

Summary

~~PeV SUSY~~  
&  
GWs

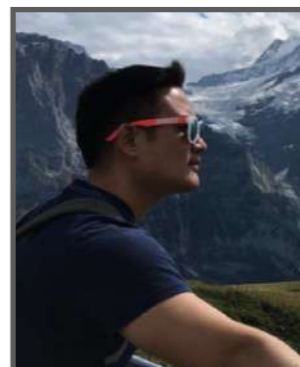
*Probing PeV scale SUSY breaking with  
satellite galaxies and primordial gravitational waves*

~~PeV SUSY~~  
&  
Small-scales

Introduction

*Ryusuke Jinno (DESY→IFT)*

*2021.11.30 @ KIAS*

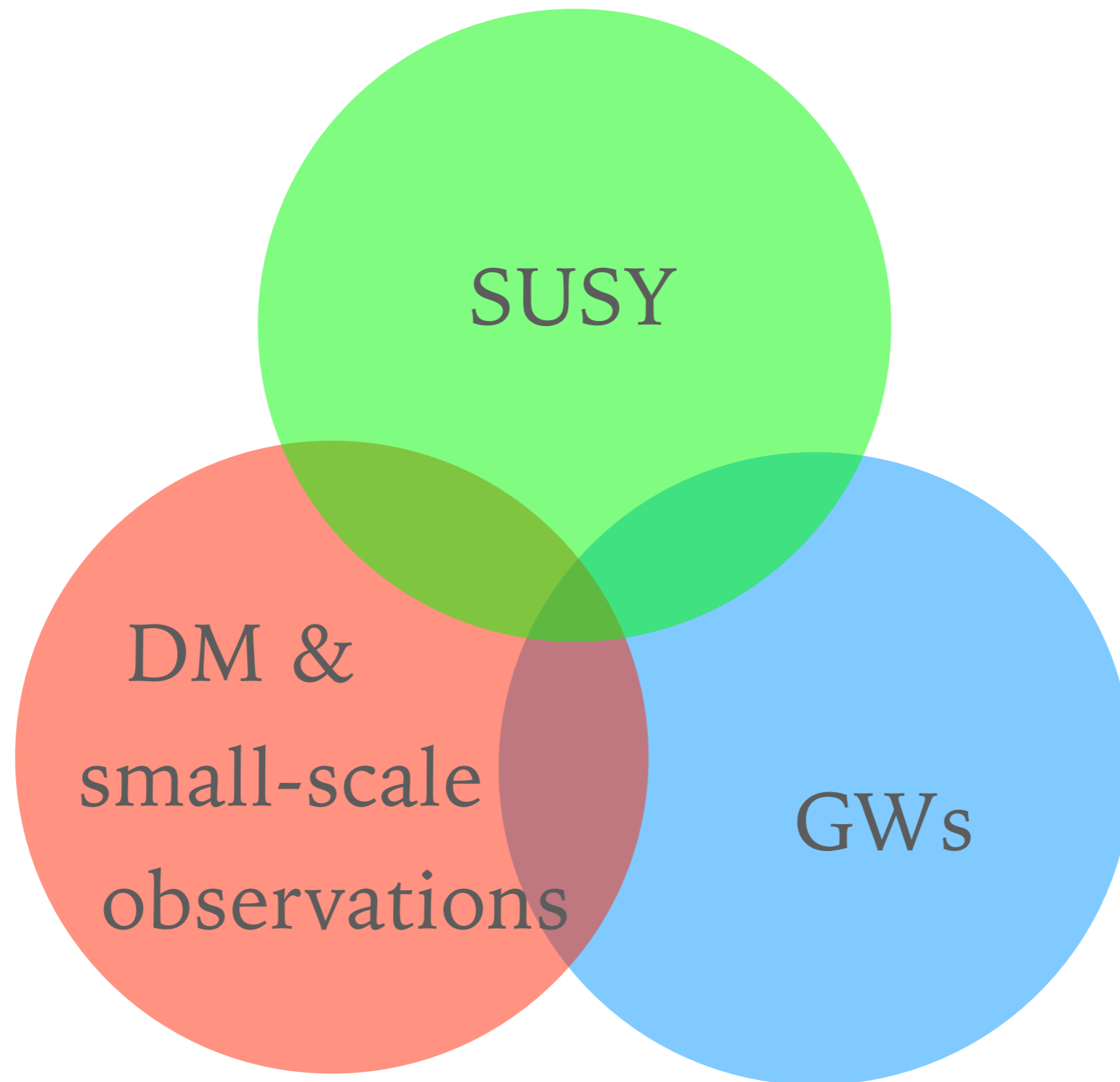


[ Choi, Jinno, Yanagida, PRD 104 (2021) 2107.12804 ]

[ Jinno, Moroi, Nakayama PRD 86 (2012) 1208.0184 ]

# INTRODUCTION

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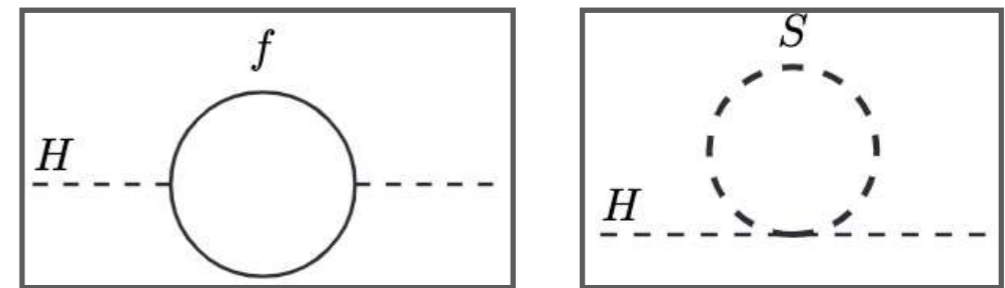


# INTRODUCTION: SUSY

## ➤ SUSY

- Symmetry btwn. bosons and fermions  $\delta\phi = \epsilon\psi$ ,  $\delta\psi = -i(\sigma^\mu\epsilon^\dagger)\partial_\mu\phi$
- Cancellation of quadratic divergence in higgs mass

→ solution to the hierarchy problem



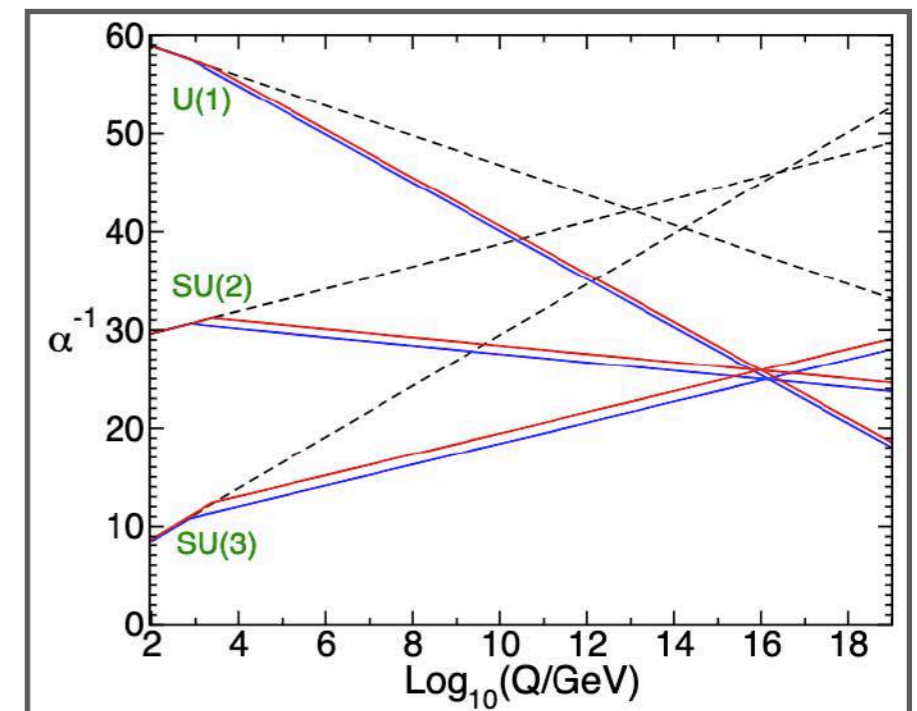
[ Martin 9709356 ]

- Unification of the gauge forces strongly suggested
- Viable dark matter candidates

higgsino, bino, wino,

gravitino,

...

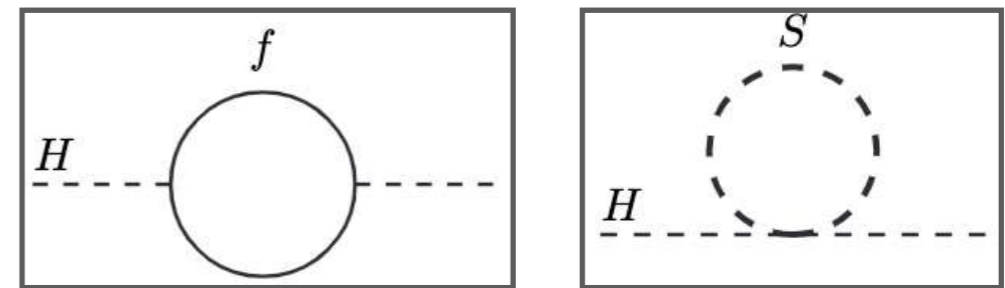


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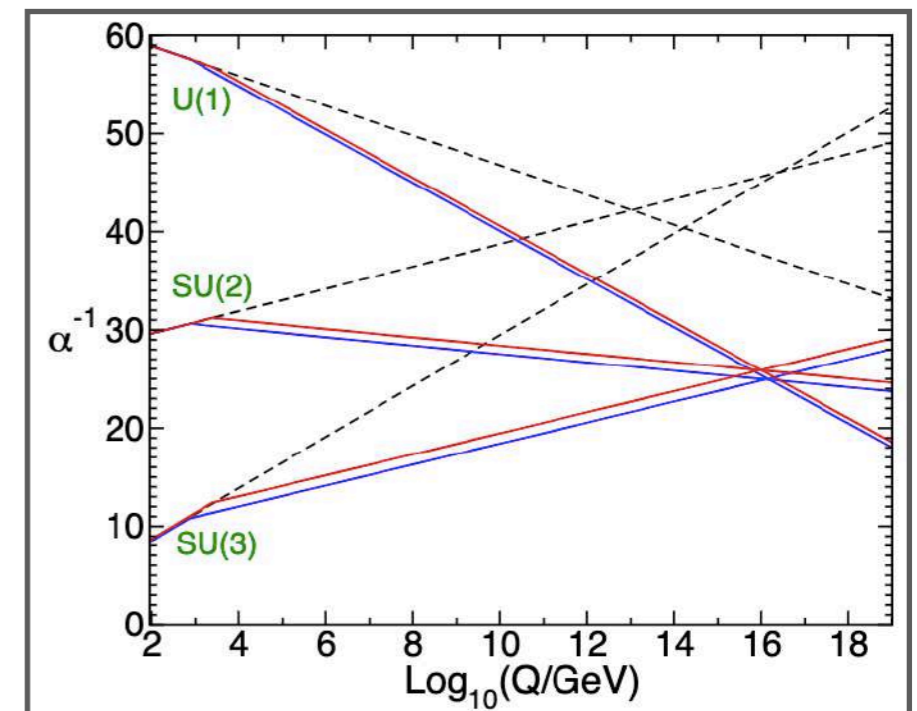
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# INTRODUCTION: SUSY

➤ Gravitino in PeV-scale SUSY breaking  $\langle F \rangle \sim (10^6 \text{ GeV})^2$

- We assume that the visible sector receives masses through gauge mediation



$$\langle F \rangle \neq 0$$

$$\text{Gluino mass } m_{\tilde{g}} \simeq N_{\text{mess}} \frac{g_c^2}{(4\pi)^2} \frac{y \langle F \rangle}{M_{\text{mess}}} \sim 10 \text{ TeV}$$

Note: The following story does not depend on the detailed mass spectrum

- When SUSY is promoted to a local symmetry, the superpartner of graviton appears

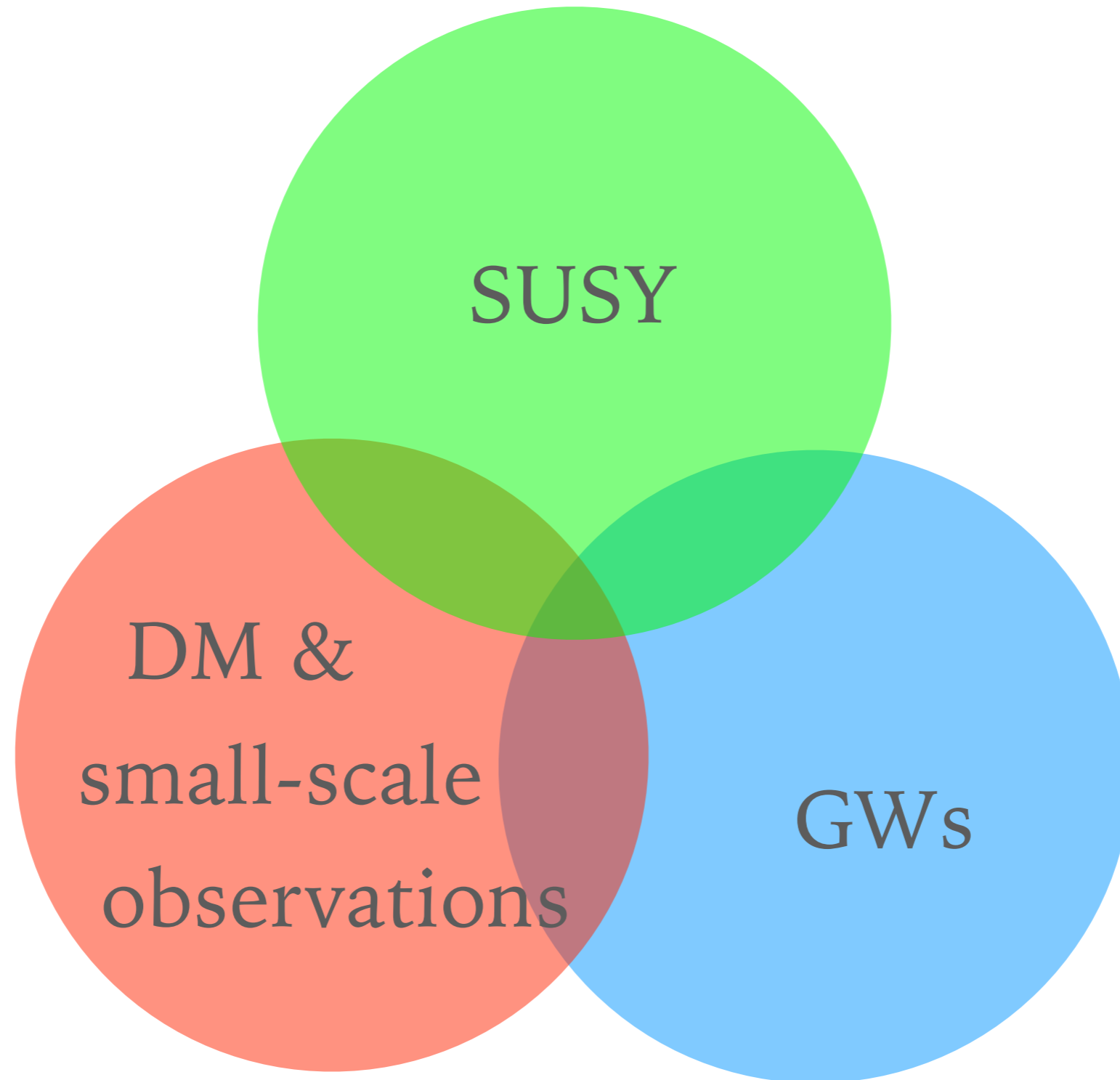
$$\text{Gravitino: } m_{3/2} \simeq \frac{\langle F \rangle}{\sqrt{3} M_P} \sim 100 \text{ eV} - 1 \text{ keV}$$

