the non-relativistic limit.

Is non-perturbativity makes significant Sommerfeld Enhancement?

Yes!

How to Solve Equation

To make Mathematica™ solving equation numerically,

1. We need to deform the equation as delay differential equation form.

2. Proper boundary conditions should be given.
   We choose Function and its derivative value at a single point.

Solution could be Partial wave analysis.

We Consider s-wave. which dominates non-relativistic case.

How to Solve Equation

We need to deform the equation as delay differential equation form.

$\alpha = g^2/4\pi \qquad a = \frac{2\nu}{\alpha}$

$$-\frac{1}{2\mu}\nabla^2\psi(\vec{r}) - \frac{g^2}{4\pi r}e^{-Mr}\psi(-\frac{m_2}{m_1}\vec{r}) = E\psi(\vec{r})$$

Separation of variable

$\psi = R(r)Y_{lm}$

Parameter Substitution

$x = \frac{\mu\alpha r}{2} = e^{-\rho}$

Adopt New Radial Function

$u_l(x) = xR(x)$

Use Logarithm to make multiplication to addtion

$\tilde{u}_l(\rho) = u_l(e^{-\rho})$

$b = m_2/m_1$

$c = \frac{2M}{\mu\alpha} = \frac{2}{\alpha}(1+b)\sqrt{2-b}$

Two Important Equations

$$\left(\frac{\partial^2}{\partial x^2} - \frac{l(l+1)}{x^2}\right)u_l(x) + \frac{4e^{-cx}}{bx}(-1)^l u_l(bx) + a^2 u_l(x) = 0$$

Imply Boundary Conditions

No absolute mass dependence

$$\frac{\partial^2}{\partial\rho^2}\tilde{u}_l(\rho) + \frac{\partial}{\partial\rho}\tilde{u}_l(\rho) - l(l+1)\tilde{u}_l(\rho) + \frac{4}{b}(-1)^l\exp\left[\rho - ce^{-\rho}\right]\tilde{u}_l(\rho - \ln b) + a^2e^{-2\rho}\tilde{u}_l(\rho) = 0$$

Numerically Solvable

How to Obtain Sommerfeld Factor

Boundary Condition determination

$$\left(\frac{\partial^2}{\partial x^2} - \frac{l(l+1)}{x^2}\right) u_l(x) + \frac{4e^{-cx}}{bx}(-1)^l u_l(bx) + a^2 u_l(x) = 0$$

$x \to 0$ ($r \to 0, \rho \to \infty$) limit : Equation became almost Coulombic.

For $l = 0$, $\tilde{u}_l(\rho) \sim Ae^{-\rho}$ form chosen

$x \to \infty$ ($r \to \infty, \rho \to -\infty$) limit : Equation became Spherical Bessel Equation

For $l = 0$, $\tilde{u}_l(\rho) \sim \dfrac{1}{a}\sin(ae^{-\rho} + \delta_0)$ form and $\tilde{u}_l'(\rho) \sim \cos(ae^{-\rho} + \delta_0)$ chosen.

Sommerfeld Factor $S_0 = |A|^2$ obtained after solving equation.

Yukawa limit Result

Numerical Sommerfeld Factor for s-wave cases

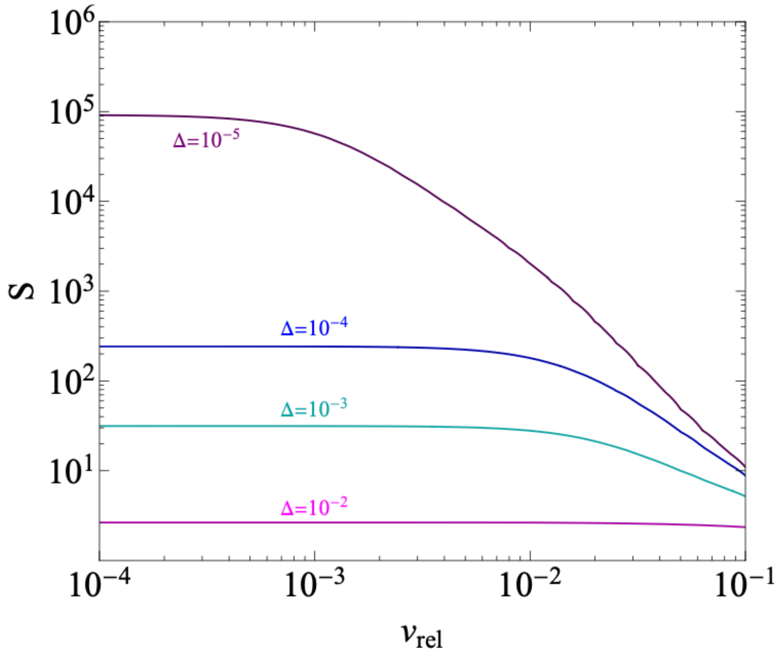


FIG. 2: Sommerfeld factor for s-wave elastic scattering, $\phi_1\phi_2 \to \phi_1\phi_2$, as a function of the relative velocity $v_{\rm rel}$. We chose $\alpha = g^2/(4\pi) = 0.1$ and $\Delta = 10^{-2}, 10^{-3}, 10^{-4}, 10^{-5}$, for $m_2 = 2m_1(1-\Delta)$, in the lines from bottom to top.