- We consider cosmology with gravitino, and point out that

inflatinoary GW spectrum has a distinct feature in PeV-scale ~~SUSY~~ scenarios

# INTRODUCTION

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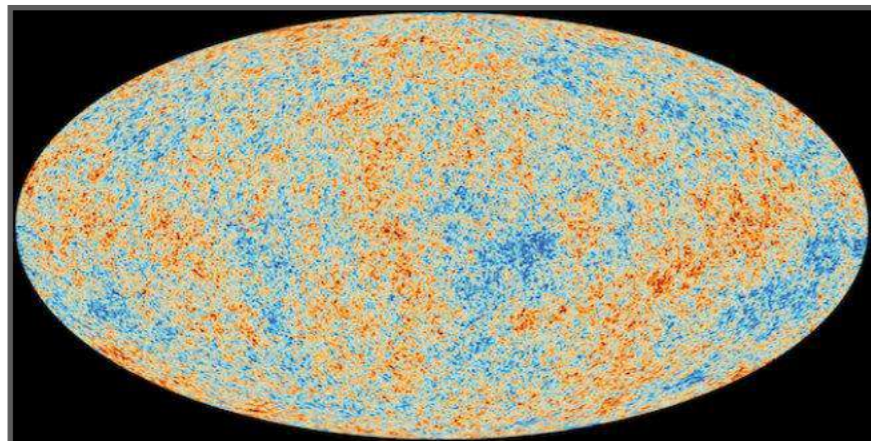
# INTRODUCTION: DARK MATTER & SMALL-SCALES

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## ► Dark matter & small-scale structures

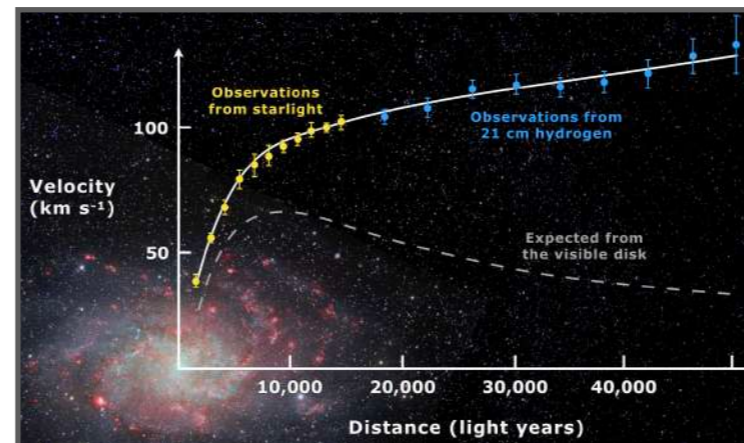
- Dark matter: unknown form of matter that accounts for  $\sim 25\%$  of the energy budget

CMB



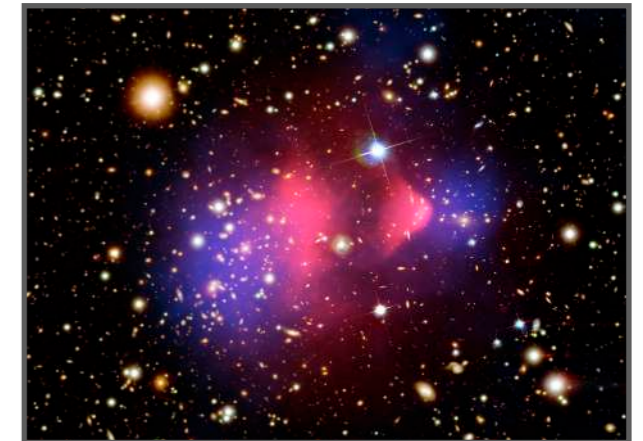
[ Planck ]

galaxy rotation curves



[ Wikipedia ]

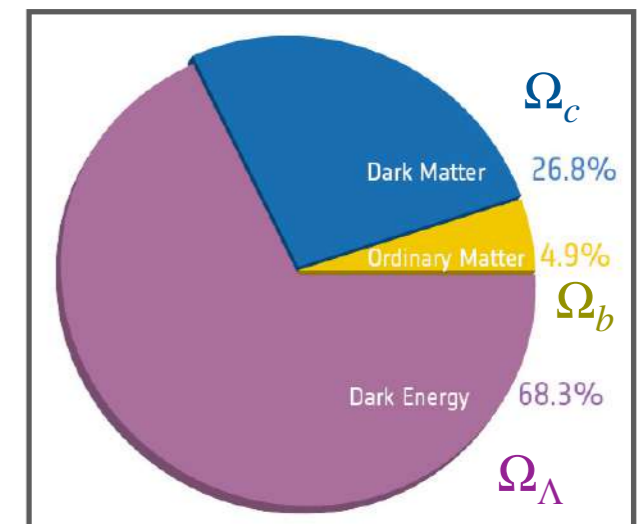
bullet clusters



[ ESA ]

- Properties:

- Long-lived
- Feeble interactions with the ordinary matter
- Not too much velocity (= not too hot)



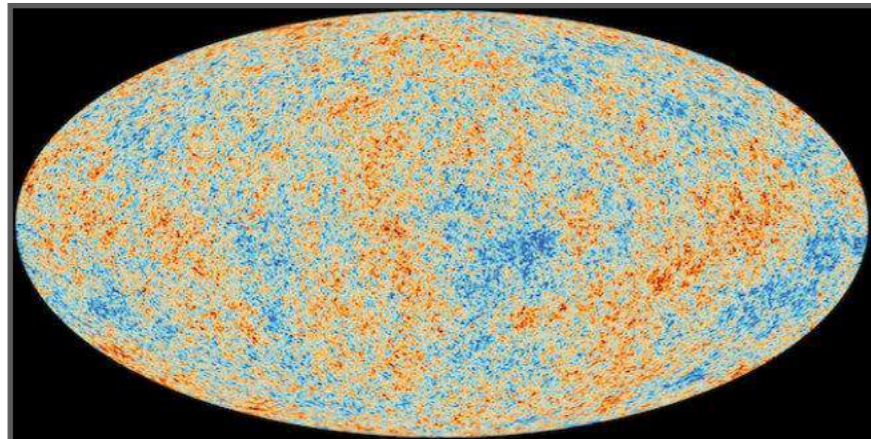
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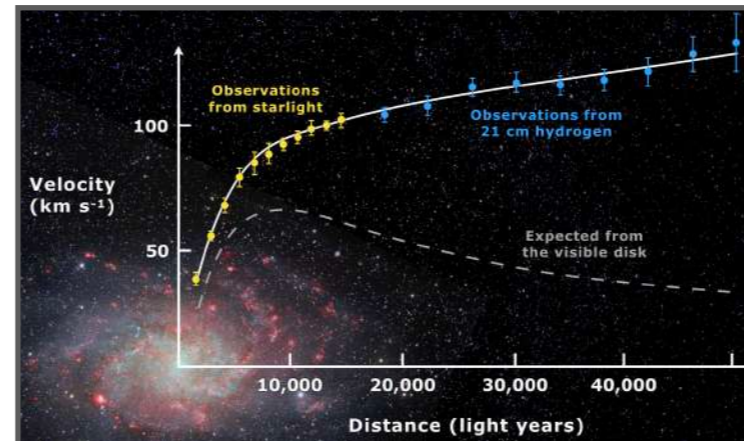
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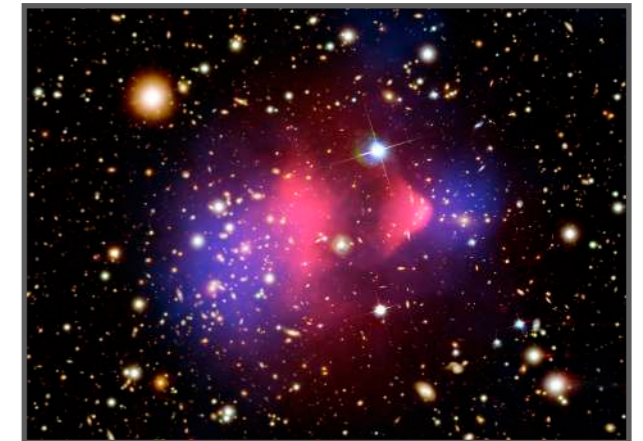
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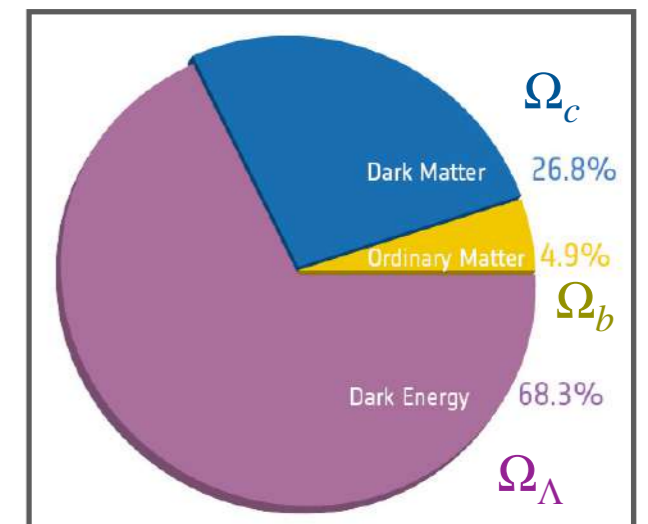
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# INTRODUCTION: DARK MATTER & SMALL-SCALES

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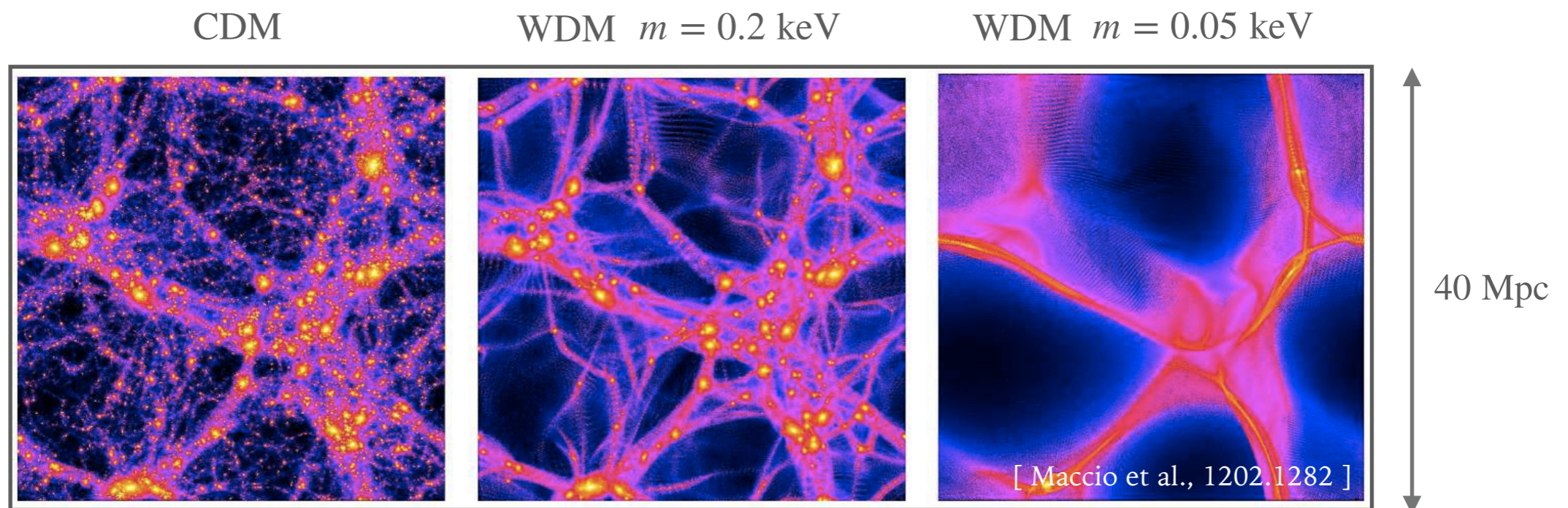
## ► Dark matter & small-scale structures

- Cold DM (= zero velocity) vs. Warm DM (= nonnegligible velocity)

In cosmology, CDM is very often assumed (e.g.  $\Lambda$ CDM model of cosmology)

However, there is no a priori reason to assume that.

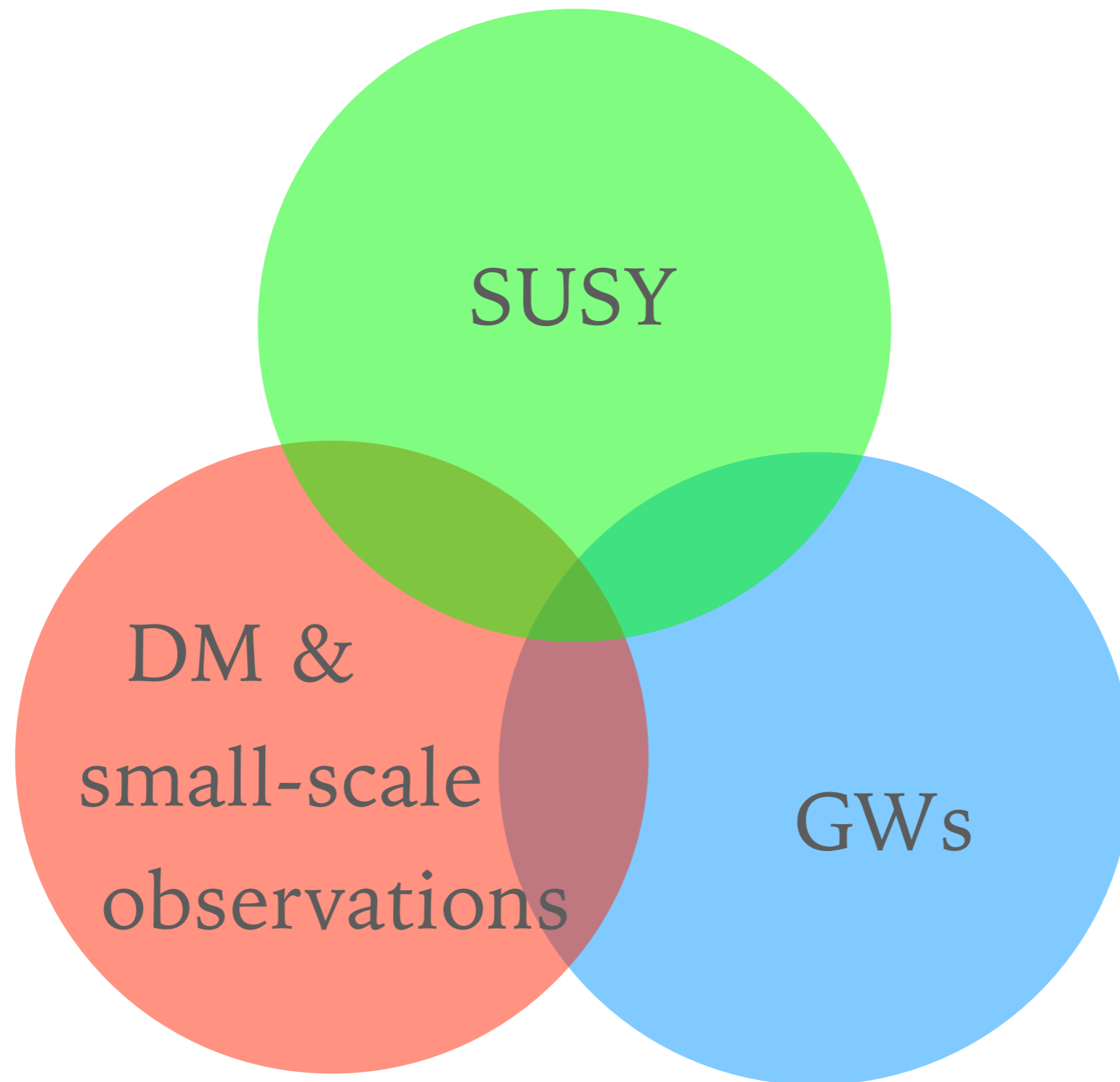
- If DM has warmth, it smears out small-scale structures  $v(z) \sim 0.012 \times (1+z) \times \left(\frac{\text{keV}}{m}\right)^{4/3}$



- Two observables will be explained as sensitive probes to the small-scale structure

# INTRODUCTION

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# INTRODUCTION: GWS

## ➤ Gravitational waves (GWs)

- Transverse-traceless part of the metric  $ds^2 = - dt^2 + a^2(\delta_{ij} + h_{ij})dx^i dx^j$

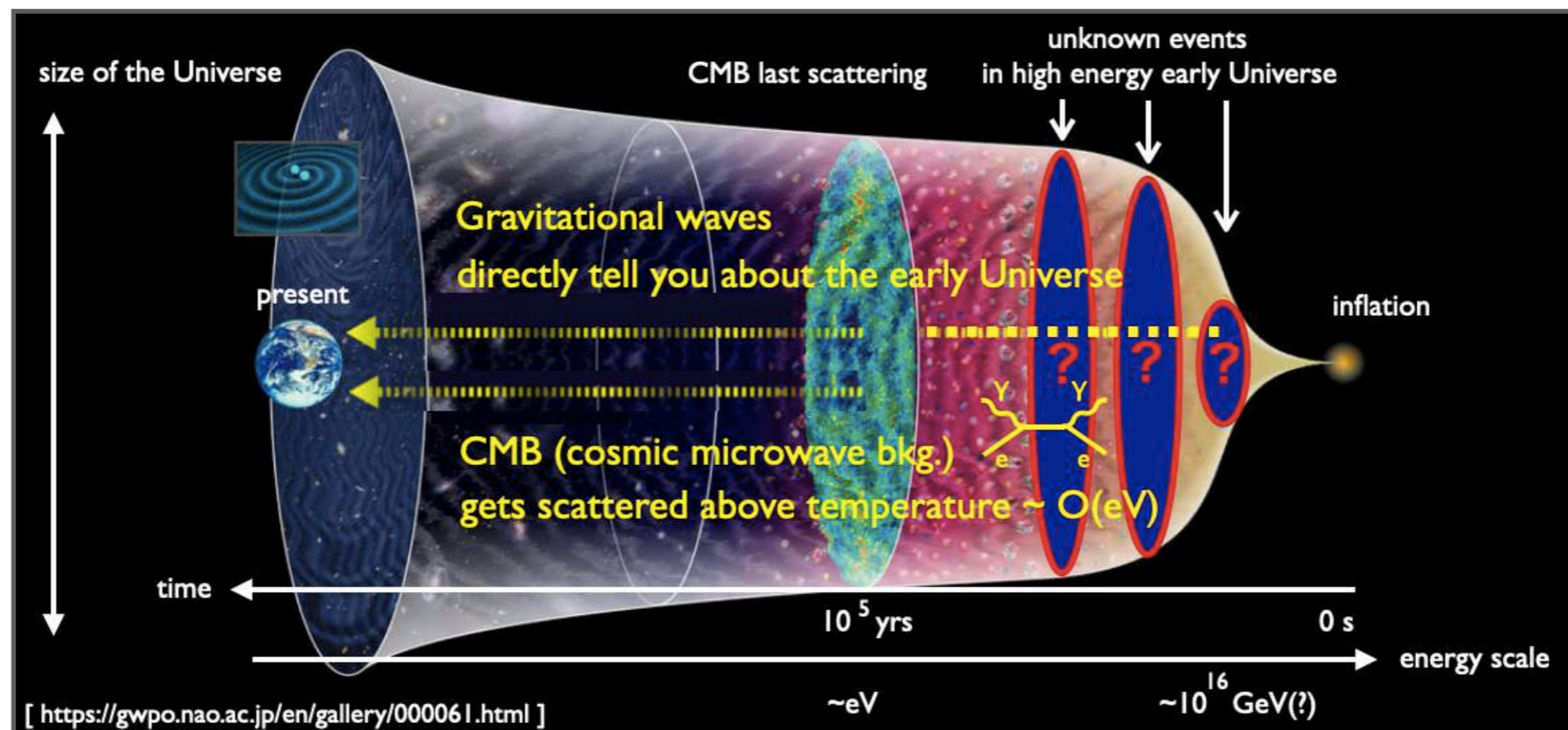
massless action

$$S_{\text{grav}} \sim \frac{M_P^2}{8} \int d^4x a^3 \left[ \dot{h}_{ij}^2 - \frac{1}{a^2} (\nabla h_{ij})^2 \right]$$

massless KG equation

$$\square h_{ij} \sim \frac{1}{M_P^2} \Lambda_{ij,kl} T_{kl}$$

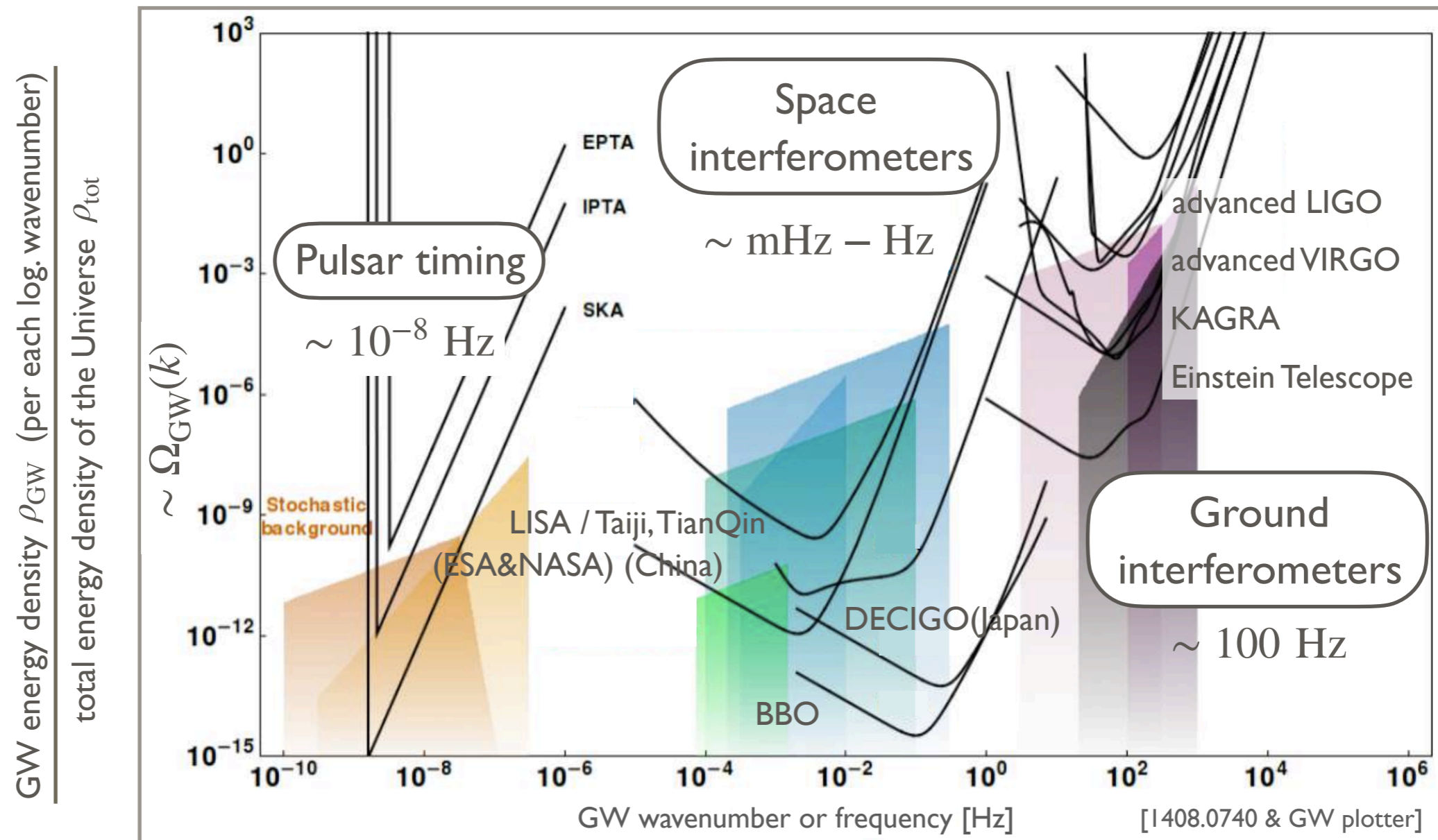
- Thanks to  $M_P$ -suppressed interactions, GWs retain the original information





# INTRODUCTION: GWS

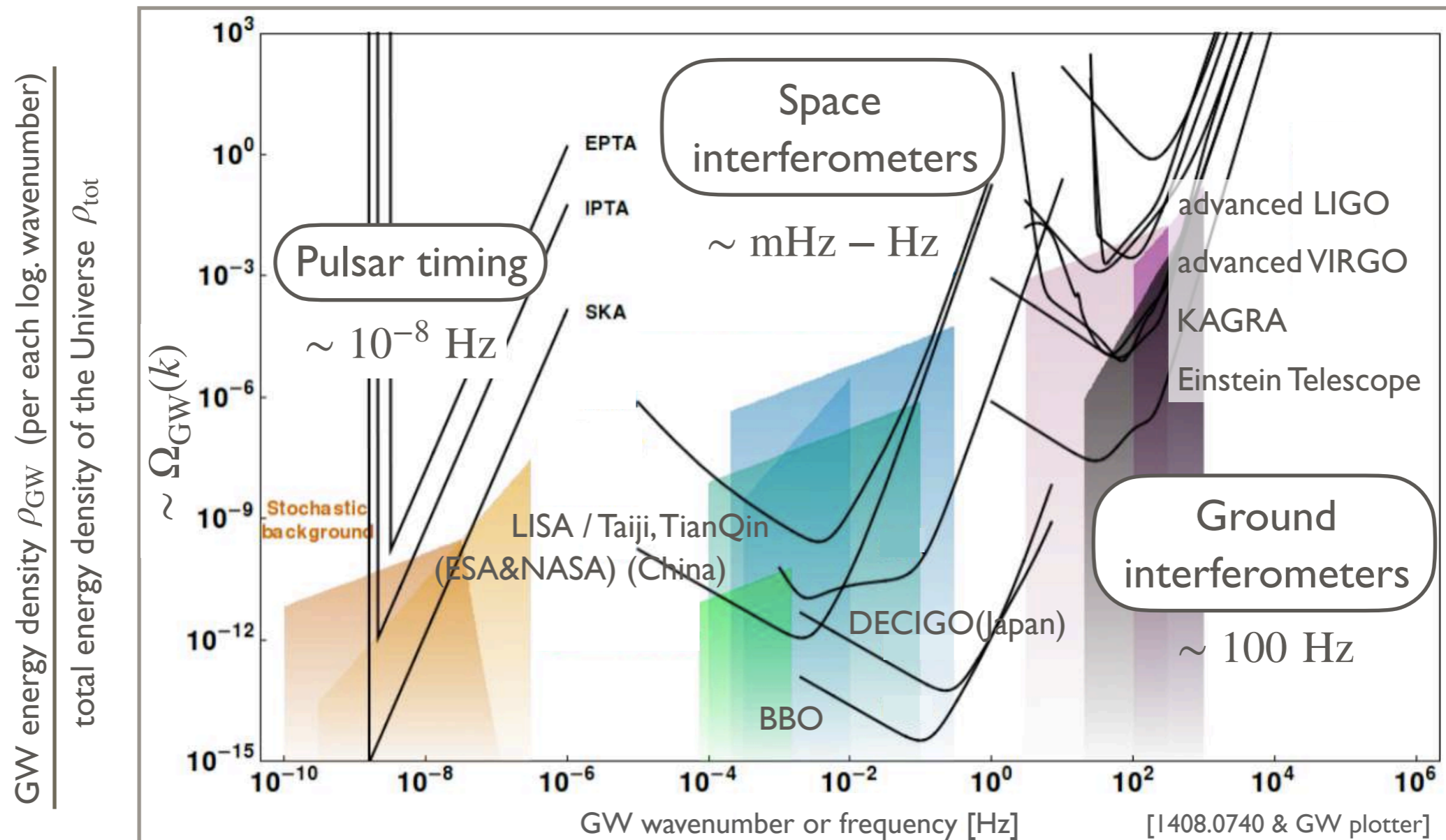
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# INTRODUCTION: GWS

- Summary of ongoing & future experiments

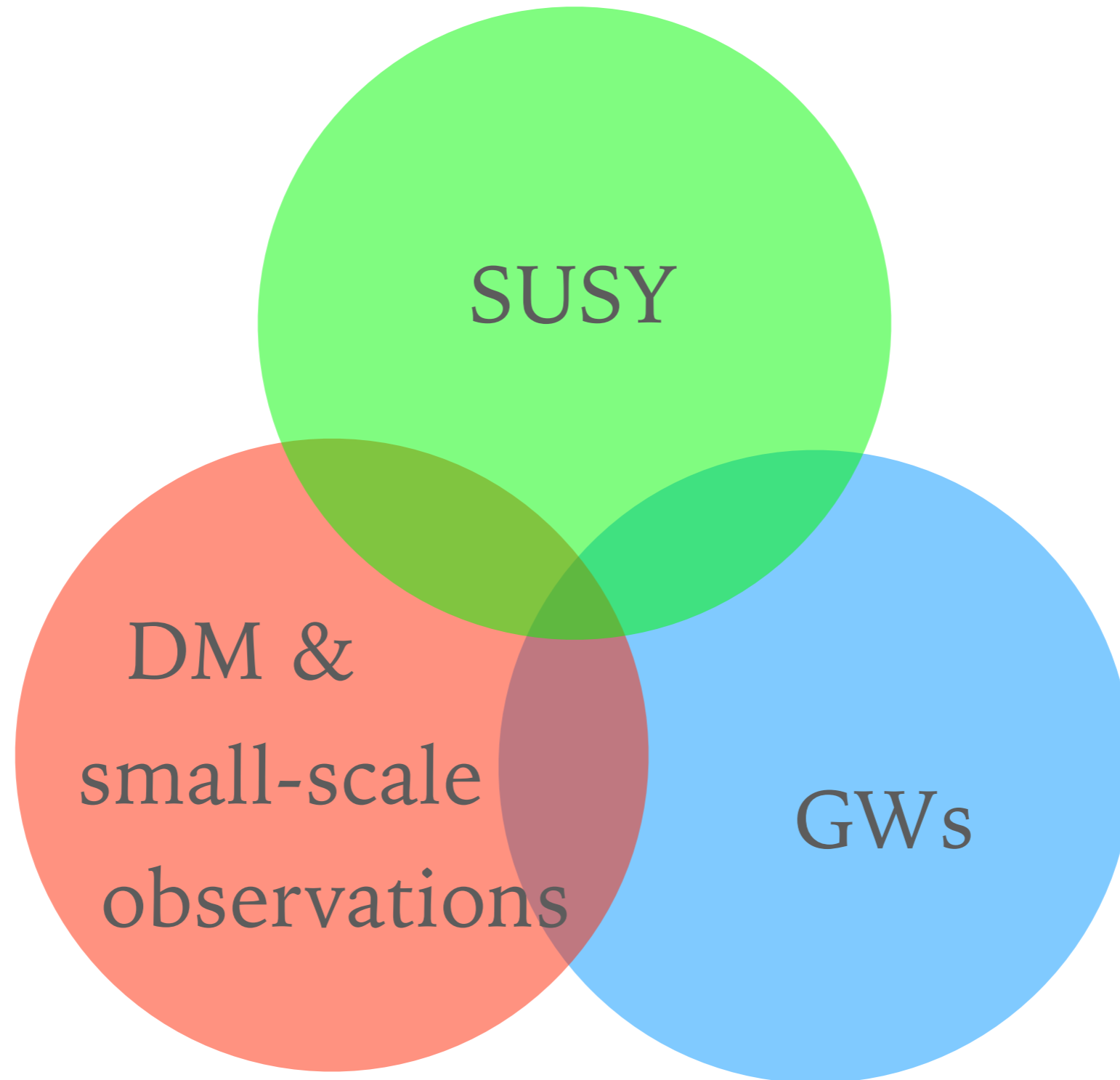


1 GeV     $10^3$  GeV     $10^6$  GeV     $10^9$  GeV     $10^{12}$  GeV

Temperature of the Universe GWs can probe (horizon-size GWs assumed)

# INTRODUCTION

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# SUMMARY

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In **PeV-scale SUSY breaking scenarios**,  
consideration on **small-scale constraints** implies  
distinct features in the **inflationary GWs**

## Punchline:

1. **small-scale constraints** on the warmness of gravitino requires entropy injection
2. entropy injection leaves an unavoidable imprint in the **inflationary GW spectrum**