We call Yukawa limit because potential's exponential suppression remains.

$$\Delta \equiv 1 - \frac{m_2}{2m_1},$$

parametrize mass difference

- Enhancement becomes larger as $|m_2 - 2m_1|$ smaller.
- There is no absolute mass dependence in Enhancement.

Yukawa limit Result

Sommerfeld-Enhanced cross-section. s-wave cases

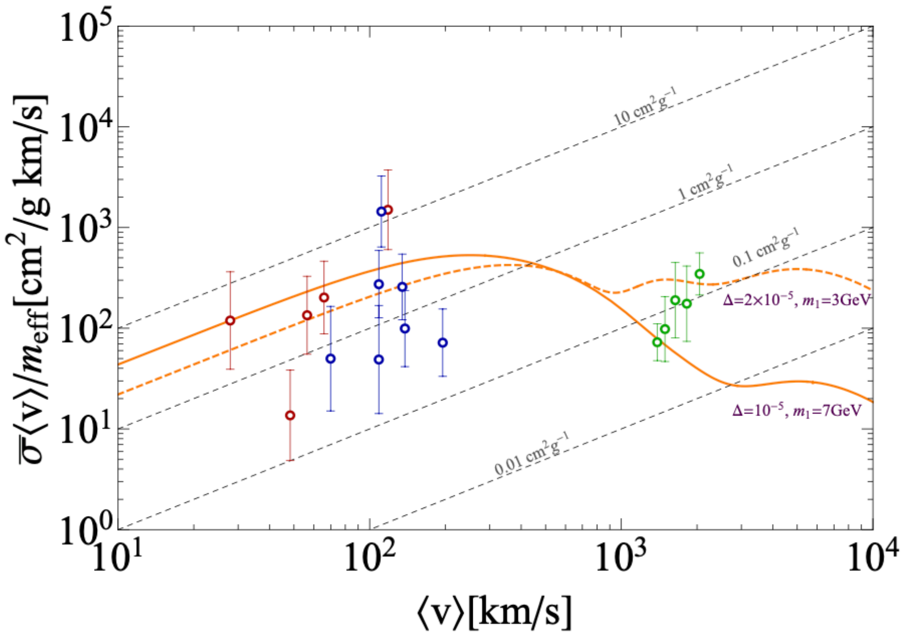


FIG. 3: Self-scattering cross section per dark matter mass for s-wave elastic scattering, $\phi_1\phi_2 \to \phi_1\phi_2$, as a function of $\langle v_{\rm rel}\rangle$. We chose $\Delta = 10^{-5}, 2\times 10^{-5}$ and $m_1 = 7, 3\,{\rm GeV}$ in orange solid and dashed lines, respectively. We took $\alpha = g^2/(4\pi) = 0.1$ and $m_{\rm eff} = 2m_1(1+b)$.

$$\Delta \equiv 1 - \frac{m_2}{2m_1}$$

Experimental Observations

- Red : THINGS dwarf galaxies
- Green : clusters
- Blue : LSB galaxies

Huge DM Mass requires a smaller Delta for larger non-perturbativity.

M. Kaplinghat, S. Tulin and H. B. Yu, Phys. Rev. Lett. 116 (2016) no.4, 041302 doi:10.1103/PhysRevLett.116.041302 [arXiv:1508.03339 [astro-ph.CO]]

Summary

- Dark Matter Scattering Cross-sections play important role in comparing observation and theory.
- Sommerfeld Effect, which is traditionally considered due to light mediator, enhances cross-section, especially lower velocity.
- We present a new Sommerfeld enhancement mechanism without a light mediator. Instead, the particle itself became a propagator.
- Our noble mechanism could be extended to the general model. The only requirement is 3 point interaction and proper mass relation.
- We Calculated Sommerfeld Factor and Enhanced Cross-section with a simple model.

Further Works

- Apply new mechanism for other models
- One More Motivation.
  Sommerfeld Enhancement for Dark Matter Annihilation.
- Solve Boltzmann Equation and obtain permissible parameter spaces that matches with observations
- 3 → 2 semi annihilation
- And more…

This is the End. Thank You!

References

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Questions are Welcome!