*Summary*

*PeV ~~SUSY~~  
&  
GWs*

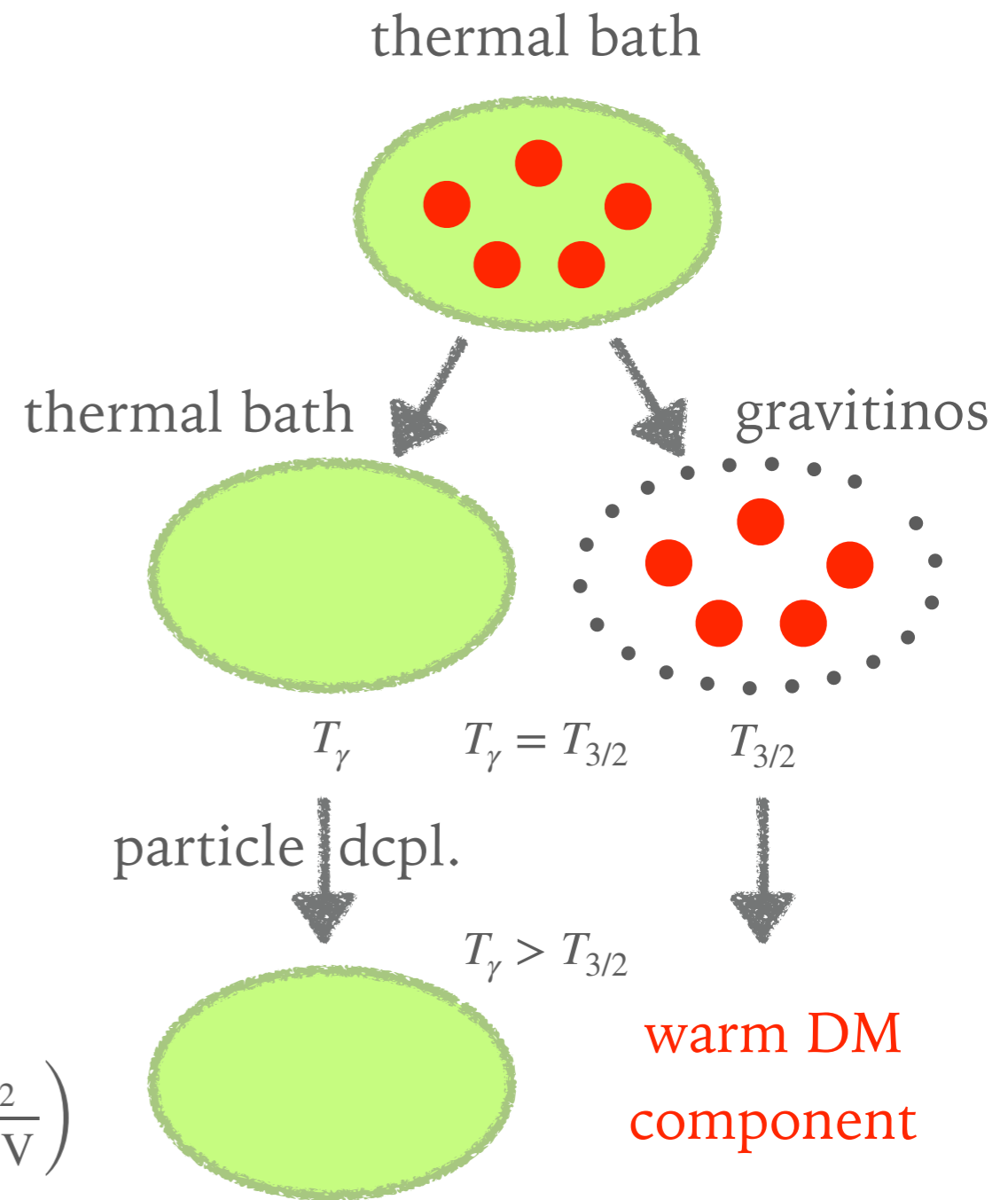
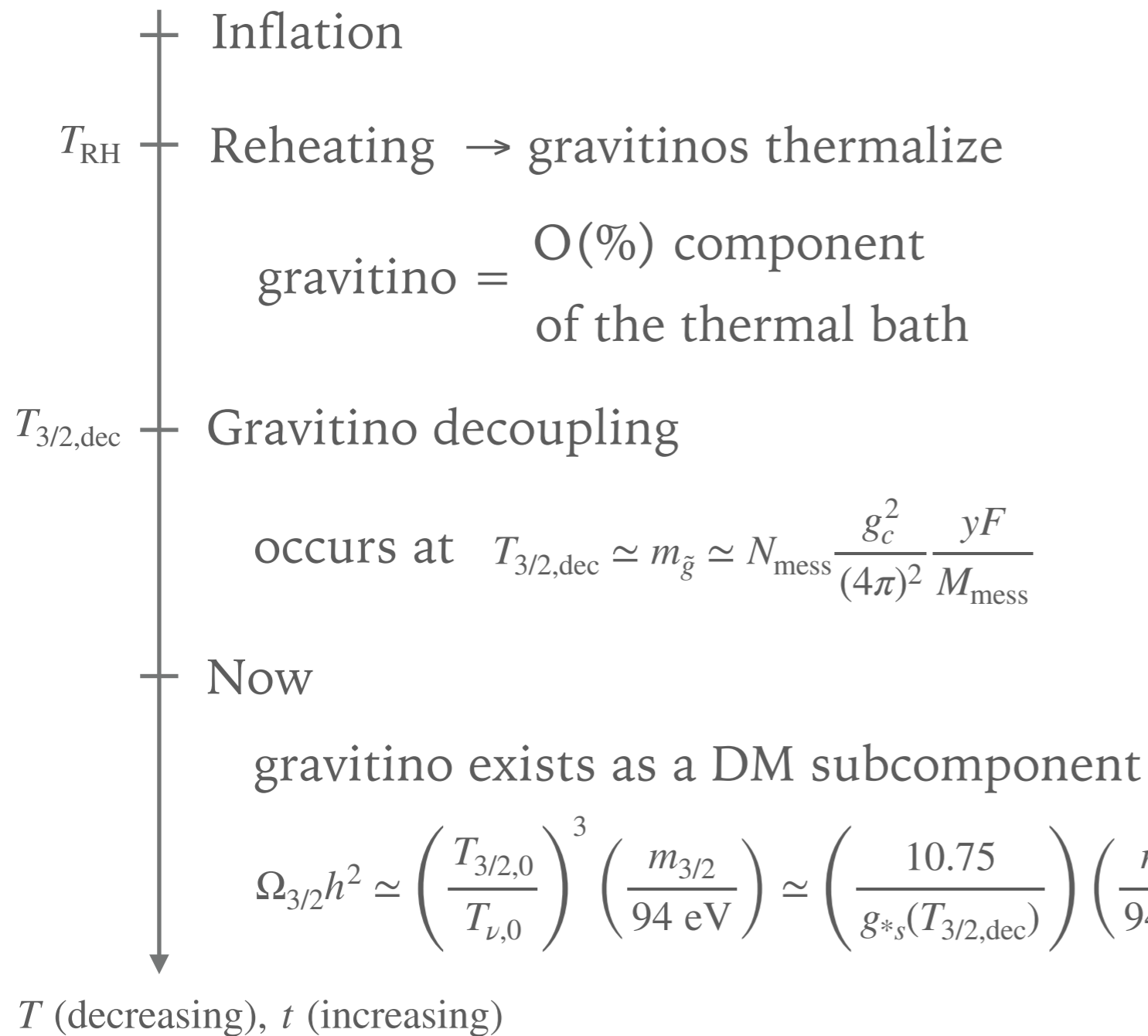
*PeV ~~SUSY~~  
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Small-scales*

*Introduction*



# SMALL-SCALE OBSERVATIONS MEET PEV-SCALE SUSY BR.

## ► Overview of the cosmic history



# SMALL-SCALE OBSERVATIONS MEET PEV-SCALE SUSY BR.

1) For  $m_{3/2} \sim 1 \text{ keV}$ , gravitino abundance should be diluted since  $\Omega_{3/2} \gtrsim 1$ .

This dilution (= entropy injection) must occur after gravitino decoupling.

2) Gravitino exists as a warm component:

$$\text{Free-streaming length } \lambda \sim \frac{2\pi}{5} \left( \frac{m_{3/2}}{1 \text{ keV}} \right)^{-1} \text{ Mpc}$$

This also requires dilution from the viewpoint of small-scale observations.

Now

gravitino exists as a DM subcomponent

$$\Omega_{3/2} h^2 \simeq \left( \frac{T_{3/2,0}}{T_{\nu,0}} \right)^3 \left( \frac{m_{3/2}}{94 \text{ eV}} \right) \simeq \left( \frac{10.75}{g_{*s}(T_{3/2,\text{dec}})} \right) \left( \frac{m_{3/2}}{94 \text{ eV}} \right)$$

particle dcpl.



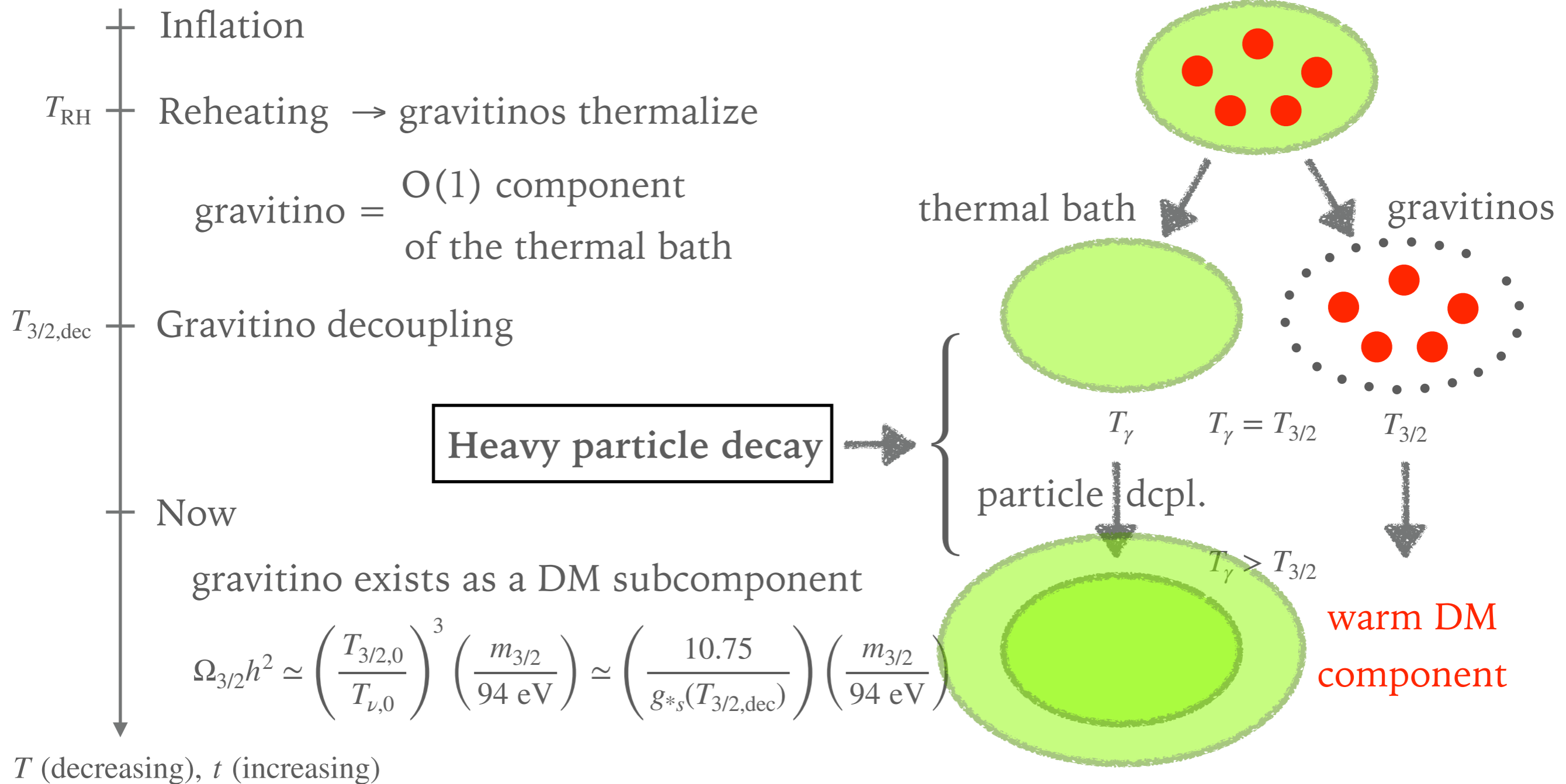
$$T_\gamma > T_{3/2}$$

warm DM component

$T$  (decreasing),  $t$  (increasing)

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How much dilution?

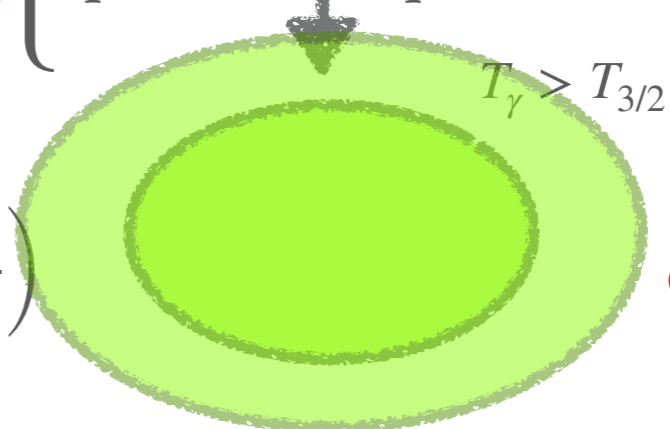
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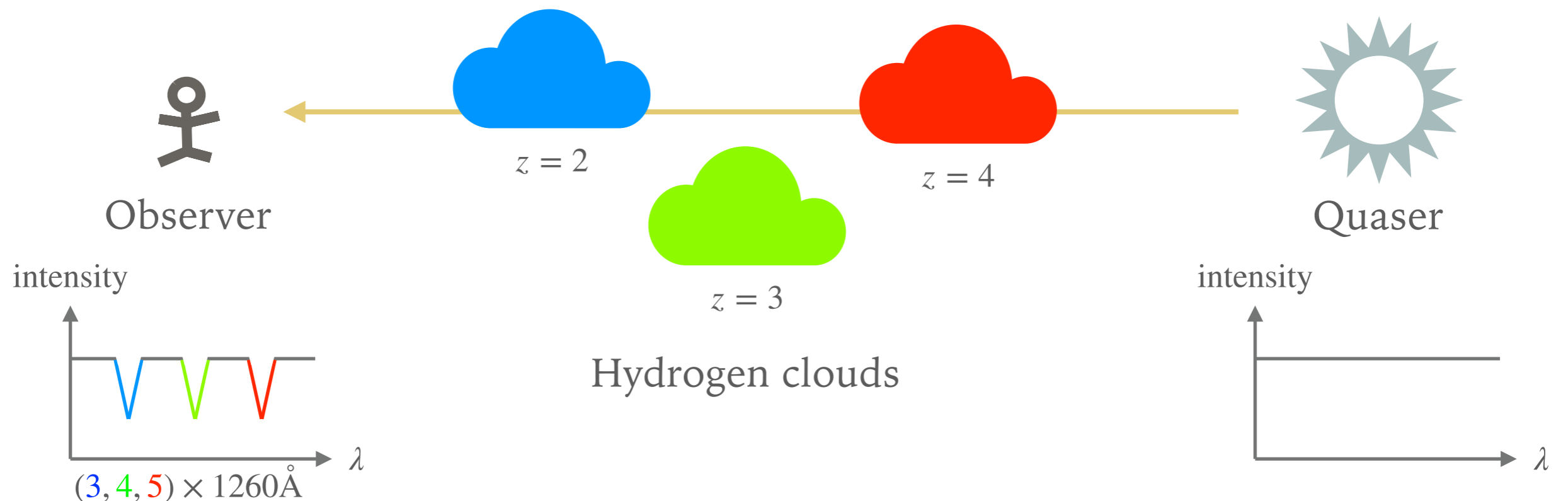


# SMALL-SCALE OBSERVATIONS MEET PEV-SCALE SUSY BR.

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## ► Small-scale observations: ① Lyman- $\alpha$ forest

- We observe the photon spectrum from quasars (also written as QSOs)
- Hydrogen clouds along the line of sight absorb photons with  $1260\text{\AA}$  at that time
- Since photons are constantly being redshifted, the absorption lines at different wavelengths correspond to different redshifts

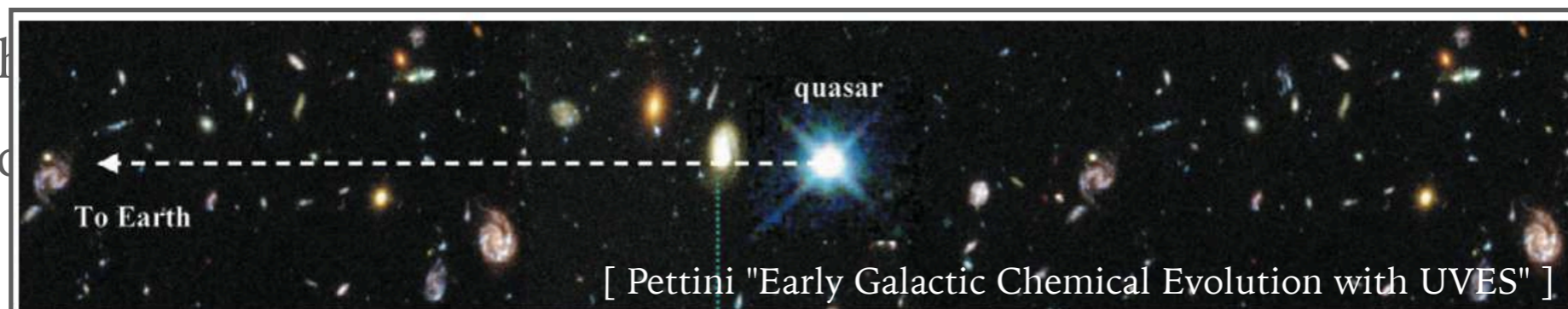


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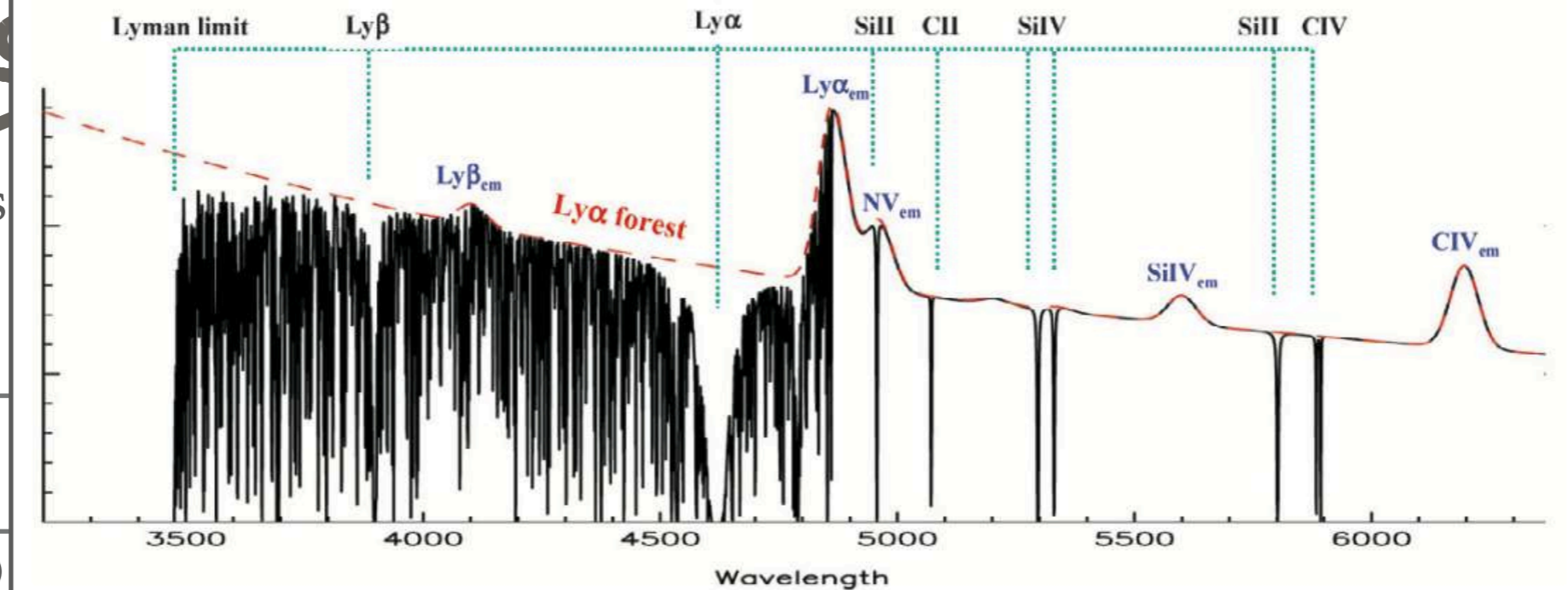
- Since ph  
the abs



shifts

Obs

intensity



quasar

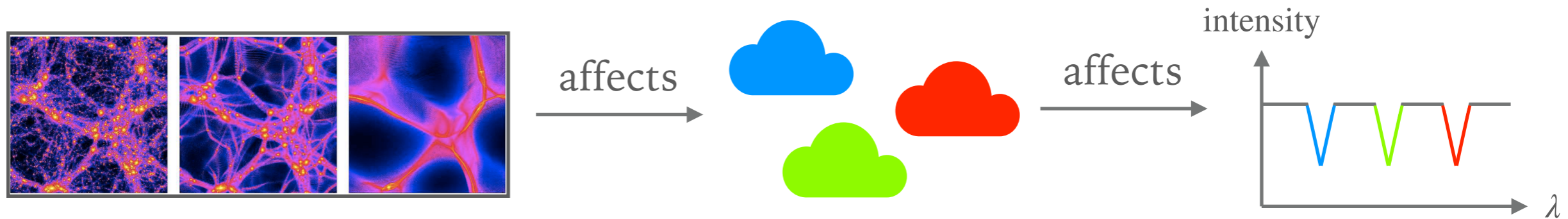
$\lambda$

# SMALL-SCALE OBSERVATIONS MEET PEV-SCALE SUSY BR.

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## ► Small-scale observations: ① Lyman- $\alpha$ forest

- Warmness affects the abundance of hydrogen clouds, and the amount of absorption



→ Constraints on warm DM can be derived

e.g. [ Irsic et al. 1702.01764 ] [ Murgia, Irsic, Viel 1806.08371 ] [ Garzilli, Ruchayskiy, Magalich, Boyarsky 1912.09397 ]

- However, we do not use Lyman- $\alpha$  constraints

Why? Typically, people assume 100% WDM and derive constraints on mass.  
But we need constraints on the allowed WDM fraction.

Anyway, Nsat (→ next slide) gives conservative bounds

# SMALL-SCALE OBSERVATIONS MEET PEV-SCALE SUSY BR.

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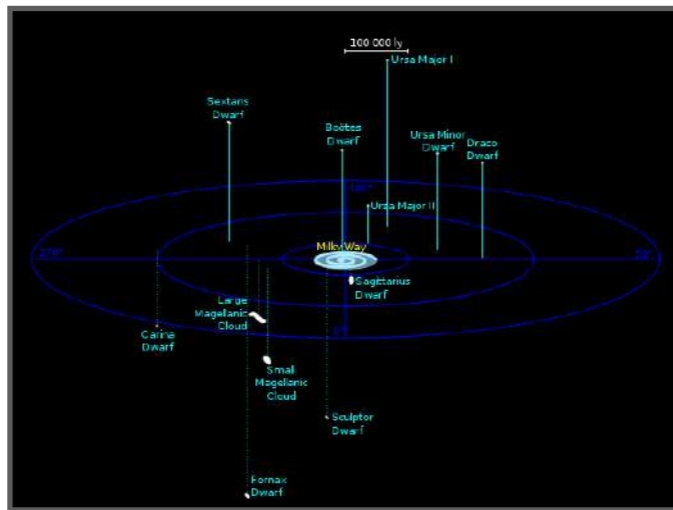
## ► Small-scale observations: ② Number of satellite galaxies

- Satellite galaxies of Milky Way (our Galaxy):

To take into account  
SDSS sky coverage  $f=0.28$

11 classical dwarf gal. + 15 ultra-faint dwarf gal.  $\times 3.5 = 63$  already 'observed'

So, any DM model should predict  $\geq 63$  satellite galaxies



Classical dwarf galaxies [ Wikipedia ]



[ Sloan Digital Sky Survey ]

- We estimate the number of satellite galaxies in PeV-scale SUSY breaking scenarios

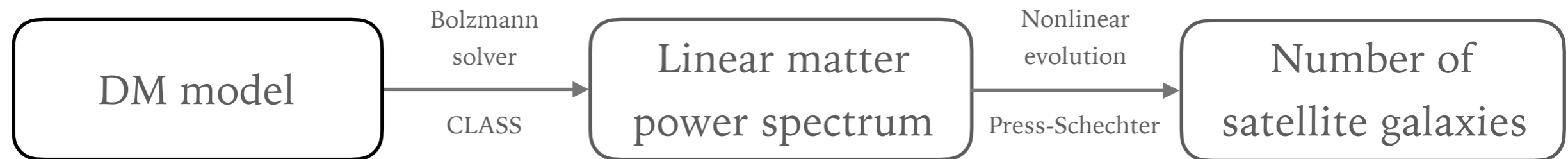
using the Press-Schechter approach [ Press-Schechter '74 ] [ Polisensky, Ricotti 1004.1459 ] [ Schneider 1412.2133 ]

# SMALL-SCALE OBSERVATIONS MEET PEV-SCALE SUSY BR.

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- Small-scale observations: ② Number of satellite galaxies

- Pipeline



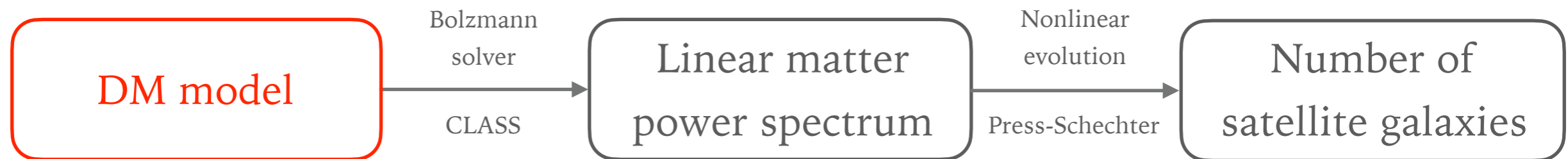


# SMALL-SCALE OBSERVATIONS MEET PEV-SCALE SUSY BR.

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## ► Small-scale observations: ② Number of satellite galaxies

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DM model?

MDM (Mixed DM) model = CDM (Cold DM) + WDM (Warm DM)

We do not specify who's this

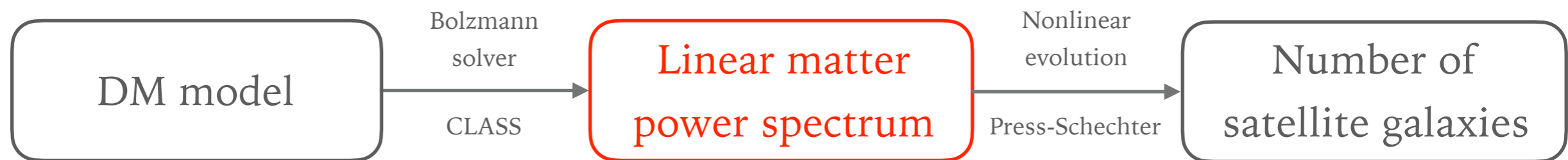
Gravitino

Free parameters: WDM mass  $m_{3/2}$  & WDM fraction  $f_{3/2}$

# SMALL-SCALE OBSERVATIONS MEET PEV-SCALE SUSY BR.

## ➤ Small-scale observations: ② Number of satellite galaxies

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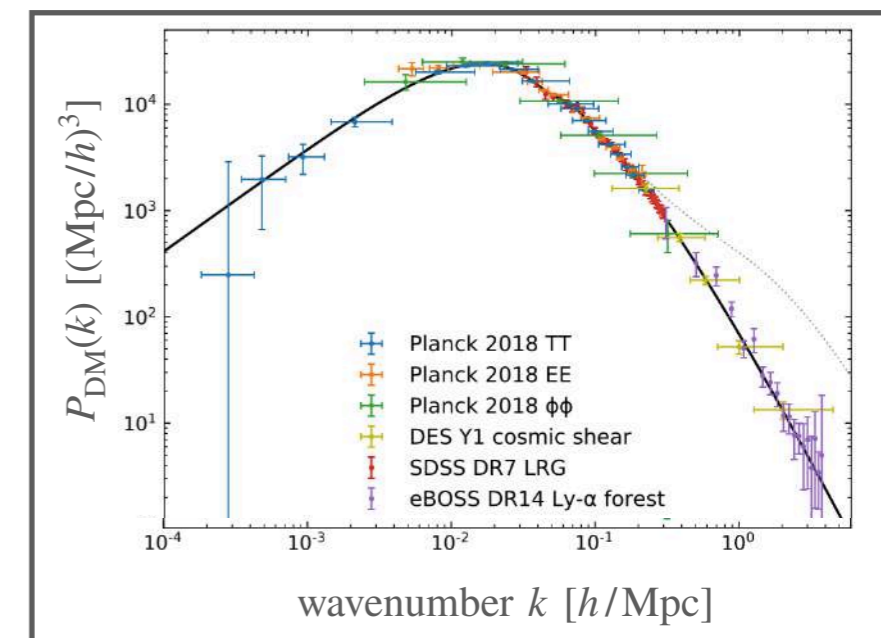
### Linear matter power spectrum?

"Power spectrum (= 2-point ensemble average) of overdensity  $\delta$  before complicated late-time nonlinear evolution sets in"

Define overdensity  $\delta(\vec{x}) \equiv \frac{\rho(\vec{x})}{\bar{\rho}} - 1$

Fourier transform  $\delta(\vec{k}) = \int \frac{d^3k}{(2\pi)^3} \delta(\vec{x}) e^{i\vec{k}\cdot\vec{x}}$

Take 2-point average  $\langle \delta(\vec{k})\delta^*(\vec{k}') \rangle = (2\pi)^3 \delta^{(3)}(\vec{k} - \vec{k}') P_{DM}(k)$



Q. Where's time label t?

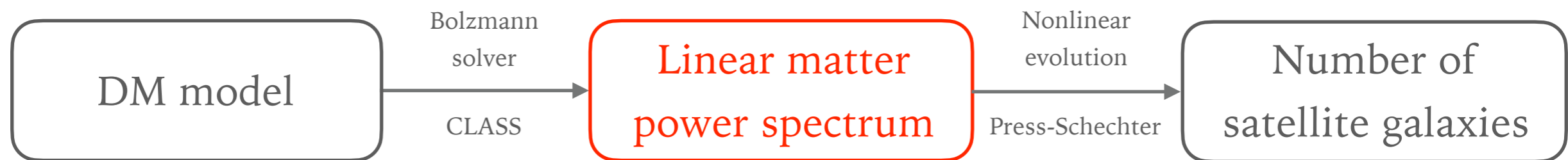
A. These quantities are evaluated well after matter-radiation equality but before nonlinear evolution starts.

More precisely, these quantities are (fictitiously) extrapolated to the present time.

# SMALL-SCALE OBSERVATIONS MEET PEV-SCALE SUSY BR.

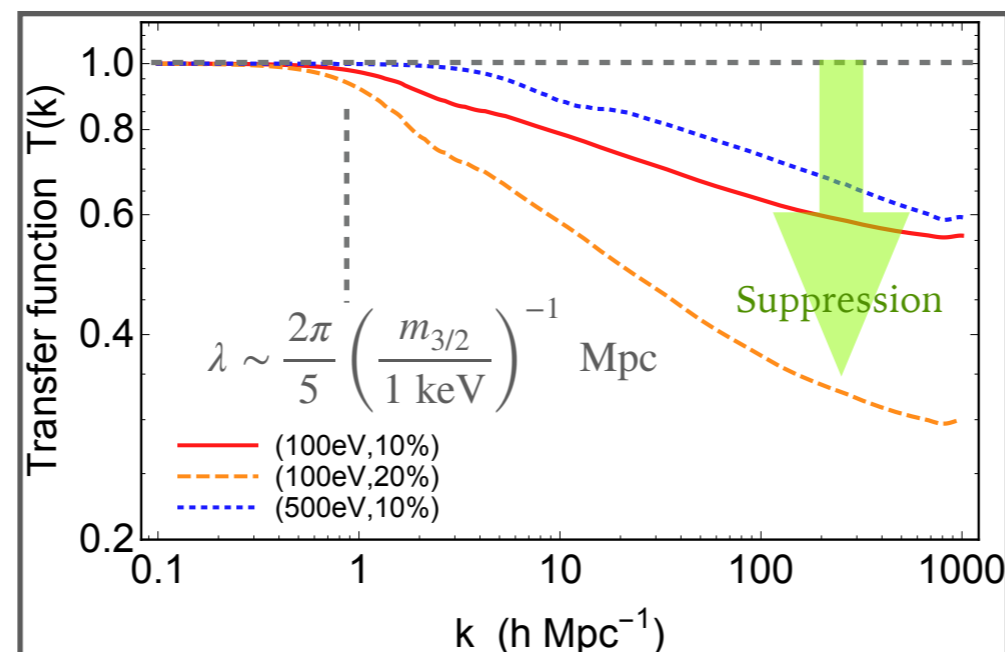
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Linear matter power spectrum?

We calculate the ratio of the linear matter power  $T(k)^2 \equiv P_{\text{MDM}}(k)/P_{\text{CDM}}(k)$

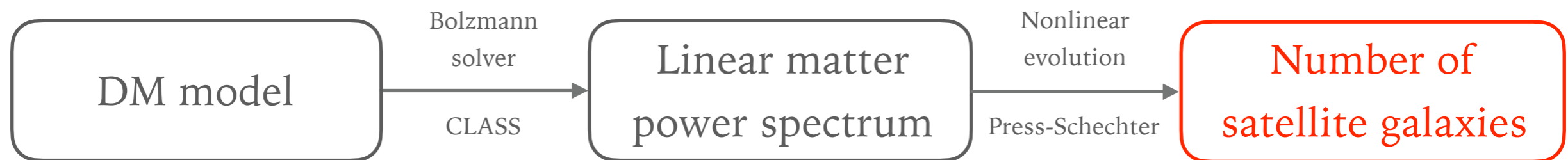


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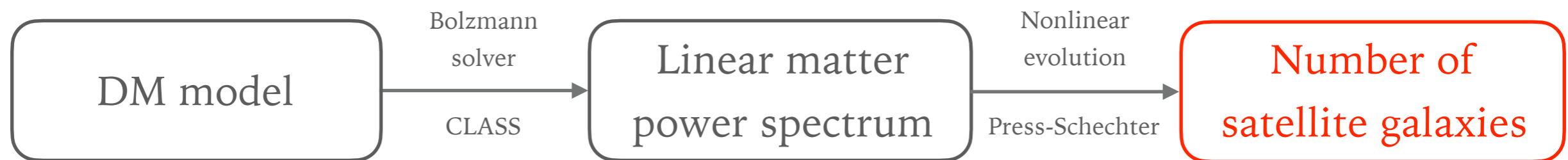
"For given overdensity  $\delta(\vec{x})$ , how many subhalos form in a Milky-way sized halo?"



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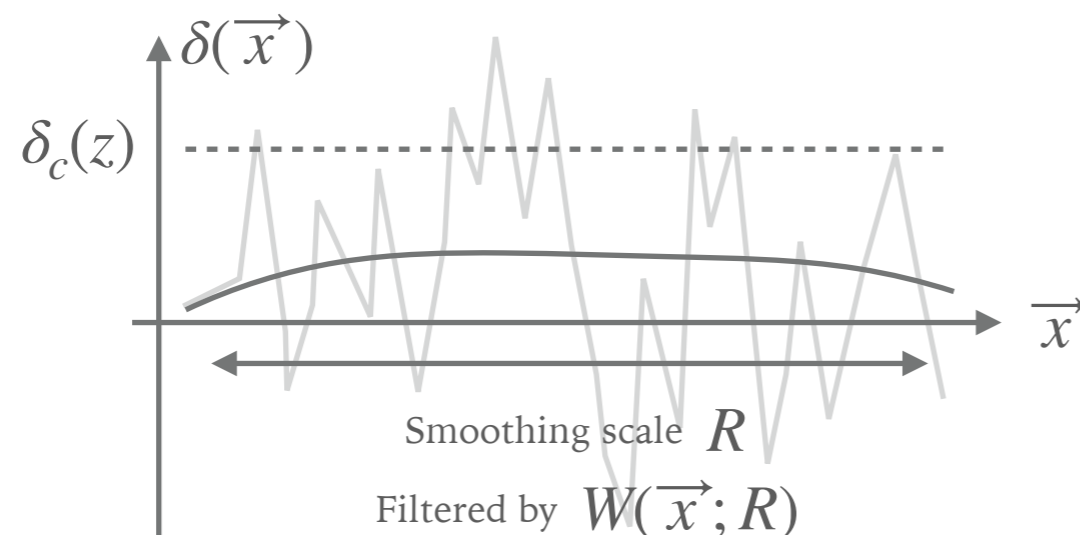
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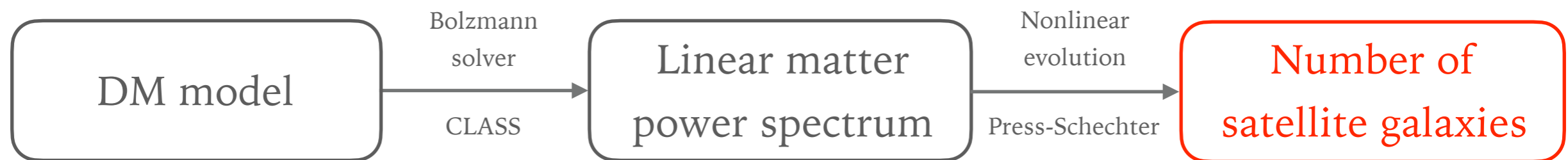




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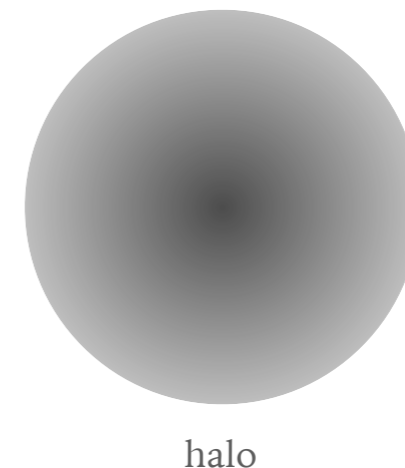
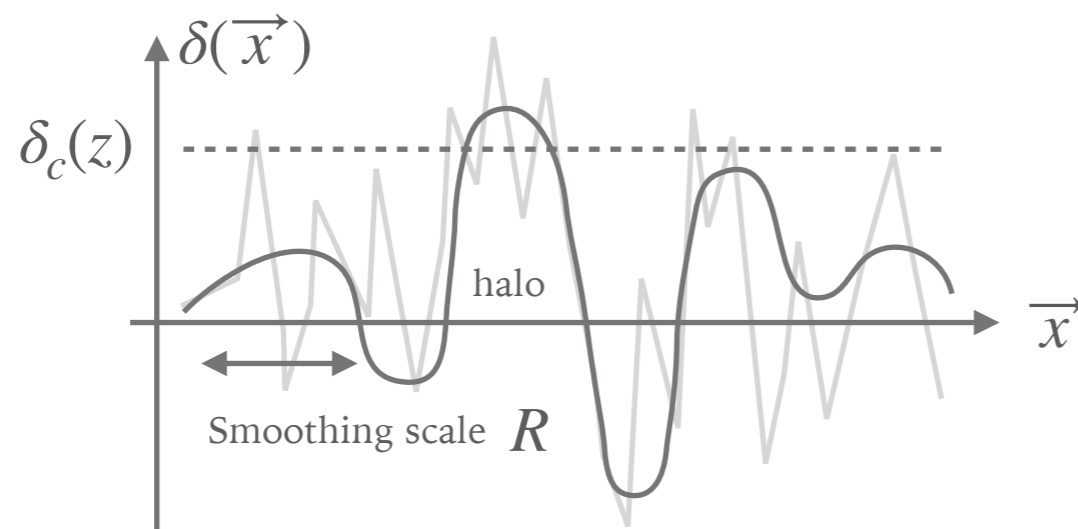
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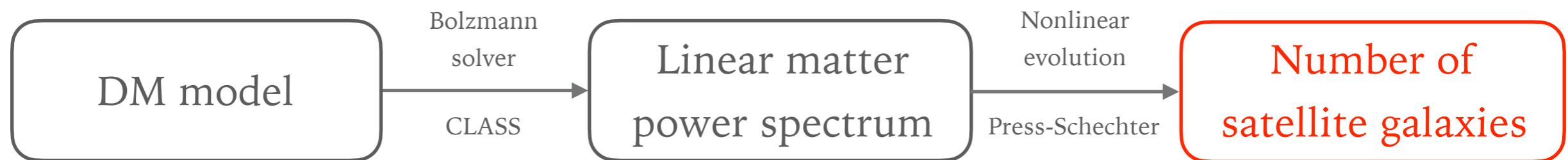
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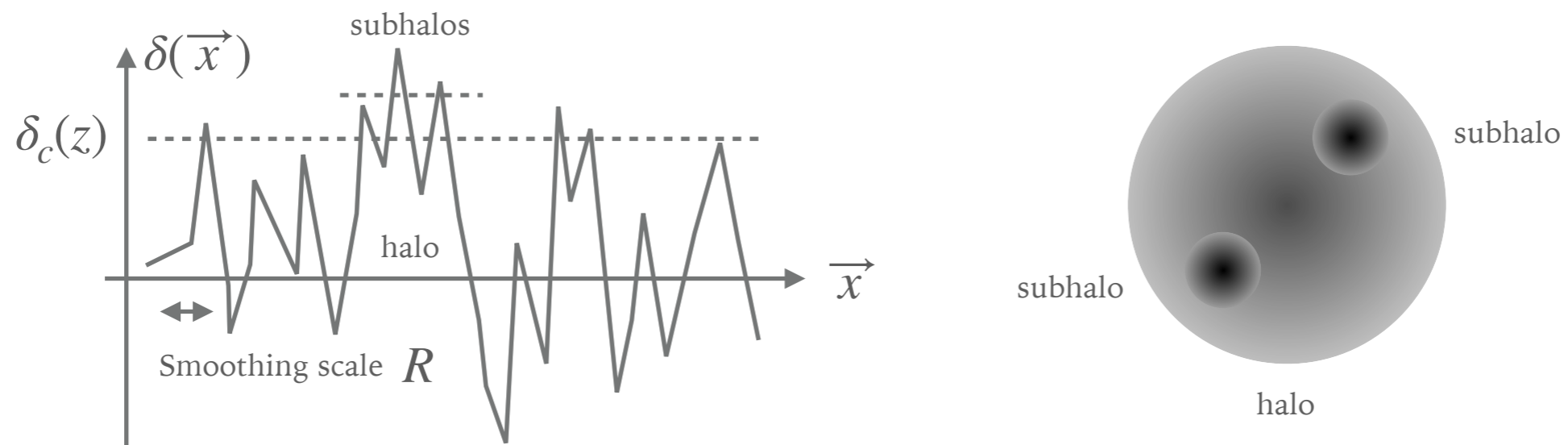
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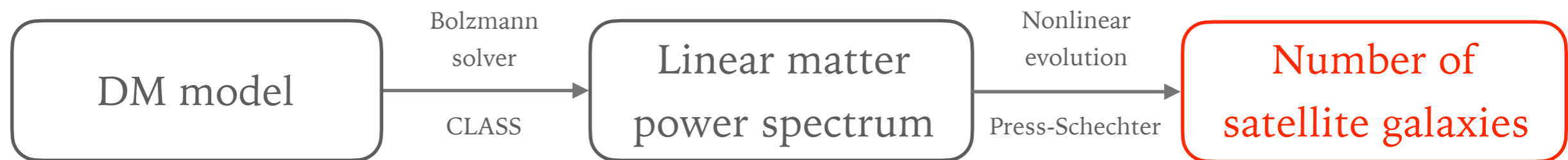
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$$N_{\text{sat}} = \int_{M_{\text{min}}}^{M_h} dM_s \frac{1}{C_n} \frac{1}{6\pi^2} \frac{M_h}{M_s^2} \frac{P_{\text{MDM}}(1/R_s)}{R_s^3 \sqrt{2\pi(S_s - S_h)}} \quad \begin{array}{l} \text{h = halo} \\ \text{s = subhalo} \end{array}$$

$M_{h,s}$  : mass of the halo (h) or subhalo (s)       $c = 2.5, C_n = 44.5$  : constants chosen to match with simulations

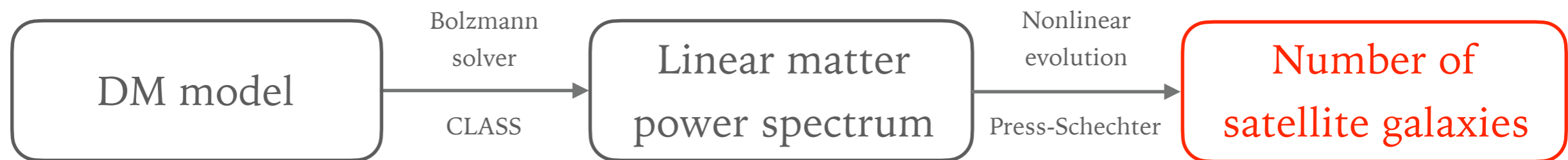
$$M_{h,s} = \frac{4\pi}{3} \times (cR_{h,s})^3 \times \rho_{m,0} \quad \text{: relation between the filter scale } R_{h,s} \text{ and enclosed mass } M_{h,s}$$

$$S_{h,s} = \frac{1}{2\pi^2} \int_0^{1/R_{h,s}} dk k^2 P_{\text{MDM}}(k) \quad \text{: variance of overdensity } \delta \text{ smoothed over the filter scale } R_{h,s}$$

# SMALL-SCALE OBSERVATIONS MEET PEV-SCALE SUSY BR.

## ➤ Small-scale observations: ② Number of satellite galaxies

- Pipeline



Number of satellite galaxies?

We use Press-Schechter approach [ Press-Schechter '74 ] [ Polisensky, Ricotti 1004.1459 ] [ Schneider 1412.2133 ]

$$N_{\text{sat}} = \int_{M_{\text{min}}}^{M_h} dM_s \frac{1}{C_n} \frac{1}{6\pi^2} \frac{M_h}{M_s^2} \frac{P_{\text{MDM}}(1/R_s)}{R_s^3 \sqrt{2\pi(S_s - S_h)}} \quad \begin{array}{l} \text{h = halo} \\ \text{s = subhalo} \end{array}$$

$M_{h,s}$  : mass of the halo (h) or subhalo (s)

$c = 2.5, C_n$

For given initial density fluctuations

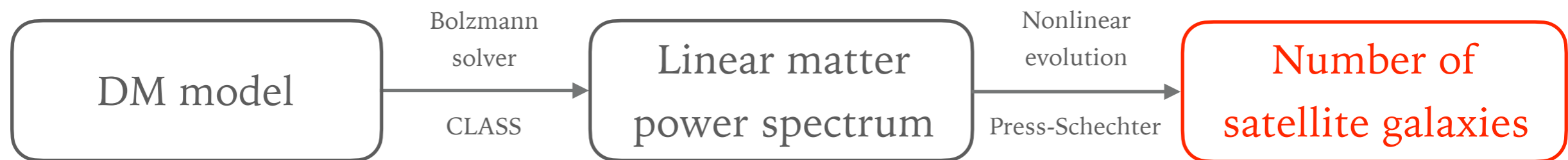
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$M_{h,s}$  : mass of the halo (h) or subhalo (s)

$c = 2.5, C_n = 44.5$  : constants chosen to match with simulations

$$M_{h,s} = \frac{4\pi}{3} \times (cR_{h,s})^3 \times \rho_{m,0} \quad \text{: rel}$$

$$S_{h,s} = \frac{1}{2\pi^2} \int_0^{1/R_{h,s}} dk k^2 P_{\text{MDM}}(k) \quad \text{: variance of overdensity } \delta \text{ smoothed over the filter scale } R_{h,s}$$

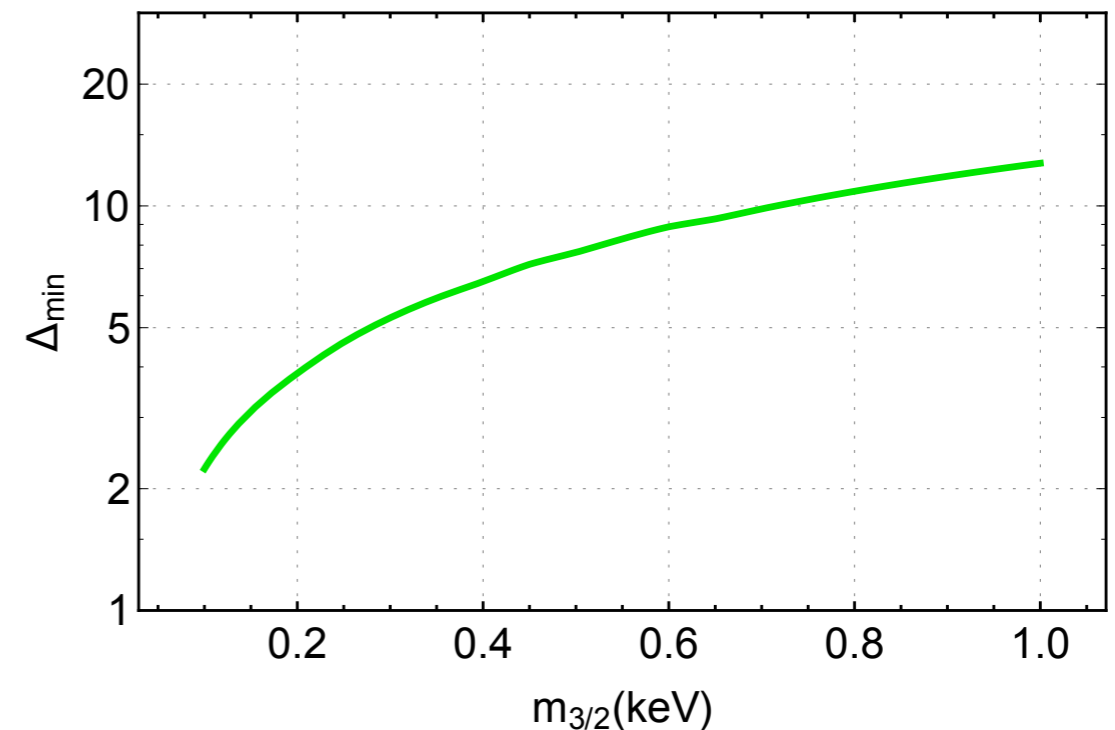
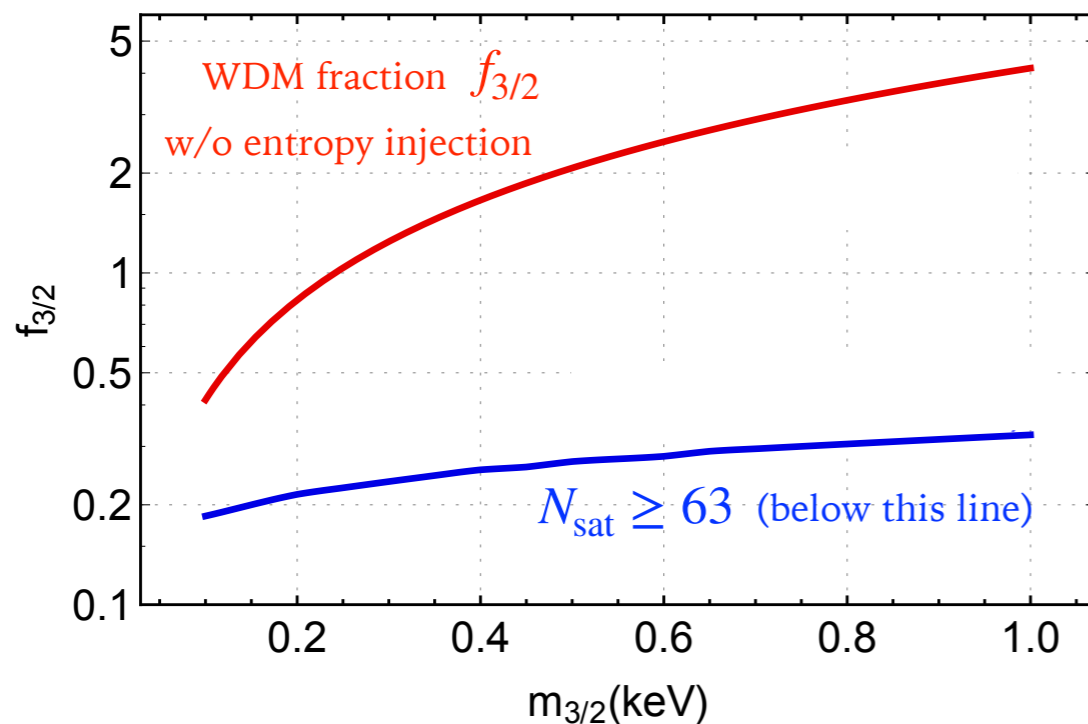
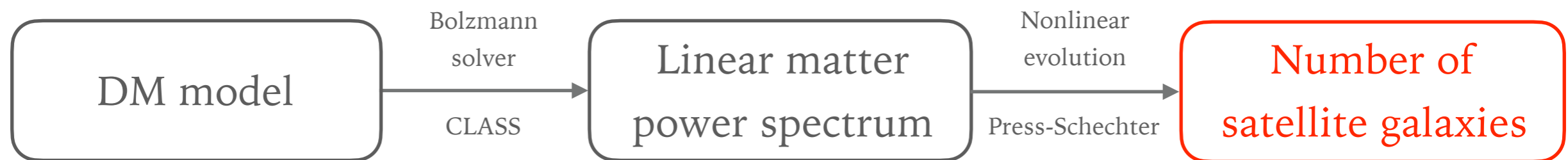
We can estimate how many subhalos form between subhalo mass  $[M_s, M_s + dM_s]$



# SMALL-SCALE OBSERVATIONS MEET PEV-SCALE SUSY BR.

## ► Small-scale observations: ② Number of satellite galaxies

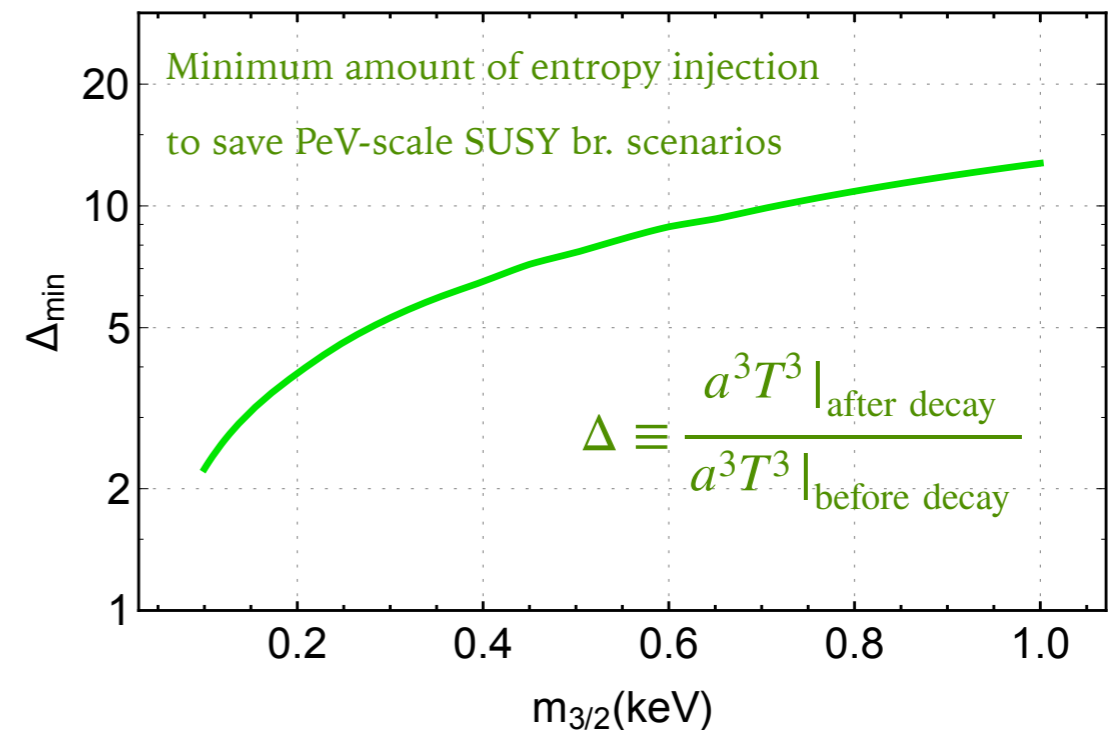
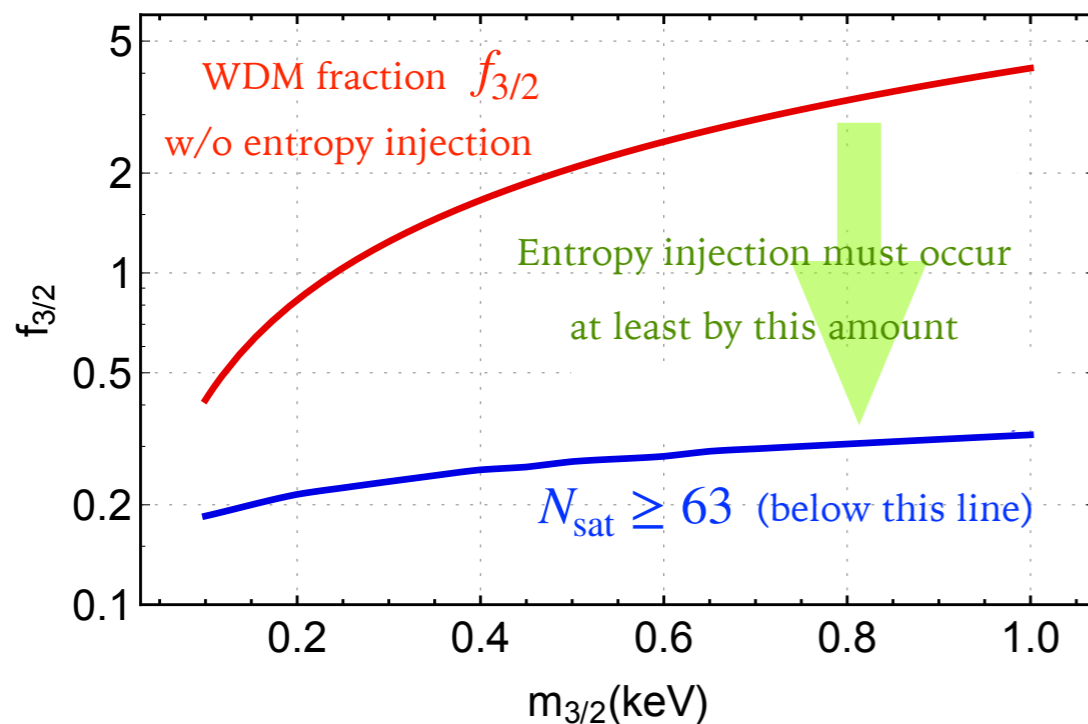
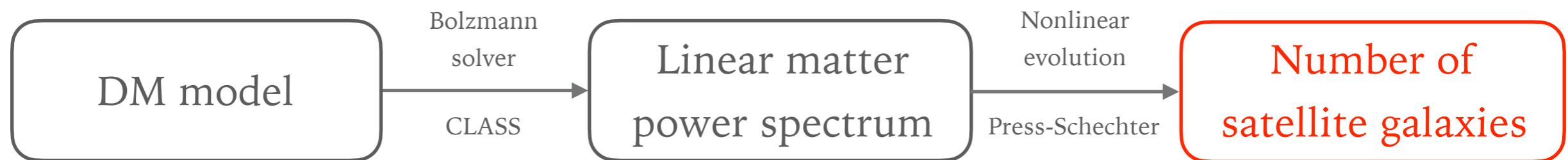
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# SMALL-SCALE OBSERVATIONS MEET PEV-SCALE SUSY BR.

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*Summary*

*PeV ~~SUSY~~  
&  
GWs*

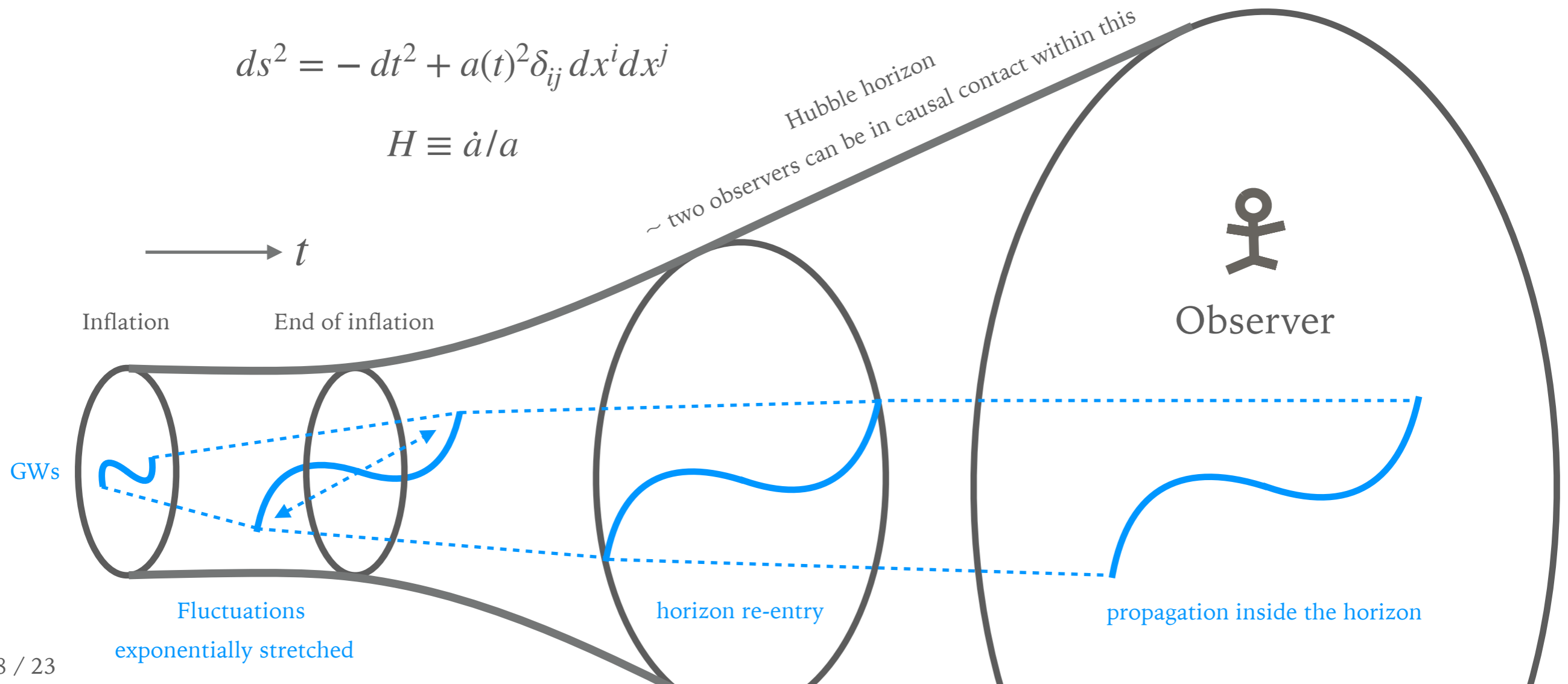
*PeV ~~SUSY~~  
&  
Small-scales*

*Introduction*

# IMPRINT OF PEV-SCALE SUSY BR. ON GRAVITATIONAL WAVES

## ► Inflationary GWs [ Starobinsky '79 ]

- During inflation, both scalar (= density) and tensor (= GWs) fluctuations are produced from quantum fluctuations, and stretched outside the horizon



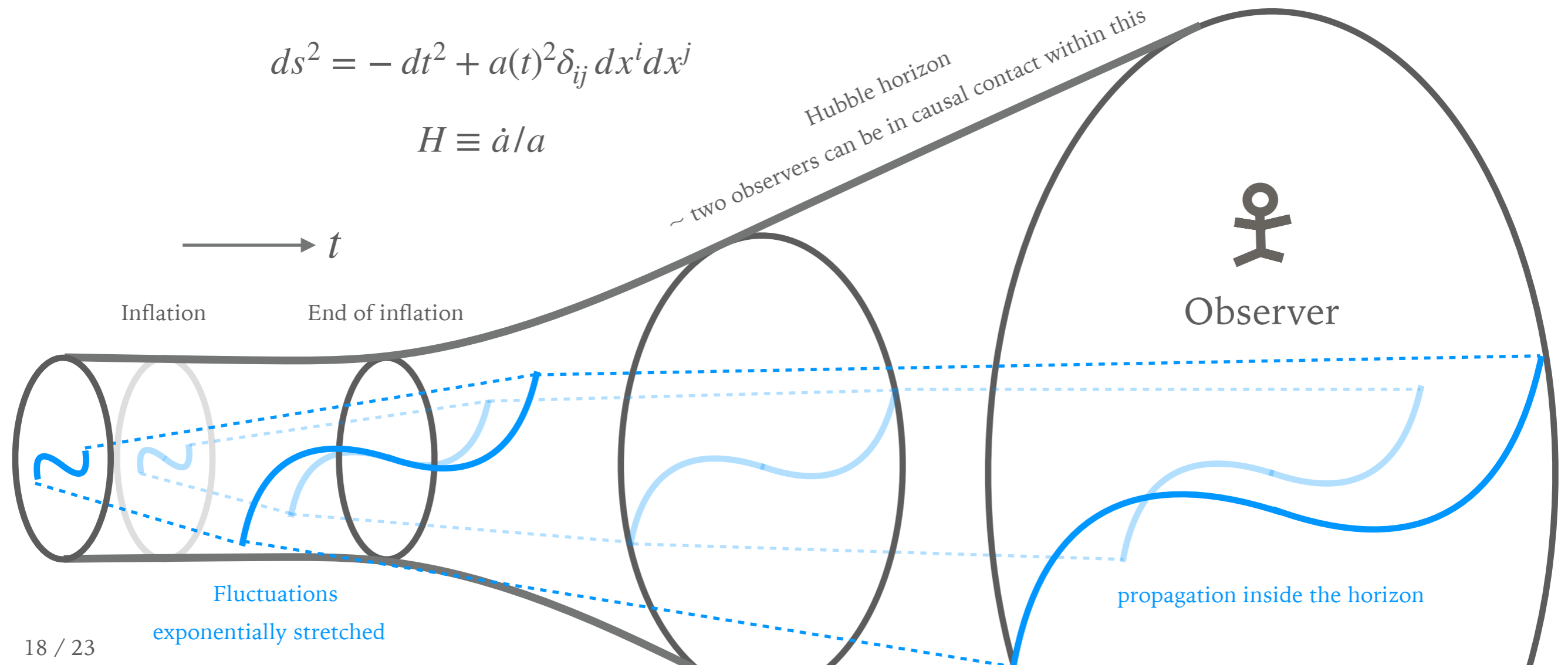
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$$ds^2 = - dt^2 + a(t)^2 \delta_{ij} dx^i dx^j$$

$$H \equiv \dot{a}/a$$



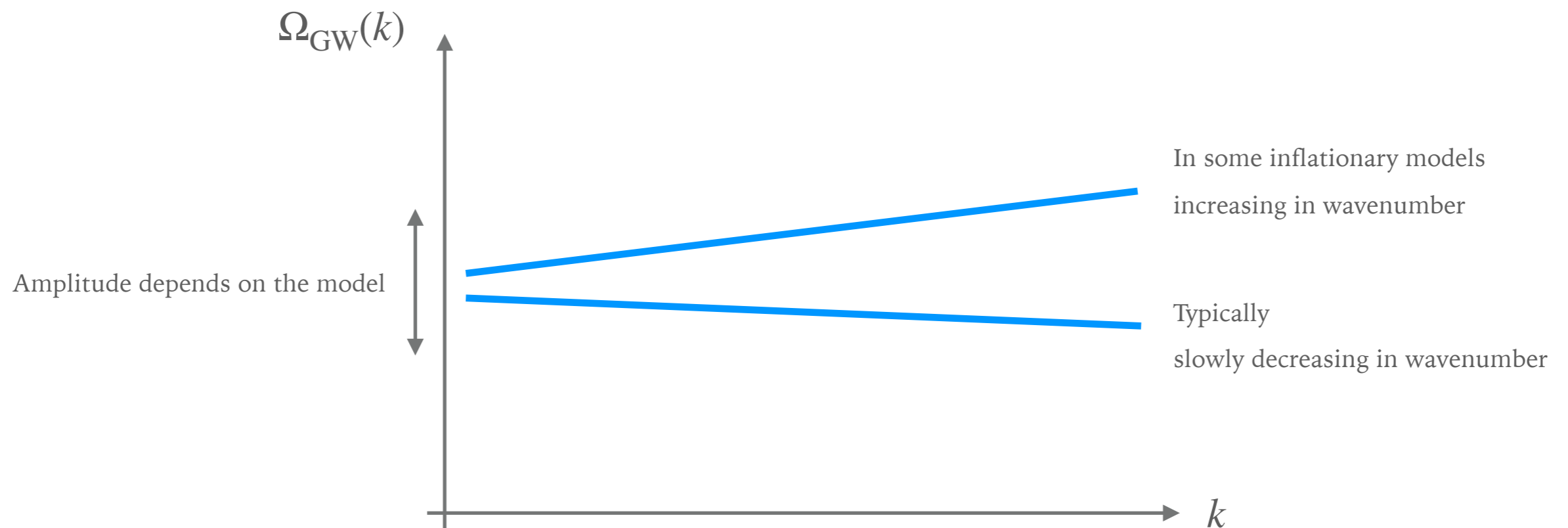


# IMPRINT OF PEV-SCALE SUSY BR. ON GRAVITATIONAL WAVES

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## ► Inflationary GWs [ Starobinsky '79 ]

- During inflation, both scalar (= density) and tensor (= GWs) fluctuations are produced from quantum fluctuations, and stretched outside the horizon
- At present, GWs of various wavenumbers (theoretically) exist around us



# IMPRINT OF PEV-SCALE SUSY BR. ON GRAVITATIONAL WAVES

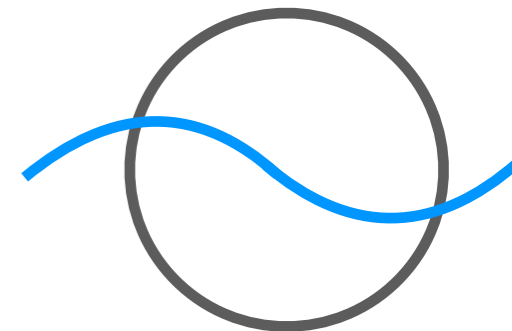
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► Entropy injection leaves a distinct feature in the GW spectrum

- Intuitively...

GWs with **small k** (large wavelengths): get stuck outside the horizon

why? Because of the equation of motion  $\ddot{h}_{ij} + 3H\dot{h}_{ij} + \frac{k^2}{a^2}h_{ij} = 0$   
friction term!



GWs with **large k** (short wavelengths): behaves as radiation within the horizon

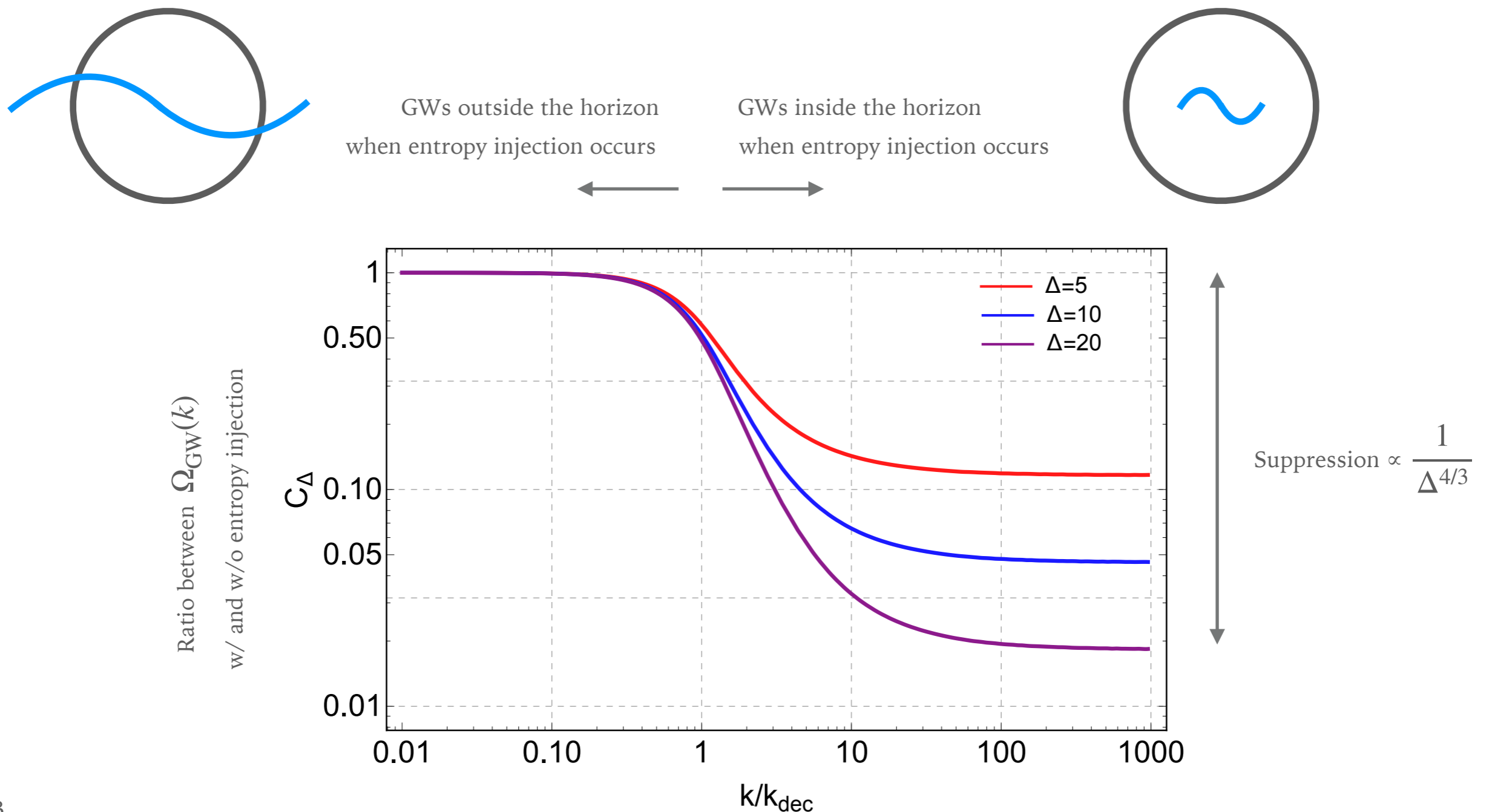
why? Because they are massless fields  $S_{\text{grav}} \sim \int d^3x \frac{a^3}{2} \left[ \dot{h}_{ij}^2 - \frac{1}{a^2} (\nabla h_{ij})^2 \right]$



- So, entropy injection dilutes only GWs with large k

# IMPRINT OF PEV-SCALE SUSY BR. ON GRAVITATIONAL WAVES

- Inflationary GW spectrum in PeV-scale SUSY breaking scenario



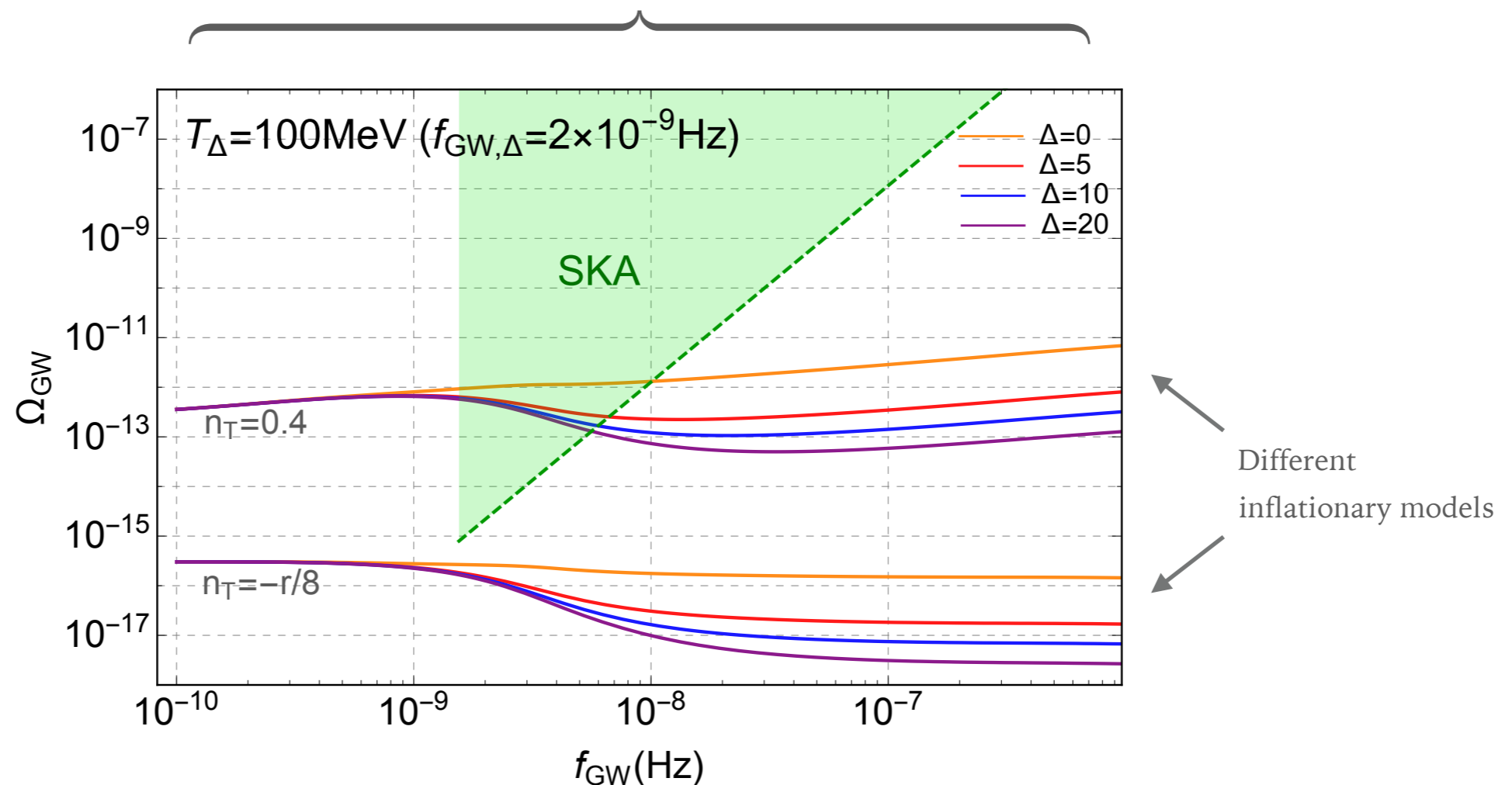
# IMPRINT OF PEV-SCALE SUSY BR. ON GRAVITATIONAL WAVES

## ► Inflationary GW spectrum in PeV-scale SUSY breaking scenario

Entropy injection must occur between

(Big Bang Nucleosynthesis)  $\sim 1\text{MeV}$  and (gravitino decoupling)  $\sim 10\text{TeV}$

→ Frequency range of the feature is  $[10^{-10}\text{Hz}, 10^{-5}\text{Hz}]$  (SKA range)



# IMPRINT OF PEV-SCALE SUSY BR. ON GRAVITATIONAL WAVES

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## ► Inflationary GW spectrum in PeV-scale SUSY breaking scenario

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→ Frequency range of the feature is  $[10^{-10}\text{Hz}, 10^{-5}\text{Hz}]$  (SKA range)

If we observe only featureless inflationary GWs between  $[10^{-10}\text{Hz}, 10^{-5}\text{Hz}]$ ,  
then PeV-scale SUSY breaking scenarios are excluded.



*Summary*

*PeV ~~SUSY~~  
&  
GWs*

*PeV ~~SUSY~~  
&  
Small-scales*

*Introduction*



# SUMMARY

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- In PeV-scale SUSY breaking scenarios, light gravitinos can be a problem

$$\langle F \rangle \sim (10^6 \text{ GeV})^2 \quad \rightarrow \quad m_{3/2} \simeq \frac{\langle F \rangle}{\sqrt{3} M_P} \sim 100 \text{ eV} - 1 \text{ keV}$$

Overclosure of the Universe / Small-scale structure

$$\Omega_{3/2} \gtrsim 1$$

$$\lambda \sim \mathcal{O}(\text{Mpc})$$

- Solution to this problem requires entropy injection at some specific era

(Big Bang Nucleosynthesis  $T \sim 1 \text{ MeV}$ )    to    (Gravitino decoupling  $T \sim 10 \text{ TeV}$ )

- This in turn implies an unavoidable feature in the inflationary GW spectrum

between a specific frequency range  $[10^{-10} \text{ Hz}, 10^{-5} \text{ Hz}]$